# Vehicle Dynamic Model-Driver Model System: Platform to Evaluate Car and Human Responses Using Double Lane Change Circuit 

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#### Abstract

Vehicle Dynamic Model-Driver Model (VDM-DM) system is developed to address the need to have a comprehensive system that can evaluate the performance of the car and the capability of the driver based on the planned trajectory. This is possible when VDM-DM system integrates the vehicle dynamic response with the driver model. The driver model determines the steer input from the geometrical properties of the intended path and this steer angle becomes the input for the vehicle dynamic response analysis. Finally, from the position of the car, the steer angle can be calculated. The position of the car will be then compared with the intended path and a new steer input can be determined by the driver model. Two case studies were carried out to demonstrate the application of the VDM-DM in evaluating the performance of the car and the capability of the driver using Double Lane Change (DLC) circuit. Based on the case studies, VDM-DM can be used as the tool to evaluate the performance of cars and capability of the drivers. This demonstrates that $V D M-D M$ is capable to simulate the behavior of different drivers and hence, VDM-DM system has the potential to bring related road safety issue to the desktop.


Keywords: Vehicle dynamic model, driver model, bicycle model, trajectory planning, double lane change circuit

### 1.0 INTRODUCTION

Trajectory planning depends on the ability of the car to steer according to the intended trajectory. Some of the works only consider the kinematic aspect without considering the vehicle dynamic effect [1-4]. Then, Kala and Warwick investigated the possibility of work on trajectory planning by taking into account the vehicle dynamic effect [6]. Based on the geometry of the racetrack, the shortest path with the least curvature and speed profile are developed. Race car driver model will maneuver through the path. Cardamone et al. used genetic algorithm to search the best racing line for the race car [7]. The method has been applied to a number of circuits. Yuan-Yuan et al. used the vehicle dynamic on the curved road with the aim to develop a theoretical support to design suitable curved road and alignment as well as management of counter flow conflicts [8].

[^0]Guo et al. focused on planning and tracking the lane changing trajectory on the curved road [9]. The simulation has shown that the intelligent car is able to perform the lane changing despite the differences in curvature of the inside and outside lane. Driver model related research involves in the parameterization of the driver has been done separately from the trajectory planning research. The outcome of the research is the driver parameters that will be used for road safety purposes.

Renski used an optimization method on the driving data to find the relationship between the driver parameters [10]. Renski went further to develop a lane avoidance system using a single aim point [11]. Instead of a single aim point, Sharp et al. used deviations of preview path, lateral position, and attitude with the actual path to establish the required steering angle [12]. The application of the works done in [10-12] is to create a human like driver in the simulation and the control system for unmanned car.

Some of the research investigate the application of the driver model in a different perspective to develop the Advanced-Driver Assistance Systems (ADAS). Le et al. developed a driver model with adjustable impairment to show the deterioration of driving performance [13]. The aim of the study is to simulate the effect of alcohol on driving performance. Lefèvre et al. looked at the problem from different view [14]. They developed a driver model that can learn from human driving behavior to predict the future input. Xiong et al. went into very specific behavior model on yellow signal indication when during drivers' indecisive zone maneuvers [15]. The decision of the driver whether to stop at a yellow signal indication was associated with and related to various parameters such as age, distraction, pedal conditions, and time to stop line. Schnelle et al. studied the characteristic of human in steering the car to develop driver steering model [16]. Based on the driver parameters, the system is able to classify the drivers according to their skill.

To develop a comprehensive system to evaluate the car and driver performance, the vehicle dynamic response and driver model must be integrated. Literature has shown that dual inputs are required to develop such system. The inputs are the path and motion. Therefore, in the development of the vehicle dynamic and driver model, both inputs should be integrated. Cubic Motion curve is one of the solutions as a Cubic Motion curve has both the path and motion properties featured in one curve.

Therefore, to develop a comprehensive system to evaluate the car and driver performance, a system must integrate the vehicle dynamic and driver model. Thus, this paper discusses the development of VDM-DM system to analyze the performance of the car and the capability of the driver based on the trajectory of the car. The methodology to develop the system will be described in the next section. Then, the result of two case studies using double lane change (DLC) circuit are presented [17].

