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Finite-Time Synchronization for a Class of Dynamical Complex Networks with Nonidentical Nodes and Uncertain Disturbance*

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Abstract This paper investigates the finite-time synchronization for a class of linearly coupled dynamical complex networks with both nonidentical nodes and uncertain disturbance. A set of controllers are designed such that the considered system can be finite-timely synchronized onto the target node. Based on the stability of the error equation, the Lyapunov function method and the linear matrix inequality technique, several sufficient conditions are derived to ensure the finite-time synchronization, and applied to the case of identical nodes and the one without uncertain disturbance. Also the adaptive finite-time synchronization is discussed. A numerical example is given to show the effectiveness of the main results obtained.

Keywords Disturbance, dynamical complex networks, finite-time synchronization, nonidentical nodes.

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1 Introduction

In the past few decades, complex networks have gained a lot of attention in various fields of science and humanity worldwide, such as food-webs, ecosystems, metabolic pathways, the World Wide Webs and so on. Their control and synchronization have been deeply investigated, and various relevant theoretical results have been established[1–7]. Also many effective control approaches have been proposed, such as adaptive control[8, 9], intermittent control[10–13], impulsive control[14–17], slide mode control[18–20] and sampled-data control[21, 22], et al.

It is frequently encountered that many significant differences exist in the relevant individual nodes. For example, in power systems, because the generators and loads are connected to buses that are interconnected via transmission lines in a network structure , the power systems can be considered as a dynamical network with nonidentical nodes[23]. When the dynamics of nodes in a complex network are nonidentical, the synchronization problems become complicated and more challenging than the case of identical nodes. By use of free matrices, [23] considered the case of synchronizing to both a common equilibrium solution of all isolated nodes and the average state trajectory with nonidentical nodes. Combining the local intermittent controller with the open-loop controller, [24] established several exponential synchronization criteria for a class of complex networks with nonidentical nodes. In [25], the pinning cluster synchronization of complex dynamical networks with time-delayed coupling and dynamic nonidentical nodes was obtained. [26] studied the finite-time synchronization problem for linearly coupled complex networks with discontinuous nonidentical nodes.

On the other hand, more and more attention has been paid to the study of complex networks with perturbations because of the wide application[27–30]. [27] investigated globally exponential synchronization for linearly coupled neural networks with time-varying delay and impulsive disturbances. The derived sufficient condition was closely related with the time delay, impulse strengths, average impulsive interval, and coupling structure of the systems. [28] addressed the scheme of cluster synchronization of overlapping uncertain complex networks with time-varying impulse disturbances. In [29], the cluster synchronization problem of coupled complex networks with uncertain disturbance was considered under an adaptive fixed-time control strategy.

Most of the related research focuses on either nonidentical nodes or uncertain disturbance. Hence it is very necessary and important, with profound theoretical and practical significance, to investigate the finite-time synchronization of complex networks subject to both nonidentical nodes and uncertain disturbance. However, two difficulties have to be faced: (i) What conditions should be proposed which are applicable to general complex networks with both nonidentical nodes and uncertain disturbance and are easy to be verified? (ii) Which kind of controller should be designed such that the nonidentical nodes and the uncertain disturbance can be well dealt with? This paper aims to overcome these two difficulties and achieve finite-time synchronization for a class of linearly coupled complex networks with both nonidentical nodes and uncertain disturbance, and further enrich the theoretical results of finite-time synchronization. The main contributions in this paper can be summarized as follows: 1) A novel discontinuous controller is designed for a class of heterogeneous networks with uncertain disturbance and the controller can overcome the influence of heterogeneous and uncertain disturbance simultaneously on the finite-time synchronization of the network; 2) Several criteria have been derived to check the finite-time synchronization of the considered networks. Different from most of the existing results, the obtained finite-time synchronization conditions are represented in the form of linear matrix inequalities, and easy to be verified; 3) The adaptive finite-time synchronization of the heterogeneous networks are addressed.

Firstly, in order to make the results obtained more easily verified and applied in practice, a model of the complex network with both nonidentical nodes and uncertain disturbance is established. After that, a set of controllers are designed. Based on the finite-time stability for the error equation, the Lyapunov function method and the linear matrix inequality technique, it is shown that the complex network considered can be finite-timely synchronized onto any isolated driving node. Secondly, several sufficient criteria are obtained to guarantee the synchronization goal, and applied to the case of identical nodes and the one without uncertain disturbance. Meanwhile, the adaptive finite-time synchronization is also discussed.

The remainder of this paper is organized as follows. In Section 2, the complex network model with both nonidentical nodes and uncertain disturbance is formulated and the finite-time synchronization problem is introduced. The finite-time synchronization conditions are obtained in Section 3. Section 4 considers the finite-time synchronization by the adaptive control method. In Section 5, a numerical example is provided to illustrate the validity of the method proposed. Some conclusions are made in Section 6, together with some potential future study.

2 Problem Formulation

Consider an array of nonlinear systems with the linear and diffusive coupling consisting of N nonidentical nodes in which each node is an n-dimensional dynamical system with uncertain disturbance as follows

$$\dot{x}_i(t) = f_i(t, x_i(t)) + h_i(t, x_i(t)) + c\sum_{j=1}^{N} G_{ij}\mathit{\Gamma} x_j(t), \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{21}(t), \cdots, x_{in}(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of the ith dynamical node; $f_i(t, x_i(t)) : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinear vector fields describing the modal self-dynamics, and $h_i(t, x_i(t)) : \mathbb{R}^+ \times \mathbb{R} \to R$ is the uncertain disturbance, $i = 1, 2, \cdots, N$; $c > 0$ is a constant and denotes the coupling strength of the whole complex network, and $\mathit{\Gamma} = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_n) \in \mathbb{R}^{n\times n}$ is the inner-coupling matrix, which is used to illustrate the way of linking the components in every pair vector of nodes with $\gamma_i \geq 0$; $G = (G_{ij})_{N\times N}$ is the constant coupling configuration matrix which represents the topological structure and may be defined to be diffusive, i.e., $G_{ij} \geqslant 0$ $(i \neq j)$ if there exists a directed connection from node j to node i; otherwise, $G_{ij} = 0$ and $G_{ii} = -\sum_{j=1, j\neq i}^{N} G_{ij}$. Here, the coupling matrices G is nor required to be symmetric or irreducible.

