# Disparate Savings Propensities and National Retirement Policy 

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The published version of this Working Paper may be found in the 1998 publication: Living with Defined Contribution Pensions.

# Living with Defined Contribution Pensions 

Remaking Responsibility for Retirement

Edited by<br>Olivia S. Mitchell and Sylvester J. Schieber

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## Chapter 12

## Disparate Savings Propensities and National Retirement Policy

## Richard A. Ippolito

In this chapter, I consider an underlying theme that is implicit in much of the discussion of national retirement policy - the disinclination of some individuals to save for retirement. Put differently, if all individuals had sufficient foresight and self-control to save for their own retirement periods, the need for a national retirement policy would be minimal. If some individuals do not save, however, they are destined to be impoverished in old age, a result that poses an important problem for society. In this chapter, I touch on some of the principles of $401(\mathrm{k})$ plans, which is the main focus of several other chapters in the volume, but my main thrust is to explore why the tax treatment of savings is important, both to national income and consumption levels during old age.

In this context, the drift away from defined benefit plans portends a growing old age problem. Historically, most pension coverage took the form of defined benefit pensions. In these plans, typically, covered workers have no choice but to earn accruals in the pension. Furthermore, these plans typically pay benefits in the form of annuities, diminishing the chances that recipients can spend down assets prior to their old age. In $401(\mathrm{k})$ plans, however, even if the firm offers workers the opportunity to participate in the plan, and though it may offer some substantial rewards for their participation (in the form of matching employee contributions), a significant portion of workers will not contribute to the pension. And these plans typically pay benefits in the form of lump sums, increasing the chances that some recipients will spend their pension monies long before they become old.

In reality, firms are not motivated to be inclusive in their pension coverage, and may have incentives to exclude workers from pensions who otherwise do not care to participate. These incentives are magnified if
the qualities of being a "spender" are correlated with attributes that make a lower-quality worker.

The trend toward $401(\mathrm{k})$ plans redoubles the need for a national retirement policy that encourages participation by all individuals in some kind of savings arrangement for old age. I do not provide specific policy recommendations, except for the sake of illustration. Rather, I concentrate on the principles that characterize an efficient solution to the "nonsaver problem."

## The Nature of the Problem

I start by separating the population into two groups, low discounters and high discounters. High discounters downplay the future. They prefer immediate gratification. Low discounters attach more value to future outcomes and thus are more likely to choose options with long-term payoffs. If two otherwise identical people are offered $\$ 100$ today or $\$ 100$ $+x$ one year in the future, the "high discounter" requires a higher value of $x$ to choose the delayed payment compared to a "low discounter." ${ }^{1}$ The differences in internal discount rates across the population can have some profound effects on the well-being of a society.

Low discounters, while young, anticipate their standard of living when they reach old age and thus save sufficient amounts to support old age consumption. High discounters attach less importance to their economic condition far in the future and thus are inclined to devote their earnings to support higher current consumption. Other things being the same, low discounters have high savings propensities; high discounters have low savings propensities. ${ }^{2}$

A conflict is predictable. High discounters will be impoverished in old age, and low discounters will be wealthy. As long as society supports old age consumption at some reasonable level, it is rational for low discounters to anticipate the need to "share" their retirement savings with high discounters. This prospect is akin to a tax on savings that diminishes low discounters' incentive to accumulate wealth for retirement.
As a general proposition, high discounters naturally gravitate toward firms that award compensation immediately, and thus tend to avoid pension firms. Unless information is perfect and workers have job choices among a large number of alternatives, however, some high discounters enter firms that offer pensions, creating incidental pension coverage of some high discounters.

To the extent that incidental coverage has characterized the pension market, it is expected to wane owing to the growing share of pension coverage by defined contribution plans (Gustman and Steinmeier 1992). These plans embody incentives to encourage high discounters to volun-
tarily quit pension firms and/or permit them to voluntarily exclude themselves from $401(\mathrm{k})$ savings (Ippolito 1998). These outcomes may be desirable from the perspective of firms trying to maximize productivity and of high discounters trying to maximize the present value of their own welfare. In the long run, however, the systemic exclusion of high discounters from pension coverage will lead to a growing portion of the population that saves "too little" for their retirement.

As long as high discounters can vote in democratic elections, and governments have the power to redistribute income, high discounters' disinclination to save poses a potential economic burden on low discounters in the long run. Unless public policies (which themselves are influenced by a democratic vote) can resolve the natural conflict between high and low discounters, the result can degenerate to an equilibrium characterized by a lower standard of living for high and low discounters, particularly in old age.

I first consider the high discounter from the corporate perspective and show that firms, individually, have little incentive to include high discounters in pension plans. I then consider the problem from the public perspective. Here, I consider how low discounters as a group can solve the high discounter problem without importantly reducing output and savings in the economy. The general idea is that instead of accepting the prospects of a tax on their own wealth when old, low discounters are better off subsidizing high discounters' savings, thereby diminishing high discounters' incentive to hold up low discounters when they reach old age.

## The Corporate Perspective

## Discount Rates and Worker Quality

Managers operate firms to maximize returns for investors. Unless pensions can increase the firm's market value, for example, by increasing labor productivity or reducing turnover and the like, ${ }^{3}$ each firm individually has no stake in its workers' savings behavior, including their pension participation. Firms, however, may have an economic interest in workers' underlying internal discount rates. If workers greatly discount the future, they may be inclined to behave in the workplace in ways that reduce productivity. In this sense, firms may have an interest in workers' overt savings behavior because it is a signal of workers' underlying internal discount rates. ${ }^{4}$

It is natural to think that individuals with low discount rates are more productive. Just as low discounters attach more value to the long-term benefits of financial savings, so do they attach more value to investments
in their human capital. These investments reveal themselves in a large array of decisions-including many workplace decisions. The behavior of individuals with low discount rates is influenced by the long-term implications of their current work performance.

For example, low discounters are less likely to take a day off on a whim or quit in a "huff," instead valuing the long-term implications of their reputation for reliability and thus economizing on firms' expenditures on duplication and hiring. They are less likely to mistreat machines and equipment, because they recognize the long-term benefit of being labeled a "low-cost" employee. They are less likely to value the short-term gains from shirking over the long-term consequences of getting caught and are more likely to be motivated to work hard to gain the benefits of promotions. In all these ways, low discounters economize on firms' monitoring costs.

In contrast, high discounters are influenced disproportionately by benefits realized in the short term. The firm either must expend resources to discipline their behavior or accept the implications of high discounters' short-term perspective. In either case, high discounters are less valuable as workers compared with low discounters. ${ }^{5}$

If values of marginal product are importantly related to internal discount rates, competition will drive firms to identify low discounters. The identification, however, is not trivially accomplished because information in the labor market is imperfect. While firms can observe some telltale signs of job applicants' discount rates, notably education attainment or other training, much of the variation in internal discount rates is unobservable. ${ }^{6}$

Firms do not all attach the same value to employing low discounters. Firms that can monitor output cheaply, and/or do not face significant costs from absenteeism, quitting, and the like, will offer compensation schemes more attractive to high discounters. Firms that attach higher value to employing low discounters will use compensation packages that put more emphasis on deferred pay. The competitive process ensures that low discounters receive compensation commensurate with their higher value of marginal product. But their compensation must be reduced by the costs incurred by firms to identify low discounters. Firms that find the most efficient mechanisms to sort low discounters attract a higher-quality work force and earn higher profits.

