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Stability of turning process with tool subjected to compression

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Abstract

This study investigates how the stability of the turning process changes when the tangential component of the cutting force is also taken into account as a compressive force acting on the cutting tool. The tool is modelled by a cantilever beam; the mathematical model is based on the Euler-Bernoulli beam theory. The effect of compression appears through the lateral stiffness of the tool. Two cases are separated in connection with the compressive force: constant and varying forces. Since compression reduces the natural frequency of the cutting tool, this also affects the stable region of the turning operation.

Keywords: turning process; regenerative effect; stability; compression

1. Motivation

During machining processes, unexpected vibrations called chatter might occur due to the regenerative effect, which generally results in noise or tool break and affects the quality of the machined surface. This is because either the cutting tool or the workpiece or both are flexible and the chip thickness varies due to the relative vibrations of the tool and the workpiece.

In case of turning, the regenerative effect is modelled by a time delayed system. The general mechanical model was constructed by Tobias [1] and Tlusty [2]. The tool cuts the surface that was machined in the previous cut, and the chip thickness is described by the current and the previous position of the tool. The time delay between two succeeding cuts is equal to the period of the workpiece rotation.

Nomenclature

c_y	modal damping
h_0	intended chip thickness
h	instantaneous chip thickness
IE	bending stiffness of the cutting tool
k_y	modal stiffness
$K_{x,y}$	cutting-force parameters
L	length of the cutting tool
m	modal mass
w	chip width
τ	time delay
ζ	damping ratio
ω_n	natural angular frequency
Ω	spindle speed

The objective of this study is to take into account the possible appearance of compressive effect on the cutting tool, which somewhat modifies the natural frequency of the bending vibration of the tool. There have been related results in the literature: Budak *et al.* [3] and Stepan *et al.* [4] have investigated how the workpiece dynamics changes due to mass removal and the variation of the tool position in turning. Bayly et al. [5] dealt with low frequency vibration in drilling to find agreement with drilling tests in the presence of large longitudinal cutting forces. Roukema and Altintas [6] and Heisig and Neubert [7] considered lateral vibration of drilling tools under compression. Beri *et al.* [8,9] have examined how the lateral stiffness of a cantilever beam changes under compression, tension and torsion, respectively.

In case of turning, there exist certain tool-workpiece arrangements where the tangential component of the cutting force acts on the cutting tool as a compressive force, which modifies the natural frequency of the system. The tool is modelled by a cantilever beam that is considered to be prismatic, homogeneous, linearly elastic and inextensible. Its mathematical model is based on the Euler-Bernoulli beam theory and the effect of compression appears through the lateral stiffness of the tool. To provide a wider picture about the influence of compression on the stability of the turning process, two cases are investigated: constant and varying forces. The latter case is in connection with the regenerative effect. Since compression decreases the

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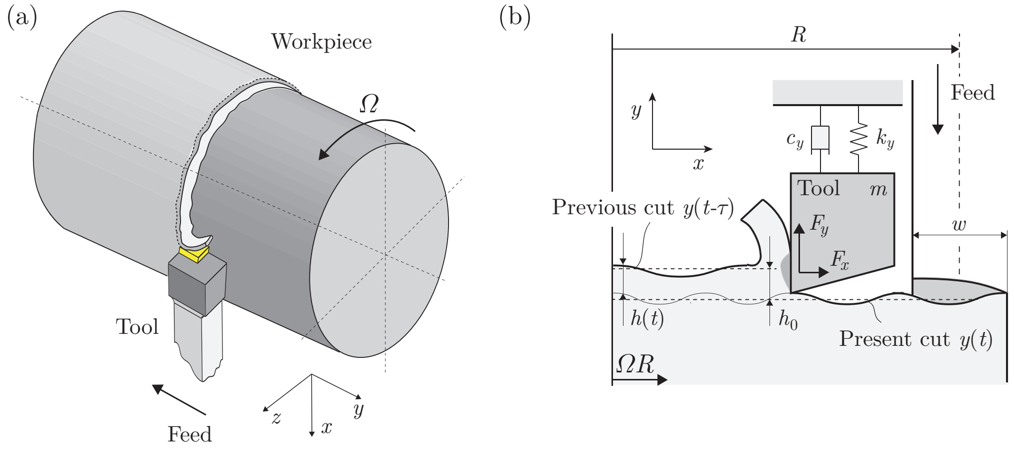

Figure 1. (a) Tool-workpiece arrangement where the tangential component of the cutting force acts on the tool as compression. (b) Mechanical model of surface regeneration in turning operation.

lateral stiffness and modifies the bending natural frequency of the cutting tool, it has a destabilizing effect, that is, it reduces the stable region of the turning process. To support our numerical results, some analytical formulas are also presented on how the lobe structure will change by the variation of the compressive component of the cutting force.