### 2.0 METHODOLOGY

Figure 1 shows the methodology adopted to develop the VDM-DM system. Firstly, the initial steer input was calculated based on the geometrical properties of the Cubic Motion curve. Due to effect of the human aspect in maneuvering the car, a new steer angle was calculated using the Renski's driver model. This new steer angle becomes the steer angle input for the vehicle dynamic model. Finally, the vehicle dynamic model system will generate the trajectory of the car. The iteration ends when all the vertices of the Cubic Motion curve have been analyzed.

### 2.1 Initial Steer Input

Cubic Motion, which is the input to the system, provides the intended trajectory and velocity of the car. Cubic Motion curve has been described in detail by Mat Ghani et al. [18].

Renski's driver model was used to calculate the initial steer input. This model was developed by Renski from Warsaw University of Technology [10]. The model is an open loop control system that replaces driver to maneuver the car based on the geometry of the path.


Figure 1: Methodology
The main reason to select the Renski's driver model for the development of VDM-DM system is related to the use of driver parameters of the model. The parameters such as sight distance ( $L_{\mathrm{a}}$ ), steering gain ( $W$ ) and delay time $\left(T_{\mathrm{k}}\right)$ reflect the human behaviour. $L_{\mathrm{a}}$ in meter is the viewing distance and $W$ is a constant that is introduced to show the rate of the steering and a value of 1 is used to represent an expert driver. Finally, $T_{\mathrm{k}}$ is the time in second to represent the delay of human action. With these parameters considered, the effect of the different types of drivers can be predicted.

The initial steer angle is based on the desired path when the car is at ( $x_{\mathrm{os}}, y_{\mathrm{os}}$ ) with heading or yaw angle ( $\psi$ ). The initial steer angle ( $\varepsilon$ ) of the car to the aim point can be calculated using Equation (1). Figure 2 describes the parameters of the equation based on the sight distance of the drivers.

$$
\begin{equation*}
\varepsilon(t)=\frac{y_{\mathrm{d}}\left(x_{\mathrm{os}}+L_{\mathrm{a}}\right)-y_{\mathrm{os}}\left(x_{\mathrm{os}}\right)}{L_{\mathrm{a}}}-\Psi\left(x_{\mathrm{os}}\right) \tag{1}
\end{equation*}
$$

where

| $x_{0}$ | $:$ | longitudinal position, the way covered by the car down the road |
| :--- | :--- | :--- |
| $y_{0}$ | $\vdots$ | lateral position of the car |
| $y_{\mathrm{d}}$ | $\vdots$ | desired path deviation from $x$-axis |
| $\psi$ | $\vdots$ | heading/yaw angle |
| $L_{\mathrm{a}}$ | $\vdots$ | aim distance in $x$-direction or sight distance (length) |

The sight distance is one of the driver parameters and its distance varies with velocity of the car. When the car travels too slow with certain sight distance, the car would experience an oversteer as far as the trajectory is concerned. Vice versa, if the car travels too fast with the same sight distance, an understeer trajectory will be produced. Figure 3 shows the relationship of the car trajectory and sight distance. In the following case studies, the median value was used as the sight distance with respect to the velocity.


Figure 2: Driver's steering control law used in the driver model [9]


Figure 3: Relationship between the car velocity and sight distance

### 2.2 New Steer Angle Based on Driver Parameter

The initial steer angle was calculated based on the sight distance of the driver. This stage will further include two more driver parameters into the steer angle equation which are the steering gain $(W)$ and delay time ( $T_{\mathrm{k}}$ ). Equation (2) equates the new steer angle. Substituting Equation (1) into Equation (2), the new steer angle is further computed in Equation (3):