Presumably, if it expends sufficient monitoring resources, the firm can align pay and productivity across the spectrum of discount rates characterizing workers in its employ. I consider a more interesting issue: whether the firm can find efficient sorting devices that allocate high and low discounters to their best uses without incurring monitoring costs.

More specifically, I pursue one part of the compensation package that is naturally suited to low discounters, namely, the pension plan.

## Selecting in Low Discounters

If information is perfect and workers can choose jobs from a large array of compensation packages, high discounters will not take jobs in firms that offer pensions. When job search is costly, some high discounters inadvertently enter pension firms.

The problem of workers entering the "wrong" firm is not easily dismissed because it is endemic to workers with high discount rates. Job shopping is inherently an investment activity. Search costs incurred early in the career result in the long-term benefits of finding the "right" job. Low discounters should thus invest more in the search process and have a greater likelihood of selecting a firm that values their long-term outlook (Lippman and McCall 1976). High discounters presumably are less careful job shoppers and thus more frequently take jobs at firms with production functions designed for low discounters.

The entry of some high discounters poses a problem for firms designed for low discounters. The firm can address the high discounter problem in several ways. It can accept the inefficiency of having some high discounters in the firm, or incur monitoring costs to identify high discounters and transactions costs to fire them, or erect a mechanism to encourage high discounters to leave the firm voluntarily early in their tenure. Plain defined contribution plans are ideally suited to perform the sorting out function.

## Sorting Out High Discounters: Defined Contribution Plans

Consider a defined contribution plan in its simplest form. The firm contributes a fixed percentage of pay, denoted by $s$, into each worker's account. Vesting is immediate. ${ }^{7}$ Upon quitting, the worker takes a lump sum (it is eligible for rollover into an individual retirement account ${ }^{8}$ ); otherwise, he has no access to his account.

Suppose that workers have identical attributes except for their internal discount rates, and that these rates have a dichotomous distribution. Low discounters have zero discount rates and a desired savings rate, $s$. High discounters have an infinite discount rate, and a zero desired savings rate.

Assume that firms know the overall proportions of high and low discounters in the labor market, but do not know individual workers' discount rates. They simply hire all workers who apply for a job and rely on sorting devices to influence the composition of their workforces. Workers
know their own discount rates, but have imperfect information about the labor market.

The firm wants to employ low discounters because they economize on monitoring costs. ${ }^{9}$ Owing to their lackluster job search, some high discounters enter the no-monitoring firm. I characterize their gain from entering as $j_{H}$ which is positive for some high discounters. ${ }^{10}$

## The Economics of Quitting a Defined-Contribution Firm

Consider the efficacy of a defined contribution pension in correcting hiring errors. At the end of period 1 , workers make a decision whether to quit the firm. Assuming they have the same knowledge of the labor market as they did at the beginning of period 1 , the economics of joining and staying are the same, with one important difference. Upon quitting, workers obtain the lump sum amount s after period 1: If they stay, $s$ is payable in some future period. For low discounters, the value of $s$ is the same whether or not it remains in the pension plan, and thus the gains from staying are the same as for joining. For high discounters, the perceived net gains from staying, $g_{H}$, are lower by the added value of obtaining the available pension amount immediately: $g_{H}=j_{H}-s$.

An economic function for defined contribution plans emerges. The lump sum they provide upon quitting encourages high discounters to select themselves for early departure from the firm. In effect, the plan continually sifts the workforce for high discounters; thereby improving the composition of the firm's workforce over time. High discounters with the smallest values of $j_{H}$ quit after period 1 . At the end of the next period, the available lump sum is $2 s$, and after the third period, $3 s$, and so on. Gradually, most of the high discounters find it economic to depart the firm.

## Paying Less to High Discounters: 401 (k) Plans

In plain defined contribution plans, the firm periodically contributes a fixed amount, often a percentage of pay, to each worker's account. In $401(\mathrm{k})$ versions of these plans, the firm might make an unconditional contribution. More often, workers choose some voluntary savings rate in the plan, and the firm often matches these contributions on an $m$-to-one basis. It is easy to understand the appeal of voluntary contributions in these plans: they facilitate a more efficient pattern of savings across workers. Explaining the matching mechanism is more problematic.

An oft-cited candidate to explain matching is the Internal Revenue Code. The Code specifies "discrimination rules" that regulate the size of contributions of higher-paid workers to the 401 (k) plan compared to
lower-paid workers. In this explanation, matching elicits more contributions from lower-paid workers, which permits more tax-favored contributions by higher-paid workers. I have shown elsewhere, however, that matching generates potential tax benefits that are inconsequentially small, and impose costs on the firm by misaligning wages and values of marginal product across its workforce (Ippolito 1998).

One could alternatively postulate that firms have a stake in the timely retirement of older workers. Even if they do not use defined benefit plans, presumably firms would attain earlier retirement patterns if its workers had access to funds sufficient to finance their earlier retirement. In this sense, the match could be interpreted as an incentive to encourage more pension savings in its workforce. But if this goal were important, the firm could use a simple defined contribution plan, thereby requiring some pension savings for all workers.

The economics of matching is captured by a simple observation: Firms pay a premium to workers solely on the basis of their decisions to contribute to 401 (k) plans. In the context of a sorting theory, a $401(\mathrm{k})$ plan with matching encourages workers to align their pay and productivity without imposing monitoring costs on the firm. Instead of encouraging high discounters to depart, $401(\mathrm{k})$ matching schemes simply pay high discounters less than low discounters.

The pay difference is not necessarily limited to the match amounts. Once the firm infers workers' internal discount rates from observing 401 (k) contribution rates, it can use this information in selecting workers for more important jobs. In this way, the implications of sorting go beyond the wage differences established from the matching amounts.

## Summary

Firms ought to be indifferent to pension participation unless it affects their market value. If premature quitting and late retirement ages reduce productivity in the firm, pension participation (especially in defined benefit plans) adds value to the firm. If firms use production functions designed for low discounters, but are not necessarily concerned with long tenure, firms may have incentives to effectively exclude high discounters from pension savings vehicles, particularly in 401 (k) plans. In this sense, the growth of defined contribution plans in place of defined benefit plans suggests lower pension coverage rates for high discounters.