We refer to the system (2) with $x_0|_{t=0} = x_0(0)$ as the driven dynamical node of (1)

$$\dot{x}_0(t) = f_0(t, x_0(t)) + h_0(t, x_0(t)). \tag{2}$$

Definition 2.1 (see [31]) The complex network (1) is said to be synchronized in finite time if there exist a designed feedback controller and a constant $t^* > 0$, which often depends on the initial state vector value $X(0) = (x_1^{\mathrm{T}}(0), x_2^{\mathrm{T}}(0), \cdots, x_N^{\mathrm{T}}(0))^{\mathrm{T}}$, such that

$$\lim_{t \to t^*} \| x_i(t) - x_0(t) \| = 0.$$

Then the synchronization performance of drive-response network are achieved in a finite-time, i.e.,

$$\| x_i(t) - x_0(t) \| = 0, \quad t > t^*, \quad i = 1, 2, \cdots, N.$$

This paper aims to design feedback controllers for the complex network (1) to achieve the finite-time synchronization.

Assumption 2.1 (see [32]) There exists a uniformly symmetric positive definite matrix $L = \mathrm{diag}(l_1, l_2, \cdots, l_n)$ such that $f_i(t, x)$ satisfies

$$(y - x)^{\mathrm{T}}(f_i(t, y) - f_i(t, x)) \le (y - x)^{\mathrm{T}} L (y - x), \quad i = 1, 2, \cdots, N \tag{3}$$

for all $x,\ y \in \mathbb{R}^n$ and $t \ge 0$.

Assumption 2.2 (see [30]) There exists a time-varying function $\mu(t) \ge 0$ such that

$$\|(f_i(t, x) - f_0(t, x)\| \le \mu(t), \quad i, j = 1, 2, \cdots, N. \tag{4}$$

Assumption 2.3 The uncertain disturbances $h_i(t, x_i(t))$ are continuous at $t, x_i(t) \ge 0$, and bounded by a given non-negative number $h_{\max}$, that is,

$$|h_i(t, x_i(t))| \le h_{\max}, \quad i = 0, 1, \cdots, N. \tag{5}$$

Remark 2.2 Assumption 2.1 is satisfied with chaotic oscillators and Rössler's systems and so on. Assumption 2.2 and Assumption 2.3 impose restrictions on the activation function, and they are widely used in literatures[23–26].

3 Finite-Time Synchronization

In this section, we design controllers for the finite-time synchronization of the complex network (1). For this, the controllers $u_i(t) \in R^n$ are constructed as

$$u_i(t) = -d_i e_i(t) - \eta(t)\overline{\mathrm{sign}}(e_i(t)) - k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta}, \quad i = 1, 2, \cdots, N, \tag{6}$$

where $d_1, d_2, \cdots, d_N$ are positive constants to be determined, $k > 0$ is a constant, $\overline{\mathrm{sign}}(e_i(t)) = (\mathrm{sign}(e_{i1}(t)),\ \mathrm{sign}(e_{i2}(t)),\ \cdots,\ \mathrm{sign}(e_{in}(t)))^{\mathrm{T}}$, $\mathrm{sign}(e_i(t)) = \mathrm{diag}(\mathrm{sign}(e_{i1}(t)), \mathrm{sign}(e_{i2}(t)), \cdots, \mathrm{sign}(e_{in}(t)))$, $|e_i(t)|^{\beta} = (|e_{i1}(t)|^{\beta},\ |e_{21}(t)|^{\beta},\ \cdots,\ |e_{in}(t)|^{\beta})^{\mathrm{T}}$, and the real number β follows $0 \le \beta < 1$.

Then the controlled complex dynamical network is given by

$$\dot{x}_i(t) = f_i(t, x_i) + h_i(t, x_i(t)) + c\sum_{j=1}^{N} G_{ij}\Gamma x_j(t) + u_i(t), \quad i = 1, 2, \cdots, N. \tag{7}$$

By defining the synchronization errors $e_i(t) = x_i - x_0$, $F_i(t) = f_i(t, x_i) - f_0(t, x_0)$ and $H_i(t, e_i(t)) = h_i(t, x_i(t)) - h_0(t, x_0(t))$, the error dynamical system can be represented by

$$\dot{e}_i(t) = F_i(t, e_i) + H_i(t, e_i(t) + c\sum_{j=1}^{N} G_{ij}\Gamma e_j + u_i(t), \quad i = 1, 2, \cdots, N. \tag{8}$$

Lemma 3.1 (see [33, 34]) *Assume that a continuous, differentiable, positive-definite function $V(t) : [0, +\infty) \to [0, +\infty)$ satisfies*

$$\frac{dV(t)}{dt} \le -\eta V^{\alpha}(t), \quad \forall t \ge t_0, \quad V(t_0) \ge 0,$$

where $\eta > 0$ and $0 < \alpha < 1$ are two constants. For any given t_0, one can have

$$V^{1-\alpha}(t) \le V^{1-\alpha}(t_0) - \eta(1-\alpha)(t - t_0), \quad t_0 \le t \le t_1$$

and $V(t) \equiv 0$, $t > t_1 = t_0 + \frac{V^{1-\alpha}(t_0)}{\eta(1-\alpha)}$.

Lemma 3.2 (see [35]) *For matrices A, B, C, D with appropriate dimensions and a scalar α, the following assertions hold.*

1) $(\alpha A) \otimes B = A \otimes (\alpha B)$;
2) $(A + B) \otimes C = A \otimes C + B \otimes C$;
3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$;
4) $(A \otimes B)^{\mathrm{T}} = A^{\mathrm{T}} \otimes B^{\mathrm{T}}$,

where $\otimes$ is the Kronecker product.

