What is the nature of the knowledge specialist teachers conceive of as deep subject and pedagogical knowledge of primary mathematics?

by

Gina Donaldson

Canterbury Christ Church University

Thesis submitted

for the Degree of Doctor of Education

2014

My thanks go to the specialist teachers who kindly gave me their time, to Robin and Mike, and particularly Viv for their support and encouragement, to Cathy, Andrew and Peter for their proof reading, and to Jamie and Peter for their patience and belief.

For Auriel and David

What is the nature of the knowledge specialist teachers conceive of as deep subject and pedagogical knowledge of primary mathematics?

Contents

Figures

Tables

What is the nature of the knowledge specialist teachers conceive of as deep subject and pedagogical knowledge of primary mathematics?

Abstract

One of the key recommendations of the Williams review of primary mathematics (2008) was for every school to have a primary mathematics specialist teacher (MaST) with 'deep mathematical subject and pedagogical knowledge' (Williams, 2008 p. 7). This knowledge would act as a 'nucleus' (p.1) for the whole school, with MaSTs supporting the teaching and learning of mathematics across the primary phase. As yet there is no model for the knowledge of these specialist teachers. This study aimed to examine the nature of this knowledge conceived of by a small sample of MaSTs, by conducting interviews as they undertook the role, and after developing it over two years and completing the Masters level training programme. The interviewer identified with the MaSTs the knowledge they conceived that they drew on in their teaching of one aspect of the mathematics curriculum and which they identified as deep subject knowledge. There were common features in this knowledge, which are argued to be indicative of the knowledge of the specialist teachers more generally. These features related to knowledge of progression across the primary phase. The MaSTs perceived that they gained new knowledge of mathematics and pedagogy which enabled them to support other staff but also impacted on their own teaching. The research found only a partial relationship between the current models which articulate the knowledge of primary classroom teachers of mathematics (Rowland et al 2009; Ball et al 2008; Ma, 1999) and the knowledge which MaSTs conceived that they drew on, and identified as deep. The research examined the relationship between the perceived knowledge of these teachers as specialists and class teachers, finding examples of case and strategic knowledge (Shulman, 1986). The MaSTs identified their new knowledge as distinct from that gained by classroom experience and valued the Masters aspects of their training programme.

Chapter 1

Introduction

In this research I aimed to explore the nature of the knowledge which Primary Mathematics Specialist Teachers conceived of as deep subject and pedagogical knowledge of primary mathematics. This is a new role in English primary schools, dating from the recommendations made by a review of primary mathematics (Williams 2008). I aimed to identify the knowledge a sample of specialist teachers conceived of as underpinning their role and, in particular, what they defined as deep subject knowledge.

My research took place in the period of 2010 to 2012, during which time two cohorts of primary teachers were nominated as specialist mathematics teachers by their head teachers and undertook a two year training programme. Prior to and during this time, primary mathematics in English schools had been portrayed as problematic. This was in response to on-going concerns for standards of children's learning of primary mathematics in England, largely the result of international comparisons (TIMSS, 2007; Mullis et al, 2008). National data had also shown a plateauing of standards of attainment for children leaving primary school, with national targets missed in 2002 and 2006 (Williams, 2008 p.16). This was despite a series of interventions in the form of The National Numeracy Strategy (DFEE, 1999) and the Primary National Strategy (DFES, 2006). It was in this context that in 2007 the Secretary of State for Education commissioned the review of primary mathematics in English primary schools and early years settings (Williams, 2008 p.2).
The response of the committee was that an emphasis on the knowledge of teachers would raise standards in children's learning:

> in-depth subject and pedagogical knowledge inspires confident teaching, which in turn extends children’s mathematical knowledge, skills and understanding (Williams, 2008 p.9).

The committee considered the demands on the knowledge of classroom teachers, arguing that raising the mathematical entry qualifications required for teaching would limit the number of teachers entering the profession, and changing the expectations and scale of Initial Teacher Education (ITE: the programme required to gain qualified teacher status in England) would not be viable. One of the final recommendations was for the training and appointment of at least one mathematics specialist teacher (MaST) in what the report defined as every reasonably sized primary school, with more than one in large schools, and access to a shared specialist teacher in small schools.

> Recommendation 3: There should be at least one Mathematics Specialist in each primary school, in post within 10 years, with deep mathematical subject and pedagogical knowledge (Williams, 2008 p.7).

Williams emphasised the role of the knowledge of the new MaSTs, claiming that both their mathematical subject knowledge and pedagogical knowledge were central in addressing concerns over plateauing standards and the attainment gap between the least and most able children. As I will explain, in choosing these particular terms of subject and pedagogical knowledge, he appeared to call on ideas first used by Shulman (1986) in discussing teacher knowledge.

The role of the MaST proposed by Williams, and used by the then Department for Children, Schools and Families (DCSF) as a basis of the training of the MaSTs, included taking a whole school leadership position in developing mathematics teaching and learning, coaching and mentoring teaching staff, offering advice on the needs of individual children including the least and more able children and leading research activity relating to mathematics. The MaST would ‘champion’ mathematics (Williams, 2008 p.1). They would provide a source of deep mathematical subject knowledge as well as pedagogical expertise, and the skills of mentoring and coaching to raise standards across the primary school. Williams proposed that the MaSTs should be provided with a part time, extended, Masters level programme to help to develop their knowledge and prepare them for their role. This programme would be provided by Universities and Local Authorities collaboratively.

Williams separated out the role of the MaST from the well-established position of the mathematics coordinator, saying that these two positions could be held by separate teachers. This would be particularly true where a school had adopted a model of one teacher taking responsibility for learning across subjects, rather than of specific subject coordinators. Williams implied that even when a school identified a mathematics coordinator or subject leader, this person was not always seen as holding specialist knowledge and expertise. Therefore he created a new role.

His particular model of the specialist primary mathematics teacher is not the only possible one. The alternative model is a specialist teacher who teaches only mathematics across the primary school, therefore releasing the primary class teacher from their teaching of mathematics. Although this has been the case in some more formal independent primary schools, English state primary schools have largely kept to the model of one class teacher providing the whole curriculum for one class each year (Alexander, 2010). Specialist teaching may occur in subjects such as modern foreign languages, music or sports coaching but not usually in core curriculum areas. Even where schools employ a system of ability setting for mathematics across a year group, sets are usually taught by class teachers who are qualified to teach the rest of the curriculum. This is despite the recommendation of Alexander et al (1992) for primary schools to consider the role of specialist teachers.

The current Coalition Government has supported the funding promised for the training of the first four cohorts of MaSTs, but have taken the opportunity at the end of this period to implement an alternative response to the continued view that mathematics remains a problem nationally (Vordeman, 2011; OCED, 2013.) Their proposal is that a number of student teachers be trained to teach solely or predominately mathematics (DFE 2011c). These specialist teachers will therefore take a different role, releasing class teachers from their teaching of mathematics. In Williams' model the expectation is that the MaST will support colleagues, both teachers and teaching assistants, in their own teaching of mathematics. Therefore this study is located in a time of exploration of the place and role of the specialist teacher of primary mathematics.

The specialist teacher is not only called on by Williams to have deep knowledge themselves, but to be a source of knowledge for their colleagues. Whereas current models for the knowledge underpinning teaching have centred around that held by the individual teacher, Williams' recommendations shifted attention to the knowledge located in and shared across the school staff, with the MaST taking a central role as a 'nucleus' (Williams, 2008 p.1) of knowledge. His focus is on knowledge distributed across a school, and across schools working together. A number of key researchers in the debate on knowledge for teaching mathematics have called for an examination of how subject knowledge is distributed in schools (Ruthven, 2011). My research examines the mathematical and pedagogical knowledge which is gained and held by one teacher, and shared with other staff.

I focus on Williams' call for deep subject and pedagogical knowledge rather than the knowledge a specialist teacher might require to take a leadership role in school. Although Williams recommended that a strand of the MaSTs' training programme prepared them with strategies for coaching and mentoring and an understanding of specific techniques such as lesson study, he gave less emphasis on this sort of knowledge.

There is currently no model or framework for the knowledge of the primary mathematics specialist teachers. The term deep subject knowledge was not fully defined in the Williams' review although it was claimed to have a pivotal role in the review's recommendations. Other documents similarly make use of the term deep knowledge, whilst not defining it. Recent guidance such as the review of professional development in mathematics in 2013 by the Advisory Committee on Mathematics Education (ACME: an English independent advisory body) states that the aim of professional development for all teachers is 'to develop deeper mathematics subject knowledge, pedagogical content knowledge and other professional learning' (ACME; 2013 p.4). The report goes on to reinforce the call for all primary schools to have access to a mathematics specialist teacher. In discussing the knowledge of these specialist teachers, it states, 'A specialist teacher is one who has received sustained post graduate training in mathematics education and who has deep subject knowledge to a level that exceeds the level to which they are teaching' (p.11). This statement is indicative of the lack of clarity around the knowledge of specialist teachers. It implies that the depth of knowledge of the specialist teacher is linked to their training, and that is distinct from that of class teachers, in that it exceeds it, but the way in which it exceeds it is not clarified. Is that in terms of depth, which is not in itself defined, or in terms of the level of complexity, relating to the curriculum for older children?

My research sits beside a large scale evaluation of the MaST programme and impact of the early cohorts of MaSTs (NFER Walker et al, 2013). Their findings suggest that the MaSTs did gain new knowledge but I will argue that it did not probe the nature of this knowledge. The evaluation left deep knowledge of the specialist teachers yet to be clarified.

Therefore my research critiques Williams' use of the term deep subject and pedagogical knowledge of primary mathematics, in order to offer the basis of an understanding of it, in the light of other frameworks for teacher knowledge. As I will explain, these include the requirements for teachers' knowledge as they engage with ITE in England and as they enter the profession and meet the Teachers' Standards for Qualified Teacher Status (the requirements to gain qualified teacher status in England). During the period of my research, the Teacher Development Agency (TDA: the body responsible for teacher education at that time in England) provided Teachers' Standards (2007) which attempted to capture how teacher knowledge might then develop through their careers and drew on the same term, deep knowledge, for Excellent Teachers and Advanced Skills Teachers. The Standards stated that these teachers should draw on 'extensive and deep knowledge of their subject' (TDA, 2007 p.11). The TDA's use of the term suggests that deep subject knowledge is a valuable and distinct degree of subject knowledge, and that there might be a continuum of the depth of a teachers' subject knowledge, as they develop from gaining Qualified Teacher Status (QTS) to the state of Excellent Teacher and Advanced Skills Teacher. Again there is no full definition of deep subject knowledge offered by the TDA, even in its attached guidance. These Standards have now been replaced with a single standard for entry into teaching (DFE, 2012a.)

In the area of primary mathematics, attention of researchers has fallen on the subject knowledge of individual classroom teachers (Rowland et al, 2009 for example). This research has resulted in models designed to articulate the knowledge classroom teachers need to teach primary mathematics, which I will consider in my study. Although research has been undertaken to explore the development of depth of subject knowledge of applicants to secondary ITE who generally do not hold a degree in mathematics (Adler et al, 2009), deep knowledge of specialist teachers has not been fully explored. In related research articles, the term is often seen in quotation marks (Ball et al, 2008 p.394), suggesting its meaning is not well understood. Neither the TDA nor Williams refer to research which shows that such deep subject knowledge is measurable, distinct from the knowledge of other teachers, or that it has a direct relationship with children's learning. This is implied.

This lack of clarity surrounding the articulation of the knowledge of specialist teachers of primary mathematics led me to formulate the following research questions:

1. What knowledge does a sample of primary mathematics specialist teachers conceive that they draw on in their approach to teaching an area of mathematics?
2. What knowledge does a sample of primary mathematics specialist teachers conceive of as deep subject knowledge?

Analysing my findings across these two questions would allow me to answer the following sub questions concerning the nature of the perceived deep subject and pedagogical knowledge:

i) Are there shared features in what a sample of MaSTs perceive as deep subject and pedagogical knowledge, or are their responses individual?

ii) If there are shared features perceived by these specialist teachers as indicative of deep subject and pedagogical knowledge, what are they? How can they contribute to an understanding of the distinctive knowledge of all primary mathematics specialist teachers?

iii) Do the current models which articulate the knowledge of primary classroom teachers of mathematics describe the perceived shared knowledge of these specialist teachers?

iv) How do these specialist teachers perceive the impact of their new knowledge on their teaching?

v) What is the relationship between the knowledge of primary mathematics, identified by these specialist teachers, as a class teacher and as a specialist?

By examining the knowledge of the specialist teachers, my research contributes to the debate on subject knowledge for the teaching of primary mathematics. I aim to articulate the deep knowledge of my sample of MaSTs, as they conceive it. I contribute original knowledge in that I focus on a new role in primary schools. I consider data gathered from the first two cohorts of teachers completing the national training programme and undertaking the role of specialist teachers: key groups of teachers for future policy development and for the mathematics education community. Furthermore, this research takes place as the government implements a new approach to mathematics specialist teachers. I begin to consider the knowledge they may develop in their training.

Summary of my methodology

My research is based on in depth interviews with eleven specialist teachers in the first and second cohort completing the training programme and undertaking the role of MaST, in 2010 – 2012. A self-selected sample of specialists teachers were interviewed at length at the beginning and at the end of the programme they undertook at the University where I am based. I set out my argument that this was a valid sample of teachers to help me to answer my research questions. In both interviews, the specialists were invited to talk in detail about their approach to the teaching of an area of mathematics of their choice. This was to mirror, to some degree, how they might share their knowledge with a member of staff, asking for advice on teaching this area of mathematics. They then discussed how they defined deep subject knowledge, as this was a relatively new term. In the second interview, the teachers had access to the transcript of the first interview, and could therefore reflect on whether their views had changed over the programme and whilst taking the role of specialist teacher for two years. The interviews therefore allowed me to identify, with the specialist teachers, examples of the knowledge they drew on in their teaching, and their more general definitions of deep knowledge: the knowledge perceived to be used and valued by the specialists. I considered whether there were identifiable, shared features of knowledge manifested by the MaSTs participating in the study, which can be argued to characterise deep subject and pedagogical knowledge of primary mathematics more generally. I analysed these features in relation to the existing models of knowledge used by classroom teachers in their teaching of primary mathematics, in order to examine how knowledge conceived by the specialist teachers to be ‘deep’, related to what has been well documented to be knowledge underpinning classroom teaching.

The practical and theoretical relevance of my research

The practical and theoretical relevance of my research concerns the nature of teachers’ professional learning for policy makers and the mathematics education community. I outline this briefly here and return to it in my final chapter.

My study provides a view of specialist teacher knowledge at a key time for mathematics education. The MaST is a new role, and this study gathers evidence from the first and second cohorts of teachers completing the training and taking the role in schools. It provides an insight into the knowledge they perceive that they develop and value during and after their training, and as they take the role for the first two years. This is important at a time when the government is considering strategies to improve the teaching of primary mathematics, introducing a new National Curriculum (DFE, 2013) and trialling alternative models of specialism.

Williams (2008) recommended that the MaST leads teaching and learning of mathematics across the school and therefore called for their knowledge of the primary mathematics curriculum across the primary phase and into Key Stage 3. The DCFS's requirements for teachers to be accepted on the programme were that they have more than two years teaching experience. Many of the teachers on the MaST programme at the university where I am based had much more than this, up to thirty years in some cases. However, they often came with much experience of teaching one or two year groups, or a section of the primary phase of three to eleven years. They typically expressed concern over their expertise in the rest of the primary phase. Specialist teachers are not required to gain experience of teaching the full primary range, but they must develop a deep understanding of the mathematics covered by the curriculum for the whole phase. Their deep subject and pedagogical knowledge cannot always be purely based on experience. My research therefore considers the nature of professional learning of teachers. Williams called for the role of the MaST to be underpinned by an extended part time programme offered and assessed at Masters degree level. The programme provided to train MaSTs was therefore validated with sixty Masters credits. Williams' recommendations were made towards the end of a period of time when the Labour government encouraged teachers to engage in Masters study (DCSF, 2007). The research spanned the end of that government and the beginning of a new one. The present government have examined alternative views of teacher knowledge, recognising craft knowledge, which is centred and learnt in schools (DFE, 2010). Although upholding the practice of countries where teaching is a Masters profession (DFE, 2010) there has been a significant cut to funding support for continued professional development at this level. My research concerned an example of the development of teachers' Masters level knowledge against this backdrop. As I will explain, I found that the sample of MaSTs perceived that the knowledge they defined as deep results not from experience alone, but from their Masters study in conjunction with reflection on their experience. In this way they perceived that they developed deep knowledge of the teaching and learning of mathematics in year groups where they had no experience. They identified that the Masters aspect of their training had been significant in their development of deep knowledge.

Furthermore, a post graduate programme, such as the one which provided training for the specialist teachers, sets learning outcomes at Masters level which relate to the curriculum for young children. My articulation of the way in which teachers develop Masters level, deep knowledge of the finite mathematical ideas included in the primary curriculum provides an example of how teachers develop understanding and knowledge of simple ideas in a masterly way. In particular, my articulation of features of deep subject and pedagogical knowledge can inform the content of the training and support for the future primary mathematics specialist teachers.

As the final data were collected for the study, the hierarchical series of standards for teachers was replaced by a single standard (DFE, 2011a). The single standard does not chart the development of teacher knowledge after gaining QTS. However, Williams required that the MaST should have at least two years' experience in schools, suggesting that the development of deep subject and pedagogical knowledge takes time and builds on experience. The final report states that the personal characteristics of potential specialist teachers should include:

> Good and secure knowledge of mathematics (this would provide a secure platform to develop a wider and deeper understanding of mathematics across the primary curriculum) (Williams, 2008 p. 23.)

This is in comparison to the new model of specialism of primary mathematics, which is supported by ITE provision, and enables newly qualified teachers to take the role of primary mathematics specialist. The term deep knowledge is in other cases used to describe the nature of the knowledge of teachers at the beginning of their careers. Research by Adler et al (2009) examines how secondary mathematics student teachers develop deep subject knowledge before their ITE as they participate in Mathematic Enhancement Courses when they do not have a degree qualification in mathematics. My research considered the nature of specialist knowledge which Williams claimed is the result of teaching experience and Masters level study. As I will explain, I explored how a sample of MaSTs perceived their knowledge to develop from that of classroom teacher to specialist, finding that the MaSTs drew on case knowledge (Shulman 1987) gained as classroom teachers in their pedagogical reasoning as MaSTs. I argue that a newly qualified teacher would not be able to draw on such depth of knowledge and that deep knowledge is often a result of reflection, typical of Masters level, on teaching experience.

My research contributes to the debate on the general nature of teacher knowledge. Primary school class teachers have traditionally met the needs of one class at a time, drawing on knowledge based around relationships with their children, gathering assessment for learning information to plan for the children's next steps. Knowledge of the specialist teachers is not based on these relationships with children. The specialist teacher advises teachers about knowledge needed to teach children she does not know and in year groups where she may have never taught.

Therefore, in examining how the specialist teachers develop their knowledge across the primary phase and in year groups where they have little or no experience, the thesis contributes to the wider debate on how new teachers develop knowledge of teaching. As I will explain, I found that this sample of specialist teachers moved between knowledge of specific examples, which they might learn as they work in a classroom, and more general overarching knowledge which they might typically learn in a university, either as part of ITE or Masters study. My research is against a back drop of a debate on teacher knowledge and whether it is best learnt in schools or Higher Education Institutions (HEIs).

My research also has implications for the current practice of teachers working together as federated schools and the introduction of Specialists Leaders in Education (DFE, 2010). This may become more prevalent given the government's new proposal for Mathematics Education

Specialist Hubs (NCETM 2013), led by Teaching Schools, which will take a lead in brokering professional development to raise standards across clusters of schools. It is increasingly common for teachers to require knowledge to champion the learning of children they have not taught themselves. In the past this has been the role of Local Authority consultants and policy makers. Teachers adopting this role bring knowledge based on their own immediate and on-going experience in their classrooms, alongside an overview of progression across the primary phase. I examine and articulate the nature of this knowledge.

The Williams' review of primary mathematics was based on a remit which portrayed primary mathematics as problematic. In recommending that the committee considered international best practice, the remit indicated the value of international comparisons, which in 2007 had suggested that children in English schools were underachieving. It has been well documented that arguments which are based on international comparisons make a number of assumptions regarding the validity of comparing practices, beliefs and curricula across countries (Askew et al 2010). However, the committee led by Williams was not asked to critique this premise, but to respond to it. One of the key messages of the review was that it was the role of the MaST, based on deep subject and pedagogical knowledge, which could address this underachievement, at least as one of a number of other recommendations. The focus of the review's recommendation was on the mathematical and pedagogical knowledge of the MaST. There was recognition of the need for MaSTs also to develop skills of mentoring and coaching, and acknowledgment of the role of attitudes to mathematics and the impact these can have on children's learning. However the key focus was on the recommendation that the MaST's deep mathematical and pedagogical knowledge could help address the perceived problem of primary mathematics. The evaluation by the NFER (Walker et al 2013) has shown that so far this has not been the case, finding limited evidence that the MaST role has impacted significantly on whole school data. Although my research has articulated that the sample of MaSTs perceived that they had developed and drawn on a distinct type of knowledge, it seems that nationally this knowledge has not yet had the impact desired by Government, of increasing the country's ratings in international comparisons. The complex nature of mathematics education has not been answered by this single recommendation.

Despite this, I believe that the role of the specialist teacher is an important one for the teaching and learning of primary mathematics. In my next chapter I go on to examine the context of my research.

Chapter 2

The context of my research

In this chapter, I examine the context of Williams' recommendation for the introduction of primary mathematics specialist teachers in English schools and in which the specialist teachers developed their knowledge.

I firstly examine the education context in 2007 which led to the commissioning of the Williams review of primary mathematics and to his particular recommendation for the role and knowledge base of the MaSTs. Secondly I consider how the Williams Review (2008) conceptualised the recommendation for the knowledge of the MaSTs. Then I review the key documents which have aimed to shape teacher knowledge during the period of my research. Finally, I consider how the role of the MaST has developed since 2008. Research which has specifically addressed the mathematical subject and pedagogical knowledge of primary school teachers will be examined in depth in my next chapter.

The context of the Williams Review

By tracing developments before the publication of the Williams review, I identify the context for the recommendation relating to the role of the primary mathematics specialist teacher.
In 2007, the Labour Secretary of State for Education charged the committee led by Williams with the remit of considering primary mathematics in England:

> Through examination of the available evidence including international best practice and through engagement with the teaching profession (Williams, 2008 p.2).

Therefore the remit of the review makes direct reference to the use of international practice as a valuable source of knowledge. There are a number of mechanisms used to compare the mathematical learning of children across nations. For example, the Trends in International Mathematics and Science Study (TIMSS) has provided on going international comparisons of performances of children on written mathematics tests. In 2008, Williams' final report was informed by data for TIMSS 2003 and 2007 which showed little progress for primary aged children in England who were ranked 7th in 2003 and 6th in 2007 amongst participating nations (Mullis et al, 2008).

The results of international comparisons are often used to fuel debate and justify changes in policy and curriculum (DFE, 2011b; Burghes, 2012). This is despite warnings of the danger of the simplistic use of international league tables (Askew et al, 2010; Smithers, 2013). However, the international comparisons were also backed up by national data. From 1998 to 1999, there had been a large increase in the percentage of children leaving primary school reaching the desired level 4 of the National Curriculum, from 59% to 69% (Williams, 2008 p.16). After that however, the percentage had steadily plateaued. The Government had set a target for 75% of children to reach level 4 by 2002 (NNS DFEE, 1999 Introduction p.2) which had been missed by 2% (Williams, 2008 p.16). A further target was set by the government for 85% of children leaving primary school to reach level 4 of the current National Curriculum in 2006, with this percentage to be maintained until at least 2008 (Slavin et al,2009). This was missed by 9% in 2006 and 8% in 2007 (Williams, 2008 p.16).

Whether or not international comparisons are sound or whether average attainment is indeed best described by level 4 of the current National Curriculum, what is important for my research is that the Williams review was a response to the belief that attainment in primary mathematics in England was problematic, based on national and international data.

The lack of significant progress by 2007 was despite a number of initiatives which had promised to improve England's national and international performance. For example there had been revisions of the National Curriculum (DES, 1988; DFEE, 1999) and the statutory requirements for the Early Years Foundation Stage (QCA, 2000; DFES, 2007). The foreword of the National Curriculum in 1999 states, 'The National Curriculum lies at the heart of our policies to raise standards' (DFEE, 1999 p.4.) When the statutory curriculum did not on its own raise standards, from 1999 onwards, increasingly detailed and prescriptive sets of guidance to support the teaching of the primary mathematics National Curriculum were commissioned by the Government. These initiatives centred round support for the classroom teachers of mathematics. The National Numeracy Strategy (NNS) in 1999 (DFEE, 1999) and the Primary National Strategy (PNS) from 2006 (DFES, 2006) provided detailed support for the planning and teaching of the daily mathematics lesson. This ranged from yearly planning to medium and lesson planning (NNS DFEE, 1999 Introduction p.2). However the initial increase of children at the end of Key Stage 2 gaining level 4 of the National Curriculum in 1998 – 1999 was not sustained (Williams, 2008 p.16).

A critique of the Williams Review (2008)

Despite these interventions the Government was concerned enough about national results for children leaving primary school, and international comparisons, to commission the Williams review.

In 2007, the Labour Secretary of State for Education charged the committee led by Williams to:

> consider and make recommendations in the following areas: …What conceptual and subject knowledge of mathematics should be expected of primary school teachers and early years practitioners, and how should Initial Teacher Training and continuing professional development be improved to secure that knowledge? (Williams, 2008 p.2).

The final report and recommendations were published in June 2008, with the claim that these were based on consultation and evidence collected from schools and early years settings. It took the form of an 88 page document with a bibliography of 44 items which included a large number of academic research findings as well as other government commissioned reports.
The focus was on teachers and practitioners in primary and early years settings, rather than the curriculum which was under review by Rose (2009). Williams identified the central place of teachers in children's learning, and the confidence and expertise which he claimed stems from deep mathematical subject and pedagogical knowledge (p.9). In using these phrases he was making at least an oblique reference to the terms used by Shulman (1986, 1987) but there is no reference to Shulman's work. Williams did not suggest that MaSTs would need to draw on any other of the domains of knowledge identified by Shulman such as curriculum knowledge. I will discuss these domains in my next chapter.
A chapter of the final report was devoted to Williams' reflections on teachers' knowledge. There was a lengthy discussion of the possibility of changing the mathematical entry requirement into ITE and the committee reluctantly rejected this on pragmatic grounds. They also acknowledged, later on in the review, that research had shown limited link between teachers' mathematical qualification and children's mathematical learning (Askew et al, 1997). A proposal for extending the scope of ITE was also rejected, on the basis that this would not be viable and also that ITE was not appropriate to support deep subject and pedagogical knowledge. In this argument Williams drew on responses to the consultation by HEIs offering ITE and from research suggesting that some of the practices typical of HEIs, auditing subject knowledge for example, do not necessarily promote depth of knowledge (Brown et al, 1999).

The key recommendation therefore, to support all teachers' knowledge, was for the role of the MaST in every school to champion the subject and to act as a 'nucleus' (p.1) for subject specific pedagogy. This role was to build on the traditional position of mathematics coordinator, and was compared to the roles of Leading Mathematics Teacher, Advanced Skills Teacher and the Scottish subject champion.

Williams also detailed the cost of the project across the country as:

> The total cost of this programme over 11 years of £187 million averages less than £20 million per annum, and should be seen as an investment in the nation's future, not as a cost. It represents an increase in the employment costs of the total primary teaching force of less than 0.15 per cent per year (Williams, 2008 p.31).

The remit for the review asked the committee to consider conceptual and subject knowledge of mathematics required by teachers of primary mathematics. Interestingly these terms were separated, suggesting therefore that they might be distinct types of knowledge and that subject knowledge is not conceptual (Rowland and Turner, 2008). In discussing the knowledge required of the MaSTs to fulfil their role, the final report usually used the terms deep subject and pedagogical knowledge, but there were examples of the use of other terms.

In referring to mathematical knowledge of the MaSTs, the report referred to 'subject mastery' (p.6), 'mathematical competence' (p.7) as well as 'deep subject knowledge' (p.7). In some sections, the report did not clearly articulate the degree of subject knowledge needed. It referred to the MaST having 'sound' knowledge… sufficient to articulate and share a clear vision for mathematics within the school' (p.20). Sound and sufficient suggest less depth. Williams says that depth of knowledge is essential but does not detail what characterises deep knowledge. The report does state that the present entry requirement for teaching, a grade C at General Certificate of Secondary Education in mathematics, does not require deep knowledge, but again does not specify why not. Similarly, where Williams suggested that deep subject knowledge was not best promoted through the auditing procedures of ITE, he implied that the promotion of deep subject knowledge was more than checking for inaccuracies, although how deep subject knowledge is typified was not fully discussed.

In the discussion of pedagogical knowledge, a number of terms were again employed. Generally the phrase 'pedagogical knowledge' p.7 was used but there was also mention of 'pedagogical skill' p.21. The final report referred to research by Hill et al (2005) to discuss mathematical knowledge for teaching, including this in its discussion of pedagogical knowledge. There was then an outline of this sort of knowledge which did include aspects of pedagogy: the use of representations, task design, and the use of mathematical dialogue. However there were also mathematical aspects listed, for example the knowledge needed to know if a child's work is correct and whether a method is generalisable. Therefore there was a blurred distinction in the report between mathematical and pedagogical knowledge.

Williams' use of the term 'deep subject and pedagogical knowledge' (p.7) also lacks clarity. It is not clear whether both subject and pedagogical knowledge are required and expected to be deep. In the final report there is only one reference to deep pedagogical knowledge (p.63) but six references to deep subject knowledge. Therefore I have assumed that the adjective deep is particularly related to subject knowledge. However this points to a further lack of clarity in the final recommendation.

The clear message of the report was that it was a combination of both subject and pedagogical knowledge which the committee believed has the most impact on children's learning.

> Hence, while in-depth mathematical knowledge and pedagogical knowledge do not separately represent sufficient conditions in their own right for successful teaching, taken together they constitute a necessary condition to progress learning for all children up to the end of Key Stage 2, which prepares them well for Key Stage 3. In this context, in-depth subject and pedagogical knowledge inspires confident teaching, which in turn extends children's mathematical knowledge, skills and understanding (Williams, 2008 p.9).

There is no direct evidence offered for this claim, despite an earlier reference to research by Hill et al (2005). The inclusion of references to research findings relating to teacher knowledge portrays an openness of the committee to this sort of evidence, but the lack of research on the qualities of deep knowledge, and its impact on learning, limited their discussion. As I have explained, this lack of clarity on the nature of specialists' subject and pedagogical knowledge fed into my choice of research question.

The review discussed mathematics professional development in some detail, commending the then Government's intention to promote Masters study for teachers and claiming that this would impact on teachers' subject and pedagogical knowledge. It noted that teachers have different balances of subject and pedagogical knowledge and that professional development should be flexible to acknowledge this. In discussing the popular cascading model of professional development, the review noted that much professional development was reliant on the quality of the subject leader in each school. It argued for an in house model of professional development, calling for MaSTs to be equipped with the skills of coaching and mentoring, and an understanding of techniques such as lesson study. This would support a distributive (Ruthven 2011) model of subject and pedagogical knowledge.

The professional development for the MaSTs themselves was discussed at length. This was to be provided by Local Authorities and HEIs working collaboratively, and would be at a Masters level. The report recognised that the Local Authority consultants would themselves require professional development. The programme would require the MaSTs to engage with mathematics which might be new to them and might involve re learning areas of mathematics if necessary. Aspects of pedagogy would also be covered by the programme, and suggestions for these aspects were listed specifically and included knowledge of problem solving, progression, children's common misconceptions, and representations of mathematical ideas.

These recommendations were the foundation of the MaST programme and role, which is the centre of my research.

Contextual factors which shaped teacher knowledge before and during my research

Next I consider the statutory requirements and reports which I identify have shaped teachers' knowledge of primary mathematics, and which have therefore impacted on the knowledge development of the sample of MaSTs I interviewed.

Teachers' knowledge of primary mathematics is shaped by the statutory National Curriculum and Early Years Frameworks. This is due not only to their statutory status, but also because they form the basis of high status end of Key Stage national testing (DES, 1999 p.18). The National Curriculum (DES, 1999) sets out the content for children aged 5 – 11 years in a largely hierarchical sequence. It represents the core knowledge of children, and therefore of teachers. The current National Curriculum (DES, 1999) promotes a view of mathematics as a way of thinking and problem solving (p. 61). It is seen as a subject of enquiry, with problem solving and investigation as its core processes, and Using and Applying mathematics at the centre of the curriculum (p.62). The calculation policy promoted by the National Curriculum clearly identifies the need to teach mental, written and calculator methods of calculation (p.69 – 70). This version of the National Curriculum was the one taught by the MaSTs during my research. Its programmes of study are presented for each Key Stage rather than for each year group. During the final year of the research the new government began a consultation on a new curriculum. Although its final version was produced in 2013 and after the research data had been collected, discussion about its form and content was active during the last year of data collection. The new curriculum has clear aims relating to fluency, reasoning and problem solving (DFE, 2013 p.3). It states that it sees mathematics as creative and highly inter-connected in nature (p.3). There is a dual emphasis on children's conceptual understanding and procedural fluency (p.3) alongside a clear acceleration and raising of the expectations for children's learning. The programmes of study, laid out in year groups, give a high priority to rapid recall of number facts, fluency and practise with formal calculation methods (for example p.32). Guidance on the use of calculators is that these should be delayed as much as possible (p.3). Children's own methods act as a temporary staging post towards formal methods. A non-statutory appendix supports teachers' knowledge of these formal methods (p.46).

The curriculum for the Early Years Foundation Stage has undergone similar changes, with new versions provided in 2000, 2007 and 2012. The detail of the curriculum has changed and has followed a similar pattern to the National Curriculum, with the newest version expecting that children in the reception year will cover mathematics previously thought to be suitable for children in Year 1, for example in doubling and halving small amounts and numbers (DFE, 2012 p.9).

Therefore over the period of the research and beyond, there have been requirements that teachers' knowledge must support them with meeting increasingly higher standards for children. Their knowledge must promote both their teaching for children's conceptual understanding and procedural fluency, with an increasing focus on knowledge to promote fluency in more formal methods of calculation. The sample of MaSTs worked within these statutory frameworks and requirements.