$$
\begin{gather*}
\delta(t)=W \varepsilon\left(t-T_{\mathrm{k}}\right)  \tag{2}\\
\delta(t)=\frac{W \mathrm{y}_{\mathrm{d}}}{L_{\mathrm{a}}}\left(t+\frac{L_{\mathrm{a}}}{U}-T_{\mathrm{k}}\right)-\frac{W \mathrm{y}_{\text {os }}}{L_{\mathrm{a}}}\left(t-T_{\mathrm{k}}\right)-W \psi\left(t-T_{\mathrm{k}}\right) \tag{3}
\end{gather*}
$$

where

| $\varepsilon$ | $:$ | angle from vehicle to aim point |
| :--- | :--- | :--- |
| $W$ | $\vdots$ | steering gain |
| $T_{\mathrm{k}}$ | $:$ | driver's time delay |
| $U$ | $:$ | velocity of the car |

Since Equation (3) and the vertex position of Cubic Motion are time-based functions, to calculate the steer angle of the car with delay time, the position of car is similar to the position of the car at $\left(t-T_{\mathrm{k}}\right)$. Therefore, a number of vertices has to be added to the curve to allow the position of the car at $\left(t-T_{\mathrm{k}}\right)$ exists when $\left(t-T_{\mathrm{k}}\right)$ is less than zero. The number of vertices to be added can then be calculated using Equation (4). The round off error can be neglected as a small-time interval, $\mathrm{d} t$ between the vertices is small $(0.001 \mathrm{~s})$.

$$
\begin{equation*}
N=\frac{T_{\mathrm{K}}}{\mathrm{~d} t} \tag{4}
\end{equation*}
$$

where
$N \quad: \quad$ number of added vertices
$\mathrm{d} t \quad: \quad$ time interval between the vertices of the Cubic Motion curve

In the second case study, three types of drivers were set. Based on Renski work, a delay time of 0.1 s was set to Driver 1 and this driver is in fact an expert driver [10]. Table 1 shows the delay time for three types of drivers.

Table 1: Delay time for specific drivers

| Type of Driver | Delay Time $\boldsymbol{T}_{\mathbf{K}}(\mathbf{s})$ |
| :---: | :---: |
| Driver 1 (Expert driver) | 0.1 |
| Driver 2 (Normal driver) | 0.2 |
| Driver 3 (Submissive driver) | 0.4 |

## $2.3 \quad$ Vehicle Dynamic Response

The vehicle dynamic response was calculated using the Bicycle Model. The model is a two DOF of VDM. Figure 4 shows the bicycle model on an XY plane. The assumptions in the development of the two DOF VDM are the car travels at constant velocity, constant normal load on tires, constant friction between road and tires, and constant longitudinal slip of tires.


Figure 4: The Bicycle Model [19]
The foundation of the Bicycle Model was derived from the lateral dynamic equation as in Equation (5) and the total moment about the center of gravity of the car (Equation (6)). Figure 4 graphically defines the parameters.

$$
\begin{equation*}
F_{\mathrm{yf}}+F_{\mathrm{yr}}=M(a+U \dot{\psi}) \tag{5}
\end{equation*}
$$

where

$$
\begin{array}{ccl}
F_{\mathrm{yf}} & \vdots & \text { front tire force } \\
F_{\mathrm{yr}} & \vdots & \text { rear tire force } \\
M & \vdots & \text { car mass } \\
A & \vdots & \text { linear acceleration } \\
U & \vdots & \text { linear velocity } \\
y \& & \vdots & \text { yaw rate } \\
& b F_{y f}-c F_{y r}=I_{z z} \ddot{\psi} \tag{6}
\end{array}
$$

where

| $b$ | $:$ | front wheelbase |
| :--- | :--- | :--- |
| $c$ | $\vdots$ | rear wheelbase |
| $I_{z z}$ | $:$ | moment of inertia |

When the car has been steered by an angle $\delta$, it causes the front and rear tires to have slip angles of $\alpha_{\mathrm{f}}$ and $\alpha_{\mathrm{r}}$, respectively. Finally, Equations (7) and (8) were derived for the total lateral forces and moment, respectively.

