TRANSMIT BEAMFORMING DESIGN WITH RECEIVED-INTERFERENCE POWER CONSTRAINTS: THE ZERO-FORCING RELAXATION

E. Lagunas[†], *A. Pérez-Neira*[⋆‡], *M.A. Lagunas*[⋆‡], *M.A. Vazquez*[⋆]

[†] Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg
29 Av. JF Kennedy, L-1855, Luxembourg, Email: eva.lagunas@uni.lu
[⋆] Centre Tecnologic de les Telecomunicacions de Catalunya (CTTC/CERCA),
Av. Carl Friedrich Gauss 7, 08860 Castelldefels, Spain
[‡] Universitat Politecnica de Catalunya (UPC)
Email:{aperez,m.a.lagunas,mavazquez}@cttc.es

ABSTRACT

The use of multi-antenna transmitters is emerging as an essential technology of the future wireless communication systems. While Zero-Forcing Beamforming (ZFB) has become the most popular low-complexity transmit beamforming design, it has some drawbacks basically related to the effort of "trying" to invert the channel coefficients towards the interfered users. In particular, ZFB performs poorly in the low Signal-to-Noise Ratio (SNR) regime and does not work when the interfered users outnumber the transmit antennas. In this paper, we study in detail an alternative transmit beamforming design framework, where we allow some residual received-interference power instead of trying to null it completely out. Subsequently, we provide a close-form non-iterative optimal solution that avoids the use of sophisticated convex optimization techniques that compromise its applicability onto practical systems. Supporting results based on numerical simulations show that the proposed transmit beamforming is able to perform close to the optimal with much lower computational complexity.

***Index Terms*—** Multi-antenna, beamforming, MISO, received-interference power.

1. INTRODUCTION

In modern wireless communication systems, the use of multi-antenna transmitters is emerging as an essential technology to sustain the increase in user data demand and the need for more spectrum efficient communications. A system where the transmitter is equipped with multiple antennas and communicates with K single antenna users is known as Multiple-Input-Single-Output (MISO) system.

The capacity of MISO channels can be achieved by Dirty-Paper Coding (DPC) [1, 2]. However, the computational complexity introduced by the non-linearities of DPC has limited its success in practical systems. In this context, significant efforts have been devoted to the design of low complexity precoding / beamforming strategies able to achieve a performance comparable to that of DPC (e.g. [3–7]).

In general, in this paper we refer to precoding when the Channel State Information (CSI) is used in the transmitter design, and we refer to *beamforming* when the transmitter design only exploits steering angular information and user location information to form the proper transmit beam pattern. However, as we will see later on, the notation is kept general and the discussion applies to both.

Among the more practical transmit beamforming strategies, the Zero-Forcing Beamforming (ZFB) has become the most popular. In particular, the performance of ZFB with proper user selection is proven to be asymptotically close to the pareto optimality of DPC [8]. In ZFB, the beamforming weights are designed to eliminate all inter-user interference and thus, decouple the multi-user interference channel. ZFB is highly related to generalized matrix inversion [9], as it basically "tries" to invert the channel coefficients. This is the reason why ZFB performs poorly in the low SNR regime, as the attempt to invert very low channel coefficients incurs an unaffordable power consumption. Moreover, ZFB cannot be applied when the number of interfered users outnumbers the number transmit antennas.

In this paper, we study a relaxation of the conventional ZFB where we allow some residual received interference instead of trying to null it out. More precisely, we formulate the MISO transmit beamforming problem as a maximization of the transmit power towards the desired direction while imposing a number of received-interference power constraint towards the undesired directions while keeping the total transmit power under certain limit. This design is closer to the final implemented solution, which is not capable of forcing "pure" zeros. Also, zero-forcing may result in a distorted beamformer with high sidelobes that increase the background interference level. The resulting optimization problem with received-interference power constraints is similar to that encountered in underlay cognitive radio literature [10, 11] and can be addressed under a Second-Order Cone Programming (SOCP) formulation. However, the resulting optimization procedure is rather inappropriate for real-time application since it requires iterative solving procedures. In this paper, we reformulate the problem by focusing on a lower-bound of the overall received-interference constraints and provide a close-form non-iterative sub-optimal solution that avoids the use of sophisticated convex optimization techniques that may compromise its applicability onto practical systems.