As I have explained, two versions of detailed guidance on teaching the National Curriculum were provided in order to meet targets for children's attainment before the Williams' review of primary mathematics. The NNS and PNS guidance are significant for my research as they previously provided support for teachers' subject and pedagogical knowledge.

The National Numeracy Strategy (NNS) (DFEE, 1999) provided guidance for the teaching of what it termed numeracy. The guidance defined numeracy as

> a proficiency which involves confidence and competence with numbers and measures. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. Numeracy also demands practical understanding of the ways in which information is gathered by counting and measuring, and is presented in graphs, diagrams charts and tables (DFEE, 1999 p.4).

Although a wide definition, there is a clear focus on number. In 2006, the Primary National Strategy (PNS) (DfES, 2006) replaced the NNS and became more complex and prescriptive in nature. The PNS used the term mathematics rather than numeracy.

The approach to calculation of the NNS and PNS clearly stated an emphasis on both mental calculation and the need for all children to be taught at least one standard written method of calculation, in each operation, in the later primary years. Progression towards these written methods was provided in detail, including part written, part mental methods (NNS DFEE, 1999 Introduction p.7).

The approach of the NNS to the Using and Applying of mathematics emphasised the teaching of word problems (NNS DFEE, Year 3 medium term planning). The PNS also promoted an interpretation of using and applying mathematics as the teaching of word problems, but increasingly supported teachers in their knowledge of more open ended problems (PNS Finding all possibilities).

Both the NNS and the PNS provided professional development materials to specifically support teachers' subject knowledge. These were provided for courses offered by Local Authorities, usually delivered by Strategy funded or part funded consultants, or by mathematics coordinators working with their colleagues. Detailed guidance in the form of a script was provided for those delivering the materials, leaving little room for individual teachers' needs to be met. Sessions tended to look at areas of mathematics briefly, such as considering ratio and proportion in 85 minutes before moving on to other areas (PNS Professional Development materials). Subject knowledge was linked closely to the NNS or PNS. It was rare to find links to other sources of ideas or mathematics outside the PNS guidance. In short, the materials were designed to support the teachers in delivering the PNS or NNS.

The NNS and PNS dictated the nature of the usual previous support for the knowledge of the sample of teachers taking the role of MaST in my research. In their first interview, I asked my sample of MaSTs to talk about their recent previous professional development in mathematics and out of 11 of these teachers, 9 stated that since their teaching qualification, their professional development in primary mathematics had been related to the NNS and PNS. Of the two remaining teachers one had received no professional development since her degree, and one, a teacher from a special school, had received professional development from a group of special schools, the only teacher to have had development not purely related to the NNS or PNS.

The Standards for Teachers, which lay out the knowledge required to gain Qualified Teacher Status (QTS) are an important part of the context of the development of knowledge of the MaSTs in my research. I will briefly review how these have developed before and during the period of my research. These standards are important for my research in that they articulate the baseline knowledge of a teacher and therefore what is seen as valuable in terms of teacher knowledge. I aimed to consider how these MaSTs conceived of their knowledge in relation to that of a class teacher. This was for two reasons. It enabled me to examine the relationship between the knowledge of a classroom teacher and that of a MaST, and furthermore how the knowledge gained by experience as a class teacher across the age range of the school is related to that of the MaST.

Since 1999 a development which has shaped teacher knowledge in England as student teachers enter the profession has been the Professional Skills tests. These have included a numeracy test for trainees gaining QTS and now for any applicant for ITE (DFE, 2011c.) Trainees are tested on the skills needed for the professional role of teaching any subject, not just primary mathematics. The Government Education website firmly distinguishes between the knowledge it tests and the subject knowledge needed for teaching (DFE Professional Tests). This test has shaped what is seen as general knowledge for teaching. It seeks to identify mathematical inaccuracies. The Standards which have been used over a period of time as a benchmark for entry into QTS have used the term secure subject knowledge, reinforcing the view that the baseline is accurate knowledge. For example, DES (2002) included in the professional standards the requirement that those awarded QTS had a secure understanding of the subjects they were trained to teach, including details of the NNS, and were aware of the curriculum of the adjacent Key Stages. The guidance stated that secure subject knowledge enabled teachers to break down concepts and sequence ideas, answer children's questions and identify misconceptions (DES 2002, p.18).

In 2007 the TDA published a new set of standards designed not only to provide a bench mark for teachers entering the profession, but to chart their development afterwards:

> The framework of professional standards for teachers set out below defines the characteristics of teachers at each career stage. Specifically it provides professional standards for:
>
> • the award of Qualified Teacher Status (QTS) (**Q**)
>
> • teachers on the main scale (Core) (**C**)
>
> • teachers on the upper pay scale (Post Threshold Teachers) (**P**)
>
> • Excellent Teachers (**E**)
>
> • Advanced Skills Teachers (ASTs) (**A**)
>
> Professional standards are statements of a teacher's professional attributes, professional knowledge and understanding, and professional skills. They provide clarity of the expectations at each career stage. (TDA, 2007 p.1).

In this progression of standards, TDA (2007), the Standards for the award of QTS again used the term secure knowledge. They required that student teachers had:

> a secure knowledge and understanding of their subjects/curriculum areas and related pedagogy to enable them to teach effectively across the age and ability range for which they are trained (p.9).

Core Standards for the end of the induction period still required:

> secure knowledge and understanding of their subjects/curriculum areas and related pedagogy (p.17).

Post threshold Standards required that teachers had;

> a more developed knowledge and understanding of their subjects/curriculum areas and related pedagogy (p.23).

The Standards for Excellent Teacher and Advanced Skills Teachers required:

> an extensive and deep knowledge and understanding of their subjects/curriculum areas and related pedagogy gained for example through involvement in wider professional networks associated with their subjects/curriculum areas (p.27).

This set of standards for teachers was current during my research. It provided a backdrop to my exploration with the specialist teachers as to how they understood their knowledge to develop as specialist teachers in relation to that of class teachers. However it did not help to distinguish fully the features of deep knowledge.

DFE (2011) set out the new standards to be implemented from 2012. Standard 3 of 8 Standards requires that teachers demonstrate

> good subject knowledge and curriculum knowledge.. have a secure knowledge of the relevant subject(s) and curriculum areas, foster and maintain pupils' interest in the subject and address misunderstandings… if teaching early mathematics, demonstrate a clear understanding of appropriate teaching strategies (DFE, 2011 p.5 – 6).

Now a single set of standards charts the entry into qualified teacher status, with no further progression. The current Teachers' Standards therefore do not contextualise the MaSTs' development of their knowledge as they take their new role.

A number of key reports shaped primary mathematics both prior to the publication of Williams' recommendation for the specialist teacher role and during the period of my research. These reports therefore represent the context of my findings. Many of these were commissioned and endorsed by the government.

At a time of accountability, inevitably the most influential of these have been provided by the Office for Standards in Education, Children's Services and Skills (Ofsted: the body charged in England with regulating education). These reports valued certain aspects of primary mathematics and therefore shaped teacher knowledge.

Ofsted reported on mathematics in 2008, 2011 and 2012, summarising their inspections of schools across the country in 2008 and 2012, and a smaller sample of schools in 2011. The key message from the reports was that significant rises in standards had slowed down, especially in Key Stage 1 (Ofsted 2012, p.9). There was an increasing gap between the least and most able children, with those appearing as less able as they begin school never catching up. Children eligible for free school meals were particularly less successful than their peers (Ofsted, 2012 p.8). All three reports called for teachers to focus on children's understanding rather than their learning of separate skills (Ofsted, 2011 p.4). Each report also noted the need for an increased focus on using and applying mathematics (Ofsted, 2008 p.9). In providing a brief summary of each of these reports separately, I have identified the points made concerning teachers' knowledge.

Ofsted 2008 employed the term 'subject expertise' (p.3) which it defined as teachers' subject knowledge and their understanding of the teaching and learning of mathematics. It provided examples of good, satisfactory and weak mathematics teaching, and in some cases these were linked to teacher knowledge. Teachers' vague subject knowledge characterised satisfactory teaching in one example (p.6), and significant inaccuracies and gaps in subject knowledge was linked to weak teaching (p.12).

Ofsted's report in 2011 summarised a review of a much smaller sample of what was judged to be successful schools, twenty in all, with ten of these being independent schools. Most of the independent schools provided specialist teaching of mathematics from Year 4 or 5 (p.7). The report states that these successful schools recognised the importance of good subject knowledge and subject specific pedagogy, and sought to enhance these (p.7). The schools often called on a subject leader with a high level of mathematics expertise. One such subject leader was a qualified MaST (p. 7).

Ofsted 2012 identified variety in the quality of teaching of mathematics, even within a school. Teaching in Year 1 was a particular concern (Ofsted, 2012 p.9). It called for robust subject specific feedback to tackle weak teaching (p.7). Again, examples were given of the impact of weak or incomplete subject knowledge of class teachers and also teaching assistants (p.34). A key recommendation of Ofsted 2012 was that the DFE promoted the enhancement of subject knowledge and subject specific teaching skills during ITE (p. 9). School based curriculum guidance and professional development was recommended to support knowledge of all staff (p.10). Specifically the report suggested that professional development might support teachers' understanding of progression of strands of the mathematics curriculum (p.10). The report noted that some subject leaders did not have depth of subject knowledge or lacked confidence or experience of teaching across the age range covered by the school (p.56). I will discuss these points later on as they link to my research findings.

Ofsted also provided exemplar descriptors for judgements of mathematics for each of their grades in 2012 and 2013. Although again towards the end of the period of my research, these descriptors indicate the thinking of Ofsted as to the features of outstanding practice and therefore the knowledge which underpins it. For example in 2012 Ofsted described outstanding quality of teaching as supporting children in making connections between topics, and between mathematical ideas and subject itself (p.3). It also noted that outstanding teaching draws on specialist knowledge and understanding of subject specific learning. Outstanding subject leadership is characterised as informed by a ‘high level of subject knowledge, subject-specific pedagogy and vision’ (Ofsted, 2012 p.6.)

I consider that a further set of reports have had an impact on the context of the MaSTs I worked with, if in a more indirect way, and therefore are significant for my research findings regarding their knowledge. The Rose recommendations (2009) for a review of the primary curriculum had been based on a large scale consultation and aimed to promote in children a love of learning (Rose 2009 p.4). There was a call for less prescription in the curriculum and room for flexibility. Rose presented a model for the curriculum which enabled cross curricular as well as subject based teaching (p.10). A change of government meant that this model was never put into practice, but many MaSTs reported having gone some way in their preparation for it.

The Cambridge Primary Review (Alexander, 2010) provided an in depth critique of primary practice and curriculum, again drawing on wide scale consultation and research. This review recognised the central place of teachers’ subject knowledge and questioned whether specialist or semi specialist teachers should support class teachers.

In 2011 a committee led by Vordeman was commissioned by the present government, just before it came into office, to advise on mathematics education. Their report outlined the failings of the English system in providing a world class mathematics education for all children and made a series of recommendations. This included the need to address the mathematical subject knowledge of primary teachers (Vordeman, 2011 p.7).

The role of MaST after 2008

Since the publication of the Williams’ review, England’s mathematical performance has continued to be portrayed as a concern. The latest TIMSS results for mathematics placed England as ninth for 9 – 10 year olds among participating nations, with the highest performing countries including those typically from the Asian Pacific Rim, led by Singapore (Sturman et al, 2012). The Organisation for the Economic Co-operation and Development (OCED) reported England as 21 out of 24 for numeracy in its survey of adult skills (OCED, 2013).

The previous government had accepted Williams' proposal that every school should have a fully trained specialist teacher of primary mathematics within the next ten years. It adopted a funding model for the first four cohorts, with a gradual withdrawal of central funding and encouragement for schools to contribute to the programme fees during this period. After the fourth cohort, the expectation was that there would be a move towards a market model, with no central funding (Walker et al, 2013 p.14). In 2010, the new government referred specifically, if rather neutrally, to the role of the MaST:

> We need more specialist mathematics teachers in primary schools and will encourage and support schools in developing this specialism (DFE, 2010 p.45).

Although the current government has maintained the existing funding model, with cohort 4 of the programme receiving funding for a proportion the first year of the programme, an alternative model of specialism was proposed in 2011.

> More schools should be able to employ primary teachers that they can deploy as specialist subject teachers in the sciences, mathematics, languages or other subjects. For the allocation of ITT places from 2012/13, TDA will prioritise primary courses that offer a specialism, particularly in the sciences, mathematics or modern languages (DFE, 2011c p.7).

New programmes have begun to train these student teachers in 2013 to take the role of primary specialists, trained to teach purely or predominately mathematics. The Government has not provided a rationale for the change of model. Further research will consider the knowledge of these specialist teachers, and I intend that my findings will contribute to this debate.

There has been little large scale academic research on and with the existing specialist teachers. Research projects have involved some MaSTs themselves in examining their own development in their role, or undertaking research into aspects of mathematics education, often as part of their training (Milik and Boylan, 2013). Studies also report how teachers are prepared to be specialists in primary mathematics in other countries (Al Zahrani and Jones, 2012.)

The most significant research on the role of the MaST was published in 2013 by the National Foundation for Educational Research (NFER). The NFER were commissioned to undertake a large scale evaluation of the programme and impact of the role of the MaST (Walker et al, 2013). This evaluation predominately focused on cohort 2 of MaSTs during 2010 to 2012, the same period as my research. It drew on evaluative data collected from MaSTs, their Head teachers, children in their schools, Local Authorities and HEIs delivering the programme. This data was largely collected from a survey, with 21 MaSTs included in case studies which involved interviews. There was comparison of MaSTs' school improvement data with that of similar schools. The report showed that although in every other instance the role had proved successful, there was, at that point, little evidence of a positive impact on whole school improvement data.

Significantly for my research, the MaSTs were asked by the NFER to rate their confidence in their mathematical knowledge after the programme as part of the survey. Results were very positive. For example, 92% of over 300 MaSTs felt very confident or confident and 73% very confident in mathematics at Key Stage 2. The reported confidence had been gained across all key stages and in specific areas of mathematics. The most improvement in confidence had been in using and applying mathematics (p.126). MaSTs noted that they had gained new knowledge of mathematics in year groups where they had little or no experience. However the NFER report uses the term confidence in subject knowledge rather than deep subject knowledge, so there is no use of this opportunity to gather the views of the large sample of MaSTs as to their conceptions of deep knowledge.

The conclusions from the survey were backed up by analysis of the smaller number of case studies.

> Several of the case-study MaSTs reported improvements in their mathematics subject knowledge as a result of participating in the programme. This applied both to the Key Stages in which MaSTs themselves taught, as well other Key Stages taught within the school. This was particularly notable in instances where MaSTs had worked with colleagues to improve continuity between Key Stages, and to draw out connections and relationships between different mathematics topics (Walker et al, 2013 p.24).

The report states that the case studies showed that the MaSTs had been supported in addressing areas of weaknesses in their own mathematical knowledge and in becoming increasingly reflective about their subject knowledge (p.129). The case studies provided little further information about how the MaSTs perceived deep subject knowledge. Therefore, although the NFER evaluation reports changes in confidence in subject knowledge of MaSTs, it does not explore what sort of knowledge they gained. My own research seeks to understand the knowledge of the MaSTs in a more detailed and nuanced way.

I have not included in this chapter the large number of research projects designed to examine and provide a model for the knowledge of classroom teachers of primary mathematics. I will examine some of the key examples of these in my next chapter as they form a basis of my methods. I use these in my analysis to consider whether the knowledge of the specialist teacher is well described by these existing models of classroom teachers' knowledge.

Chapter 3

Models of classroom teachers' knowledge of primary mathematics

Within the context which I outlined in my previous chapter, researchers have identified models of the mathematical and pedagogical knowledge of classroom teachers of primary mathematics (Hill et al, 2008a; Rowland et al, 2009). My study aimed to explore how such models are related to the nature of the knowledge identified by the MaSTs in their new role. I used the existing models as comparative frameworks for my analysis of the shared features I identified of the MaSTs' knowledge. I examined whether the categories of the existing models can be used to articulate the features of the knowledge of the MaSTs. In doing so I considered the possible progression from the secure knowledge of classroom teachers gaining QTS, to deep knowledge of a specialist teacher. I also examined with my sample of MaSTs whether their new knowledge, which covers the whole of the primary phase, was comparable to that gained by a class teacher who has taught every year group across the primary phase. My review of the existing models underpinned these reflections.

This chapter is therefore devoted to a discussion of a selection of the existing models of classroom teachers' knowledge for teaching primary mathematics, including knowledge which is related to both subject and pedagogy as recommended by Williams. Firstly I will review the model proposed by Shulman (1986, 1987) for teacher knowledge generally, which appears to underpin Williams' terms for teacher knowledge, and is a basis for other studies in this area. Critiques of Shulman's model will also be considered as they impact on my study. Then I will review work by three researchers, or teams of researchers, who have provided comprehensive models for classroom teachers' knowledge of primary mathematics. I acknowledge that I may have selected other models, but my choice is backed up by other researchers in the field (Turner, 2012). Finally the literature review will go on to consider three themes of features of knowledge for teaching primary mathematics represented in a wider selection of existing literature. None of the literature I examine specifically considers the knowledge of the primary mathematics specialist teachers.

Shulman (1986, 1987): a general model for teacher knowledge

Shulman (1986, 1987) and Wilson, Shulman and Richert (1987) began the charting of knowledge for teaching. They initiated a debate about how the knowledge for teaching could be conceptualised, and to some extent measured and assessed. Shulman (1986, 1987) proposed seven categories of teacher knowledge:

- subject matter knowledge or content knowledge which best matches Williams' use of the term subject knowledge, and which includes the substantive and syntactic aspects of the subject as described by Schwab (1978).
- pedagogical content knowledge, which is subject specific, the most distinctive form of teacher knowledge and which best links to Williams' use of the term pedagogical knowledge
- curricular knowledge including vertical knowledge of progression in the complexity of ideas and lateral knowledge of links to other subjects and ideas
- general pedagogical knowledge
- knowledge of learners and their characteristics
- knowledge of educational contexts
- knowledge of educational end, purposes and values

Shulman also offered a model of forms of knowledge which, he argued, could describe how these categories of knowledge are organised. These forms of knowledge have been critiqued, for example as static by Petrou and Goulding (2011) and Warburton (2012).

My study is based on Williams' (2008) recommendation that 'There should be at least one Mathematics Specialist in each primary school, in post within 10 years, with deep mathematical subject and pedagogical knowledge' (p.7).

Williams' language appears to be based on Shulman's categories. He separates subject and pedagogical knowledge but recognises that the MaST needs both saying,

> ‘The main thrust of this review, therefore, is that a combination of deep subject knowledge and pedagogical skill is required to promote effective learning’ (Williams, 2008 p.7).

Shulman’s categories have been the basis of many other writers’ thinking and have been re-examined, critiqued and re-conceptualised (Turner-Bisset, 2006.) In some cases these critiques and re-conceptualisations have applied specifically to the teaching of primary mathematics. For example, the categories of knowledge have been critiqued for the model of learning which underpins them (Aubrey, 1997; Meredith, 1995; Poulson, 2001). This appears to be mainly of a transmission orientation, largely rejected in primary mathematics education (Askew et al, 1997). The re-examination of Shulman’s use of the terms subject and pedagogical knowledge are particularly significant for my research as Williams chose to use these terms. The distinction between pedagogical content knowledge and subject matter content knowledge has been questioned (Aubrey, 1997; Poulson, 2001; McNamara, 1991). The argument has been that the boundary between these categories is at least blurred. The study led by Baumert et al (2010) however claimed it is possible to theoretically and empirically distinguish between content and pedagogical content knowledge. Their study looked specifically at secondary teachers’ knowledge of mathematics. They found that both subject and pedagogical knowledge were required for teaching and argued that teachers’ mathematical knowledge remains inert unless accompanied by knowledge of pedagogy, learning and curriculum. It was pedagogical content knowledge which explained differences between children’s learning in their study, rather than content knowledge, although teachers’ content knowledge provided a boundary for their development of pedagogical content knowledge.

Davis and Renert (2012) report on-going work with a number of teachers in Canada, exploring the idea from Baumert et al’s review that mathematical knowledge can lie inert and that pedagogical knowledge can activate mathematical knowledge. They consider the impact of developing such knowledge through a longitudinal study of teachers studying at Masters level.

Despite these examples, current models of teachers' knowledge of primary mathematics, as I will explain, have generally avoided the separation of subject and pedagogical knowledge and use more collective terms such as 'mathematics for teaching' (Ball et al, 2008 p.394) which includes both domains. Similarly Rowland et al's model (2009) includes aspects of subject and pedagogical knowledge. In my research I examine if the MaSTs I interviewed drew on, and conceived of as deep, mathematical and pedagogical knowledge, and whether these are combined, as Williams recommended.

A number of writers have questioned whether a set of parallel categories best describe teachers' knowledge, as Williams and Shulman suggest. Davis and Simmt (2006) argue that teachers' knowledge is usually tacit, and it is more appropriate to see domains of their knowledge as nested, rather than in parallel categories, as this oversimplifies the model. They claimed that there at least four intertwining or nested categories of teachers' mathematics – for – teaching. Two of these are relatively stable: curriculum structures and mathematical objects and two are dynamic: subjective understanding and classroom collectivity. Davis and Renert (2012) argued that mathematical knowledge for teaching is best viewed as a disposition, focusing on the dynamic nature of teacher knowledge. The dynamic nature of teacher knowledge was also identified by Ainley and Luntley (2005) who claimed that a significant part of teachers' knowledge is what they termed attention based knowledge which enables them to respond to learning as it takes place in the classroom. Watson and Barton (2011) argue that the use of mathematical modes of enquiry is an important component of mathematical knowledge of teachers, offering this as an alternative or complement to categories of knowledge for teaching mathematics. They claim that teachers enact mathematics and therefore their knowledge can be seen as a contextual application of mathematical thinking.

The critiques of Shulman's model raise key questions about the ability of any model of teacher knowledge which is organised in categories, to conceptualise the fluid and dynamic aspects of teacher knowledge. This is a point I will consider in relation to my findings in my conclusions.

Although Shulman's model appears to underpin Williams' use of terms for teacher knowledge, it is designed to capture teacher knowledge generally. Some re-conceptualisations of Shulman's work have explicitly focused on the knowledge for mathematics teaching and these models have been more useful in underpinning my study than Shulman's more general categories, acting as a comparison for the knowledge the MaSTs use in their teaching and conceive of as deep.

Ball et al (2008): A model for mathematical knowledge for teaching

Work by Ball and a number of colleagues at the University of Michigan (Hill et al, 2004; Hill et al 2005, Hill et al 2008a; Ball et al 2008) has elaborated the original categories suggested by Shulman (1986). They have focused on what they term mathematical knowledge for teaching, critiquing and refining the categories of Shulman for the specific teaching of mathematics. For example they analysed the mathematical demands of teaching mathematics in large scale and longitudinal studies (Hill et al, 2005). They considered the mathematics involved in planning, marking, devising tasks and activities, and tackling errors and misconceptions. This, they argued, allowed the analysis of teacher knowledge in context. They concluded that the teaching of mathematics makes substantial mathematical demands. They claimed that Shulman's original categories can be sub divided, for mathematics, into distinct sub categories and that teachers may have knowledge of these separately (Hill et al, 2008a).

They claim that Shulman's category of subject matter knowledge or content knowledge for mathematics can be subdivided into:

- common content knowledge, not unique to teachers but which can be held by members of any profession who draw on similar mathematical knowledge
- specialised content knowledge, which they claim is pure as it is not connected to children or pedagogy and specialised as it is not used in any other profession than teaching. For example, it might include the knowledge needed to recognise when an idiosyncratic method of calculation can be used generally or to evaluate a mathematical argument. Mathematics is seen as a compressed subject where ideas are communicated in an efficient and elegant manner but the teaching of mathematics requires unpicking

concepts and making key mathematical ideas more transparent. The fact that this form of knowledge is not linked to particular learners is significant for my research, as the knowledge of the MaST is of the whole primary range and therefore may not be based on teaching experiences.

- horizon knowledge, which they define as 'an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. It also includes the vision useful in seeing connections to much later mathematical ideas' (Ball et al, 2008 p.403) The example given is children's use of the number line and the teacher's knowledge that children will later be able to fill in more numbers as they gain a full understanding of the number system.

The researchers claim that Shulman's category of pedagogical content knowledge can be subdivided into:

- knowledge of content and students, for example knowledge of common misconceptions experienced by children (this was included generally by Shulman as a generic category of teacher knowledge but Ball et al, 2008 and Aubrey, 1997 argue that this is content specific)
- knowledge of content and teaching, for example the knowledge needed to select resources, images and examples to respond to common misconceptions.
- Knowledge of curriculum

For example, a teacher may use knowledge of content and students to anticipate common errors in a particular aspect of mathematics. The teacher might draw on common content knowledge to recognise an incorrect answer in a child's workings. They might use specialised content knowledge to analyse the error, and knowledge of content and teaching to select a resource or approach to tackle the error.

Several articles by this team offer the following diagram to show the relationship between the categories of subject matter knowledge and pedagogical content knowledge of Shulman and their own sub categories (Ball et al, 2008 p. 403)

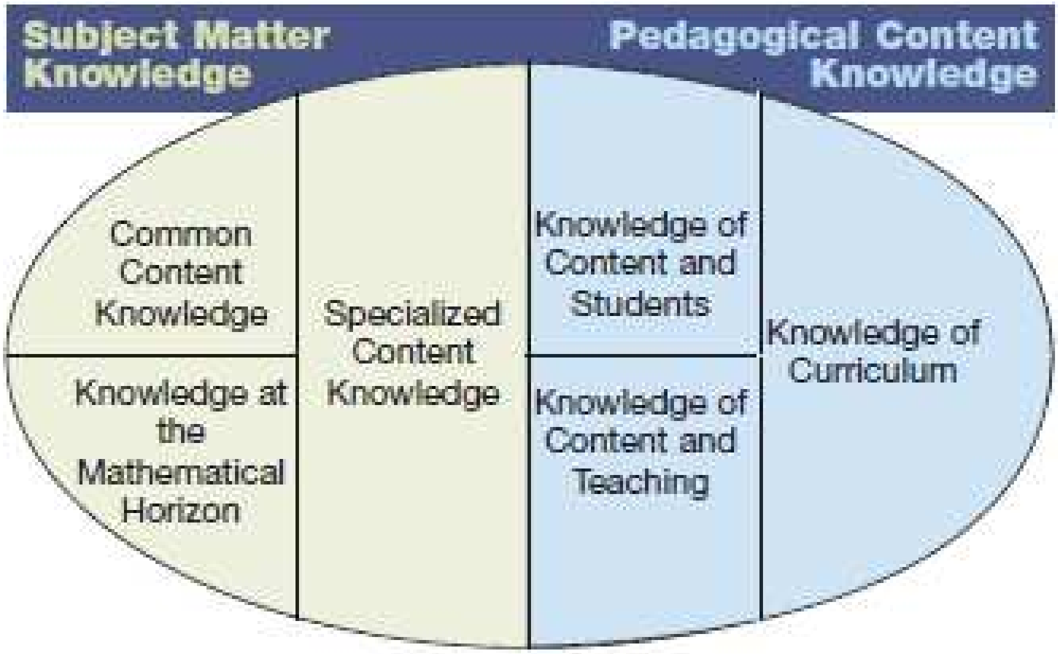


Figure 1: Model of teacher knowledge Ball et al, 2008 p. 403

The sub categories are in the process of exploration and refinement with Ball et al (2008) recognising themselves that boundaries between them are not well distinguished and that static categories cannot chart the flexible and fluid processes of reasoning and decision making used by teachers.

The same team have, alongside and as part of their articulation of these sub categories, devised measures for these aspects of teacher knowledge in the form of written tests (Hill et al, 2008). They developed measures of content knowledge and knowledge of content and students to see if it is possible to isolate these forms of knowledge and identify their relationships to children's learning. They found that teachers do draw on these as distinct categories of knowledge and that there is some evidence that content knowledge is a predictor of children's learning (Hill et al, 2004), although they cannot argue that this is purely the effect of content knowledge and not general pedagogical knowledge. They also claim some success in isolating knowledge of content and students, although again it is difficult to argue that this is distinct from content knowledge (Hill et al, 2008a). Similarly Hill et al (2008b) conducted a study to measure the correlation between mathematical knowledge for teaching, which was based on their categories of common and specialised content knowledge, and what they term Mathematical Quality of Instruction. They found that the two generally aligned, with strong mathematical knowledge for teaching associated with teachers' use of mathematical language, the richness of their teaching, use of multiple methods and opportunities for conceptual meaning making. However, not all cases in their sample of ten teachers showed a close correlation, with teachers' beliefs and use of curriculum guidance and materials mediating the effect of mathematical knowledge for mathematics. Further work (Hill et al, 2012) linked positive and negative performances on measures for mathematics knowledge for teaching with quality of instruction and children's learning, but there were weaker correlations for teachers scoring in the middle range on these measures.

Ball et al's categories offer me a more focused and detailed set of domains of knowledge to use as a form of comparative analysis of the responses of the MaSTs as they reflect on their knowledge in my interviews. My methodology section explains clearly how these domains of knowledge were used in my study. However, there are some difficulties with the categories. Hill et al (2008a) included, for example, in the measures of knowledge of content and students questions which covered a wide set of aspects of knowledge. Ruthven (2011) refers to the issues with working with categories which are amalgams and are therefore too wide ranging to be valuable. Furthermore, the definitions of the categories are still being refined. The category of horizon knowledge is unconnected with pedagogy in that it lies outside pedagogical content knowledge but Ball et al 2008 do connect it to children when they refer to teachers' knowledge of children in different grades (p.403). Ball et al 2008 state that they are unsure whether this category relates to mathematical understanding only or whether it may run across other categories of pedagogical understanding. Petrou and Goulding (2011) argue that there is no clear distinction between specialised content knowledge and pedagogical content knowledge, and that there is no recognition of the impact of teachers' beliefs.

However the model suggested by Ball and her colleagues is significant for my research because it refines Shulman's model, offering more detailed categories to use as a comparison for my findings relating to the knowledge of the MaSTs.

Ball et al's categories draw on a methodology which is set in the context of teaching to explore teacher knowledge and it aims to consider knowledge needed to teach mathematics well. This is sometimes stated specifically (Ball and Bass, 2000) so there is an element of judgement. The model describes what the researchers believe is the knowledge needed to teach primary mathematics effectively. Therefore the model can be seen as more than just a base line for teaching mathematics and can be compared with the notion of deep subject and pedagogical knowledge of the MaSTs. However it is a model for classroom teachers of mathematics, rather than specialist teachers of mathematics, taking a whole school role to champion the subject.

Rowland et al (2005, 2009): a model for teacher knowledge of primary mathematics

The work of Shulman and Ball et al in articulating the knowledge base of teachers has been further refined in the area of primary mathematics by a team of researchers based at the University of Cambridge. Their empirically based model has been influential in my study.

Their research aimed to identify ways in which student teachers' mathematical and pedagogical content knowledge could be seen to 'play out' (Turner and Rowland, 2011 p.197) in their teaching (Rowland et al, 2003; Rowland et al, 2005; Thwaites et al, 2005; Rowland et al, 2009.) Twelve student teachers undertaking the ITE Post Graduate Certificate volunteered to have two lessons each videoed and analysed, with reference to their planning and in comparison to an audit of their mathematical knowledge. Incidences where knowledge could be identified were coded using a grounded approach, and grouped into four dimensions, collectively named the knowledge quartet. The model is claimed by the team to be an empirically comprehensive framework for considering the impact of subject and pedagogical knowledge on teaching. This is argued to respond to the call for subject specific feedback to teachers following lesson observations and to support their own reflections on their teaching, called for most recently by Ofsted (2012). The model has been further used as a tool in a longitudinal study to support the reflection of student teachers (Thwaites et al, 2005) and to investigate the developing and deepening knowledge of practising early career teachers (Turner, 2012).

The nature of the four domains of the knowledge quartet will be outlined here as they form part of the comparative analysis:

- The foundation domain

  This is usually the first of the domain to be listed. It includes the knowledge and understanding which the researchers argue is fundamental to decision making when planning and teaching mathematics (Rowland et al, 2009). It includes mathematical knowledge, general understanding of pedagogy, teachers' beliefs regarding the nature of mathematics and its purpose in the curriculum, knowledge of how children learn mathematics and study into mathematics education. Knowledge in this area of the quartet is propositional in nature, using Shulman's term (1986). It is evident both in teachers' planning and their teaching. Rowland et al (2003) provide the example of a student teachers' knowledge of the distinct structures of subtraction as part of the foundation dimension.

- The transformation domain

  This is perhaps the closest to Shulman's pedagogical content knowledge as it relates to teachers' choice and use of representations, examples, demonstrations, resources and activities. It is knowledge which again is evident in both planning and teaching. Rowland et al (2009) discuss representations chosen by teachers and their use of sequences of examples to support learning.

- The connection domain

  This includes the knowledge which underpins decisions about the sequencing of mathematical ideas across an activity, lesson or a series of lessons. This knowledge therefore relates to coherence in the order of ideas and teachers' ability to anticipate the cognitive requirements of activities. It is knowledge evident in both teaching and planning. Rowland et al (2009) also suggest that this part of the quartet includes the knowledge needed to connect alternative meanings of concepts or ways of representing the same idea or connecting a procedure to a concept.

- The contingency domain

  This domain is only evident in teaching and not in planning. It involves teachers' responses to children's ideas which have not been anticipated. This area of knowledge underpins their decision to act on or ignore the unexpected, and their ability to modify planning unexpectedly.

Rowland et al (2009) offer this link between the knowledge quartet and Shulman's categories

Shulman 1986 , 1987	Rowland et al 2009
Knowledge of purpose Subject matter knowledge	Foundation
Pedagogical content knowledge	Transformation
Subject content knowledge as connected knowledge Pedagogical content knowledge in making connections with and for children	Connection
All seven of Shulman's categories could be called on to make decisions in the moment of teaching	Contingency

Table 1: Link between the models of Rowland (2009) and Shulman (1986, 1987)

The research aimed initially to consider the evidence of the impact of mathematical knowledge, or Shulman's subject matter or subject content knowledge. However the researchers found this focus limited their analysis of the full range of incidences of the impact of teacher knowledge and that the distinction between subject and pedagogical knowledge was blurred and unhelpful, agreeing here with other writers such as McNamara (1991) and Aubury (1997). Petrou and Goulding (2011) found that the knowledge quartet did not address curriculum knowledge, and in particular the extended use of text books which is a common feature of teaching in some countries.