If pensions do not affect performance, the only reason firms offer pensions is to confer tax advantages to workers. ${ }^{11}$ In a pure tax model, firms are disinterested savings agents. Firms individually have no inherent interest in their workers' living standard during their retirement. Similarly, low discounters individually have no stake in programs that
force high discounters to engage in long-term savings. Each low discounter wants flexibility to attain his desired pension savings even if it means zero participation by high discounters.

From a broader perspective, firms as a whole-as corporate taxpay-ers-and low discounters as a whole-as income tax payers-have a stake in the savings behavior of high discounters. If high discounters do not save for retirement then, as long as society provides public monies to support high discounters' retirement, taxpayers face the prospects of transferring part of their wealth to finance high discounters' retirement consumption. It is inevitable that, in a free market, high discounters will pose problems for society in their old age. In this sense, there is a need for some public savings mechanism that is inclusive; otherwise, the incentives for low discounters to save may also wane, as I will show below.

In the next section, I develop a simple model of a democracy with a government function that can be used to redistribute income. The model, though highly simplified, illustrates some of the implications of high and low discounters coexisting in the same society, and illustrates how cooperative solutions can be found to diminish the adverse consequences that arise when high discounters do not save for retirement.

## The Public Perspective

If an economy is comprised of either all high discounters or all low discounters, the results are predictable. Economies dominated by high discounters will have low levels of investment in human and physical capital, and thus will be poor. Economies dominated by low discounters will have high levels of investment in human and physical capital, and thus will be rich. ${ }^{12}$

The interesting question is whether a democratic society with a mix of high and low discounters can become rich. Individuals with disparate internal discount rates accumulate different wealth positions. A government can perform a valuable function in this society by establishing property rights and providing police and court functions to ensure stability of ownership. At the same time, through enactment of comprehensive income or wealth taxes, the government also can "legitimately" tax private property, or some stream of income derived from property. If high discounters can gain more votes than low discounters, a government can be transformed from a protector of property rights to a facilitator of transfers from voters who have wealth to those who do not.

The conundrum for this society is simply put: The incentive to save and invest must be sufficient to encourage low discounters to follow their natural inclinations to save; yet, the unequal distribution of wealth that inevitably ensues must be sustainable. In this section, I pursue the nature
of contracts between high and low discounters that can provide for a selfenforcing equilibrium level of savings and transfers.

## Natural Outcomes

Consider a two-person, two-period model. Both individuals have identical attributes except for their internal discount rates. ${ }^{13}$ The low discounter has a zero discount rate. The high discounter has some positive discount rate, $r \gg 0$.

The individuals are born to zero wealth, have no children, ${ }^{14}$ and produce output worth $\$ 1$ in period 1 when they are young. Owing to a natural deterioration of productive capacity, both can produce zero in period 2 when they are old. Death occurs at the end of period 2. Both individuals have full information. The only reason to save is to support retirement consumption. In general, savings can generate productivity improvements, but initially, I assume that the only benefit of savings is smoothing lifetime consumption; thus, savings are goods set aside in period 1 for consumption in period 2.

I assume that utility is a $\log$ function of consumption that is identical and separable across individuals and periods. Unless some output is saved in period 1 , consumption and utility in period 2 is zero. When the individuals live separately, each chooses a savings rate S , that maximizes discounted lifetime utility:

$$
\begin{equation*}
U=\log \left[1+C_{j 1}\right]+\log \left[1+C_{j 2}\right] /\left(1+r_{j}\right), j=L, H, \tag{1}
\end{equation*}
$$

subject to the constraints:

$$
\begin{equation*}
C_{i 1}=1-S_{j} ; C_{i 2}=S j, \text { and } r_{t}=0 ; r_{H}=r \gg 0 . \tag{2}
\end{equation*}
$$

where $C_{j t}$ is consumption by the $j$ th individual in period $t$, and the subscripts $L$ and $H$ denote low and high discounter.

The optimal savings rates for the low and high discounter, $S_{L}$ and $S_{H}$ respectively, are:

$$
\begin{array}{ll}
S_{L}=1 / 2 & \Rightarrow C_{L 1}=C_{L 2}, \quad \text { and }  \tag{3}\\
S_{H}=[1-r] /[2+r] \geq 0, & \Rightarrow C_{H 1}>C_{H 2} \geq 0 .
\end{array}
$$

The low discounter equalizes consumption in both periods, and thus has a 50 percent savings rate. The high discounter consumes output disproportionately in period 1: If his discount rate is at least 100 percent ( $r=1$ ), he consumes all his income in period one and thus saves nothing. In this case, consumption in period 2 is zero, and thus, the individual is impoverished in old age.

## Mixing High and Low Discounters

I now consider savings decisions when high and low discounters are mixed in a democracy. For simplicity, I assume that production is unaffected by commingling workers in the same economy. I assume that the only function of the government is to assign and protect property rights, and that this function is effected at zero cost. The assignment of property rights depends on groups' abilities to influence voting outcomes.

I employ a model of political economy that mimics the essence of a government transfer mechanism in a democratic society, but which is free of unnecessary complexities. The model reflects the principle that the majority is favored to win each vote but that political outcomes are uncertain due to other factors (for example, nonlinear utility functions, asymmetric information, different production functions to influence voting, and so on). Thus, minorities can win voting outcomes, albeit with a lower probability.

More specifically, I use a simple stochastic voting outcome model. The "government" is an urn filled with $p$ balls marked $H$ and $1-p$ balls marked $L$. Conflicts between the high and low discounter that cannot be resolved to their mutual self-interest result in a draw from the urn. Either individual can make the draw, but there can be only one draw per period. If an H -ball is drawn, all wealth existing in that period goes to the high discounter. The all-or-nothing outcome simplifies the problem but is not critical to the basic thrust of the model. ${ }^{15}$ If an $L$-ball is drawn, all wealth goes to the low discounter. The outcome is enforced at zero cost. ${ }^{16}$

## First-Round Effects

Savings patterns that prevail when the high and low discounter live in separate societies may not be sustainable in a democracy. To illustrate, assume that the high discounter's discount rate is sufficiently high so that his savings rate is zero. ${ }^{17}$ Thus, the high discounter consumes all his wealth in period 1 , and faces dire prospects in period 2.

In period 2, the high discounter has an incentive to try to take the low discounter's wealth, and thus to make a draw from the urn: he has nothing to lose, and some probability $p$ that he can take the low discounter's wealth.

The low discounter must decide whether to make an offer to the high discounter to dissuade him from playing the lottery. If the low discounter does nothing, he faces some chance (with probability p ) of having zero consumption in period 2. His alternative is to give some portion, $t$, of his wealth to the high discounter so as to dissuade the high discounter from making a draw. The transfer, $t S_{t}$, must be sufficiently large so that the
high discounter's utility from certain consumption of this amount is higher than expected utility of attempting to take the low discounter's entire wealth:

$$
\begin{equation*}
\log \left[1+t S_{L}\right] \geq p \log \left[1+S_{L}\right], 0<t<1 . \tag{4}
\end{equation*}
$$

When condition (4) is satisfied, the high discounter has no incentive to risk losing the transfer amount, $t S$, in exchange for a chance, $p$, of obtaining the low discounter's cache $S_{L}$; and thus, the lottery is not used.

