Set Theory and Proofs for Engineering Education

Martha Pieper[§] and Shih-Feng Chou[†]

Author Affiliation:
Department of Mechanical Engineering
The University of Texas at Tyler
3900 University Blvd, Tyler, TX, 75799, USA
[§]Presenting author: mpieper@patriots.uttyler.edu
[†]Corresponding author: schou@uttyler.edu

Abstract

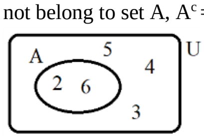
Through evaluations of learning objectives on several Engineering courses, the majority of students at some point will struggle with demonstration of proof of a principle in their homework assignments, quizzes, and exams. An early introduction of "Set Theory and Proofs" to engineering students can enrich their intuition and ability to solve comprehensive problems. As illustrated in this paper, set theory can be recognized by students as a simple and unnecessary topic. However, the understanding of principles in set theory and its derived concepts are essential to engineering students so they can improve their problem-solving skills when approaching a more complex problem using mathematics. Set Theory is a vast field of study which includes: Operations and algebra with sets, power sets, product sets, relations, functions, quantifiers, family of sets, index sets, just to name a few [1]. At The University of Texas at Tyler, the authors experienced set theory embedded in the learning objectives of Manufacturing Systems (MENG 5318) course offered by the Mechanical Engineering Department to its graduate students. In Fall 2016 and 2017, most of the students in the class failed to apply some of the principles in set theory. Overall, set theory is an important topic to engineering students where an understanding of the principal will ensure the success in completing advanced level courses.

1. Introduction

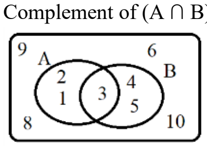
A set is any collection of define, distinguishable objects. These objects are called the elements or member of a set, e.g.
A = {a, b, c, 2, 3} and B = {1, 2, 3}. Union (U), intersection (∩), and subtraction (-) are the basic operation concepts in set theory [2].
A U B = {a, b, c, 1, 2, 3}   * AUB: the union of set A and set B
A ∩ B = {2, 3} * A∩B: the intersection of set A and set B
A – B = {a, b, c}   B – A = {1}   * A - B: set A minus set B
b is element of A: $b \in A$
1 is element of B: $1 \in B$
1 is not element of A: $1 \notin A$
a is not element of B: $a \notin B$

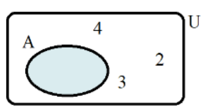
2. A conditional, a universal set (U), a subset (⊆), a complement (^{c}) of a set, and an empty set (∅)[3].

A conditional, e.g. If I don't water my plant, then my plant will die.
M = {a, b, c}   and   N = {a, b, c}
We have that, M = N
If M = N, then $M \subseteq N$ and $N \subseteq M$ and
If $M \subseteq N$ and $N \subseteq M$, then M = N
Universal set: U = {2, 3, 4,5,6} Set A = {2, 6} Set A is subset of set U: $A \subseteq U$ The complement of set A: all the objects that do not belong to set A, A^c = {3,4,5}

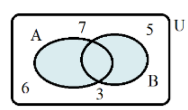


Complement of (A U B): $(A \cup B)^c$ = {6, 8, 9, 10}, the
Complement of (A ∩ B): $(A \cap B)^c$ = {1, 2, 4, 5, 6, 8, 9, 10}

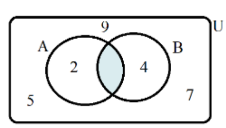




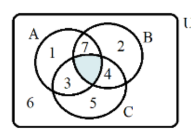
Set A is an empty set: A=Ø



A U B = Ø



A ∩ B = Ø



A ∩ B ∩ C = Ø

3. Conditionals

3.1 Understanding conditionals, negation (~), the concepts "*or* (∨)", "*and* (^)" and subsets in set theory [4].

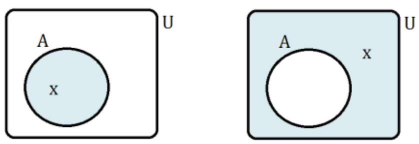
Let $x \in A \cap B$

If $x \in A \cap B$, then $x \in A$ *and* $x \in B$. See Fig. 1(a)

Let $x \in A \cup B$, implies that $x \in A$ *or* $x \in B$. See Fig. 1(b)

CASE 1: $x \in A$ If $x \in A$, then $x \notin B$. See Fig. 1(c)

