
**CHARACTERIZING INTERSTATE CRASH RATES BASED ON TRAFFIC
CONGESTION USING PROBE VEHICLE DATA**

Michelle M. Mekker

Purdue University
550 Stadium Mall Drive
West Lafayette, IN 47907-2051
(765) 496-7314
mmekker@purdue.edu

Stephen M. Remias

Purdue University
sremias@purdue.edu

Margaret L. McNamara

Purdue University
mcnamar0@purdue.edu

Darcy M. Bullock, Corresponding Author

Purdue University
550 Stadium Mall Drive
West Lafayette, IN 47907-2051
(765) 494-2226
darcy@purdue.edu

ABSTRACT

Crash reduction factors are widely used by engineers for prioritizing safety investments. Work zones are routinely analyzed by the length and duration of queues. Queue detection warning technology has been growing in availability and reliability in recent years. However, there is sparse literature on the impact of freeway queueing on crash rates. This paper analyzes three years of crash data and crowd-sourced probe vehicle data to classify crashes as being associated with queueing conditions or free flow conditions. In 2014, only 1.2% of the distanced-weighted hours of operation of Indiana interstates operated at or under 45 MPH. A three-year study on Indiana interstates indicates that commercial vehicles were involved in over 87% of back-of-queue fatal crashes compared to 39% of all fatal crashes during free flow conditions. A new measure of crash rate was developed to account for the presence and duration of queues: crashes per mile-hour of congestion. The congested crash rate on all Indiana interstates in 2014 was found to be 24 times greater than the uncongested crash rate. These data were also separated into both rural and urban categories. In rural areas, the congested crash rate is 23 times the uncongested crash rate. In urban areas, the congested crash rate is 21 times the uncongested crash rate. Queues are found to be present for five minutes or longer prior to approximately 90% of congestion crashes in 2014. Longer term, this information shows the importance in the development of technology that can warn motorists of traffic queues.

Keywords: Queue, Congestion, Crash Rate, Exposure, Safety, Probe Data

INTRODUCTION AND MOTIVATION

Congestion impacts both safety and mobility on the roadway. There is a debate whether congestion improves safety by causing lower speed or degrades safety by increasing the number of potential conflict points or opportunities for crashes. This paper studies historical crash data to determine crash rates during congested and uncongested traffic conditions on the interstates of Indiana. The purpose of this paper is to provide better understanding of safety risks due to congestion and help engineers prioritize and evaluate safety and mobility improvements.

There are two main types of crashes that occur in association with congestion. First, there are the low speed crashes that occur within a queue. It is generally accepted that these crashes are of low severity. The second type of congestion-related crash is the back-of-queue crash, which involves a vehicle traveling at a higher speed striking a vehicle traveling at a lower speed. These crashes are often high severity. Figure 1 is an image of a back-of-queue crash on I-65 near mile marker 215 on the morning of February 2, 2015. The queue had grown from the initial crash of a jack-knifed trailer and existed for over 90 minutes before the occurrence of the secondary, back-of-queue crash. The driver that struck the back of the queue was injured with a fractured arm.



FIGURE 1 Back-of-queue crash on I-65 on February 2, 2015

This paper discusses different factors prevalent in back-of-queue fatal crashes and compares crash rates during congested and uncongested traffic conditions. A new measure of exposure was developed for calculating these crash rates, to be discussed in a later section. Only crashes on the main lanes of interstate travel in Indiana are considered.

LITERATURE REVIEW

Agencies are concerned with the effect of the roadway and traffic conditions on safety since these are factors that can potentially be impacted via infrastructure improvements and changes. When safety is a concern, crash rates are the most common performance measure used by agencies and researchers. The *Highway Safety Manual (1)* defines crash frequency as the number of crashes over a period of time, usually a year. Crash rate is defined as the crash frequency of a period of time divided by the exposure in that same time period. Exposure is the total of all opportunities for a crash to occur, whether or not a crash actually occurs. The *Highway Safety Manual* refers to exposure as a measure of volume but, over the years, researchers have used a number of different ways to measure exposure, such as induced exposure (2-6) and volume-based exposure (7- 20). The volume-based exposure techniques and variations on those are the most relevant to this study.

A volume measure of some sort is the most common basis for exposure. Some studies use traffic counts recorded by infrastructure technology. Other studies use annual average daily traffic (AADT).

Mensah and Hauer (7) advise caution when using AADT as a measure of exposure. AADT is an aggregate measure and is not appropriate when considering the traffic conditions at the time of a crash. Specifically, when studying the effect of congestion on safety, an average measure of volume does not adequately represent the traffic conditions.

Regardless of the source of the volume data, there are three types of volume-based measures that are the most common in safety studies. One study used volume for calculated crash rates for different levels of severity, finding that property-damage-only and injury crash rates were highest when traffic was lightest (8). Another study used AADT-based hourly volumes to estimate the potential for conflicts (9). A third study modeled crash severity using flow as a variable in addition to speed and delay caused by congestion. (10). Vehicle-miles traveled (VMT) is a widely accepted and often used measure of exposure when calculating crash rates (11, 12, 13, 14). Lastly, density is frequently used in safety studies directly concerned with the effects of congestion on crash rates (15, 16, 17, 18). A common finding amongst safety studies using density as exposure is the parabolic, or U-shaped, relationship between density and crash rates, where the highest crash rates occur at low densities (mostly single vehicle crashes) and high densities (mostly multi-vehicle crashes). Some less common but no less viable measures of exposure are the standard deviation of speed between vehicles (19) and the volume-to-capacity ratio at the time of the crash (20).

Recently, with the greater availability and reliability of real-time traffic condition data, queue detection and alert systems are becoming more common. One system focused on specific highway sections designated as high-crash locations (21). This detection system used a number of factors, such as average speed, different forms of traffic density, headway variability, acceleration noise, etc., to calculate the crash likelihood in real-time. The combination of crash likelihood model and detection algorithm succeeded in detecting 58% of crashes during the study. Another detection system was developed for the Indiana Department of Transportation (INDOT) and covered the entire interstate system instead of small sections (22). This system uses only the difference between the space mean speeds of two adjacent roadway segments. If the average speed of an upstream segment is significantly higher than the average speed of the immediate downstream segment, an alert is made visible to dispatchers and emergency responders.

Of most relevance to this paper is a study by University of California-Berkeley's Transportation Research and Education Center (13, 14). In this study, four different traffic states are considered. The four traffic states are based on speeds upstream and downstream of a crash and use 50 MPH as a threshold for congestion, using VMT and vehicle-hours traveled (VHT) as exposure. In this study, the researchers found that crash rates for the three different congestion states were about 5 times greater than the crash rate for the free flow state.

RESEARCH OBJECTIVE

There is wide interest and need to understand crash rates associated with work zones and queued traffic. Historically, it has been very challenging to associate crash data with queued traffic. This paper looks at opportunities to fuse new crowd sourced probe data with crash reports to develop improved crash factors.