## Second-Round Effect

Knowing in period 1 that a hold-up is inevitable, the low discounter does not save the amount $S_{L}$. He views the hold-up in period 2 as a de facto tax on savings, where the tax rate is $t$. If the low discounter saves the amount $S_{L}$, he will consume the amount $(1-t) S_{L}$ in period 2 and transfer the amount $t S_{L}$ to support the high discounter's period-2 consumption. When the low discounter sets his savings rate in period one, he maximizes his utility in expression (2) subject to the following income constraints:

$$
\begin{equation*}
C_{l, 1}=1-S_{l} ; C_{l 2}=[1-t] S_{l} . \tag{5}
\end{equation*}
$$

The low discounter's optimal savings rate is either positive or zero depending on the tax rate:

$$
\begin{align*}
& S_{L^{*}}=1-1 /[2(1-t)]>0, \text { all } t<1 / 2, \text { and }  \tag{6}\\
& S_{L^{*}}{ }^{*} 0, \\
& \text { all } t \geq 1 / 2 .
\end{align*}
$$

If the de facto tax rate is zero ( $t=0$ ), the savings rate is 50 percent, which reflects the low discounter's natural preference. If the tax rate is positive, the low discounter's savings rate falls. If the tax rate is sufficiently high ( $t=$ $1 / 2$ in the illustration), the solution degenerates, so that savings are zero: the low discounter is better off by consuming all his wealth in period 1 . Thus, in this simple model, a necessary condition for positive savings is that the tax rate is not "too high." In addition, the condition in (4), which ensures a sustainable income distribution, must be satisfied. In this model, it turns out that there is no tax rate, $t$, and consequent savings rate, $S_{L}$, that satisfies (4): a period-2 hold-up attempt is inevitable. The low discounter therefore saves nothing, ${ }^{18}$ and both the high and low discounter are impoverished when old.

The solution is not general in the sense that a savings tax always leads to zero savings. The general result is that systems that depend on a tax on savings to redistribute income lead to lower savings levels. While the tax is designed to alleviate a savings deficiency for part of the population, in fact, it leverages the problem to the entire population: it exacerbates the
low savings problem. If instead income is redistributed by subsidizing the high discounter's savings, a cooperative solution can be found that is characterized by lower transfers, making the low and high discounter better off.

## Subsidizing the High Discounter's Savings

I now pursue a savings subsidy solution to the high discounter problem. In place of penalizing low discounters from following their natural inclination to save, high discounters are offered a subsidy to participate in savings. Low discounters still award a transfer to high discounters, but because high discounters save some portion of their earnings in response to the subsidy, the transfer is lower, and the distribution of income during retirement is more likely to be sustainable.

To keep the illustration simple, I consider a solution where low discounters subsidize the high discounters. In the model, these types of individuals are readily identified. In reality, discount rates are not observable, and so the savings subsidy must be offered to everyone, including low discounters. This complicates the solution, but the overall thrust of the outcome is the same. ${ }^{19}$

Suppose that for each dollar saved by the high discounter, the low discounter adds the amount $s$. The high discounter chooses a savings rate, $S_{H}^{o}$, that maximizes his lifetime utility in (1) subject to the constraints:

$$
\begin{equation*}
C_{H 1}=1-S_{H} \text {, and } C_{H 2}=S_{H}(1+s) . \tag{7}
\end{equation*}
$$

In general, $S_{H}$ is weakly positive in $s{ }^{20}$
The low discounter chooses his savings rate, $S_{L}^{o}$, so as to maximize lifetime utility in (1), subject to his new income constraints. ${ }^{21}$ The low discounter's optimal savings rate is:

$$
\begin{equation*}
S_{L}^{O}=1 / 2\left(1-s S_{H}^{O}\right) . \tag{8}
\end{equation*}
$$

## Equilibrium Conditions

Consider the equilibrium subsidy rate. The low discounter must set the subsidy rate at least at the minimum that guarantees a sustainable income distribution, say, $s^{\circ}$. In period 2, the high discounter must find it in his interest not to try to take the low discounter's wealth. ${ }^{22}$ Beyond this level, the equilibrium subsidy rate cannot be determined without putting more structure on the model. To avoid the complexities of gaming solutions, I simply assume that the low discounter can choose the subsidy rate so as to maximize his utility, subject to the period-2 stability condition. ${ }^{23}$ Hence,

Table 1 Comparing Outcomes When Savings Are Either Taxed or Subsidized

| Category | Consumption period I |  | Consumption period 2 |  | Savings |  | $\begin{aligned} & \text { Utility } \\ & \text { period } 2 \end{aligned}$ |  | $\begin{gathered} \text { Utility } \\ \text { period } 2 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LD | HD | LD | HD | LD | HD | LD | $H D$ | LD | $H D$ |
| Isolation | . 50 | 1.0 | . 50 | 0.0 | . 50 | 0.0 | . 81 | . 69 | . 40 | . 00 |
| Democracy: <br> Tax on savings (degenerate solution) | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 | . 69 | . 69 | . 36 | . 28 |
| Democracy: $s^{\sigma}=.5$ (savings subsidy solution) | . 44 | . 77 | . 44 | . 33 | . 44 | . 33 | . 73 | . 72 | . 36 | . 28 |

Note: The table shows the consumption rates, savings rates, and utility levels for period 1 (work period) and period 2 (retirement period) for low and high discounters. Illustrative solutions are based on the assumptions that there is one low discounter (LD) with a zero discount rate, one high discounter (HD) with a discount rate equal to 100 percent ( $r=1$ ), and a zero productive return on savings. The variable $s^{"}$ is the minimum level of the subsidy rate offered by low discounters on each dollar saved by high discounters that ensures a stable distribution of income.
the low discounter's optimal choice of subsidy rate also is the minimum necessary to stave off a period-2 hold-up, namely, $s^{0 .}{ }^{24}$

It turns out that when the high discounter's discount rate is 100 percent, the value of $s^{a}$ is 50 percent. In this solution, the high discounter voluntarily saves 22 percent of his income. He receives a subsidy from low discounters equal to one-half of this amount, so that his savings rate, gross of the subsidy, is .33 , which finances his period-2 consumption. Notably, the high discounter ends up with higher income (namely, 1.11) than the low discounter (namely, 89). Clearly, the high discounter is better off compared to living in isolation, particularly in "old age."

The low discounter is always worse off compared to living in isolation because some transfer is required to ensure sustainable property rights. In this sense, there is an inherent cost to being a low discounter. The low discounter is better off, however, offering to subsidize the high discounter's savings, compared to a solution that depends on the threat of a holdup in period 2. The gains to a cooperative solution are always positive in this sense, but the gains are especially large when the hold-up solution produces a degenerate equilibrium.

