

# Ensemble Data Assimilation

## Algorithms – Applications – Software

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Lars Nerger

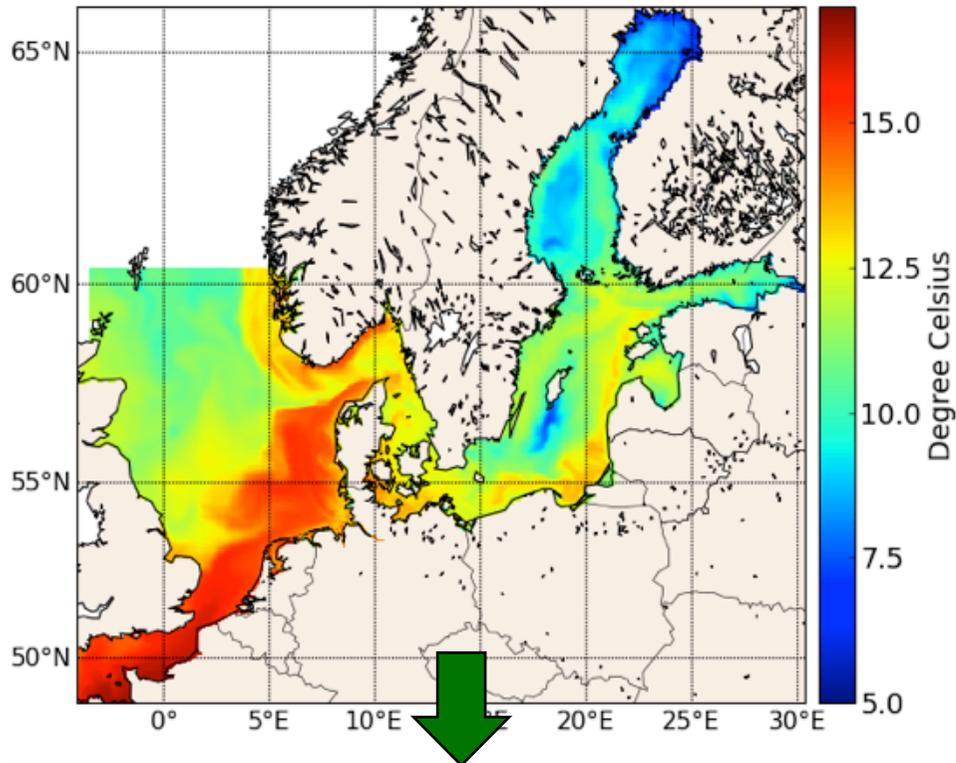
Alfred Wegener Institute Helmholtz Center for Polar and Marine Research  
Bremerhaven, Germany

Acknowledgements:

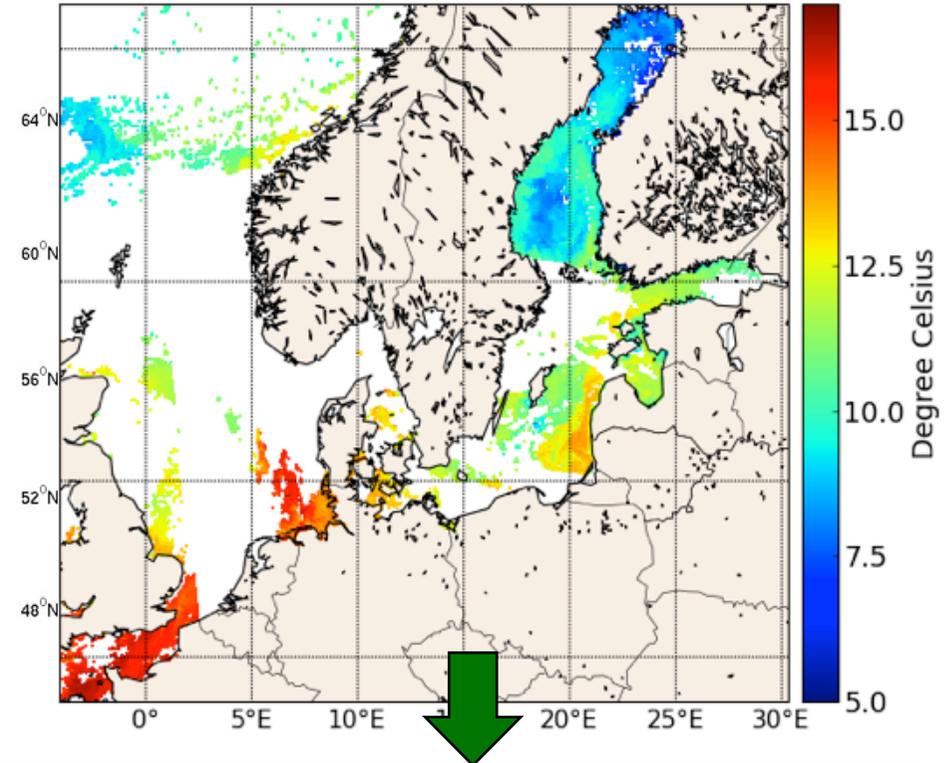
W. Hiller, J. Schröter, S. Losa, A. Androsov,  
H. Pradhan, M. Goodliff, Q. Tang, Q. Yang, L. Mu

# Motivation

*Model* surface temperature



*Satellite* surface temperature



Combine both sources of information  
quantitatively by computer algorithm  
→ Data Assimilation

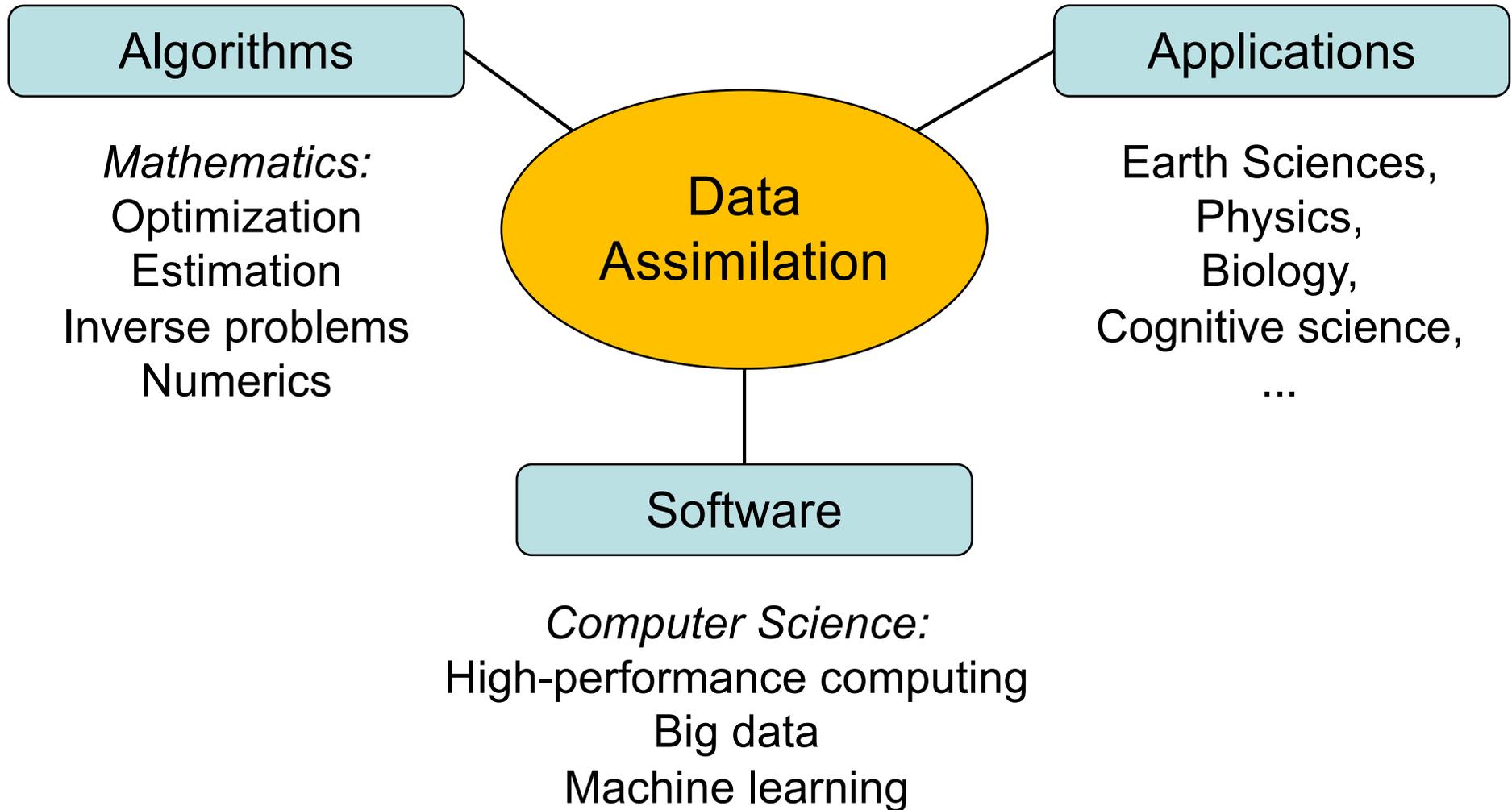
# Data Assimilation

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Methodology to combine model with real data

- Optimal estimation of system state:
  - initial conditions (for weather/ocean forecasts, ...)
  - state trajectory (temperature, concentrations, ...)
  - parameters (ice strength, plankton growth, ...)
  - fluxes (heat, primary production, ...)
  - boundary conditions and ‘forcing’ (wind stress, ...)
- More advanced: Improvement of model formulation
  - Detect systematic errors (bias)
  - Revise parameterizations based on parameter estimates

# Interdisciplinarity of Data Assimilation



# Outline

## Ensemble Data Assimilation

### Algorithms / Methodology

- Efficient methods for high-dimensional nonlinear systems

### Applications

- Examples of what one can expect to achieve

### Software

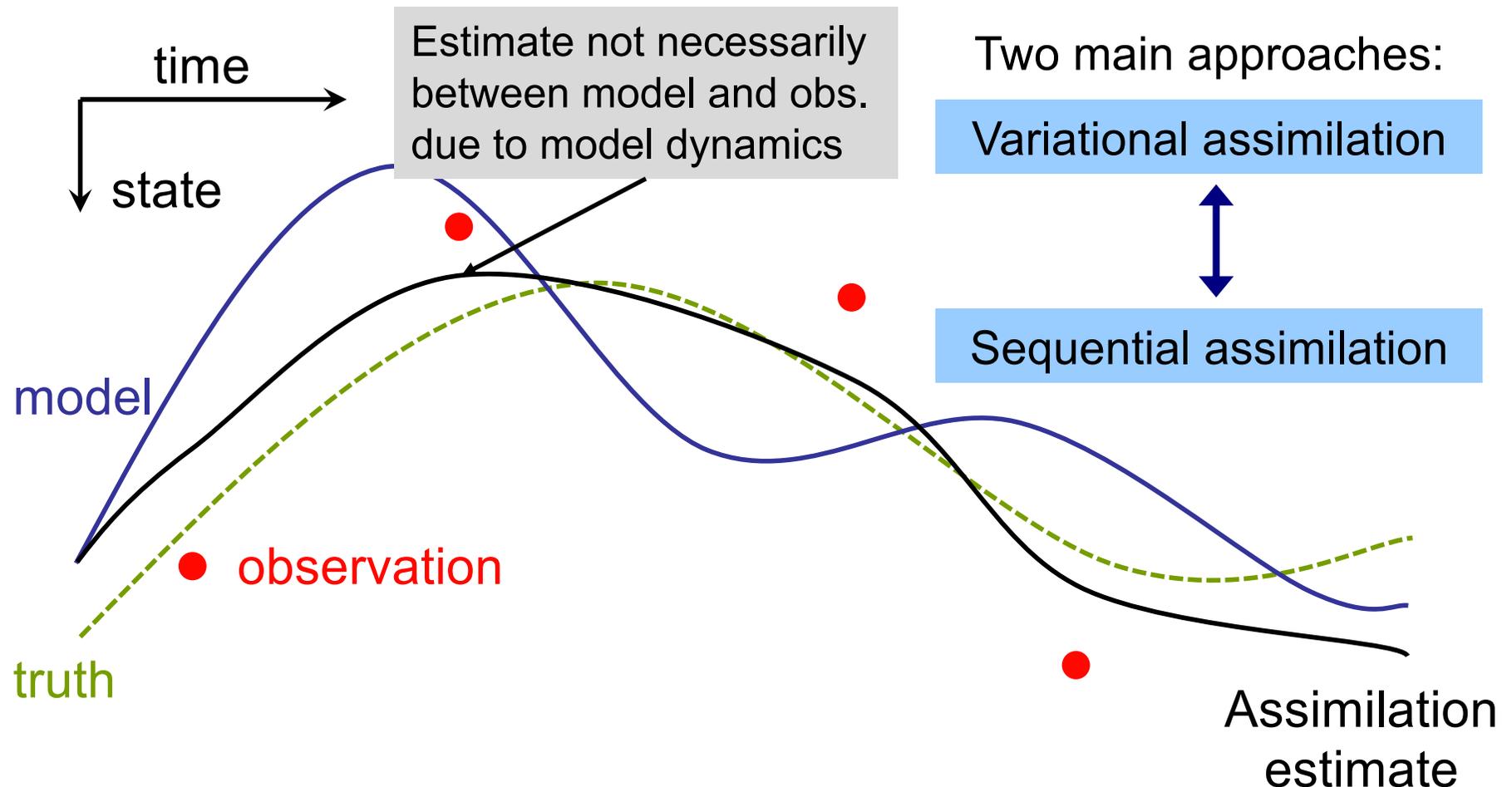
- Make ensemble data assimilation easily usable
  - Parallel Data Assimilation Framework (PDAF)

# Methodology

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# Data Assimilation – a general view

Consider some physical system (ocean, atmosphere, land, ...)