The initial research which identified the four dimensions of knowledge was with student teachers, and they were not necessarily judged to be effective. The model describes the impact of knowledge as it is observed in classroom teaching and planning. It is not necessarily a model of knowledge of specialist teachers. However it allows me to consider how the subject and pedagogical knowledge of the MaST compares to the Knowledge Quartet. Are all four dimensions equally well developed in the MaSTs or do a number of dimensions develop more than others?

Ma (1999): a model for profound knowledge

A fourth piece of work which has informed my research is by Ma (1999). She aimed to explore why Chinese students typically outperformed students from the United States of America (USA) whereas teachers in the USA had generally experienced more education themselves than teachers in China. She claimed that this is at least partly explained by differences in teachers' understanding of mathematics and the structure of their knowledge of mathematics in the curriculum. She began by looking for examples of sufficient subject matter content and found that the Chinese teachers displayed a more comprehensive and differently structured knowledge of the mathematics taught in elementary (primary) schools. From this she defined what she called profound understanding of fundamental mathematics taught in elementary schools.

Ma worked with twenty three teachers from the USA who had been identified as experienced and effective, and seventy two Chinese teachers. She used an interviewing technique related to that used by Ball (1988) which drew on the context of teaching. Teachers were asked to consider the teaching of the topics of subtraction with regrouping, multi digit multiplication, division by fractions and the relationship between perimeter and area. They discussed how they would respond to children's errors, how they would represent the topic to children and how they would respond to a novel idea raised unexpectedly by a child. She also compared their discussion of the knowledge package which structures their teaching of each topic. Here the teachers were asked to identify the key ideas of the topic, and ideas of lesser importance, connections and sequences between these ideas, which key ideas will need to have been understood previously and 'concept knots' (p.115) which link the most important ideas. Ma argued that the knowledge package captures the structure of the teacher's knowledge of the topic, claiming that,

> the knowledge packages reveal the teachers' understanding of the longitudinal process of opening up and cultivating such a field in students' minds (Ma, 1999 p. 113 – 114).

Ma's analysis of the teachers' responses led her to conclude that there were distinct patterns in the nature of the knowledge that teachers from each country drew on.
Although most teachers from the USA drew on secure understanding of subtraction and multiplication, some found division by a fraction and the relationship between area and perimeter difficult. The Chinese teachers were generally competent in all four areas. Typically the teachers from the USA demonstrated a focus on arithmetic competence. In comparison, the Chinese teachers demonstrated arithmetic competence and conceptual understanding. When they referred to a procedure they were likely to discuss a rationale for why the procedure works. Such rationales led them to identify key ideas on which mathematics is based. For example in their discussion of the written procedure for subtraction they would discuss the key ideas of place value which underpin the base ten number system. They would often support their discussion for the rationale for procedures with written arguments drawing on sophisticated use of symbols and correct mathematical terminology, referring for example to the distributive law and its place in the rationale for long multiplication. They made more connections in the interviews than the US teachers, demonstrating a coherent understanding of mathematics. They would typically consider and compare more than one procedure in each area of mathematics, including standard and alternative methods. Whereas the US teachers tended to focus on the

standard procedure for each operation, the Chinese teachers focused on the operations themselves, considering the relationships between operations and the meaning of operations across different groups of numbers. For example they discussed how multiplication is a form of repeated addition so it can be seen as a deriving from addition and how addition is the inverse to subtraction. The teachers from the USA demonstrated knowledge which appeared to be fragmented. Few connections were made in their interviews between mathematical ideas. This could also be evidenced from the discussion of knowledge packages. The Chinese teachers' discussion of knowledge packages tended to include groups of ideas rather than single ideas, in comparison to those of the US teachers. Teachers who demonstrated conceptual knowledge and procedural knowledge referred to knowledge packages which were structured in different ways to those who had procedural knowledge only. They included more conceptual links as well as procedural ones, whereas those from teachers with procedural knowledge included generally just procedural links, and fewer ideas and connections in total. The Chinese teachers often included connections to basic mathematical principles which underpin the area and represent part of the structure of mathematics.

Ma concluded that typically the Chinese teachers demonstrated what she termed profound understanding of fundamental mathematics. She identified the features of the early ideas of mathematics taught at primary, or elementary, school as fundamental in that they are foundational for later mathematical ideas, primary in that they include the rudiments of more advanced ideas and elementary in that they are seemingly simple enough to be grasped by young minds (Bruner 1977). She explores the idea of depth of understanding, defining it as connecting to more conceptually powerful principles of the subject.

> The closer an idea is to the structure of the discipline, the more powerful it will be, consequently, the more topics it will be able to support. (Ma 1999, p. 121)

She also discusses the notion of breadth of understanding, suggesting that this refers to connections of one idea to others of similar or less conceptual power. She argues that it is possible to have a broad and deep understanding of mathematics because it is a broad and deep subject.

She concludes with defining the characteristics of profound understanding of fundamental mathematics as:

- Connected

  A teacher with such understanding is able to make connections and avoids promoting fragmented knowledge in children
- Multiple perspectives

  The teacher has access to and promotes a variety of meanings of ideas and methods of calculation
- Basic ideas

  The teacher identifies basic powerful mathematical ideas and their teaching engages children with these ideas
- Longitudinal Coherence

  The teacher is not limited to the mathematics curriculum provided for a particular class; they can review concepts which should have been learnt earlier and lay foundations for later learning.

Ma's construct has been widely well received by the mathematics education community (Petrou and Goulding, 2011 p.17) but critiqued as presenting teachers' knowledge of mathematics as fixed and static. For example Davis and Renert (2012) argue that although curriculum knowledge may be relatively stable, teachers' and children's understanding of mathematics is dynamic. They question and extend Ma's interpretation of fundamental mathematics as foundational, primary and elementary, arguing it is best thought of as emerging mathematics. In critiquing Ma's work, they draw on the view already noted (Ball and Bass, 2000; Davis and Simmt, 2006) that mathematics is a subject which compresses and packs ideas into succinct concepts and statements, and that teachers of mathematics need to expand or unpack these concepts and statements. They argue that no teacher can fully master the range of interpretations of ideas that the children they teach will offer. Teachers' knowledge therefore needs to be responsive and evolving; 'a flexible vibrant category of knowing' (Davis and Renert, 2012p.247).

Nevertheless, Ma's study was important for mine as it was the only study which specifically defined profound or deep subject and pedagogical knowledge of primary mathematics. Therefore it was a key comparison of the deep subject knowledge the MaST. However Ma's conclusions referred to classroom teachers rather than specialist teachers. Would their deep subject knowledge match all four elements of Ma's definition? Furthermore, Ma claimed to identify factors which supported the Chinese teachers' development of profound understanding. The MaSTs have developed their deep knowledge in a very different education system.

Additional themes

Writers such as Ma (1999), Ball et al (2008) and Rowland et al (2009) aimed to articulate models of classroom teachers' knowledge employed in the teaching of mathematics. In this study I drew on these in my analysis of what the MaSTs recognised as significant in their deep subject and pedagogical knowledge of mathematics. However, I also drew on a wider selection of literature.

Across these studies, and others in the field, key themes can be identified which, I shall argue, characterise mathematical subject and pedagogical knowledge, and therefore can be used to analyse the knowledge that the MaSTs recognised they developed in their role. Three key themes will now be considered. These themes draw on the works discussed above and on a wider field of literature. They served to ensure that my research was based on a wide conception of knowledge for teaching of mathematics, resulting in a full and rich analysis of the responses of the MaSTs. It is important to note that the studies contributing to these themes do not necessarily refer to subject and pedagogical knowledge of specialists, but to the knowledge of classroom teachers. Therefore they acted as a baseline to identify the nature of the knowledge of the MaSTs.

Three key themes in the current debate will be identified and discussed. I acknowledge that the selection of themes are based on my own views. Whilst I cannot claim that any other researcher would have selected the same themes as representative of the current debate, I draw on my own experience and understanding in this selection and provide references to numerous texts to back up my argument that these are valuable themes, relevant to my research.

Connected knowledge: connecting equivalent and similar ideas

Hiebert and Carpenter (1992) and Skemp (1989) offered definitions of desirable mathematical understanding for all learners, both of which drew on the idea of connected knowledge. A more recent work by Barmby et al (2009) describes mathematical understanding as being able to reason between ideas. The ideas may be memories or mental representations which the learner connects with lines of reasoning. ACME (2012) recommended that able children in mathematics should experience a curriculum characterised by rich connections between topics and ideas, rather than by acceleration. They called for professional development for all teachers of mathematics to develop connections between topics. Connected knowledge is therefore a characteristic identified in the existing literature for the knowledge of both children and teachers. Research in the USA has widely drawn on this idea (Ball et al, 2008; Resnick et al, 1992; Leinhardt and Smith, 1985; Brophy, 1991). In the UK, Askew et al (1997) found that the most effective teachers made connections in their teaching between mathematical ideas, between different representations of the same mathematical idea and between what children have learned already and what is yet to be learned. Connection is also part of the knowledge quartet (Rowland et al, 2009).

A key feature of the literature and research findings on teachers' mathematical subject knowledge is that it must be characterised as dynamic, responding to the immediate needs of the learners (Ainley and Luntley, 2005; Poulson, 2001; Meredith, 1995). This is also an integral feature of the frameworks offered by Davis and Simmt (2006) and Rowland et al (2009). Teachers need to make connections to children's changing interests, contexts and learning needs.

Goulding et al (2002) and Murphy (2006) found that student teachers often begin their ITE with fragmented knowledge. Indeed DFEE (1998) listed knowledge of equivalences as necessary for student teachers. The term equivalence relates to the way in which, within mathematics, ideas can be notated in different forms. For example, 2+ 4 is the same as 6 or 10- 4, and 12 x 7 is the same as (10 x 7) + (2 x 7). ACME (2008) suggests that equivalence is one of the four big ideas of the primary curriculum. Gray and Tall (2007) discuss compression as a key feature of mathematics. As a subject, it compresses a number of situations, problems and questions in to a simple model or concept such as 2 + 3 = 5. It notates ideas in a short, abbreviated way. This is its strength. The art of a mathematics teacher is to do the opposite, (Davis and Simmt, 2006), to expand and connect 2 +3 = 5 to its various forms for children to develop a full understanding of its meaning.

Teachers' subject knowledge therefore can support them in presenting the same idea in various representations and connecting different but linked ideas. Wilson et al (1987) discussed a teacher who felt he needed to know mathematics in 150 ways in order to teach it. However Ainsworth (2006) found that the ability to access multiple representations does not in itself lead to understanding (Barmby et al, 2009). The ability to make valid connections between representations and to select these purposefully to meet the needs of children seems to be more significant. The use of representations is a key part of the transformation dimension of the knowledge quartet and Rowland et al (2009) argue that they provide a scaffolding link between the physical and the abstract in mathematics. They correspond to the iconic phase of Bruner's phases of learning (1966). The research by Ma (1999) found that teachers with what she termed profound understanding were able to make specific links, and articulate these clearly, for example between manipulatives and mathematical ideas, or between standard and non-standard methods.

The ability to draw connections between mathematical ideas, and between mathematics and other areas of the curriculum (Rose, 2009) therefore appears to be a valid part of a definition of teacher subject and pedagogical knowledge, and will be used as part of the analysis of the knowledge of the MaSTs. This goes beyond being aware of multiple equivalent ideas or representations of the same idea, to the ability to reason clearly between them and perhaps to know which connections are more important than others.

<u>The ability to identify key mathematical principles and trace these through the curriculum</u>

This theme draws together references in existing literature and research on connections which support progression in children's understanding of mathematical ideas. Typically these are vertical connections rather than the sideways connections discussed above. These can be mathematical connections based on progression in mathematical complexity, or pedagogical connections based on understanding of children's learning and how it progresses. These ideas stem from Bruner (1996) who claimed that any subject can be taught to any child in an honest form and then developed through a spiral curriculum. In Rowland et al's knowledge quartet (2009), this aspect is included in the domain of connections, where teachers anticipate complexity and match conceptually appropriate content to children's needs which progress over a series of teaching episodes.

Ma (1999) found that teachers with profound understanding of mathematics were able to identify the single mathematical principle underlying topics in the primary curriculum. They discussed these principles in their simplest, mathematical, technical terms. They could also trace these principles through the curriculum. Given a topic, they could identify the particular underlying mathematical principle, and be able to state when this should have first been introduced to children, how it had already developed and how the principle will continue to develop in later stages of the curriculum.

The existing literature suggests that teachers' knowledge can support them in judging the key mathematical principles and distinguishing these from ideas which lie on the periphery of the subject (Goulding et al, 2002; Shulman, 1986, 1987; ACME, 2008). This requires relating ideas to the subject itself. Twiselton (2000) found that strategies such as the National Literacy Strategy tend to obscure the rationale for the subject itself, which should provide coherence and cohesion to teaching. The work of Freudenthal (1973, 1991) backs this notion. He reflects on the link between learning about the 5th triangle number and learning about triangular numbers more generally.

Mathematics by its nature lends itself to abstraction (Ginsburg and Amit, 2008.) An understanding of mathematics, its key ideas and structure, can support teachers in arranging examples and demonstrations which make visible the particular attributes to abstract (Gray and Tall, 2007). For example, when introducing children to the concept of a triangle, teachers can present examples which allow children to understand the attributes which are significant, the number of straight and closed sides and corners, rather than the attributes of colour, size and orientation. Children who struggle with mathematics may well attend to different, trivial attributes and abstract incorrectly. Bruner (1978) defined scaffolding as reducing the degrees of freedom so that learners can focus on a particular area of learning. Teachers' knowledge can support them in providing differentiated and scaffolded access to the particular key mathematical principle embedded in a topic, whilst minimalizing aspects which are on the periphery. Barmby et al (2009) in their argument for understanding being the ability to reason between representations, claim that some representations and forms of organisation are particularly fundamental. Simon (2006) called these key development understandings.

So existing literature suggests that teachers' knowledge can enable them to state the key principles of the primary curriculum, where these are introduced to children, and how they progress through the curriculum. This sort of knowledge enables teachers to consider any learning objective or topic to be taught in relation to the rationale of mathematics itself. This theme was used as a tool for comparative analysis of the knowledge identified by the MaSTs.

Using and Applying Mathematics

The final theme employs the term using and applying mathematics which describes the part of the current primary National Curriculum (DFEE, 1999) which relates to children's investigation and problem solving, or their work as mathematicians, thinking and reasoning mathematically. In this study, the term will also refer to the processes of teachers engaging with investigations and problems, undertaking mathematical enquiry which requires them to think mathematically. This is opposed to solving closed problems, where there might be a set procedure to follow. A key theme in the literature on mathematical learning of both children and teachers suggests that the ability to use and apply mathematics might characterise a desirable feature of teachers' knowledge.

Shulman (1986, 1987) and Rowland et al (2009) draw on Schwab's (1978) distinction between substantive knowledge of the way concepts and principles are organised to incorporate facts; and syntactic knowledge of the sets of ways in which truth, value and validity are established. Both are needed, argued Schwab, if education is concerned with both imparting knowledge and the art and skill of the subject. Both are reflected in the aims of the new National Curriculum (DES 2013).

The argument then is that teachers and children need to engage in processes of solving non routine problems which require mathematical thinking; generating general statements and constructing mathematical arguments in order to understand how these are constructed by mathematicians. Ma (1999) found that teachers with profound understanding of mathematics were more likely to investigate mathematics than those with procedural knowledge. They were familiar with procedures for establishing proofs and were more likely to act as mathematicians. Reflecting on Ma's model, Davis and Renert (2012) argue that mathematics knowledge for teaching is dynamic because of the nature of mathematical understanding which they describe as emerging. Teachers need to respond to children's interpretation of mathematical ideas. Because it responds to children's sense making of mathematics, teachers' knowledge therefore includes elements of syntactic knowledge.

A further argument for the place of using and applying mathematics as a feature of the knowledge of teachers stems from the way the nature of mathematics is presented to children in school. If mathematics is typified by enquiry, then only by modelling enquiry can teachers present a full picture of mathematics. Banks et al (2005) discuss the relationship between teachers' knowledge and pedagogy, and argue that the nature of school knowledge is a different version of the subject itself. The subject is restructured to make it teachable and accessible to learners. It is 'codified, partial, formalized and ritualized' (Banks et al 2005 p334). It has a beginning and an end. Subjects should not be packaged, or fragmented (ACME 2008), as this does not present them in their full form. Freudenthal (1973, 1976) describes how 'ready-made' (Freudenthal, 1973 p114) mathematics can be presented to children, which conceals the subject. Teachers should instead guide children in the activity of mathematics. For example, Freudenthal claims that text books generally present mathematical vocabulary, such as the term parallelogram, as a given. However, the reaching of a definition such as parallelogram is the result of a human activity (Freudenthal, 1973 p.106). Learners need to grasp the true meaning of the term, what it does and does not include, by their own activity. By engaging with shapes which are and are not parallelograms, they learn both the definition and the act of defining for themselves. Teachers should therefore present the activity of mathematics, arguably captured by the terms using and applying in the current National Curriculum (DFEE, 1999), to children rather than the product of mathematics.

> The only didactically relevant element, the analysis of the subject matter, is dropped, the student is confronted with the result of the analysis and may watch the teacher who knows the result, putting the things analysed together again (Freudenthal, 1973 p103).

Teachers' knowledge can therefore support them in guiding children's using and applying mathematics. For this to be so, they need to engage themselves in using and applying mathematics. Teachers can too easily choose the right term, the right method and conceal the reasoning they needed to do to find them. Freudenthal states that even professional mathematicians tend not to do this.

> A mathematician is used to objectivating. Rather than giving his course of thought, he edits an objective elaboration: definition, theorem and proof. If he had to publish some of the ideas that led him to the result, he would feel as if he had been put out in the street in his underclothes. (Freudenthal, 1973 p.105).

Mathematics is therefore presented as a ready-made subject (Freudenthal, 1973 p.114), like a ready-made meal, prepared by someone who already knows how to make it. It can only be reproduced by learners. This section of literature suggests that teachers' knowledge should include the ability to predict errors in reasoning and guide children through the processes of making sense of them. Teachers can allow children to make mistakes in the belief that this will further their understanding. This involves engaging with problems and grappling with errors in reasoning themselves. Freudenthal argues that children need to engage with mathematics in contexts, organise the mathematical ideas therein at a low level, and then reach a higher level of understanding. This is what he describes as reinventing mathematics (Freudenthal, 1973 p.126). He contrasts it to discovering mathematics, which is uncovering what has been covered up by someone else, like hidden Easter eggs (Freudenthal, 1991 p.46). Engaging with mathematics as a mathematician is essential, for children and teachers.

Teachers' knowledge can therefore be argued to include the ability to learn mathematics as yet unknown to them; to investigate and problem solve, to think logically, to be playful with number and ideas, to abstract and reach generalisations, and construct arguments and proofs. To do this, they need to draw on an understanding of the processes which are valid for establishing mathematical truths and proofs. Watson and Barton's (2011) claim that teachers of mathematics engage with and model modes of mathematical enquiry, such as conjecturing, abstracting and proving, backs up the argument that this is an important component of teacher knowledge.

Teachers thus provide a mathematical environment for children's thinking and experience of mathematics. The act of teaching mathematics mirrors the act of mathematising.

Conclusion

My study provides a new contribution to the existing debate by considering the degree to which the existing models for classroom teachers' knowledge for teaching mathematics and the three themes which draw from a wider field, provide a framework for analysis of the knowledge of the MaSTs.

I expected that the knowledge of the MaST would not exactly fit the existing models. This was for the following reasons.

None of the models I have considered in this chapter have been for specialist teachers, as there is no framework yet offered for their knowledge. Shulman, Ball et al, Rowland et al's taxonomies and the themes in the existing literature concern classroom teachers of mathematics. Their knowledge is unlikely to require them at any one time to consider the learning of children across more than three or four year groups, although the ability spread in a class could be wider than this. The knowledge of the MaST must support them in maintaining an overview of the learning of children as they progress throughout the whole school and into Key Stage 3. For example, they must provide a range of resources for the whole school and identify the particular mathematical ideas shown by each to ensure that their use supports progression in children's learning. They must be able to analyse errors in one part of the school and use these to perhaps question a school's over reliance on a calculation method or resource in other year groups. They might for example recommend the array as a model for multiplication in Year 1 knowing that it will support children in their learning of grid multiplication and then long multiplication at the end of Key Stage 2. They must see the whole primary phase when considering part of it, and indeed consider learning well beyond Year 6. These skills may be required of classroom teachers too, but to a lesser extent.

Therefore the existing models for classroom teachers might not exactly match the knowledge of the MaSTs but might throw light on their knowledge. I considered with my sample of MaSTs how they conceived of their deep knowledge in comparison to that of their experience as a class teacher and that of a class teacher with experience in each of the primary years for children from three to eleven years. I considered whether the subject and pedagogical knowledge of primary mathematics is the sum of good knowledge, as already described by the literature, of each part of the primary phase. The knowledge of the MaST must enable them to support the learning of children who may not be well known to them. I reflected on how this knowledge fed from and into their own classroom teaching.

The models I have reviewed in this chapter are not always for effective teachers of mathematics. In particular, Rowland et al's model is analytical, and does not seek to prescribe how teachers' knowledge should be evidenced in their teaching, rather to offer a framework to describe how it was observed in the cases in their research. However Turner and Rowland (2011) and Turner (2012) show how the Knowledge Quartet supported teachers' development of both mathematical and pedagogical knowledge over a longitudinal study of 4 years. The other researchers imply or say specifically that they are considering the knowledge of effective teaching of mathematics (Ball and Bass 2000 p. 89). These models link more closely therefore to the recommendation of the knowledge of the MaSTs. The implication from Williams is that the knowledge of the MaST will have a positive impact on the learning of all the children across a school, 'in-depth subject and pedagogical knowledge inspires confident teaching, which in turn extends children's mathematical knowledge, skills and understanding' (Williams, 2008 p.9).

In my next chapter I move on to consider how I set about deciding on a methodology to enable me to identify features of the knowledge which the MaSTs identified that they drew on and conceived of as deep, and to compare these features to the models I have reviewed in this chapter.

Chapter 4

My role in researching subject and pedagogical knowledge

In this chapter I reflect on myself as a researcher. I begin by articulating my own values and go on to explain how these values influence my research. I discuss my choice of theoretical framework and inquiry strategy. Finally I explain the premises of my argument.

My role in the research

Throughout my study I draw on my own knowledge and beliefs. As these play a part in the questions I ask, the methods I use to collect data and my analysis of it to draw conclusions, I reflect on them here.

My research is based on my belief that mathematics is a body of knowledge, fascinating and powerful, which changes due to the work of mathematicians who explore its limits. I see investigation, proof and argument as key mathematical processes or ways of thinking. Children actively construct their understanding of this body of knowledge, and are inducted into investigation, proof and mathematical argument. In this way they act as mathematicians, exploring the mathematics at the limits of their own current understanding. Furthermore I assume the agency of teachers and MaSTs, and myself, in the process of understanding of mathematics. Teachers of mathematics actively seek to assess children's previous understanding and choose questions, resources, images and models to promote children's new learning. In turn, MaSTs actively develop their knowledge and understanding of their role, of their own mathematics teaching, and of mathematics in other year groups across their school. As a researcher, I too build my understanding of the knowledge specialist teachers draw on. The interviews aim to reach a shared understanding of features of the MaSTs' conceptions of deep subject and pedagogical knowledge. Both the participating MaSTs, and myself have an active role in this.

I believe that this research is morally worthwhile (Christians, 2005) in that it strives to articulate effective teaching of mathematics for young children. It aims to enhance the work of teachers in the learning of all children. In this sense it promotes social transformation, empowering MaSTs, and the teachers they support, to enable children to learn and enjoy mathematics. The MaST has a new voice and role in this.

Walford states 'all research is researching yourself' (2001, p.98). I am a passionate teacher of mathematics. My own history as a teacher of primary mathematics, mathematics subject leader in several schools and now primary mathematics lecturer charts my own constant striving to understand primary mathematics in a deeper way and to find more effective ways to teach it to children so that they understand and are fascinated by it. My research is part of this process. The beginning of the research focussed mostly on the term deep subject knowledge as a new term and one which attracted me as it emphasised the mathematics in primary mathematics education. As I will explain in my analysis, I found that a focus on purely mathematical subject knowledge was not rich enough. Mathematical and pedagogical knowledge are too closely entwined, as already claimed by, for example, McNamara (1991). Widening my focus from mathematical knowledge to include pedagogical knowledge mirrors the changes in the focus of the research for example by Ma (1999).

I am the programme director and tutor for the Primary Mathematics Specialist Teacher Post Graduate Certificate which the sample of MaSTs completed. This role has a particular impact on my findings. I am not a silent voice in this research, and I cannot claim that the research would be the same if someone else conducted it, although I do argue that my argument is both logical and worthwhile. I have 'authentic resonance' (Christians, 2005 p. 156) with the MaSTs due to my experience and knowledge, and my relationship with them. As Dey (1993 p.63 – 64) states, 'there is a difference between an open mind and empty head'. I bring experience and understanding to this research. However, my role as MaST programme director and tutor has an impact on my relationship with the MaSTs involved in the research. As programme director, and in some cases their tutor I had, or would have in the case of the first interviews, marked and judged assignments. I also acted as a referee for one of the teachers following the research. I consider that as a tutor I take a stance which is of facilitator of professional learning rather than transmitter of knowledge, but inevitably I am seen in the position of expert. My role as tutor and programme director changed to researcher through my initial emails negotiating the participation in the research and agreeing the practical details. This email can be seen in Appendix i. Here I saw the MaSTs as expert, holding knowledge that I sought to share.

Our relationship during the interviews was not reciprocal. I did not give any inkling of my knowledge of the particular area of mathematics which was discussed, even when the MaSTs asked for my view. I withheld information yet expected the MaSTs to be open. I actively sought to gain and understand their perceptions. I assumed that the teachers volunteered to participate in the research because they recognised they would benefit from a one to one discussion with a tutor, and that they were interested in the role of the MaST.

In summary the advantages of my role can be laid out in the following way. I acknowledge my mathematical knowledge and understanding, my experience as a primary teacher of mathematics, and as a lecturer specialising in mathematics education. This is well known to the MaSTs. I bring an informed view of primary education and their role; although not of the particular schools they work in. I am able to identify and understand mathematical nuances and ask probing questions. My role enables us to work in a relationship, which I endeavour to use to reach shared understandings. We generally share a common understanding of terms.

As I interviewed more than one MaST, I was in a position to spot commonalities, which the MaSTs themselves could only do in a group together. Being interviewed together might change what they were prepared to disclose. I also brought knowledge of other theory and previous research with which to analyse their responses after the interviews. My position as lecturer requires me to take a critical stance to the analysis of data, literature and research findings. This critical stance supports my thinking as a researcher.

However, the disadvantages of my role are also clear. In my position as programme director, I want the programme to remain recognised and funded, and therefore am less prepared to publicise research which disputes the existence of distinct and valuable knowledge of the MaSTs. As a doctoral student, there is a pressure on me to summarise as an act of closure, finding the answers to my research questions.

My knowledge and experience enabled me to understand and analyse together with the MaST, but pre-disposed me to identify familiar issues in the open coding. There was a possibility of assumed shared understanding leading from my previous knowledge and understanding, which could lead me to miss features and make wrong assumptions. I constantly asked myself the question identified by Ely et al (1991) ‘Am I talking about them or am I talking about me?’ (p.125). Drawing on my experience as a teacher and teacher educator in primary mathematics, I realised as I completed this research that I held assumptions about the beliefs and practices of the sample of MaSTs, in that I assumed that these would be similar to my own. As I will outline in my conclusions, I was surprised to find this was not the case in a significant area, which led me to reflect on the assumptions I unconsciously hold.
My role in marking and moderating assignments in the position as tutor and programme director, and acting as a referee, had an impact on what the MaSTs were likely to disclose to me. The relationships I sought to establish aimed to balance empathy and distanced judgement. I have laid out these disadvantages to show how I have considered and acknowledged them. My methods involve me in a constant cycle of comparing interviews, alternating data analysis and collection of data, and reflecting on my findings. I reflect on the selections I make and the decisions I take, to hold my argument up for scrutiny.

Many teachers tell me that it is only when they teach mathematics that they fully understand it. In some ways, this study is an attempt to understand my own subject knowledge of mathematics and to consider its depth. Furthermore, my own child is now at primary school, and as I observe his mathematics education, it is important to me that his teachers promote a love for mathematics based on deep knowledge.

Next I will consider how my beliefs and values have fed into my choice of theoretical framework and inquiry strategy, and therefore the type of knowledge which I make claim to.

My chosen theoretical framework and inquiry strategy

In the light of my beliefs and values stated above, I draw from a largely constructivist paradigm for my research (Denzin and Lincoln, 2005), where I believe that knowledge is constructed actively by the MaSTs, and by teachers they support and the children they teach.
My approach is interpretivist (Denzin and Lincoln, 2005) in two senses. Firstly, I encourage the MaSTs to interpret their conceptions of the knowledge they draw on, discussing examples of practice from their classrooms and considering in particular their conception of deep subject knowledge. Secondly, I interpret the responses of MaSTs individually, and across cases and in the light of existing literature. The new knowledge I make claim to is drawn from the responses of MaSTs selected and understood by them and by me, and my interpretation of the current debate. I acknowledge that my selection of features from the interviews and the current literature is based on my interpretation, particularly for the two cases where the MaSTs did not wish to be tape recorded and where I took notes instead. Whilst recognising my research rests on my understanding, I aim to show in my argument that this is a 'subjective but disciplined' interpretation (Stake, 2005 p. 459).

My research was exploratory or descriptive in nature (Robson, 1993) rather than explanatory. In particular I used the interview as a negotiation of understanding, an 'inter- view' (Kvale, 1996) or 'co- elaboration' (Miles and Huberman, 1994 p. 8). In our discussions we negotiated shared understandings of the MaST's teaching of mathematics and conceptions of deep subject knowledge. The interviews were a transactional process (Ely et al, 1991), where I asked the participants to share knowledge with me. By asking the questions, I impacted on this knowledge. Ely et al (1991) claim, 'once the habit of reflection is introduced into a setting, the setting has already changed' (p. 196 – 197). In this case, the habit of reflection had already been introduced through the programme, which had required the specialist teachers to be critically reflective. However, I accept that the articulation of deep subject and pedagogical knowledge evolves as it is researched and conceptualised. I claim that the first question of the interview is representative of the sort of question asked of a MaST in their role, on a day to day basis, fulfilling what Williams asked of the MaST when he explained:

> This specialist teacher would fulfil the following personal and job specification: …
>
> - Act as peer-to-peer coach and mentor and support the mathematical professional development of serving teachers, NQTs, ITT students on placement and teaching assistants within the school
>
> (Williams, 2008 p. 20)

Williams also called for the knowledge of the MaST to be a ‘nucleus’ (Williams, 2008 p.1) for the school, therefore formed for communication with teaching staff. In conversation with colleagues, and also with me in the interviews, the MaSTs became aware of their knowledge base and therefore developed it. The interviews were a ‘construction site for knowledge’ (Kvale, 1996). In this way the data I collected can be seen as temporary, but part of the normal development of specialist teacher knowledge. My study analysed the knowledge identified by MaSTs in the interview on that day. Tomorrow it might be different, shaped by the interview and the next mathematics lesson. As I aim to capture features of deep subject and pedagogical knowledge held by each MaST, I cannot escape developing it. The features I identify therefore are different, slightly, to the ones which characterise the deep subject and pedagogical knowledge of the MaSTs before each interview. Research transforms the knowledge itself. It is interactive and generative. Therefore it is important that the research is in the context of the role, as I try to capture the knowledge perceived by the MaSTs.

I recognise the central position of the MaSTs in the research and try to use their voices when possible. The style of questioning was designed to enable a joint investigation of the nature of teacher knowledge. The interviews are best described as a shared striving to understand. For example my understanding of the language the MaSTs used in the interviews was important. I used probing questions to ensure that I did understand terms the MaSTs used, even if at first I might assume this was the case. The interviews often included refinements of definitions of features, and I asked probing questions to check my interpretation of the MaSTs' responses, and to show that I was genuinely unsure of the nature of deep subject knowledge myself and recognised the MaSTs' central role in defining it.

I aimed for a discussion in the interviews, but recognise that myself and the MaSTs did not have equal participation. The interviews were a negotiation of meaning (Fontana and Frey, 2005). Although I use the term negotiation, this has a feeling of equal power (Pring, 2000). Stronach and Maclure (1997 p.35) might prefer the term 'struggle'. My view was privileged in that I questioned, selected, analysed, chose categories, illustrative quotes, and summarised. However the MaSTs in the research exercised power in their choice of what to disclose and their agreement of the transcript before it was analysed. Using the categories from Chase (2005), I avoided an authoritative voice. I may have been seen at first as the expert, but I was asking for their insights. Without these I would not be able to complete my research. They were in the unique position of being able to identify features of their own knowledge.

Next I consider the nature of the data I collected. Collecting qualitative interview data allowed me to gather and interpret views of MaSTs on the knowledge they used and developed in their role. Then I was able to analyse quantitative patterns across the MaSTs' interviews. Denzin and Lincoln (2005 p.4) use the term 'bricoleur' or quilt maker, for the qualitative researcher. I brought together the responses of each MaST during a lengthy interview to negotiate a picture of the knowledge they perceived they used in the role of MaST, and then across the examples of MaSTs to see if there were common threads.

My interview questions invited storytelling. My questions made their account of practice 'story worthy' (Chase, 2005 p. 661), and therefore valuable. The MaSTs' responses included narratives or 'retrospective story telling' Chase (2005 p. 656), to exemplify their practice, ordering it and justifying choices made. In this way it was a performance, for me. These stories were not only interesting in their content but because the MaST chose them to exemplify their knowledge. The MaST became the narrator, often speaking for a long period of time, articulating responses which may establish their identity as MaST. In many cases the MaSTs told oppositional stories (Goodson, 1995; Coffey and Atkinson, 1996) in that they recounted things they did which were different to previous practice or challenged perceived established guidance. In my research I analyse examples of this.