Lemma 3.3 (see [36]) *Suppose that $a_i \ge 0$ for $i = 1, 2, \cdots, n$, $0 < p \le 1$ and $0 < q < 2$, it follows that $(\sum_{i=1}^{n} a_i)^p \le \sum_{i=1}^{n}(a_i)^p$ and $\sum_{i=1}^{n}(a_i)^q \ge (\sum_{i=1}^{n} a_i^2)^{q/2}$.*

Theorem 3.4 *Consider the complex network* (1) *under the set of controllers* (6). *If Assumption* 2.1, *Assumption* 2.2 *and Assumption* 2.3 *hold, and*

$$\eta(t) \ge \mu(t) + 2h_{\max}, \quad I_N \otimes L + c(G^s \otimes \Gamma) - D \otimes I_n < 0 \tag{9}$$

with $D = \mathrm{diag}\{d_1, d_2, \cdots, d_N\} > 0$, $G^s = (G + G^{\mathrm{T}})/2$, L being given by Assumption 2.1, *then* (7) *can be synchronized in a finite time with*

$$t^* = \frac{V(0)^{\frac{1-\beta}{2}}}{2^{\frac{\beta-1}{2}} k(1-\beta))}, \tag{10}$$

where $V(0) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(0)e_i(0)$ and $e_i(0)$ is the initial condition of $e_i(t) = x_i(t) - x_0(t)$ for $i = 1, 2, \cdots, N$.

Proof For (8), a Lyapunov function is constructed by

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t),$$

whose derivative along the trajectory of System (8) can be calculated as

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) \\
&= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\left\{F_i(t, e_i(t)) + H_i(t, e_i(t)) + c\sum_{j=1}^{N} G_{ij}\mathit{\Gamma} e_j(t) + u_i(t)\right\} \\
&= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\left\{-d_i e_i(t) - \eta(t)\overline{\mathrm{sign}}(e_i(t)) - k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta}\right\} \\
&\quad + I_1(t) + I_2(t) + I_3(t), \qquad (11)
\end{aligned}$$

with $I_1(t) = c\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\sum_{j=1}^{N} G_{ij}\mathit{\Gamma} e_j(t)$, $I_2(t)= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)H_i(t, e_i(t))$, and $I_3(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)F_i(t, e_i(t))$.

From Lemma 3.2 and noticing that $\mathit{\Gamma}$ is a diagonal matrix, we have

$$\begin{aligned}
I_1(t) &= ce^{\mathrm{T}}(t)(G \otimes \mathit{\Gamma})e(t) \\
&= ce^{\mathrm{T}}(t)\frac{G \otimes \mathit{\Gamma} + (G \otimes \mathit{\Gamma})^{\mathrm{T}}}{2}e(t) \\
&= ce^{\mathrm{T}}(t)(G^s \otimes \mathit{\Gamma})e(t). \qquad (12)
\end{aligned}$$

By use of (5) in Assumption 2.3, one can get

$$\begin{aligned}
I_2(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)(H_i(t, x_1(t)) - H_0(t, x_0(t))) \\
&\leq \sum_{i=1}^{N} \|e_i(t)\|_2\|H_i(t, x_1(t)) - H_0(t, x_0(t))\|_2 \\
&\leq \sum_{i=1}^{N} \|e_i(t)\|_2(\|H_i(t, x_1(t))))\|_2 + \|H_0(t, x_0(t))\|_2) \\
&\leq \sum_{i=1}^{N} \|e_i(t)\|_2(2h_{\max}). \qquad (13)
\end{aligned}$$

Also it can be seen that

$$\begin{aligned}
I_3(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)(f_i(t, x_i(t)) - f_0(t, x_0(t))) \\
&= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)((f_i(t, x_i(t)) - f_i(t, x_0(t))) + (f_i(t, x_0(t)) - f_0(t, x_0(t))))
\end{aligned}$$

$$= I_{31}(t) + I_{32}(t) \tag{14}$$

with $I_{31}(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)(f_i(t, x_i(t)) - f_i(t, x_0(t)))$ and $I_{32}(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\ (f_i(t, x_0(t)) - f_0(t, x_0(t)))$. From (3) in Assumption 2.1, it is known that

$$I_{31}(t) \le \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) L e_i(t) = e^{\mathrm{T}}(t) I_N \otimes L e(t). \tag{15}$$

By (4) in Assumption 2.2, one can get

$$\begin{aligned} I_{32}(t) &\le \sum_{i=1}^{N} \|e_i(t)\|_2 \|f_i(t, x_0(t)) - f_0(t, x_0(t))\|_2 \\ &= \sum_{i=1}^{N} \|e_i(t)\|_2 \mu(t). \end{aligned} \tag{16}$$

Submitting (12)–(16) into (11) yields that

$$\begin{aligned} &\dot{V}(t) \\ \le\ & e^{\mathrm{T}}(t)(I_N \otimes L)e(t) + c e^{\mathrm{T}}(t)(G^s \otimes \mathit{\Gamma})e(t) + \sum_{i=1}^{N} \|e_i\|_2 \mu(t) + 2\sum_{i=1}^{N} \|e_i(t)\|_2 h_{\max} \\ &+ \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) \left\{ -d_i e_i(t) - \eta(t)\overline{\mathrm{sign}}(e_i(t)) - k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta} \right\} \\ =\ & W_1(t) + W_2(t) + W_3(t), \end{aligned} \tag{17}$$

where

$$\begin{aligned} W_1(t) &= e^{\mathrm{T}}(t)(I_N \otimes L)e(t) + c e^{\mathrm{T}}(t)(G^s \otimes \mathit{\Gamma})e(t) - \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) d_i e_i(t), \\ W_2(t) &= \sum_{i=1}^{N} \|e_i\|_2 \mu(t) + \sum_{i=1}^{N} \|e_i(t)\|_2 (2h_{\max}) - \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)(\eta(t)\overline{\mathrm{sign}}(e_i(t))), \\ W_3(t) &= -\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta}. \end{aligned}$$