DATA SOURCES

Two different data sources were used in this study. Crowd-sourced probe vehicle data were used as data for traffic conditions. Crash data were retrieved from state crash records.

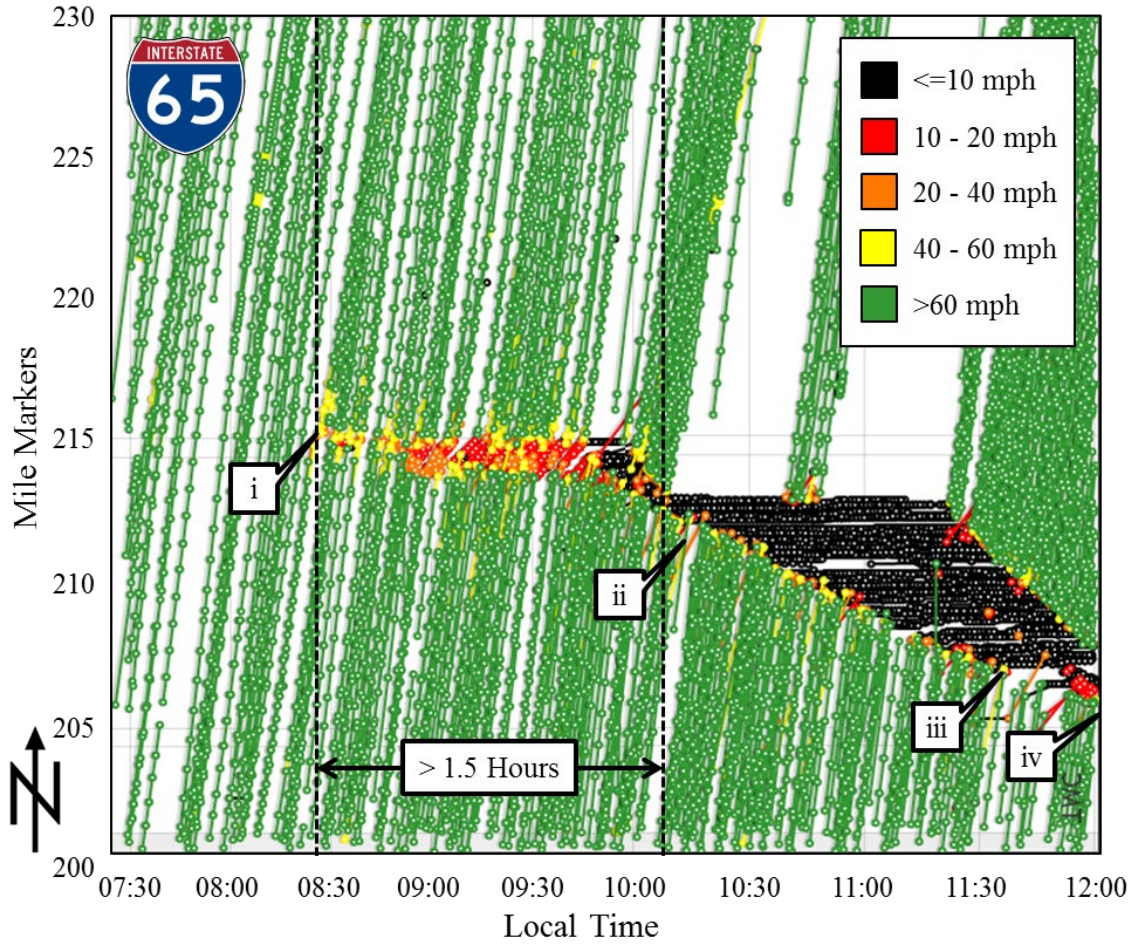
Crowd-Sourced Probe Vehicle Data

Speed data from probe vehicles are used in this study to assess traffic conditions when a crash occurs. Speed and trajectory information is collected from millions of probe vehicles by a third party vendor and aggregated into space mean speeds every minute for specified roadway segments. The vendor has two possible segmentation schemes. The first is based on Traffic Message Channels (TMC) and is the older of the two segmentation schemes. The TMC segments range from 0.5 to 15 miles in length. The second

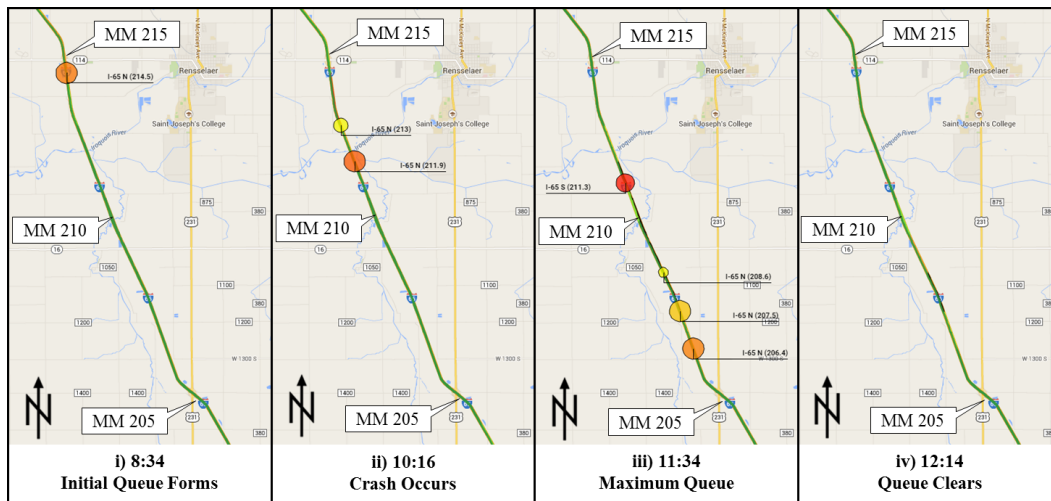
segmentation scheme, XD, is proprietary, with segment lengths ranging from 1 to 2 miles. The XD scheme has greater resolution but is only available from January 2014.

To illustrate the foundation of this probe data, Figure 2a shows a sample of speed and trajectory data before it is aggregated into space mean speeds. Specifically, these time space diagrams are for probe vehicles passing through a section of I-65 Northbound on February 2, 2015, before, during, and after a crash (Figure 1). The incident began when a trailer jack-knifed due to slick road conditions at about 8:30 AM (labeled *i* in Figure 2). A queue began to form, with vehicles in the queue moving at 10 to 20 MPH until 9:40 AM when lanes were restricted to facilitate clean up. Within this queue, vehicles moved at less than 10 MPH, if at all. At approximately 9:55 AM, the queue began to dissipate quickly and was almost cleared when a passenger vehicle struck a trailer at the back of the queue at 10:16 AM (*ii*). Prior to and upstream of the crash, the queue existed for more than 90 minutes. The back-of-queue crash (*ii*) caused the queue to reform with speeds of less than 10 MPH lasting for more than 2 hours after the crash and extending nearly 10 miles behind the crash. Figure 2b shows the development of the queue using the real-time shockwave boundary detection tool on the INDOT web page (22).