The new knowledge I make claim to is based on data gathered from case studies. I chose this approach as it allowed me to understand in some depth the knowledge identified by individual MaSTs. As I will explain, I felt this could not be understood using other approaches.

I draw on Robson's (1993) discussion of the features of case study as research in context. Each MaST provided situated responses, in their own setting. The argument for researching in a natural setting has been widely made (Lincoln and Guba, 1985.) This lent authenticity to the data. I acknowledge that I used predominately one dominant method of data collection which is unusual for case studies (Robson, 1993).

Firstly I gathered and considered detailed data concerning the knowledge identified by each individual MaST, in the specific episode of the interview. My emphasis was initially on understanding this knowledge, with 'intensity in the examination of the particular' (Pring, 2000 p.41). The analysis was of the language used by participants themselves. I was not able to predict if there would be any shared commonality or if there was, whether it would correspond to previously identified features of teacher knowledge. Each response was complex and open coding initially allowed me to begin to capture this complexity. Therefore my initial focus was on the 'epistemology of the particular' (Stake, 2005 p. 454). In this stage of the research, I considered what Stake calls 'instrumental case' study (2005 p. 445) as I analysed each study in depth. The next stage of the inquiry was to look across cases to identify patterns and therefore make suggestions about features of knowledge which may be common. My approach at this stage matches Stake's 'collective case study' (2005 p.445) or multiple case studies (Robson,1993) in order to identify both specific and generic features of the knowledge identified by the MaSTs.

I sought to understand if there was a common pattern in the knowledge identified by each individual MaST and have reflected at length as to whether I have any claim to identify commonality.

I anticipated that there may be what Guba and Lincoln (2005) describe as 'universal commonness' p.155, drawing on the view for example of Satre (1981) that every example bears some stamp of the general, although unique in its setting and detail. A single case of knowledge identified by a MaST demonstrated a possible universal characteristic. During the research I moved from analysis of the particular to consider possibilities for the general, (Lincoln and Guba, 1985; Robson, 1993). I considered the degree to which deep subject and pedagogical knowledge might manifest itself in different ways in individual teachers, and whether there were common features, or collective stories (Kvale, 1996). A belief in the ability to articulate common features of knowledge is the basis of the models and frameworks reviewed in my previous chapter and which form a tool for my analysis. It also underpins Williams' claim that a degree of subject and pedagogical knowledge is distinct from other forms of teacher knowledge and which enables a MaST to support staff in their teaching of mathematics.

I have reflected at length on my sample of eleven cases of MaSTs. The sample of cases studied is a small self-selecting set of a complex and diverse population. I do not claim it is randomly or strategically selected. The sample represents a range of convenience variety in teaching experience, age range taught, type of school and other roles held. These details can be found in Appendix ii. The sample is accessible but not necessarily typical. Schostak (2002) argues that we cannot claim that cases are representative of a sample, a single instance of a homogenous population, only that each case can be considered as an example of a complex idea. This limits my claim to general statements about the knowledge of all MaSTs. I follow Davis and Renert's (2012 p.249) advice, when discussing learning of mathematics generally, to adopt 'tentativeness towards overarching statements and attentiveness to the contribution of participants'. The value of my research lies in the fact that each case is a valuable voice. Each case provided me with insight. Whilst recognising the limitations of my sample in terms of reaching general definitions of the knowledge of the MaSTs, I argue that any random or strategic sample would not allow me to make completely generalisable claims.

In considering firstly local knowledge, and then whether there are grounds for drawing general conclusions about the knowledge of other MaSTs, my sample of eleven MaSTs is large enough to allow me to consider threads across examples, whilst small enough to explore the individuality of each example. They are a sample of an interesting, unique and relevant group: the first and therefore significant MaSTs, important for future policy. I claim my conclusions have analytical if not statistical claims towards generalisation (Yin, 1989; Stake, 1994; Kvale, 1996). Rather than reaching a theory of the MaSTs' deep subject and pedagogical knowledge as a set of propositions with claims to generalisation and precision, I offer an understanding of the knowledge of the specialist teacher.

Finally for this section, I will reflect briefly on my choice of focus on teacher knowledge rather than skills, attitudes and beliefs. Williams called for the role of MaST to be underpinned by knowledge. This was his recommendation with respect to specialist teachers, although he stated that pedagogy is often associated with knowledge and beliefs. He recognised the place of confidence, beliefs and attitudes to mathematics, as well as knowledge. The final report stated,

> It is widely recognised that a teacher's own enthusiasm for, and knowledge of, mathematics, as well as their beliefs about teaching and learning, will impact on their classroom practice, regardless of the external constraints on curriculum and lesson design (Williams, 2008 p.63).

He called for the MaST to support colleagues in their beliefs in teaching and learning, their knowledge of mathematics and their enthusiasm for mathematics. However the recommendation was about MaSTs' knowledge and the provision to be made to support them in developing this. My research aimed to consider Williams' focus on the MaSTs' knowledge. My methods were designed to allow me and the MaSTs I work with to explore the knowledge which underpinned their teaching approach to one area of mathematics and what they considered to characterise deep subject knowledge. This allowed me the opportunity to interpret aspects of knowledge which MaSTs used and which they defined as deep. I recognised that MaSTs chose to share with me these aspects of their knowledge, and that these choices were based on their attitudes and beliefs. My sample of MaSTs' attitudes and beliefs related to, for example, mathematics, the teaching of mathematics, their role as MaSTs and their relationships with me as an interviewer. These therefore modified the knowledge they shared with me. For example in Chapter 3 I selected the theme of using and applying mathematics as a feature of knowledge needed for the teaching of primary mathematics. However I acknowledge that teachers' beliefs in the nature of mathematics and mathematics education impacted on whether or not they conceived of this sort of knowledge as valuable (Ernest, 1989). Knowledge and beliefs are clearly intertwined and in exploring the knowledge of MaSTs I do not attempt to separate this from their wider beliefs (Askew et al, 1997; Hill et al, 2008). In my analysis I did identify data relating to beliefs but my focus was on the knowledge which was developed and conceived of as deep. Therefore, following Williams' recommendation, my study was of MaSTs' knowledge, whilst acknowledging that their attitudes and beliefs were part of this knowledge.

Finally I set out the premises of my argument with respect to the knowledge of my sample of specialist teachers.

The premises of my argument

My research was based on the argument that the teachers interviewed had the subject and pedagogical knowledge typical of that recommended by Williams. I do not aim to prove this, but argue that these teachers can be assumed to have such knowledge.

My argument rests on these premises. These teachers were nominated by the head teachers under the detailed expectations for MaSTs set out by the then DCSF included in Appendix iii. By the second interview, they had undertaken and passed the requirements of the programme including the learning outcomes referring to deep subject and pedagogical knowledge (although at this stage it was not fully defined and this is an argument for the value of my research.) As deep subject and pedagogical knowledge is not well articulated, there is no 'test' of it. As this was a new programme and a new role in school, there was no other group with an equal claim to deep subject and pedagogical knowledge of primary mathematics.

I do not argue that the teachers did not have deep subject and pedagogical knowledge before the programme, in the first interview, and had developed it during the programme by the second interview. This study is not a programme evaluation, although it will inevitably impact on my future practice. This is a new programme and a new role, and there was no assumption by Williams 2008 that teachers with this sort of knowledge did not exist already.

> Our recommendation therefore acknowledges that in many schools the equivalent post to the Mathematics Specialist advocated here already exists; indeed, many have been encountered during this review (Williams, 2008 p.21).

However, the second interviews are the most valuable in that they show the knowledge of teachers already established in the role of MaST. The first interviews helped to validate the interview schedule and methods used. They also prompted the MaSTs to consider the development of their knowledge as they undertook the role, by considering their responses from the first interview, during the second.

I consider that the features I identified in the interviews are indicative of specialists' subject and pedagogical knowledge rather than the characteristics of any good primary teacher. This study does not make use of a control group, but quotes from the MaSTs themselves suggest that they see specialist subject and pedagogical knowledge in their new role as something different. For example:

'… I can't put it in to words, having read around the subject a bit more, you kind of understand… it is very hard to explain, … it's …it's understanding the reason why you are

doing things as well, because in our MaST role we have to tell staff or ask staff or explain *things to staff I am able to explain it … a lot more confidently because I am able to give some reasoning and theory and background to it whereas before I wouldn't have been able to stand* up in a staff meeting and say this is what we are doing, and the theory and understanding *behind it maybe… in that respect maybe my subject knowledge is deeper because am able to* jus*tify …some approaches, maybe..'*B2

'Since doing the MaST, as I said I think my subject knowledge has definitely deepened now, where I thought I had subject knowledge when we spoke at the beginning of the course, I thought I had, it was knowing what addition is and knowing how to do it yourself and then being able to teach it to children, I thought that was your subject knowledge, but now I realise actually it is really deeper than that, it is knowing the progression of it, knowing the skills you *need before you teach that… Skills that the children need to have before they can go.. H2*

(Throughout my thesis I present quotes from the MaSTs in italics.)

In my next chapter I explain which methods I used to identify the knowledge which the sample of MaSTs conceived that they drew on in their role and which they defined as deep.

Chapter 5

My methods of research

In this chapter I outline my methods of collecting and analysing data which enable me to answer my research questions, and explain how these follow from my beliefs and chosen inquiry strategy which I have already set out.

Then I reflect on my methods. I consider studies which use similar and alternative methods to research mathematics education and explain that my method is the most appropriate to answer my research questions. I identify the limitations of my methods and explain how I have addressed these. I set out the argument for the worth and quality of my research. Finally I lay out my ethical considerations.

My methods

The new knowledge I make claim to is a function of my methods (Davis et al, 2007). I required methods which would enable MaSTs to identify features of the knowledge they used in their role and which they conceived of as deep, and to allow me to understand this, across examples and compared to existing models. The most effective methods to do this were extended interviews.

Teachers participating in the Primary Mathematics Specialist Teacher Programme at the university where I am based were informed of my research and asked via email (Appendix i) at the beginning of their programme to agree to two interviews. The interviews were repeated at the beginning and at the end of the programme. I invited two cohorts to participate.

The interviewed MaSTs were all at the same stage, beginning and then completing the programme, although cohort 1 interviews took place 9 months before cohort 2. As I have explained, during the time of the research there were significant changes in the education climate due to a general election in May 2010. This took place between the first interviews of cohort 1 and 2.

The initial email resulted in positive responses from the following participants:

Number of interviewees cohort 1 interview 1	5
Number of interviewees cohort 2 interview 1	10
Number of interviewees cohort 1 interview 2	5
Number of interviewees cohort 2 interview 2	6

Table 2: Number of interviewees

Two teachers left the programme during cohort 2 shortly after the first interview due to changes in their role in school or changes in schools. Two more teachers in cohort 2 chose not to participate in the second interview but did complete the programme. One cited changes in school and family circumstances resulting in lack of time and the other did not offer an explanation. Their first interviews were therefore discarded. A total of eleven MaSTs were interviewed twice and my research is based on these interviews. The sample of MaSTs is bounded by time, in that all the teachers were in the first two years of taking the role of MaSTs, and undertaking the MaST programme at the University where I am based.

In my research I refer to my MaSTs as female. This is not to ignore the importance of their gender, and this is something perhaps for future research, but to protect the identity of the one male MaST interviewed in my study. Details of the interviewees and their settings can be found in Appendix ii. Details which would act to identify them have not been disclosed. I labelled the MaSTs by letters, beginning with those in cohort 1.

The chronology of my interviews allowed each stage to challenge and inform the next, and to allow patterns to emerge (Strauss and Corbin, 1998). In this way the research is recursive, acting as a series of ‘circles within circles’ (Ely et al, 1991 p. 227). This can be seen from the following table.

Method employed	Timing	My reflections at that point in the study
Cohort 1 first interviews	January 2010	Analysis led me to consider the schedule for the first interview cohort 2
Cohort 2 first interviews	September 2010	Analysis led me to consider the schedule for the final interview cohort 1
Cohort 1 final interview	January 2012	Analysis led me to consider the schedule for final interview cohort 2
Cohort 2 final interview	September 2012	Final analysis

Table 3: Chronology of interviews

Although, as can be seen above, my research with each cohort overlapped, my methods involved me in:

1. Conducting the first and second interviews
2. Coding each interview openly
3. Comparing coding across interviews
4. Comparing coding with respect to existing frameworks, individually and across interviews
5. Coding stories in the interviews

The next section of this chapter will consider each of these in detail.

The interviews were semi structured. This was to recognise the tension acknowledged in my research between the nature of individual's knowledge and shared features of knowledge. They were **semi** structured to provide depth of data for each specialist teacher, and semi **structured** to enable me to compare across interviews.

A number of standardised closed questions were initially asked in each first interview. These acted as means of putting the MaST at ease. They provided me with information about the teacher's individual setting which was important for me to gain as rich an understanding as possible of the context of their responses.

- How many years have you been teaching?
- Which year groups have you taught?
- When did you qualify?
- What was the subject of your degree?
- What was your last mathematics qualification?
- What professional development have you had in mathematics so far?
- Are you the mathematics coordinator/subject leader in your school?
- Do you hold any other position in your school?
- What sort of school do you teach in, infant, special, primary, junior?

The main body of the interview centred around two questions.

Firstly the teachers were asked to identify an area of mathematics which they had taught recently to their present class and which they would be teaching around the time of the second interview. They were then invited to talk at length about how they approached this area of mathematics.

Secondly they were asked to describe their understanding of the term deep subject knowledge.

Supplementary questions in the second interview asked MaSTs to reflect on how they found their knowledge as MaST to be different to that of any other teacher, if at all, and specifically how it compared to the knowledge of a teacher who had taught all year groups.
In the final interview a transcript of the first was available to facilitate reflection on possible changes over the two year period.

The rationale for the questions was as follows.
The first question asked for a reflection on current approaches towards the teaching of one area of mathematics chosen by the interviewee and possible changes in approach in the second interview. This was a long and in depth discussion. Teachers were asked to 'substruct' (Davis and Renert 2012 p.252) their teaching of this area of mathematics by providing in depth detail of their approach, and their reasons for taking it. This question promoted discussion typical of that of a MaST with their colleague.
I argue that the responses to this question allow both myself and the MaST to identify features of the knowledge base of the MaST. This is a view shared with other researchers (Simon, 1995; Ma, 1999; Bednarz and Proulx, 2009). I base this argument on the following thinking. I assume agency of the MaSTs, in that they are not blindly following published schemes or prescriptive guidance. The question itself, and following probing questions, force them to articulate why they take such an approach. My use of the transcript from the first interview during the second promoted reflection on and articulation of knowledge. Furthermore, I believe practice is underpinned by knowledge, and this belief is the basis of Williams' recommendations (2008) and Standards for Teaching such as TDA 2007 and DFE 2012.
The first question asked teachers to give an example of their knowledge base. The second asked them to articulate how they perceived their knowledge more generally. It asked for more direct reflection on the nature of the MaSTs' knowledge and articulation of the term deep subject knowledge which was particularly used by Williams (2008). The question was placed second to enable the MaSTs to firstly reflect on details of examples of their classroom teaching, and then consider this more abstract and defining part of their role. I did not ask them to define

pedagogical knowledge generally or the pedagogical knowledge they developed and drew on as a MaST as well as subject knowledge. In answering my question about deep subject knowledge, the teachers often naturally discussed aspects of pedagogy. A focus on mathematical knowledge maintained my initial interest in mathematics, whilst the first question had allowed discussion of pedagogy. I also wanted to see if they distinguished between subject and pedagogical knowledge.

For both questions, probing questions were designed to clarify meanings, especially where these might be assumed as shared meanings, and to invite a full reflection. This mirrors my aim of negotiation and shared understanding of knowledge. For example, probing questions included:

- Can you tell me more about…?
- What do you mean by…?
- Can you give me an example of…?
- I'm sorry, I am not sure exactly what you meant by…

The full interview schedules can be found in Appendix iv.

The interview schedule underwent peer review and was piloted with a colleague. The interview schedule was used with Cohort 1 and this confirmed the approach was suitable for Cohort 2 and for the second interview for both cohorts.

The interviews were generally conducted in the MaST's school, in a staffroom, or the MaST's classroom at the end of the day, or another quiet shared staff area. I have argued that this replicates to some degree the way in which a MaST needs to draw on their knowledge in their role. It mirrors how specialist teachers can be asked to help colleagues, making their knowledge explicit by articulating, justifying and unpicking their own teaching approach. These occasions can take place outside of lessons, in shared areas such as staffrooms. So the interviews give an indication of how MaSTs use their knowledge and therefore provide a legitimate reason to use the interviews to reflect on it.

The interviews were recorded systematically. Participants were asked to state a preference for taping or note taking. All the second interviews were taped, and all but two of the first interviews. The interviews were transcribed and the transcripts sent to participants for verifying. This gave the MaSTs chance to remove anything they felt identified them, but they rarely made any changes at all. I also noted my own or interviewees' use of nonverbal information such as proximity, seating, pausing when others came in to the room, silences, and gestures. This was to add depth to my understanding of each interview. There was only one case where this non-verbal information appeared to be significant. My reflections on this case can be seen in Appendix v (MaST E).

Each interview was analysed in depth, twice to ensure that I could gain as rich an understanding as possible of the meaning of the views expressed. My analysis of the content of interviews began with question 1, relating to the specialist teachers' approach to teaching one area of mathematics. I analysed the responses in the second interview only, that is at the end of programme and when the MaSTs had been in their role for two years. This was because I wanted to engage in the knowledge that the MaSTs drew on in their role, which was best captured in the second interview. I did not design my research to be a programme evaluation, where I would survey teachers' practice before and after it. The programme had included specific sessions on aspects of teaching such as the use of models and images for the teaching of fractions, and the value of mathematical talk. Analysing both interviews might have led me into reflecting whether MaSTs had engaged with and embedded these particular aspects of practice. Analysing just the second interview gave me a more holistic picture. Encouraging the MaSTs to engage with the transcript of the first interview during the second interview enabled them to reflect on changes in practice, if these had occurred.

In my analysis I firstly searched and recorded any mathematical references. This was because initially I had focussed solely on mathematics and the deep subject knowledge of the MaSTs. However as I coded, I felt that much of interest was pedagogical in nature and I widened my focus.

Secondly I coded any references to the MaST's wider beliefs, the professed ones and actual ones. This was important for my rich understanding of the responses of each MaST.

Finally I searched systematically (Thomas, 2013) each of the interviews for the aspects MaSTs identified as different in their practice after reflecting on the first interview, or typical of their present practice, as indications of the knowledge they drew on in their role. All of these were recorded with codes, and were stored using Nvivo software, summarised with a heading for the code. For later interviews I would use the same code if it matched the content of the interview or form new codes if not. Some phrases were coded more than once where they contained references to more than one code. This was because I wanted to use phrases rather than odd words out of context. This impacted on the percentages of coverage. So I recorded the percentage of each coded phrase of the whole interview, including my words, as well as the number of occurrences of codes. In this way I analysed the words of the MaSTs, the number of times they spoke about a particular issue and how long they spent discussing it. This qualitative and numerical information can be seen in Appendix v for each MaST.

The following table shows the resulting codes and explains how I decided the content would be categorised.

Code Name	Explanation and examples of what was included
Connecting similar mathematical ideas, making horizontal connections	Used when MaSTs: link mathematical ideas presented in different ways eg 'counting session *every day and linking it to tables' A2* link mathematical operations eg multiplication and division link number facts eg 3 x 4 and 4 x 3, or discuss families of facts link different structures of operations eg grouping and sharing link different areas of mathematics eg data handling, and using and applying

	mathematics
Whole school approach	Used when MaSTs talk about an approach they have adopted across their school eg MaST A discusses the introduction of multiplication fact competitions in whole school assemblies
Rapid recall, basic skills	Used when MaSTs talk about their work in promoting rapid recall of and use of number facts. Also includes reference to mental strategies for operations 'I think I focus more on mental division now than I did before' F2
Progression tracking forwards	Used when MaSTs: Discuss ideas which become mathematically more challenging *'you've done the sevens, you do the seventies, and the 0.7s and then using that to work the grid…' A2* Note impact of what is taught to younger children on later learning *'but having done the course now and spent time with early year foundation specialists and key stage 1 teachers, you see actually what goes on at that level, and the impact it has coming up through the school..'A2* Move on their current class more quickly than before because of their knowledge of the next steps *'I think I am braver in my teaching because I am more confident, I do know it, I do know where they are going so if it is appropriate than it is OK to do it, 'B2* Use knowledge of later ideas in the curriculum in differentiation for more able children Advise other staff who teach children younger than the MaST's current year group on how to ensure quicker progress drawing on knowledge of the content of later learning Discuss changing activities so they don't limit learning Alter teaching so that they don't contribute to a misconception in later learning

	Discuss their views that some calculation strategies build into a coherent progression better than others and so claim these strategies have a positive impact on later learning Discuss how they consider whether to introduce children to conceptually harder ideas in order to promote conceptual understanding *'So another point is it is important not to stop at certain things, so some people might say Ok they are muddling up those teen numbers so they just put a cap on twenty and they don't look at the higher numbers until they have got this but it is thinking well if they understand the structure then they understand those connections, so I think it was not having that fear as well…' J2*
Progression supporting staff with policies etc	Used when MaSTs refer to guiding other members of staff to ensure their teaching promotes progression in learning, usually in the form of the calculation policy or guidance or children's target setting or verbally in a staff meeting or in coaching or mentoring.
Use of resources and images	Used when MaSTs refer to resources or images generally *'I still start with the practical basis because I think they still need to have the really hands on experiences and we probably try to do more real life experiences as oppose to going straight to doing pencil and paper ways'* B2 specifically eg discussion of counting sticks, number lines, empty number lines, arrays, smarties, balloons, mirrors, numicon, hundred squares. Some resources are also mentioned in the code 'Context for mathematics' where for example realistic resources are identified as a sources for mathematical learning eg water play
Use of ICT resource	Used when MaSTs refer to specific ICT packages by name or more generally to using ICT as a resource.

Law or structure, basic mathematical idea	Used when MaSTs refer to: Mathematical laws such as the commutative law and the distributive law Structures of operations eg subtraction as take away and difference or division as grouping and sharing Identify basic ideas which underpin more complex ideas 'just simple things *like one to one correspondence… trying to explain to my TAs what one to* one correspondence actually means, so I say to them for children it is really *quite hard because they have to touch one thing each time' D2 'then we did a thing actually talking about what is an angle, turns, they didn't know that'* C2 Explore different aspects of basic ideas such as column and quantity value within the general idea of place value
Progression tracking backwards, often from error	Used when MaSTs: Discuss changes in how they begin topics by tracking back to more basic and less complex ideas and then work forward Show that they find a common error in their classroom and trace back to tackle its origin in their teaching of their own class, often moving out of the recognised mathematics for that year group Show that they find a common error in their classroom and trace back to tackle its origin in their guidance for the whole school Say that they are more knowledgeable now about the curriculum which has gone before that which they are teaching to their current class and this has changed their practice Knowing the approaches thought to be good practice in years below their current class and making use of these approaches in their current class When advising colleagues, being able to trace back to identify gaps and misconceptions and suggest way to tackle these

	Structure work for a week or a lesson around the progression of learning which should have gone before the current learning '(before) *I'd go straight* in with this is what they are supposed to be doing, I will teach them what the curriculum says for this year group, and then coming unstuck and discoveri*ng the gaps in children's knowledge and then having to go back to* try and fill those gaps, whereas I find that by doing a recap, show me what you know, helps me understand where they are and helps them to review *what they know' E2* Talk about changing the sequence of teaching after analysing the skills needed to undertake particular calculations and making sure these skills are learned first Use their knowledge of previous curriculum to change how they start a topic of work, using their knowledge as a guide to assessment for learning Break down statements in the National Curriculum knowing what learning needs to go before each statement. Talk about their modifying their approach when they change year groups to an earlier year group
Reference to theory, literature, research findings, assignments	Used when MaSTs refer to the following as sources of their knowledge: Readings undertaken as tasks or for assignments, referenced by the programme, referred to by name 'so I looked at what I call the bible, the H*aylock book' B2* Research findings which not named *'I know the research would suggest that setting only really has impact on the top, it doesn't here' A2* Official documents eg the National Curriculum, the Primary National Strategy Websites the programme has directed them to General references to theory which are not specified

	Also included is one reference where the MaST feel she no longer needs to use documents to guide her as her knowledge has increased 'Yes, definitely gained more knowledge because although you would look things up, if someone came to me I probably would have looked it up as oppose to now if they come to me, generally the knowledge is there to help them, whereas *before I would have had to go away and look up before I could help' H2* This code also includes reference to plans to do further research H2
Unique to my school	Used when MaSTs talk about their role in meeting a particular need, typical of their school and which is perceived to be different to other schools
Change in differentiation	Used when MaSTs discuss changes in the way they differentiate to meet individual needs
Context for mathematics	Used when MaSTs discuss how they put a particular area of mathematics into a context, which might be another curriculum area or a realistic problem
Using and Applying mathematics: open ended investigations	Used when MaSTs refer to open ended investigations or problem solving
Vocabulary	Used when MaSTs discuss the way they introduce mathematical vocabulary to children, and often how this has changed as they develop their role
Maths in play	Used when MaSTs refer to children learning mathematically as they play
Mathematics in a non-mathematical context	Used when MaSTs talk about a context where they are able to identify potential for mathematical learning (as opposed to the code identifying context for mathematics where MaSTs begin with a particular area of mathematics and search for a context for it) or when MaSTs identify mathematical learning in an activity which is not necessarily mathematical or is used for non-mathematical learning objectives *'I could say actually that*

	child is doing maths but you might not realise that they are, I can help her to *identify what she is already doing' C2*
Key idea and those on periphery	Used when MaSTs identify a key mathematical idea and distinguish this from those ideas on the periphery *'The most impor*tant thing is reading off the information, it is not even about the making the pictograms, that is just a nice task to do, you want them to be able to tell you how many more, cats than dogs, it is information finding, not just a pretty picture, it is infor*mation finding,.. ' D2*
Meeting children's individual needs, not progression	Used when MaSTs discuss identifying and meeting needs, usually as a form of assessment for learning, but not related to progression which is coded separately 'my children have got *ASD and their minds work very differently… usually I* try to think of a tactile idea and a non tactile idea.. for different needs, some *children need the systematic work. Some need the multi sensory stuff more'* D2
Talk	Used when MaSTs discuss the use of talk in their lessons
Games	Used when MaSTs discuss the use of games in their mathematics lessons
Early mark making	Used when MaSTs discuss their knowledge of and use of Early Mark Making
Supporting other staff other than progression	MaSTs' discussion of how and when they support other staff, in the area of mathematics they are discussing

Table 4: Codes for question 1

I also coded any examples of figurative language used by MaSTs to describe their knowledge, and any examples of stories, where MaSTs talked about a sequence of events.
After ensuring all aspects had been recorded I then collated what was left of the MaSTs' words so that I could see what had not been coded so far, and consider if anything had been missed. It also gave me a numerical feel for what had been excluded from the coding so far. Schostak (2002) calls these 'strategic cuts' p.66. The data which was not fitting into my preconceptions was significant. Identifying these parts of the interviews and reflecting on them challenged me to consider information I may have missed. In Appendix vii I show how much was not coded from responses to question 1, as a percentage of the whole interview, including my words.

I then reflected on the interviews of each MaST in turn, to complete my understanding of their responses. The result was a pen portrait for each one, or vignette (Miles and Huberman, 1994). These pen portraits describe the key themes and draw largely on direct quotes from the interviews. I began this with the final interview, as I felt that always beginning with the first MaST to be interviewed would mean that I might simply repeat what I had found before, and close my analysis down to new features.
These pen portraits can be seen in Appendix v.

I then conducted a similar analysis of the responses to question 2, relating to the MaSTs' conceptions of deep subject knowledge. The table in Appendix vi shows the codes I created and gives examples of the sort of phrases in each. I analysed the answers to the second question in a separate N Vivo file as this enabled me to code the responses openly. In other words I did not use the codes already established in my analysis of question 1, although these inevitably informed in my thinking. This was because I wanted to be open in my analysis and not make assumptions that what I found would reflect responses to the first question. As before, I coded responses first for mathematical references and then for phrases which indicated wider beliefs. Then I searched for features identified by each MaST of deep subject knowledge. The responses were coded by phrases, trying to maintain the sense and context of each one to ensure that meaning could be understood within the code. I began with no codes at all, and then slowly built them up as I worked through the interview for each participant, chronologically working my way through in the order that the first interviews took place. In several cases I noticed an area for coding in a later interview, for example the identification of the importance of pattern in the first interview of MaST J and went to back to check if I had missed this in earlier interviews. In all cases I had not.

I did not distinguish between mathematical and pedagogical knowledge, but collected and coded any phrase which referred to a specific aspect of the MaSTs' approach to their teaching for question 1 and an aspect of knowledge conceived as deep in question 2. I did not judge it necessary to code these as positive or negative as the MaSTs indicated that these aspects were all desirable features of their teaching in question 1 and aspects of deep subject knowledge in question 2.

I wanted to explore the meanings of each interview openly first, and the sequence of my coding allowed me to do this. At the risk of reducing the data and separating it from the words of the MaSTs, I compared the occurrences of codes, and percentages of coverage out of the whole interview of each code, across interviews to identify any shared features of knowledge between the MaSTs. I began to get a feel for shared features and unique features, and how they might relate to the teacher's setting, experience, and beliefs. Matrices allowed me to explore these, which are included in Chapter 7 and 8 (Miles and Huberman 1994).

For both questions, coding was a dynamic and fluid process, a form of 'conceptual ordering' (Strauss and Corbin, 1998, p19). Identifying features as categories acts to reduce it but also illuminates it and allows for reflection and analysis. The codes were used as a retrieval, comparative and organising device. Although coding de-contextualised the MaSTs' words (Strauss, 1987) it allowed me later to re-contextualise them, to bring similar phrases from across interviews together. I then compared my coding with respect to existing frameworks and themes I had selected as significant from the existing debate and which are presented in Chapter 3. I compared which of these had occurred in each interview and across all the interviews. I also compared my coding against the content of the programme the specialist teachers had completed.

The final coding considered the specialist teachers' use of narratives, accounts or story telling in the interviews. Coding whole stories or accounts served to look at data in a different way, which was less fragmentary than the other codes. I firstly identified stories or narratives. The first question in particular tended to invite these, but some MaSTs used them to exemplify their response to the second question. I then considered the function of the story or narrative, and whether it appeared to be to justify a change in teaching approach or conceptions of knowledge, and therefore provide a sense of identity (Coffey and Atkinson, 1996). Where MaSTs reflected on the transcript of the first interview and identified changes, I analysed examples of contrastive rhetoric (Hargreaves, 1981, 1984), looking at the language devices such as figurative language used by MaSTs to express why their current practice was better.

I then summarised my findings and sent them to my sample of participating MaSTs to ask for their views on whether I had correctly interpreted their interviews. My email can be seen in Appendix viii.

Reflection on my methods

In the next section of this chapter I will consider why I chose my particular method of interviewing individual specialist teachers and disregarded any other method of answering my research questions. I lay out the possible limitations of my methods and the strategies I have used to overcome them. Finally I discuss the worth of my research and its ethical considerations.

My survey of current research into the knowledge of teachers relating to primary mathematics, outlined in Chapter 3, reveals that the most used methods of collection of data are: interview, either of individual or groups of teachers, the collection of teachers' narratives, observation of lessons including joint reflection on video footage and mathematical tests. I chose to undertake interview of individual specialist teachers in line with researchers such as Ma (1999), Alder et al (2009), Compton and Edwards (2011) and Turner (2012). My use of individual interviews aimed to re-create, to some degree, a context where a specialist teacher might be asked to discuss their teaching with a colleague, as in my first question. Compton and Edwards (2011) asked a similar question of their eleven teachers, inviting them to discuss their teaching approach but to a specific area of mathematics rather than one of their choice. I asked my sample of interviewees to choose their own area of mathematics to allow for the particular age range of the children they taught. Some teachers were teaching nursery children and some eleven year olds. I also wanted to examine the nature of their knowledge as displayed through their teaching approach rather than its specifics. Adler et al (2009) used an interview schedule similar to my second question on teachers' conceptions of deep knowledge. They worked with student teachers preparing to undertake their secondary ITE programmes, so represented a different type of participant. Turner (2012) and Kristindottir (2013) made use of group interviews. I chose not to interview specialist teachers together as I felt that the nature of the knowledge they identified as a group would be different to the knowledge they drew on to discuss teaching in a one to one context, similar to their role in school. I felt they would reveal different types of knowledge to each other, and that a group meeting might re-create the setting of the programme, where my role had been established as leader. I aimed instead for my role to be more equal with the specialist teachers. A number of researchers use interview to collect teachers' narratives (Ryan and Williams, 2011; Brown, 2013; Oslund, 2008). My open questions did engage my interviewees in this sort of account.

Video footage of teachers' lessons has been used to promote rich discussions in interviews with teachers (Turner, 2012). Other researchers observe lessons and video footage of lessons as part of their collection of data (Rowland et al, 2009; Hill et al, 2008b; Newell, 2011; Adler and Pillay, 2007). I chose not to ask the specialist teachers to allow me to observe their lessons. This was for the following reasons. My experience with the MaSTs showed me that their schools were rarely able to allow teachers time to observe the specialists in their classroom, as this involved covering the teacher's own class. This is backed by the finding of the NFER evaluation:

> the activity that MaSTs were most frequently engaged in was 'offering advice to colleagues on mathematics-specific pedagogies' with 45 per cent reporting they did this half-termly or more (Walker et al, 2013 p. 26).

Therefore the interview provided a closer context to the way in which the specialist teachers share their knowledge. I also felt that I needed to balance the potential extra burden for the participating teachers of an observation, from a former tutor, against the possible additional research findings.

I chose not to use other strategies such as a scrutiny of specialist teachers' planning, as on its own this would not provide the depth of information about the conceptions of the MaSTs. Researchers also use mathematical tests as a form of data. Some research refers to subject knowledge audits undertaken as part of ITE (Turner and Rowland, 2011; Newell, 2011) and other researchers design and use tests to measure knowledge for classroom teaching (Hill et al, 2008b; Warburton, 2012). However, I was not in a position to draw up a mathematical test on a form of knowledge which I had yet to identify. Other researchers attempt to use mathematical tests for the children of the teachers they work with, to link teacher knowledge to children's learning (Askew et al, 1997; Hill et al, 2008b). As the knowledge base of the MaST is conceptualised, this approach would be a logical next step for my research.