Table 1 summarizes these solutions for the simple case. In the appendix, I consider the results when high discounters have discount rates different than 100 percent, and when the numbers of high and low dis-
counters are unequal. The results confirm intuition that solutions with positive savings are easier to find when low discounters are more numerous than high discounters, and when high discounters are more like low discounters (that is, when their discount rates are closer to zero than 100 percent).

## A Positive Return on Savings

In the above solutions, the value of savings stems solely from diminishing returns to consumption. A productive return to savings raises the stakes to finding a cooperative solution.

Suppose that if an individual sacrifices consumption in the amount $S$ in period 1, he is rewarded with consumption in period 2 in the amount ( $1+$ i) $S$, where $i$ is the productive return to savings. ${ }^{25}$ For illustration, I set $i$ equal to 50 percent. I set the high discounter's discount rate to 200 percent to ensure a zero savings rate for this level of return. In isolation, the high discounter saves nothing, and thus has a zero period-2 consumption. The low discounter naturally saves two-thirds of his period-1 output, and enjoys consumption in period 2 equal to $1.0 .{ }^{26}$

I now reconsider the hold-up solutions (tax on low discounters' savings) and the cooperative solutions (subsidy on high discounters' savings). The new solutions are shown in Table 2; they are comparable to those in Table 1 except national income is a function of the savings rate. ${ }^{27}$

The first row shows the outcomes when both individuals live in isolation. The second and third row shows the solutions under either the tax or subsidy approach. Recall that I award all the bargaining power to the low discounter. Thus, the low discounter chooses the tax or subsidy rate that maximizes his utility, subject to the condition that the period- 2 wealth distribution is sustainable. In this solution, the savings tax rate is 45 percent ( $t^{*}=.45$ ). The low discounter saves 39 percent of his period-1 product and transfers 18 percent of his income to the high discounter.

Alternatively, in the savings subsidy solution, the equilibrium subsidy rate is 80 percent $\left(s^{\circ}=.8\right)$. The high discounter saves 22 percent of his income and, inclusive of his subsidy, has gross savings of 38 percent of income. The low discounter sets his own savings rate to 56 percent of income. Per capita savings per capita are twice as high as the in the tax solution; per capita income increases from 1.10 to 1.23 . Compared to a redistributive tax on savings, the low discounter clearly is better off offering to subsidize high discounters' savings.

Figure 1 shows how the results change with different proportions of high and low discounters. (Details are presented in the appendix.) The figure shows per capita savings and output under the tax and subsidy solutions. There are two striking features of the results.

Table 2 Tax and Subsidy Solutions, Given a Positive Return on Savings

|  | Tax/ <br> subsidy <br> solution | Savings <br> LD | Savings <br> $H D$ | Transfer <br> per LD | Utility <br> LD | Utility <br> HD | Output <br> per <br> capita | Savings <br> per <br> capita |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Category | - | .66 | .00 | .00 | .98 | .69 | 1.16 | .33 |
| Isolation <br> outcome | - |  |  |  |  |  |  |  |
| Democratic <br> solution | $t^{*}=.45$ | .39 | - | .18 | .75 | .77 | 1.10 | .20 |
| $L=H=1$ | $s^{\circ}=.80$ | .58 | .22 | .16 | .86 | .73 | 1.24 | .48 |

Note: $L$ is the number of low discounters and $H$ is the number of high discounters. The high discounter (HD) has a 200 percent discount rate ( $r=2$ ), the low discounter (LD) has a zero discount rate, and the return on savings is 50 percent $(i=.5)$. The variable $t^{*}$ denotes the minimum tax rate on the low discounter's savings that ensures a stable distribution of income; the variable s ${ }^{0}$ is the minimum level of the subsidy rate offered by low discounters on each dollar saved by high discounters that ensures a stable distribution of income. Lifetime utility is discounted to period 1 using each individual's discount rate.

First, the equilibrium outcomes depend importantly on the share of low and high discounters in the population. If there are "too many" high discounters, no interior solution exists: low and high discounters consume all their output in period 1 and are impoverished in period 2. In the illustration, the critical low discounter share is approximately 30 percent. If low discounters are a smaller minority, savings are zero and per capita consumption in period 1 is 1 .

As low discounters increase their proportions beyond this level, interior solutions are feasible, which dramatically affect per capita output. ${ }^{28}$ Per capita income increases with the proportion of low discounters in the economy.

Second, per capita savings and income are always higher using a subsidy solution to the high discounter problem rather than a tax solution. Solutions that encourage high discounters to act more like low discounters lead to more savings and output than solutions that discourage low discounters from following their natural inclinations to save. The difference in these outcomes is always positive, but is largest when low discounters do not dominate the population.

Figure 2 shows high and low discounters' utility levels under the savings subsidy solutions. Lifetime utility is discounted to period 1 using each individual's personal discount rate. High discounters do not attach a high value to the prospect of higher period-2 consumption, and thus their lifetime utility is dominated by consideration of period-1 consumption.

Viewed from the perspective of period 2, however, high discounters' utility is substantially higher in the cooperative solution. Compared to zero savings and zero period-2 consumption in the isolated solution, high


Figure 1. How per capita income and saving vary with the share of low discounters. High discounter's discount rate is taken as $2.0 \%$, low discounter's as $0.5 \%$.
discounters enjoy higher consumption in old age. In return for generating relatively high period-2 consumption for high discounters, low discounters enjoy a relatively high (and sustainable) consumption level in period 2.

Starting from equilibrium solutions when low discounters dominate the population, high discounters clearly are better off if they can increase their relative numbers in the population. Their higher share gives them

Lifetine Utility (Evaluation Period 1)


Figure 2. How utility levels vary with the share of low discounters (assuming a savings subsidy cooperative solution). Utility $=\log \left(1+C_{1}\right)+\log \left(1+C_{2}\right) /(1+r)$; $r=0$ for LD and 2.0 for HD; return on savings is 0.5 .
more leverage to collect transfers from low discounters who attach substantial value to ensuring a sustainable income distribution. As their numbers grow beyond some critical point, however, there are "too few" low discounters to support economical transfers, and a degenerate equilibrium is inevitable.

Low discounters are always somewhat worse off compared to a world in which they dominate the population. But as long as there are not "too
many" high discounters, low discounters can have a high standard of living when young and old that is not threatened by an unsustainable distribution of wealth.

## Conclusion

When low discounters are mixed with high discounters, different wealth positions develop, giving rise to a "high discounter problem:" low discounters accumulate wealth, and high discounters do not. The ability of high discounters to vote to effect a redistribution of income gives rise to prospects for a hold-up in later periods. To thwart the hold-up, low discounters can "share" their wealth - that is, incur a wealth tax to redistribute income to high discounters. This outcome is problematic because the threat of a redistributive tax discourages savings. In effect, the defacto tax on savings encourages low discounters to act more like high discounters.