These new data sets provide the ability to precisely characterize traffic flow regimes with fidelity that has historically only been discussed in an academic context. Figure 3 depicts the shockwave diagram (23) developed from the time-space diagram (Figure 2). Before the back-of-queue crash at 10:16 AM, the queue had a frontal stationary boundary, a backward forming boundary propagating at approximately 1 MPH, and a backward recovery boundary with a speed of 12 MPH. Just before the back-of-queue crash and because of the lane restrictions, the backward forming boundary speed increased to 3.78 MPH. Before the first accident was cleared, the frontal stationary boundary existed at mile marker 215, the site of the initial crash. However, with the back-of-queue crash, a new frontal stationary boundary was formed at the 213 mile marker. In addition to backward forming and backward recovering boundaries, the queue from the secondary crash also had a rear stationary boundary for a short time. Table 1 shows the duration and speeds of each of the 7 boundaries of the queue for this incident.



(a) Time-space diagram created with probe data



(b) Queue location over time

FIGURE 2 INRIX trip trace from February 2, 2015, crash on I-65

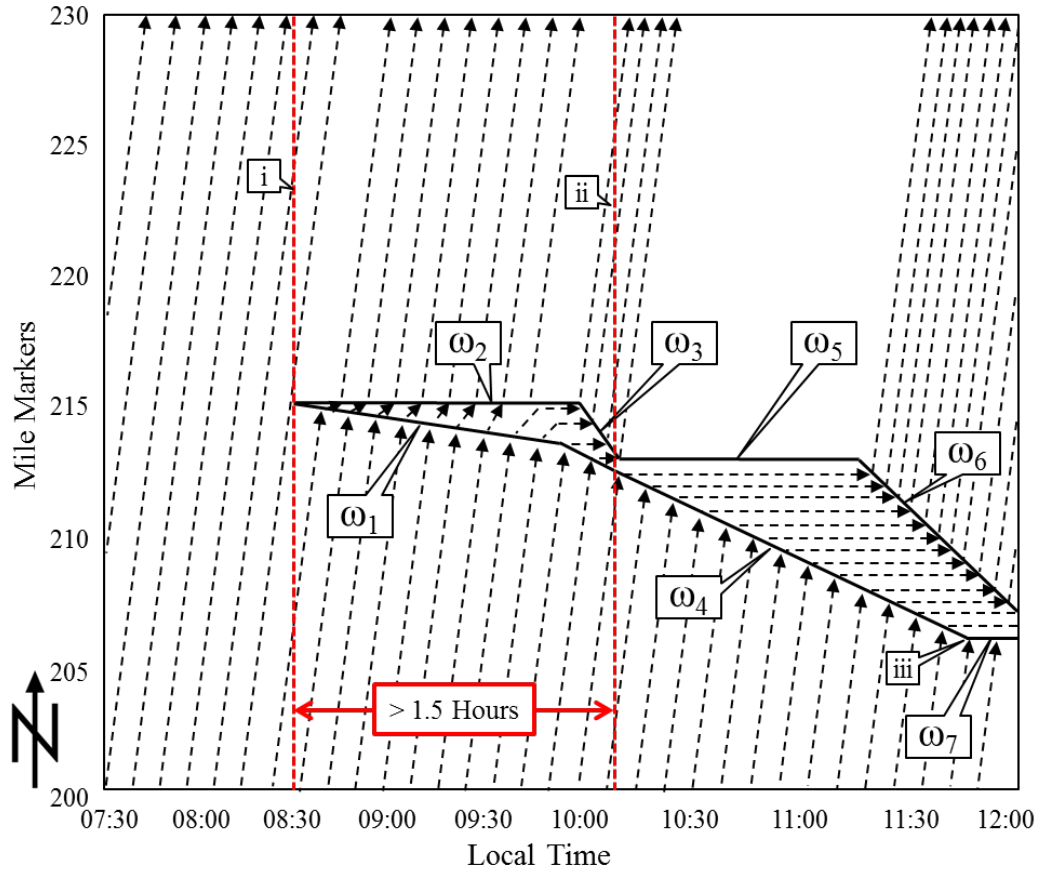


FIGURE 3 Shock wave diagram from February 2, 2015, crash on I-65

TABLE 1 Shock Wave Boundaries from February 2, 2015, crash on I-65

	Class	Duration (minutes)	Speed (mph)
ω_1	Backward Forming	87	1.03
ω_2	Frontal Stationary	95	--
ω_3	Backward Recovery	10	12
ω_4	Backward Forming	115	3.78
ω_5	Frontal Stationary	40	--
ω_6	Backward Recovery	50+	7.2
ω_7	Rear Stationary	15+	--

Crash Database

Crash data were retrieved from the state crash database. Only crashes defined as being within the specified time frame (2012-2014) and as having occurred on an interstate were retrieved. Personal information, such as names and license plate numbers, were omitted. The crash data included the number of vehicles involved, the number of trailers involved, the number of injuries and deaths, whether or not construction was associated with the crash, the primary factor or cause, the manner of collision, information on the geometry of the road, etc. It should be noted that these crash data did not use the KABCO (K = fatal, A = incapacitating injury, B = non-incapacitating injury, C = possible injury, O = property damage only) scale of severity.

Before being used in this study, the raw crash data had to be cleaned, which required extensive reading of the narrative to verify and correct database attributes. Any crash with an unknown or unreliable location was eliminated from the study data. Any crash that did not occur in the interstate travel lanes, such as ramps, was also eliminated. Lastly, only crashes that occurred on interstates of the Interstate Highway System in Indiana were used, which includes I-265, I-465, I-469, I-64, I-65, I-69, I-70, I-74, I-80, I-865, I-90, and I-94. Interstate 275 was not included in this study due to lack of probe vehicle data and that its length in Indiana is only 3 miles.

INTERSTATE CRASHES

This crash analysis was conducted in two parts. For the first part of this study, only fatal crashes that occurred in 2012 through 2014 were considered. The second part of the study looked in more detail at the 2014 crashes. This paper looked at the data in two cohorts. The longitudinal analysis of fatal crashes from 2012-2014 used the legacy TMC probe data that was available from 2012 onwards. Automated classification of 2014 crashes as occurring during congested or uncongested conditions was done using the newer, higher fidelity XD data with segment lengths approximately 1 mile in length.

Fatal Crashes

There were 230 fatal crashes total over this three year period on Indiana interstates. For each fatal crash, speed data from the crowd-sourced probe vehicles prior to and upstream of the crash were analyzed to ascertain whether or not the crash occurred at the back of a queue. The probe data were augmented by the crash report narratives. Using this method, 30 of the fatal crashes were determined to be back-of-queue crashes. Figure 4 shows a Pareto chart of the durations of queues as seen in the probe vehicle data before each of the 30 fatal back-of-queue crashes. The durations range from not seen in the data at all (5 crashes) to 6 hours. The chart also shows which back-of-queue crashes were associated with construction and which involved commercial vehicles (trucks with trailers).