Researchers in this field have rarely made use of questionnaires, with the few examples using these to collect preliminary data to follow up with smaller scale research using other methods (Burgess and Mayes, 2008). The NFER (Walker et al, 2013) evaluation made use of questionnaires of a large number of MaSTs and followed these up with interviews of a smaller number. However the interviews were not used to draw detailed conclusions on the nature of the knowledge of these MaSTs. I considered the use of a questionnaire, but my focus was on the nature of deep knowledge, not easily captured in this way. I did not feel that questionnaires would reveal conceptions of the nuances of knowledge which I hoped to collect.

The following table outlines what I identify as the potential limitations of my methods and how I have addressed these.

How can I balance the tension between individual MaSTs' knowledge, perceived by them in their particular setting and my aim of exploring features of MaSTs' knowledge generally	The interviews allow individual responses to be explored in depth. The sample is large enough to allow the possibility of identifying shared features but small enough to hold extended interviews. I do not claim to make generalisations but to suggest shared features.
If deep subject knowledge is a personal response, how can I claim to understand it? If I am interpreting teachers' interpretations, there is room for misinterpretation.	My role and the knowledge and understanding it entails enables me to engage and reach a shared understanding with each MaST. The sample size helps to address this.
The quotes from the interviews are descriptive claims but might not be accurate. Denzin and Lincoln (2005) state 'individuals are seldom able to give full explanations of their action or intentions; all they can offer are accounts, or stories, of what you have done and why' p.21.	The number of interviews and the use of probing questions attempt to minimise this. Questions are in the context of the role of specialist teacher, therefore the findings are indicative of the knowledge they will use in their mentoring and coaching.
The sample may not be representative	Although a self-selecting group they can be argued to be a valid group. They all are employed in one Local Authority but the

	largest one in the country and varied in its schools. They are all enrolled in the same programme but the programme is based on national parameters provided from the DCSF, required of all programmes across the country, and the university's validation procedure checked this. So although the MaSTs were all following a particular programme, this programme is characterised by the parameters of any MaST programme.
Were the MaSTs simply repeating what had been covered in their training because they wanted to please me?	They are reassured that there is no right answer and I am conducting the research because I do not know the answers of my research question
Were the MaSTs simply repeating what had been covered in their training?	My comparison of my findings against the outline of the programme is set out in Chapter 8
Influences on knowledge may not be to do with the role of MaST, it could be partly or wholly due to for example a change of year group, head teacher, climate, change of school, other professional development courses or mentoring and coaching	These are all normal part of teacher development and therefore I do not aim to isolate the knowledge gained by MaSTs from these aspects to focus solely on that developed as a MaST. All MaSTs would engage in learning through these forms of professional development and this adds to the knowledge of the MaST.
I used no control groups, am I just identifying knowledge of good teachers?	These are good teachers taking the role of MaST and so still relevant and interesting

	The teachers themselves say this knowledge is different to the knowledge they used before.
The knowledge identified with me during the interviews may be different to the knowledge identified by MaSTs in discussion with their colleagues (Miles and Huberman 1994)	I constantly reassured the interviewees that the reason for the interviews was that I did not know the sort of knowledge they had developed, and that there was no right or wrong answer My findings were verified by the MaSTs.
Early identification of features of knowledge creates a bias for following interviews and analysis	I pay careful consideration to negative cases and report the percentages of occurrences of shared and empty codes or codes limited to one interview.
Some data might be weighted, for example some teachers might spend longer making the same or a similar point to others, some entries into codes might be lengthier but constitute only one entry	Both the percentage of coverage and the number of utterances is reported.
The capturing of teacher knowledge in categories might not be possible	I am open to this as much as possible and look at stories and narratives as an alternative way of analysing the interview data

Table 5: Limitations of my methods and how they have been addressed

Next I consider my arguments relating to the worth and quality of my study, and my ethical considerations.

I claim my research is valid from the systematic way I have laid out my review of relevant issues, context and methods, and clarified my own values and their role in the research (Charmaz, 2005). I take a systematic approach to dealing with the data to show the merit of my argument and the claims made for individual cases and across cases. My procedures have been laid out clearly and were informed by regular debriefs with my doctoral tutor. I deal with the data rigorously, providing the words of MaSTs to exemplify my claims. I analyse stories as well as other more fragmentary codes. Credibility is also claimed from my commitment to negative case analysis. I examine contrary evidence, for example where there were unique codes found in one interview but no other, or where open coding did and did not match the existing frameworks for classroom teacher knowledge. My use of quantification is overt.

I also claim the reliability of my research. I was persistent in my collection of data, engaging with two cohorts over a total period of over two years. Therefore there is consistency in my findings across time, with each cohort being interviewed over a two year period. Furthermore, after each interview, the MaSTs were emailed with the transcript to enable them to make any alterations and to give a final permission for my use of their interviews. This allowed the MaSTs to have prolonged engagement with the interview content. As I completed the final write up they were sent a summary of my findings. The sample of MaSTs were consistent at each stage in confirming that the interviews gave a clear representation of their views. Therefore I claim that both I and the sample of MaSTs have had a prolonged engagement with the data. The sustained period of research allowed me to gain a suitable sample size to answer my research questions.

I searched for consensus between MaSTs to underpin any claims I make for conclusions about the knowledge of MaSTs generally, and my use of quantification makes this clear. I also collected data across two cohorts and found consensus. In the nine months between the interviewing of cohort 1 and cohort 2, a change of government had produced a series of changes, as I have explained in Chapter 2. However results for each cohort were comparable.

My findings resonate with those already in the debate, and therefore my findings complement and supplement accepted views of teacher knowledge (Schwandt, 1989). I claim that my findings are coherent with existing models of knowledge of primary classroom teachers. I find that the knowledge of the MaSTs matches some aspects of these models, but not all.

My conclusions strive towards contributing to society, through the development of teacher knowledge and children's learning. They are relevant to the work of MaSTs and therefore applicable to any school. They are based on ethically sound research as outlined below. I claim the research has authenticity as the MaSTs stand to benefit from it. Firstly the participating MaSTs benefited from the discussion itself, as an opportunity for shared reflection and therefore a form of professional development. They will also benefit from the articulation of the knowledge base as part of the population of MaSTs. A test of the research is that the participating MaSTs themselves are convinced of its worth (Lincoln and Guba, 1985).

In this final section of the chapter I present my thinking relating to the ethics of my research. My ethical considerations are informed by the principles and procedures outlined in the University documents: An Introduction to Ethics Issues and Principles in Research Involving Human Participants, The Code of Conduct: Practice for Research Involving Human Participants, Ethical Procedures for the Conduct of Research Involving Human Participants and Ethics Policy for Research Involving Human Participants. The methods underwent the scrutiny of peer and doctoral tutor review, and were approved by the Faculty of Education Ethics Review Panel. The following are the underlying principles for the strategies I adopted. I consider each one in the light of my research and reflect on the extent to which my research is ethically justifiable.

I had no right to enquire about the knowledge of the MaSTs, and therefore I sought to gain their informed consent. Potential participants were provided with the information to make an informed choice relating to the nature of the research. They were informed of the procedures to protect identity. This can be seen in Appendix ix. In fact they gave informed consent four times in the project, at the beginning of each interview, as they were two years apart, and when they sent back the verified transcripts. Therefore there was an on-going dialogue about their participation in the project. However I accept that the participants were not fully informed as the expected findings of the study were at that stage unclear. They had undertaken the programme's content on research procedures and the related ethics, and therefore were able to draw on this understanding. I did not rely on this though, and I did my utmost to ensure they were able to give consent based on understanding. I recognise that although there was no overt coercion to particpate, there was some emotional and moral persuasion. The MaSTs probably wanted to please their tutor. We entered a contract via email before and after the interview through the signing of the consent form at the beginning of each interview. This can be seen in Appendices i and x.

I aimed for relationships with my MaST participants which are what Christian (2005) described as collaborative and trusting. At all times I avoided deceiving the participants, acting in line with the values of the University, and my own beliefs. However, I recognise that without wanting to deceive the MaSTs, I was not entirely truthful with them. To prevent contaminating my data, when they asked for my views, I did not respond and diverted the conversation. I expected them to be open with me, but I was not with them. Their responses often suggested they are concerned with providing what I might think are the right answers. I reassured them that there was no right answer and this is the value of the research. I considered possible risks to the interviewees in terms of their vulnerability and to the University in terms of liability.

To ignore the power relationships within my research is to argue that it is neutral and value free. I recognise it was not. There were issues of approval, vulnerability and possible shame (Christians, 2005). The MaSTs risked embarrassment and loss of standing by participating in the interviews. They also stood to benefit from the interview and were aware of this from the programme which was based on the premise that reflection on the teaching and learning of mathematics is a learning opportunity. I recognised my responsibility of care for the MaSTs, both as their tutor and as a researcher. I built a relationship of trust (Buber, 1958; Christians

,2005), based on that already in place as tutor, by explaining my procedures and allowing for voluntary withdrawal. Two MaSTs did withdraw before the second interview.

My analysis of the findings was rigorous but maintained the dignity of the MaSTs. An element of judgement in my analysis was essential for my argument. I endeavoured to do this maintaining anonymity and with respect. I considered the consequences of the publication of my research for the individual MaSTs. To identify anything negative might be harmful to an individual's self-esteem, although avoiding it would compromise my argument. In fact I did not find negative aspects of pedagogy or major misconceptions in teachers' knowledge.

In terms of fairness (Guba and Lincoln, 2005) I strove to ensure that all MaSTs' voices were equally well represented in the analysis. I demonstrated clearly what percentage of each interview was coded. I ensured marginality was prevented to take an inclusive view.

I aimed to protect the identity of my participants and to ensure them of this. The procedures I undertook are detailed in Appendix ix. I negotiated my descriptions of the MaSTs' other roles in school and the type of school they were based in with each MaST. Therefore the details I have included in Appendix ii have been agreed with each MaST. However, I recognise the difficulty of ensuring that each individual specialist teacher is not identifiable. In order to answer my research questions, I analysed responses of individual MaSTs in depth and this analysis was often enriched by considering the context of the MaST, for example if they worked with a particular year group. There was only one teacher in the group of participants from a special school setting. This makes her identifiable in the group, although not personally identifiable.

The research promotes social transformation (Christian, 2005) in that it aims to lead to better practice and could therefore be argued to have 'catalytic authenticity' (Guba and Lincoln, 2005 p207). Its aims are to enhance the mathematical learning of children through the role of the MaST.

In my next chapter I briefly introduce all the specialist teachers in my sample and present one in more detail.

Chapter 6

Introducing the sample of Specialist Teachers

In this chapter I introduce my eleven participating MaSTs, outlining their setting, previous teaching experience and professional development as they undertook the role as specialist teacher. I begin with MaST K and consider them in reverse order to the one in which they were first interviewed. I then present the mathematical topics they chose to discuss. Finally I present one of my case studies in more detail, MaST C. Pen portraits of all of the MaSTs can be found in Appendix v.

The eleven MaSTs

MaST K

MaST K qualified as a teacher in 1990, and had taught for 13 years before taking the role of MaST, mostly in Years 4 to 6 and Year 2. As she began the programme she was working with a Year 2 class. She took the lead for mathematics in her primary school. Her first degree was in Education and Mathematics. She had completed a Masters degree in Education which included an element of mathematics education. This was her last mathematics qualification. Her professional development in mathematics had taken the form of subject leader courses led by her Local Authority. In her first two years as MaST she taught Year 2, and then moved to Year 3.

MaST J

MaST J was a Key Stage 1 teacher in a primary school. She had taught for 11 years as she took the role of MaST, after qualifying in 1998. She had taught Years 1 to 6 and as the programme began she was working with a Year 1 - 2 class. The subject of her degree had been education and mathematics, and this was her last mathematics qualification. She has received and provided Local Authority professional development in mathematics prior to the programme. She was the mathematics subject leader in her school. During the first two years of the role she had taught a mixed Year 1 and 2 class, teaching mathematics to a group of Year 2 children in the second year. She then moved on to teach Year 2.

MaST I

MaST I was a Key Stage 2 teacher and mathematics subject leader in a junior school. She began the role of MaST in her 13th year of teaching. She had a wide range of teaching, from Year 1 to GCSE, but had most experience in Years 4 and 6. She had completed an Education degree, including a mathematics specialism which was her last mathematics qualification. Her most recent mathematics professional development had been in the form of Local Authority led maths courses, attending leading teacher and subject leader courses. She was the subject leader for mathematics in the school, as well as head of Year 4, acting deputy in the first year of the programme and Basic Skills coordinator. She taught Year 4 throughout the programme.

MaST H

MaST H was acting deputy head of a primary school in the first year of the programme, as well as leading Well Being and Modern Foreign Languages across the school. She had taught for ten years as she took the role of MaST, mostly in Key Stage 2 teaching Year 4, mixed Years 4 and 5, and Year 6. In the first two years of the role she taught Year 6 and then moved to Years 4 – 5. She had never taught below Year 4. Her degree was related to mathematics. She had undertaken work as a Primary National Strategy consultant for mathematics before taking the role of MaST, and this had been her main source of gaining professional development in mathematics, and delivering it. She was the mathematics subject leader for her current school which was a primary school, including a nursery.

MaST G

MaST G was an Early Years teacher and mathematics subject leader in a primary school. As she took the role of MaST she was in her 6^{th} year of teaching after qualifying in 2005 and completing a degree in Philosophy. Her last mathematics qualification had been her GCSE and she had attended Local Authority mathematics and early years courses as her latest forms of mathematics professional development. During the programme she changed schools but continued to teach Reception and nursery children throughout the two years.

MaST F

MaST F was a Key Stage 2 teacher and maths subject leader in a primary school. She took the role of MaST in her 12th year of teaching after completing a Natural Sciences degree which included some mathematical learning such as engineering. This was her last mathematics qualification. She had undertaken Local Authority courses as her last mathematics professional development. She was the leader of learning as well as mathematics subject leader. During the two years MaST F moved to another primary school where she took the role of mathematics subject leader. MaST F had taught predominately from Years 4 to 6. In the two years of the programme she taught Year 4 and then moved to year 6.

MaST E

MaST E was a Key Stage 2 teacher in a primary school. She began the role of MaST in her third year of teaching, qualifying in 2007, three years previously, after completing a degree in Natural Sciences with Biology. This degree included a module relating to mathematics which was her last mathematics qualification and mathematics professional development. In the first interview, she was not the mathematics coordinator in her present school but was shortly to change schools where she would take this position. Before the programme she had taught Years 4 and 5 and undertaken work as a supply teacher across the primary phase. During the programme she taught Years 4 to 6, with most time in Years 4 and 5.

MaST D

MaST D was a teacher at a special school who had taught for 13 years before taking the role as MaST, qualifying in 1997. (Special schools in England are used for children judged to be not best educated in mainstream schooling.) She had completed a mathematics degree which was her last mathematics qualification. She had led mathematics professional development for a group of schools and had attended special school subject leader courses. She was the mathematics coordinator in the primary phase of her school. During the programme she had taught Years 1 and 2, and then took a mixed Years 3, 4, 5 and 6 class of children with severe needs relating to the Autism Spectrum Disorder.

MaST C

MaST C was Mathematics coordinator and Key Stage 2 teacher in a primary school. She had four years' teaching experience as she took the role of MaST, qualifying in 2006 with a BA Ed degree in primary education and geography with QTS. Her last mathematics qualification had been her A level. She had attended professional development courses such as the Local Authority subject leader courses. During the programme she taught Years 5 and 2 and then moved to Year 6 where she taught top set for mathematics.

MaST B

MaST B was a Key Stage 1 primary school teacher who had been teaching for seven years as she took the role of MaST. She had experience of teaching Years 3, 5, 1 and 2 after qualifying in 2003. She had gained a degree in geography and tourism. Her last qualification in mathematics was her GCSE. She had attended Local Authority subject leader courses and mathematics courses. She took the role of mathematics and geography coordinator, head of Key Stage 1 and NQT mentor. By the second interview MaST B was acting deputy head of the school, as well as Early Years and Key Stage 1 leader. She was by then teaching Year 2 and had taught in Key Stage 1 throughout the two year programme.

MaST A

MaST A was a Key Stage 2 teacher in a primary school. She had been teaching for 12 years when she took the role of MaST, after qualifying in 1997. She had completed a B Ed with a specialism in Physical Education. Her last mathematics qualification was her GCSE. She had attended courses relating to her role as subject leader but these tended to cover management rather than mathematics. She was the school's mathematics coordinator. During the programme she taught mathematics to Years 5 and 6 in sets.

Mathematical topics chosen for discussion

MaST	Mathematical topic chosen for discussion
A	Multiplication: rapid recall of facts
B	Division
C	Division
D	Using and applying mathematics
E	Calculations
F	Division
G	Number
H	Calculation
I	Fractions, decimals, percentages, ratio and proportion
J	Fractions
K	Place value

Table 6: Mathematical topics discussed in interviews

The Case of MaST C

MaST C chose to discuss her teaching of division in response to my first question. Over the two years she had reflected on how to ensure children in her school understood division as both grouping and sharing, and how she could successfully support progression to understanding and proficiency in the written methods. She articulated the different structures of division and how they linked to the written method. She had worked with colleagues in modelling the chunking procedure practically and recording thinking on a number line, refining her whole school calculation guidance over the two years.

In her second interview, she demonstrated application of her knowledge of progression in her teaching of her own class, 'I have now got top set year 6 so my target audience that I am working with is completely different to what I had before, so with my years 6s we have talked about division that if it is dividing by tens, TU and HTU divided by units then they are more than happy with bus stop method and with our calculation policy that is the final stage if they *are confident and I've got 12 levels 5s and I am hoping to put in 2 to the level 6 paper, so they* are really confident with the understanding, and then obviously when it comes to division by TU *they are using chunking because they don't k*now their 24 times tables, but even with the *confident top year 6 set they still didn't understand chunking at the start of the year…it is really interesting that they understood the quick fix bus stop method but they didn't understand the* chunking so I had to work a lot on division facts so that if I know that 20 divided by 2 is 10 then 200 divided by 2 is 100 so I had to work on a lot on that then I had to go on to decimals so if it *was I don't know 3.6 divided by 6 is 0.6 that actually they could just d*o 36 divided by 6 and *then manipulate the answer and so that is the level I am at with my years 6s' C2.*

As she reflected on her teaching of division there were numerous examples of her knowledge of connections and links, 'manipulating what 10 times, 20 times and 30 times and 40 times is, so I have had to do a lot of work on times tables and linking it to if we know that number fact, so if *we know that, what other number facts…' C2. 'in the mental maths test when it says wri*te 7 *tenths as a decimal they couldn't do that and I said that is really easy it is just like writing 7* units and I had to break it down what all the columns stood for and then link that to their *division' C2.* Most of these links were hierarchical, in that they connected increasingly complex mathematical ideas or procedures.

She summed this approach up as 'I think it is kind of unpicking where they were and then *narrowing the gaps' C2.* Her aim was understanding *'I think the problems with that is with* divi*sion, they didn't fully understand why they were dividing, they knew they had to take away*

10 lots of something but they didn't really understand why and if they did understand why they got confused with the subtracting….'C2.

She identified that knowledge of progression had come from both the MaST training and her experience of different year groups. *'But I think before the maths specialist course I am not sure I really understood, because to be honest I started teaching in year 5, didn't fully understand* where the children had come from, I knew I had to teach them this chunking method, and if it killed me I was going to teach them, take away ten times, take away ten times and I think going down to year 2 and then going back to year 6 has made me more aware of the stages of *learning… so in order to help, to be able to help those year 3 teachers I can only do that* because I taught it in year 2, now I know where they are going in year 6 so I can see the many stepping stones but, maybe when I was in year 5 havi*ng not taught lower down, I didn't may be fully appreciate why the children had those gaps and why there was misunderstanding' C2.*
'I think may be just from the MaST course really unpicked where the transition, like I've never really thought before' C2.
'I'd like to think a maths specialist would be able to … fully understand what happens from the moment they enter school to when they go to secondary school.. so to be able to identify , look at a group of children or a class and identify where their gaps are and how to move them forward, by understanding the progression, but not having necessarily taught all those year groups *…'C2.*

MaST C provided several examples of her ability to take an example of learning and apply her knowledge of how this learning might progress. She used this knowledge to support her colleagues, such as in the following discussion of her support for a reception teacher.
'We talked about this child, I said what about in your water area could you give them some containers that you co*uldn't fill from one to the other and then she started to say well the other day there was this child who was trying to fill up the watering can from the tap but he couldn't* get the watering can to fit underneath the tap and he struggled and struggled and I watched him

and in the end he flooded the whole area and I told him off! So I said could you suggest or say to him to go and get another container which could fit under the tap which that would have been to do with shape and space.. that would have been *perfect… but she just got really cross and he just kept turning the tap further up and she said she didn't have the patience, and I think* that is what is hard in foundation, how do you get them to use that initiative without telling them *or getting cross…I* know what in year 1 and 2 what they are going to need to know, so I know the skills they are going need to have and she thinks that is just flooding the classroom but actually when they are in year 1 and 2 when they are looking at capacity if he had gone to get another container which was smaller, he would have had some grasp of smaller and larger *instead of being told to turn off the tap' C2.*

There were also several examples of MaST C's application of her knowledge of progression where she identified a problem or gap and tracked back to curriculum and learning which should have been secure previously, *'and now, even with my year 6s the top set,.. like in fractions I mentioned fractions and they all said oh no we don't want to do it and I said right* we are going to do a quick ten minute practice and I said right, in year 2 this is what you would have done and in year 3 this is what you would have done and along the line I managed to pick them all up and now they can do representing the quotient as a fraction or a decimal so somewhere along the line I clicked where they were and where they had some *misconceptions,… and maybe where you have got teachers… have weaknesses in year 3 and 4 they don't know what they cover in year 1 and 2 and they probably don't* have an inclination to find out and they are too scared to find about year 5and 6 and they are just in their little bubble *and they don't understand the full picture' C2.*

*'I think in year 6 constantly I am constantly doing test based questions and constant*ly saying, for example we did a sats paper before Christmas and none of my children could do the shape with the coordinates and the parallelogram and it had two points and they had to identify what the missing one was and none of my group could do that so I had to go.. this week I have built up.. so the lady in my class today could do that question, so I had to go right back to Ok what

are the properties of quadrilaterals, we had to a lesson on parallelograms, looking at acute and obtuse angles, then we ha*d to a lesson on, then I built on well, actually they didn't know how to* use a protractor so we had to do that when we was there, and then we did a thing actually *talking about what is an angle, turns, they didn't know that, then we talked about parallel* and perpendicular lines and how we could test what a parallel and perpendicular lines look like and then investigate them round the classroom and then we had to do coordinates to build up to today so they could recognise that is a parallelogram and those angles are the same so therefore those lengths must be the same and so that is where the point is, so that is a whole *week's worth of work to build up to do that one question..*

.... It took quite a long time to plan actually, because I was thinking well they can't do that, so they can't do that, and they didn't really know what a quadrilateral was so they didn't know what a parallelogram was .. and it took me a week, well obviously they came up with other *things.. they couldn't estimate angles,.. they didn't* know that in that parallelogram there was going to be two acutes and two obtuses, and yet they were level 4 but there were so many gaps in their understanding for them to be able to answer one question because of what had gone before,.. and I think in year *6 teaching like that… me and my partner, we have to do that all the* time in order to achieve one thing, those little mini steps have got to be in place

… And I don't think I would have done that if I hadn't done the maths specialist course, I think I would have gone, oh come on guys, that line is the same as that line,

…Well I would never have understood, they didn't even know what a parallelogram was, if I didn't understand all of that shape and space topic, I would have just have said, well they don't know coordinates'C2.

MaST C's knowledge of progression led her to question the appropriateness of expecting children's learning to be at the required level for their year groups. *'If had we only taught in year 4 and 5, …I didn't really care where the chil*dren had come from, I just knew what I had to *get them to,' C2.*

'and maybe where you have got teachers…have weaknesses in year 3 and 4 they don't know what they cover in year 1 and 2 and they probably don't have an inclination to find out and they are too *scared to find about year 5and 6 and they are just in their little bubble and they don't understand the full picture' C2*

MaST C reflected on the need to explore mathematical ideas in real contexts in her teaching. She also discussed how she supported her reception teacher in identifying mathematical learning as it happened in contexts which were unplanned and not primarily mathematical: 'she laid out all her maths and I could say actually that child is doing maths but you might not realise that they a*re I can help her to identify what she is already doing' C2*. The example of her reflections on the water activity is another example of MaST C's awareness of mathematical learning in a context.

In defining the features of deep subject knowledge, MaST C discussed the idea that specialist teachers need to understand mathematics. This was a view expressed in both interviews. She defined understanding as knowing why, 'it is not just well I know that 12 divided by 2 is 6, you need to be able to explain to the children why that happens, explain in different ways. I think *some teachers who struggle with numeracy would argue that you can't explain it if you can't do it your self. I think it's a mixture of the two, you need to do it yourself and you need to be able t*o *explain it. If I'm teaching something like science, I'll be half way through and then I'll think oh that's why it is because I've broken it down so much, in detail, I think oh actually, I'v*e kind of explained it to myself.*' C2.*

Knowledge of progression was another important feature of deep subject knowledge, 'So when my class were in year 4, they were terrible at place value and we had to go right back to tens and units, but when we were doing adding on a number line, with partitioning, adding on a chunk, because they knew how to do chunks on the number line, they knew how to do partitioning on the grid. So it is taking what they already know and trying to get to the next step. *And knowing it yourself…'C1.*

MaST C said she had knowledge of progression in areas not covered on the programme because of, '*the amount of time I spent looking at the national curriculum, the amount of time I spent looking at the PNS and all of those progressions of all the areas…'C2.*

Pen portraits of the other MaSTs can be found in Appendix v.

In my next chapter I begin to lay out my findings.

Chapter 7

What knowledge does a sample of primary mathematics specialist teachers conceive that they draw on in their approach to teaching an area of mathematics?

In this chapter, I will look across the interviews and analyse my findings in response to my first interview question: how do you approach your teaching of your chosen area of mathematics? I aimed to use this question to identify, with my sample of MaSTs, the knowledge that they conceived that they drew on.

Question 1 asked for an example of the MaST's knowledge in action in the context of their discussion of their teaching of an area of mathematics. This contrasted with question 2 which had asked MaSTs to consider their conceptions of general defining features of deep subject knowledge. The interviews were complex and in some cases general statements about deep subject knowledge were made in responding to question 1, and more specific statements about teaching were made in response to question 2, as MaSTs returned to the earlier topic. I decided in my analysis to separate the general and specific responses, so that in my discussion of question 1 I consider examples of discussion of particular approaches to teaching and in my analysis of question 2 I analyse the general discussion of deep subject knowledge.

The responses given by the MaSTs in the interviews were not uniform. As they discussed their approach to their chosen area of mathematics, the MaSTs made references to some aspects of teaching which were not shared across interviews. Therefore my findings for question 1 led me to conclude, as I had expected, that the MaSTs suggested that they were drawing on different types of knowledge, or privileged different parts of their knowledge in deciding on their approach to teaching, and what to include in the interview. However there were strongly shared features of knowledge in responses, to both questions, which leads me to claim that I had identified evidence of common features indicative of deep subject and pedagogical knowledge of MaSTs.

For each question I consider first the aspects which were individual and not shared across the sample of MaSTs and then continue to examine these which were common to all or most MaSTs. Finally I consider examples of the MaSTs' reasoning or stories, usually expressed in much longer passages of speech.

This chapter examines my findings from my analysis of responses to my first question, and the next chapter for the second question. I refer to examples from my pen portraits in Appendix v. Appendix xii includes extracts from exemplar interviews. In Chapter 9, I go on to compare these findings to the models in the existing literature, discussed in Chapter 3.

Features of MaSTs' approaches which were not shared across the interviews

First I considered features which were not widely shared but were identified by one or two MaSTs. I chose to look at features shared only by one or two MaSTs because that would give me a sense of how deep subject and pedagogical knowledge are indeed individual. I chose to look at examples mentioned by two MaSTs as this still seemed to be a small proportion of the sample, around 18%. I reflected why these features identified by the MaSTs were not shared more widely. I do not claim that the MaSTs who did not mention these features did not therefore think them an important aspect of their teaching, only that in their consideration of their approaches to teaching this particular area of mathematics, they did not include them. This may be due to the area of mathematics they chose to discuss. Some areas of mathematics may lend themselves more or less strongly to the features listed below.

The table below identifies the codes which occurred for one or two MaSTs for question 1.

	MaST	Number of utterances	Percentage of coverage of the whole interview	Example The context for each example can be seen in Appendix v.
Using ICT packages in the teaching of their chosen area of mathematics.	A	2	2.25%	*we've got ICT packages its multiplying* monkeys when you put them into sets and you look at 3 lots of 4 and 4 lots of 3 and you can look at the relationships
	E	1	1.66%	I am also more inclined to use ICT, we have My Maths here which we also had in my old school but I now more familiar and confident with it so rather than just using it as.. what I have been using it as well as

				independent work is taking some of the lessons, using it as a basis for starting the teaching, so I am more inclined to experiment with things like that..
Games	F	1	0.26%	I do play division games
Mathematics in play	B	1	2.3%	I know we had a discussion with a year 5 teacher last week and she has some children who are finding the very early concept of counting on quite tricky so we talked about simple things she could do, even simple things like games of snakes and ladders and the concept of counting on a*nd counting back and she said I haven't* even thought about those sort of things for *a maths lesson.. for her it hadn't even* occurred to her that was maths learning but just a free time playing and some children need to have experience of physically counting and recognising..
	C	2	5.4%	yes through doing all the reading and the understanding where children are going, I think you have a better understanding and may be able to identify mathematics aspects of what is going on in that child initiated play, may be better than someone *else, but I don't know if you have had to do* those certain things to be able to identity

				that
Early mark making	G	1	3.47%	because I did one of my essays on Worthington and Carruthers and the mark making and that is really interesting because sometimes I think we disregard what they have done, but I try now to think about whether it has got any meaning for *that child, even if it doesn't look like* anything to me
Talk	E	1	0.92%	we do an awful lot of talking and paired work and if I ask a question I usually ask them to talk about it, 30 seconds to talk about it and then I will ask you, rather than *put your hand up if you know…*

Table 7: Question 1: Features noted by 1 or 2 MaSTs

As can be seen from this table, to a degree the MaSTs were individual in their responses, with some MaSTs choosing to highlight as typical of their teaching aspects not usually noted by other MaSTs. This is given the fact that they teach the same National Curriculum, and have largely been provided with professional development to support their subject knowledge based on the NNS and PNS. However, the discussion of features identified by one or two MaSTs tended to be short and mentioned only once or twice in each interview. They represent some generic issues in the current climate which are for example mentioned in government documents and promoted by Local Authorities (for example talk for learning Alexander, 2010, the use of ICT Ofsted, 2008, early mark making Williams, 2008, use of play QCA, 2000), although these are not necessarily always specific to mathematics.

Some of the less common features related to settings the MaSTs were working in. For example, three MaSTs made references to approaches which I considered to be particularly relevant to Early Years: mathematics in play and early mark making. Of these, two MaSTs were teaching children in the Early Years or held the role of Early Years team leader. The third was a Year 6 teacher, reflecting on her support for teachers in the Early Years setting.
Therefore, from considering the codes identifying approaches to teaching which occurred in the interviews of only one or two MaSTs, I argue that MaSTs develop knowledge which is not entirely uniform in nature, can relate to generic aspects of current debates and which may be related to their setting. MaST D's interview stood out as different to the others in nature, as I will discuss later on in this chapter.

Features of MaSTs' approaches which were shared across larger numbers of interviews

Then I considered which features were shared by 5 or more MaSTs. I chose this number as it is close to half of my participating sample. I could then consider why these features were more likely to be mentioned. I begin my analysis with the features which appeared in 5 interviews and work my way through to the feature in fact mentioned in every interview. Therefore the number of MaSTs identifying each feature increases, apart from where I have grouped similar codes as they relate to each other, for example codes relating to using and applying mathematics.

I found that the shared features of knowledge that the MaSTs conceived that they drew on, in order of increasing occurrence, related to:

- Connections between ideas of similar complexity
- Rapid recall and fluency skills
- Using and applying mathematics
- Mathematical laws, structures and principles
- Theory and reading

- Resources, models and images
- Progression between ideas of differing levels of complexity

I consider each of these below. For each one I include a table showing how many times each code occurred, in which MaST's interview and the percentage of coverage out of the MaST's whole interview. Examples of quotes from the MaSTs are given in my discussion. The context for the quotes can be seen in Appendix v.

<u>Connections between ideas of similar complexity</u>

In the reflections of five MaSTs I identified instances which indicated that the MaSTs connected similar or linked mathematical ideas.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Connecting similar mathematical ideas, making horizontal connections	A	3	2.74%
	B	2	3.17%
	C	3	3.07%
	D	6	4.27%
	J	3	5.41%

Table 8: Question 1: Horizontal connections

This code recorded instances of what I termed horizontal connections. References to connections can be separated into those which are hierarchical or vertical in that they connect ideas of different complexity eg '*you've done the sevens, you do the seventies and the 0.7s and then using that to work the grid…'* A2, and those which are to sideways or horizontal connections, connecting ideas of equal mathematical complexity or equivalent ideas shown in different representations (as in the examples below). References to vertical connections were often linked to MaSTs' discussion of progression, and which I therefore coded separately and will explore them later on in this section. These occurred much more frequently.

This code collated incidences where MaSTs referred to actual examples of connections, or made references to the importance of making such connections. The MaSTs referred to connecting similar areas of mathematics, connecting number facts to establish the commutative law, connecting representatives of mathematical ideas and the idea itself, and connecting structures of operations. There were in total seventeen references to making these sorts of connections across the five MaSTs, suggesting that when they found it important enough to mention in their responses, they did so more than once. Examples of phrases collated in this code were:

'counting session every day and linking it to tables' A2

'an understanding of multiplication to be able to use division and vice versa because they are connected' B2

*'it is perhaps seeing all those connections, I didn't see so much before and how it is all linked, interlinked… so I would say my knowledge before, it wasn't sort of the spokes on a wheel it was more of a spider's web with the connections in between but n*ow it has almost become like a piece of fabric with all the connections within the connections, so yes I can see that whole

mosaic of knowledge and how it does all connect together so I can then just say Ok we are doing this but actually I can take you in *this way,'* J2

The responses of MaST D, from the special school, stood out. She made six references to connections, more than the other four MaSTs. She talked at some length about her approach to using and applying mathematics and how this was incorporated into her teaching of data handling. Whereas other MaSTs tended to make links as part of their teaching, this was the basic starting point of MaST D's approach and one which she endeavoured to share with her colleagues.