Goal: Obtain optimal estimate of system  
constrained by model dynamics and observations

# Needed for Data assimilation

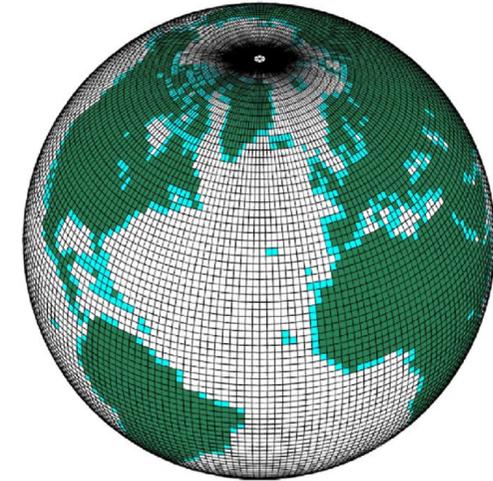
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1. Model
  - with some skill
2. Observations
  - with finite errors
  - related to model fields
3. Data assimilation method

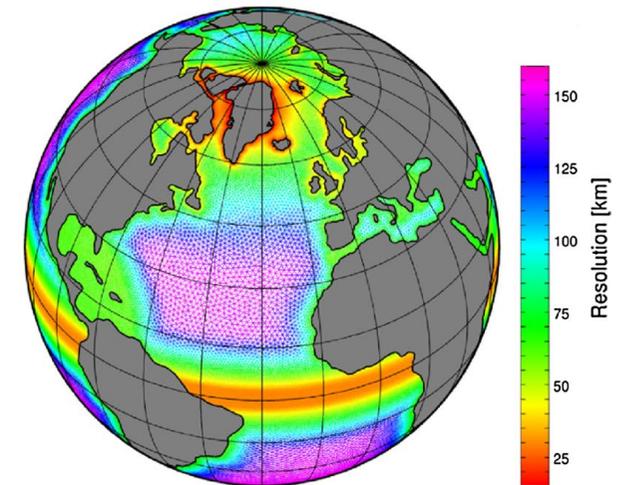
# Models

Simulate dynamics, e.g. the ocean

- Numerical formulation of relevant terms
- Discretization with finite resolution in time and space
- “forced” by external sources (atmosphere, river inflows)
- Uncertainties
  - initial model fields
  - external forcing
  - in predictions due to model formulation



*Uniform-resolution mesh*



*Variable-resolution mesh  
(ocean model FESOM)*

# Observations

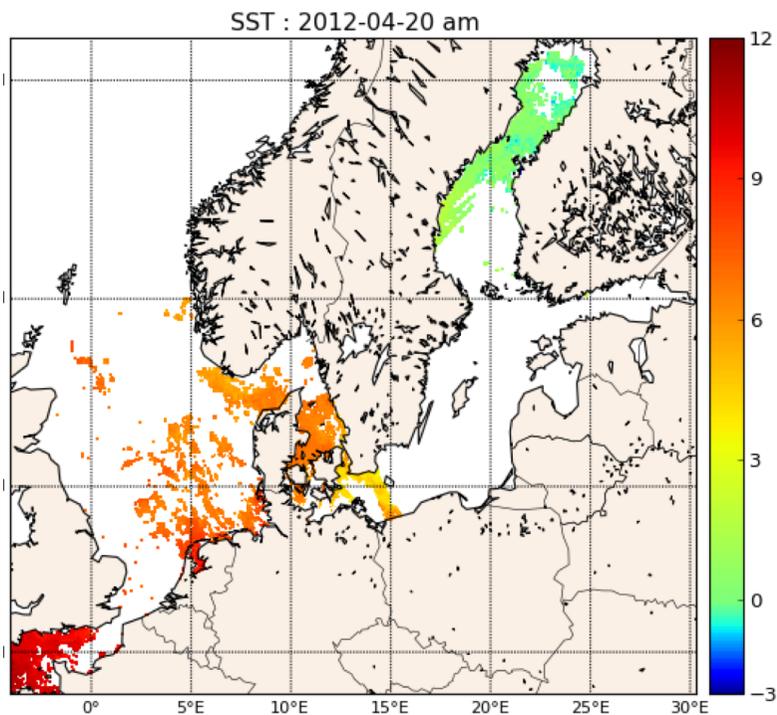
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Measure different fields ... for example in the Ocean

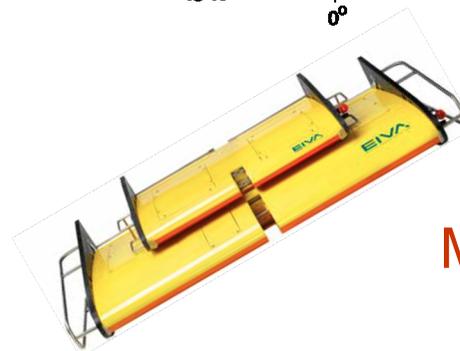
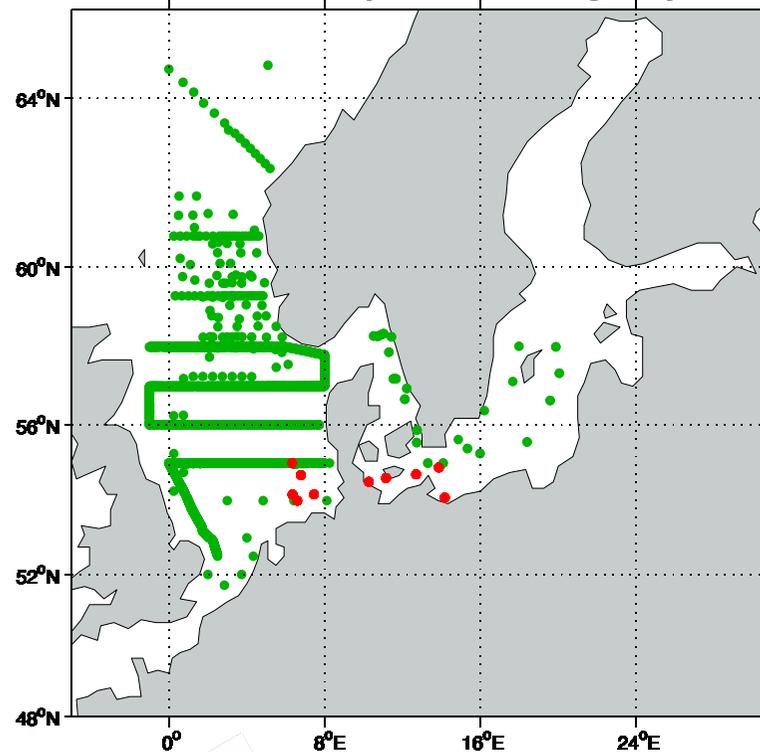
- Remote sensing
  - E.g. surface temperature, salinity, sea surface height, ocean color, sea ice concentrations & thickness
- In situ (ships, autonomous vehicles, ...)
  - Argo, CTD, Gliders, ...
- Data is sparse: some fields, data gaps
- Uncertainties
  - Measurement errors
  - Representation errors:  
Model and data do not represent exactly the same  
(e.g. cause by finite model resolution)

# Example: Physical Data in North & Baltic Seas

Satellite surface temperature  
(12-hour composite)



Available T and S profiles during July 2008

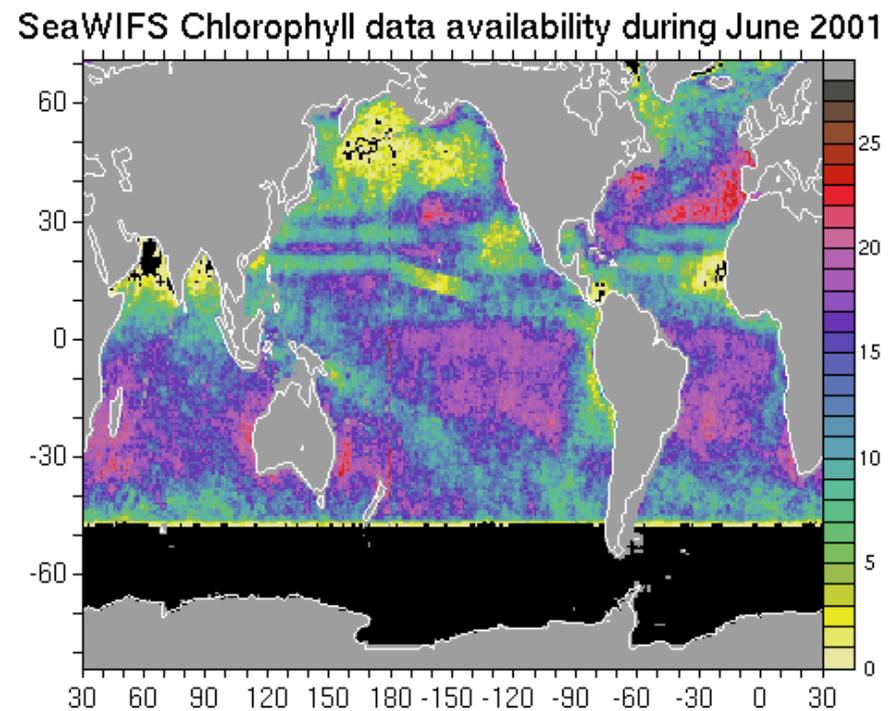
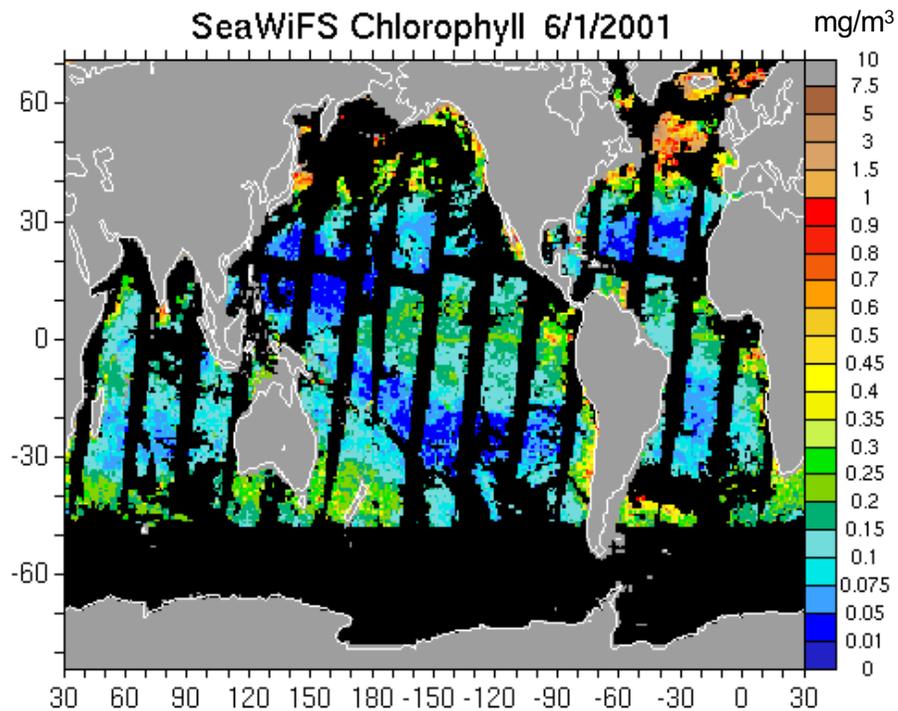


Scanfish and  
CTD profiles

MARNET  
stations



## Example: Chlorophyll-a observations (SeaWiFS)



### Daily gridded SeaWiFS chlorophyll data

- gaps: satellite track, clouds, polar nights
- On model grid: ~13,000-18,000 data points daily (of 41,000 wet grid points)
- irregular data availability

# Observation Error Estimates

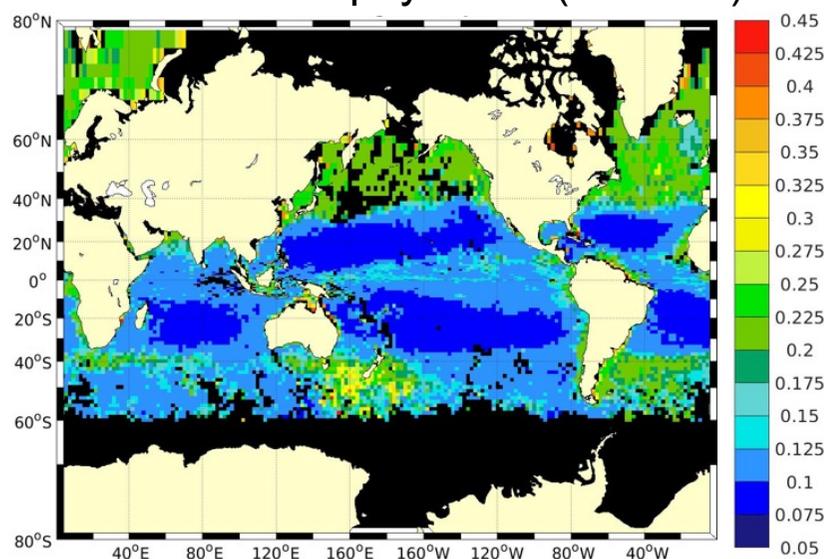
If observation errors available:

- they are typically usable
- usually do not account for representation errors (might be too low)

If no observation errors available:

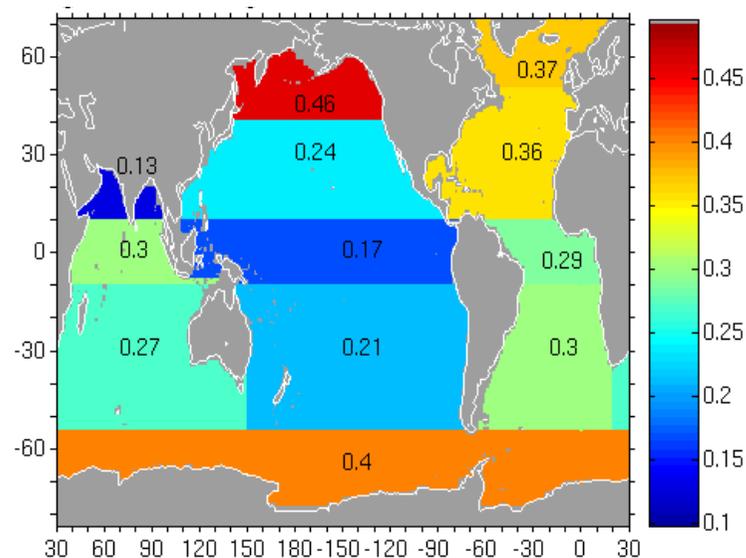
- need to estimate them

logarithmic data errors provided with satellite chlorophyll data (OC-CCI)



Pradhan et al, JGR 2019

data errors from comparison with 2186 collocation points of in situ data (SeaWiFS)



Nerger & Gregg, JMS 2007

# Data Assimilation Methods

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## Combine observations and model state estimate

- Account for uncertainty in observations
- Account for uncertainty in model state estimate
- Account for relations (correlations) between observed part of the model state and unobserved parts