$$
\begin{align*}
L_{F}= & {\left[\frac{-C_{\alpha f}-C_{\alpha r}}{U}\right] V+\left[\frac{-b C_{\alpha f}+c C_{\alpha r}}{U}\right] \dot{\psi}+\left[C_{\alpha f}\right] \delta } \\
& L_{F}=L_{V} V+L_{r} \dot{\psi}+L_{\delta} \delta \tag{7}
\end{align*}
$$

where

$$
\begin{align*}
& \begin{array}{lll}
L_{\mathrm{F}} & : & \text { total lateral force } \\
C_{\alpha \mathrm{f}} & : & \text { front tire cornering stiffness } \\
C_{\alpha \mathrm{r}} & : & \text { rear tire cornering stiffness } \\
\delta & : & \text { steer angle } \\
N= & {\left[\frac{-b C_{\alpha}+c C_{\alpha \mathrm{r}}}{U}\right] V+\left[\frac{-b^{2} C_{\alpha \mathrm{f}}-c^{2} C_{\alpha \mathrm{r}}}{U}\right] \dot{\psi}+\left[b C_{\mathrm{af}}\right] \delta} \\
& N=N_{\mathrm{V}} V+N_{\mathrm{r}} \dot{\psi}+N_{\delta} \delta
\end{array}
\end{align*}
$$

Therefore, the total lateral acceleration and yaw acceleration are shown in Equations (9) and (10), respectively.

$$
\begin{gather*}
\dot{V}=\left[\frac{L_{\mathrm{V}}}{M}\right] V+\left[\frac{L_{\mathrm{r}}}{M}\right] \dot{\psi}+\left[\frac{L_{\mathrm{v}}}{M}\right] \delta  \tag{9}\\
\ddot{\psi}=\left[\frac{N_{\mathrm{V}}}{I_{\mathrm{zz}}}\right] V+\left[\frac{N_{\mathrm{r}}}{I_{\mathrm{zz}}}\right] \dot{\psi}+\left[\frac{N_{\delta}}{I_{z z}}\right] \delta \tag{10}
\end{gather*}
$$

Using ODE, the yaw rate, yaw acceleration, lateral velocity and acceleration were solved. With the steering angle $\delta$, the car will turn by a yaw angle $\Psi$. The relationship between the first derivative of the position of the car, yaw angle, longitudinal and lateral velocities is shown in Figure 5. Therefore, Equations (11) and (12) were derived and the integration of the velocities in $x$ and $y$ axes compute the position of the car.

$$
\begin{align*}
\dot{x} & =U \cos (\psi)+V \sin (\psi)  \tag{11}\\
\dot{y} & =U \sin (\psi)+V \cos (\psi) \tag{12}
\end{align*}
$$



Figure 5: Relationship between the longitudinal and lateral velocities in $x$ and $y$ axes
The detailed derivation of the Bicycle Model can be found in Abe [19]. In the following case studies, three types of cars have been used. The parameters of these cars are shown in Table 2 for commercial cars.

### 2.4 Comparison with Actual Trajectory

Previous step determines the position of the car and the heading angle. Then, the system will compare the $x$ coordinate of the current position with the $x$ coordinate of the intended trajectory to check whether the car has reached the destination. If not, step 1 will be executed with the current position as ( $x_{\mathrm{os}}, y_{\mathrm{os}}$ ) and heading angle as $(\psi)$. The process continues until all the vertices on the Cubic Motion curve has been examined.

Table 2: Car parameters

| Parameter | Unit | Car A | Car B | Car C |
| :--- | :---: | :---: | :---: | :---: |
| Total mass | kg | 1500 | 1218 | 1251 |
| Yaw moment inertia | $\mathrm{kgm}^{2}$ | 2500 | 2250 | 2027 |
| Front wheelbase, $b$ | m | 1.167 | 1.200 | 1.251 |
| Rear wheelbase, $c$ | m | 1.333 | 1.600 | 1.201 |
| Front cornering stiffness <br> for two tires, $C_{\alpha \mathrm{f}}$ | $\mathrm{N} / \mathrm{rad}$ | 50000 | 50000 | 50000 |
| Rear cornering stiffness for <br> two tires, $C_{\alpha r}$ | $\mathrm{~N} / \mathrm{rad}$ | 50000 | 50000 | 50000 |

### 3.0 RESULTS AND DISCUSSION

Two case studies were carried out. The first case study was to study the trajectory of three different cars on a double lane circuit (DLC). The second case study was to examine the trajectory of the car when it was driven by three different drivers. The first case study focused on the effect of the VDM in generating the trajectory, whilst the second case study was to evaluate the effect of the drivers' parameters of the driver model in generating the trajectory.