The resulting beamformer presents a design similar to that of the virtual Signal-to-Interference-plus-Noise Ratio (SINR) transmit beamformer [12] or to the so-called MMSE transmit beamformer [13]. The main difference between [12, 13] and our proposal resides in the regularization factor. While in [12,13] the regularization factor is obtained by resorting to the non-trivial uplink-downlink duality framework, which was originally developed for characterizing the sum-capacity of the Gaussian broadcast channel in [14], in this work we resort to a more natural and intuitive transmit design: the control of the generated interference power.

Received-power constraints have been considered in previous authors' works [15] and [16]. Both [16] and [15] present distributed power control techniques for multi-objective optimization applied to Time-Area-Spectrum (TAS) licensed system considering multiple multi-antenna transmit stations. In this paper, we focus on a single multi-antenna transmit beamforming design and we generalize the conventional ZFB by relaxing the interference-nulling constraints and converting those into maximum received-interference power constraints. The main goal of this paper is to provide a more general framework for transmit beamforming design applicable to any MISO scenario.

The rest of the paper is organized as follows: Section 2 introduces the general MISO system model and presents the optimization framework. Section 3 presents two proposed transmit beamforming design with received-power constraints. Supporting simulation results are presented in Section 4, and finally, concluding remarks are provided in Section 5.

2. SYSTEM MODEL

We consider the multi-user MISO scheme illustrated in Fig. 1, where a N-antenna transmitter communicates K single antenna users. The transmitted vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is a linear transformation of the information symbols s_k, $k = 1, \ldots, K$. In particular, the transmitted signal is formed as

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{b}_k \cdot s_k, \tag{1}$$

where $\mathbf{b}_k \in \mathbb{C}^{N\times 1}$ is the beamforming vector associated to user k. Assuming that the information symbols have average unit energy, i.e. $\mathbb{E}\left\{|s_k|^2\right\} = 1$, the limitation on the available power at the transmitter side is given by $P_{\text{TX}} = \sum_{k=1}^{K} p_k \mathbf{b}_k^H \mathbf{b}_k$.

The signal observed by user k can thus be modeled as,

$$y_k = \sqrt{p_k}\mathbf{h}_k^H \cdot \mathbf{x} + w_k, \tag{2}$$

where $\mathbf{h}_k$ is the N-length channel vector from the transmitter to the k-th user and w_k denotes the zero-mean unit-variance complex Gaussian noise samples at the k-th user. For a given beamforming matrix $\mathbf{B}$, the Signal-to-Interference-plus-Noise Ratio (SINR) at the k-th user can be expressed as,

$$\text{SINR}_k = \frac{p_k \left|\mathbf{h}_k^H \mathbf{b}_k\right|^2}{\sum_{\substack{i=1\\ i\neq k}}^{K} p_k \left|\mathbf{h}_k^H \mathbf{b}_i\right|^2 + 1}. \tag{3}$$

The system sum-rate is given by $\sum_{k=1}^{K} \log\left(1 + \text{SINR}_k\right)$.

Among the most popular low-complexity beamforming strategies, ZFB designs the beamformer vectors $\mathbf{b}_k$, $k = 1, \ldots, K$, to enforce zero inter-user interference, i.e. $\mathbf{h}_k^H \mathbf{b}_i = 0$, for $k \neq i$ [17]. As mentioned in the introduction, the goal of completely decoupling the MISO interference channel may become very challenging, particularly in the low SNR regime, as the ZFB solution would require $P_{\text{TX}} \to \infty$. In addition, forcing zeros may result in significant background noise due to the raising side-lobes and does not work when $(K - 1) > N$. In the next section, we propose an alternative beamforming design where instead of zero-forcing the undesired interference, we allow certain interference levels that the system is suppose to be able to tolerate.