A more efficient solution arises if low discounters can alter the form of a transfer from a tax on their savings to a subsidy on high discounters' savings. The subsidy encourages some savings by high discounters and gives them a stake in the economy's wealth position. In effect, high discounters are encouraged to act more like low discounters. Per capita savings and output increase, partly financing the transfers to high discounters. Old age consumption is higher for both low and high discounters compared to solutions that depend on a hold-up potential.

The model has implications for tax, regulatory, and national retirement policy. Existing pension policy is partly driven by an attempt to expand coverage (for example, vesting rules, anti-discrimination rules, and participation rules). The model suggests that this approach is unlikely to be fruitful: high discounters have a natural tendency to avoid firms that defer part of compensation; and firms that offer pensions more likely seek to hire and retain low discounters. In this sense, profitmaximizing incentives in the market work in tandem with high discounters' natural aversion to save. The rise of $401(\mathrm{k})$ plans further diminish the prospects for forced coverage in the private market.

The thrust of the comprehensive income tax in the United States essentially conforms to the tax solution discussed above. The general rule in the tax system is that income diverted to savings are "doubly taxed"; effectively income devoted to savings is taxed at higher rates than income devoted to immediate consumption. ${ }^{29}$ By penalizing savings, the tax code discourages low discounters from following their natural inclination and redoubles high discounters' natural aversion to savings.

Exceptions in the tax code are made for pension vehicles, but even here, defined contribution plans are treated more favorably than de-
fined benefit plans. The distortion occurs because the latter plans are denied the opportunity to fund their plans at the same level as defined contribution plans. ${ }^{30}$ Moreover, so-called antidiscrimination rules in the Internal Revenue Code are chiefly designed to prevent highly-paid workers from saving "too much," rather than toward encouraging more participation by those otherwise not inclined to save.

An unfunded social security system such as that used in the United States has some of the elements suggested by a cooperative solution: there is a requirement to participate in the program, but formulas are set up to subsidize those least likely to voluntarily accumulate wealth for old age. The same principles could apply to a funded system, whether defined benefit or defined contribution. Nevertheless, the social security system penalizes those who save and rewards those who save little or nothing for old age. ${ }^{31}$ In a broad sense, it is far more likely that a consumption tax approach - which eliminates the special added taxes on savings - combined with more subsidies to some levels of savings for retirement, would generate higher national income, more resources for consumption in old age, and a more sustainable distribution of income across society.

## Appendix: The Role of the High/Low Discounter Mix and Discount Rate Spreads

The chapter considers a case where there is one low discounter and one high discounter. When the proportions of high and low discounters vary, the transfers from each low discounter are aggregated and spread over the population of high discounters. As such, the conditions for a sustainable income distribution are altered, and this is the subject of this appendix. Here, I recalculate the minimum tax rates and subsidy rates that generate a sustainable equilibrium on the assumption that the probability of winning a voting outcome is proportional to high discounters' proportion in the voting pool $(p=H /[L+H]) .{ }^{32}$

## No Productive Return to Savings

Appendix Table 1 shows the outcomes for the low and high discounter under several conditions when the productive return savings is zero. The top portion of the table shows the expected outcome when each individual lives in isolation. The bottom portion shows various outcomes in a democratic society. The values in the second column are the minimum tax rate $\left(t^{*}\right)$ and minimum subsidy rate $\left(s^{\circ}\right)$ that generate a sustainable income distribution under either a savings tax approach or a subsidy approach.

Appendix Table 1 Comparing Savings and Tax Outcomes When There Is No Productive Return on Savings

| Category | Tax/subsidy solution | $\begin{aligned} & \text { Savings } \\ & \text { LD } \end{aligned}$ | Savings $H D$ | Transfer per $L D$ | Utility $L D$ | Utility $H D$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Isolation outcome | - | . 50 | . 00 | . 00 | . 81 | . 69 |
| Democratic outcomes |  |  |  |  |  |  |
| $\begin{aligned} & L=1 \\ & H=1 \end{aligned}$ | $t^{*}=\mathrm{ds}$ | . 00 | - | . 00 | . 69 | . 69 |
| $r=1$ | $s^{\circ}=.5$ | . 44 | . 22 | . 11 | . 73 | . 72 |
|  | $t^{*}=.2$ | . 37 | - | . 07 | . 75 | . 79 |
| $L=3$ |  |  |  |  |  |  |
|  | $s^{\circ}=.45$ | . 48 | . 21 | . 03 | . 79 | . 71 |
|  | $t^{*}=.15$ | . 42 | - | . 06 | . 76 | . 74 |
| $r=1 / 2$ |  |  |  |  |  |  |
|  | $s^{\circ}=.2$ | . 47 | . 30 | . 06 | . 77 | . 73 |
|  | $t^{*}=\mathrm{ds}$ | . 00 | - |  |  |  |
| $H=2$ |  |  |  |  |  |  |
|  | $s^{\circ}=\mathrm{ds}$ | . 00 |  |  |  |  |
|  | $t^{*}=\mathrm{ds}$ | . 00 | - |  |  |  |
| $r=11 / 2$ |  |  |  |  |  |  |
|  | $s^{\circ}=.8$ | . 43 | . 17 |  | . 72 | . 71 |

Note: The table shows the savings rates, utility levels, and transfers from low to high discounters, assuming alternatively that high discounters' retirement savings are enhanced by either a tax on low discounters' savings or a subsidy on high discounters' savings. In either case, the tax and subsidy rates are chosen to maximize low discounters' utility, subject to a stable distribution of income. L is the number of low discounters; H is the number of high discounters. The high discounter (HD) has a discount rate equal to $r>0$, the low discounter (LD) has a zero discount rate, and the productive return on savings is zero. The variable $t^{*}$ denotes the minimum tax rate on the low discounter's savings that ensures a sustainable income distribution. The variable $s^{\circ}$ is the minimum level of the subsidy rate offered by low discounters on each dollar saved by high discounters that ensures a sustainable distribution of income. Lifetime utility is discounted to period 1 using either the low or high discounter's discount rate, as appropriate. The notation ds depicts a degenerate solution: There is no tax (or subsidy) solution that makes low discounters better off, and ensures a sustainable income distribution. Thus, overall savings are zero.

The other columns report the savings rates of the low and high discounter, the amount transferred from the low discounter to the high discounter, and the discounted utilities for both individuals under these solutions, evaluated in period 1. In the cooperative solution, the transfer is the matching amount, and thus is part of aggregate savings.

The first row of democratic outcomes shows the solution discussed in the text when there is one low discounter with discount rate zero and one high discounter with a discount rate of 100 percent $(r=1)$. The coopera-
tive solution is clearly superior to the potential for a period-2 hold-up. The subsidy rate is set to 50 percent. The high discounter voluntarily saves 22 percent of his income. The low discounter has a net savings rate of 44 percent, and transfers 11 percent of his income to the high discounter. Aggregate savings in this solution is 38 percent of aggregate period-1 output, which exceeds the solution where individuals live in isolation. ${ }^{33}$ Both individuals are better off compared to the hold-up solution; the high discounter is better off in the democracy (he consumes 55.5 percent of total output, inclusive of the transfer).