Rapid recall and fluency skills

Five MaSTs reflected on the importance of the teaching of rapid recall and basic fluency skills in their teaching approach.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Rapid recall and reference to basic skills	A	2	4.18%
	B	2	1.66%
	C	2	0.84%
	E	1	3.49%
	F	1	0.65%

Table 9: Question 1: Rapid recall and basic skills

These references seems to be brief, with the most from MaST A who choose to discuss rapid recall of multiplication facts as her area of mathematics. The others included a focus on fluency with and knowledge of facts as part of their teaching approach. MaST B reflected that her emphasis on rapid recall had increased during the two year period. It is not clear whether this increase in her practice was due to the role she has taken as MaST and her corresponding new knowledge, or the message from the government in terms of the aims of the new National Curriculum (DFE, 2013). Given this message I was surprised that there were not more and lengthier references.

Using and applying mathematics

I linked some of the codes used to categorise the teaching approaches shared by the MaSTs to the mathematical topic Using and Applying mathematics. These codes included five MaSTs making references to open ended investigations, seven MaSTs discussing their use of problem solving in a context which usually took place in a mathematics lesson, and five referring to beginning with a non-mathematical context usually outside a mathematics lesson and identifying sources of mathematical learning.

From the five MaSTs who used open ended investigations in their approach to their teaching in the second interview, there were nine references in all.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Using and	B	4	6.76%

applying mathematics (open ended tasks)	C	1	2.35%
	E	1	0.23%
	F	1	0.86%
	K	2	5.71%

Table 10: Question 1: Open ended tasks

In the case of MaST B this focus on open ended investigations was a new approach, as can be seen from Appendix v. The scale of it was a consequence of her role and knowledge as MaST.

'*I'm not so scared now to do problems solving with division as well because it doesn't have to be.. it can be quite open ended and I'm trying to get them to think about division… there was a question or a challenge I've given when you've got a certain amount and you can find different* ways to share equally, like the smarties but it was more open ended and they have to find all the *different .. ' B2*

In other cases it was a reconfirmation of previous practices and beliefs. MaST K directly referred to knowledge gained from work for her assignment for the MaST programme in leading to a change in her approach to using and applying mathematics.

'(I've always) been strong on using and applying maths…because it came into my second assignment, because I was trying to solve the problem of why *our girls weren't making* progress, and making maths look a bit more like English, because that is where the girls are happy, so I guess I have turned my using and applying a bit on its head in that I am starting to use more literature skills in maths, trying to get the children to write their thoughts down, trying *to model writing in maths…' K2.*

Seven MaSTs referred to using a context for teaching their chosen area of mathematics, where the learning outcomes where primarily mathematical and set in a mathematics lesson.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Context for mathematics	B	3	6.34%
	C	1	5.09%
	D	3	3.38%
	E	3	5.63%
	F	2	4.42%
	H	1	2.28%
	J	4	9.76%

Table11: Question 1: Context for mathematics

In some cases this was a general reference, and in others the MaSTs gave specific examples, referring to for example science or geography. MaST J reflected that she had changed her approach to the use of context after the two year period, from teaching skills and then applying them, to teaching skills through application.

Five MaSTs discussed how they used non- mathematical contexts for mathematical learning. Here the learning outcomes had not necessarily been primarily mathematical, but the teacher could identify additional opportunities for mathematical learning, usually outside of a mathematics lesson.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview

Mathematics in a non-mathematical context	C	2	4.81%
	D	3	3.33%
	E	1	2.75%
	G	1	0.56%
	J	2	7.31%

Table 12: Question 1: mathematics in context

Three of the nine references came from MaST D in her special school. Three references came from MaSTs working with young children or supporting colleagues who work with young children, suggesting this is a practice as being particular to the setting of the MaST.

Taken together, every MaST but two, MaSTs A and I, referred to either open ended investigations or using some sort of context for mathematics in their responses to question 1. The background for their discussion can be seen in Appendix v.

<u>Mathematical laws, structures and principles</u>

Eight MaSTs discussed a mathematical law such as the distributive law or the commutative law, structures of mathematical ideas such as of subtraction and division, and underlying principles such as angle as a measurement of turn. Further examples can be seen in Appendix v.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Law, structure or basic mathematical idea	A	2	1.85%
	B	2	3.9%
	C	3	3.02%
	D	3	6.25%

	F	2	1.62%
	I	1	3.49%
	J	2	2.67%
	K	1	6.45%

Table 13: Question 1: mathematical law, structure or basic idea

'Where they have to understand the distributive law and I've told them, I use that language with them…and they like… I say to them you don't know have to use that word but this is what it is, but I modelled it with that group by showing then 7 x 5 is the same as 5 x 5 and 2 x 5 and we drew it out and I showed them pictures of 5 x 5 and 2 x 5 and recombined it and going right *back to understanding why the grid method works…and I have never done that before, I thought… I have never actually done it.. I've don*e it by showing 24 as two lots of ten and 4 but I have never done it other than partitioning by place value, I have never partitioned with the 5 and the 2 and I am sure that is something come from MaST.. so that is something that I have *used…' I2.*

MaST B directly relates her ability to teach the structures of division to knowledge gained from her assignment for the MaST programme. In the second interview she distinguishes clearly between grouping and sharing, whereas she reflects that in the first interview she had not clearly understood each one.

<u>Theory and reading</u>

Nine of the eleven MaSTs discussed how they drew on knowledge gained from theory, literature, research findings and other documentation.

	MaST	Number of utterances	Percentage of coverage of the

			MaST's whole interview
Use of theory, literature, research findings and other documentation	A	3	6.48%
	B	7	12.15%
	C	3	3.46%
	D	2	2.41%
	E	2	5.36%
	G	2	1.05%
	H	5	8.82%
	I	2	4.42%
	K	9	22.35%

Table 14: Question 1: References to theory and reading

There were 25 references in all to what could be termed academic reading, for example reading included in the bibliography for the programme, and 10 references in total to documentation such as the National Curriculum, the National Numeracy Strategy and the Primary National Strategy materials. This suggests that the MaSTs acknowledge the knowledge gained from the Masters nature of the programme.

Academic Reading Some MaSTs name this reading or talk generally about reading academic texts for their assignments as part of the programme	National Curriculum, National Numeracy Strategy, Primary National Strategy
25 references	10 references

Table 15: Question 1: types of literature referenced

MaST K discussed the importance of research findings at length, covering 22.35 % of her interview. For example, she reflected on how she had changed her teaching of place value based on her reading of the research by Thompson and Bramald (2002)

'so we could juggle with big numbers in key stage 1 quite happily as long as we talk about their *value, like 20 and 4, but we can't juggle big numbers when we are talking about 2 tens and 4* units, because that adds another layer, and there are probably other areas of maths where that is also the case and I *just haven't met them yet' K2*

<u>Resources, images and models</u>

Ten of the eleven MaSTs referred to their use of resources, images and models in their teaching approaches.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Use of resources, images and models	A	2	3.1%
	B	7	10.51%
	C	4	7.71%

	D	5	19.15%
	E	4	7.01%
	F	5	12.67%
	G	2	6.28%
	I	1	1.77%
	J	3	3.42%
	K	1	0.72%

Table 16: Question 1: Use of resources, images and model

This code was included in all but one MaSTs' interviews. Many incidences tended to be brief and discussed how the role of MaST had reconfirmed their belief in and understanding of how to use such tools in their teaching.

*'I still l agree with what I said before about all the practical equipm*ent, and the importance of teaching place value ' K2

'so I worked with year 3 last year, to help them use and apply… and subtraction, and making it visual, so they could have them physically doing things,… realising perhaps we don't do enough of that in the school, not just with white boards, but practical division with different situations *and different objects, sharing and grouping, so it has maybe helped me intervene.' F2*

When changes had occurred, they were connected to new knowledge. For example her knowledge of the structure of number affected the way MaST G used the image of the number line.

'I think, I would think a lot more about the misconceptions now, about how , about what I would need to teach, or un teach, when I should pick up those things and when I can let them… and I am really conscious of things like zero is not the last number, th*ere is numbers below that… I know it is little things, the kids might not pick up on but to me it is quite important that we don't limit… , because the question might come up one day and you have taught them all your life that* it starts at zero and it goe*s upward… so I think about that in terms of my teaching practice* about how I have got to be conscious about what I might be setting them up for, and just making sure they have got a really firm understanding of the basic principles, otherwise they are going *to struggle further up the school' G2*

The only MaST not to refer to resources, models and images was MaST H in her discussion of calculations. One MaST talked about considering the image of the array in her assignment but felt it had not been 'transla*ted into practice by the staff'* A2, the only phrase collated which referred to not using a resource or model, rather than using it.

Progression between ideas of differing levels of complexity

All eleven of the MaSTs made some reference to progression in their discussion of their approach to their chosen area of mathematics. This was the most common code or set of codes. I had expected MaSTs to refer to knowledge of progression as part of their understanding of deep subject knowledge, and therefore in response to question 2, but not as part of their own teaching.

	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Change in differentiation based on knowledge of progression	B	1	0.35%
	I	1	1.77%
Progression: supporting other members of staff, eg policies and guidance, staff meetings	A	2	6.36%
	B	1	2.4%
	C	3	4.17%
	E	1	1.06%
	H	3	3.79%
	I	1	1.88%
Progression tracking back	A	1	7.13%
	B	5	10.46%
	C	9	27.73
	D	5	6.68%
	E	5	8.98%
	F	3	10.84%
	G	1	4.06%
	H	7	17.56%
	I	4	13.65%
	J	3	7.69%
	K	6	14.38%
Progression tracking forwards	A	3	6.86%
	B	8	16.23%
	C	8	14.04%

	D	4	5.14%
	E	5	8.53%
	G	8	24.85%
	H	5	19.98%
	I	2	3.08%
	J	6	8.79%
	K	2	4.94%

Table 17: Question 1: progression

References to progression took several forms, as can be seen from Appendix v. This included examples of the MaSTs identifying that they had acquired knowledge of the curriculum below that which they were teaching. This use of knowledge of earlier curriculum and learning was often used in responses to children's errors and in differentiation for less able children.

'.. I think what is different is I have a better appreciation of where the children have come from, or should come from …. I think what's changed is my understanding…why children struggle' F2.

It was also used to establish conceptual understanding for example by MaST D, working in her special school. She would take children's learning back to a key principle if she did not judge it to be secure enough. MaST C and E also tracked learning back to key mathematical principles, to a varying degree. I will discuss these examples a little later.

Other MaSTs tracked back to sources of misconceptions or what they termed as gaps in understanding. This might address the misconception or error but not necessarily establish understanding of the underlying principle. For example MaST H reflected on the difficulties of learning written calculation methods. She discussed the need to establish that the skills for each calculation are in place first, rather than tracing back to the conceptual understandings of the operation.

'Skills that the children need to have before they can go.. so for example if you are teaching multiplication you know that the children need to be able to add the numbers back together *'' H*2

Another MaST had challenged her perceptions of the curriculum for younger children. Gaining new knowledge of earlier year groups made her consider how she had thought she could 'drag down' the curriculum for Key Stage 2 into Key Stage 1.

*'I guess I th*ought I could drag everything down... rather than start at the other end and building *up...'* K2

Other MaSTs reported how knowledge of progression had changed how they began units of work. They did not assume they could begin with the objectives generally expected for their year group, but would consider the conceptual progression before this and used their knowledge to act as a form of assessment for learning. When working with children towards the end of Key Stage 2, two MaSTs reported that they would cover the whole progression which should have been learned previously. They used this approach to ensure the children were secure in less complex but key ideas before they moved on to what was the expected curriculum for the year group. These two MaSTs gave three examples, two of which took a period of days, and one which was done in a matter of minutes.

'and now, even with my year 6s the top set,.. like in fractions I mentioned fractions and they all said oh no we don't want to do it and I said right we are going to do a quick ten minute practice and I said right, in year 2 this is what you would have done and in year 3 this is what you would have done and along the line I managed to pick them all up and now they can do representing the quotient as a fraction or a decimal so somewhere along the line I clicked

where they were and where they had some miscon*ceptions,… and maybe where you have got teachers… have weaknesses in year 3 and 4 they don't know what they cover in year 1 and 2 and they probably don't have an inclination to find out and they are too scared to find about* year 5and 6 and they are just i*n their little bubble and they don't understand the full picture'* C2.

A further example was from MaST J who had rejected guidance set out in year groups such as the PNS, preferring instead to work for the National Curriculum which is set out in Key Stages rather than year groups. She showed how she took a statement from the National Curriculum programme of study and interpreted, often mathematically in her examples, the learning which was needed to lead up to achieving that statement.

'so even though in some way the national curriculum… with my knowledge I can break that down to what the statement means, so if we are looking at number sequences we are looking at multiples and we are looking at odd and even numbers and we are looking at hundred squares and the patterns within the hundred squares and we are looking at, putting multiples and what *patterns could they see and discussing what that looks like…*' J2

When asked how they supported other staff in their teaching of their chosen area of mathematics, MaSTs also drew on knowledge of curriculum below their usual year group. One MaST used an analysis of common errors in her own Year 6 class to trace back the progression needed in the rest of the school to ensure secure learning in the future. This example can be seen in Appendix v MaST A. Another MaST analysed an example of possible learning through water play in the reception class to guide the teacher there, by drawing on her knowledge of learning in Key Stage 1, even though she usually works in Key Stage 2.

I said what about in your water area could you give them some containers that you couldn't fill from one to the other and then she started to say well the other day there was this child who was trying to fill up the watering can from the tap but he c*ouldn't get the watering can to fit* underneath the tap and he struggled and struggled and I watched him and in the end he flooded the whole area and I told him off! So I said could you suggest or say to him to go and get another container which could fit under the tap which that would have been to do with shape *and space.. that would have been perfect… I know what in year 1 and 2 what they are going to* need to know, so I know the skills they are going need to have and she thinks that is just flooding the classroom but actually when they are in year 1 and 2 when they are looking at capacity if he had gone to get another container which was smaller, he would have had some grasp of smaller and larger instead *of being told to turn off the tap' C2.*

Other examples of knowledge of progression used in the MaSTs' own teaching approaches involved MaSTs in reflecting on the acquisition of knowledge of the curriculum and learning which was above their current year groups. This affected their differentiation and teaching of more able children, and their use of models such as the number line to support later learning of the structure of the number system (Appendix v MaST G). Knowledge of how learning will progress to more complex ideas changed the way they sequenced their teaching where they analysed skills children will need to perform calculations and consider carefully how to order their teaching. MaST K directly related this to a piece of research which challenged her to change the sequence of her teaching of place value (Appendix v MaST K). Another MaST led her school in rejecting certain mental methods of calculation which did not support later progression (Appendix v MaST H).

In considering their knowledge of the curriculum for children older than their own class, some MaSTs used the figurative language of emotion, using terms such as having less 'fear' (J2) of moving children through the curriculum, feeling 'braver' or 'not so scared now' (B2). Therefore there were suggestions that MaSTs were not only gaining knowledge but also confidence. MaST G used language of limits, and discussed how her increased knowledge of later learning made her identify ceilings she had made for children previously. She uses terms such as ‘limiting’ to discuss her previous practice (Appendix v MaST G).

A key theme which all of the MaSTs identified, related to knowledge of other year groups and how this enabled MaSTs to teach learning outcomes outside the ones expected for their particular year group. This theme was expressed in terms of new knowledge of curriculum and learning in year groups above and below that of their current class, new confidence to include this curriculum and learning in their teaching, and a belief that they should encourage other teachers to do the same.

I will discuss this in more detail in a later section.

Knowledge of pedagogy and mathematics above and below the MaSTs’ current year group was identified by MaSTs as impacting on their knowledge of progression of the children they teach, and was the most common set of codes for responses to question 1. Therefore although I have concluded that there is no entirely uniform approach to teaching taken by all these MaSTs, there are similarities between the teaching approaches which I argue may be typical therefore of the knowledge base of MaSTs more generally.

This knowledge may have been acquired in the MaSTs’ role to support other staff across the school, but in fact impacted on the MaSTs’ own teaching in a way I had not expected. Although I was aware that this knowledge matched some of the categories already in the literature, I began to see these examples as types of pedagogical reasoning or deliberation, drawing on knowledge. I chose to analyse more closely some of these examples of pedagogical reasoning. In this case I analysed longer accounts, or stories, rather than shorter coded phrases. They gave me an insight into how MaSTs appeared to use their knowledge of progression in teaching their own class. MaSTs generally attributed this knowledge as drawn from the role of MaST or the training programme.

Examples of MaSTs’ reasoning in response to question 1

I begin with an examination of an example of one of MaST D's accounts. MaST D held the position of mathematics specialist teacher in a special school. She discussed at length examples where in her own classroom, and in her support for her colleagues, she identified mathematical activities which detracted from what she considered to be the key mathematical area to be learnt. Her ability to do this appeared to draw on her knowledge of underlying mathematical principles, for example one to one correspondence in the learning of counting and comparison of sets of information in the making of a pictogram.

*'she (another teacher) was doing a pictogram, and she wanted.. they couldn't associate a cat with a picture of a cat, so why don't you … do a 3D graph, ,… it is just little simple things like that, she hadn't looked at their devel*opment level and worked out.. and what I tend to say if someone is P4, I say if you think about that in normal child development that child would be about 2 and a half, so if you think about your child at 2 and a half, could he have done a pictogram, no, but he could have told you how many animals there were, and if you asked him to put the cats in a line he would have done, and if you had asked him to put the dogs in a line he would have done and he would have said oh look that is higher than that, but you are *introducing all those grid lines and he doesn't need it, if you read a pictogram it is to read information from, it doesn't matter if it is not, the lines aren't all straight, as long as the animals are at the same level and the same height …' D2*

Further examples can be seen in Appendix v MaST D.

This became an important example of a theme which I identified throughout my analysis. In my argument I use the term 'principle' to represent the most basic and powerful form of a mathematical concept, rather than an idea which might link to a principle. Ma (1999 p.121) referred to power of principles as relating to how close they lie to the structure of the discipline of mathematics. MaST D identified mathematical principles and how these could be hidden by other ideas. Here the drawing of a pictogram obscured its key function. A further example, in her first interview, is her discussion of a colleague's teaching of finding a fraction of a shape, which had almost entirely focused on the children's use of a ruler. This she felt had been a barrier for the children's understanding. Her knowledge was not replicated to the same degree in all other MaSTs' interviews but there was a sense that when they discussed progression, they were aware of the need to go back to misconceptions or gaps in understanding if not to key mathematical principles. However the other MaSTs in mainstream schools did not discuss the stripping back of peripheral ideas and activities as MaST D did. As I will explain, they tended to see the key principle as an important part of progression forwards and backwards. MaST D however considers teaching of ideas which might go alongside the key principle and which might detract from it or form a barrier to learning. I began to reflect on the significance of the difference in MaST D's knowledge. If she is typical of other MaSTs in special schools, this sort of knowledge may be influential in mainstream primary schools. A move to harness and share the knowledge located in special schools has been signified by the government in meeting the needs of all children (DFE, 2011c). MaST D clearly provided an additional way of considering progression which might complement the understanding of progression of other MaSTs.

MaST C also provided a clear example of tracing learning back to a key mathematical principle, although not of removing distracting ideas, when she discussed how she identified errors to do with a shape question with her Year 6 class. She discussed her rather lengthy approach to tackle a practice SATs question relating to marking the vertex of a parallelogram on a coordinate grid, given the location of the three other vertices, which her Year 6 children struggled to answer.

*'I think in year 6 constantl*y I am constantly doing test based questions and constantly saying, for example we did a sats paper before Christmas and none of my children could do the shape with the coordinates and the parallelogram and it had two points and they had to identify what the missing one was and none of my group could do that so I had to go.. this week I have built up.., so I had to go right back to Ok what are the properties of quadrilaterals, we had to ... a lesson on parallelograms, ... looking at acute and obtuse angles, then we had to a lesson on,

then I built on well, actually they didn't know how to use a protractor so we had to do that when we was there, and then we did a thing actually talking about what is an angle, turns, they didn't know that, then we talked about parallel and perpendicular lines and how we could test what a parallel and perpendicular lines look like and then investigate them round the classroom and then we had to do coordinates to build up to today so they could recognise that is a parallelogram and those angles are the same so therefore those lengths must be the same and *so that is where the point is, so that is a whole week's worth of work to build up to do that one* question..

…. It took quite a long time to plan actually, because I was thinking well they can't do that, so they can't do that, and they didn't really know what a quadrilateral was so they didn't know what a parallelogram was .. and it took me a week, well obviously they came up with other *things.. they couldn't estimate angles,.. they didn't know that in that parallelogram there was* going to be two acutes and two obtuses, and yet they were level 4 but there were so many gaps in their understanding for them to be able to answer one question because of what had gone *before,.. … And I don't think I would have done that if I hadn't done the maths specialist* course, I think I would have gone, oh come on guys, that line is the same as that line,

…Well I would never have understood, they didn't even know what a parallelogram was, if I didn't understand all of that shape and space topic, I would have just have said, well they don't know coordinates'C2.

She analysed the mathematics within this question and created a progression of learning which aimed to ensure the children could not only answer the question but also had a firm and full understanding of all connected areas. Firstly she returned to the topic of quadrilaterals which would normally be taught before Year 6. Here she demonstrated knowledge of the wider definition of shapes. She did not rectify the problem with just a lesson on parallelograms, but moved first to the conceptually more powerful topic. Then she moved to consider parallelograms. I envisaged the topic of parallelograms as underpinning the question, and the topic of quadrilaterals as underpinning the example of parallelograms. I began to see how she moved between layers of knowledge, between those of more and less conceptual power.

After parallelograms, and presumably because of a further gap in the children's understanding, she moved to acute and obtuse angles. This might be described as a sideways step from parallelograms. Presumably on finding other gaps she then moved across to a skill of similar complexity, of using the protractor, and then downwards to the underlying principle of angle as turn. This could be argued to be unrelated to the question as it represents a different model of angle, a dynamic aspect of angle instead of a static one. However it is an underlying concept of angle, and angle underpins all aspects of the work and the initial question. She then went on to look at more complex ideas of parallel and perpendicular lines.

So I can see in MaST C's reasoning a complex zigzag path between ideas of more and less, and similar complexity. When a gap was found she moved to a more basic but more powerful idea and then worked upwards again, unless another gap was found and she moved back down again or sideways to tackle the gap she had identified. In doing this she touched on an underlying principle, and one of most conceptual power, in her teaching of the definition of angle.

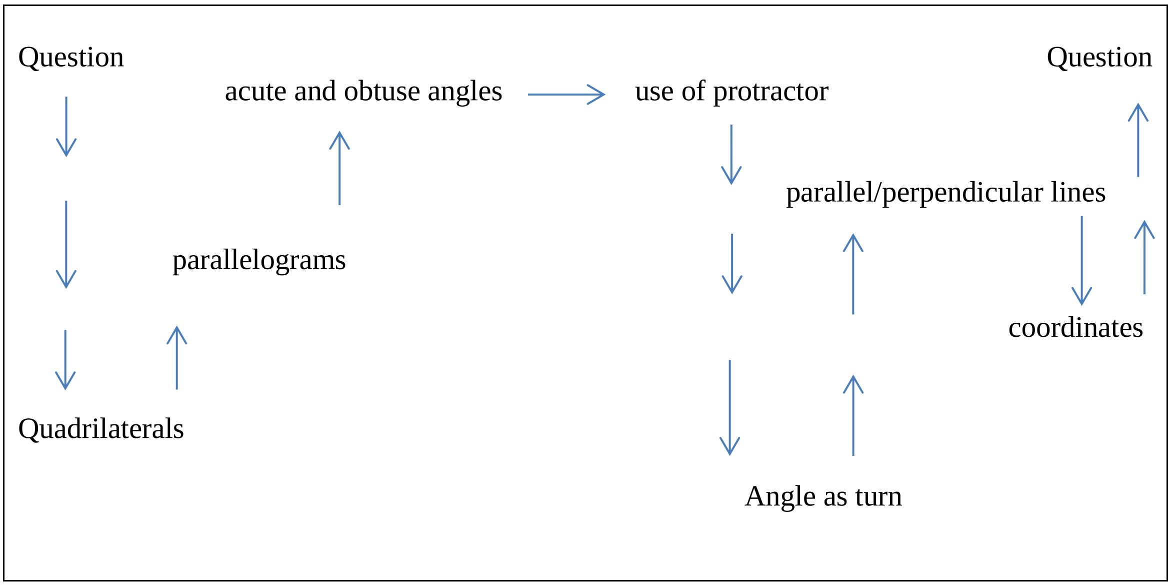
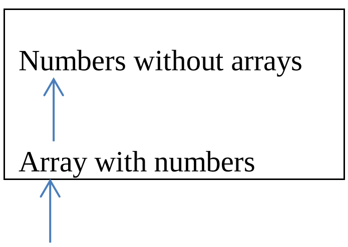


Figure 2: MaST C's reasoning

MaST E also demonstrated an ability to go downwards to a basic principle and trace forwards to a given learning objective. The pathway here was less complex but directly 'vertical', between related ideas of differing complexity. For example in beginning a week's topic of work on multiplication with her Year 4 and 5 class, MaST E began with the array, more associated with children in Key Stage 1 and which can be argued to underpin understanding of multiplication (Barmby et al, 2009). During the week she then moved progressively through the curriculum to where the children might be expected to be for their year group (Appendix v MaST E). Her approach to the most basic of the ideas, the array, was open ended. She did not teach the children the array, but used open ended investigation of it. This allowed her the opportunity for assessing the children's understanding and for the children to engage with the structure and features of the array rather than repeated practice of more closed problems concerning it. She expressed a concern that the children in her class might find this approach not 'grown up' and twice in her second interview expressed pleasure that the children had not commented that they were covering work they had done before. From arrays she moved steadily through the curriculum, adding numbers to the array and then removing the array and working solely with numbers. She talked about individual children and groups of children 'peeling off', finding their 'ceiling' or 'limit'. Then she was able to teach more directly the next steps in the progression. She called this 'progression in a week'. Thus her interview presented her planning as directly vertical, through increasingly complex stages of understanding of and fluency with multiplication, but beginning with ideas far more basic but conceptually more powerful than she would have done before her experience as MaST. This was a similar pathway, with fewer sideways moves than that of MaST C's example.



Array

Figure 3: MaST E's reasoning

In these examples there was a sense of the MaSTs' understanding of the way mathematical ideas progress in a recursive nature through the curriculum (Davis and Simmt 2006; Watson and Mason 2002). MaSTs drew on understanding of the recursive nature mathematically, in MaST C's use of the relationships between the definitions of shapes, and in terms of pedagogy, in MaST E's use of the array. In the case of MaST K this was based on her interpretation of the research by Thompson and Bramald (Appendix v MaST K).

Other MaSTs, such as MaST F and H, talked about tracking back to the sources of errors or gaps in understanding but not necessarily to a key underlying mathematical principle. For example, MaST H analysed the skills involved in the formal written calculations, such as the use of addition in long multiplication, and tracked back to make sure these were fully secure before teaching the formal method, thus altering the sequencing of her teaching (Appendix v MaST H). MaST H and other MaSTs tracked progression back but not so far as to conceptually underlying principles.

As I have previously described, I also found that MaSTs tracked learning forward. MaST G removed ceilings from the learning of nursery and reception children she taught as she explored the number system with them. She used the term 'limit' to express her practice before taking the role as MaST.

'I think more carefully about now, rather than limiting…little examples like rather than limiting, if I am doing a game where they have number cards and they have to choose the right number, I

might put out white boards and pens so they can write the numbers as well, and just trying to think of ideas to expand their options and definitely ideas that are not limiting their thinking by the activities that I do.. I did a number line activity that I might have done two years ago with zero to ten or zero to twenty, and I would have had the zero to twenty cards and they would take it in turns to pick one up and order them, but now I give them blank paper, ask them to write a *number that they know and then to order those numbers on a number line and it doesn't matter* if they have got the same ones or if they have got a million or if they have got 3, it is them knowing those numbers and taking ownership, so definitely think I have changed that view,.. just thinking a bit more deeply about how I give them the opportunity to show what they *know…' G2*

This is quite different to what MaSTs saw as typical of their previous practice of teaching the objectives for their year group:

'But I think before the maths specialist course I am not sure I really understood, because to be honest I started teaching in *year 5, didn't fully understand where the children had come from, I* knew I had to teach them this chunking method, and if it killed me*…I didn't really care where* the children had come from, I just knew what I had to get them to,' C2

'No I'd go straight in with this is what they are supposed to be doing, I will teach them what the curriculum says for this year group, and then coming unstuck and discovering the gaps in *children's knowledge and then having to go back to try and fill those gaps, whereas I find* that by doing a recap, show me what you know, helps me understand where they are and helps them *to review what they know' E2*

There were examples where MaSTs typify their previous approaches as a horizontal model, working through the learning objectives for the given year group, or a deficit model, beginning with the objectives and going back to fill gaps, so that the children have to struggle before they experience appropriate teaching.

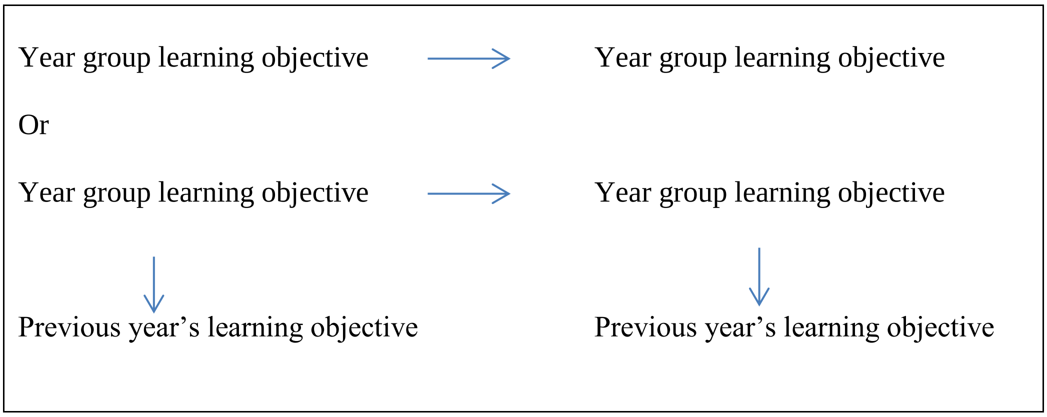


Figure 4: Typical reasoning before role of MaST

Instead these specialist teachers used their knowledge of progression of mathematical ideas, such as the number system and the definitions of shapes, and their knowledge of pedagogy, for example the way in which the array builds understanding of multiplication, in their reasoning about their approaches to teaching. There were examples of MaSTs creating a trajectory which was finely graded in terms of mathematical thinking and pedagogy (for example MaST D). They stated this was new practice, based on new knowledge of mathematics across the primary phase gained from the programme and role. As well as new knowledge relating to mathematics and pedagogy, there was a strong element of curriculum knowledge in their responses. In some cases MaSTs referred specifically to the curriculum in other year groups, for example MaST E.

In summary, my findings from MaSTs' responses to question 1 show that although not uniform, the specialist teachers drew on common aspects of knowledge relating to progression. The interviews included examples of MaSTs narrating their reasoning about changes in their practice based on new knowledge of both mathematics and pedagogy across the primary phase.

Next I examine my findings from my analysis of the MaSTs' responses to question 2.

Chapter 8

What knowledge does a sample of primary mathematics specialist teachers conceive of as deep subject knowledge?

In this chapter I analyse my findings from my second interview question relating to the MaSTs' conceptions of deep subject knowledge. As I have previously explained, I chose to include in my analysis of responses to question 2 any comment MaSTs made in response to question 1 which was about their knowledge generally. Although I do not report a full analysis of both interviews, in some cases I refer to MaSTs' conceptions of deep knowledge in the first interviews at the beginning of the two year period. This helps me to examine how their conceptions of deep subject knowledge developed as they undertook the programme and developed their role in school. In Appendix xi I present a table showing the incidences of codes across the responses of MaSTs to question 2 in both the first and second interview. In my analysis I draw on my argument presented earlier in Chapter 4 that I can assume that these specialist teachers have a claim to understand the nature of deep knowledge.

In this chapter I will firstly consider features MaSTs identified as typical of deep subject knowledge which occurred in only one or two MaSTs' interviews. Then I will consider features which were shared by five or more MaSTs. I will then examine the nature of the knowledge I had interpreted as identified by the MaSTs as deep.

Finally I draw some conclusions across my analysis of both questions.

Features used by MaSTs to define deep subject knowledge which were not widely shared across the interviews

There were a number of rarely occurring features, mentioned by one or two MaSTs, which suggests that there is no entirely common understanding of deep subject knowledge, as I had expected. The definition of deep subject knowledge to some degree is individual to the MaST herself and her setting. I suspect that this is also related to her existing knowledge and beliefs.

	MaST	Number of utterances	Percentage of coverage of the whole interview	Example
Ability to identify key mathematical ideas and those on the periphery	D	4	13.16%	*Sometimes we're told to bring so much into* a lesson, and sometimes you've got to step *back and say you can't bring it* all in *because you're just muddling and confusing* them.
Ability to identify mathematics in a non-mathematical context	D	1	0.65%	I think it is being able to unpick the maths in something and look at every little aspect of what they are doing there and be able to find something.

Being able to support other staff with the mathematical structures and laws	F	1	1.76%	(advising staff) make sure at some point you do finding the difference as well because here you are just taking away
Provide access to other support	G	2	3.26%	It is getting back in the swing of using the library to research that sort of thing, I could probably bring out some articles which might be helpful
Being flexible	G	1	0.51%	it think it is being able to be flexible with what you know
Being passionate about mathematics	J	1	1.5%	Deep subject knowledge is about passion for maths
Ability to challenge others	G	1	0.54%	to be able to challenge people by what they see in your lessons

Table 18: Question 2: Features noted by 1 or 2 MaSTs

MaST D's interviews stood out, as in my findings for question 1. She chose to exemplify her definition of the knowledge she drew on as MaST with examples where she had guided staff on the ability to distinguish between key principles and those on the periphery. This had been a feature of her answer to question 1 and she felt it also characterised her deep subject knowledge. She also was the only MaST to discuss deep knowledge as including the ability to identify mathematical opportunities in what might usually be thought of as non- mathematical contexts.