By virtue of Lemma 3.2 and (9), it is known that

$$\begin{aligned} W_1(t) &= e^{\mathrm{T}}(t)(I_N \otimes L)e(t) + c e^{\mathrm{T}}(t)(G^s \otimes \mathit{\Gamma})e(t) - e^{\mathrm{T}}(t)(D \otimes I_n)e_i(t) \\ &= e^{\mathrm{T}}(t)(I_N \otimes L + c(G^s \otimes \mathit{\Gamma}) - D \otimes I_n)e(t) \\ &\le 0. \end{aligned} \tag{18}$$

Since $e_i^{\mathrm{T}}(t)\overline{\mathrm{sign}}(e_i(t)) = (e_{i1}(t), e_{i2}(t), \cdots, e_{in}(t))(\mathrm{sign}(e_{i1}(t)), \mathrm{sign}(e_{i2}(t)), \cdots, \mathrm{sign}(e_{in}(t)))^{\mathrm{T}} = \sum_{j=1}^{n} |e_{ij}(t)|$ and $\|e_i(t)\|_2 - e_i^{\mathrm{T}}(t)\overline{\mathrm{sign}}(e_i(t))) = (\sum_{j=1}^{n} e_{ij}^2(t))^{\frac{1}{2}} - \sum_{j=1}^{n} |e_{ij}(t)| \le 0$ according to Lemma 3.3, one can give

$$W_2(t) = \sum_{i=1}^{N} (\mu(t) + 2h_{\max}) \|e_i(t)\|_2 - \eta(t) \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\overline{\mathrm{sign}}(e_i(t))$$

$$
\begin{aligned}
&\leq (\mu(t) + 2h_{\max}) \sum_{i=1}^{N} \|e_i(t)\|_2 - \eta(t) \sum_{i=1}^{N} \|e_i(t)\|_2 \\
&= (\mu(t) + 2h_{\max} - \eta(t)) \sum_{i=1}^{N} \|e_i(t)\|_2 \\
&\leq 0. \qquad (19)
\end{aligned}
$$

Considering that $|e_i(t)| = (|e_{i1}(t)|, |e_{i2}(t)|, \cdots, |e_{in}(t)|)^{\mathrm{T}}$ and $|e_i(t)|^{\beta} = (|e_{i1}(t)|^{\beta}, |e_{i2}(t)|^{\beta}, \cdots, |e_{in}(t)|^{\beta})^{\mathrm{T}}$, we have

$$
\begin{aligned}
W_3(t) &= -\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) k \mathrm{sign}(e_i(t)) |e_i(t)|^{\beta} \\
&= -k \sum_{i=1}^{N} |e_i(t)| |e_i(t)|^{\beta} \\
&= -k \sum_{i=1}^{N} \sum_{j=1}^{n} |e_{ij}(t)|^{1+\beta}. \qquad (20)
\end{aligned}
$$

By Lemma 3.3, it can be obtained that

$$
\left(\sum_{i=1}^{N} \sum_{j=1}^{n} |e_{ij}(t)|^{1+\beta} \right)^{\frac{1}{1+\beta}} \geq \left(\sum_{i=1}^{N} \sum_{j=1}^{n} |e_{ij}(t)|^{2} \right)^{\frac{1}{2}},
$$

and

$$
\left(\sum_{i=1}^{N} \sum_{j=1}^{n} |e_{ij}(t)|^{1+\beta} \right) \geq \left(\sum_{i=1}^{N} \sum_{j=1}^{n} |e_{ij}(t)|^{2} \right)^{\frac{1+\beta}{2}} = \left(\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} e_i(t) \right)^{\frac{1+\beta}{2}},
$$

which together with (20) implies that

$$
\begin{aligned}
W_3(t) &\leq -k \left(\sum_{i=1}^{N} e_i(t)^{\mathrm{T}} e_i(t) \right)^{\frac{1+\beta}{2}} \\
&= -k (2V(t))^{\frac{1+\beta}{2}} \\
&= -2^{\frac{1+\beta}{2}} k V(t)^{\frac{1+\beta}{2}}. \qquad (21)
\end{aligned}
$$

Submitting (18), (19) and (21) to (17), we can get

$$
\dot{V}(t) \leq W_1(t) + W_2(t) + W_3(t) \leq -2^{\frac{1+\beta}{2}} k V(t)^{\frac{1+\beta}{2}}.
$$

According to Lemma 3.1, $V(t)$ converges to zero in a finite time, and the finite time t^* is given by

$$
t^* = \frac{V(0)^{1-0.5(1+\beta)}}{2^{\frac{1+\beta}{2}} k (1 - 0.5(1+\beta))} = \frac{V(0)^{\frac{1-\beta}{2}}}{2^{\frac{\beta-1}{2}} k (1-\beta))}.
$$

Hence, the error vector $e_i(t)$ converges to zero within t^* for $i = 1, 2, \cdots, N$, and (1) under (6) is finite-timely synchronized in the finite time t^*. The proof is completed. ∎

Remark 3.5 Theorem 3.4 also provides a method how to select controllers for getting the finite-time synchronization of a linearly coupled heterogeneous complex network (1) when the Laplacian matrix G is asymmetric and the nodes are nonidentical. The controller is divided into three parts: The first part $-\eta_i e_i(t)$ overcomes the linear condition of nonlinear function, the second part $-d(t)\overline{\text{sign}}(e_i(t))$ is used to offset the difference between state functions $f_i(t, x_i(t))$ and the uncertain disturbance $h_i(t, x_i(t))$, and the last one $-k\text{sign}(e_i(t))|e_i(t)|^\beta$ is used to make the network achieve the finite-time synchronization.