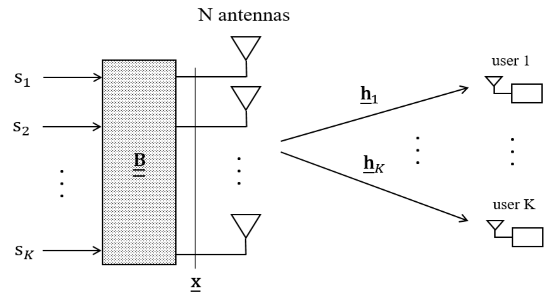

Fig. 1. Scheme of a multiuser MISO downlink system

3. TRANSMIT BEAMFORMING DESIGN WITH RECEIVED-INTERFERENCE POWER CONSTRAINTS

Let us assume a desired user d whose channel vector is denoted by $\mathbf{h}_d$. The transmit beformer associated to the symbol of the desired user should be designed to maximize the link gain towards user d, while satisfying received-power constraints towards the existing unintended users of the system. In other words, the design of each of the beamformer vectors $\mathbf{b}_k$, $k = 1, \ldots, K$, shall be given by the solution to the following optimization problem,

$$\begin{aligned} \max_{\{\mathbf{b}_k, p_k\}} \quad & p_k \mathbf{b}_k^H \mathbf{R}_d \mathbf{b}_k \\ \text{s.t.} \quad & p_k \mathbf{b}_k^H \mathbf{R}_j \mathbf{b}_k \leq P_j, \quad j = 1, \ldots, J \quad \text{(C1)} \\ & p_k \mathbf{b}_k^H \mathbf{b}_k \leq P_{\text{max}}, \quad \text{(C2)} \end{aligned} \tag{4}$$

where $\mathbf{R}_d = \mathbf{h}_d \mathbf{h}_d^H$ and $\mathbf{R}_j = \mathbf{h}_j \mathbf{h}_j^H$ for $j = 1, \ldots, J$, where J denotes the unintended users whose received interference we are trying to limit. Note that $p_k \mathbf{b}_k^H \mathbf{R}_d \mathbf{b}_k$ is the received power at the desired user d, and $p_k \mathbf{b}_k^H \mathbf{R}_j \mathbf{b}_k$ is the received power at the unintended user j. Clearly, the constraint (C1) denotes the received-power limits imposed to the unintended users, where P_j denotes the maximum received power that the j-th unintended user can tolerate. Finally, the constraint (C2) restricts the total transmit power associated to the beamformer $\mathbf{b}_k$ to be below P_{max}, which can be fixed based on the total available power.

While ZFB requires $J < N$ so that the channel is invertible. In the proposed beamforming design (4), this is no longer a necessary condition, as it will depend on the value that we introduce in P_j, $j = 1, \ldots, J$.

The problem in (4) can be cast as a SOCP [18] and solved with standard optimization software, e.g. [19]. In what follows, we will present a sub-optimal solution with a closed-form expression that can drastically reduce the computation time of the SOCP procedure. For this, we first convert the constraints (C1) and (C2) into the following expressions,

$$p_k \leq \frac{P_j}{\mathbf{b}_k^H \mathbf{R}_j \mathbf{b}_k}, \quad j = 1, \ldots, J \tag{5}$$

$$p_k \leq \frac{P_{\text{max}}}{\mathbf{b}_k^H \mathbf{b}_k}. \tag{6}$$

From (5) and (6) it is not obvious to determine which of the constraints on p_k is the most restrictive one. Therefore, to obtain a single constraint that encompasses (5) and (6) we make use of the following inequality,

$$\left(\sum_{q=1}^{Q} \frac{1}{x_q}\right)^{-1} \leq x_{\min}, \tag{7}$$

where $x_{\text{min}} = \min\{x_1, x_2, \ldots, x_Q\}$. Therefore, we can replace (C1) and (C2) in (4) with the following constraint,

$$p_k \leq \frac{P_{\text{max}}}{\mathbf{b}_k^H \left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right) \mathbf{b}_k}. \tag{8}$$

Taking the equality in (8) and substituting the value of p_k into the objective function of the main problem (4) results in the following optimization problem,

$$\max_{\{\mathbf{b}_k\}} \quad \frac{P_{\text{max}} \mathbf{b}_k^H \mathbf{R}_d \mathbf{b}_k}{\mathbf{b}_k^H \left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right) \mathbf{b}_k}. \tag{9}$$

Using the *Rayleigh Quotient*, we know that (9) is equivalent to,

$$\begin{aligned} &\max_{\{\mathbf{b}_k\}} \quad \mathbf{b}_k^H \mathbf{R}_d \mathbf{b}_k \\ &\text{s.t.} \quad \mathbf{b}_k^H \left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right) \mathbf{b}_k = 1 \quad \text{(C3)} \end{aligned} \tag{10}$$

The solution to (10) is given by,

$$\mathbf{R}_d \mathbf{b}_k = \lambda \left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right) \mathbf{b}_k, \tag{11}$$

where λ denotes the Lagrangian multiplier associated to the constraint (C3). From (11) it is clear that the solution to (10) corresponds to a generalized eigenvalue problem, with the optimal beamforming vector $\mathbf{b}_k^*$ given by the eigenvector associated to the largest eigenvalue λ_{max} of $\left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right)^{-1} \mathbf{R}_d$.