# Ensemble Data Assimilation

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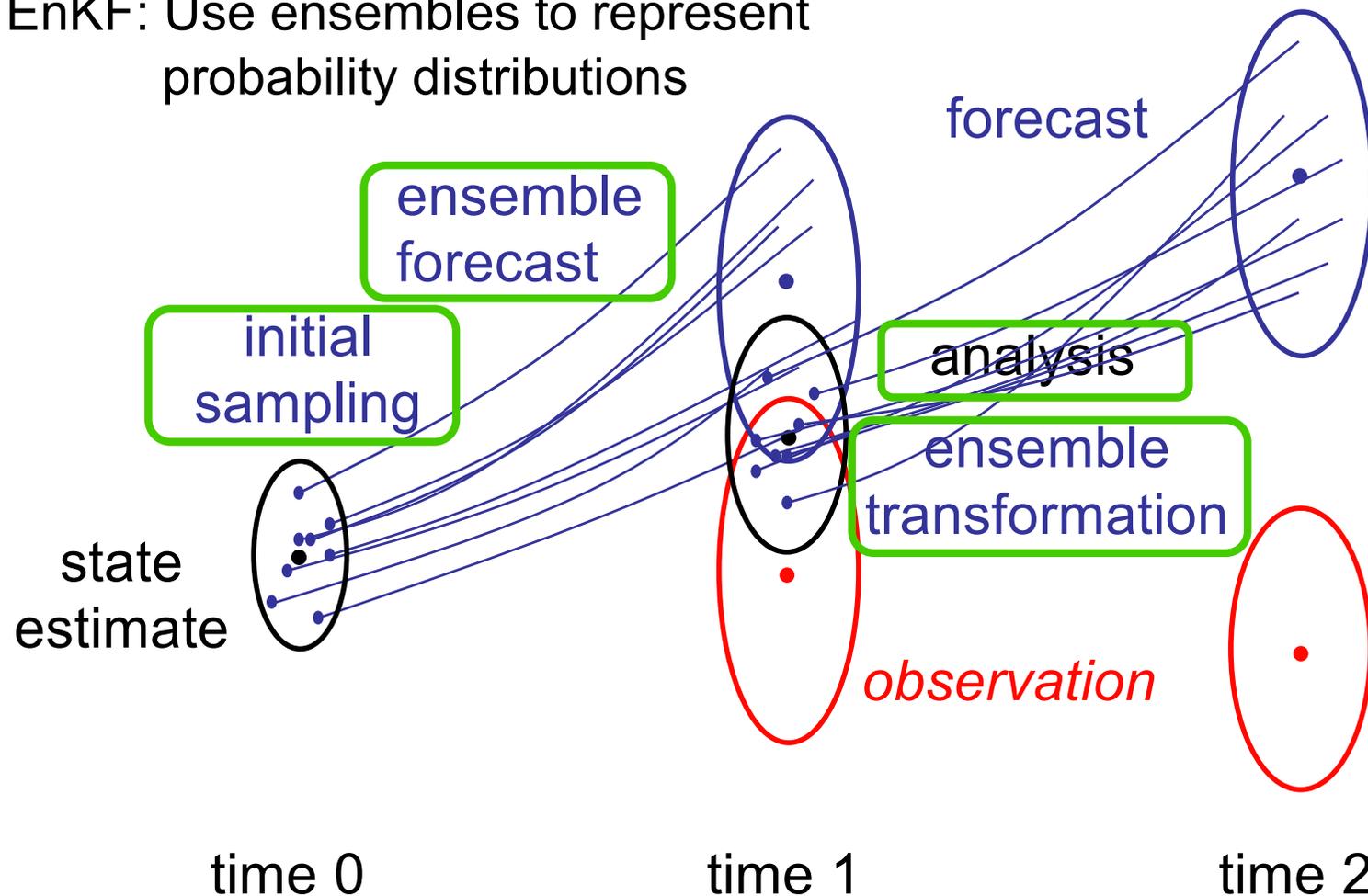
**Estimate uncertainty**

# Ensemble Kalman Filters

First formulated by G. Evensen (EnKF, J. Geophys. Res. 1994)

Kalman filter: express probability distributions by mean and covariance matrix

EnKF: Use ensembles to represent probability distributions



There are many possible choices!

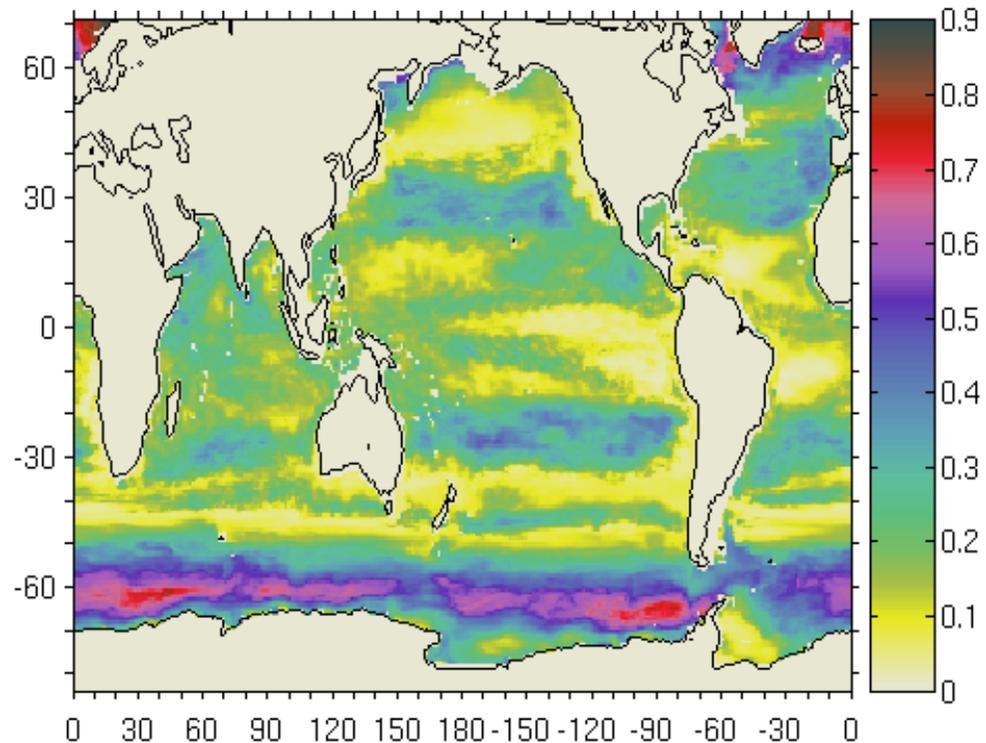
What is optimal is part of our research

Different choices in PDAF

# Ensemble Covariance Matrix

- Provide uncertainty information (variances + covariances)
- Generated dynamically by propagating ensemble of model states

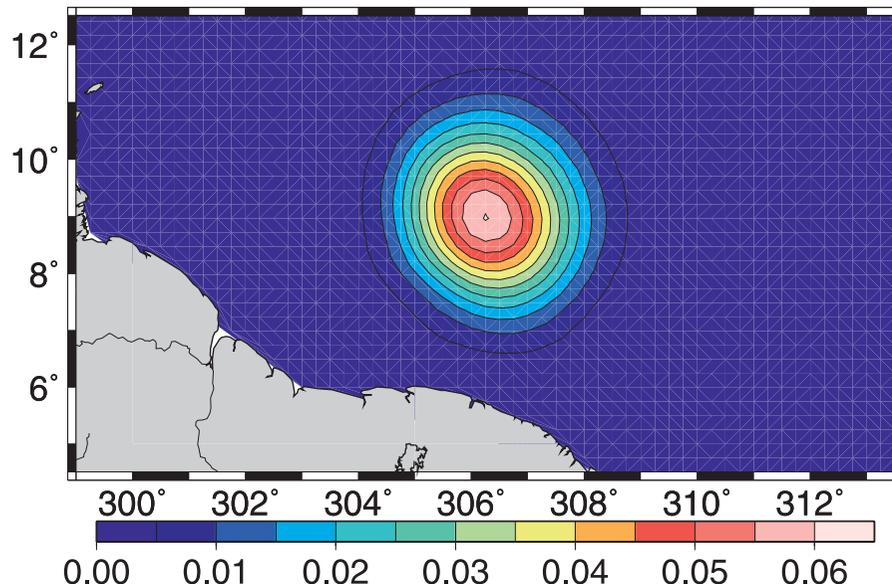
Uncertainty: Standard deviation of log Chlorophyll



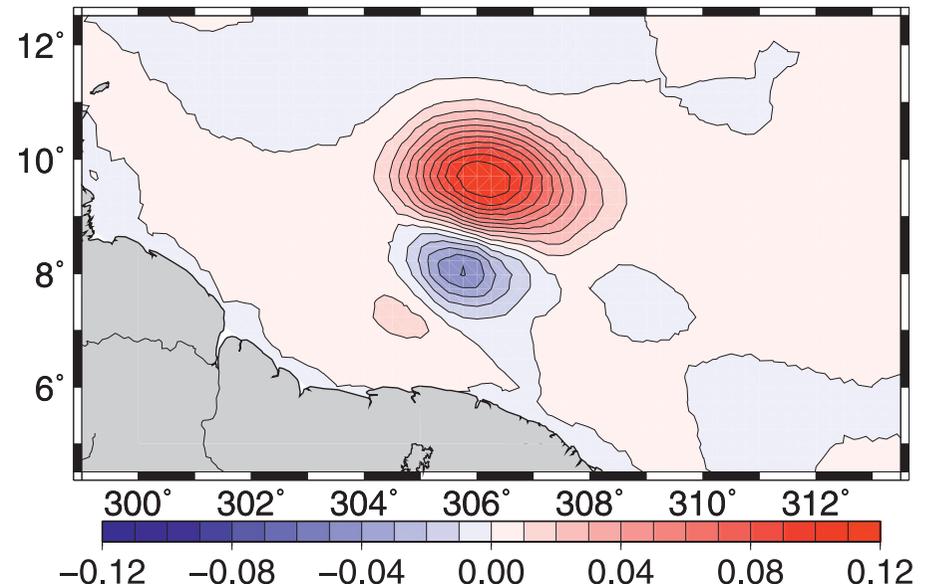
## Ensemble Covariance Matrix (II)

- Also:  
Provide information on error correlations  
(between different locations and different fields)
- Example: Assimilation of sea surface height  
(Brankart et al., Mon. Wea. Rev. 137 (2009) 1908-1927)

Assimilation increment in sea surface height

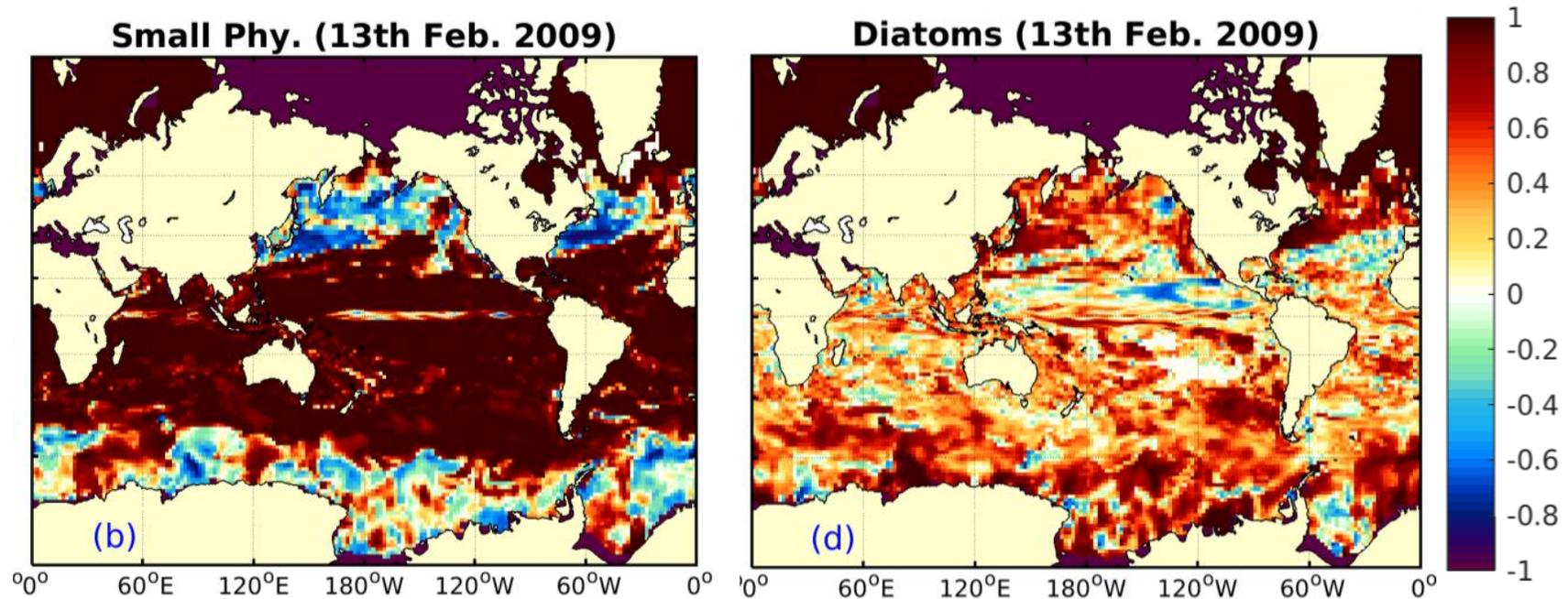


Induced change in zonal velocity



# Ensemble-estimated Cross-correlations

Cross correlations between total chlorophyll and chlorophyll in phytoplankton groups



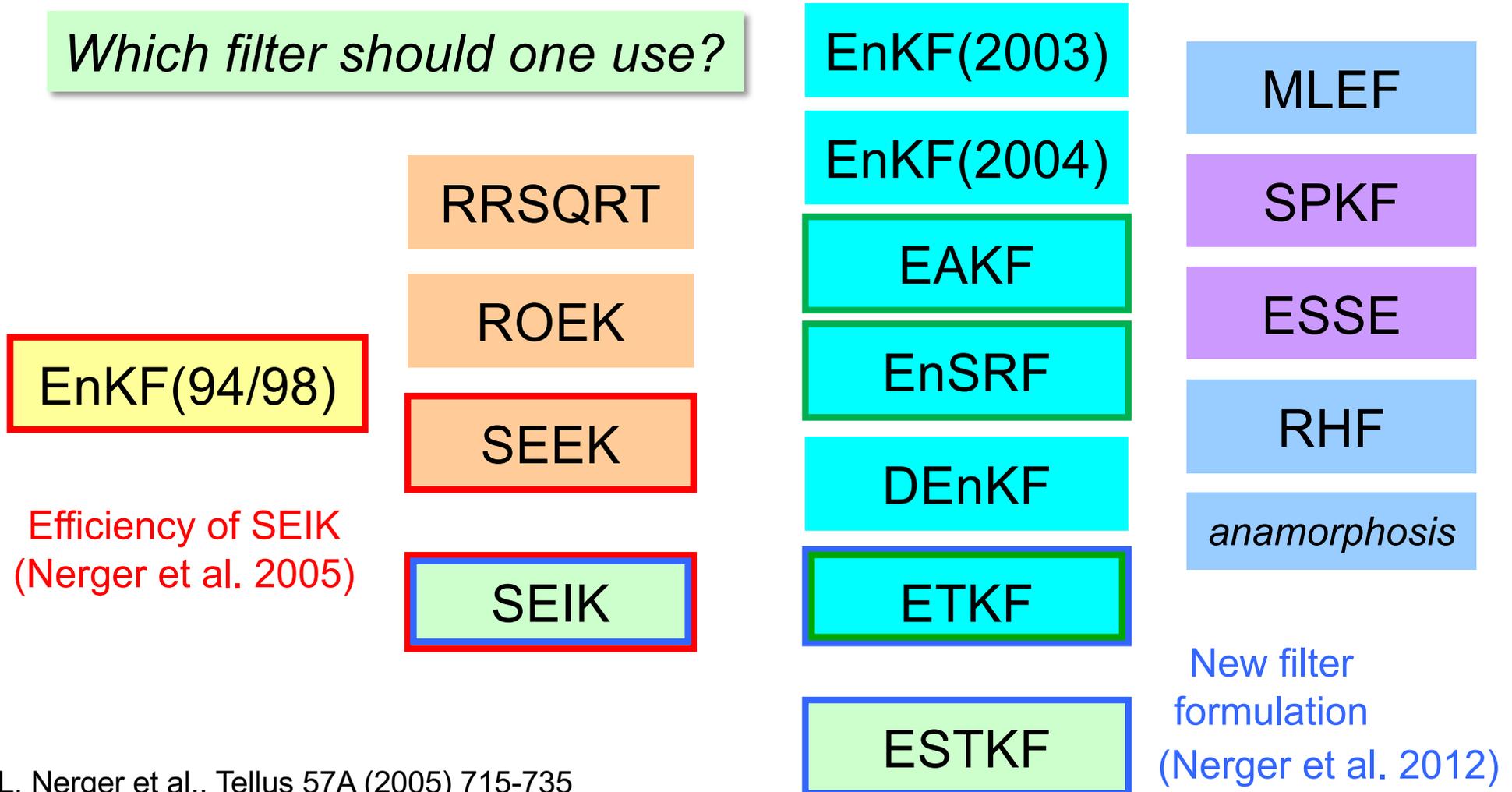
Cross-correlations are used to correct non-observed quantities from observed ones