If the nodes are identical which indicates that $f_1, f_2, \cdots, f_N$ are equal to the same function denoted by f, then (1) becomes

$$\dot{x}_i(t) = f(t, x_i) + h_i(t, x_i(t)) + c\sum_{j=1}^{N} G_{ij}\Gamma x_j(t), \quad i = 1, 2, \cdots, N. \tag{22}$$

The drive dynamical node with $x_0|_{t=0} = x_0(0)$ is given by

$$\dot{x_0}(t) = f(t, x_0(t)) + h_0(t, x_0(t)) \tag{23}$$

and it can be seen that the error dynamical system is

$$\dot{e}_i(t) = F(t, e_i(t)) + H_i(t, e_i(t) + c\sum_{j=1}^{N} G_{ij}\Gamma e_j + u_i(t),$$

where $F_i(t) = f(t, x_i(t)) - f(t, x_0(t))$ and $H_i(t, e_i(t)) = h_i(t, x_i(t)) - h_0(t, x_0(t))$.

Let

$$u_i(t) = -d_i e_i(t) - \eta(t)\overline{\text{sign}}(e_i(t)) - k\text{sign}(e_i(t))|e_i(t)|^\beta, \quad i = 1, 2, \cdots, N, \tag{24}$$

where $d_i \geq 0$ is constant to be determined for $i = 1, 2, \cdots, N$; $k > 0$ is a constant; $\overline{\text{sign}}(e_i(t)) = (\text{sign}(e_{i1}(t)), \text{sign}(e_{i2}(t)), \cdots, \text{sign}(e_{in}(t)))^{\mathrm{T}}$, $|e_i(t)|^\beta = (|e_{i1}(t)|^\beta, |e_{i2}(t)|^\beta, \cdots, |e_{in}(t)|^\beta)^{\mathrm{T}}$, sign $(e_i(t)) = \text{diag}(\text{sign}(e_{i1}(t)), \text{sign}(e_{i2}(t)), \cdots, \text{sign}(e_{in}(t)))$; the real number β satisfies $0 \leq \beta < 1$. Then, we can get a criterion on the finite-time synchronization of the complex network with identical nodes.

Corollary 3.6 *Consider the complex network* (22) *under the set of controllers* (24)*. If Assumption* 2.1 *and Assumption* 2.3 *hold, and*

$$\eta(t) \geq h_{\max}, \quad I_N \otimes L + c(G^s \otimes \Gamma) - D \otimes I_n < 0$$

with $D = \text{diag}(d_1, d_2, \cdots, d_N) > 0$*, then* (22) *can be synchronized to the state of the node* (23) *in a finite time*

$$t^* = \frac{V(0)^{\frac{1-\beta}{2}}}{2^{\frac{\beta-1}{2}}k(1-\beta))}.$$

Furthermore, if $h_i = 0$ for $i = 0, 1, \cdots, N$ in (22), that is, all nodes not only are identical but also have no uncertain disturbance, then (1) is reduced to

$$\dot{x}_i(t) = f(t, x_i(t)) + c\sum_{j=1}^{N} G_{ij}\Gamma x_j(t), \tag{25}$$

and the drive dynamical node of the complex network model (25) with $x_0|_{t=0} = x_0(0)$ is

$$\dot{x_0}(t) = f(t, x_0(t)).$$

In this case, the finite-time synchronization of (7) is changed to the problem of the complex network (25) with identical nodes. A criteria for the finite-time synchronization is given in the corollary below.

Corollary 3.7 *Consider the complex network* (25) *under the controllers*

$$u_i(t) = -d_i e_i(t) - k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta}, \quad i = 1, 2, \cdots, N,$$

where k is a positive constant, $0 \le \beta < 1$, $|e_i(t)|\beta = (|e_{i1}(t)|^{\beta}, |e_{i2}(t)|^{\beta}, \cdots, |e_{in}(t)|^{\beta})^{\mathrm{T}}$ *and* $\mathrm{sign}(e_i(t)) = \mathrm{diag}(\mathrm{sign}(e_{i1}(t)), \mathrm{sign}(e_{i2}(t)), \cdots, \mathrm{sign}(e_{in}(t)))$. *If Assumption* 2.1 *holds and*

$$I_N \otimes L + c(G^s \otimes \Gamma) - D \otimes I_n < 0$$

with $D = \mathrm{diag}(d_1, d_2, \cdots, d_N) > 0$, *then* (25) *can be synchronized in a finite time*

$$t^* = \frac{V(0)^{\frac{1-\beta}{2}}}{2^{\frac{\beta-1}{2}}k(1-\beta))}.$$

4 Adaptive Finite-Time Synchronization

This section discusses the adaptive finite-time synchronization of (1). We design an adaptive controllers as follows

$$\begin{aligned} u_i(t) &= -cd_i(t)\Gamma e_i(t) - \eta(t)\overline{\mathrm{sign}}(e_i(t)) - k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta}, \\ \dot{d}_i(t) &= q_i e_i^{\mathrm{T}}(t)\Gamma e_i(t) - k\left(\frac{c}{q_i}\right)^{\frac{\beta-1}{2}} \mathrm{sign}(d_i(t) - d)|d_i(t) - d|^{\beta}, \end{aligned} \tag{26}$$

where d is a constant to be determined; $k > 0$ is a constant; $|e_i(t)|^{\beta} = (|e_{i1}(t)|^{\beta}, |e_{i2}(t)|^{\beta}, \cdots, |e_{in}(t)|^{\beta})^{\mathrm{T}}$, $\overline{\mathrm{sign}}(e_i(t)) = (\mathrm{sign}(e_{i1}(t)), \cdots, \mathrm{sign}(e_{in}(t)))^{\mathrm{T}}$, $\mathrm{sign}(e_i(t)) = \mathrm{diag}(\mathrm{sign}(e_{i1}(t)), \cdots, \mathrm{sign}(e_{in}(t)))$; the real number β satisfies $0 \le \beta < 1$.