Let us analyze the case where all the tolerable interference levels P_j, $j = 1, \ldots, J$ are fixed and equal to P_I. Under this assumption, (11) converts into,

$$\mathbf{h}_d \mathbf{h}_d^H \mathbf{b}_k = \lambda_{\text{max}} \left(\frac{P_{\text{max}}}{P_I} \sum_{j=1}^{J} \mathbf{h}_j \mathbf{h}_j^H + \mathbf{I} \right) \mathbf{b}_k \tag{12}$$

from which it can be observed that,

$$\mathbf{b}_k^* \propto \left(\frac{P_I}{P_{\text{max}}} \mathbf{I} + \sum_{j=1}^{J} \mathbf{h}_j \mathbf{h}_j^H \right)^{-1} \mathbf{h}_d. \tag{13}$$

Note that (13) corresponds to the regularized version of ZFB with the main difference being on the value of P_I, which is usually heuristically taken as the noise level in the ZFB designs that exists in the literature. Therefore, the obtained close-form optimal solution (11) represents a generalized transmit beamforming framework comprising the particular case of the conventional ZFB. Note that (12) when $P_I \to \infty$ results in $\mathbf{b}_k^* \propto \mathbf{h}_d$, which is commonly known as Matched Beamformer (MB). MB is the preferred option when the scenario is dominated by noise rather than interference.

Note that substituting the optimal transmit beamformer $\mathbf{b}_k^*$ into (8) (assuming the equality) gives the following expression,

$$p_k = \frac{P_{\text{max}}}{\mathbf{h}_d^H \left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right)^{-1} \mathbf{h}_d}. \tag{14}$$

Now substituting (14) into the objective function $p_k \mathbf{b}_k^H \mathbf{R}_d \mathbf{b}_k$ provides the following received power level at the desired user,

$$P_{\text{max}} \mathbf{h}_d^H \left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right)^{-1} \mathbf{h}_d. \tag{15}$$

3.1. An Alternative Transmit Beamforming Design

An alternative optimization criteria providing a slightly different solution is the one presented in (16), which considers the maximization of the desired received power minus the sum of unintended received powers. The latter adds additional penalization to the generated interference.

$$\begin{aligned} &\max_{\{\mathbf{b}_k, p_k\}} \quad p_k \mathbf{b}_k^H \mathbf{R}_d \mathbf{b}_k - p_k \mathbf{b}_k^H \left(\sum_{j=1}^{J} \mathbf{R}_j \right) \mathbf{b}_k \\ &\text{s.t.} \quad \text{(C1), (C2)} \end{aligned} \tag{16}$$

Following a similar optimization strategy as the one presented in section 3, we can reach the conclusion that the optimal beamforming is given by,

$$\left(\mathbf{R}_d - \sum_{j=1}^{J} \mathbf{R}_j \right) \mathbf{b}_k = \lambda \left(\sum_{j=1}^{J} \frac{P_{\text{max}}}{P_j} \mathbf{R}_j + \mathbf{I} \right) \mathbf{b}_k, \tag{17}$$

which again corresponds to the eigenvector associated to the maximum eigenvalue solution. Assuming $P_j = P_I$, $j = 1, \ldots, J$, we can express (17) as,

$$\left(\mathbf{R}_d - \sum_{j=1}^{J} \mathbf{R}_j \right) \mathbf{b}_k = \tilde{\lambda} \left(\sum_{j=1}^{J} \mathbf{R}_j + \frac{P_I}{P_{\text{max}}} \mathbf{I} \right) \mathbf{b}_k. \tag{18}$$

