Energy Conservation Analysis of Human Body Locomotion Modelled as an Inverted Quadruple Pendulum Dynamical System

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Abstract— **Human body parts move when they are involved in an activity. In this paper an attempt is made to analyse the motion of the parts in line with the conservation of energy principle. Human body was modelled longitudinally as an inverted Quadruple pendulum. Analytical approach was adopted in analysing the potential and kinetic energies of the systems segments. The findings are consistent with the ones in the literature. Specifically, there is a positive correlation between the height of segments of the system and mechanical energy at the different segments of the system.**

Index Terms **— Energy Conservation, Human Body Locomotion, Inverted Pendulum, Dynamical System.**

I. INTRODUCTION

ENERGY is usually release by different parts of human body, especially when involved in an activity. Energy can be defined as the capacity for doing work. Energy is the work needed to accelerate a body of a given mass from rest to its stated velocity. It may exist in a variety of forms and may be transformed from one type of energy to another.[1] However, these energy transformations are constrained by a fundamental principle, the Conservation of Energy principle [2]. Total energy of an isolated system remains constant. The conservation laws are exact for an isolated system [2,3]. An isolated system implies a collection of matter which does not interact with the rest of the universe at all - and as far as we know there are really no such systems [3]. The law of conservation of energy states that the total energy of an isolated system remains constant—it is said to be *conserved* over time [3,4]. Energy can neither be created nor destroyed; rather, it transforms from one form to another. For instance, chemical energy can be converted to kinetic energy in the explosion of a stick of dynamite [4]. A consequence of the law of conservation of energy is that a perpetual motion machine of the first kind cannot exist. That is to say, no system without an external energy supply can deliver an unlimited amount of energy to its surroundings [4,5]. For equations of motion which do not have time translation symmetry, the conservation of energy may not be able to be defined [5]: In a hydroelectric plant, water falls from a height on to a turbine causing it to turn. The turbine turns a coil in a magnetic field, thereby generating a electric current. Therefore, potential energy of the water is converted into kinetic energy of the turbine, which is converted into electrical energy [6]. Also in a bicycle pump, mechanical energy is converted into heat energy. Thus the pump gets hot of a gas is compressed, the mechanical work done gets converted into heat energy. Alternatively, if a gas is allowed to expand, it does work and its temperature falls as its energy is used up. If we rub our hands, heat is produced due to friction. Another example is in places where there are strong winds; the winds turn the blades of a wind mill, the shaft of which turns a coil in a magnetic field, generating an electric current. Thus the energy of motion of the wind is converted into mechanical energy of the wind mill, which is converted into electrical energy as a coil is made to turn in a magnetic field.[6,7]

If one knows the kinetic and potential energies that act on an object, then one can calculate the mechanical energy of the object. Imagine a roller coaster car traveling along a straight stretch of track. The car has mechanical energy because of its motion: kinetic energy [7].

The principle of conservation of energy was adopted and analysed. One way to state the conservation of energy principle is "Energy can neither be created nor destroyed".[7]

Inverted pendulum is the one with its centre of mass is above its pivot point. Quadruple pendulum is like four pendulum joined together.

This system is constantly unstable. In other to stabilise it some forces have to be applied. Body Mechanics plays a very important role in the movement of human body part when involved in an activity [8]. An ordinary pendulum is one with the pivot at the top and the mass at the bottom. An inverted pendulum is the opposite way round. The pivot is at the bottom and the mass is on top. The inverted pendulum is a mechanism for carrying an object form one place to another and this is how it functions during walking. The "passenger unit" as Perry would call it is carried forward by the outstretched leg as it pivots over the foot [8]. So an inverted pendulum is a pendulum that has its centre of mass above its pivot point. It is often implemented with the pivot point mounted on a moving body that can move horizontally. Unlike a normal pendulum which is always stable when hanging downwards, an inverted pendulum is

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inherently unstable, and must be actively balanced in order to remain upright. This can be done in different ways including the pivot point horizontally as part of a feedback system [8,9].