Theorem 4.1 *Consider the complex network* (1) *under the set of adaptive controllers* (26). *If Assumption* 2.1, *Assumption* 2.2 *and Assumption* 2.3 *hold, and*

$$\eta(t) \ge 2h_{\max}, \quad d \ge \frac{1}{c}\lambda_{\max}\left(I_N \otimes (L\Gamma^{-1}) + cG^s \otimes I_n\right),$$

then (22) *can be synchronized in a finite time*

$$t^* = \frac{V(0)^{\frac{1-\beta}{2}}}{2^{\frac{\beta-1}{2}}k(1-\beta))}, \tag{27}$$

where $\lambda_{\max}(\cdot)$ *represents the largest eigenvalue of a matrix,* $V(0) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(0)e_i(0) + \sum_{i=1}^{N} \frac{c}{2q_i}(d_i(0) - d)^2$, *and* $e_i(0)$ *is the initial condition of* $e_i(t) = x_i(t) - x_0(t)$ *for* $i = 1, 2, \cdots, N$.

Proof For (1) under (26), a Lyapunov function is constructed as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}\frac{c}{2q_i}(d_i(t)-d)^2.$$

Along the trajectory of the error system (8), the derivative of $V(t)$ can be

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) + \sum_{i=1}^{N}\frac{c}{q_i}(d_i(t)-d)\dot{d_i}(t) \\
&= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\left\{F_i(t,e_i(t)) + H_i(t,e_i(t)) + c\sum_{j=1}^{N} G_{ij}\mathit{\Gamma} e_j(t) + u_i(t)\right\} \\
&\quad + \sum_{i=1}^{N}\frac{c}{q_i}(d_i(t)-d)\left\{q_i e_i^{\mathrm{T}}(t)\mathit{\Gamma} e_i(t) - k\left(\frac{c}{q_i}\right)^{\frac{\beta-1}{2}}\mathrm{sign}(d_i(t)-d)|d_i(t)-d|^{\beta}\right\} \\
&= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)F_i(t,e_i(t)) + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)H_i(t,e_i(t)) + c\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\sum_{j=1}^{N} G_{ij}\mathit{\Gamma} e_j(t) \\
&\quad + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\left\{-cd_i(t)\mathit{\Gamma} e_i(t) - \eta(t)\overline{\mathrm{sign}}(e_i(t)) - k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta}\right\} \\
&\quad + c\sum_{i=1}^{N}(d_i(t)-d)e_i^{\mathrm{T}}(t)\mathit{\Gamma} e_i(t) \\
&\quad - \sum_{i=1}^{N} k\left(\frac{c}{q_i}\right)^{\frac{1+\beta}{2}}(d_i(t)-d)\mathrm{sign}(d_i(t)-d)|d_i(t)-d|^{\beta}) \\
&= W_1(t) + W_2(t) + W_3(t),
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
W_1(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)F_i(t,e_i(t)) + c\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\sum_{j=1}^{N} G_{ij}\mathit{\Gamma} e_j(t) \\
&\quad - \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)cd_i(t)\mathit{\Gamma} e_i(t) + c\sum_{i=1}^{N}(d_i(t)-d)e_i^{\mathrm{T}}(t)\mathit{\Gamma} e_i(t), \\
W_2(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)H_i(t,e_i(t)) - \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)(\eta(t)\overline{\mathrm{sign}}(e_i(t))), \\
W_3(t) &= -\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)k\mathrm{sign}(e_i(t))|e_i(t)|^{\beta} \\
&\quad - \sum_{i=1}^{N} k\left(\frac{c}{q_i}\right)^{\frac{1+\beta}{2}}(d_i(t)-d)\mathrm{sign}(d_i(t)-d)|d_i(t)-d|^{\beta}).
\end{aligned}$$

Letting $d \geq \frac{1}{c}\lambda_{\max}(I_N \otimes (L\Gamma^{-1}) + cG^s \otimes I_n)$, and similar to the proof of Theorem 3.4, we can obtain

$$\begin{aligned} W_1(t) &= e^{\mathrm{T}}(t)(I_N \otimes L)e(t) + ce^{\mathrm{T}}(t)(G^s \otimes \Gamma)e(t) - c\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)d\Gamma e_i(t) \\ &= e^{\mathrm{T}}(t)(I_N \otimes L + c(G^s - dI_N) \otimes \Gamma)e(t) \leq 0, \end{aligned} \tag{29}$$

$$\begin{aligned} W_2(t) &= \sum_{i=1}^{N} \|e_i(t)\|_2(2h_{\max}) - \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)(\eta(t)\overline{\mathrm{sign}}(e_i(t))) \\ &\leq \eta(t)\biggl(\sum_{i=1}^{N} \|e_i(t)\|_2 - \sum_{i=1}^{N} |e_i(t)|\biggr) \\ &\leq 0 \end{aligned} \tag{30}$$

and

$$\begin{aligned} W_3(t) &= -k\biggl(\sum_{i=1}^{N} |e_i(t)|^{1+\beta} + \sum_{i=1}^{N} \Bigl(\frac{c}{q_i}\Bigr)^{\frac{1+\beta}{2}} |d_i(t) - d|^{1+\beta}\biggr) \\ &\leq -k\biggl(\sum_{i=1}^{N} \| e_i(t) \|_2^2 + \sum_{i=1}^{N} \frac{c}{q_i}|d_i(t) - d|^2\biggr)^{\frac{1+\beta}{2}} \\ &= -2^{\frac{1+\beta}{2}} kV(t)^{\frac{1+\beta}{2}}. \end{aligned} \tag{31}$$

Submitting (29)–(31) into (28), we can get

$$\dot{V}(t) \leq W_1(t) + W_2(t) + W_3(t) \leq -2^{\frac{1+\beta}{2}} kV(t)^{\frac{1+\beta}{2}}.$$

By Lemma 3.1, $V(t)$ converges to zero in a finite time, and the finite time t^* is given by

$$t^* = \frac{V(0)^{1-0.5(1+\beta)}}{2^{\frac{1+\beta}{2}} k(1 - 0.5(1 + \beta))} = \frac{V(0)^{\frac{1-\beta}{2}}}{2^{\frac{\beta-1}{2}} k(1 - \beta))}.$$

