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A Procedure for Estimating the Combustion Noise Transfer Matrix of a Diesel Engine

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A PROCEDURE FOR ESTIMATING THE COMBUSTION NOISE TRANSFER MATRIX OF A DIESEL ENGINE

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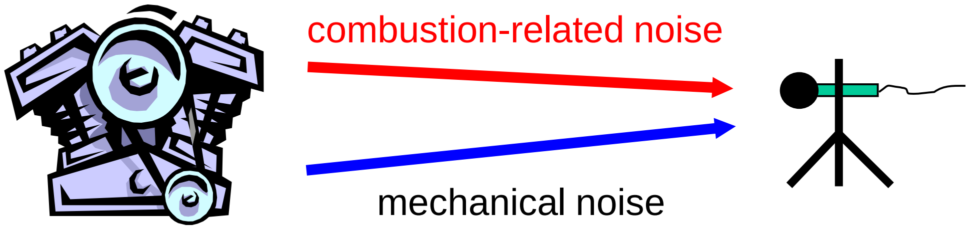
Background

- Two source mechanisms (i.e., combustion-related and mechanical noise) contribute to the total noise generation in engines

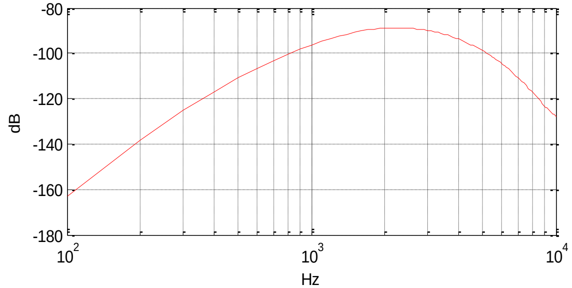


- It is important to identify the contribution of each source mechanism for the purpose of achieving optimized NVH performance
- Objective
  - To define a procedure for estimating an engine-platform-dependent transfer matrix that relates multi-cylinder pressures to radiated sound pressures resulting from the combustion
- Methods
  - Multi-input/multi-output (MIMO) system analysis
  - Cross-spectral procedure

Current approach

- To-date, the empirical prediction of diesel engine combustion noise has usually been achieved by combining a cylinder pressure with a single, smoothed structural attenuation function
- Lucas combustion noise meter
  - Designed by averaging the characteristics of several diesel engines from various manufacturers
  - Simple measurement setup
  - Sacrifice in accuracy
    - Represents only the averaged behavior of engines
    - Assumes that the characteristics of each cylinder are the same
    - Considers only the autospectral amplitude, thus resulting in error when cylinder pressure signals are correlated with each other, i.e., does not account for inter-cylinder correlation effects

< Lucas meter transfer function >



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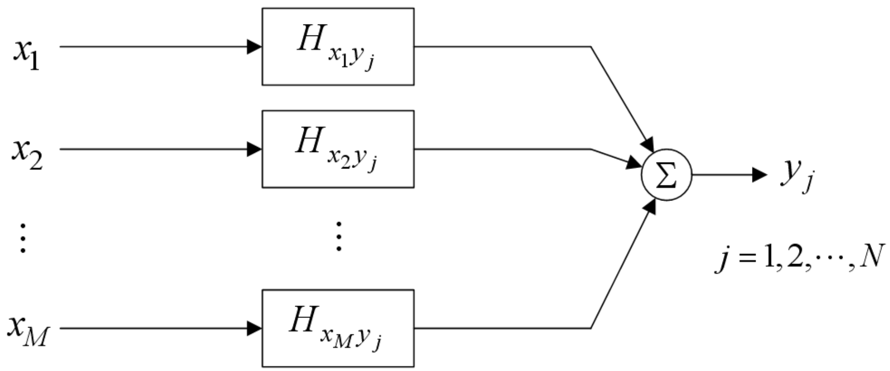
Multi-input/multi-output (MIMO) system representation

- Description of a multi-input/multi-output system model as a combination of multi-input/single-output system models