# Ensemble-based/error-subspace Kalman filters

A little “zoo” (not complete):

Filter instability  
(Nerger 2015)

*Which filter should one use?*



L. Nerger et al., Tellus 57A (2005) 715-735

L. Nerger et al., Monthly Weather Review 140 (2012) 2335-2345

L. Nerger, Monthly Weather Review 143 (2015) 1554-1567

S. Vetra-Carvalho et al., Tellus A 70 (2018) 1445364

# Assessing Ensemble Kalman Filters

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Mathematical assessment of ensemble Kalman filters limited by

- optimality only proven for Gaussian error distributions
- convergence properties only clear for large ensemble limit

but

- models are nonlinear -> non-Gaussian distributions
- only small ensemble feasible to run for high-dimensional models

A practical approach

- compare and characterize behavior of different methods
- reach general conclusions from analyzing differences mathematically

Further: Ensemble Kalman filters don't work in 'pure' form

- Need adaptations ('fixes')

# Essential “Fixes” for Ensemble Filters

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**Covariance Inflation**

**Localization**

# Covariance inflation

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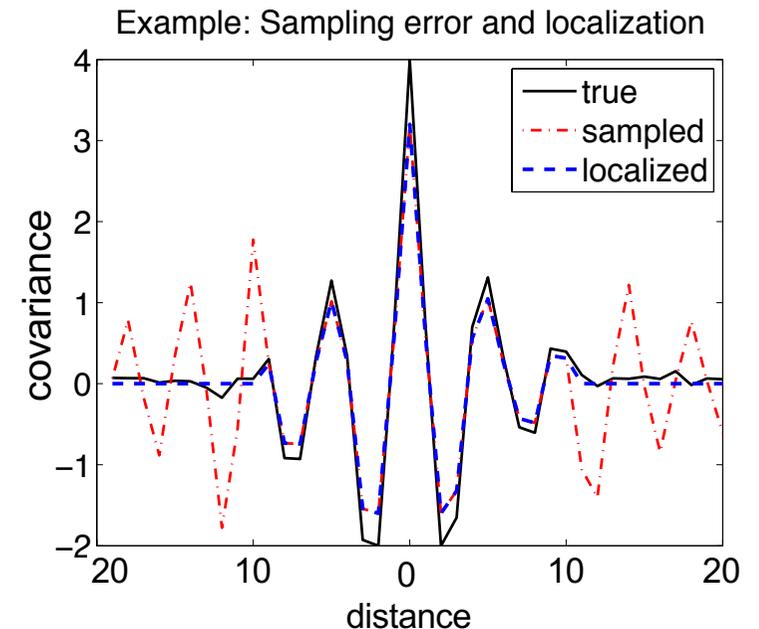
- True variance is always underestimated
  - small ensemble size
  - sampling errors (unknown structure of P)
  - model errors

→ can lead to filter divergence
- Simple remedy
  - Increase error estimate before analysis
- Inflation
  - Increase ensemble spread by constant factor
  - Some filters allow multiplication of a small matrix (“forgetting factor”  $\leq 1$ ; computationally very efficient)
  - Needs to be experimentally tuned

(Mathematically, this is a regularization)

# Localization: Why and how?

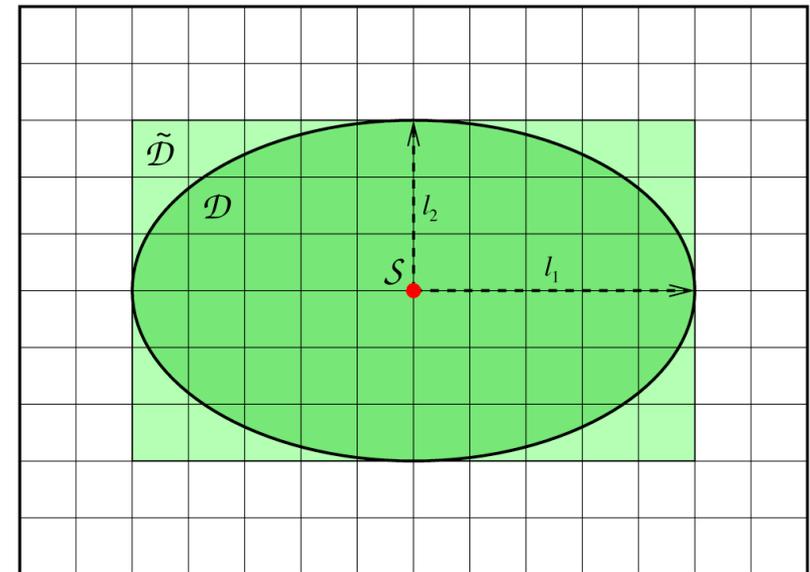
- Combination of observations and model state based on ensemble estimates of error covariance matrices
- Finite ensemble size leads to significant sampling errors
  - errors in variance estimates
    - usually too small
  - errors in correlation estimates
    - wrong size if correlation exists
    - spurious correlations when true correlation is zero
- Assume: long-distance correlations are small in reality
- Localization: damp or remove estimated long-range correlations (Houtekamer & Mitchell, 1998, 2001)



# Observation Localization

## Local Analysis:

- Update small regions (like single vertical columns) allows to define distance
- Use only observations within some distance around this region
- State update and ensemble transformation fully local



S: Analysis region

D: Corresponding data region

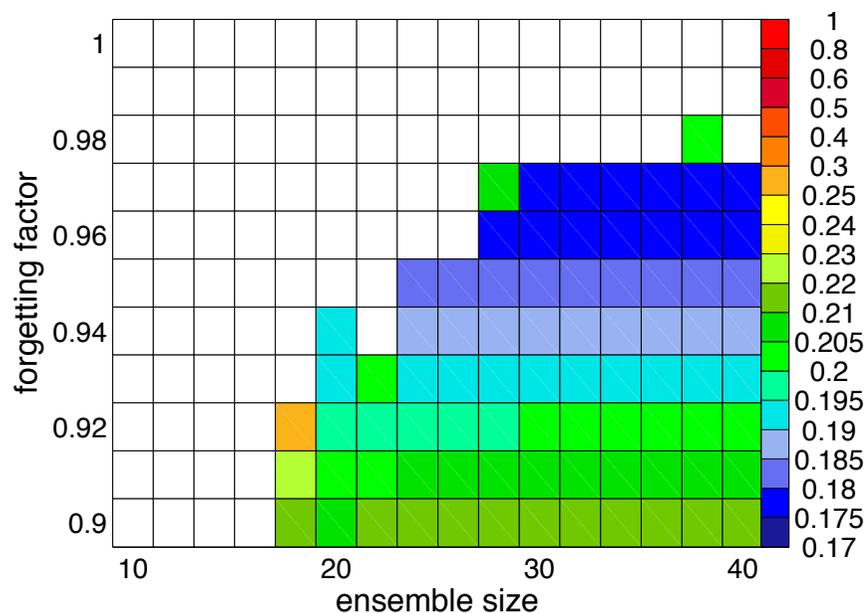
## Observation localization:

- Down-weight observations with increasing distance

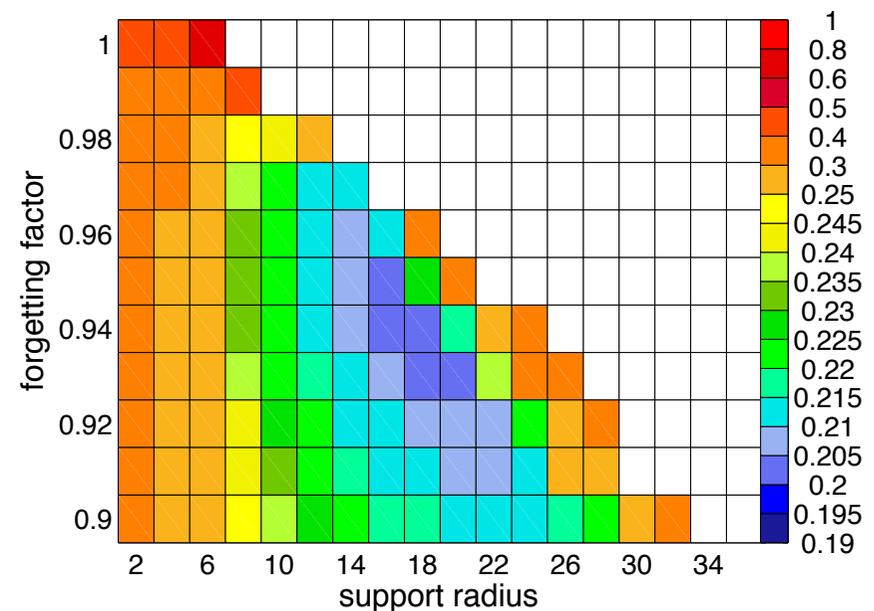
# Impact of inflation and localization

## Experiments with Lorenz96 model

Global filter



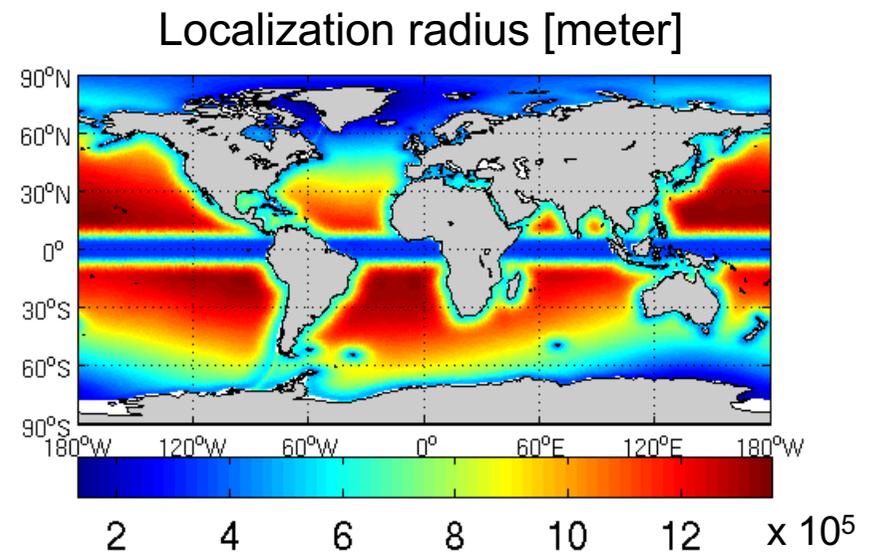
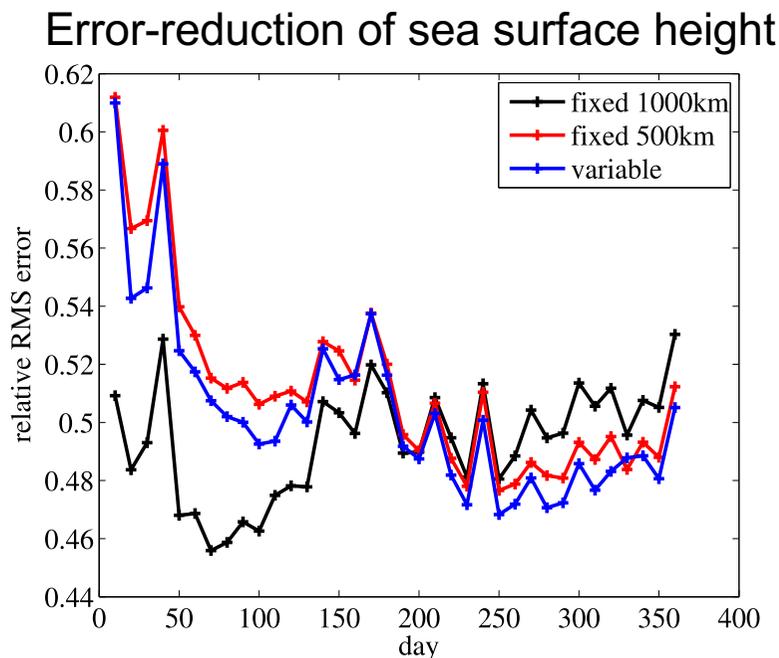
Localized, ensemble size 10



- smaller ensemble usable with localization
- optimal combination of forgetting factor and support radius

# Adaptive localization radius in global ocean model

- Localization radius is usually hand-tuned
- Numerical analysis in small models shows:  
errors minimal when localization radius chosen such that  
*local sum of observation weights = ensemble size*
- Application with FESOM (Finite Element Sea-ice Ocean Model):
  - Fixed 1000km radius leads to increasing errors in 2nd half of year
  - Lower RMS error in sea surface height than fixed 500km radius



# Current developments

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# Current developments

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- Ensemble Kalman filters (and standard variational methods) are current ‘work horses’
  - With various ‘fixes’ like localization
- Aim: Better account for nonlinearity
- Fully nonlinear: Particle filters
  - still no established method for high-dim.
- Hybrid methods
  - Hybrid ensemble-variational
  - Hybrid ensemble Kalman – particle filters
- Iterative filters

# Linear and Nonlinear Ensemble Filters

- Represent state and its error by ensemble  $\mathbf{X}$  of  $N$  states
- Forecast:
  - Integrate ensemble with numerical model

- Analysis:

- update ensemble mean

$$\bar{\mathbf{x}}^a = \bar{\mathbf{x}}^f + \mathbf{X}'^f \tilde{\mathbf{w}}$$

- update ensemble perturbations

$$\mathbf{X}'^a = \mathbf{X}'^f \mathbf{W}$$

(both can be combined in a single step)