Hence, the error vector $e_i(t), i = 1, 2, \cdots, N$, can converge to 0 within t^*. Consequently, under the controllers (26), the complex network (1) is synchronized in the finite time t^*. The proof is completed. ▌

5 Numerical Example

Consider a five-pendulum coupled nonlinear system[37] with linearly and diffusively coupling in which the dynamics of the ith node is described by

$$\dot{x}_i(t) = f_i(t, x_i) + h_i(t, x_i) + c\sum_{j=1}^{5} G_{ij}\Gamma x_j(t), \quad i = 1, 2, \cdots, 5 \tag{32}$$

with initial values $X(0) = (x_1^{\mathrm{T}}(0), x_2^{\mathrm{T}}(0), \cdots, x_5^{\mathrm{T}}(0))^{\mathrm{T}} = (0.6\ 0.9\ 1.2\ {-0.3}\ {-0.9}\ 0\ {-1.2}\ 1.5\ 1.05\ {-}$ $1.2)^{\mathrm{T}}$. The inner-coupling matrix Γ and the Laplacian matrix G are

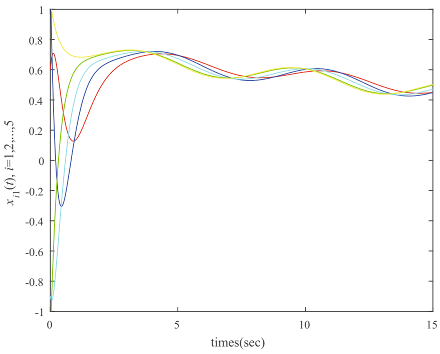
$$\varGamma = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Its activation function $f_i(t, x_i(t)) = (x_{i2}, -qx_{i2} - r_i \sin(x_{i1}))^{\mathrm{T}}$ and its uncertain disturbance $h_i(t, x_i) = (0, \phi_i(t))^{\mathrm{T}}$, where $q = 3.15$, $r_i = 0.02$ for $i = 1, 2, \cdots, 5$, and $(\phi_1, \phi_2, \cdots, \phi_5)^{\mathrm{T}} = (0.1 \sin(t), -0.15 \sin(t), -0.2 \sin(t), 0.15 \sin(t), 0.2 \sin(t))^{\mathrm{T}}$. The drive dynamical node is

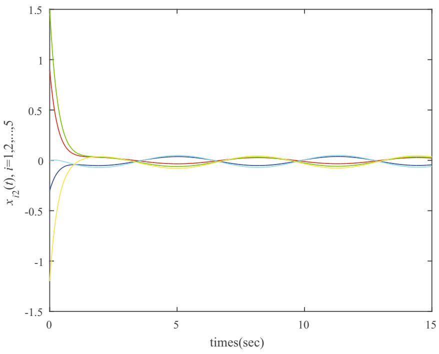
$$\dot{x}_0(t) = f_0(t, x_i) + h_0(t, x_i)$$

with initial values $x_0(0) = (1, 1)^{\mathrm{T}}$, where $f_0(t, x_0) = (x_{02}, -qx_{02})^{\mathrm{T}}, h_0(t, x_0) = (0, 0)^{\mathrm{T}}$, and the coupling strength $c = 2.0$.

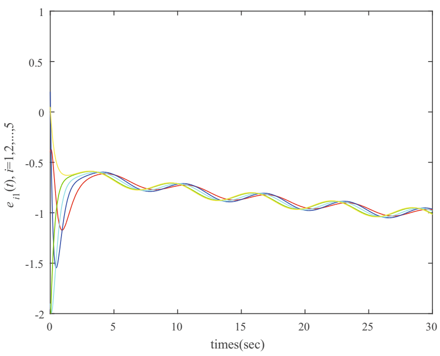
As a comparison, the state response trajectory and the state error response track of the heterogeneous complex dynamic network (32) without control are given. It can be seen from Figure 1 that before the application of the control the synchronization errors of each node do not tend to 0, and then the states of nodes have not been synchronized.



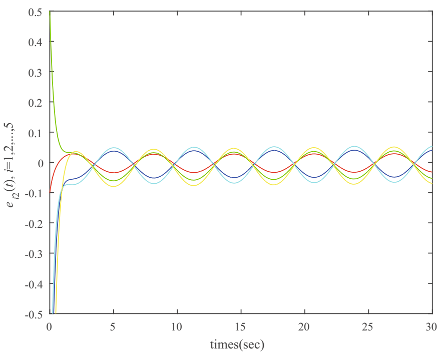
(a) State response x_{i1}



(b) State response x_{i2}



(c) Synchronization error e_{i1}

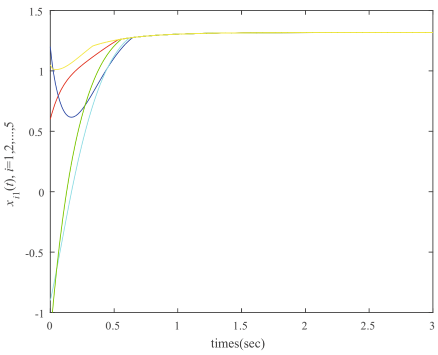


(d) Synchronization error e_{i2}

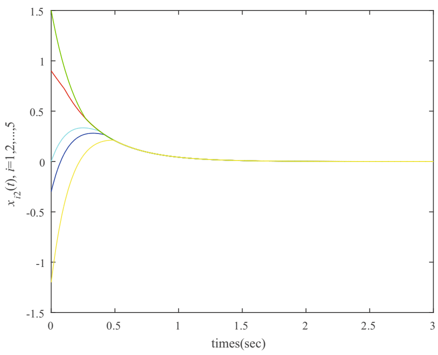
Figure 1 State responses and synchronization error without control

Let $L = I_2$, $\mu(t) = 0.1$ and $h_{\max} = 0.5$, which implies that Assumption 2.1, Assumption 2.2 and Assumption 2.3 hold since $\|f_i(t, x) - f_0(t, x)\| = \|(0, -r_i \sin(x_1)\| = r_i \leq 0.1$. With $D = \mathrm{diag}(2, 2, 2, 2, 2)$, it is known that $\lambda_{\max}(I_N \otimes L + c(G^s \otimes \Gamma) - D \otimes I_n) = -0.3592$, which indicates that (9) can be satisfied. According to Theorem 3.4 with $\eta(t) = \mu(t) + h_{\max} = 0.6, k = 1, \beta = 0.6$, by use of (6), (32) can be synchronized to the drive node (33). From Figure 2, it can be seen that the synchronization error of each node is reduced to 0 within $t^* = 4.3766$, and the node state is synchronized within $t^* = 4.3766$. In fact, the time of the numerical simulation actually synchronizes is 0.6090 seconds.