- A particular output signal can be expressed as a linear combination of input signals multiplied by the appropriate transfer functions

$$Y_j(f) = \sum_{i=1}^{M} H_{x_i y_j}(f) X_i(f) \quad , \quad j = 1, 2, \cdots, N$$

$$x_i(t) \xrightarrow{\text{Fourier Transform}} X_i(f)$$

$$y_j(t) \xrightarrow{\text{Fourier Transform}} Y_j(f)$$

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Signal relations

- The input and output signals can be related in matrix form as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} H_{x_1 y_1} & H_{x_2 y_1} & \cdots & H_{x_M y_1} \\ H_{x_1 y_2} & H_{x_2 y_2} & \cdots & H_{x_M y_2} \\ \vdots & \vdots & \ddots & \vdots \\ H_{x_1 y_N} & H_{x_2 y_N} & \cdots & H_{x_M y_N} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix} \Rightarrow \boxed{\mathbf{Y} = \mathbf{H}_{xy}^{\mathrm{T}} \mathbf{X}} \quad (1)$$

- Cross-spectral representation

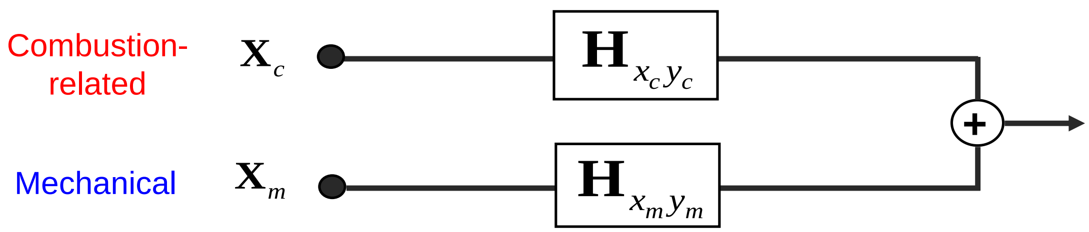
expectation operator

  - Cross-spectral matrix: $\mathbf{C}_{xx} = \mathrm{E}\left\{\mathbf{X}^* \mathbf{X}^{\mathrm{T}}\right\}$ $\mathbf{C}_{xy} = \mathrm{E}\left\{\mathbf{X}^* \mathbf{Y}^{\mathrm{T}}\right\}$
  - From Eq. (1) $\boxed{\mathbf{C}_{yy} = \mathbf{C}_{xy}^{\mathrm{H}} \mathbf{C}_{xx}^{-1} \mathbf{C}_{xy} = \mathbf{H}_{xy}^{\mathrm{H}} \mathbf{C}_{xx} \mathbf{H}_{xy}}$ (2)

$\boxed{\mathbf{H}_{xy} = \mathbf{C}_{xx}^{-1} \mathbf{C}_{xy}}$ (3)

Diesel engine noise analysis

- Definitions of the two noise components
  - Combustion-related noise: noise component that is correlated with cylinder pressure signals resulting from the combustion process of an engine
  - Mechanical noise: the remaining part (i.e., the noise component that is not correlated with cylinder pressure signals)
- Assume that source signals related to those two mechanisms can be measured independently, the MIMO model for diesel engine noise can be represented by



$$\mathbf{Y} = \mathbf{H}_{x_c y_c}^{\mathrm{T}} \mathbf{X}_c + \mathbf{H}_{x_m y_m}^{\mathrm{T}} \mathbf{X}_m$$
$$= \mathbf{Y}_c + \mathbf{Y}_m$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_c \\ \mathbf{X}_m \end{bmatrix} \quad \mathbf{H}_{xy} = \begin{bmatrix} \mathbf{H}_{x_c y_c} \\ \mathbf{H}_{x_m y_m} \end{bmatrix}$$

By definition, $\mathbf{C}_{x_c x_m} = \mathbf{C}_{x_m x_c} = \mathbf{C}_{x_c y_m} = \mathbf{C}_{x_m y_c} = \mathbf{0}$

Estimation of combustion-related properties

- Cross-spectral matrices

$$\mathbf{C}_{xx} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{X}_c^* \\ \mathbf{X}_m^*\end{bmatrix}\begin{bmatrix}\mathbf{X}_c^{\mathrm{T}} & \mathbf{X}_m^{\mathrm{T}}\end{bmatrix}\right\} = \begin{bmatrix}\mathbf{C}_{x_c x_c} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{x_m x_m}\end{bmatrix} \qquad \mathbf{C}_{xy} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{X}_c^* \\ \mathbf{X}_m^*\end{bmatrix}\left(\mathbf{Y}_c^{\mathrm{T}} + \mathbf{Y}_m^{\mathrm{T}}\right)\right\} = \begin{bmatrix}\mathbf{C}_{x_c y_c} \\ \mathbf{C}_{x_m y_m}\end{bmatrix}$$