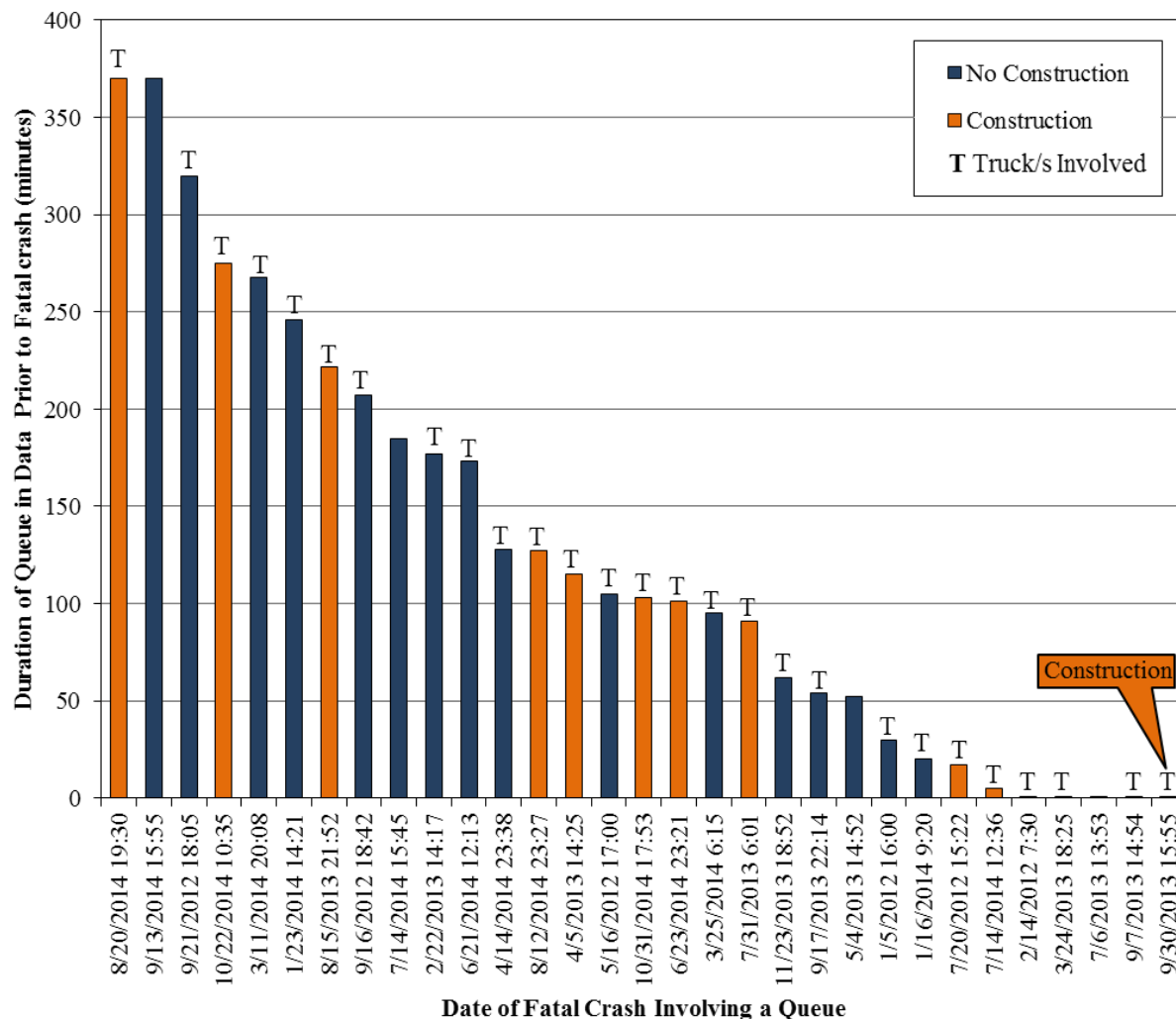


FIGURE 4 Duration of queue before fatal back-of-queue crash

Figure 5 shows the total fatal crashes and number of fatal back-of-queue crashes by year. The number of back-of-queue crashes increases over the three year period but the total number of fatal crashes does not. This could be attributed to the randomness in crash occurrence or perhaps influenced by increasing congestion. However, with only a three year sample, there is insufficient data to reach a conclusion.

In this part of the study, different possible trends in back-of-queue fatal crashes were considered and evaluated. For example, a larger percentage of back-of-queue crashes than non-back-of-queue were associated with construction. This trend is perhaps influenced by the fact that work zones cause queuing more so than non-work zones. The most significant trend found in fatal back-of-queue crashes is the involvement of one or more trucks with trailers (Figure 6). Out of all fatal back-of-queue crashes over the three year period, 87% involved at least one truck. In comparison, only 39% of the non-back-of-queue fatal crashes involved at least one truck.