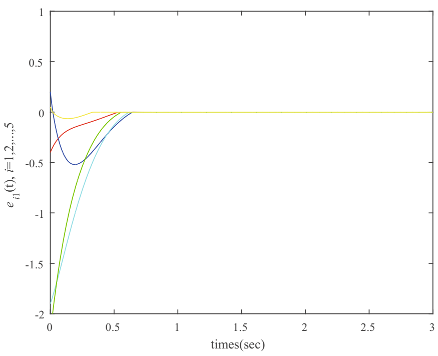
Furthermore, we use (26) with $d = 5$ and the initial values $X(0) = [x_1^{\mathrm{T}}(0), X_2^{\mathrm{T}}(0), x_3^{\mathrm{T}}(0), x_4^{\mathrm{T}}(0), x_5^{\mathrm{T}}(0)]^{\mathrm{T}} = [2\ 3\ 4\ -1\ -3\ 0\ -4\ 5\ 3.5\ -4]^{\mathrm{T}}$, $x_0(0) = [1\ 1]^{\mathrm{T}}$. From Theorem 4.1 and (27), it can be seen that (32) can be synchronized within the finite time $t^* = 10.4051$ and the actual synchronization time is 0.1074 seconds, which is illustrated by Figure 3.



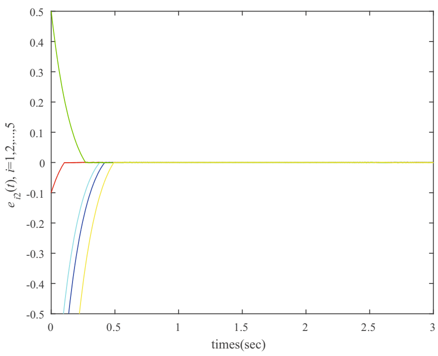
(a) State response x_{i1}



(b) State response x_{i2}

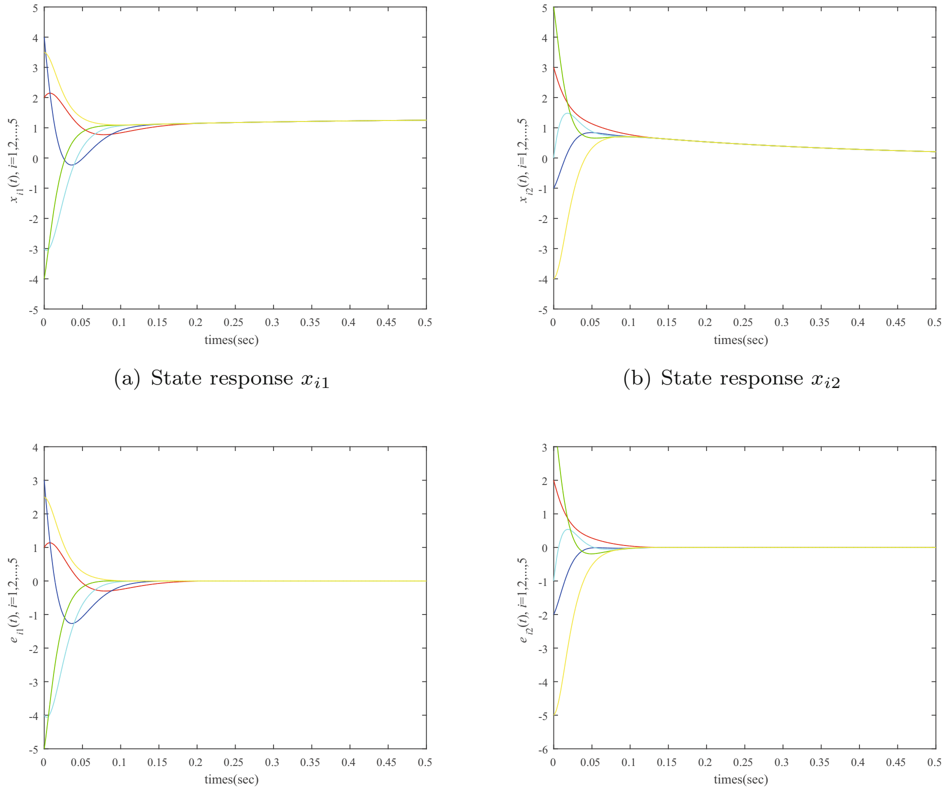


(c) Synchronization error e_{i1}



(d) Synchronization error e_{i2}

Figure 2 Finite-time synchronization



(a) State response x_{i1} (b) State response x_{i2}

(c) Synchronization error e_{i1} (d) Synchronization error e_{i2}

Figure 3 Adaptive finite-time synchronization

6 Conclusions

This paper has discussed the finite-time synchronization for a class of dynamical complex network with nonidentical nodes and uncertain disturbance, some sufficient conditions have been proposed by using the Lyapunov function method and the linear matrix inequality technique, and they have been applied to the case of identical nodes and the one without uncertain disturbance. Also the adaptive finite-time synchronization has been studied. Future work can focus on the fixed-time synchronization for complex networks with nonidentical nodes and uncertain disturbance.

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