By adding $\frac{P_I}{P_{\text{max}}} \mathbf{b}_k$ to both sides and rearranging the resulting components, we obtain the following expression,

$$\left(\mathbf{R}_d + \frac{P_I}{P_{\text{max}}} \mathbf{I} \right) \mathbf{b}_k = \left(\tilde{\lambda} + 1 \right) \left(\sum_{j=1}^{J} \mathbf{R}_j + \frac{P_I}{P_{\text{max}}} \mathbf{I} \right) \mathbf{b}_k. \tag{19}$$

Note that (19) is also the solution of (21) below.

$$\max_{\{\mathbf{b}_k\}} \frac{\mathbf{b}_k^H \left(\mathbf{R}_d + \frac{P_I}{P_{\text{max}}} \mathbf{I} \right) \mathbf{b}_k}{\mathbf{b}_k^H \left(\sum_{j=1}^{J} \mathbf{R}_j + \frac{P_I}{P_{\text{max}}} \mathbf{I} \right) \mathbf{b}_k} = \tag{20}$$

$$\max_{\{\mathbf{b}_k\}} \frac{\mathbf{b}_k^H \left(\mathbf{R}_d - \sum_{j=1}^{J} \mathbf{R}_j \right) \mathbf{b}_k}{\mathbf{b}_k^H \left(\sum_{j=1}^{J} \mathbf{R}_j + \frac{P_I}{P_{\text{max}}} \mathbf{I} \right) \mathbf{b}_k} + 1. \tag{21}$$

Comparing (21) with (9), the main difference between both is that the numerator of (21) is penalized by the amount of generated interference to the unintended users.

4. SIMULATION RESULTS

We initially test the proposed design in (11) with a scenario considering $N = 5$ elements Uniform Linear Array (ULA) with half wavelength as inter-element spacing at 2 GHz, and the presence of a single desired user at $\theta_d = 20°$. Therefore, the channel vector corresponding to the desired user can be obtained as,

$$\mathbf{h}_d = \begin{bmatrix} 1 & e^{-j\pi \sin(\theta_d)} & \cdots & e^{-j(N-1)\pi \sin(\theta_d)} \end{bmatrix}^T. \tag{22}$$

On top of the desired user, there are $J = 6$ unintended users also, whose angular location is given by $\theta_i = (-70°, -45°, -30°, 0°, 35°, 50°)$. Clearly, ZFB cannot be applied as $J > N$.

We have simulated the optimal solution of (4), the proposed transmit beamforming solution (11), the conventional matched filter given by $\mathbf{b}_k = \mathbf{h}_d^H$ and the virtual SINR beamformer [12]. Note that

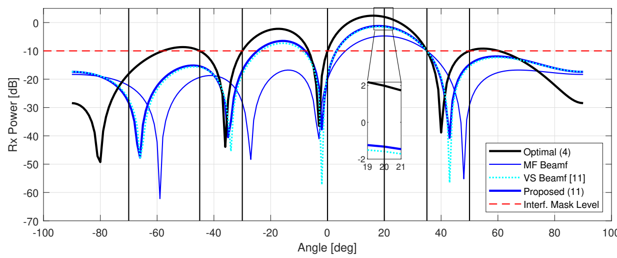

Fig. 2. Received power versus angle for different transmit beamforming techniques with $P_{\max} = 2$ W

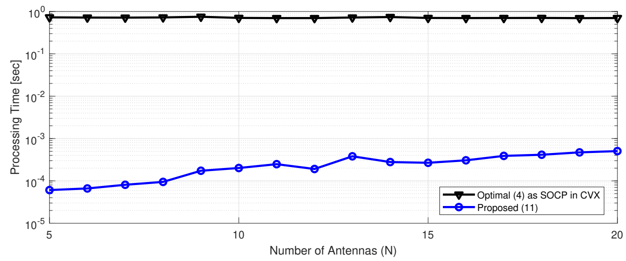

Fig. 3. Processing time versus number of antennas

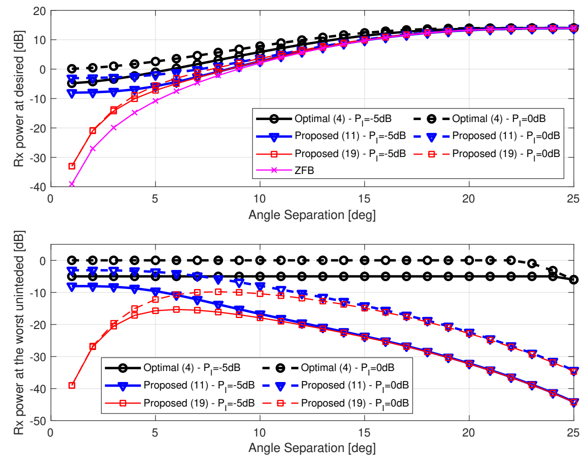

Fig. 4. Achievable received power at the desired user (top), and received power at the worst unintended user (bottom)