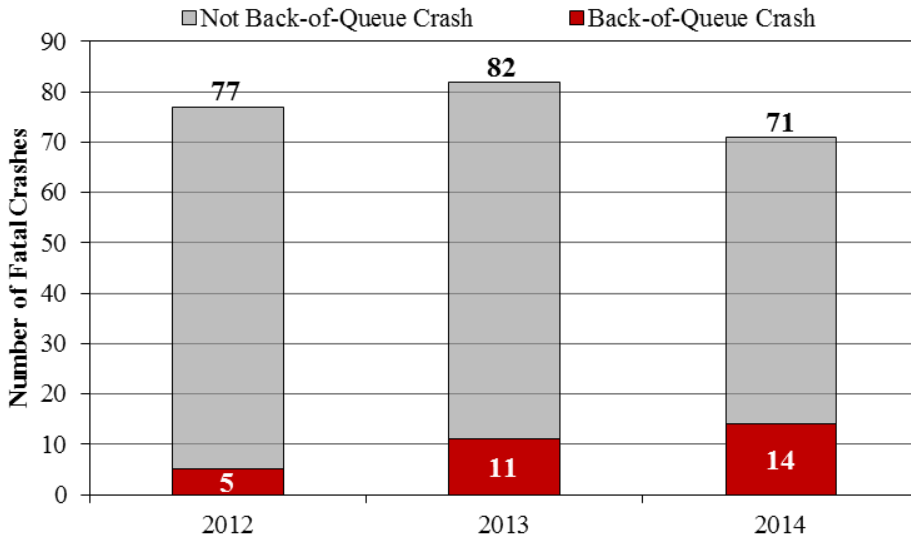


FIGURE 5 Number of fatal crashes on Indiana interstates by year

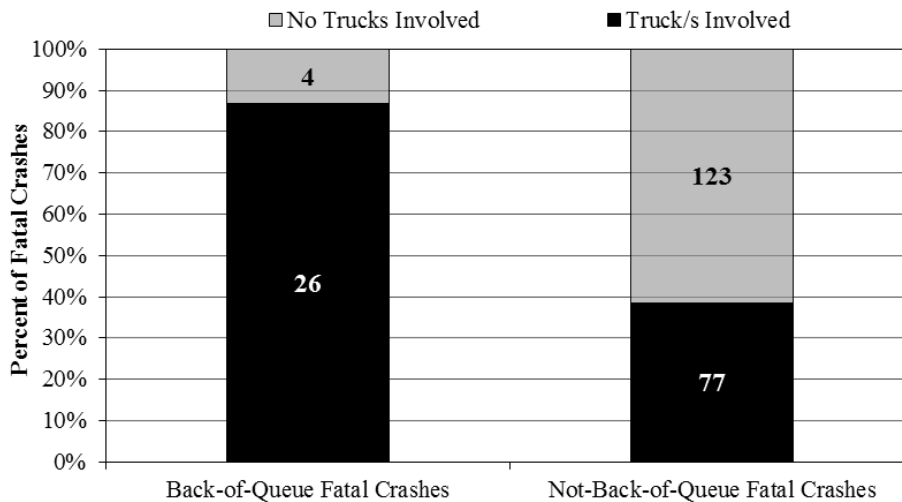


FIGURE 6 Percent of fatal crashes that involved trucks, 2012-2014

All Interstate Crashes in 2014

In the second portion of this study, crashes of all severities in 2014 were analyzed. Crash rates in congested and uncongested traffic conditions were the focus. In 2014, over 15,000 crashes occurred in the main lanes of travel on interstates in Indiana. Of these crashes, 3,448 were designated as being involved in a queue. The following subsection will describe how a new unit of exposure was developed in order to define crash rate. Then, the process for determining whether a crash was associated with a queue is discussed. Lastly, the different crash rates will be discussed.

Mile-Hours as Unit of Exposure

As discussed in the literature review, the vast majority of crash rates use volume, or some form of volume, as the unit of exposure. Many safety studies use AADT to derive volume. However, an aggregate measure of volume would be insufficient in this case since congested conditions are not adequately

represented by average measures. Some safety studies use count data as measured by ITS (intelligent transportation system) infrastructure, such as detector loops. However, no agencies have statewide coverage, particularly in work zones or rural areas. Count stations are located infrequently enough that any volume measure would still be too aggregated. Also, even if ITS devices are installed near work zones, the temporary lane use patterns often degrade the quality of data. Therefore, a new unit of exposure was developed for this study that uses crowd sourced probe data.

A mile-hour of congestion is a measure of exposure that combines the duration of a condition with the length of roadway that the condition covered. For this study, the probe vehicle data were used in calculations of mile-hours of exposure. As described above, each segment has a length and an average speed every minute. A threshold of 45 MPH was used for defining congestion. The sum of hours when the segment operated at or under 45 MPH multiplied by the segment's length is defined as the exposure of that segment to congestion. For example, a queue of 1 mile in length that lasted for 1 hour would equate to 1 mile-hour of congestion.

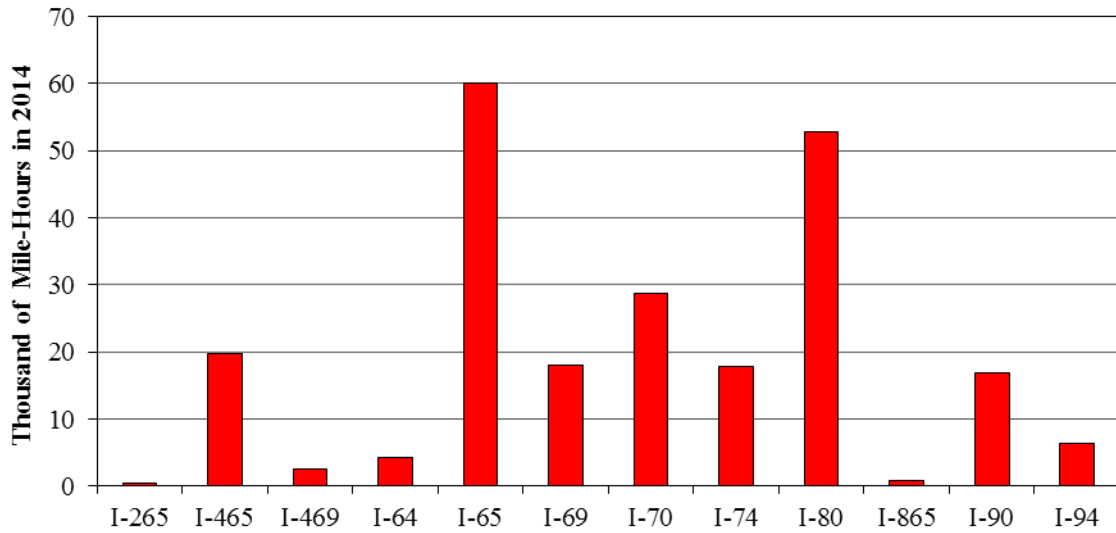
Following this idea, the crash rate is defined by the number of crashes that occurred during a certain condition and the mile-hours of exposure to that condition. In this case, the uncongested crash rate (Equation 1) uses mile-hours of uncongested conditions and the congested crash rate (Equation 2) uses mile-hours of congested conditions.

$$\text{Uncongested crash rate} = \frac{\text{Number of crashes in uncongested traffic conditions}}{\sum_{n=1}^N \text{Segment length} \times \text{Number of uncongested hours}} \quad (1)$$

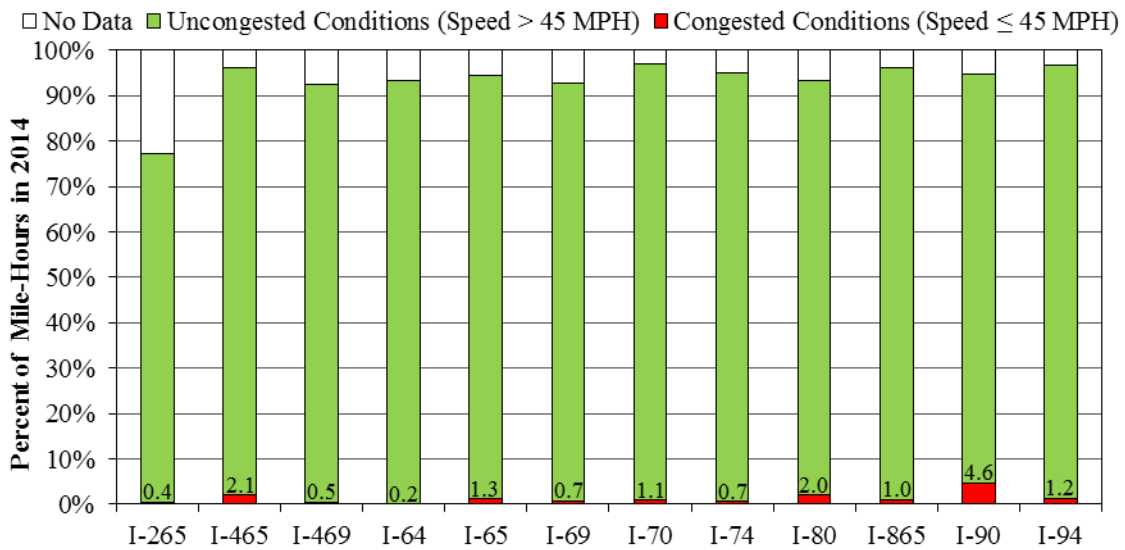
$$\text{Congested crash rate} = \frac{\text{Number of crashes in congested conditions}}{\sum_{n=1}^N \text{Segment length} \times \text{Number of congested hours}} \quad (2)$$