Features used by MaSTs to define deep subject knowledge which were shared across the interviews

In considering common features amongst the MaSTs' definitions of deep subject knowledge, as with question 1, I explored features which were mentioned in around half the interviews. I start with those represented in five of the eleven MaSTs' interviews and then work up to the most common feature. Example quotes are given in my discussion.

I found that the shared features of knowledge that the MaSTs conceived of as deep, in order of increasing occurrence, related to:

- Using and applying mathematics
- Understanding mathematics
- Progression

<u>Using and applying mathematics</u>

Five out of eleven MaSTs included knowledge of using open ended investigations as part of teaching Using and Applying mathematics as a defining feature of deep subject knowledge.

Interview 2	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Using and Applying Mathematics: open ended investigations	D	1	1.47%
	E	1	0.7%
	F	3	11.85%
	G	3	2.06%
	K	2	5.35%

Table 19: Question 2: Using and Applying Mathematics

For example, MaST G talked about exploring, investigating and meaningful, purposeful mathematics.

'*I don't see the point of doing something without a reason, particularly for children… why do they need to do it… ' G2*.

Only MaST F made links between using and applying mathematics and the sort of reasoning which might be expected to be learned as a result.

'was really interested in the articles about mathematical reasoning, my guess is, I don't know what deep subject knowledge is *but without it, you can't teach that way…you can't encourage children to reason about maths' F2*

Otherwise references tended to be brief. Three MaSTs included knowledge of providing contexts for mathematics and only one MaST referred to knowledge of being able to identify mathematical learning in what was usually considered to be non-mathematical contexts.
In all, seven MaSTs out of eleven made some reference to one of the codes which relate to using and applying mathematics, which were open ended tasks, using a mathematical and non-mathematical context for mathematics.

<u>Understanding mathematics</u>

Understanding was the next most frequently perceived feature of deep subject knowledge, occurring in seven out of eleven interviews.

Interview 2	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Understanding	B	1	0.43%
	C	2	0.98%
	E	2	3.25%
	F	1	12.67%
	H	1	0.59%
	J	2	2.12%
	K	2	1.1%

Table 20: Question 2: Understanding

This is summed up by MaST H.

*'But I would see deep subject knowledge as really understanding a lot of the areas of maths, being able to support colleagues in areas of maths if they don't have the understanding, sometimes it is not that they can't teach it, it is not knowing what the s*ubject is really about, *being able to then teach it' H2.*

MaST E referred to Skemp's (1989) model of understanding in her response, an example of her use of academic texts.

References to understanding were often short and I found this was a less useful code. The term understanding seemed to be similar to deep subject knowledge in its lack of precision. I had not accepted it as a full answer but probed its meaning. I found I could analyse what MaSTs meant by understanding by using other related codes such as knowledge of gaps and misconceptions, being able to explain in different ways and knowing why.

Three MaSTs recognised knowledge of children's common gaps and misconceptions as part of deep subject knowledge and stated that an aspect of their role was to support other staff in this type of knowledge. One MaST had also become aware of misconceptions in her own staff's mathematical knowledge relating to bar charts and block graphs, and had provided professional development on this.

Similarly, relating to the idea of deep subject knowledge as understanding, four MaSTs discussed deep subject knowledge as 'knowing why'. MaSTs discussed the importance of knowing why procedures work or why number facts are linked. MaST B directly related her growth of understanding 'why' to the MaST course. She explained that it helped her in convincing other staff when she wanted to implement change. MaST I and MaST H summed up the differences between knowledge for QTS and deep subject knowledge as knowing why.

'what can you find and why, it is like the 9 times on your fingers, but why does it work? It is being able to explain why it works, lots of teachers at school know how to do it but why, it is not knowing, not knowing 9 x 7 but why does 9 x 7 on your fingers work, so that is the difference *between deep subject knowledge and having subject knowledge' I2.*

Being able to explain in different ways and take different approaches was included as part of deep subject knowledge in five out of eleven interviews. MaSTs talked about deep subject

knowledge as having access to alternative practices. This was not in an uncritical way, as they discussed how they judged whether alternative practices such as those from other countries, could be effective in their schools. MaST D talked particularly about the need to present the same idea in different ways to children at her special school. MaST E discussed how staff needed support with presenting ideas in different ways when their confidence was low. MaST G attributed this knowledge of alternative practices to the academic part of the programme.
This code links to the MaSTs' conceptions of the way in which their deep subject knowledge compared with a teacher who had taught every year group. MaST F stated that experience in each year group would provide knowledge of how things were done in that school, but not of the way they might be done.

*'I don't think it is just the practical things for teaching each parti*cular year group, yes you see *more and you would know the areas of weakness but you wouldn't necessarily know what to do about them or how to approach them differently or how it could be taught differently…' F2*.

Therefore there were a number of references to 'understanding' across interviews, suggesting that the MaSTs had explored in the interviews what understanding meant in the context of their role.

Progression

The most common code related to examples where MaSTs argued that deep subject knowledge was characterised by knowledge of progression. This was mentioned at least once by every one of the eleven MaST across the two interviews, and by 9 out of eleven MaSTs in the second interview.

Interview 2	MaST	Number of utterances	Percentage of coverage of the MaST's whole interview
Progression	A	1	6.19%

	C	6	24.84%
	D	4	9.23%
	E	1	6.57%
	F	2	2.77%
	G	3	6.44%
	H	5	2.45%
	J	5	14.33%
	K	2	1.86%

Table 21: Question 2: Progression

For example MaSTs talked about knowledge of steps of learning, and the need to know the step children were on, the next step, and the step before it in case their understanding was not secure. Deep subject knowledge included being able to do this across the primary phase.

Key themes were being able to express the progression, knowing it in small steps, and use it to analyse errors and gaps and to move children forwards along the progression. This idea was expressed both in terms of pedagogy of mathematics and with specific mathematical examples. For MaST D deep subject knowledge was about when to move the children in her special school on but also when to slow down their learning.

'So deep, I would say it is knowing what to do and when, having the experience of knowing when to stop and change something and make some*thing a bit different' D2.*

Several MaSTs talked about guiding staff with issues of progression in staff meetings, monitoring the plans of colleagues or providing written progression guidance. In the first interview, MaST A had provided 'ladders of progression' to support other staff in this sort of knowledge, implying it was not knowledge all teachers in her school shared. By the second interview she had begun to recognise the complexities of learning captured by the 'ladders' and notes,

'because all these children come in at different levels and different experiences, in the nursery you can talk to some when they are playing with water about halves and full and some of them look at you completely blankly so you need to know what the end result is, what the finished result is and the different paths or tracks you take to get there…' A2.

Therefore there is recognition by the second interview that learning may take different routes. In responding to question 2 and discussing progression, some MaSTs began to relate what they had suggested in question 1 about moving outside the learning objectives for their year group to the idea of deep subject knowledge. The knowledge of mathematics and pedagogy across the whole primary phase provided confidence, which was expressed as a feeling of empowerment.

Knowledge of progression was considered as part of deep subject knowledge in both interviews 1 and 2, as can be seen in Appendix xi. Although I have not reported a full analysis of interview 1 here, I could see that there were differences between the ways it was discussed in each interview. Progression in interview 1 was significant knowledge for MaSTs but it was discussed in shorter phrases, rarely illustrated with examples, and expressed generally or in more conventional terms such as in National Curriculum levels. By interview 2 it was still a concern but MaSTs unpicked issues using mathematical and mathematically pedagogical examples. There were also many more incidences in the second interview, compared to the first, of MaSTs discussing specific examples of tracking backwards and forwards mathematically, and identifying knowledge of such tracking as indicative of deep subject knowledge.
Other differences lay in the number of references to MaSTs' view that their deep subject knowledge was linked to theory and research, which did not occur in the first interview but was identified in four of the second interviews. Knowledge of structures and laws of mathematics was not identified in interview 1 but only in interview 2. Interview 2 also included increased

references to deep subject knowledge as ‘knowing why’ and being able to explain in different ways.

Therefore, in summary, deep subject knowledge is identified by around half or more MaSTs as characterised by features of knowledge of progression, understanding (illustrated by knowledge of gaps and misconceptions, understanding why and knowledge to be able to explain in different ways) and knowledge of using and applying mathematics. Features related to progression were most prevalent. (Interestingly, these findings match the categories used by Adler et al (2009) in their work with secondary student teacher’s perceptions of deep knowledge.)

What was the nature of the knowledge which MaSTs conceptualised as deep?

In examining my findings, I reflected on the nature of the knowledge which MaSTs had perceived as deep. Was the MaSTs’ characterisation of deep subject knowledge purely mathematical? Although Williams had emphasised the importance of each type of knowledge and the combination of both, did the MaSTs take the question as an opportunity to look only at mathematical knowledge?

In their discussion three MaSTs raised points, often as questions, about whether deep subject knowledge was about knowing mathematics significantly beyond the primary curriculum. For example, MaST G wondered if deep subject knowledge meant the knowledge which an accountant might have. Nevertheless she went on to discuss deep subject knowledge as knowledge of using and applying mathematics, knowing how to use contexts for mathematics, understanding of progression, being able to identify misconceptions, explain ideas in different ways and connecting mathematical ideas. These are features of knowledge not normally associated with accountants. So that although these three MaSTs reflected on further and more advanced knowledge of mathematics, their responses were in fact firmly rooted in the primary curriculum. Three MaSTs talked about deep subject knowledge as knowing more advanced mathematics, such as that covered in Key Stage 3, so that they know the next mathematical steps for children as they left the primary phase.

Three MaSTs discussed deep subject knowledge as pedagogical, for example:

*'I think it was as much the peda*gogical knowledge, the way, how children learn maths and how *best to present it to them, I think that is what I most value' F2.*

Otherwise, as I might have expected from the discussion from the literature, the MaSTs described features of deep subject knowledge which were mathematical and pedagogical, often taking a pedagogical approach to mathematics, and vice versa. I will go on to explore this combination of mathematical and pedagogical knowledge later in the light of the models reviewed in Chapter 3.

I was particularly interested in the relationship between what the MaSTs identified as deep subject knowledge and the usual secure subject knowledge needed by all teachers to gain QTS. I explored this by asking the MaSTs whether their knowledge as MaST was similar to that of a class teacher and specifically one who had taught every year group. Could they have gained knowledge for their role as MaST from this experience rather than undertaking the programme? MaSTs were not asked about the value of the knowledge gained from each, but whether it was the same in nature. MaST A was not asked this question. Hers was the first of the second interviews and the question resulted from analysis of the first interviews and her second interview. Her responses made me consider how deep subject knowledge might relate to the knowledge gained for QTS, and then develop as teachers gain experience of year groups. Out of the ten MaSTs asked, all of them felt that their knowledge gained from undertaking the programme and the role of MaST, was not the same as that gained if they had taught each year group. Knowledge and experience were not seen as identical. Although not asked about the value of each, all the MaSTs except MaST B privileged the knowledge gained from the programme and role over that gained by classroom experience. MaST B was the only MaST who felt that classroom experience was at least as important as the knowledge gained as MaST. She felt that reading and more academic knowledge would never be valuable if not put to the test as a classroom teacher. So that in her role she had felt it essential to work with children from other year groups to validate the knowledge she had developed.

The other nine MaSTs felt that the knowledge they had gained was more valuable than that acquired from being a class teacher of each other year group. In these cases, examples of the reasons MaSTs gave were:

'*I think there is experience and there is knowledge and that is the difference…*' D2

'I think the difference would be, for me anyway.. the specific focus on different areas, it is actually made me unpick exactly what I am doing, but if someone had taught year 1, year 2 they *wouldn't necessarily have gone through the process of unpicking what they are doing, they just* pick up the curricu*lum and off we go'* E2

'I think it is the readings, … I don't think it is just the practical things for teaching each particular year group, yes you see more and you would know the areas of weakness but you *wouldn't necessarily know what to do about them* or how to approach them differently or how it could be taught differently ' F2

'I think you just need time to reflect on your practice… and when you are teaching in your classroom, in all those year groups, I think you would just build up a… not always…I think you just concentrate on what you are doing, and I think sometimes just spending that time thinking about where they have come from and where they are going to...' G2

I think there is a difference, why I think there is a difference is that you would have, again like perhaps it is like a quilt and that you can sometimes when you teach a year group you have your square of quilt, a colour of quilt and you might just touch *the edges… but you don't think of it as* the whole quilt.. whereas the course has just kind of encouraged us to think of it as the whole *quilt and think about where you are and how the pieces get together' J2*

'otherwise I am just sort of making the most of my limited experience and not gaining from the experience of all these people who have done these big research projects because some things *are not common sense' K2*

Nevertheless there is a relationship between the year groups where the MaST had most experience and their understanding of other year groups. In some examples, MaSTs used their experience in certain year groups in their role to support others. MaST C used her experience in Years 2 and 6 to guide staff in each Key Stage as she was aware of the desired outcomes. MaST A used her analysis of a consistent subtraction error in her year 5 and 6 classes to promote a focus on counting throughout the school. MaSTs with experience of younger children promoted practices based on child initiated learning and learning through play and practical activities across their school. It seems that it is knowledge of progression and of specific year groups which MaSTs can use in their role. The experience of MaSTs such as MaST C and A at the end of key stages led me to consider whether there is a key year group for a MaST's experience, which I will discuss later on.

MaSTs were also asked directly, as a probing question after question 2, to explain how deep subject knowledge compared to subject knowledge required for QTS**.** MaST J talked about concentric circles of knowledge, where knowledge for QTS was the smaller circle and deep knowledge a larger circle round it. This larger circle included the features of knowledge which she identified as particular to deep subject knowledge. Other MaSTs referred to a significant feature they had used to define deep subject knowledge as distinct from the knowledge for QTS. For MaST D, this was the knowledge of mathematical connections; MaST I sums up the differences between knowledge for QTS and deep subject knowledge as knowing why. These answers, alongside their reflections on their knowledge compared to that gained from experience, suggest that they see the two types of knowledge as distinct in nature.

I considered if the MaSTs would feel that they had deep subject knowledge only in the areas covered by the programme. In considering my findings in the light of the programme content, I wanted to ascertain the degree to which the MaSTs were repeating aspects of the programme which might indicate that they were tailoring their responses to meet the expectations of their tutor as the interviewer. It would also give me a sense of the nature of their knowledge as specialist teacher. The Williams review had made recommendations for the content of the programme (Williams, 2008 p. 25). These recommendations were the basis of the DCSF's requirements for the programme and Appendix x shows how they were interpreted into module learning outcomes and assignments for the programme at the university where the sample of MaSTs undertook their training. As can be seen in the appendix, the programme had three learning outcomes, with one of these relating to mathematical knowledge and one to pedagogical knowledge. Progression is not mentioned in these learning outcomes, but was cited specifically as a focus of one third of one of the two assignments. Therefore although a key part of their programme, the MaSTs had selected progression from a number of other aspects it covered.

The MaST programme did not cover all areas of the curriculum. When asked if they were able to support staff in areas not covered by the programme, they were all confident to say so. They explained that they gained knowledge of areas of the curriculum not covered by the programme by: reading (2 MaSTs), asking advice of other MaSTs via email (1 MaST), using their ability to unpick and analyse (1 MaST), transferring knowledge from assignments on other areas of the mathematics curriculum (1 MaST), working with other staff in the role of MaST (1 MaST), inert knowledge which was there all the time made active from reading and assignments (2 MaSTs.) Therefore deep subject knowledge appears to be seen by these MaSTs as transferable, given the resources of the MaSTs in terms of their contacts, access to reading and their skills of analysis. It also possibly lies inert and is activated by study.

I then considered the degree to which the responses of the MaSTs described deep subject knowledge as characterised by features of Masters level study. This helped me to consider how the MaSTs saw their deep subject knowledge as different to that of any teacher with secure subject knowledge. The programme was validated by the university at Masters level, as a requirement from the DCSF in response to the call from Williams for Masters level training. MaST K and MaST F already had Masters degrees. MaST B had Masters credits as part of her PGCE. For the others, it was their first introduction to study at this level.

In exploring this question I considered The Quality Assurance Agency for Higher Education (2011) descriptors for Masters level work. I then considered qualitative examples of responses from MaSTs which might suggest that they valued the Masters aspects of the programme or where their reflections appeared to be typical of this level of study. I was not expecting to find pure examples of thinking which might be judged to be at Masters level, but rather indications of thinking which may or may not match some of these descriptors.

In referring to the nature of their knowledge, the MaSTs recognised the impact of the level of the programme. Four MaSTs included references to the use of theory and research as indicative of their knowledge. They talked about knowing the 'theory behind' ideas B2, and referring to international comparisons:

'this country has tried this, this other country has done this… so is this a better way, and knowing that research about the areas as well and knowing all of that' C2.

These results correspond to some degree to the findings from question 1 where 25 out of the 35 references to using literature or documentation to support their knowledge were to academic literature.

'I suppose when I think about the assignments I think about…I was thinking at some point … I can't put it in to words, having read around the subject a bit more, you kind of understand… it

is very hard to explain, … it's …it's understanding the reason why you are doing things as well, because in our MaST role we have to tell staff or ask staff or explain things to staff I am able to *explain it …a lot more confidently because I am able to give some reasoning and theory and background to it whereas before I wouldn't have been able to stand up in a staff meeting and* say this is what we are doing, and the theory an*d understanding behind it maybe… in that respect maybe my subject knowledge is deeper because am able to justify …some approaches, maybe..' B2*

'It knocked me flying in one way because I thought I was really good with my maths, I thought I understood it all, and I guess I thought am I going to learn very much from this MaST? And I *have…. You need some people with a bit of space, to think of questions, to think I wonder if… and to look into that… I really enjoy looking into that and being able to feed that* information into school and that is my worry now I am a MaST, how do I keep up with that because it takes *time, and we were forced to do it on the course and how do I keep that discipline up?' K2.*

There were other examples of MaSTs' use of academic aspects of their programme. When asked how they gained knowledge of areas of the curriculum not covered by the programme, six out of the eight responses appeared to be connected with the more academic part of the role of MaST.

MaST F explained that if she only taught other year groups and did not have the overview of the MaST she would not have access to alternatives not practised in the school at that time. She illustrated this with the use of problems at the beginning of a topic rather that at the end, a practice she felt she would have missed if knowledge had only been gained through her current school. She therefore recognised the potential of study to offer alternatives. A further example of the recognition of the application of theory in their schools can be seen when two MaSTs referred to setting by ability in their school and both talked about the research into setting. They referred to discussions which had taken place in their school and the references to research, although general and not specific, were a natural part of their dialogue.

An analysis of MaST I's discussion of how she supported other members of staff demonstrates the level of her self-reflection.

*'If I support a teacher and give he*r some ideas that I have not used, missed, it definitely makes *me go away and think why haven't I done that myself, I am going to and give that ago myself in* class and I do do that often, so I think it makes my own teaching, it improves my own teaching, su*pporting others as well as hopefully helping them, it makes me more analytical… and more self … I self-assess my own teaching more than if I didn't have that role, and I know I do that less in other subjects because I don't have the responsibility in that subject … like in English where I think I have secure subject knowledge but I don't think have anything like the deep knowledge that I do in maths, so I don't think I evaluate my teaching in quite the same way…'*
I2

MaST K used a piece of research to challenge her teaching of place value and progression in the teaching of place value across her school. MaST K, who already had a Masters degree and a mathematics background, said she had not expected to be challenged by the content of the programme but it had 'knocked her flying'. She used language of 'surprises'.

'I think the MaST pulls apart what you think you know, and it does take you places where you *think oh I don't know any more and you have to construct your understanding again, and I think* that is *a really healthy thing to do, I don't think you get opportunities in school, in your* training, in any of the local training I have done to pull something apart at quite an intellectual *level before you can then put it back together' K*2.

MaST K's response to question 1 had centred on her reading of research by Thompson and Bramald (2002). In her discussion of deep subject knowledge she expressed concern about the ability to continue to keep up to date with research findings after the programme had finished, saying that what she valued from research was the ability to see everyday classroom practice from a different view point. Clearly this was important to her in the role of MaST, to challenge her thinking and that of the school, so that she could say that her practices and those of her colleagues were the best of the alternatives. There was a sense of criticality, of wanting to critique present ideas in the light of new ones, even if the new ones are rejected in favour of the current practices. MaST K wanted to be able to look at 'things from a different view point'. She felt that the MaST programme and role had given her the habit and disposition to seek out research.

*'When I have supported teachers before I would never have thought of going and looki*ng at *maths research, whereas now, that might be one of my first port of calls now' K2*

She expressed some frustrations in her role before that of MaST, and her recycling of ideas. She explained that she intended to continue to read research articles, as did MaST I. Her reason for this was:

*'otherwise I am just sort of making the most of my limited experi*ence and not gaining from the experience of all these people who have done these big research projects because some things *are not common sense… You need* some people with a bit of space, to think of questions, to *think I wonder if… and to look into that… 'K2*

The notion of a researcher, having the freedom and space to ask and answer question, was compared to her former inward looking position.

So the MaSTs perceived their deep knowledge as different from that required for QTS, and experience as a class teacher across the primary phase. It was sometimes characterised by features demonstrating some aspects of Masters level study. It was not dependent on having specific input from a programme, with some MaSTs stating that it was transferable. Some MaSTs claimed it was inert knowledge, activated by the programme and role.

My main conclusions based on analysis across both questions:

My findings indicated no entirely uniform approaches to the teaching of mathematics or general perceptions of deep subject knowledge, suggesting the knowledge of the specialist teachers is a complex concept, not easily captured.

However, there were common themes manifested in the MaSTs' reflections on their teaching approaches in their responses to question 1. These themes mostly related to knowledge of progression in children's learning. MaSTs used their knowledge gained of the whole primary phase to modify their own classroom practice. They demonstrated a relationship between their knowledge of their own class and that of the whole primary school. They sometimes promoted whole school changes based on practice deemed effective in their own classrooms or in response to consistent errors in their own classes. Most MaSTs expressed new knowledge of other year groups and felt empowered to teach their classes objectives outside their year groups, particularly those objectives below their year group.

When MaSTs discussed a general definition of deep subject knowledge they also identified knowledge of progression as key. They conceived the most important aspect of their knowledge as allowing them to support a smoother and more secure progression in children's learning across their school. MaSTs explained that deep subject knowledge was different to that gained from experience and needed for QTS. There was some evidence that they conceived of deep knowledge as characterised by features demonstrating some aspects of Masters level study. Several examples suggested that their knowledge was demonstrated as a disposition to reason, often around progression, sometimes in a Masterly way calling on academic reading.

In the summer of 2013 I summarised my findings and sent them to the sample of MaSTs I had interviewed to gain their views. This was an important part of my argument that my conclusions were reliable. The summary email I sent to them can be seen in Appendix viii.

Ten of the eleven MaSTs replied, with no response from the one MaST who had moved to another area. All replies indicated that the ten specialist teachers agreed with my findings. I took this as an endorsement that my findings matched the main views of my sample of MaSTs.

I then compared my conclusions to the existing models for classroom teachers' knowledge of primary mathematics. I record my analysis in my next chapter.

Chapter 9

Comparison of my findings to the existing models for class teachers' knowledge of primary mathematics

Finally I considered how my findings compared to the existing models for subject and pedagogical knowledge for teaching primary mathematics. These models are for the knowledge of classroom teachers of primary mathematics and not necessarily for the role of the specialist. I aimed to consider the relationship between these models and the knowledge which MaSTs identified that they drew on in their teaching, and captured in their own definitions of deep subject knowledge.

I did not find that any of my codes significantly stood outside the existing models in the literature, but I was surprised to find that MaSTs did not appear to evidence evenly use of knowledge across the models, or to define deep subject knowledge as including all aspects of knowledge in the models.

In this chapter I will consider my findings in the light of the models for teacher knowledge of Shulman (1986), Ball et al (2008), Rowland et al (2009), and Ma (1999). I will then compare them to the three wider themes identified in Chapter 3.

Shulman (1986, 1987)

Firstly I considered whether the MaSTs drew on types of knowledge identified by Shulman (1986 1987). I considered in particular his domains of subject matter content knowledge, pedagogical content knowledge, and curriculum knowledge, which corresponded to all of my codes. These first two terms were similar to those used by Williams in his recommendation for specialist teachers. As I have previously discussed, in responses to question 1, the MaSTs did make reference to ideas which were mathematical and I coded separately any specific calculations or use of mathematical terminology or vocabulary. Every MaST made references to mathematical terms, totalling 382 references in all. There were also 16 actual calculations discussed, across 6 MaSTs' interviews. Each of these though was in the context of teaching. Of course, my question had invited discussion of knowledge in the context of teaching. Similarly there were few mentions of purely pedagogical issues which were not discussed in terms of mathematics. These tended to be the features identified by only one or two MaSTs. In defining what they perceived as the features of deep subject knowledge in response to my second question, MaSTs did still include clear links to pedagogy.

Therefore it seems that both subject knowledge and pedagogical knowledge did in fact underpin the MaSTs' responses. My findings also back up claims by for example McNamara (1999) that the distinctions between pedagogical and subject knowledge are blurred, and Williams' (2008)

call for a combination of these types of knowledge. However my sample of MaSTs also conceived of deep knowledge as including aspects of curriculum knowledge.
In discussing curriculum knowledge, Shulman defined curriculum as:

> represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instruction materials available in relation to the programs and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or programme materials in particular circumstances (Shulman, 1986 p.10).

He goes on to distinguish between lateral curriculum knowledge enabling teachers to relate content to other topics of a similar level of difficulty, and vertical curriculum knowledge as knowledge of previous and future learning in the same area. MaSTs did not necessarily refer to programs or instruction materials. They did however refer to increased knowledge of the mathematics and pedagogy associated with the curriculum across the primary phase, and used this to reflect on and evaluate their own teaching, which links to Shulman's vertical curriculum knowledge. There were some examples of horizontal curriculum knowledge, particularly in response to question 1 but the most significant curriculum knowledge was vertical in responses to both question 1 and 2.

Shulman also proposed three forms of knowledge, propositional, case and strategic knowledge. These forms of knowledge have been critiqued, for example as static by Petrou and Goulding (2011) and Warburton (2012). However I found examples of MaSTs using two of these forms in an active and deliberate way. There were instances of MaSTs using case knowledge, for example generalising from cases of errors found in older children to promote changes across the whole school. I became increasingly interested in these examples. Knowledge of the whole primary phase cannot, for most MaSTs, be based on experience. Therefore case study knowledge is key as it enables MaSTs to have benchmarks. For example, MaST C who had experience in both Years 2 and 6 used this to help her to support progression in both Key Stage 1 and 2. MaST A used a common error in her Year 5 and 6 sets to promote a whole school focus on counting.

There were also examples of strategic knowledge in the form of MaSTs' reasoning. An example of this was the recurring theme of MaSTs teaching outside their expected learning outcomes for their year groups. MaSTs justified their teaching of curriculum outside of their year group, questioning the accepted implementation of the curriculum. There were examples of evaluation, reflection, and deliberation. MaSTs drew on knowledge of progression to do this.

Baumert et al (2010) claimed pedagogical knowledge can be limited by subject knowledge. I did not find examples of limited or limiting subject knowledge, apart from one instance where a MaST referred in interview 1 to a simplified understanding of division but which had widened by interview 2. This was perhaps to be expected given the requirements of previous knowledge and experience of teachers nominated to take the role of MaST. Baumert et al also claim that subject content knowledge, which is unrelated to children's learning, can lie inert and can be activated by pedagogical knowledge. I found two examples where MaSTs claimed themselves that deep knowledge similarly was inert but activated by the academic side of the role.

'But I suppose the knowledge was there of the different things you needed but I never really thought of it, I know it sounds silly, … I think the knowledge was there but I never really used it in that wa*y and that has come from the years of teaching, and knowing and thinking… I think it* has all been there but I have never really thought about it before, I would just teach it, I would *never think about it, oh actually… and it was through the MaST when I th*en started teaching *things afterwards, actually….' H2.*

'because it made me kind of evaluate my subject knowledge, so I may have had it somewhere lurking in the depths but it has made me really think about what I was doing, but also the reading I did for *the course, for the assignments, I haven't done academic reading since I left university' I2.*

These MaSTs claimed that reflection and study typical of Masters work activated previously acquired knowledge. This is a similar finding to that of Davis and Renert's (2012) study of teachers on Masters level courses.

Therefore Williams was justified in suggesting that the role of the MaST would be underpinned by subject and pedagogical knowledge, but this sample of MaSTs also drew on aspects of what Shulman termed curriculum knowledge.

Ball et al (2008)

I then considered how my findings related to the categories suggested by Ball et al (2008).

Ball et al sub categorised subject content knowledge as including common content knowledge, specialised content knowledge and horizon knowledge. They suggest that these forms of knowledge are not connected to specific children or pedagogy. For example specialised content knowledge is pure as in unrelated to particular children or pedagogy but specialised as it is not held by any other profession. As explained above I did find mathematical references but these were never unconnected to pedagogy. However, my questions had invited reflections on teaching so this was to be expected. The mathematical knowledge the MaSTs drew on in question 1 and reflected on in question 2 was related entirely to teaching. For example MaST G reflected on her knowledge of the number system and avoided calling zero or one the 'last number' on the number line which she used with her reception class. MaST C traced back to the mathematical topic of quadrilaterals to ensure that children had a full understanding of the properties of parallelograms. MaST H rejected the addition strategy of partitioning both numbers as it would not support the children's understanding of subtraction. These are deliberations of teachers, relating their subject knowledge to pedagogy. Therefore it was difficult to locate examples of pure specialised content knowledge, unrelated to particular children or pedagogy. My findings supported Petrou and Goulding's (2011) claim that there was a blurred boundary between pedagogical and specialised content knowledge.

However I could interrogate my findings to explore specialised content knowledge, referring to pedagogy but unrelated to particular children's learning. MaSTs reflected on supporting staff in year groups where they have had no experience, and distinguished between their deep subject knowledge gained from the programme and developing the role over two years, and that of a teacher with experience of teaching each year group across the primary phase. MaSTs' knowledge, although it is related to their experience and knowledge of nearby year groups, is often unrelated to actual children.

Ball et al define horizon knowledge as:

> an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. It also includes the vision useful in seeing connections to much later mathematical ideas (Ball et al, 2008 p.403).

Again the boundaries between this and pedagogical knowledge are not clear and the authors recognise that this is the case. This quote suggests that horizon knowledge is linked to pedagogy, although this sub category lies outside pedagogical knowledge. Therefore it was difficult to match my codes relating to progression to horizon knowledge which is purely mathematical. My findings suggested that the sample of specialist teachers had developed horizon knowledge which linked both pedagogical and mathematical knowledge.

Ball et al define pedagogical content knowledge as including the sub categories of knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum.

Each sub category was wide and it proved difficult to match my codes distinctly to each one. The MaSTs' discussion of progression could be included in each of the three sub categories. For example, MaST A discussed a consistent error in her Year 5 and 6 sets to do with subtracting a one digit number from a near multiple of one hundred. In analysing this error, she recognised the importance of progression in counting skills and implemented a whole school policy of a short counting session every day. She supported staff in planning for progression in children's counting. In this discussion she drew on knowledge associated with content and students (knowledge of common errors), content and teaching (knowledge of strategies to respond to errors), and knowledge of curriculum (support for staff across the primary phase at different stages in the curriculum).

Therefore I found some links between my findings and Ball et al's sub categories. Ball et al state that their sub categories of subject content knowledge are specialised and pure, not connected to particular learners or pedagogy, but they do use examples of pedagogy to illustrate them. My examples of references to mathematics were specialised as they were in the context of teaching, but I did find examples which were not referenced to particular children. Similarly my sample of specialist teachers valued what appeared to be horizon knowledge but discussed this in terms of pedagogy as well as mathematics. Therefore my findings do not fit neatly into Ball et al's categories for subject content knowledge. The sub categories for pedagogical content knowledge did not provide a framework for me to distinguish the MaSTs' conceptions of deep subject knowledge clearly, no one category captured what the MaSTs had perceived and demonstrated as different.

Rowland et al (2009)

I also found that none of the codes I had identified particularly lay outside the Knowledge Quartet (Rowland et al, 2009).

The foundation domain was the widest ranging, and covered a number of my codes. In my first coding of the interviews I had included a code labelled beliefs which included any phrases which seemed to indicate a particular belief. Of course, most of the interviews indicated beliefs, but the time spent considering these phrases gave me an insight into each MaST's views. I had some indication of the areas Rowland et al included in the foundation aspect of the Knowledge Quartet throughout the interviews, although none of these seemed to stand out as particularly significant or matched features widely shared across the MaSTs. However I found the sub sections of the foundation aspect wide ranging and it was difficult to get a sense of the extent to which specialist teachers drew on each one.

There was evidence that MaSTs had reflected on knowledge of the transformation domain when discussing their teaching approach, choosing to highlight for example their use of resources and images. I could identify some examples of MaSTs reflecting implicitly on sequences of examples as part of their discussion of progression in learning. MaST H considered whether the optimum sequence of calculations would be to tackle the most difficult first, discussing for example subtracting from a zero when introducing decomposition. MaST D also reflected on children's activities and the need to focus on underlying principles rather than distracting activities. However, MaSTs generally did not identify this knowledge as part of deep subject knowledge, other than including knowledge needed to explain why, and to explain in different ways.

The aspect of the knowledge quartet most evidently identified by MaSTs in their discussion of the chosen area of mathematics, and as characterising deep subject knowledge, was related to the connection domain. There were a number of examples of MaSTs identifying the making of connections in their discussion of their teaching and as a key feature of deep subject knowledge. However, the majority of connections were between ideas of different levels of complexity rather than to linked or equivalent ideas. A number of MaSTs also included consideration of connections to key underlying mathematical principles. A more extreme example of this was

MaST D's knowledge of key principles and those on the periphery, and her support for colleagues in this.

My interviews did not allow me to consider in detail the contingency domain of the Knowledge Quartet. However I did find that MaSTs talked about responding to children as a key part of their own teaching and a focus of their support for other staff.

Of the four aspects of the Knowledge Quartet, it was a sub section of the connections domain which appeared to be the most developed by the MaSTs. This suggests that my sample of MaSTs gained knowledge to enable them to sequence learning outside their usual year groups and across the primary phase. This was the type of knowledge they were more likely to conceive of as deep.