CASE 2: $x \in B$ If $x \in B$, then $x \notin A$. See Fig. 1(d)



Let $x \in A$ If $x \in A$, then $x \notin A^c$

Let $x \in A^c$ If $x \in A^c$, then $x \notin A$. See figures above

Let $x \in A - B$

If $x \in A - B$, then $x \in A$, and $x \notin B$. See Fig. 2(a)

Let $x \in B - A$

If $x \in B - A$, then $x \in B$, and $x \notin A$. See Fig. 2(b)

Let $x \in A$

If $x \in A$, then $x \in A \cup B$. See Fig. 2(c)

Let $x \in B$

If $x \in B$, then $x \in (A \cup B)$. See Fig. 2(d)

Let $x \in (A \cap B)$

If $x \in (A \cap B)$, then $x \in A$, *and* $x \in B$. See Fig. 2(e)

Negation: ~

$\sim (x \in A \cup B)$ *x is not element of (A U B). See Fig. 4(a)

$x \in (AUB)^c$ *x is element of the complement of AUB. See Fig. 4(b)

$x \in (A \cup B)^c \Rightarrow x \notin (AUB) \Rightarrow \sim (x \in A \cup B)$ It reads, x is element of the complement of AUB implies x is not element of AUB implies the negation of $x \in (AUB)$.

$\sim (x \in A \cup B)$ implies $\sim (x \in A \vee x \in B)$ implies $(x \notin A \wedge x \notin B)$ *x is not element A *and* x is not element of B

Thus, $\sim (x \in A \cup B) \Rightarrow x \notin (A \cup B)$

If $x \notin (AUB)$, then $x \in (A \cup B)^c$

$\sim (x \in A \cap B)$ * x is not element of (A ∩ B)

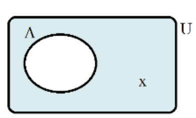
Implies, $\sim (x \in A \wedge x \in B)$ implies, $(x \notin A \vee x \notin B)$ *x is not element A *or* x is not element of B

$\sim (x \in A \cap B) \Rightarrow x \notin (A \cap B)$. See Fig. 4(c)

If $x \notin (A \cap B)$, then $x \in (A \cap B)^c$

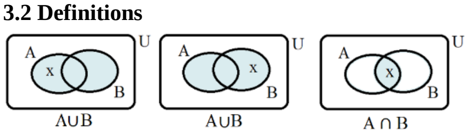
Knowing the concepts "or (∨)", "and (^)" in set theory is important, and it can be very easy to get confuse with them when doing proofs.

x is not element A, implies $x \in A^c$, implies x is element of the complement of A: $\sim (x \in A) \Rightarrow x \in A^c \Rightarrow x \notin A$. If $x \notin A$, then $x \in A^c$



$B \subseteq A$ *B is subset of A. Let $x \in B$. If $x \in B$, then $x \in A$, see Fig. 3(a). Now, let $x \in A$. If $x \in A$ it does not imply that $x \in B$. See Fig. 3(b)

3.2 Definitions



$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$, it reads: A U B = {x is element of the universal set, such that x is element of set A *or* x is element of set B}. See AUB figures above.

$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$, it reads A ∩ B = {x is element of the universal set such that x is element of A and x is element of B}. See A ∩ B figure above.

Let $x \in (A \cup B \cup C)$ implies that $x \in A$ *or* $x \in B$ *or* $x \in C$. See Fig. 6(a)

Let $x \in A \cup (B \cap C)$ implies $x \in A$ *or* $x \in (B \cap C)$. See Fig.6(b)

Let $x \in (A \cup B) \cap (A \cup C)$ implies $x \in (AUB)$ *or* $x \in (A \cap C)$. See Fig.6(c)

4. Proofs[1]

*Prove that, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Proof:

Let $x \in A \cup (B \cap C)$, implies that $x \in A$ *or* $x \in B \cap C$

CASE 1: $x \in A$

If $x \in A$, then $x \in (A \cup B)$ *and* $x \in (A \cup C)$

Thus, $x \in (A \cup B) \cap (A \cup C)$

CASE 2: $x \in B \cap C$

If $x \in B \cap C$, then $x \in B$ *and* $x \in C$

Since $x \in B$ *and* $x \in C$, $x \in A \cup B$ *and* $x \in A \cup C$

Thus, $x \in (A \cup B) \cap (A \cup C)$

In either case $x \in (A \cup B) \cap (A \cup C)$

Therefore, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

*Prove that $A - B = A \cap B^c$ See fig. 5

Step 1. Prove that $A - B \subseteq A \cap B^c$

Proof:

Let $x \in A - B$

If $x \in A - B$, then $x \in A$ *and* $x \notin B$

If $x \notin B$, then $x \in B^c$

We have that $x \in A$ and $x \in B^c$

$\Rightarrow x \in A \cap B^c$

Thus, $A - B \subseteq A \cap B^c$

Step 2. Prove that $A \cap B^c \subseteq A - B$

Proof:

Let $x \in A \cap B^c$

If $x \in A \cap B^c$, then $x \in A$ *and* $x \in B^c$

If $x \in B^c$, then $x \notin B$

We have that $x \in A$ and $x \notin B$

$\Rightarrow x \in A - B$

Thus, $A \cap B^c \subseteq A - B$

Therefore, since $A - B \subseteq A \cap B^c$ and $A \cap B^c \subseteq A - B$,

$A - B = A \cap B^c$

5. Summary

Set theory is an interesting subject and an essential tool for doing proofs. It is encouraged that students need to be introduced to set theory early in their mathematical education. Set theory is a wide field of study, and its introduction to students should be started with the basic principles.

Acknowledgement

This work was performed as "classroom projects" funded and supported by the University of Texas at Tyler.

References

[1] Pieper, M. (2016). Set theory and proofs. The Hammerhead Shark Publisher House. ISBN 978-0984615339.
[2] Fletcher, P. and Patty, C.W. (1995). Foundations of higher mathematics. Cengage Learning. IBSN 978-0534951665.
[3] Smullyan, R.M. and Fitting M. (2010). Set theory and the continuum problem. Dover Publications. IBSN 978-0486474847.
[4] Levy, A. (2002). Basic set theory. Dover Publications. IBSN 978-0486420790.

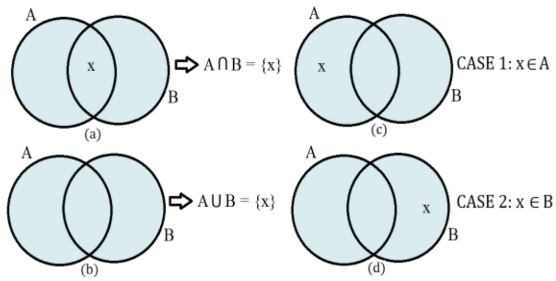


Fig. 1 (a), (b), (c), (d)

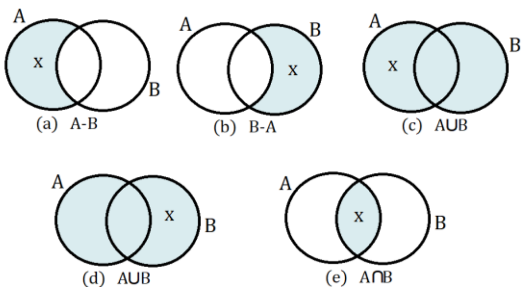


Fig. 2(a), (b), (c), (d)

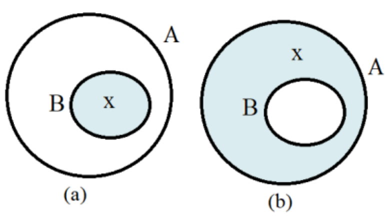


Fig. 3 (a), (b)

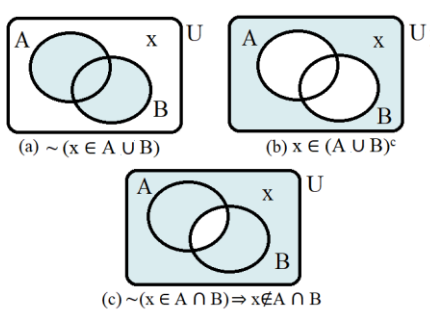


Fig. 4 (a), (b), (c)

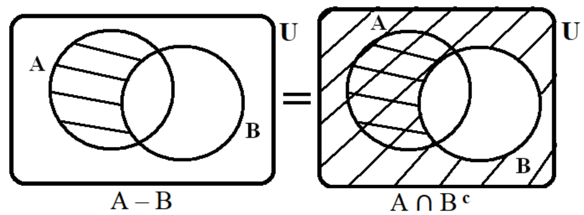


Fig. 5