Figure 7a shows the total number of congested mile-hours on Indiana interstates in 2014. Congested conditions make up only 1.2% of the total possible mile-hours of operation. Figure 7b shows the percent of mile-hours operating under uncongested and congested conditions for each interstate. Interstates in Indiana experience congested conditions for a very small portion of yearly operation.

Differences between rural and urban crash rates and traffic conditions were also considered. Using the metropolitan statistical areas, based on the United States Census, of Chicago, Indianapolis, and Louisville, the interstate segments and crashes were designated as either rural or urban. Four interstates were contained entirely in urban areas: I-265, I-465, I-865, and I-90. Interstate 469 was the only interstate that was entirely defined as rural. Figure 8 shows the mile-hours of congestion as seen in Figure 7a split between rural and urban interstate segments.

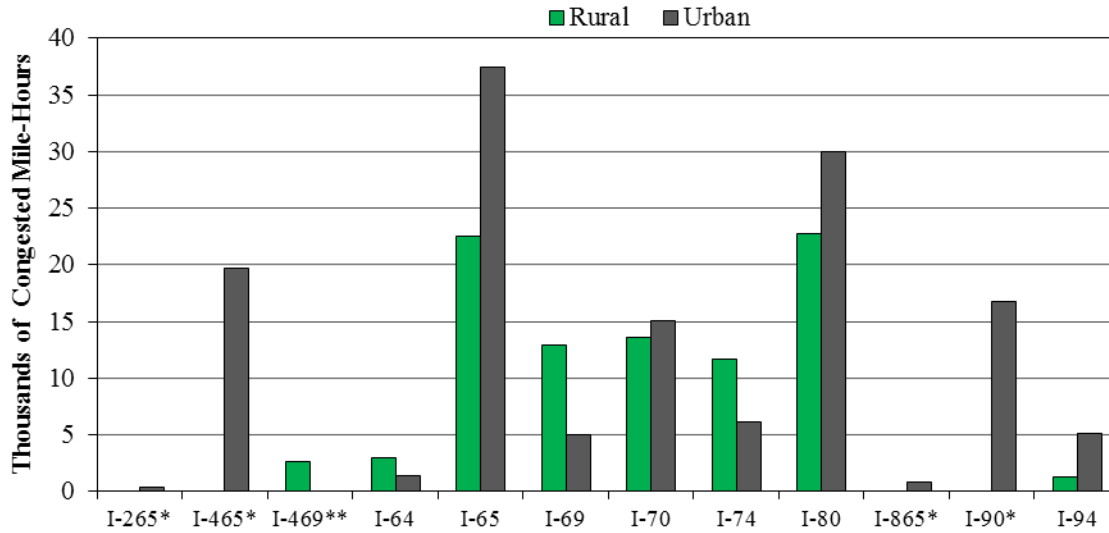


(a) Mile-hours of congested conditions (speed ≤ 45 MPH) by interstate in 2014



(b) Percent of congested vs. uncongested mile-hours

FIGURE 7 Summary of congested conditions by interstate in 2014



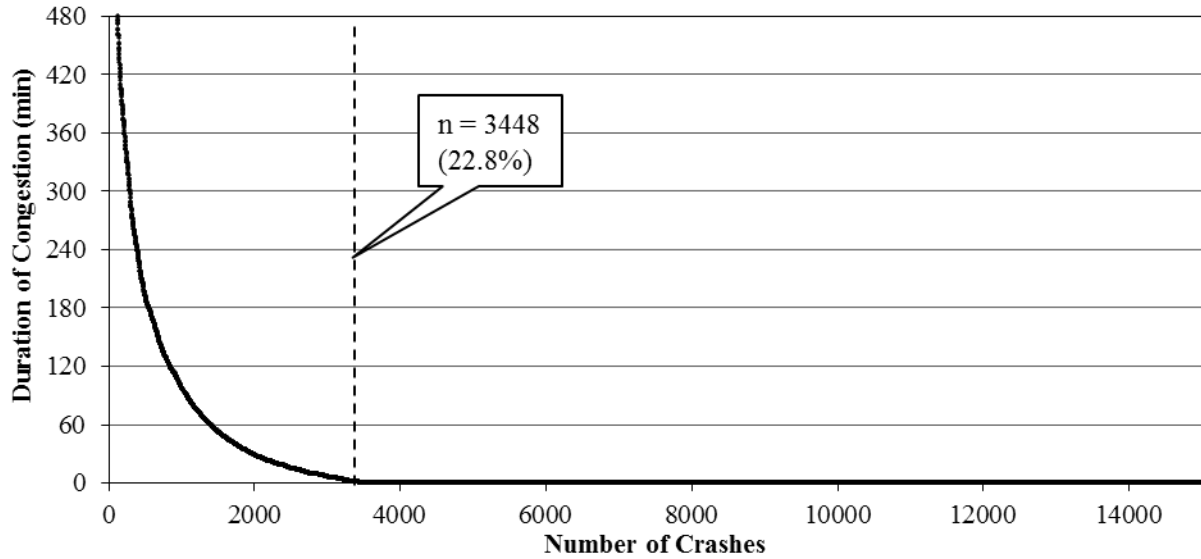
*Has only urban roadway segments
 **Has only rural roadway segments

FIGURE 8 Mile-hours of rural vs. urban congested conditions by interstate in 2014