- Ensemble Kalman & nonlinear filters: Different definitions of
  - weight vector  $\tilde{\mathbf{w}}$
  - Transform matrix  $\mathbf{W}$

## ETKF (Bishop et al., 2001)

- Ensemble Transform Kalman filter
  - Assume Gaussian distributions
  - Transform matrix

$$\mathbf{A}^{-1} = (N - 1)\mathbf{I} + (\mathbf{H}\mathbf{X}'^f)^T \mathbf{R}^{-1} \mathbf{H}\mathbf{X}'^f$$

- Mean update weight vector

$$\tilde{\mathbf{w}} = \mathbf{A}(\mathbf{H}\mathbf{X}'^f)^T \mathbf{R}^{-1} \left( \mathbf{y} - \mathbf{H}\overline{\mathbf{x}}^f \right)$$

(depends linearly on  $\mathbf{y}$ )

- Transformation of ensemble perturbations

$$\mathbf{W} = \sqrt{(N - 1)} \mathbf{A}^{-1/2} \mathbf{\Lambda}$$

(depends only on  $\mathbf{R}$ , not  $\mathbf{y}$ )

## NETF (Tödter & Ahrens, 2015)

- Nonlinear Ensemble Transform Filter

- Mean update from Particle Filter weights: for all particles  $i$

$$\tilde{w}^i \sim \exp \left( -0.5 (\mathbf{y} - \mathbf{H}\mathbf{x}_i^f)^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x}_i^f) \right)$$

(Nonlinear function of observations  $\mathbf{y}$ )

- Ensemble update

- Transform ensemble to fulfill analysis covariance (like ETKF, but not assuming Gaussianity)
- Derivation gives

$$\mathbf{W} = \sqrt{N} \left[ \text{diag}(\tilde{\mathbf{w}}) - \tilde{\mathbf{w}}\tilde{\mathbf{w}}^T \right]^{1/2} \Lambda$$

( $\Lambda$ : mean-preserving random matrix; useful for stability)

# ETKF-NETF – Hybrid Filter Variants

## 1-step update (*HSync*)

$$\mathbf{X}_{HSync}^a = \overline{\mathbf{X}}^f + (1 - \gamma)\Delta\mathbf{X}_{NETF} + \gamma\Delta\mathbf{X}_{ETKF}$$

- $\Delta\mathbf{X}$ : assimilation increment of a filter
- $\gamma$ : hybrid weight (between 0 and 1; 1 for fully ETKF)

## 2-step updates

**Variant 1 (*HNK*):** NETF followed by ETKF

$$\tilde{\mathbf{X}}_{HNK}^a = \mathbf{X}_{NETF}^a[\mathbf{X}^f, (1 - \gamma)\mathbf{R}^{-1}]$$

$$\mathbf{X}_{HNK}^a = \mathbf{X}_{ETKF}^a[\tilde{\mathbf{X}}_{HNK}^a, \gamma\mathbf{R}^{-1}]$$

- Both steps computed with increased  $\mathbf{R}$  according to  $\gamma$

**Variant 2 (*HKN*):** ETKF followed by NETF

# Choosing hybrid weight $\gamma$

- Hybrid weight shifts filter behavior
- How to choose it?

Possibilities:

- Fixed value
- Adaptive
  - According to which condition?

- Base on effective sample size  $N_{eff} = \sum_i 1/(w^i)^2$

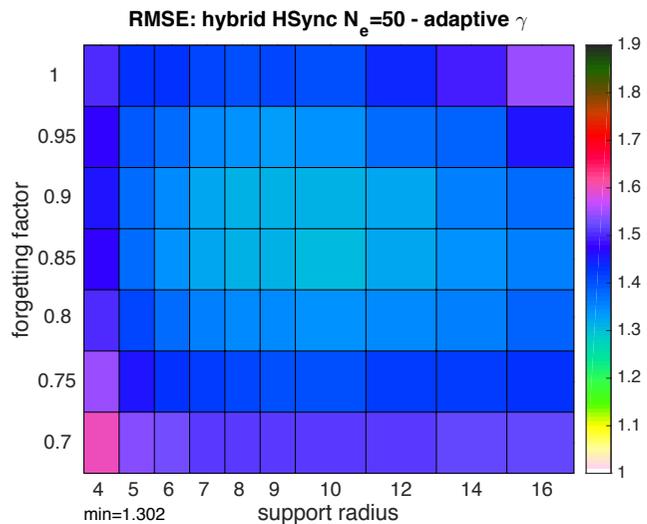
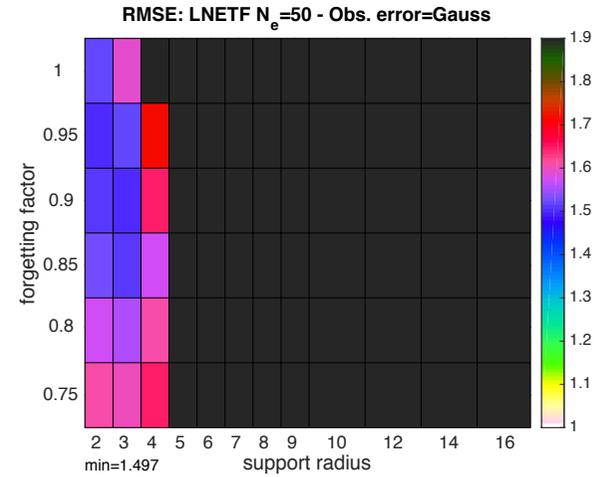
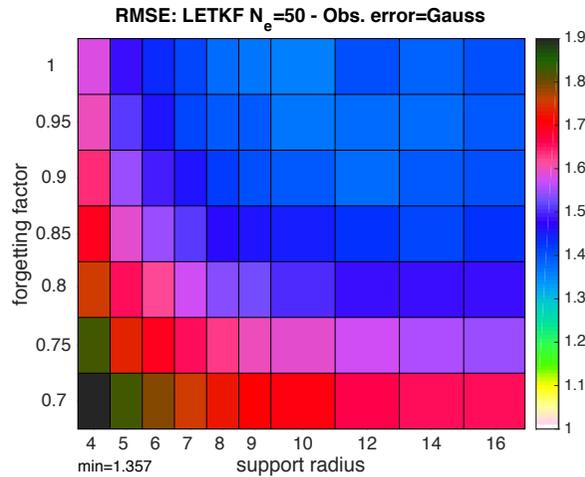
set

$$\gamma_{adap} = 1 - N_{eff}/N$$

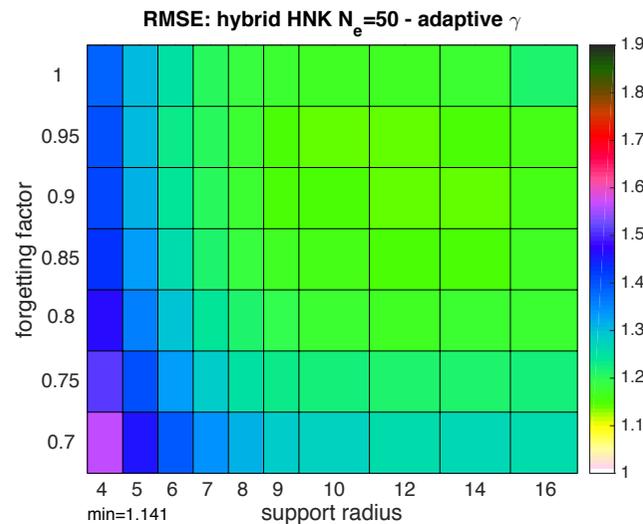
(close to 1 if  $N_{eff}$  small, i.e. small contribution of NETF)

# Test with Lorenz-96 Model (ensemble size N=50)

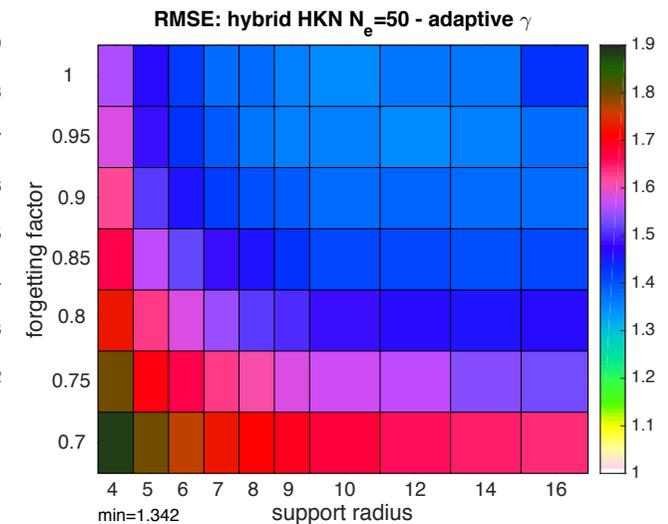
## Ensemble size N=50



4% improvement



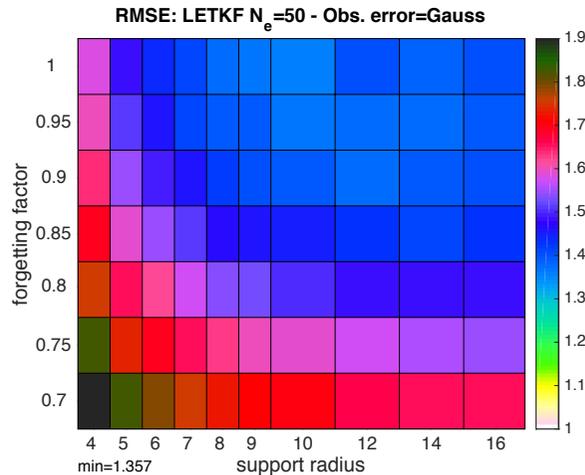
16% improvement



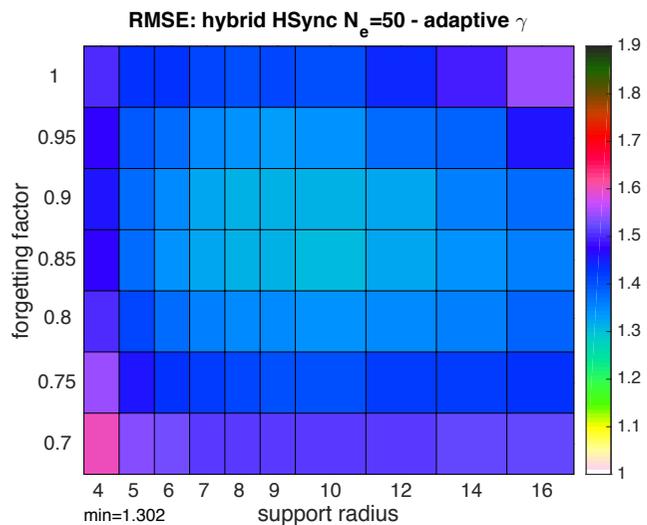
1% improvement

# Test with Lorenz-96 Model (ensemble size N=50)

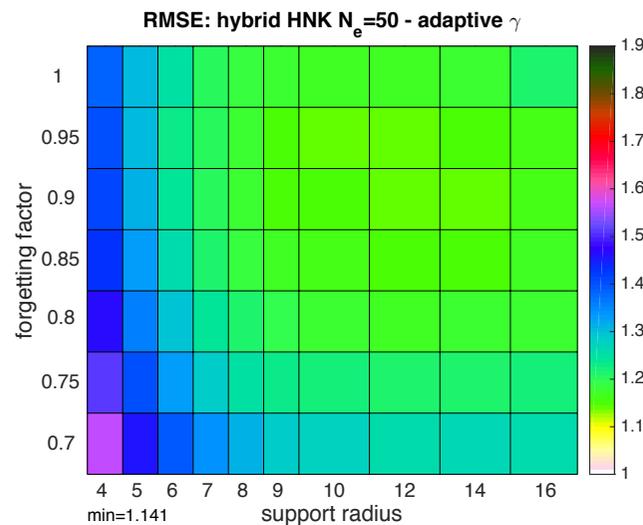
## Ensemble size N=50



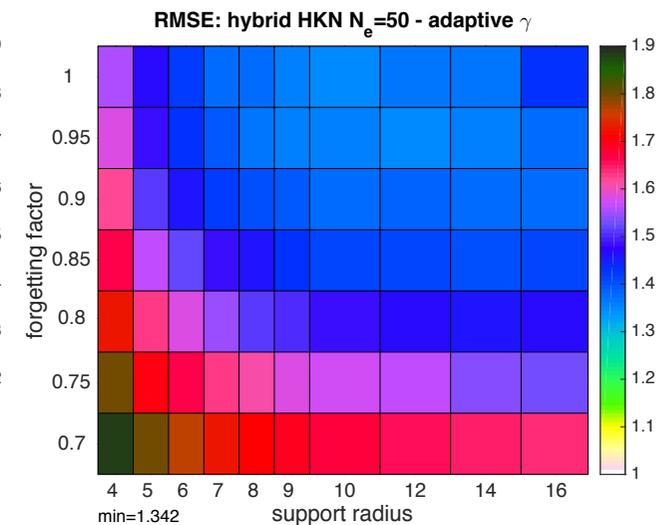
- All hybrid variants improve estimates compared to LETKF & NETF
- Dependence on forgetting factor & localization radius like LETKF
- Similar optimal localization radius
- Largest improvement for variant HNK (NETF before LETKF)
- Currently testing in a larger model ...