A multiple pendulum can be referred to as a combination of several simple pendulums. Dissipative and driven forces can be accounted for by splitting the external forces into a sum of potential and non-potential forces.[9]. A simple demonstration of this is achieved by balancing an upturned broomstick on the end of one's finger. The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies [9,10]. An inverted quadruple pendulum is an inverted pendulum with four segments.[10] Proper functioning of human body results in a good balance. Human body movement modelling and analysis using mechanics and other Mathematical concepts is constantly expanding and becoming very important in human performance and rehabilitation studies [10]. Biomechanics of rhythmic movements and cycles are -to a large extent- the application of Newtonian mechanics to the physiology and neuromuscular skeletal systems [10,11]. Human locomotion appears to be a learned process. No one can watch the struggles of an infant as he first attempts to stand, holding onto the edge of a chair or tightly grasping in his hand the supporting fingers of a doting parent, without feeling that this is pure experimentation rather than the maturation of an inborn reflex. After the first few faltering steps, with the many inevitable fails, greater stability and precision are rapidly acquired [12,13,14]. One of the first things a human being learns to do is to walk. Walking is always a very simple matter for us. Before walking, we have never asked ourselves such questions as, “What angle should I place my feet at?” “Will I lose my balance if I step this way?” or “Will I fall over if I lift my feet up high?” However, the “walking movement” we perform so easily, in fact, takes place as the result of a highly complex system.[15,16,17]. Let us consider the energy requirements of a normal adult male, walking at varying speeds on the level. If energy expenditure, calculated on the basis of oxygen consumption and expressed as calories/m./kg. of body weight, is plotted against speed of walking, a curve results which passes through a minimum[17]. Walking faster or slower requires more energy per step.[18,19] All living organisms exhibit a special characteristic feature of moving whole part or a part of the body from one place to other. This act of exhibiting various motions such as running, walking, jumping, crawling, swimming, etc. by the body is known as locomotory movements. [19,20] Movement is one among the characteristic feature of all living organisms.

Locomotion helps us to move from place to other. The locomotory movement in the humans involves the body mechanics.. Movement is one of the significant features of human beings. Such voluntary movements that results in change of position are called locomotion. Walking, running, climbing, flying, swimming are all some forms of locomotory movements. We use limbs for changes in body postures and locomotion as well. Movements and locomotion cannot be studied separately. The two may be linked by stating that all locomotions are movements but all movements are not locomotions.[20]

In this paper, human body is generally divided into four segments, represented by a quadruple pendulum. These segments exact different level of energy during human activities. The total energy involves both the potential and kinetic energy of the different segments added together.

II FORMULATION OF PROBLEM

An object’s mechanical potential energy derives from work done by forces, and a label for a particular potential energy comes from the forces that are its source. For example, the roller coaster has potential energy because of the gravitational forces acting on it, so this is often called gravitational potential energy. The roller coaster car’s total mechanical energy, which is the sum of its kinetic and potential energies, remains constant at all points of the track (ignoring frictional forces). The combination of the kinetic and potential energies does vary, however. When the only work done on an object is performed by conservative forces, its mechanical energy remains constant, whatever motions it may undergo. Say, for example, that you see a roller coaster at two different points on a track — Point 1 and Point 2 — so that the coaster is at two different heights and two different speeds at those points. Because mechanical energy is the sum of the potential energy:

$$\left(\text{mass} \times \text{gravity} \times \text{height}\right)$$

and kinetic energy

$$\left(\frac{1}{2}\text{mass} \times \text{velocity}^2\right),$$

That is,

$$PE = \text{mgh} \qquad (1)$$

$$KE = 1/2\text{mv}^2 \qquad (2)$$

The total mechanical energy at Point A is

$$ME_A = mgh_A + \frac{1}{2}mv_A^2 \qquad (3)$$

At Point B, the total mechanical energy is

$$ME_B = mgh_B + \frac{1}{2}mv_B^2 \qquad (4)$$

What’s the difference between ME_A and ME_B? If there’s no friction (or another non-conservative force), then $ME_A = ME_B$, or