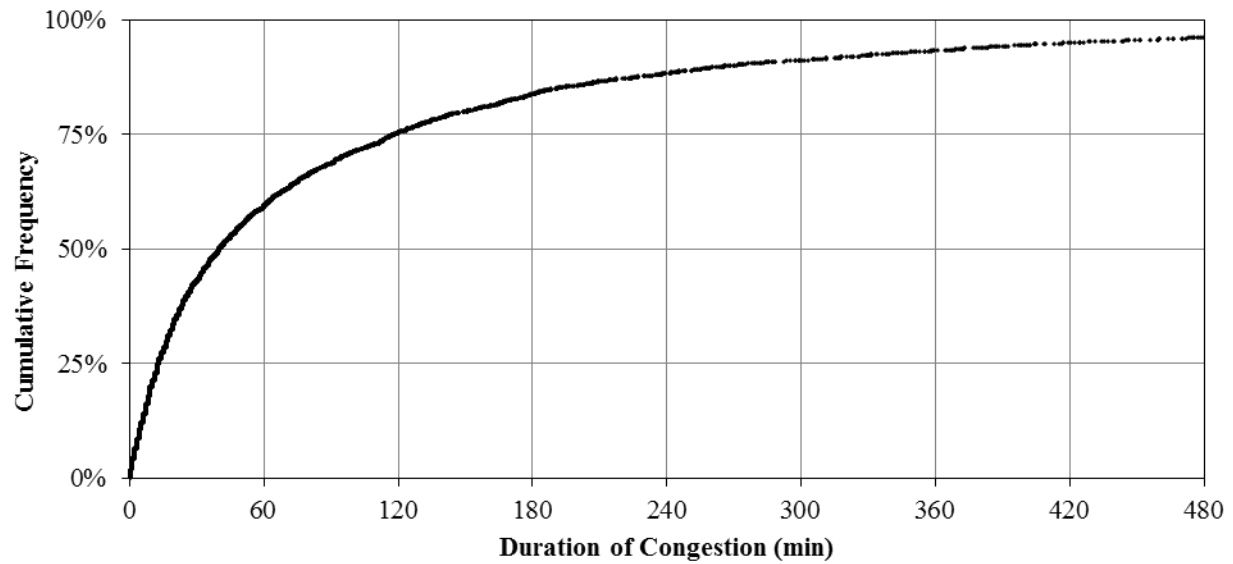
Queue Duration Algorithm

In the first part of this study, the speed data and crash reports were analyzed in-depth manually. However, this process proved to be time-consuming and would not be feasible for the 15,000+ crashes that were considered for the second part of the study. Therefore, an algorithm was developed that would analyze the speed data from a large number of crashes and provide the duration of a queue in the data as an output.

The algorithm needs only the date, time, and location (roadway, direction, and mile marker) of each crash. It takes into account the movement of a shock wave boundary between interstate segments, which may cause minor fluctuations in average speed. For example, as a shock wave enters a segment, the average speed of that segment could vary between 50 MPH and 40 MPH. While 50 MPH is above the congestion threshold, it does not mean that the queue has disappeared. A buffer period of 10 minutes is used to account for shock waves passing between segments and allows the algorithm to see a queue that exists across several segments. In summary, the algorithm evaluates the speed prior to a crash in the segment that the crash occurred. If a queue is found, the algorithm evaluates consecutive roadway segments upstream of the crash until the origin time and location is found. The difference between the origin time of the queue and the time of the crash is taken as the queue duration. As stated above, 3,448 crashes were found to have been involved in a queue. Figure 9a shows a Pareto chart of the queue durations for all crashes in 2014, similar to Figure 4 for the fatal back-of-queue crashes. Of the 15,117 total crashes, 3,448 or 22.8% were associated with congestion prior to the crash itself. Figure 9b is a cumulative frequency diagram of the duration of congestion prior to crash for each of the 3,448 congestion crashes. Approximately 90% of congestion crashes have a queue duration of 5 minutes or longer and 75% have a queue duration of 14 minutes or longer.



(a) All crashes (n = 15,117)



(b) Congestion crashes (n = 3,448)

FIGURE 9 Distribution of congestion duration before crashes on all interstates in 2014

Crash Rates

Using Equations 1 and 2, uncongested and congested crash rates were calculated for each interstate and overall in 2014. Figure 10a shows both crash rates side-by-side for each interstate. The dotted lines represent the overall crash rates. Figure 10b and Figure 10c show the crash rates segmented by rural and urban interstate segments, respectively.