4% improvement



16% improvement



1% improvement

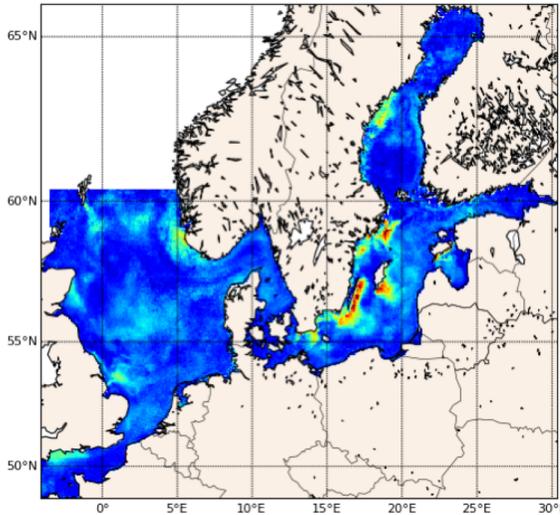
# Applications

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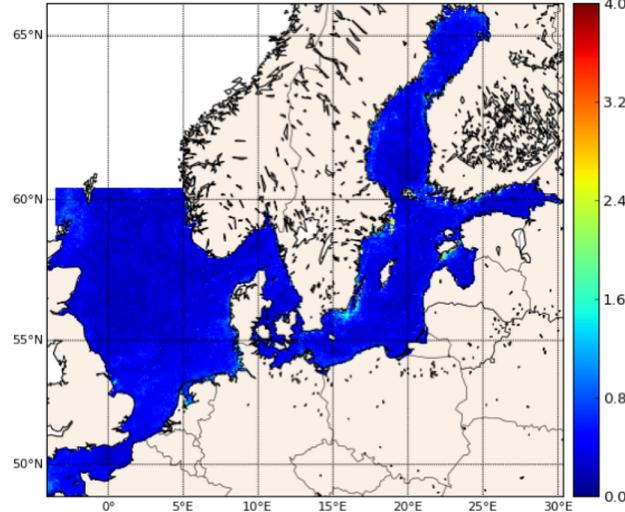
# Assimilation effect on Temperature (September 2012)

## RMS (root-mean-square) deviation

Free run



Assimilation (analysis)



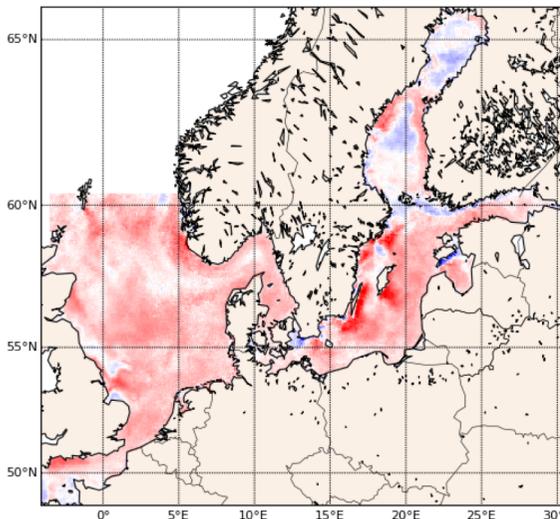
Assimilate surface temperature each 12 h

Compare assimilated estimate with assimilated surface temperature data (monthly average)

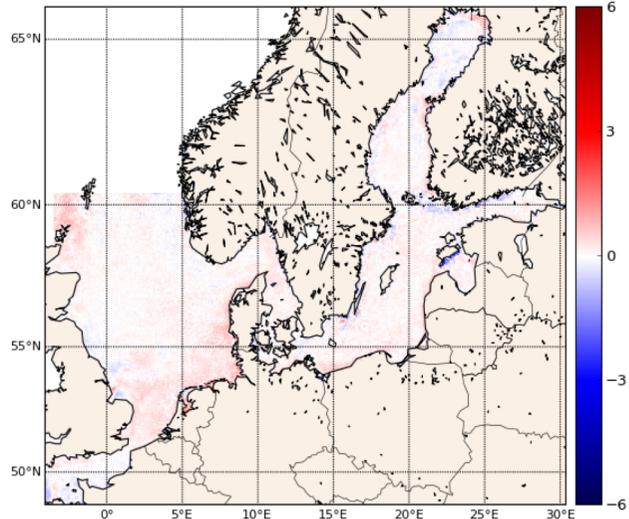
Reduce RMS deviation and mean deviation (bias)

## Mean deviation (observation – model)

Free run

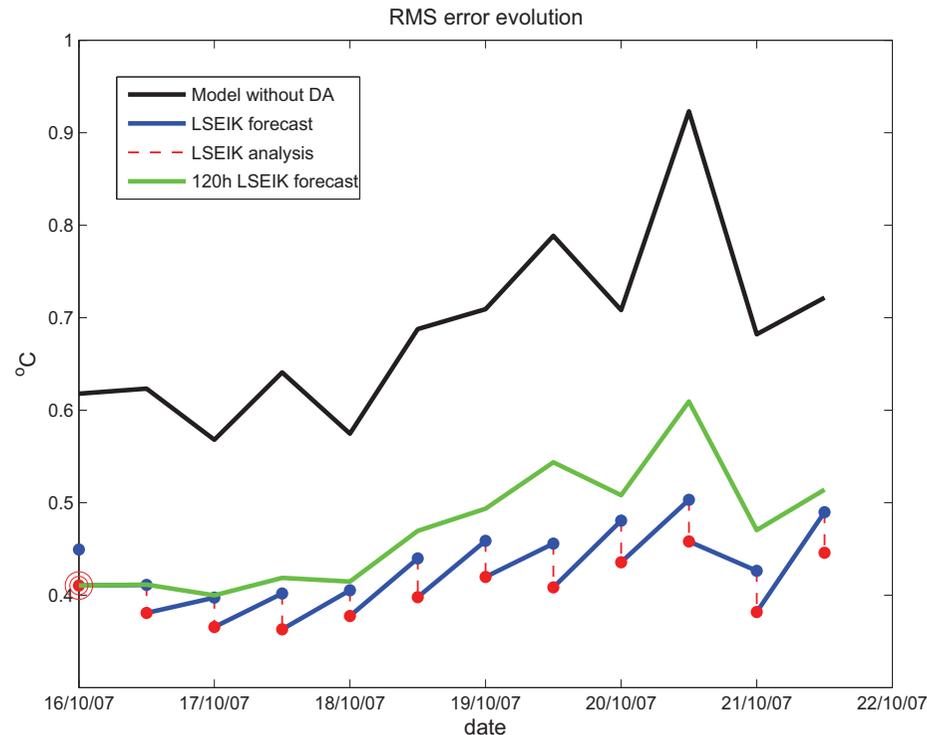


Assimilation (analysis)



→ necessary effect

## Impact of Assimilation for temperature forecasts (North & Baltic Seas)



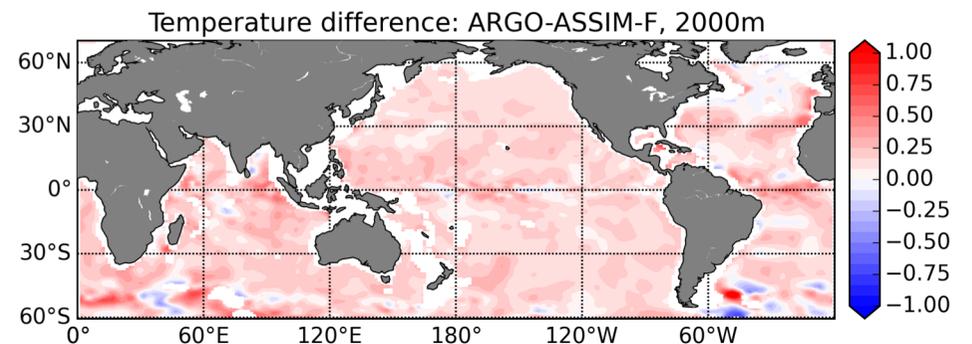
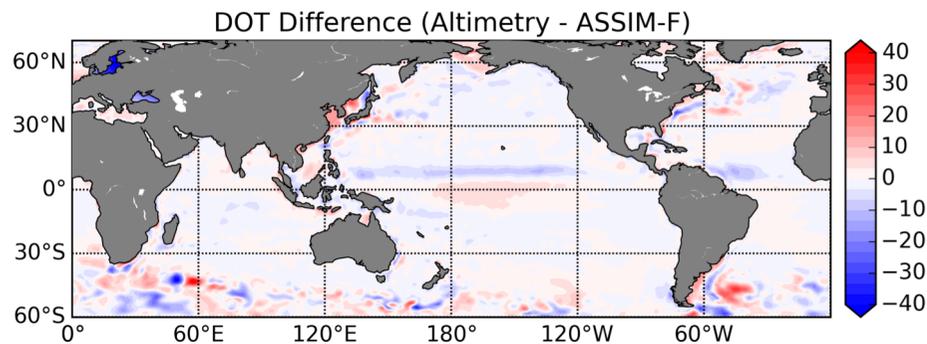
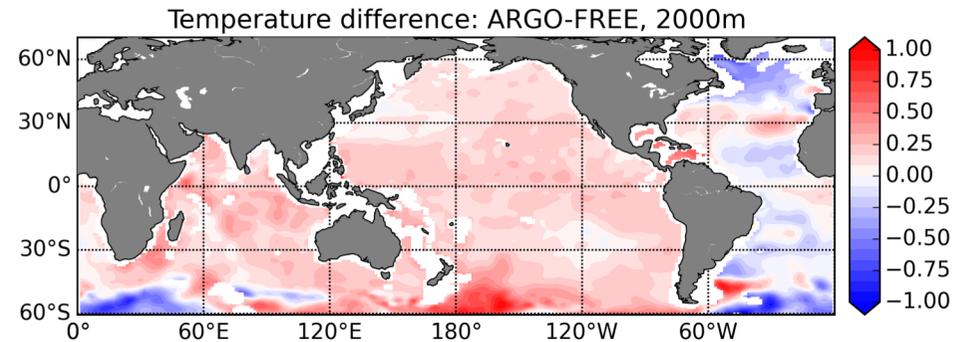
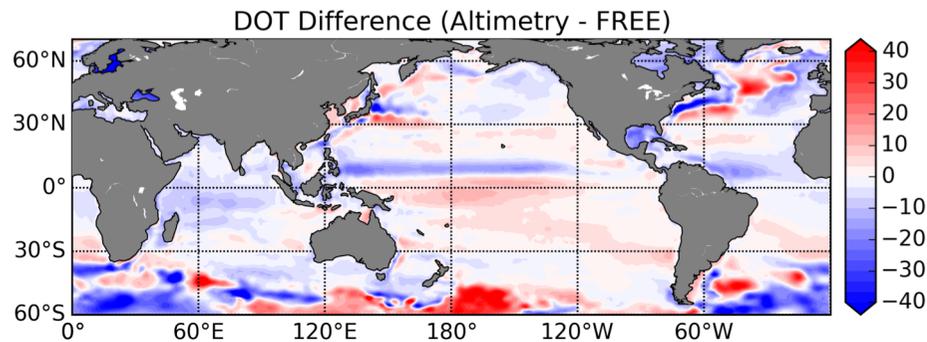
- Very stable 5-days forecasts
- At some point the improvement might break down due to dynamics

# Longe-range effect

**Example:** Assimilate satellite sea surface height data (DOT)

Reduce difference to assimilated data (necessary)

Improve also temperature at 2000m depth



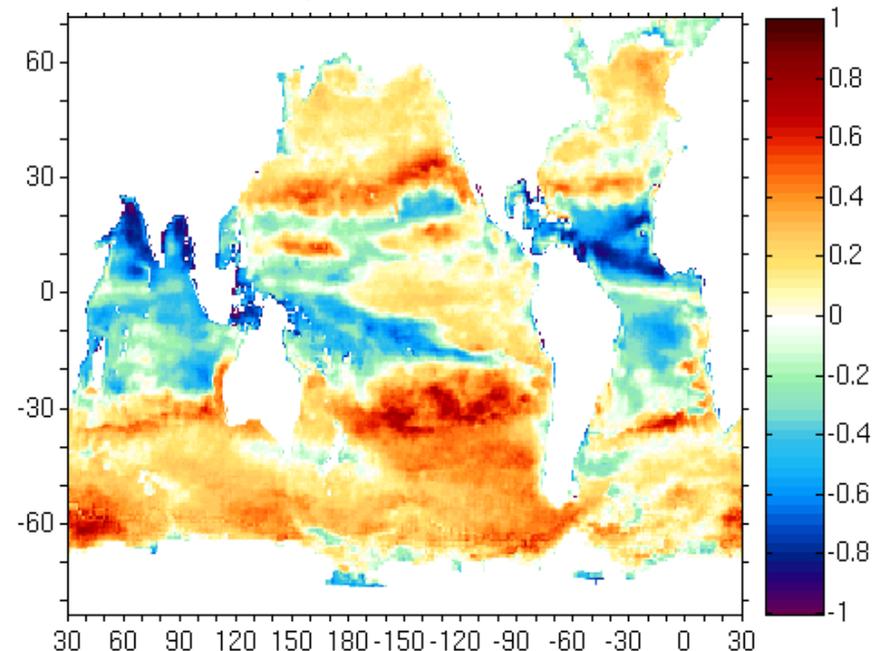
# Bias Estimation

**Example:** Chlorophyll bias of a biogeochemical model

Bias = systematic errors

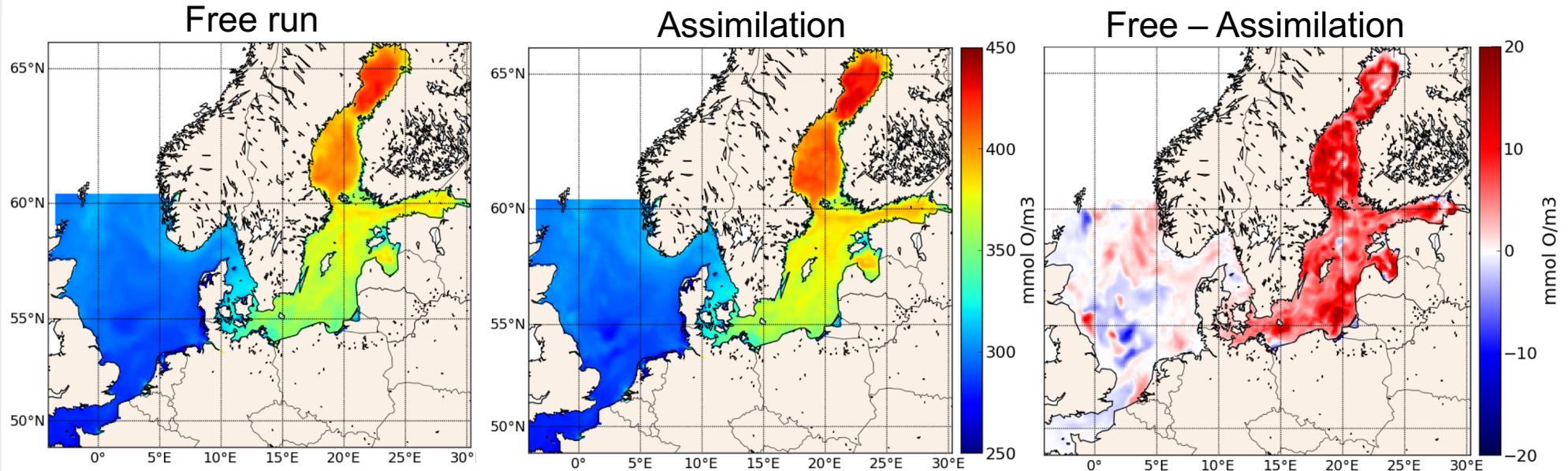
- *un-biased system:*  
random fluctuation around true state
- *biased system:*  
systematic over- and underestimation  
(common situation with real data)
- *Bias estimation:*  
Separate random from systematic deviations

Logarithmic bias estimate  
April 15, 2004



# Biogeochemistry: Coupled data assimilation effect

Surface oxygen mean for May 2012 (as mmol O / m<sup>3</sup>)