- Substitution of the relations shown above into Eqs. (2) and (3) gives

Combustion-related noise transfer matrix

$$\begin{bmatrix}\mathbf{H}_{x_c y_c} \\ \mathbf{H}_{x_m y_m}\end{bmatrix} = \begin{bmatrix}\mathbf{C}_{x_c x_c}^{-1}\mathbf{C}_{x_c y_c} \\ \mathbf{C}_{x_m x_m}^{-1}\mathbf{C}_{x_m y_m}\end{bmatrix}$$

Combustion-related noise

$$\begin{aligned}\mathbf{C}_{yy} &= \mathbf{C}_{x_c y_c}^{\mathrm{H}}\mathbf{C}_{x_c x_c}^{-1}\mathbf{C}_{x_c y_c} + \mathbf{C}_{x_m y_m}^{\mathrm{H}}\mathbf{C}_{x_m x_m}^{-1}\mathbf{C}_{x_m y_m} \\ &= \mathbf{H}_{x_c y_c}^{\mathrm{H}}\mathbf{C}_{x_c x_c}\mathbf{H}_{x_c y_c} + \mathbf{H}_{x_m y_m}^{\mathrm{H}}\mathbf{C}_{x_m x_m}\mathbf{H}_{x_m y_m} \\ &= \mathbf{C}_{y_c y_c} + \mathbf{C}_{y_m y_m}\end{aligned}$$

Independent measurement of the mechanical noise source signals is not necessary so long as source signals directly related to the combustion-related noise components (i.e., $\mathbf{X}_c$) can be measured accurately

Measurement Scheme

- Setup
  - Microphones to measure sound pressures at a number of external locations 1-m away from an engine (i.e., **Y**)
  - Pressure transducers flush-mounted to each of cylinders to measure cylinder pressure signals (i.e., $\mathbf{X}_c$)
- The cylinder pressure signals can be used successfully in place of the source signals of the combustion-related noise
  - Cylinder pressure transducers measure only the combustion-related components
  - Each pressure transducer measures only the pressure signal of the cylinder in which it is mounted
- What measured
  - Cross-spectral matrix between cylinder pressures, $\mathbf{C}_{x_c x_c}$
  - Cross-spectral matrix between cylinder pressures and total sound pressure, $\mathbf{C}_{x_c y}$
- Since $\mathbf{C}_{x_c y} = \mathbf{C}_{x_c y_c} + \cancel{\mathbf{C}_{x_c y_m}} = \mathbf{C}_{x_c y_c}$, the combustion-related noise transfer matrix can be estimated from the measured cross-spectral properties by

$$\boxed{\mathbf{H}_{x_c y_c} = \mathbf{C}_{x_c x_c}^{-1} \mathbf{C}_{x_c y}} \qquad (4)$$

Source Signal Requirement

- For the transfer matrix to be estimated properly, it is required that the inverse of $\mathbf{C}_{x_C x_C}$ be well defined
- The ordinary inverse of a square matrix exists if the matrix is fully-ranked
- The rank of a matrix can be identified by counting non-zero singular values

Singular value decomposition

$$\mathbf{C}_{x_c x_c} = \mathbf{U\Sigma V}^{\mathrm{H}} = \mathbf{U}\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_M \end{bmatrix}\mathbf{V}^{\mathrm{H}}$$

where $\mathbf{C}_{x_c x_c}$ is an *M*-by-*M* matrix

- It is more appropriate to check the effective rank since no zero singular value can be observed in practical cases
- For a matrix to be effectively fully-ranked

$$\text{condition number [dB]} = 10\log_{10}\left(\frac{\lambda_1}{\lambda_M}\right) < \varepsilon$$