2. Modelling and Analysis

In order to perform the stability calculation, the mechanical model of the turning operation of a flexible cutting tool is considered as shown in Fig.1. The governing equation of the 1 DoF model assumes the form

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \frac{F_y(t)}{m}\,, \tag{1}$$

where $\zeta = c_y/(2m\omega_n)$ and $\omega_n = \sqrt{k_y/m}$. Here, $k_y = k_y(F_x)$ is the lateral stiffness of the cutting tool in the y direction depending on the tangential component of the cutting force F_x, which acts as a compressive force. Based on the Euler-Bernoulli beam theory, the stiffness k_y of the clamped tool under compression F_x is expressed by [8] in the form

$$k_y = \frac{\alpha^3 IE}{\tan(\alpha L) - \alpha L} \tag{2}$$

where $\alpha = \sqrt{F_x/IE}$. Equation (2) can be approximated by the power series

$$k_y = \frac{3IE}{L^3} - \frac{6}{5L}F_x - O(F_x^2)\,, \tag{3}$$

where only the first two components are considered since even the Euler buckling load $F_{cr} = \pi^2 IE/(4L^2)$ is satisfactorily approximated by $k_y = 0$.

In Eq. (1), the widely used cutting force characteristic [1] is applied:

$$F_x(t) = K_x w h^q(t)\,, \tag{4}$$

$$F_y(t) = K_y w h^q(t)\,, \tag{5}$$

where q is the cutting force exponent, which represents a strong nonlineraity and plays significant role in determining the chatter free cutting conditions. In the literature, there exist many suggestions for the value of exponent q (see [10–16]) to provide an accurate estimation for the cutting force characteristics.

In the mechanical model depicted in Fig.1.(b), it is assumed that the tool does not leave the surface, that is, the instantaneous chip thickness $h(t) > 0$ during the turning operation. The chip thickness

$$h(t) = h_0 + y(t-\tau) - y(t) \tag{6}$$

can be obtained by taking into account the regenerative effect, that is, the cutting tool meets the surface that was formed in the previous cut (see Fig.1(b).) Here, τ is the regenerative time delay that is approximated by

$$\tau = \frac{60}{\Omega} \tag{7}$$

where the constant spindle speed Ω is given in (rpm).

2.1. Type of Compression

In connection with the compressive force F_x, two cases are separated: constant and varying forces. In the first case, F_x is considered to be constant, thus it affects only the lateral stiffness of the cutting tool by a constant value through Eq. (3). In the latter case, let us consider that the tangential force F_x is proportional to the normal force F_y:

$$F_x(t) = \frac{F_y(t)}{\sigma} \tag{8}$$

where $\sigma = 0.3$ is a typical value in the literature (see for example [18]). Substitution of Eqs. (5) and (6) into (8) yields

$$F_x(t) = \frac{K_y w}{\sigma}(h_0 + y(t-\tau) - y(t))^q \tag{9}$$

where F_x is proportional to the chip width w and the regenerative effect also appears. We note that the expansion of the lateral stiffness (see Eq. (3)) gives

$$k_y = k_{y0} - \frac{k_{y1}}{\sigma}K_y w(h_0 + y(t-\tau) - y(t))^q\,, \tag{10}$$

where $k_{y0} = 3IE/L^3$ and $k_{y1} = 6/(5L)$. By means of Eqs. (1), (5) and (6), the equation of motion of the system can be written in the form

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \frac{K_y w}{m}(h_0 + y(t-\tau) - y(t))^q\,. \tag{11}$$

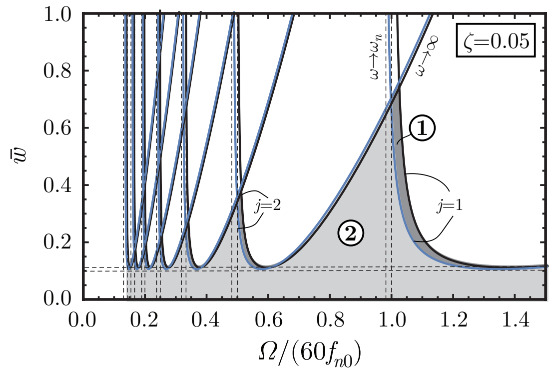

Figure 2. Dimensionless stability diagram where notation ① shows the stable region of the system subjected to no compression and notation ② shows the stable region of the system subjected to large constant compression (F_x = 8000 N). The damping ratio is $\zeta = 0.05$ and the length of the tool is $L = 0.10$ m. Here, $f_{n0} = \omega_{n0}/(2\pi)$ is the *basic* natural frequency.