$$mgh_A + \frac{1}{2}mv_A^2 = mgh_B + \frac{1}{2}mv_B^2 \qquad (5)$$

These equations represent the principle of conservation of mechanical energy. The principle says that if the net work done by non-conservative forces is zero, the total mechanical energy of an object is conserved; that is, it doesn’t change. Another way of rattling off the principle of conservation of mechanical energy is that at Point A and Point B,

$$PE_A + KE_A = PE_B + KE_B \qquad (6)$$

You can simplify that to the following:

$$ME_A = ME_B \qquad (7)$$

where ME is the total mechanical energy at any one point. In other words, an object always has the same amount of

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energy as long as the net work done by non-conservative forces is zero..

Equation (5) can be written as

$$gh_A + \frac{1}{2}v_A^2 = gh_B + \frac{1}{2}v_B^2 \quad (8)$$

The potential energy of the system is given as [7]:

$$PE = m_1gy_1 + m_2gy_2 + m_3gy_3 + m_4gy_4 \quad (9)$$

and the kinetic energy is given by [7]:

$$KE = \frac{1}{2}\mathrm{m}_1\mathrm{v}_1^2 + \frac{1}{2}\mathrm{m}_2\mathrm{v}_2^2 + \frac{1}{2}\mathrm{m}_3\mathrm{v}_3^2 + \frac{1}{2}\mathrm{m}_4\mathrm{v}_4^2 \quad (10)$$

Therefore equation (6) becomes:

$$(m_1gy_1 + m_2gy_2 + m_3gy_3 + m_4gy_4)_A + (\frac{1}{2}\mathrm{m}_1\mathrm{v}_1^2 + \frac{1}{2}\mathrm{m}_2\mathrm{v}_2^2 + \frac{1}{2}\mathrm{m}_3\mathrm{v}_3^2 + \frac{1}{2}\mathrm{m}_4\mathrm{v}_4^2)_A = (m_1gy_1 + m_2gy_2 + m_3gy_3 + m_4gy_4)_B + (\frac{1}{2}\mathrm{m}_1\mathrm{v}_1^2 + \frac{1}{2}\mathrm{m}_2\mathrm{v}_2^2 + \frac{1}{2}\mathrm{m}_3\mathrm{v}_3^2 + \frac{1}{2}\mathrm{m}_4\mathrm{v}_4^2)_B \quad (11)$$

The positions of the bobs are given respectively as: (x_1,y_1), (x_2,y_2), (x_3,y_3) and (x_4,y_4)

where,

$$x_1 = l_1 sin\theta_1 \quad (12)$$
$$x_2 = l_1 sin\theta_1 + l_2 sin\theta_2 \quad (13)$$
$$x_3 = l_1 sin\theta_1 + l_2 sin\theta_2 + l_3 sin\theta_3 \quad (14)$$
$$x_4 = l_1 sin\theta_1 + l_2 sin\theta_2 + l_3 sin\theta_3 + l_4 sin\theta_4 \quad (15)$$

$$y_1 = -l_1\cos\theta_1 \quad (16)$$
$$y_2 = -l_1\cos\theta_1 - l_2\cos\theta_2 \quad (17)$$
$$y_3 = -l_1\cos\theta_1 - l_2\cos\theta_2 - l_3\cos\theta_3 \quad (18)$$
$$y_4 = -l_1\cos\theta_1 - l_2\cos\theta_2 - l_3\cos\theta_3 + l_4\cos\theta_4 \quad (19)$$