### 3.1 Case Study 1

In this case study, three cars, namely, Car A, Car B and Car C travel at a specific velocity on a double lane change (DLC) circuit. The steering gain $W=1$, and delay time $T_{\mathrm{k}}=0 \mathrm{~s}$ were set to be the same values for all first case study simulations.

Figure 6 shows the trajectory of the cars traveling at $10 \mathrm{~m} / \mathrm{s}$ with the most suitable sight length, 5 m . All three trajectories are mostly the same. The trajectories are perfectly generated on the same line because they are able to follow the desired path without any major deviation from the intended path.

Figure 7 shows the trajectories of these three cars cruising at $20 \mathrm{~m} / \mathrm{s}$ with the sight length of 10 m . At the first corner, all the cars can follow the path. After the second corner while approaching the third corner, trajectories of Cars A and B show larger deviations than the trajectory of Car C. Then, all the cars can follow the path to the last corner. At the last corner, the highest deviation of all car trajectories is shown compared to the trajectories of the previous corners. Car C's trajectory shows the least deviation from the desired path compared to Cars A and B.

Then, all the cars were simulated at a speed of $25 \mathrm{~m} / \mathrm{s}$, and the results are shown in Figure 8. The cars started to take the corner earlier than the previous simulations. This is because the sight length is 15 m , which is longer than those of the two previous simulations.

From Figure 9 at the first corner of the intended path, the trajectories of all the three cars are mostly similar. However, Cars A and B trajectories have slight deviation from Car C path when exiting the first corner. From the second corner to third corner, Car C has the smallest deviation from the intended path compared to the paths of Cars A and B. Car A is most deviated from the desired path. The last corner shows all cars deviate from the intended path and struggle to get back on track. However, the pattern of the deviation is similar to the deviation at the second and third corners.


Figure 6: Different cars tested and traveling at $10 \mathrm{~m} / \mathrm{s}$


Figure 7: Different car parameters tested and traveling at $20 \mathrm{~m} / \mathrm{s}$


Figure 8 Different car parameters tested and traveling at $25 \mathrm{~m} / \mathrm{s}$

Based on previous results of Figures 6 and 7, Car C has the least deviation visually as expected, followed by the trajectories of Cars B and A that have the largest deviations. Figure 8 presents the trajectories of the cars and these outcomes are as expected.

### 3.2 Case Study 2

The second case study was to investigate the trajectories when Car A was driven by three different drivers. The delay time to react was used to categorize the driver. Table 1 shows the categorization of the driver and delay time. The expert driver (Driver 1) has the lowest delay time which is 0.1 s because the driver was assumed to have high mental concentration and very experienced in driving. The other drivers have delay times of 0.2 s and 0.4 s . Other driver parameters, $L_{\mathrm{a}}$ and $W$ remain the same for all the drivers.

The first simulation was conducted with $T_{\mathrm{k}}=0.1 \mathrm{~s}$ according to the expert driver delay time, and the car is traveling at $10 \mathrm{~m} / \mathrm{s}$ using Car A parameters and the trajectory is shown in Figure 9. Other driver parameters were set to default values which are $L_{\mathrm{a}}=5 \mathrm{~m}$ (referred to as the most suitable sight distance with respect to the car speed) and the steering gain, $W$ $=1$. The result shown indicates that the car can follow the desired path and deviation occurs after leaving the second and last corners.