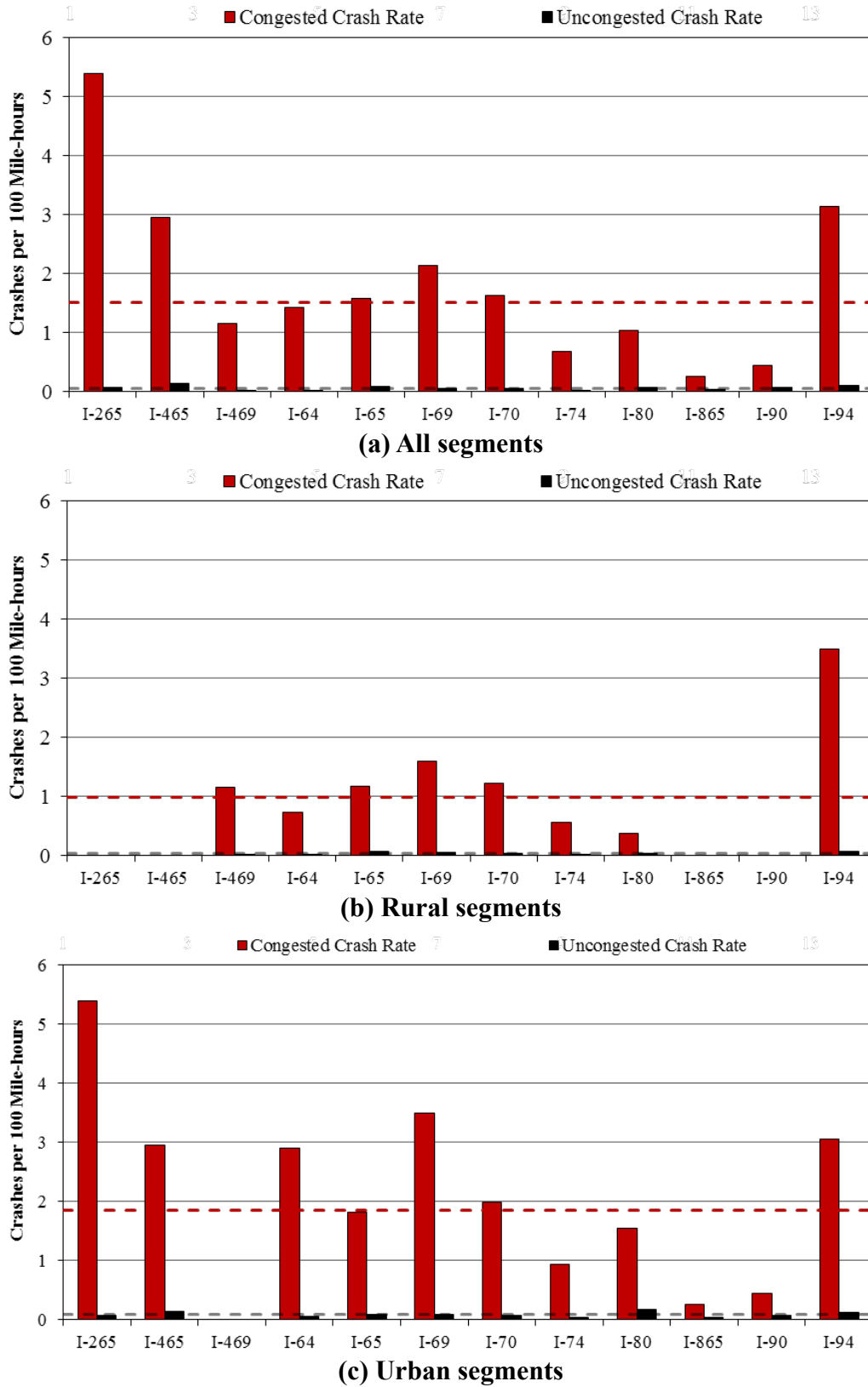


FIGURE 10 Uncongested vs. congested crash rates by interstate in 2014

The ratios between the uncongested and congested crash rates are significant. In this paper, the crash rate ratio is defined as the congested crash rate divided by the uncongested crash rate. Figure 11a shows the crash rate ratios for each interstate in 2014. The ratios range from 6 for I-865 to 69 for I-265. The overall congested crash rate is 24.1 times the overall uncongested crash rate. Figure 11b and Figure 11c show the crash rate ratios for rural and urban segments, respectively. For rural interstate segments, the congested crash rate is 23.8 times the uncongested crash rate. For urban interstate segment, the congested crash rate is 20.7 times the uncongested crash rate. The total crash rate ratio is higher than both the urban and rural crash ratios due to the congested crash rate being influenced heavily by urban conditions, while the uncongested crash rate is equally influenced by urban and rural conditions. This is expected because congested conditions are primarily located in urban environments, while uncongested conditions are shared in both urban and rural environments.

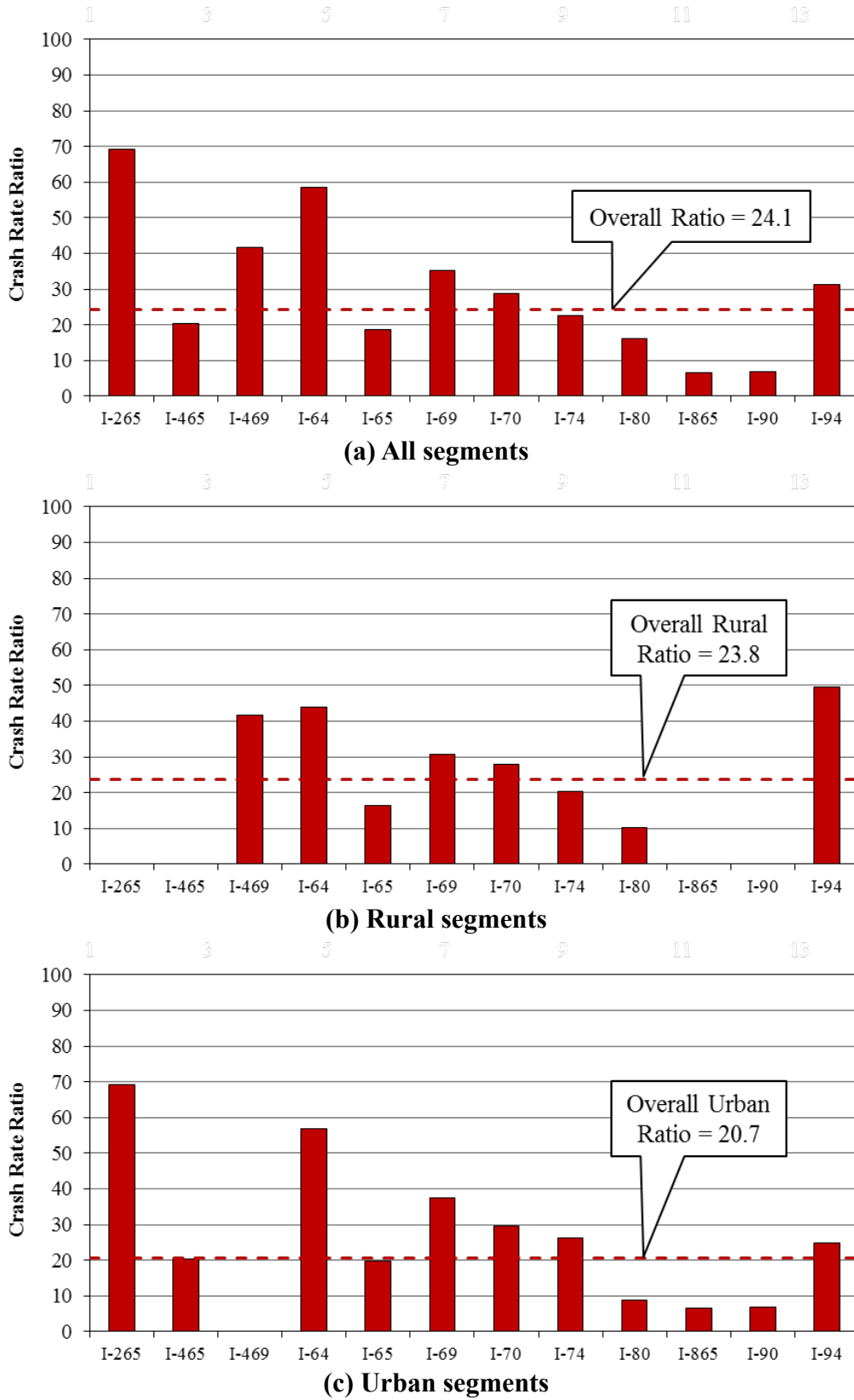


FIGURE 11 Congested/uncongested crash rate ratios by interstate in 2014

CONCLUSIONS AND RECOMMENDATIONS

The impact of congestion on crashes is quite evident from the data presented in this paper. Using crash and probe vehicle data, the following trends were found:

- Over the 3 years studied, 13% of fatal crashes occurred at the back of a queue.
- 87% of fatal back-of-queue crashes involved at least one commercial vehicle.
- Only 1-2% of the total mile-hours of interstate operated under congested conditions.
- 90% of congested crashes in 2014 had a queue duration ≥ 5 minutes
- 75% of congested crashes in 2014 had a queue duration ≥ 14 minutes
- Overall congested crash rate was 24.1 times greater than the uncongested crash rate
- Rural congested crash rate was 23.8 times greater than the rural uncongested crash rate
- Urban congested crash rate was 20.7 times greater than the urban uncongested crash rate

The data reported in this paper may be useful to designers in performing alternative analysis of mobility enhancements and work zone traffic management designs. Special consideration should also be given to congestion- and queue-management in the design of work zones. Though this study is specific to interstates in Indiana, it can be assumed that similar results would be found for interstates across the country. Longer term, this information is important to communicate to decision makers on the importance of advancing connected vehicle technology that warn motorists of queued traffic on the interstate.

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