The velocity v can be expressed as [];

$$v_i^2 = \dot{x}_i + \dot{y}_i \quad (20)$$

In this case i = 1,2,3,4

Substituting equations (12) – (20) into equation (11) and rearranging gives:

$$[-(m_1+m_2+m_3+m_4)gl_1\cos\theta_1 - (m_2+m_3+m_4)gl_2\cos\theta_2 - (m_3+m_4)gl_3\cos\theta_3 - m_4gl_4\cos\theta_4]_1 + [\frac{1}{2}\mathrm{m}_1\left(\dot{x}_1^2+\dot{y}_1^2\right) + \frac{1}{2}\mathrm{m}_2\left(\dot{x}_2^2+\dot{y}_2^2\right) + \frac{1}{2}\mathrm{m}_3\left(\dot{x}_3^2+\dot{y}_3^2\right) + \frac{1}{2}\mathrm{m}_4\left(\dot{x}_4^2+\dot{y}_4^2\right)]_A$$
$$=[-(m_1+m_2+m_3+m_4)gl_1\cos\theta_1 - (m_2+m_3+m_4)gl_2\cos\theta_2 - (m_3+m_4)gl_3\cos\theta_3 - m_4gl_4\cos\theta_4]_2 + [\frac{1}{2}\mathrm{m}_1\left(\dot{x}_1^2+\dot{y}_1^2\right) + \frac{1}{2}\mathrm{m}_2\left(\dot{x}_2^2+\dot{y}_2^2\right) + \frac{1}{2}\mathrm{m}_3\left(\dot{x}_3^2+\dot{y}_3^2\right) + \frac{1}{2}\mathrm{m}_4\left(\dot{x}_4^2+\dot{y}_4^2\right)]_B \quad (21)$$

Equation (21) shows the conservation of mechanical energy of an inverted multiple pendulum with four segments. Both oscillating vertical motion and non-.conservation forces are neglected. Thus the mechanical energy at point 1 is the same as the mechanical energy at point 2 and by extension at any other point n.

Now considering the mechanical energy at one point, say point A, and equating it to zero, gives:

$$-(m_1+m_2+m_3+m_4)gl_1\cos\theta_1 - (m_2+m_3+m_4)gl_2\cos\theta_2 - (m_3+m_4)gl_3\cos\theta_3 - m_4gl_4\cos\theta_4]_1 + [\frac{1}{2}\mathrm{m}_1\left(\dot{x}_1^2+\dot{y}_1^2\right) + \frac{1}{2}\mathrm{m}_2\left(\dot{x}_2^2+\dot{y}_2^2\right) + \frac{1}{2}\mathrm{m}_3\left(\dot{x}_3^2+\dot{y}_3^2\right) + \frac{1}{2}\mathrm{m}_4\left(\dot{x}_4^2+\dot{y}_4^2\right) = 0 \quad (22)$$

Equation (22) can be written as:

$$-(m_1+m_2+m_3+m_4)gl_1\cos\theta_1 - (m_2+m_3+m_4)gl_2\cos\theta_2 - (m_3+m_4)gl_3\cos\theta_3 - m_4gl_4\cos\theta_4 + \frac{1}{2}\mathrm{m}_1\left(v_1^2\right) + \frac{1}{2}\mathrm{m}_2\left(v_2^2\right) + \frac{1}{2}\mathrm{m}_3\left(v_3^2\right) + \frac{1}{2}\mathrm{m}_4\left(v_4^2\right) = 0 \quad (23)$$

III NUMERICAL EXAMPLE

For the purpose of numerical results the following values are used for the parameters, with acceleration due to gravity (g) taken to be 10:

Table 1: Randomly selected parameter values for the system

Parameters	Values (1)	Values (2)	Values (3)	Values (4)
m_1	3	2	5	4
m_2	3	5	6	6
m_3	4	6	7	3
m_4	7	5	5	3
l_1	4	5	6	4
l_2	6	9	4	5
l_3	6	4	2	5
l_4	4	9	7	6
θ_1	30	45	30	60
θ_2	60	60	60	45
θ_3	45	45	30	60
θ_4	45	30	60	45
V_1	?	?	?	?
V_2	?	?	?	?
V_3	?	?	?	?
V_4	?	?	?	?