A suggested value of ε is a value between 10 and 15 dB

Application to a multi-cylinder diesel engine

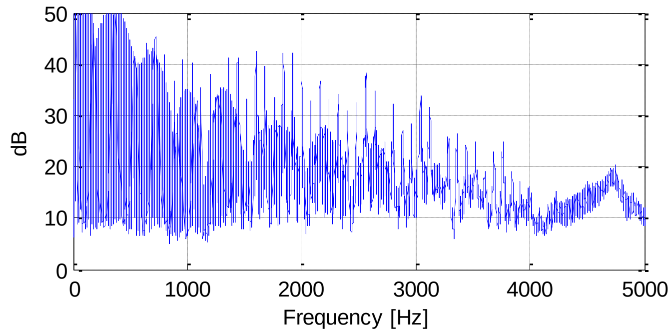
- Test engine: inline, turbocharged six-cylinder diesel engine of 6.7 liter displacement
- Six, flush-mounted pressure transducers (Kistler Type 6043A)
- Microphones (Brüel & Kjær Type 4189) at four 1-m locations (front, left, right, and top)
- Measurements were performed in a semi-anechoic chamber

- Engine operating conditions
  - Stationary speeds between 1200 rpm and 2800 rpm
  - Engine speed sweep from 1200 rpm and 2800 rpm
  - Various load conditions
  - Normal and randomized combustion process

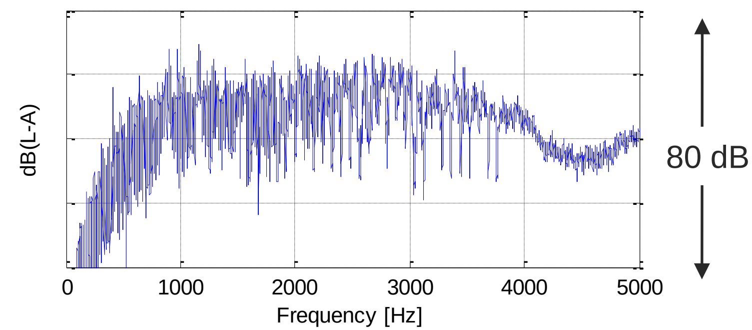
Results with a normal combustion process

< At a stationary engine speed of 1600 rpm with half load applied >

Condition number of the cylinder pressure cross-spectral matrix



Estimated combustion-related noise transfer matrix



- Since cylinder pressure signals are highly correlated with each other, the inverse of the cylinder pressure cross-spectral matrix is poorly determined
- The use of the latter signals resulted in erroneous estimation of the transfer matrix

- Definition of the amplitude plot of the transfer matrix

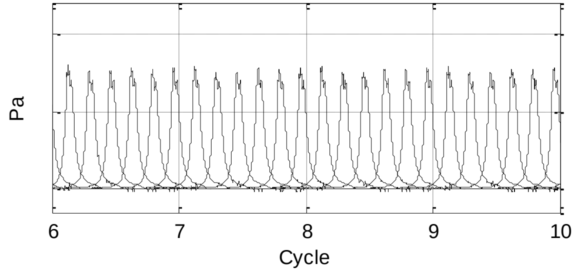
$$\left|\mathbf{H}_{x_c y_c}\right| = \sqrt{\frac{1}{4}\sum_{j=1}^{4}\sum_{i=1}^{6}\left|H_{x_{c_i} y_{c_j}}\right|^2}$$

Randomization of a combustion process

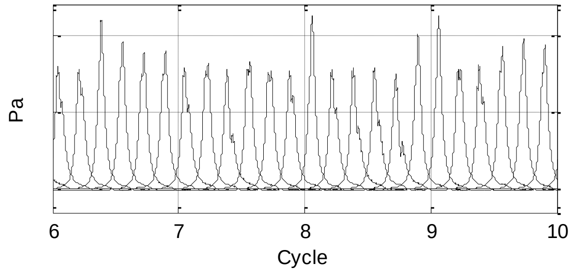
- Randomization results in cycle-to-cycle and cylinder-to-cylinder variations of combustion events
- The correlation between cylinder pressure signals can be decreased, thus providing signal characteristics more desirable for the estimation of the transfer matrix
- Randomization scheme considered
  - Start-of-injection (SOI) timing: found to be more effective
  - Amount of fuel injected in each cycle