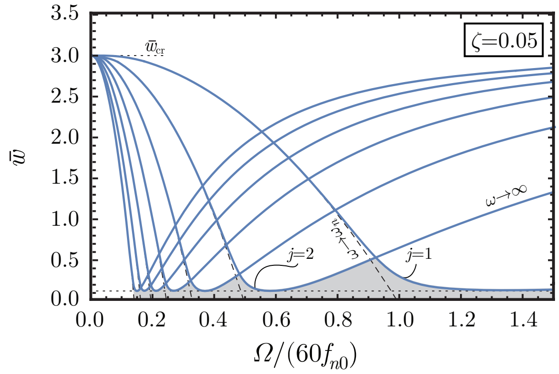

Figure 3. Dimensionless stability diagram of the system subjected to varying compression. The numerical values $\sigma = 0.3$, $\zeta = 0.05$, $q = 0.6$, $h_0 = 0.005$ m and $L = 0.1$ m are used. Here, $f_{n0} = \omega_{n0}/(2\pi)$ is the *basic* natural frequency.

By considering the equilibrium position $y(t) = y_0$ in Eq. (11), one obtains

$$y_0 = \frac{K_y w h_0^q}{k_{y0} - \frac{k_{y1}}{\sigma} K_y w h_0^q} . \tag{12}$$

This constant steady-state corresponds to the deflection of the tool when no vibrations occur during the operation, that is, it means the stationary case. The linearisation about the equilibrium position gives the variational system [18]

$$\ddot{\eta}(t) + 2\zeta\omega_n\dot{\eta}(t) + (\omega_n^2 + H)\eta(t) = H(\eta(t-\tau) - \eta(t)) , \tag{13}$$

where η means the small perturbation about the equilibrium. Here, $\omega_n^2 = (k_{y0} - k_{y1}K_y w h_0^q/\sigma)/m$ and the specific cutting coefficient $H = K_y w q h_0^{q-1}(1 + k_{y1}y_0/\sigma)/m$.

The stability analysis of the system Eq. (13) is performed by the D-subdivision method [17], which gives two formulas

$$H = \frac{(\omega^2 - \omega_n^2)^2 + 4\zeta^2\omega_n^2\omega^2}{2(\omega^2 - \omega_n^2)} \tag{14}$$

and

$$\Omega = \frac{30\omega}{j\pi - \arctan\left(\frac{\omega^2 - \omega_n^2}{2\zeta\omega\omega_n}\right)}, \quad j \in \mathbb{N}_0 \tag{15}$$

where ω is the frequency of the arising regenerative vibrations. These two expressions provide parametric stability boundary curves, called D-curves, that represents dynamic loss of stability (Hopf-bifurcation).

3. Results

When the tangential force F_x is either constant or zero, the natural angular frequency ω_n is also constant in accordance with Eq. (3) and H is simply equal to $\bar{H} = K_y w q h_0^{q-1}/m$ in Eq. (13) (see in Ref.[18]). In the stability charts, the dimensionless chip width $\bar{w} = \bar{H}/\omega_{n0}^2$ is used where $\omega_{n0} = \sqrt{k_{y0}/m}$ denotes the *basic* natural angular frequency.

The dimensionless D-curves can be constructed by using the analytical expressions Eqs. (14) and (15) shown in Fig.2. $\bar{w}$ is proportional to the chip width w, thus only the practically relevant domain $\bar{w} > 0$ is depicted. The vertical dotted lines (see Fig.2) are asymptotes, which can be obtained from Eqs. (3) and (15)

$$\Omega_{\mathrm{asy}}^0 = \lim_{\omega\to\omega_n} \Omega = \frac{1}{j}\frac{60}{2\pi}\sqrt{\frac{k_{y0} - k_{y1}F_x}{m}} . \tag{16}$$

The horizontal dotted lines describe the minimum points of the lobes where the turning process is still stable independently to the cutting speed. These are determined by Eq. (14) using the method of local minima. It can be seen that the constant compressive force provides a shifted stability map. The map moves left and down because the lateral stiffness of the cutting tool decreases, which affects the natural angular frequency, too. Accordingly, even a large constant compression force has negligible destabilizing effect.