Substituting these value into equation (23), four equations are obtained. The equations are solved to obtain the values of V1, V2, V3 and V4.

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$$-1673.58+1.5v_1^2+1.5v_2^2+2v_3^2+3.5v_4^2=0 \quad (24)$$

$$-2057.24+v_1^2+2.5v_2^2+3v_3^2+2.5v_4^2=0 \quad (25)$$

$$-1738.78+2.5v_1^2+3v_2^2+3.5v_3^2+2.5v_4^2=0 \quad (26)$$

$$-1021.54+2v_1^2+3v_2^2+1.5v_3^2+1.5v_4^2=0 \quad (27)$$

Solving equations (24) – (27) gives:

$$v_1^2=-1983.01, v_2^2=-334.01, v_3^2=-233.73, v_4^2=-287.45$$

From the principle of conservation of energy it was evident that the mechanical energy at different points is the same. This implies that it is constant, if non-conservative forces are zero.

$$PE+KE=C \quad (28)$$

where C is a constant.

Let C = 0, then

$$mgh+\frac{1}{2}mv^2=0 \quad (29)$$

$$gh+\frac{1}{2}v^2=0 \quad (30)$$

For the first segment of the inverted quadruple pendulum system equation (30) becomes:

$$gh_1+\frac{1}{2}v_1^2=0 \quad (31)$$

$$10h_1+\frac{1}{2}(-1983.01)=0 \quad (32)$$

$$h_1=99.15 \quad (33)$$

Similarly for the other segments of the system,

$$h_2=16.7, h_3=12.24, h_4=14.37 \quad (34)$$

From equation (9), the potential energy of the system at a particular point is given as, (taken $y_i = h_i$).

$$PE_1=2974.5+501+489.6+1005.9 = 4971 \quad (34)$$

Similarly at the second point

$$PE_1=PE_2=4971 \quad (35)$$

From equation (10), the kinetic energy of the system at a particular point is given as:

$$KE_1=-2974.52-501.02-489.46\ -1006.08$$
$$=-4971.08 \quad (36)$$

Similarly the kinetic energy at point 2

$$KE_1=KE_2=-4971.08 \quad (37)$$

From equations (34) – (37), equation (6) is confirmed. This is in line with the principle of conservation of energy.

IV. RESULTS DISCUSSION

Using the randomly selected values of the parameters, the values of the square of velocities of different segments are given
as

$$v_1^2=-1983.01, v_2^2=-334.01, v_3^2=-233.73, v_4^2=-287.45$$

This shows negative values because of the inverted pendulum. The kinetic energy at the segments are also negative. The absolute value of the kinetic energy of the first segment is the highest followed by that of fourth segment. The second segment has the least kinetic energy value of 501.02 absolutely. There is a direct positive correlation between the kinetic energies at different segments of the pendulum and the values of $v_1^2, v_2^2, v_3^2, v_4^2$ at the different segments. For the inverted pendulum at rest, the height of each segment are given as

$$h_1=99.15, h_2=16.7, h_3=12.24, h_4=14.37$$

This shows that the first segment of the system has the highest height followed by the second segment. The third segment has the least value. In all, the principle of conservation of energy was obeyed by the inverted quadruple pendulum dynamical system considered.

V. CONCLUSION

This paper set out to analyse the concept of conservation of energy of an inverted quadruple pendulum dynamical system. Different random parameter values were chosen to carry out the analysis. Four equations were formed, from where the values of the square of the velocities at the four segments were sought. the mechanical energy of the system at point A is the same as that at point B. The principle of conservation of energy was confirmed by the system.

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