<u>Cylinder pressure time history</u>

Normal combustion process



Randomized combustion process

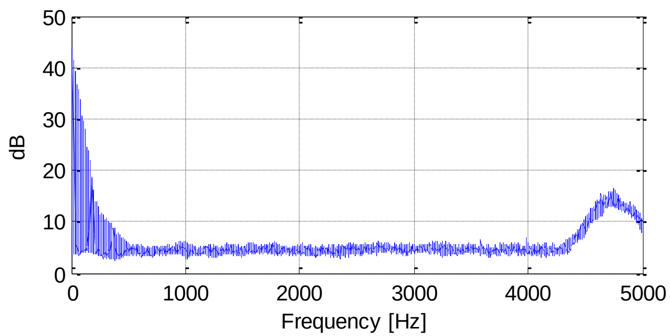


Six cylinder pressure signals are overlaid

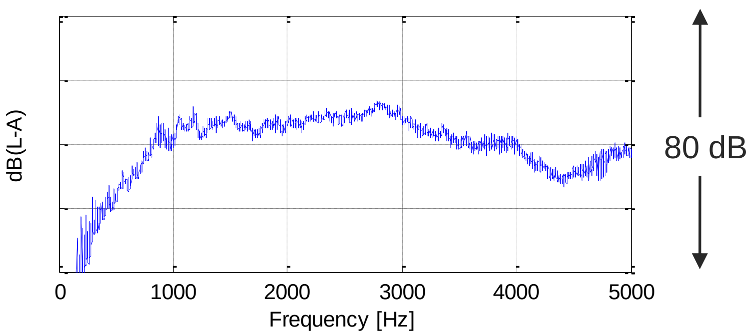
Results with a randomized combustion process

<u>At a stationary engine speed of 1600 rpm with half load applied</u>

Condition number of the cylinder pressure cross-spectral matrix



Estimated combustion-related noise transfer matrix



- The condition numbers dropped to acceptably low levels after randomizing the combustion process
- A good estimation of the transfer matrix could therefore be achieved
- Still poor at very low frequencies. However, those frequencies are not of great importance since structural attenuation is relatively large

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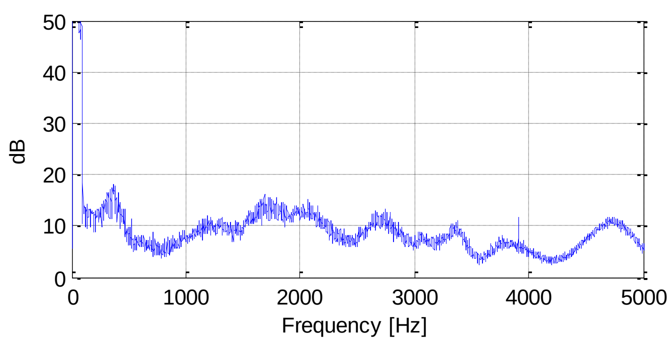
Run-up measurement

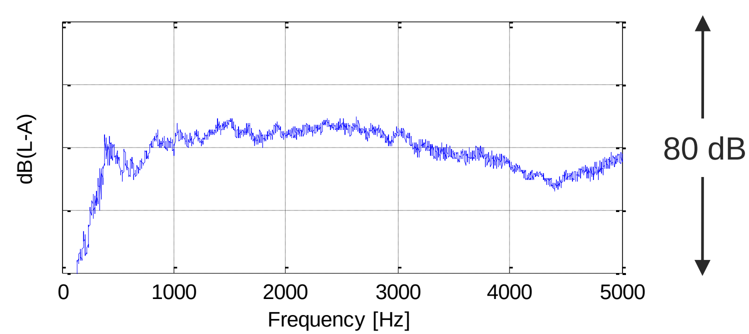
- Randomization of a combustion process involves a complicated engine control scheme
- Alternatively, the effect of inter-cylinder correlation can be mitigated by sweeping through a range of engine speeds during a measurement

<u>Run-up from 1200 to 2800 rpm with a normal combustion process</u>

Condition number of the cylinder pressure cross-spectral matrix



Estimated combustion-related noise transfer matrix



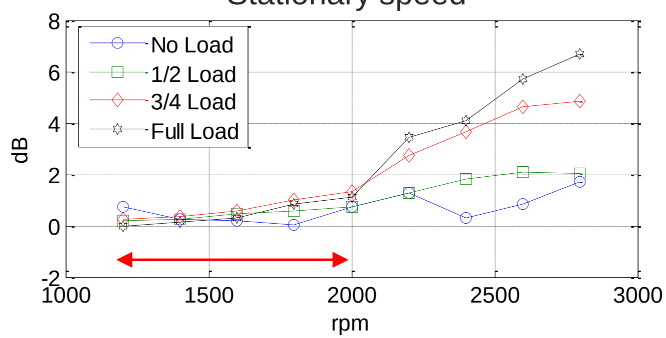
- The condition numbers dropped to acceptably low levels after randomization
- A good estimation of the transfer matrix could therefore be achieved over the frequency range of interest