Moving down the table, the solutions show that low discounters tend to fare better, the higher their proportions in the population, and the more closely high discounters resemble low discounters in the level of their discount rates. The second and third democratic solutions reflect situations where low discounters are three times more numerous than high discounters; and high discounters have a more modest discount rate (namely $1 / 2$ instead of 1 ).
In the last two rows, I show solutions less favorable to low discounters, namely, when high discounters are twice as numerous as low discounters, and when high discounters have an even higher discount rate ( $r=1.5$ in place of $r=1$ ). Except for a subsidy solution in the latter case, these cases are dominated by degenerate solutions (that is, there is no positive savings rate that makes low discounters better off and ensures a sustainable income distribution).

## Productive Return to Savings

Appendix Table 2 shows similar solutions when the productive return on savings is 50 percent and the high discounter's discount rate is 200 percent. The solutions are reported for various portions of the population that are low discounters (where $a$ denotes the low discounter percent of total population).

The results show that when high discounters outnumber low discounters two to one ( $a=.33$ ), the savings rate is negligible in a tax solution. But the savings subsidy solution produces a savings rate of approximately 40 percent. Compared to the tax solution, per capita income increases from 1.01 to 1.21. Finding conditions that generate a sustainable solution with positive savings is an important determinant of income and utility in the model.

Per capita income increases with the proportions of low discounters. If there are two low discounters for every three high discounters, interior solutions can be found for both the savings tax and subsidy approaches, but the subsidy outcome produces higher savings and per capita income.

Appendix Table 2 Tax and Subsidy Solutions When There Is a Positive Return on Savings

| Category | Tax/subsidy solution | Savings $L D$ | Savings $H D$ | Transfer per LD | Utility $L D$ | Utility <br> HD | Output per capita | Savings per capita |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Isolation outcome | - | . 66 | . 00 | . 00 | . 98 | . 69 | 1.16 | . 33 |
| Democratic outcomes |  |  |  |  |  |  |  |  |
|  | $t^{*}=.65$ | . 05 | - | . 04 | . 70 | . 69 | 1.01 | . 00 |
| $a=.33$ |  |  |  |  |  |  |  |  |
|  | $s^{\circ}=.75$ | . 50 | . 21 | . 32 | . 72 | . 73 | 1.21 | . 41 |
|  | $t^{*}=.60$ | . 17 | - | . 10 | . 71 | . 74 | 1.03 | . 07 |
| $a=.4$ |  |  |  |  |  |  |  |  |
|  | $s^{\circ}=.75$ | . 54 | . 21 | . 24 | . 79 | . 73 | 1.22 | . 44 |
|  | $t^{*}=.45$ | . 39 | - | . 18 | . 75 | . 77 | 1.10 | . 20 |
| $a=.5$ |  |  |  |  |  |  |  |  |
|  | $s^{\circ}=.75$ | . 58 | . 22 | . 16 | . 86 | . 73 | 1.24 | . 48 |
|  | $t^{*}=.30$ | . 52 | - | . 16 | . 83 | . 79 | 1.16 | . 31 |
| $a=.6$ |  |  |  |  |  |  |  |  |
|  | $5^{\circ}=.70$ | . 61 | . 21 | . 10 | . 91 | . 73 | 1.26 | . 51 |
|  | $t^{*}=.25$ | . 55 | - | . 14 | . 85 | . 81 | 1.18 | . 37 |
| $a=.67$ |  |  |  |  |  |  |  |  |
|  | $s^{\circ}=.65$ | . 63 | . 20 | . 06 | . 93 | . 72 | 1.26 | . 53 |
|  | $t^{*}=.15$ | . 61 | - | . 09 | . 90 | . 81 | 1.23 | . 45 |
| $\begin{array}{lllllllll}a=.75 & s^{0}=60 & .65 & .19 & .04 & .95 & .72 & 1.28 & \\ & \text { c }\end{array}$ |  |  |  |  |  |  |  |  |
|  | $5^{0}=.60$ | . 65 | . 19 | . 04 | . 95 | . 72 | 1.28 | . 56 |

Note: The table shows the solutions when a savings tax or savings subsidy approach is used to address the high discounter problem. $a$ is the percent of low discounters in the population ( $a=L /(L+H)$, where $L$ is the number of low discounters and $H$ is the number of high discounters. The high discounter (HD) has a 200 percent discount rate $(r=2)$, the low discounter (LD) has a zero discount rate, and the return on savings is 50 percent $(i=, 5)$. The variable $t^{*}$ denotes the tax rate on the low discounter's savings that generates a sustainable income distribution. The variable $s^{0}$ is the level of the subsidy rate offered by low discounters on each dollar saved by high discounters that ensures a sustainable distribution of income. Lifetime utility is discounted to period one using the individual's internal discount rate.

The difference between the tax and subsidy outcomes is greater, the more the population is dominated by high discounters.

The views expressed in this chapter do not reflect the official views of the Pension Benefit Guaranty Corporation.

## Notes

1. The individual is indifferent between $\$ 100$ now and $\$ 100+x$ in one year when $\$ 100=(\$ 100+x) /(1+r)$ where $r$ is the individual's discount rate. Thus,
the higher that $x$ needs to be to create the equality, the higher must be the discount rate he attaches to payoffs one year into the future.
2. The implications of heterogeneous savings propensities on wealth accumulation can be found in Poterba, Venti, and Wise (1995) and Engen and Gale (1996). Savings propensities also can differ across workers owing to different amounts of information about the consequences of savings behavior (Bernheim and Garrett 1995).
3. For example, the firm may wish to use defined benefit pension plans to discourage quitting and to encourage earlier retirement.
4. This idea is developed more fully in Ippolito (1995).
5. The argument makes the assumption that capital markets are imperfect, that high discounters cannot borrow against the promise of higher income in the future. As long as borrowing is not possible, productivity decisions will be influenced by their internal discount rate.
6. Some low discounters may not have opportunities to pursue education; some high discounters may attain education because it is financed by parents, and so on.
7. In the United States, cliff vesting can be no more than five years. While fiveyear cliff vesting is almost universal in defined benefit plans, it typically is much shorter in defined contribution plans. The firm can and sometimes does use immediate vesting.
8. If the individual does not roll over the monies, he pays an excise tax of 10 percent and takes the lump sum into immediate taxable income.
9. The firm does not use a defined benefit plan because it does not find a deferred wage scheme to be economic; that is, the benefits of deferred wages are outweighed by the indenture premium.
10. For example, suppose that the high discounters's imperfect information is described by a perceived alternative wage in a no-monitoring firm of $1+e$, where $e$ is an error term with mean zero. For high discounters with large, negative values of $e$, the perceived alternative wage is less than the wage in the monitoring firm, and thus, the gains from entering the monitoring firm are positive: $j_{I I}=w_{0}-s-$ $[1+e]>0$, for some $e<0$.
11. The full tax advantages of pensions are afforded only in plans offered by firms. Individual retirement accounts permit only $\$ 2,000$ per annum in contributions.
12. The mobility of physical capital from low- to high-discount economies is hampered because property rights are more tenuous in economies dominated by high discounters. That is to say, high discounters perceive the immediate gains of holding up low discounters without appreciating the long-run consequences on lower future capital investment. Thus, if rich nations make specialized investments in economies dominated by high discounters, they face the risk that their capital will be expropriated.
13. I later expand the results to allow varying proportions of high and low discounters.
14. Thus, there are no prospects for intergenerational transfers.
15. The qualitative results are similar if a significant portion of wealth is transfer upon winning a draw.
16. In reality, government outcomes need not be all or nothing, and production functions for influencing outcomes generally goes beyond a random draw concept (Becker 1983; Peltzman 1976).
17. In terms of condition 4, zero savings occur if the discount rate is 100 percent or higher.
18. To be complete, I need to show that the low discounter prefers certain consumption of all his income in period 1 compared to saving some portion to be contested in a period-2 lottery: $\log (2)>\log \left(2-S_{t}\right)+(1-p) \log \left(1+S_{l}\right)$. In fact, this condition is satisfied.
19. If the subsidy is made available to all, then low discounters also respond to the incentive, creating some transfers from low discounters to themselves, and creating an inefficiency from encouraging low discounters to "too much" savings. In practice, the solution might involve a subsidy over some initial levels of savings, diminishing at higher levels of savings.
20. Maximizing (1) subject to the constraints in (7), the high discounter's optimal savings is:

$$
S_{H}^{\circ}=(2 z-1) /(1+s+z), \text { where } z=(1+s) /(1+r) \text {, }
$$

and thus consumption in period 2 is

$$
C_{H 2}^{o}=S_{H}^{o}[1+s] .
$$

Consider the case when the high discounter's discount rate is 100 percent ( $r=1$ ). Without the subsidy ( $s=0$ ), the high discounter saves nothing. If the discount rate is sufficiently high, the high discounter will maintain zero savings within the range of subsidy rates that makes the low discounter better off.
21. These constraints are: $C_{L 1}=1-S_{L}-s S_{H}$; and $C_{L 2}=S_{L}$.
22. The minimum value of $s$ that guarantees stability is $s^{\circ}: \log \left[1+\left(1+s^{\circ}\right) S_{H}^{\circ}\right]=p$ $\log \left[1+\left(1+s^{o}\right) S_{H}^{o}+S_{L}^{o}\right]$.
23. See note 22.
24. I also make this assumption in developing savings tax solutions that compete with the subsidy scheme.
25. For example, suppose work in period 1 takes the form of expending effort to search for food. Savings in this simple model might take the form of using some period-1 time to develop plants that yield food without an expenditure of effort. This process may yield a superior solution compared to spending all of period 1 collecting food for both periods.
26. The new income constraints are:

$$
C_{j 1}=1-S_{j} \text { and } C_{j 2}=S_{j}(1+i) .
$$

Maximizing (1) subject to these constraint gives the optimum savings rates:

$$
S_{j}=\left[2(1+i)-\left(1+r_{j}\right)\right] /(1+i)\left(2+r_{j}\right), j=L, H .
$$

27. In the case of the tax solution, the income constraints for the low discounter are:

$$
C_{L 1}=1-S_{L ;} ; C_{L 2}=d S_{L}, d=(1-t)(1+i),
$$

and thus the low discounter's savings rate is:

$$
S_{L}^{*}=1-1 / 2 d
$$

The stability condition is the same as expression (4) except that $S_{L}^{*}$ is replaced by ( $1+i$ ) $S_{t}{ }^{*}$.

In the case of the savings subsidy to the high discounter, the high discounter's constraints are

$$
C_{H 1}=1-S_{H} \text { and } C_{H 2}=S_{H} \lambda, \text { where } \lambda=(1+i)(1+s) \text {, }
$$

where $s$ is the per dollar subsidy offered by low discounters to high discounters' savings. Thus, the high discounter's optimal savings rate is:

$$
S_{H}^{o}=[2 \lambda-(1+r)] / \lambda(2+r) .
$$

The low discounter faces the constraints:

$$
C_{L 1}=1-S_{L}-s S_{H} \text { and } C_{L 2}=S_{L}(1+i),
$$

where $s$ is the subsidy rate offered to high discounter's savings. Thus, the low discounter's optimal savings rate is

$$
S_{t .}^{n}=\left[(1+i)\left(2-s S_{H}\right)-1\right] / 2(1+i) .
$$

The stability condition is the same as in footnote 19 except that $S_{j}^{*}, j=L, H$ are replaced by $(1+i) S_{j}^{*}$.
28. In this model, when the return on savings is .5 and the high discounter's discount rate is 200 percent, equilibrium degenerates when the share of low discounters in the population falls below 30 percent of the population.
29. The double tax refers to the income tax applied at the time wages are earned; then again as the individual tries to save these monies to finance future consumption. See Ippolito (1990) for a fuller discussion.
30. A special full funding limit was enacted in 1987 that sharply limits funding of defined benefit pensions, particularly those that have lots of workers and few retirees. See Ippolito (1990).
31. These rules include special taxes on social security benefits to individuals who have other sources of income during retirement, the availability of nursing home care to those without assets, and so on. A menu of reforms for social security and pensions along the lines discussed here are provided in Ippolito (forthcoming).
32. In the tax solution, the expression for the low discounter's savings choice is the same, given some tax rate $t$ against savings (see expression 6). The high discounter's period-2 consumption becomes:

$$
C_{l R 2}=(1+i)\left[l t S_{l}+S_{H}\right], l=L / H .
$$

Thus, the period-2 stability condition is

$$
\log \left[1+(1+i)\left(l t S_{L}+S_{H}\right)\right] \geq p \log \left[1+(1+i)\left(l S_{l}+S_{H}\right)\right], p=H /(L+H)
$$

In the subsidy solution, for any subsidy rate, $s$, high discounters solve for the same level of savings. Since the subsidy amount to H high discounters, $\mathrm{HsS} \mathrm{S}_{H}$, is spread across L low discounters, the optimum savings rate for each low discounter is

$$
S_{L}=\left[(1+i)\left(2-s S_{H} / l\right)-1\right] / 2(1+i) .
$$

The period-2 stability condition becomes:

$$
\log \left(1-\lambda S_{H}\right) \geq p \log \left[\left(1+\lambda S_{H}+l(1+i) S_{L}\right), \lambda=(1+i)(1+s) .\right.
$$

33. Savings are .44 for the low discounter, .22 for the high discounter, plus the transfer amount of .11; since total output is 2 then $(.44+.33+.11) / 2=.38$.

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