the last two are *not* designed to meet the received-interference power constraints (C1) and (C2) of (4). In addition, the setting of the regularization factor of the virtual SINR in [12] is not a trivial task. Here, we have taken the regularization factor equal to $P_{\max}$. For the sake of visual comparison, the beamforming solutions presented in Fig. 2 have been normalized (i.e. $\mathbf{b}_k^H \mathbf{b}_k = 1$) and the transmit power p_k as $p_k = \min\left(P_{\max}, \frac{P_j}{\mathbf{b}_k^H \mathbf{R}_j \mathbf{b}_k}\right)$, which essentially tries to adjust the resulting beam pattern to meet the most strict received-interference power constraint.

Fig. 2 shows the comparison of the aforementioned transmit beamforming techniques in terms of received power at different angular directions, for $P_{\max} = 2$ W and $P_j = -10$ dB, $\forall j$. As expected, the optimal solution provides the highest power at the desired users while satisfying the received-interference power constraints. The proposed solution provides a lower gain in the desired user's direction with respect to the optimal, but still is able to satisfy the constraints (C1) and (C2) as well. After the normalization and the proposed power assignment, the virtual SINR beamforming provides a solution slightly worse than the proposed one. However, it should be noted that the virtual SINR in [12] requires a complex procedure in order to define the regularization factor involved in the final expression, while our solution has a simple and intuitive close-form expression. Finally, the normalized matched filter provides the worst result since the received power at the desired user is much lower than that provided by all the others.

In order to assess the complexity of the optimal solution and the proposed transmit beamforming in (11), we make use of the MATLAB function "timit", which measure the time required to run function by running it several times and providing the average processing time. Assuming the same scenario as before with $J = 6$ unintended users, we obtain the numbers provided in Fig. 3. Clearly, the proposed technique is much faster as it avoids the use of iterative procedures to solve a SOCP problem and executes a close-form expression. In particular, the processing time is reduced from seconds to few microseconds (by a factor of 10^{-6}).

Focusing on the optimal solution of (4), and the two proposed alternatives in (11) and (19), we analyzed a scenario with one desired user at $\theta_d = 20^\circ$ and two unintended users located at $\theta_i = \theta_d \pm \Delta$.

Fig. 4 compares the three solutions in terms of the received power at the desired users (objective function in (4)) and the maximum received power at the unintended users, for $P_{\max} = 5$ W and for different values of P_I. As expected, we observe that the proposed alternative (11) attains a suboptimal solution in terms of received power at the desired but achieving a lower generated interference seen at the unintended users. This is because the proposed design considers the lower bound resulting from the harmonic mean of the interference constraints and, therefore, is enforcing lower interference than that of the required tolerance level P_I. The proposed alternative (19) behaves similarly to (11) for wide separation between desired and unintended while significantly reducing the generated interference when this is located closer to the desired user. In general, the solution in (19) is found to provide higher discriminance between the received power levels at the desired and the unintended users, and this becomes more significant in low SINR regimes. When comparing the received power at the desired user in Fig. 4, we also include the results with ZFB, which nulls-out the interference directions but provides the lowest power in the desired direction when the unintended users are in close angular proximity.

5. CONCLUSIONS

In this paper, we presented a new transmit beamforming design framework that considers received-interference power constraints at the unintended users, in an attempt to relax the design of conventional ZFB. By allowing some residual received interference instead of trying to null it out, the proposed design is able to deal with scenarios where there are more interferers than antennas. Instead of using convex optimization software tools that invoke iterative optimization procedures, we provide a close-form non-iterative optimal solution that is more suitable for practical systems. Unlike other popular designs available in the literature, the proposed technique does not rely on tunable regularization parameters. Finally, we validated and compared the proposed design through numerical simulation experiments, showing that it performs close to the optimal while significantly reducing the computational complexity.

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