The second simulation was set for Driver 2 (normal driver) with a delay time of 0.2 s and the result is shown in Figure 10. Other parameters were set to default values. The trajectory follows the intended path and it appears to be very unsteady. The driver struggled to follow the track. The oscillating trend in the trajectory is increasing throughout the simulation. There are several factors that contribute to this trajectory formation.

The factor that produces this result is due to inappropriate sight distance value. As the delay time of the driver changes, then the sight distance should be changed. The car is traveling at $10 \mathrm{~m} / \mathrm{s}$ and the delay time is 0.2 s , the aim distance will be 2 m short. Thus, the actual sight distance for this simulation is only 3 m ; this is not a suitable sight distance for a car traveling at $10 \mathrm{~m} / \mathrm{s}$. The sight distance should be changed to 7 m to compensate for the delayed aim distance.

The second factor that affects the simulation is the steering gain. The steering gain represents the driving style of driver [20]. If the steering gain is high, it means that the driver will steer swiftly. In contrast, if the steering gain is low, the driver will steer steadily. For the simulation to be corrected, the driver parameters must also be corrected. According to Renski, for the interdependence relation of driver parameters, a time delay of 0.2 s should have a steering angle with a value of 0.6 when the sight distance is 7 m [10]. The corrected driver parameters through the simulation is shown in Figure 11. It shows the car trajectory for Driver 2 with the corrected driver parameters with respect to the time delay. The trajectory is smoother than the trajectory in Figure 10. This is because the steering gain has improved the drivers steering ability not to steer the car swiftly at the corner. The longer sight distance improves the ability of the driver to retrieve information on the desired path.

Finally, the ability of the Driver 3 (submissive driver) is simulated on the DLC circuit. The delay time is 0.4 s . The other parameters were set as the default setting to see the effect of the delay time on the simulation. Driver 3 failed even to take the first corner. The trajectory at the first corner is shown in Figure 12. For 0.4 s delay time when traveling at $10 \mathrm{~m} / \mathrm{s}$, the delayed sight distance was 4 m , while the sight distance of the driver was set to 5 m . Hence, the actual aim point distance for this simulation is only 1 m . The actual aim point relative to the speed of the car is not sufficient for the driver to maneuver the car to follow the path. Therefore, the driver failed to steer the car.

The maneuvering can be improved by correcting the driver parameters. With the delay time $T_{k}=0.4 \mathrm{~s}$, the sight distance $L_{\mathrm{a}}=9 \mathrm{~m}$ and steering gain $W=0.25$, the trajectory of the car driven by Driver 3 is shown in Figure 13. In all, it can be concluded that Driver 3 cannot maneuver the DLC circuit when the speed of the car is $10 \mathrm{~m} / \mathrm{s}$.


Figure 9: Car trajectory for Driver 1 with $T_{\mathrm{k}}=0.1 \mathrm{~s}$


Figure 11: Car trajectory for Driver 2 with corrected driver parameters


Figure 10: Car trajectory for Driver 2 with $T_{\mathrm{k}}=0.2 \mathrm{~s}$


Figure 12: Car trajectory for Driver 3 with $T_{\mathrm{k}}=0.4 \mathrm{~s}$


Figure 13: Car trajectory for Driver 3 with corrected driver parameters

### 4.0 CONCLUSION

VDM-DM system is a system that integrates the vehicle dynamic response and driver model. The key point of the integration is driver model that provides the steer input to vehicle dynamic analysis and the iteration continues until all the vertices has been visited. The case studies have shown that VDM-DM system has the capability to evaluate the car
and driver performance using the intended path as the input. In the case of the driver type, VDM-DM system is able to simulate different types of drivers. This system can be further used to analyze the driver behavior in order to follow the intended path. VDM-DM system has the ability to bring the assessment of the car and driver to the desktop. This system has the potential to be used as an assessment tool for the car and driver. More importantly, the system can be used to predict the failure to follow the path that is due to the car or driver. This system has been developed based on 2D trajectory and two DOF Bicycle Model. A 3D trajectory to include the road vertical elevation which requires a six DOF VDM model should be developed.

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