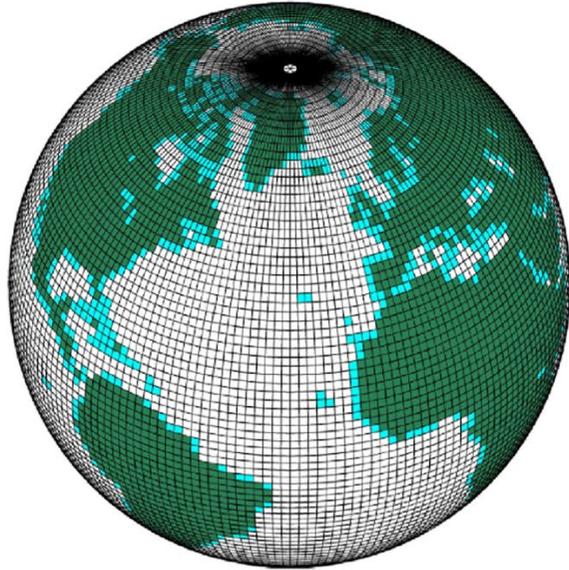


Coupled data assimilation case: physics and biogeochemistry

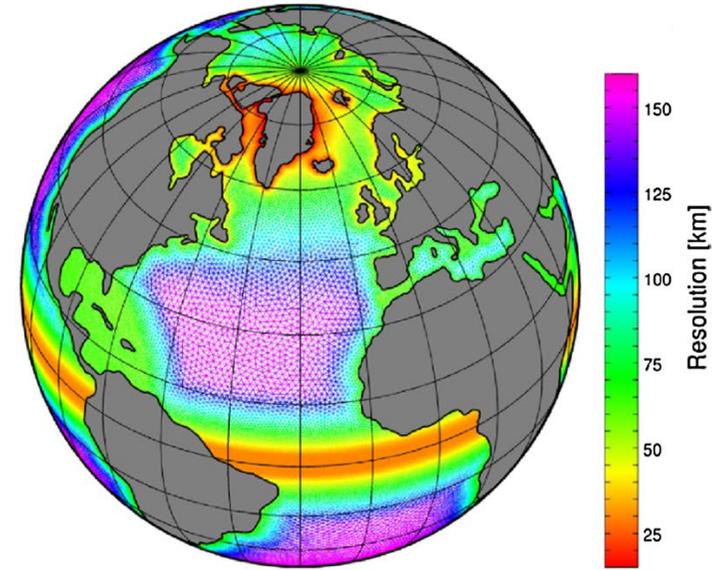
- Assimilate satellite sea surface temperature observations
- Assimilation directly changes Oxygen and other biogeochemical variables (strongly-coupled assimilation)

# Assimilation into coupled model: AWI-CM

Atmosphere



Ocean



OASIS3-MCT

fluxes



ocean/ice state

**Atmosphere**

- ECHAM6
- JSBACH land

**Coupler library**

- OASIS3-MCT

**Ocean**

- FESOM
- includes sea ice

Two separate executables for atmosphere and ocean

**Goal: Develop data assimilation methodology for cross-domain assimilation (“strongly-coupled”)**

# Assimilation Effect on Surface Temperature

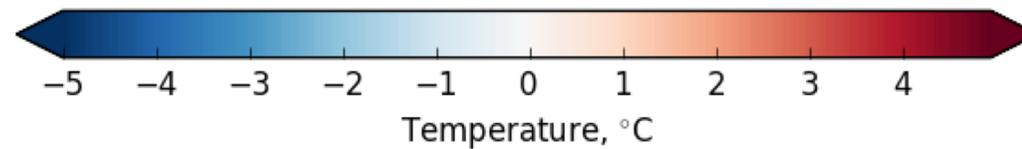
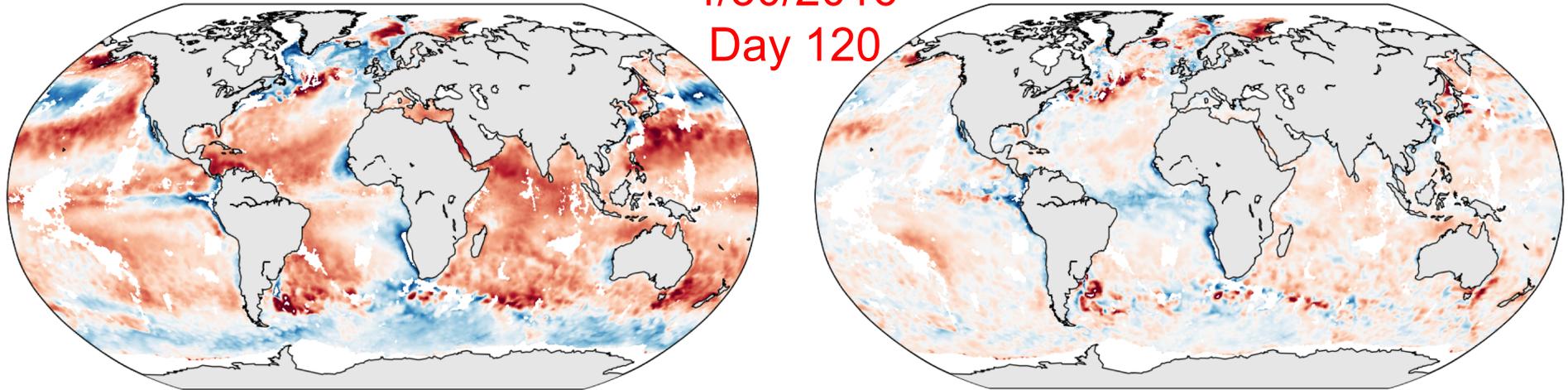
Assimilate subsurface temperature profile data

Difference between model simulations and observations

No Assimilation

4/30/2016  
Day 120

Assimilation



Qi Tang @ AWI

- Also subsurface temperature is improved

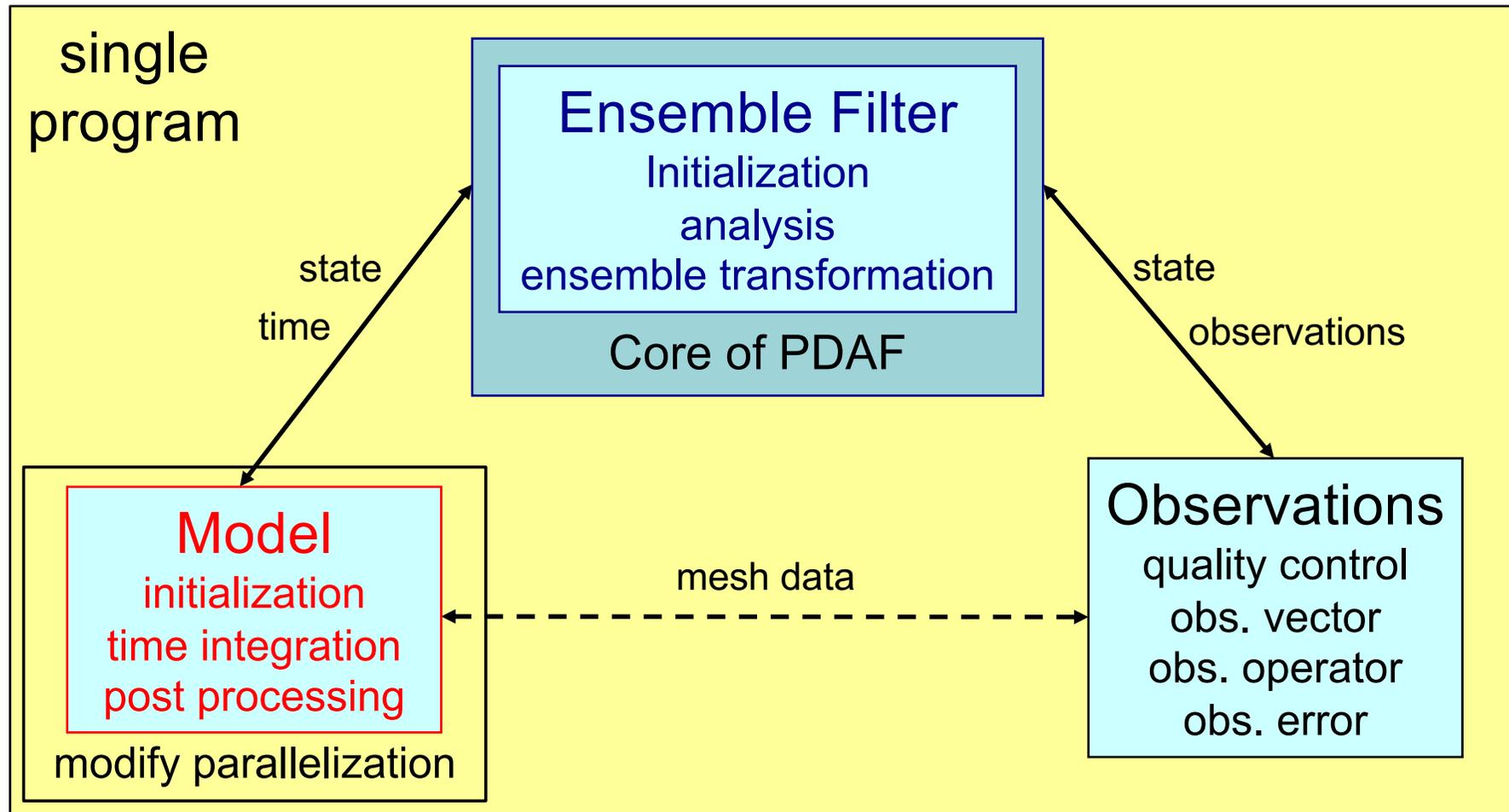
Current work

- Assess effect on atmosphere
- Final aim: strongly-coupled assimilation (e.g. assimilate oceanic observation into atmosphere)

# Software

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# Components of an Assimilation System



↔ Explicit interface

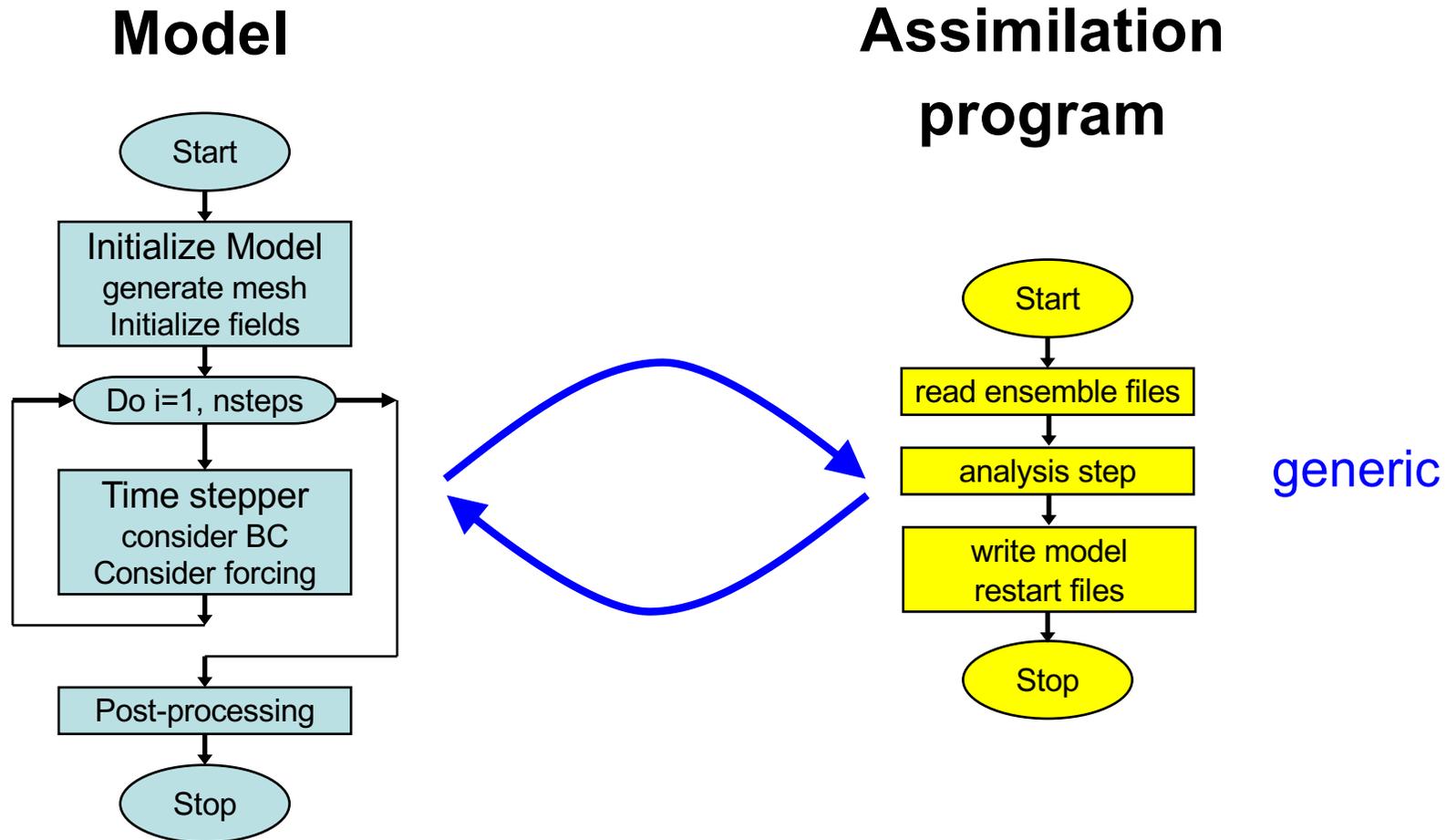
⋯ Indirect exchange (module/common)

## PDAF - Parallel Data Assimilation Framework

- a program library for ensemble data assimilation
- provide support for parallel ensemble forecasts
- provide fully-implemented & parallelized filters and smoothers (EnKF, LETKF, NETF, EWPF ... easy to add more)
- easily useable with (probably) any numerical model (applied with NEMO, MITgcm, FESOM, HBM, TerrSysMP, ...)
- run from laptops to supercomputers (Fortran, MPI & OpenMP)
- first public release in 2004; continuous further development
- ~370 registered users; community contributions

Open source:  
Code, documentation & tutorials at  
<http://pdaf.awi.de>

# Offline coupling – separate programs



- For each ensemble state
- Initialize from restart files
  - Integrate
  - Write restart files

- Read restart files (ensemble)
- Compute analysis step
- Write new restart files

# Offline coupling - Efficiency

Offline-coupling is simple to implement but can be very inefficient

## Example:

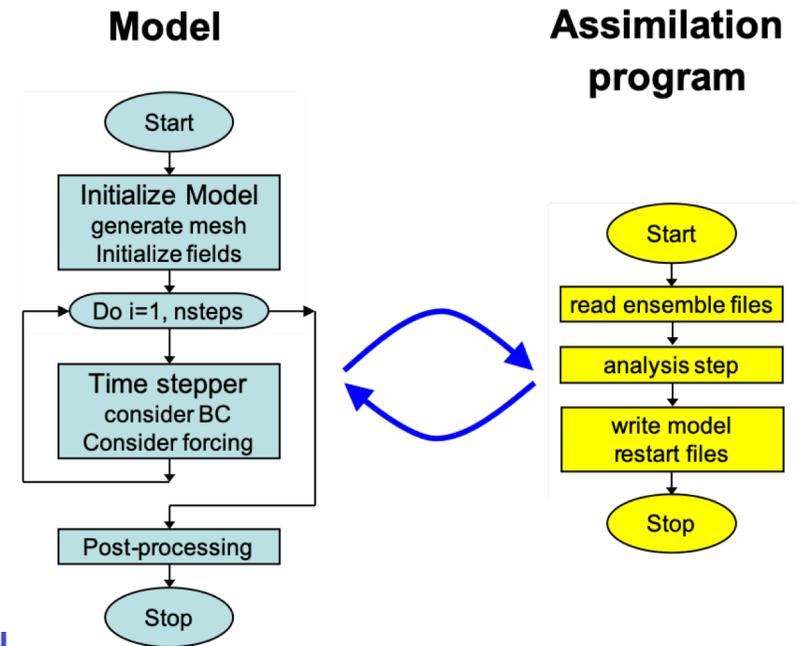
Timing from atmosphere-ocean coupled model (AWI-CM) with daily analysis step:

Model startup:	95 s	} overhead
Integrate 1 day:	28 s	
Model postprocessing:	14 s	

Analysis step: 1 s

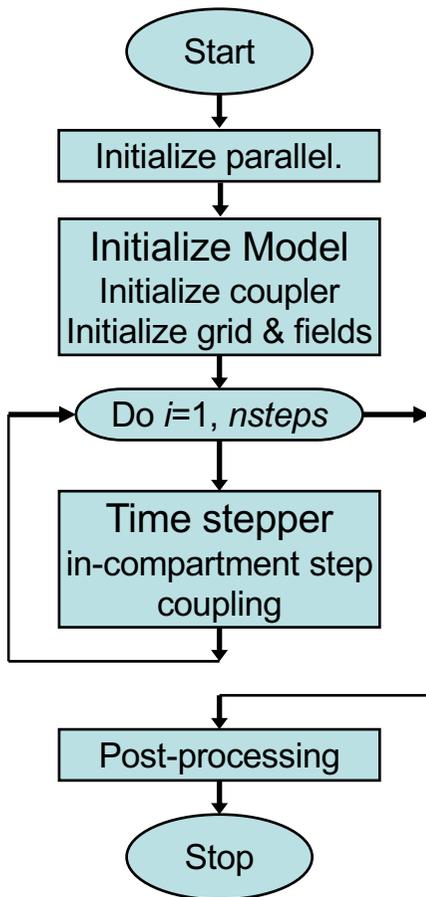
Restarting this model is ~3.5 times more expensive than integrating 1 day

→ avoid this for data assimilation

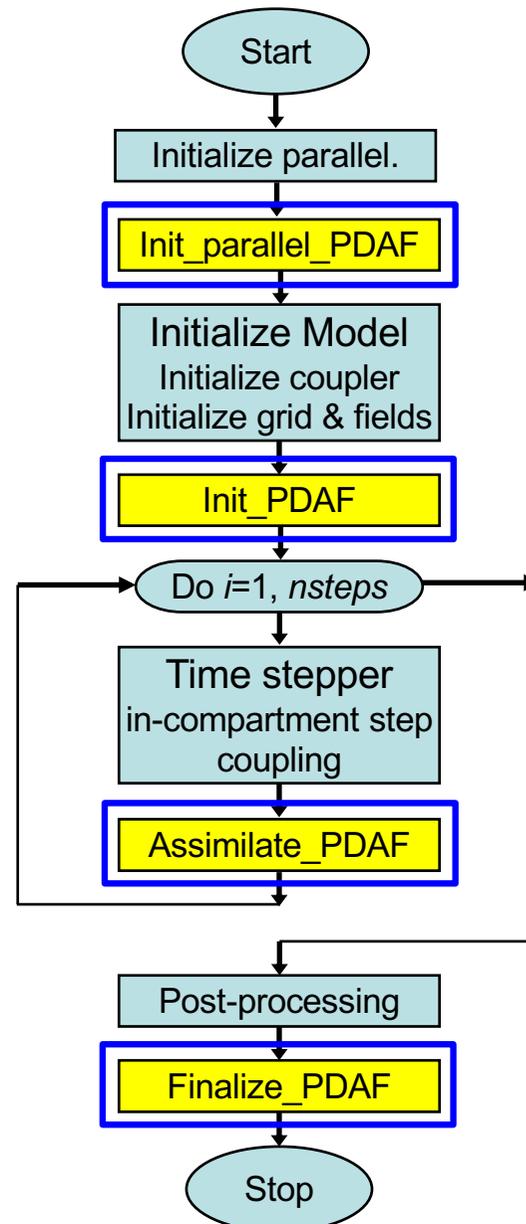


# Extending a Model for Data Assimilation

Model  
*single or multiple executables*



revised parallelization enables ensemble forecast



Extension for data assimilation

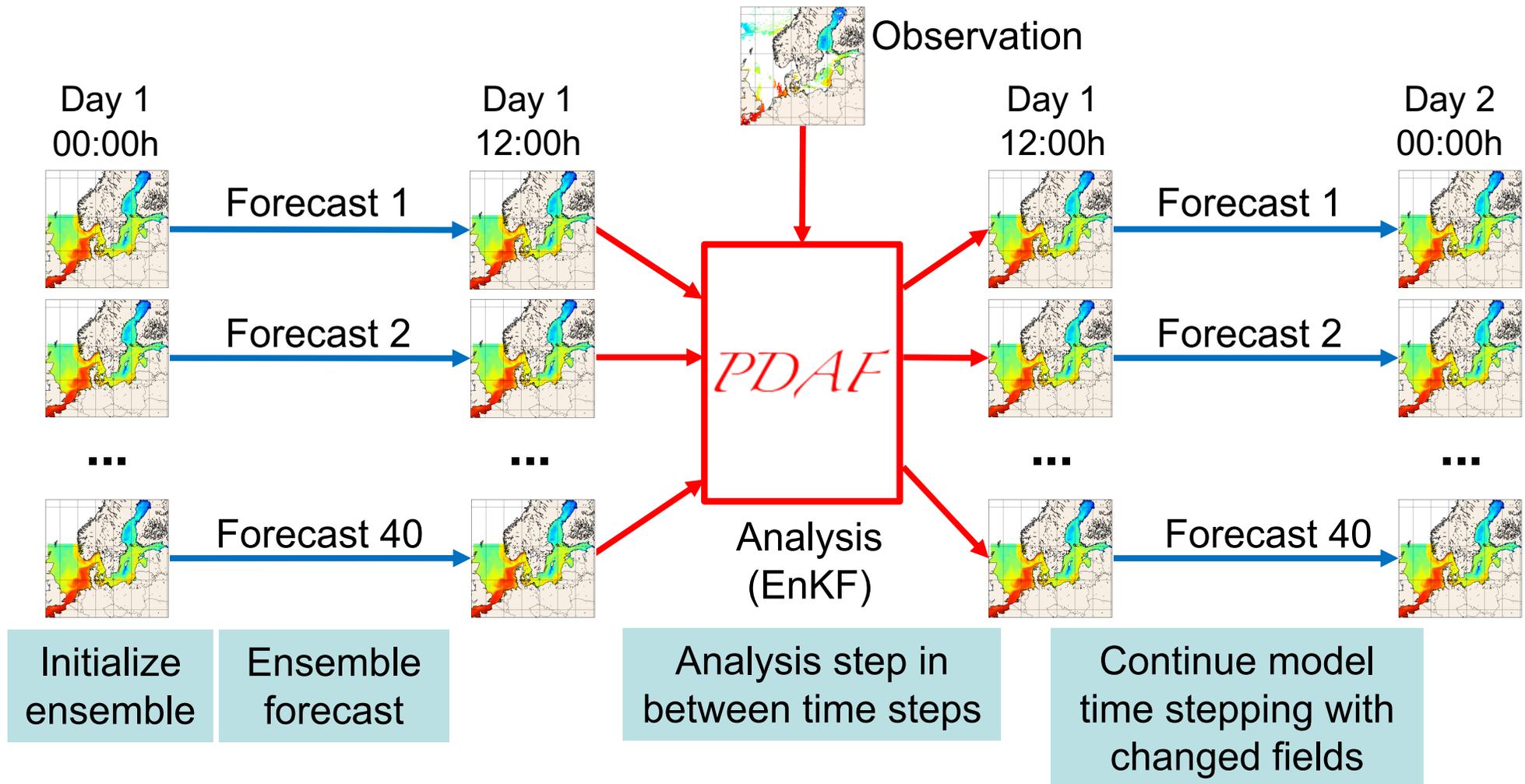
plus:  
Possible model-specific adaption



# Augmenting a Model for Data Assimilation

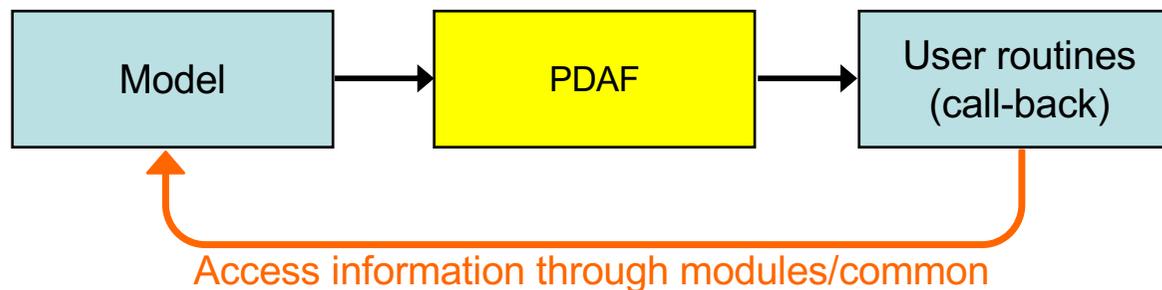
Couple PDAF (Parallel Data Assimilation Framework) with model

- Modify model to simulate ensemble of model states
- Insert correction step (analysis) to be executed at prescribed interval
- Run model as usual, but with more processors and additional options



## PDAF interface structure

- Interface routines call PDAF-core routines
- PDAF-core routines call case-specific routines provided by user (included in model binding set)
- User-supplied call-back routines for elementary operations:
  - field transformations between model and filter
  - observation-related operations
- User supplied routines can be implemented as routines of the model  
(for MITgcm: Fortran-77 fixed-form source code)



Assumption: Users know their model

→ let users implement assimilation system in model context

For users, model is not just a forward operator

→ let users extend their model for data assimilation

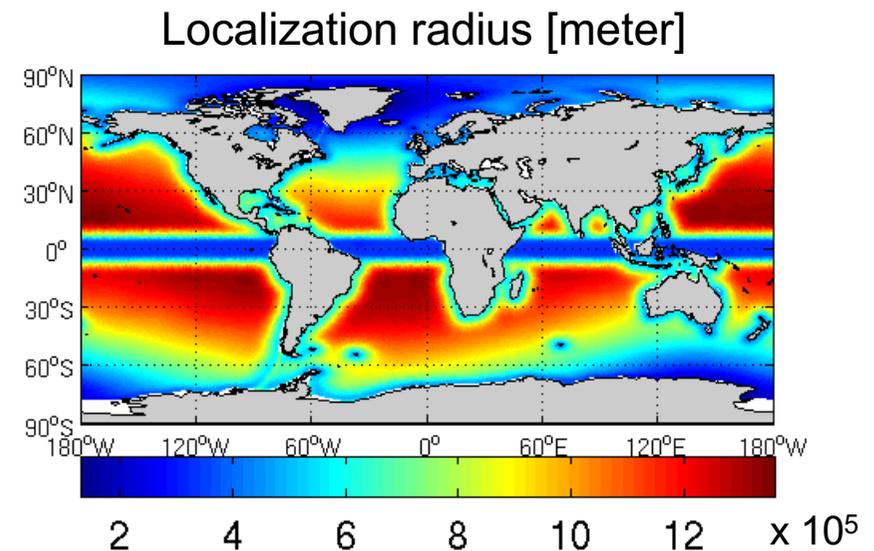
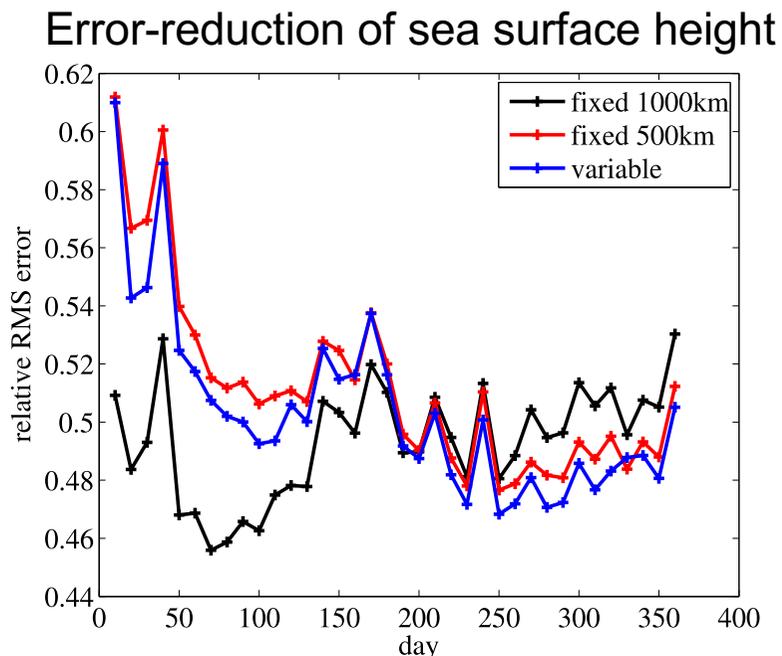
Keep simple things simple:

- Define subroutine interfaces to separate model and assimilation based on arrays
- No object-oriented programming (most models don't use it; most model developers don't know it; not many objects would be involved)
- Users directly implement observation-specific routines (no indirect description of e.g. observation layout)

## Adaptive Localization (Kirchgessner et al, 2012)

- Original study done with small models (Lorenz-96, shallow water)
- Paper reviewer asked to apply it with full-scale forecast model
- FESOM with PDAF was fully coded without adaptivity
  - Update PDAF library (just when recompiling)
  - Adding adaptivity routine and running experiment

} 1 day!



# Summary

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## Ensemble data assimilation

- Quantitative combination of model and observational data
- Improve observed and non-observed fields, fluxes, parameters, and predictions

PDAF simplifies the implementation and application of data assimilation

- Get faster to the application and results

## Tomorrow's Tutorial:

- Implementation of PDAF with simple model
- Experiments with an ensemble Kalman filter

## References

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- <http://pdaf.awi.de>
- Nerger, L., Hiller, W. *Software for Ensemble-based DA Systems – Implementation and Scalability.* Computers and Geosciences 55 (2013) 110-118
- Nerger, L., Hiller, W., Schröter, J.(2005). *PDAF - The Parallel Data Assimilation Framework: Experiences with Kalman Filtering*, Proceedings of the Eleventh ECMWF Workshop on the Use of High Performance Computing in Meteorology, Reading, UK, 25 - 29 October 2004, pp. 63-83.