Consideration of engine speed and load conditions

- The transfer matrices estimated for various engine speed and load conditions exhibited similar frequency characteristics in their overall shape, but not in their amplitudes

The RMS amplitudes of the transfer matrices across an overall frequency band

Stationary speed



Run-up

Load	dB
No	0.3
60%	1.4
Full	2.4

$$\text{RMS amplitude} = \sqrt{\frac{1}{N_f}\sum_{i=1}^{N_f}\left|\mathbf{H}_{x_c y_c}(f_i)\right|^2}$$

N_f: the number of spectral lines

The levels are normalized to the smallest one

- Strictly, $\mathbf{C}_{x_c y} = \mathbf{C}_{x_c y_c} + \mathbf{C}_{x_c y_m} = \mathbf{C}_{x_c y_c} + \Delta$

The error term becomes larger as the contribution of mechanical noise increases (when the engine speed and load are high)

- Thus, it is suggested that measurements be performed at an engine speed lower than a certain speed, which was found to be 2000 rpm in this case

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Validation (1)

- The procedure was validated by comparing mechanical noise, which is considered to remain the same regardless of combustion process
- Three engine operating conditions considered
  - Normal combustion process with pilot off
  - Normal combustion process with pilot on
  - Randomized combustion process with pilot on
- Prediction of the combustion-related noise
  - Choice of the transfer matrix
    - Run-up from 1200 rpm to 1800 rpm
    - 30% throttle load
    - With randomization
  - Performed based on the use of $\mathbf{C}_{y_c y_c} = \mathbf{H}^{\mathrm{H}}_{x_c y_c} \mathbf{C}_{x_c x_c} \mathbf{H}_{x_c y_c}$
- Mechanical noise = measured total noise – predicted combustion-related noise: i.e., $\mathbf{C}_{y_m y_m} = \mathbf{C}_{yy} - \mathbf{C}_{y_c y_c}$

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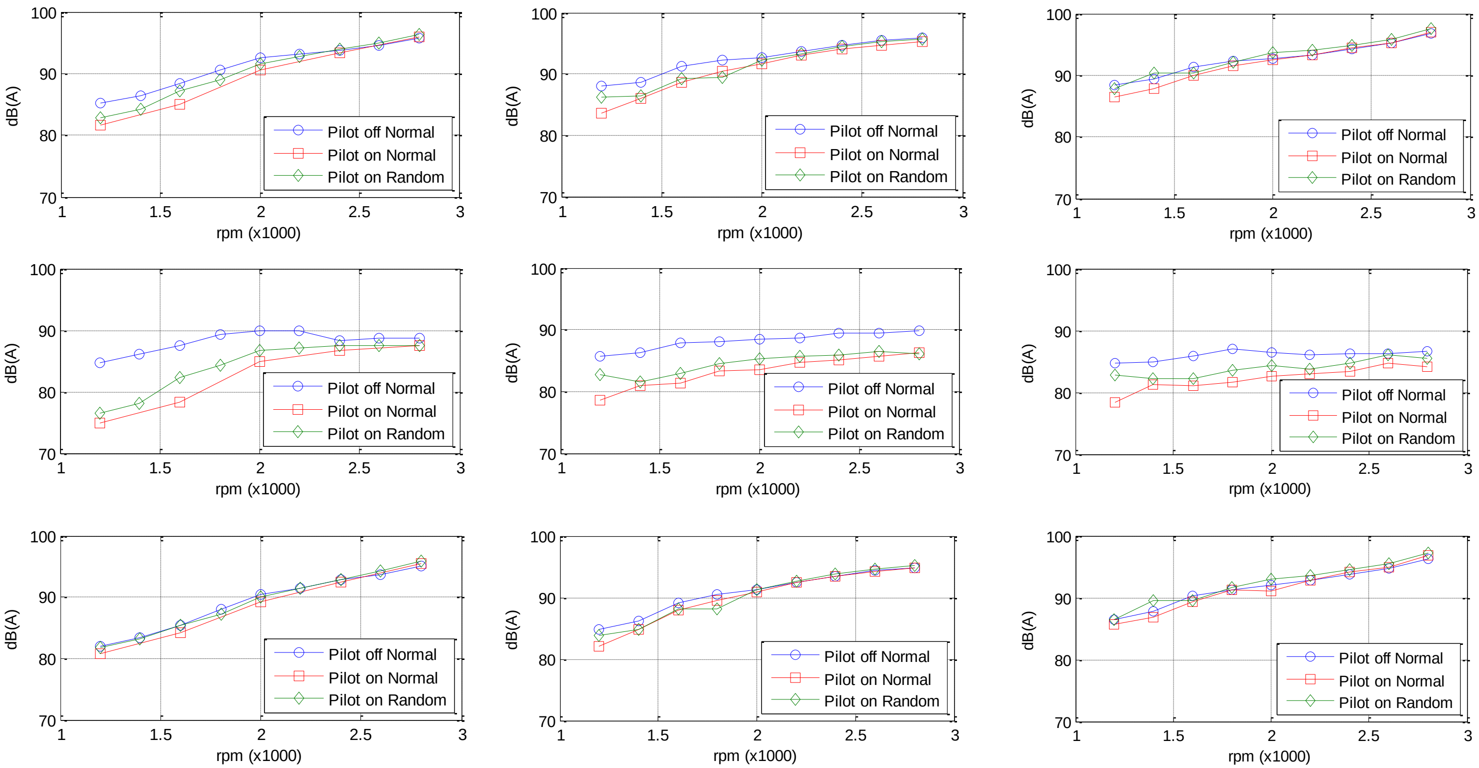
Validation (2) – Comparison of OASPL