In contrast, in case of varying compression (see Subsec.2.1.), H and the natural angular frequency ω_n also depend on the chip width w, which leads us to a nonlinear expression in w (see Eq.(14)). Hence, we are only able to provide the D-curves numerically. The dimensionless stability map can be seen in Fig.3. Similarly to Eq. (16), the asymptotes can easily be calculated. Since the natural angular frequency is a function of the chip width w, the asymptotes will also depend on w described by the closed form analytical formula

$$\Omega_{\mathrm{asy}}^{\mathrm{v}} = \frac{1}{j}\frac{60}{2\pi}\sqrt{\frac{k_{y0} - \frac{k_{y1}}{\sigma}K_y w h_0^q}{m}} . \tag{17}$$

Since the natural angular frequency decreases as $\bar{w}$ increases (see in Fig.3), the asymptotes are bent to the left. There is a critical value where both the asymptotes and the D-curves meet, which can be expressed by

$$\bar{w}_{\mathrm{cr}} = \frac{5L\sigma q}{6h_0} . \tag{18}$$

This value means the static loss of stability of the cutting tool, that is, the lateral stiffness of the tool decreases to zero, which results in the buckling of the tool and tool breakage.

In Fig.4., the stability diagrams of the varying and the zero compression cases are merged for different damping ratio values. The lobe structure is shifted down and deflected both to the left and also somewhat to the right compared to the non-compressed case. Basically, the varying compressive force has a large scale destabilizing effect but it might also broaden the reference (non-compressed) stable region slightly at relatively

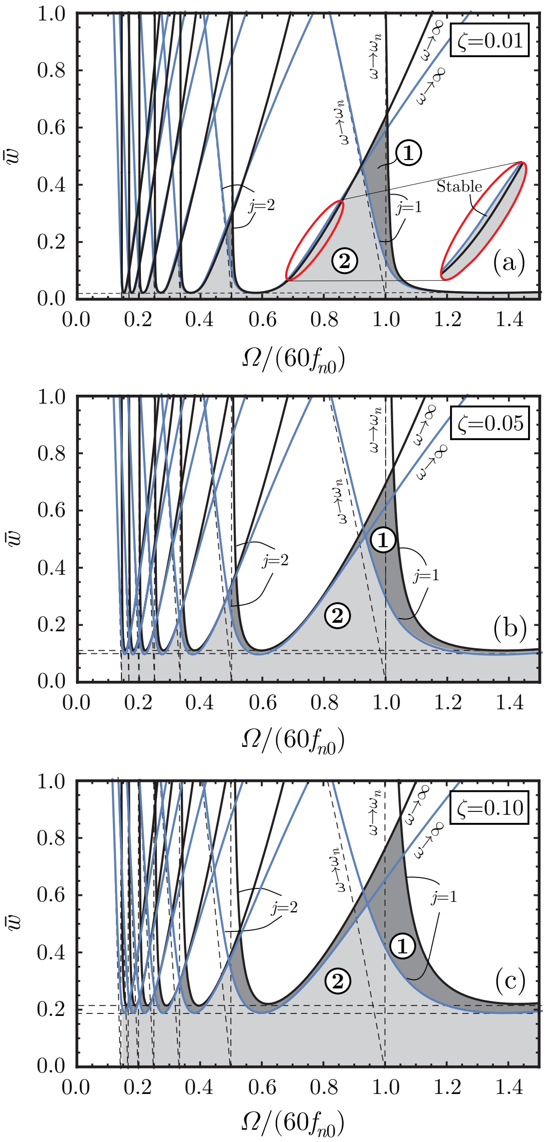

Figure 4. Dimensionless stability charts where notation ① shows the stable region of non-compressed system and notation ② shows the stable regions of the system subjected to varying compression. The numerical values $\sigma = 0.3$, $q = 0.6$, $h_0 = 0.005$ m and $L = 0.1$ m are used. Here, $f_{n0} = \omega_{n0}/(2\pi)$ is the *basic* natural frequency.

small damping ratios as shown in the enlarged panel of Fig.4 (a). As the damping ratio grows, the stable region significantly reduces in contrast to the reference case. This means, for example, that the available maximum values of the chip width w are reduced in the stable pockets of the charts.

4. Conclusion

The paper brings up the classical topic of the stability of the turning operation, which can be described by a 1 DoF dynamical model. The cutting force can be resolved to a normal force F_y and a tangential force F_x that acts on the cutter as compression in certain tool-workpiece configurations. This study investigated two cases: constant and varying compressive forces.

Based on the Euler-Bernoulli beam theory, the lateral stiffness of the cutting tool can be calculated, which also describes the variation of the natural angular frequency of the tool. By using constant compression, the lobe structure is slightly shifted to the left and down compared to the non-compressed case.

In contrast, under the effect of the varying compression, the stability map is shifted down and deflected both to the left and to the right compared to the non-compressed case. This predicts a non-negligible reduction of the size of the stable pockets, which serves as a basis for future experimental verification. These phenomena may become relevant when slender tools are used for turning inside lengthy tubes.

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