No Load | Half load | Full load

Total

Combustion-related

Mechanical



- The change in the combustion process has an impact mainly on the combustion-related noise levels while the mechanical noise levels remain almost the same

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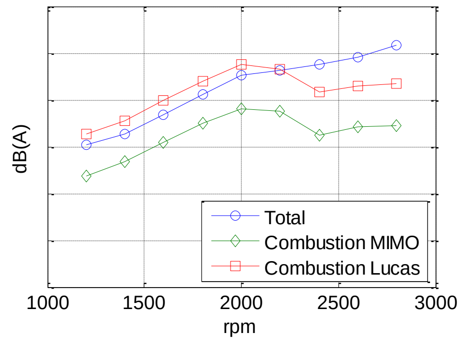
Comparison to Lucas meter

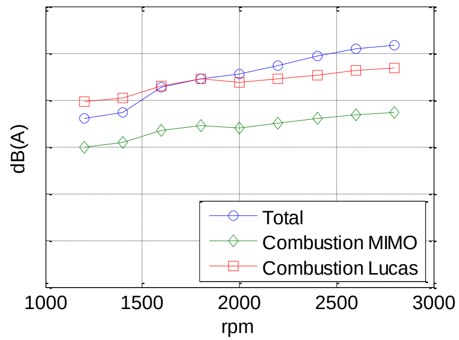
Comparisons of the overall sound pressure levels

- Measure total noise levels
- Combustion-related noise levels predicted by using the transfer matrix
- Combustion-related noise levels predicted by using the Lucas meter transfer function

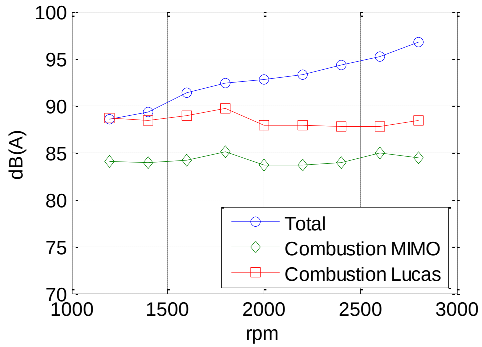
No Load



Half load



Full load



- The use of the Lucas meter resulted in a significant overestimation of the combustion-related noise for the test engine

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Conclusions

- A procedure for estimating the combustion-related transfer matrix based on a MIMO procedure was described
- To satisfy the input (cylinder pressure) signal requirement related to the use of the MIMO procedure
  - Randomization of a combustion process
    - Achieved by randomizing the start-of-injection timing
    - Particularly necessary in measurements at a stationary engine speed
  - Run-up measurement
    - All frequency components can be excited evenly
    - May make the randomization of the combustion process unnecessary
- An engine operating condition should be chosen so that combustion noise contributes significantly to the total sound field during the measurements
- The procedure proposed was validated by applying to a six-cylinder diesel engine
- It was shown that the use of the Lucas meter resulted in significant errors for the test engine