Ma (1999)

Ma's (1999) model was the only framework from the existing literature to use a term similar to deep. She defined profound understanding as deep as it concerned knowledge connected to conceptually powerful principles and broad as connected to other ideas of similar or less conceptual power. She claimed that such knowledge is characterised as connected, including multiple perspectives, is linked to conceptually underpinning principles and includes longitudinal coherence.

These wider descriptions of knowledge included loosely all the codes from the findings from my interviews. Some MaSTs had demonstrated Ma's depth of knowledge where in their teaching they had tracked learning back to a key mathematical principle such as the examples of MaST D's tracking back to the key function of pictograms, and MaST C's progression back from a Year 6 question on parallelograms to the concept of angle. There were other examples where MaSTs began from conceptually powerful ideas in their teaching of topics such as MaST E's use of simple arrays in the beginning of her teaching of multiplication to older children and

MaSTs K's delay of certain aspects of place value to ensure the most basic was secure. MaST G used models of the number line to avoid contributing to misconceptions around key principles the children would encounter later on in their learning.

Other MaSTs tracked learning back to the sources of errors or gaps in understanding, and so to conceptually underlying ideas, if not to underpinning mathematical principles. This appeared to show knowledge of longitudinal coherence, if not depth in Ma's terms.

In fact although uninvited, MaSTs gave several examples of what could be described in Ma's terms as knowledge packages. In some cases these were in terms of unpicking and moving backwards from an error or from learning outcomes assumed to be appropriate for the year group to those for other year groups, or forwards in terms of starting with more basic ideas and moving towards the assumed appropriate learning objectives. Ball and Bass (2000) referred to knowledge packages as 'pre parcelled' (p.3) and lacking in flexibility for the demands of the classroom. In contrast, the examples indicated by the MaSTs were in response to children's learning. They appeared to be active rather than static examples of pedagogical reasoning, situated in the immediacy of teaching and planning.

In their own teaching and in their discussion of deep subject knowledge some MaSTs included the term connected knowledge, and gave some examples of making connections sideways, or to ideas of similar or less conceptual power. They also discussed new knowledge to explain ideas in different ways, or in Ma's terms, they offered and recognised multiple perspectives. This was sometimes linked to knowledge gained from the programme and expressed in terms of criticality,

'(Before) *you wouldn't necessarily know what* to do about them or how to approach them *differently or how it could be taught differently' F2*

'you know what has more merit, what doesn't work quite so well,' H2.

Therefore when matched to Ma's framework, the strongest link with my findings was to Ma's longitudinal coherence. MaSTs used and discussed their knowledge of how to link mathematical ideas to those of more conceptual power, and in some cases to underlying mathematical principles. There were also some examples of MaSTs drawing on and valuing their knowledge of multiple perspectives and knowledge of connections to conceptually similarly ideas. So there was evidence of breadth and some examples of depth, as described by Ma.

Before going on to examine my findings in the light of the themes I identified in Chapter 3, I reflected on the appropriateness of conceptualising knowledge perceived as deep by the MaSTs in categories. A number of writers considered in my literature review suggest that teachers' knowledge is unsuited to simple categorisation. Davis and Simmt (2006) identify the dynamic nature of teachers' knowledge and argue that although it can be described in nested categories, it is often implicit. Davis and Renert (2012) claim that mathematics for teaching is best seen as a disposition. I found examples of MaSTs sharing their reasoning with me, generally around progression. They offered opportunities for me to see how they drew on new knowledge to self-evaluate, make changes and reflect on the impact on children's learning. I could attempt to categorise this new knowledge but I was interested to see its use in teachers' thinking. Watson and Barton (2011) talk about teachers' engagement in mathematical modes of enquiry. However, I found very few examples which indicated that the MaSTs were engaging in what could be termed mathematical modes of enquiry. They did not talk about exploring mathematical ideas themselves, in a purely mathematical way, or rehearsing investigations at the children's level to see the sorts of pathways children might take. However they did reason about their mathematical pedagogy in a logical way, drawing on new knowledge of the curriculum across the primary phase.

Additional Themes

I also considered my findings in the light of what I had termed additional themes in my review of existing debates on knowledge for teaching primary mathematics.

The first theme was related to connected knowledge. The literature discusses how connections can be made to the same idea represented differently (ACME, 2008) and between different but linked ideas (Askew et al, 1997), between representations of concepts and concepts themselves (Ma, 1999). I considered that I had found some examples of these in the way MaSTs depicted their teaching as including sideways connections in response to my first question. Some MaSTs also made references to these sorts of connections, although not explained explicitly in these terms, as desired features of deep subject knowledge. In particular, some MaSTs identified knowledge to explain in different ways as part of deep subject knowledge. However there were fewer examples to sideways connections than to vertical connections.

As I have explained, in many examples, MaSTs identified their knowledge as including vertical connections. When children struggled with a compressed idea, MaSTs tracked back to an expanded representation of the concept and worked back to the compressed representation. Some MaSTs clearly identified how as a result of their role and the programme, they spent more time working with expanded representations before moving to the contracted forms (for example MaST E's use of the array).

My second theme had identified vertical connections as links to key underpinning mathematical principles. I found some examples of MaSTs making such links in their discussions of their teaching approaches, with others tracking back, but not to these underpinning principles. In discussing the features of deep subject knowledge MaSTs talked about progression, but did not usually discuss the role of underlying principles in this progression. MaST D, from a special school, was the only MaST to illustrate her ideas of deep subject knowledge with examples of these sorts of connections to underlying principles. She gave examples in response to both questions where she identified key principles and distinguished them from ideas on the periphery, in which I interpreted an implicit recognition of the need to support children in abstracting the key attributes of an idea. Therefore although there were many examples of tracing ideas through the curriculum, fewer MaSTs identified key principles and traced these through the primary phase.

My final theme was that of using and applying mathematics. In this theme I had identified the importance of teachers' reasoning, their knowledge of how mathematics is established, their syntactic knowledge (Schwab, 1978) and their own engagement with mathematical enquiry (Watson and Barton, 2011). I was able to find examples, in all but two interviews, of MaSTs including using and applying mathematics in their own approaches to teaching, referring to the use of contexts to teach mathematics, including what might be thought of as non-mathematical contexts, and open ended investigations. Just over half of the sample included knowledge of aspects of using and applying mathematics as features which characterise their deep subject knowledge. However these features were never discussed in depth and were not as prevalent as references to progression. MaSTs did not give examples of their own mathematical enquiry.

The conclusions of my comparisons of my findings to the existing models

In the light of Shulman's conceptualisation of teacher knowledge, I found that MaSTs drew on examples of knowledge which were across the categories of subject and pedagogical knowledge. They defined deep subject knowledge in terms of pedagogy. However, there were also numerous examples of MaSTs drawing on vertical curriculum knowledge and including it as a feature of deep subject knowledge. MaSTs provided examples of what appeared to be case study knowledge, based on experience from their own teaching to transfer knowledge to year groups where they had little or no experience. They also used strategic knowledge, where their reasoning was explained and often drew on this vertical curriculum knowledge.

Ball et al's categories were more difficult to match to my findings. I found examples where MaSTs used what appeared to be specialist content knowledge and horizon knowledge but this was connected to pedagogy, if not to specific children. My codes mapped across the three types of knowledge included as sub categories of pedagogical content knowledge. Therefore it was difficult to say that deep subject knowledge, as identified by the MaSTs, linked to any distinct sub category.

In the light of Rowland et al's (2009) Knowledge Quartet, MaSTs drew on knowledge across the quartet, although it was not possible to make a judgement on the contingency domain. There were examples of transformation knowledge developed in their own teaching but fewer mentions of this as part of their deep subject knowledge. The connections aspect of the quartet was the most represented both in the MaSTs' own teaching and in what they conceived of as deep knowledge. In particular connections were made between ideas of different levels of complexity rather than between alternative meanings of concepts or ways to represent the same idea.

My findings linked to aspects of Ma's definition of profound knowledge. There was most evidence of knowledge related to longitudinal coherence. In fewer cases this was linked to conceptually more powerful ideas and sometimes to the most powerful underlying principle, which Ma claimed to be indicative of depth of knowledge. There was also some evidence of knowledge related to making connections to ideas of similar or less conceptual power and to multiple perspectives in the MaSTs' identification of features of deep subject knowledge, and examples in MaSTs' discussion of their own teaching.

I was surprised that not all elements of each framework appeared to be evenly identifiable in the MaSTs' own teaching and in their articulation of deep subject knowledge. I was particularly interested to find that there was not stronger representation of using and applying mathematics throughout, and use of resources, models and images in MaSTs' conceptions of deep knowledge. I was also surprised that connections were rarely sideways in the MaSTs' teaching approach.

In the light of the three themes I had identified across a wider range of literature, I was able to see that there was some evidence of connected knowledge, stronger in a vertical sense. In some cases MaSTs identified key mathematical principles and made connections to these, with MaST D standing out in this knowledge. There were references to using and applying mathematics made in response to both questions but these were not discussed at length, and MaSTs rarely mentioned engaging with mathematical enquiry themselves.

Comparing my findings to the existing debate led me to draw more detailed conclusions in response to my research questions, which I will lay out in my next chapter.

Chapter 10

My conclusions

My research set out to examine the nature of the knowledge specialist teachers conceive of as deep subject and pedagogical knowledge of primary mathematics.

As I stated in my introduction, my research has resulted in conclusions in response to my research questions:

1. What knowledge does a sample of primary mathematics specialist teachers conceive that they draw on in their approach to teaching an area of mathematics?
2. What knowledge does a sample of primary mathematics specialist teachers conceive of as deep subject knowledge?

Analysing my findings across these two questions would allow me to answer the following sub questions:

i) Are there shared features in what a sample of MaSTs perceive as deep subject and pedagogical knowledge, or are their responses individual?

ii) If there are shared features perceived by these specialist teachers as indicative of deep subject and pedagogical knowledge, what are they? How can they contribute to an understanding of the distinctive knowledge of all primary mathematics specialist teachers?

iii) Do the current models which articulate the knowledge of primary classroom teachers of mathematics describe the perceived shared knowledge of these specialist teachers?

iv) How do these specialist teachers perceive the impact of their new knowledge on their teaching?

v) What is the relationship between the knowledge of primary mathematics, identified by these specialist teachers, as a class teacher and as a specialist?

I will consider my conclusions to these questions in turn, examining the broad implications for policy makers and the mathematics education community.

My initial two questions were addressed directly in my interviews and have been discussed in Chapters 7 and 8. In response to my first question, the teachers reflected on their teaching approaches to one area of mathematics, and whether these had developed over the first two years of taking the role of specialist teacher. I found that the knowledge of the specialist teacher was in some respects individual in that it was dependent on the teacher themselves and their setting. However from an analysis across a number of interviews, I found that the specialist teachers demonstrated some commonalties as they discussed their own teaching, The specialist teachers drew on common aspects of knowledge relating to progression. As I have explained, the interviews included examples of MaSTs narrating their reasoning about changes in their practice based on new knowledge of both mathematics and pedagogy across the primary phase.

In response to my second question, as I have explained, deep subject knowledge was identified by around half or more MaSTs as characterised by features of knowledge of progression, understanding (illustrated by knowledge of gaps and misconceptions, understanding why and knowledge to be able to explain in different ways) and knowledge of using and applying mathematics. Features related to progression were most common. To some degree this reflected the educational climate of the period of the research.

Analysis across these two questions has enabled me to consider my five sub questions relating to the nature of the perceived knowledge underpinning the role of MaST.

In answering my first sub question, when considering the responses of this small number of specialist teachers in interview, it is clear that the knowledge they conceive that they draw on is to some degree individual. I can identify features of the knowledge they use to justify their approach to teaching an area of mathematics, and which they conceive of as deep subject knowledge, which are particular to them and often reflect their school setting. However there are also clear common features which I suggest are worthy of considering as indicative of the knowledge of primary mathematics specialist teachers generally.

My methods of identifying the knowledge of the MaSTs recognise the socially constructed and constantly evolving nature of teacher knowledge. Therefore my claims are to understanding the knowledge of the sample of these MaSTs, as they were shared in my interviews. However I can also make valuable, but not precise, claims about the knowledge of all primary mathematics specialist teachers. The existing models I have examined in my research suggest that there are commonalities in the knowledge held by individual teachers, and that it is possible to articulate these shared features so as to understand them better. My findings are in line with this notion. My conclusion supports Williams' call for the role of the specialist teacher to draw on a distinct knowledge base. My sample did identify, and conceive as deep, similar sorts of knowledge. Although my research did not draw on a control group of comparable teachers who were not specialists, this knowledge was identified by the MaSTs themselves as distinct as they reflected on their former knowledge base and practices.   My research has focused on the knowledge of the specialist teachers. Whilst acknowledging the importance of skills, attitudes and beliefs, I have found it possible to identify shared knowledge of the sample of MaSTs, as Williams suggested.

I have not simply repeated the findings of the NFER evaluation (Walker et al, 2013) which suggested that a large number of teachers reported increased subject knowledge, in that I have considered whether a small number of MaSTs reported increase in the same sort of knowledge. My findings also show that this knowledge is indeed, as Williams recommended, both mathematical and pedagogical, with the two rarely separated out in the conceptions of the MaSTs in response to my questions. However I also found the sample of MaSTs conceived of deep subject and pedagogical knowledge as including curriculum knowledge and in particular progression through the curriculum. Curriculum knowledge was not a key focus of William's recommendation. MaST D stood out from my sample as drawing on a slightly different type of knowledge in her role in a special school. My findings relating to her knowledge cannot be generalised. Nevertheless I suggest that the contribution of specialist teachers like her could complement the knowledge of MaSTs in mainstream schools. This is significant in the light of the proposed role of teachers in special schools in providing support for teaching in mainstream schooling (DFE 2011a p.20).

What my research has not done, is linked this knowledge to children's learning. This would be the next step.

Therefore, in response to my second sub question, as a result of my research I can articulate indicative features of deep subject and pedagogical knowledge of MaSTs more generally. However, I offer this conclusion as a contribution to the understanding of the specialists' knowledge rather than a general definition. My findings relate to the conceptions of a small sample of MaSTs who had completed the same programme, which can be seen, not as evidence of generalisability, but as evidence for further enquiry. Further research with MaSTs undertaking programmes in other Universities would contribute to my findings.

The shared features I identified relate to knowledge of progression. I found that all of the specialist teachers I worked with drew on knowledge over the primary phase. They used this knowledge to develop an understanding of the whole primary phase in terms of mathematical ideas, pedagogy and curriculum. They perceived that this had enabled them to support other staff as a 'nucleus' of knowledge (Williams, 2008 p.1) but also had led them to modify their own classroom practice. New knowledge of progression challenged them to reconsider their differentiation for less and more able children. The interviews included examples of changes in how teachers planned, how they began topics and how they perceived the knowledge needed for moving year groups. Changes also occurred in the identification of key ideas and the ability to track back to them, and the use of models to demonstrate key ideas met later on in the curriculum. In some examples MaSTs analysed children's errors and traced learning back to underlying mathematical principles. In others, they traced back to the sources of gaps or misconceptions.

In my analysis, I was able to separate examples of specialist teachers' discussion of progression into those which related predominately to mathematics, pedagogy and curriculum. Examples were rarely related solely to one of these three, but often focused more on one than the others. My questions had invited reflections on the classroom, so I had expected that all knowledge would centre around teaching and learning.

There were most examples of progression relating to mathematical ideas, and how these could be ordered in terms of complexity, and in some cases connected to more powerful underlying mathematical principles. For example, MaST C traced progression in the mathematical definitions and properties of quadrilaterals, linking the properties of parallelograms to the properties of quadrilaterals. MaST G discussed her understanding of the structure of number and the possible misconceptions she may be contributing to by calling one the last number on the number line with reception children. MaST H considered the mathematical anomalies between using a strategy which partitions both numbers for subtraction as well as addition. MaST D distinguished between mathematical principles and peripheral ideas which might distract from them. These examples of reasoning about progression linked well to Shulman's (1986) concept of vertical curriculum knowledge and Ball et al's (2008) horizon knowledge as they were mathematical in nature.

In other examples, there was a clear focus on progression relating to pedagogy, the teaching and learning of mathematics, and the types of pedagogy best associated with different parts of the primary phase. Here the specialist teachers discussed the use of models and images or activities designed by teachers to support learning. Examples include MaST E's approach to teaching multiplication to children at the end of Key Stage 2 using the array, at that time more commonly used with children in Key Stage 1. MaST B advised her colleague in Year 6 to use a snakes and ladders board game to reinforce children's understanding of counting on where this was not secure, employing a strategy used by her with younger children but which had not occurred to her colleague teaching the older age range. MaST H reflected on the optimum sequence of examples to ensure children's understanding of subtraction by decomposition.

In other examples, specialist teachers made reference to specific curriculum for particular year groups in their discussion of progression. For example both MaSTs E and C discussed with their classes learning which took place in previous year groups, referring to what the children had covered in specific year groups, as part of their review of learning as they began a new topic.

In all of these three types of examples, specialist teachers reasoned about trajectories of learning, drawing on new knowledge of parts of the primary phase previously less familiar to them, but linked to year groups where they had most experience. In each case the trajectory was expressed predominately in terms of mathematics, pedagogy or curriculum. This was, in some cases, closely linked to academic research on trajectories in children's learning. For example MaST K's understanding of progression in children's learning of place value stemmed from her engagement with research by Thompson and Bramald (2002).

This finding relating to the shared knowledge of the sample of MaSTs enables me to make a new contribution to the debate on primary mathematics and in particular to the way in which specialist teachers are trained and supported. This is a key conclusion at a time when proposals (DFE, 2011c) have resulted in the new ITE training of mathematics specialist teachers and a call for new hubs to enable specialist knowledge to be shared across schools (NCETM, 2013). I have exemplified the meaning of deep subject and pedagogical knowledge to feed into ITE and professional development of specialist teachers of primary mathematics.

In response to my third sub question, I have argued that there is only a partial relationship between the current models which articulate the knowledge of primary classroom teachers of mathematics and the perceived shared knowledge of these specialist teachers. This claim is based on my analysis of what the small sample of specialist teachers chose as appropriate responses to my questions, what they perceived as their deep knowledge, and what they were willing to disclose to me as a former tutor.

Knowledge of progression is a key aspect of the models and themes which I identified at the beginning of my research and which aim to articulate the knowledge of classroom teachers of primary mathematics. My findings link to Shulman's vertical curriculum knowledge (1986) which the MaSTs explored in terms of subject and pedagogical content knowledge, Ball et al's horizon knowledge (2008) although this was expressed in terms of mathematics and pedagogy rather than just mathematics as in Ball et al's original work, Rowland et al's knowledge of connections (2009), Ma's longitudinal coherence (1999) with some examples of depth of understanding where children's learning was traced to underlying mathematical principles. Therefore I have shown that these MaSTs conceived that they had developed a particular part of each of the models for knowledge of classroom teaching of primary mathematics. Similarly, compared to the themes I identified across a wider selection of literature, my findings related to knowledge of key mathematical ideas and the ability to trace these through the curriculum. Other aspects of the existing models and themes were not as well represented in my findings. Fewer of the sample of MaSTs characterised deep subject knowledge with features relating to horizontal than vertical connections. Therefore although these specialist teachers did draw on and conceive of a deep knowledge which was connected, it was characterised by connections between ideas of different complexity, relating to progression, rather than connections between linked ideas of similar complexity or equivalent ideas expressed in different forms. This aspect of the models and themes in the literature was not as developed in the shared knowledge of the MaSTs. This is interesting when compared to the research findings for effective classroom teachers of mathematics such as those of Askew et al (1997) and more current advice such as that from ACME (2012) which emphasise the importance of horizontal connections. The broader implication is that this could be an area to develop in future professional development for MaSTs.

Similarly, although nine MaSTs identified aspects of using and applying mathematics in their own teaching, only seven of them conceived of this as a feature of deep subject knowledge. Across the interview, there were limited references to mathematical reasoning which comes from problem solving and engaging with open ended investigations. The MaSTs did not discuss how their own knowledge might relate to this area, how they might solve problems and develop investigative reasoning. In short, there was less evidence of Schwab's (1978) syntactic knowledge, than of substantive knowledge. I consider this to be of some concern given that the new National Curriculum (DFE, 2013) has one of its three aims that children should be taught to:

> reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language (p.3)

This aim is clearly stated but the programme of study does not reflect how it will be met. An implication therefore of my findings is that this is an area of knowledge which MaSTs will need to develop in order to meet this aim in their own teaching, and to support teaching staff across their school.

In identifying other mismatches between the models of classroom teacher knowledge and my findings for the MaSTs' knowledge, I have further concerns for the role of the MaST in supporting the implementation of the new National Curriculum. Knowledge of resources and images was mentioned by all but one MaST in their discussion of their teaching but there were only three references in total to the use of resources and none to images in the MaSTs' conceptions of deep subject knowledge. It may be that the MaSTs did not discuss resources and images as part of subject knowledge, but would have included this in a discussion of pedagogical knowledge. However there were other instances of references to pedagogy in their discussion of deep subject knowledge, such as to the use of context to teach mathematics. Therefore I suggest that in terms of the requirements of the new National Curriculum for children's ability to calculate using formal methods and to work with fractions, the MaSTs may need to call on increased knowledge of resources and images if they are going to meet the expectations for children's conceptual and procedural understanding (DFE, 2013). This conclusion has relevance for the continued support and the training of future MaSTs.

Therefore my findings showed that this sample of specialist teachers perceived that they drew on, and defined as deep, knowledge which did not exactly match the existing models of knowledge for classroom teaching of primary mathematics and themes in the current debate. Some aspects of these models were particularly prevalent in the knowledge identified from my interviews, but not all of them. The model proposed by Ma was the only one to include the term depth, and my findings do correspond to her definition of profound understanding to some degree. Ma claims that teachers in China developed profound understanding based on their own mathematical education and the culture of professional development in their schools. The sample of MaSTs did not have these experiences, yet the programme and the role itself developed similar knowledge. Further research might consider how profound knowledge, in Ma's terms, might be developed in primary schools.

My findings led me to consider whether the knowledge of the MaSTs is best described in parallel and separate categories. Ruthven (2011) identified how the emphasis of writers such as Watson and Barton (2011, p. 91) on teachers' reasoning is 'challenging an apparent focus on frozen mathematical content at the expense of fluid mathematical process'. Similarly, Hill et al's (2008a) work on measures to assess teachers' knowledge of mathematics for teaching recognise that there is a distinction between what teachers' know and what they 'figure out' (p.396). My sample of specialist teachers drew on particular aspects of existing models for classroom teacher knowledge of primary mathematics. They then used this knowledge in their thinking. I felt that I was witnessing a form of contingency knowledge (Rowland et al, 2009) for MaSTs, as they responded to the request of the interviewer, arguably like a colleague, asking how they approach their teaching of a particular area of mathematics.

This could be compared to Shulman's (1986, 1987) framework of forms of knowledge, describing how teacher knowledge can be represented and organised. The interviews included examples of stories told by specialist teachers in response to the request to share their approaches to teaching an area of mathematics, which can be seen as examples of case knowledge. They promoted changes across their school based on practice deemed effective in their own classrooms or in response to consistent errors in their own classes. They also changed their practice in their own classroom based on new knowledge of mathematics and pedagogy outside of their usual year group. They could '*see the whole quilt'* J2. These stories were often told as 'precedents' (Shulman, 1986 p.11), a means of solving practical problems but also as 'parables' (p.11) with some reference to the MaSTs' belief that they were the right thing to do. The MaSTs told stories which justified their approach and identified the knowledge they drew on and conceived of as deep. The stories were an act of metacognition, given high status by the interview. They were in response not to children, but in a situation similar to their role in responding to colleagues, where they have to answer unanticipated questions. My findings examine knowledge which sits in certain categories of the existing models, but also concerns the way in which this knowledge is used. I claim that I found examples of strategic knowledge, where case studies were used to make professional judgements and provide reasoned rationale (Shulman, 1986 p.13). They were examples of active reasoning, rather than static forms of knowledge (Petrou and Goulding, 2011).

In identifying how MaSTs drew on case knowledge to develop strategic knowledge, I considered whether there was a key year group for MaSTs to develop case knowledge. For example MaST A used her experience in Years 5 and 6 to make changes across her school. Similarly MaST B drew on her experience in Year 2 to make changes across Key Stage 1 and to promote a more practical approach to teaching in Key Stage 2. MaST C referred to her experience in Year 2 and 6, the end of both Key Stages, to underpin her support for teachers across Years 1 to 6 but had less confidence in the Foundation Stage. Strategic placing of specialist teachers could increase their fund of case knowledge. In an education climate which values end of Key Stage measurements of learning and progress, this would be in Years R, 2 and 6. However, in a different education climate, say one which is centred on the way young children learn, the MaST may be better placed with the youngest children.

The use of case knowledge of my sample of MaSTs leads me to question whether specialist teachers trained to teach only mathematics and who would take the role as NQTs would need time to build this bank of case knowledge. I suggest that the strategic placing of these teachers in certain year groups within the school would support them in gathering case knowledge. Shulman (1987) might recognise this use of case and strategic knowledge as a form of pedagogical reasoning, where specialist teachers use new knowledge of mathematics and pedagogy. My sample of specialist teachers displayed a disposition towards mathematics and reasoning, claimed by Davis and Renert (2012) as a more appropriate way to articulate teacher knowledge. A conceptualisation of the knowledge of specialist teachers must recognise not only the types of knowledge they draw on, but the way they use their knowledge in their own classroom, and share it in their role to support the collective knowledge across their school. Again this implication has broad relevance for the professional development and support for specialist teachers. There are also implications for the knowledge of teachers generally, both in terms of ITE and continued professional development. This sample of teachers valued the knowledge gained from academic study to help them to make sense of the knowledge gained from experience in school. It seems that in their view, both schools and Universities have a role in teachers' professional learning.

My fourth sub question related to the impact of the knowledge of specialist teachers on their teaching. Here, I considered an example of reasoning identified across my sample of specialist teachers. I identified from my findings how the knowledge these teachers developed when undertaking the role of specialist provided them with confidence to reconsider their use of the staged nature of the mathematics curriculum.

In their second interview, after taking the role of MaST for two years, the specialist teachers talked in some way about teaching mathematics which was out of the expected range for their year groups. They did so as if this was a change in their practice, based on reasoning which was related to the curriculum, mathematics and pedagogy. Before gaining this knowledge, the specialist teachers seemed to work horizontally through the curriculum. They reported that it would have been usual to assume they would begin with the expected curriculum for the year group, and work through it methodically. This surprised me. However teachers are supported in this horizontal view of learning by guidance such as the Primary National Strategy (2006) and published schemes. This guidance is usually arranged in year groups. The focus is on beginning at the expected curriculum and moving children forward within that stage of the curriculum, although Ofsted (2012) recommended that teachers consider progression across a strand of the curriculum. These MaSTs drew on new knowledge to increase their agency to adapt the curriculum to meet the needs of their present class. I felt that I had witnessed a ‘reconstruction’ (Ruthven, 2011 p.87) of both the MaSTs’ knowledge and identity, as a teacher who could use knowledge to justify their reasoning when they chose to teach in a way which might be outside the accepted guidance. As I write this thesis, a new National Curriculum has been produced, with programmes of study detailed by year groups, although with the suggestion that teachers might use these flexibly. This leads me to argue that such a curriculum model, even given the freedom for some adaptation by schools, will reinforce this focus on the curriculum for a particular year group, whether this is appropriate for the children or not. This conclusion has implications for the way the curriculum is presented and guidance provided to ensure that all teachers feel confident in interpreting the programme of study flexibly.

Finally, I conclude that although there is a relationship between the sample of specialist teachers' knowledge of primary mathematics as a class teacher and as a specialist, the teachers described these forms of knowledge as distinct in nature.

This finding is based on the following argument. If we assume that the models I reviewed in Chapter 3 accurately portray the knowledge of classroom teachers, then I have shown that these MaSTs did not develop the same sort of knowledge evenly. Furthermore, the teachers themselves conceived their knowledge to be different to that of a class teacher, even one who had happened to work with all year groups across the primary phase. My finding that they did not draw on knowledge in the current debate evenly, and perceived their knowledge as distinct from that of a classroom teacher, questions the notion that knowledge for QTS develops seemlessly into the deep knowledge of the specialist teacher. Writers such as Davis and Renert (2009) argue that mathematics for teaching is a distinct body of mathematics. My findings suggest that the MaSTs' knowledge of mathematics for teaching is a distinct branch of this body of knowledge.

All the MaSTs in my sample who were asked, differentiated between knowledge gained as MaST and as a classroom teacher, even with experience across the whole primary phase. They suggested that this sort of knowledge based on experience would only ground them in the current practice of their school, and would not offer them alternatives to reflect on. Two MaSTs claimed that their deep knowledge had been previously gained, and had been inert but activated by the programme and role. It seems that they recognised that it was not just new knowledge but that they were accessing and using this knowledge in a particular way in their role.

Their deep knowledge was not therefore conceived as wholly based on experience with children. They are required in their whole school role to make strategic decisions about children they do not teach. Yet their role is of a practising teacher within the school setting, and therefore different to a non-teaching senior leader or Local Authority consultant or policy maker. What these specialist teachers bring is a unique blend of current practice in the classroom in the heart of the actual school, specialist knowledge and a whole school role. The teachers in my sample used their case knowledge strategically to do this.

There is some evidence that the sample of MaSTs saw aspects of Masters level study as helping them to gain this distinct form of knowledge.

'so for me reading that paper and finding out, it was a very convincing paper, … that I shouldn't be talking about columns with the younger children, because they would find that more difficult to understand. So I was trying to refer to the values of the numbers, so rather than saying 2 tens and 4 units I wo*uld say 20 and 4. For me that was a huge change in my language'* K2

MaSTs discussed reading for and writing assignments as transforming their knowledge, and valued time for critical reflection. It appears that the deep knowledge identified by these MaSTs is not dependent on having specific input from a programme, with some MaSTs stating that the specific knowledge gained from the programme is transferable. Although there was no comparison with a control group, it seems that the requirement of the programme in terms of reflection, within the frameworks of academic reading, supported them in developing the ability to use case knowledge strategically.

I also considered how the mathematical ideas suitable for 3 – 11 year olds can indeed be understood at Masters level. The mathematical ideas captured in the requirements for the Foundation Stage and National Curriculum for 5 to 11 year olds are finite. However the possible understandings which children, and teachers, may hold of these ideas are not.

My findings led me to believe that Williams was correct in insisting on the Masters level of the programme for the specialist teachers, although I acknowledge that my findings are based on the perceptions of teachers as they talk to a former tutor on such a Masters level programme. This conclusion has broad implications for policy makers determining the required level of teachers' qualifications and training. My articulation of the knowledge of these specialist teachers has provided an example of the way teachers perceive that they understand the curriculum for young children at a Masters level.

My research has allowed me to understand the knowledge of this sample of specialist teachers. In realising that the shared features of their knowledge did not match the categories of the existing models for classroom knowledge, I reflected on why this might be. My research took place at a time when accountability is measured by progression through a set of levels and sublevels of the National Curriculum (DFEE, 1999). I believe that the current education climate has led these teachers to value progression more than any other domain of knowledge represented in the current debate. The way in which children's learning is measured through a new curriculum will change (DFE, 2013). The knowledge which specialist teachers draw on and conceive as deep may well change in its focus. As the new National Curriculum is implemented, MaSTs may draw on and value other aspects of these models. For example, the expectations for children's learning of formal calculations and fractions may lend itself to a focus on Rowland et al's (2009) domain of transformation and an increased focus on resources and images. The National Curriculum (DFE, 2013) aim relating to reasoning and proof may lead to increased need for syntactic knowledge represented in Ball et al's (2008) common content knowledge and Rowland et al's (2009) foundation knowledge. Reflections on my findings have led me to believe that it is the nature of the specialists' knowledge which is more significant that its content, which is situated in and a direct function of the educational climate the teachers act in. Repeating my research in another five years' time would I believe also result in the identification of a shared knowledge base of these specialist teachers, but it may be slightly different to the one I have identified here. However it is the use of knowledge of progression which was significant for this sample of specialist teachers in 2010 to 2012.

Consideration of specialist knowledge marks part of a shift from a focus on the individual teacher to how knowledge is distributed across a school and between schools. Specialist teachers engage with problems ranging from those which are specific to their own class to those which are whole school, including those which relate to a class they might never have taught in a year group where they have no experience. To solve those problems they need to reason both mathematically, in terms of the curriculum, and pedagogically. To do so they use their own experience as case study knowledge or a benchmark. Experience in some year groups spread across the primary phase, and particularly those where key assessments take place such as at the end of Key Stages, increases their access to significant case study knowledge. My sample of MaSTs also drew on academic knowledge. Both case and academic knowledge were used to develop strategic knowledge, or strategic pedagogical reasoning. These specialist teachers valued the academic aspects of their study, implying that Masters level training should have a place in the induction of teachers into the role. The particular strategic use of case knowledge of the sample of MaSTs leads me to suggest that NQT specialists would need time to build up their experience before they could play an equivalent role.

I believe that the MaSTs have a role in leading improvements in primary mathematics education. Mathematics is still perceived as an issue for England based on international tests (OCED, 2013), however much the use of these is contested (Smithers, 2013). The findings of the NFER evaluation (Walker et al, 2013) show that the impact of the specialist teacher has not yet been evident on school data, as it is currently measured. Williams' call for the role of the specialist teacher has not in itself impacted on the perceived problem of primary mathematics. Nevertheless, this sample of MaSTs report changes in their schools which are based on mathematical and pedagogical knowledge, used in reflective reasoning.

This research has led to my own learning. I have developed my understanding of research and of myself as a researcher. I have reflected on the place of interpretation in my own thinking and on the way in which I select and understand key ideas from the current debate. Whilst it has been a privilege to begin to understand the knowledge of these specialist teachers, I have been made uncomfortable by the way the process of undertaking research has involved the unpicking of my own beliefs and knowledge. I have confronted assumptions I have previously made about the knowledge and practices of teachers. I realise that I had expected these to align with my own, as I set them out in Chapter 4. In this way the research has taught me about my own understanding of mathematics and the way it is taught and learned. This will impact on my role as teacher educator. I have been surprised at the way in which experienced teachers had not felt able to teach what they judged to be appropriate mathematics, if this was below the expected level for the year group. My findings have led me to reflect on my own work with student and practising teachers. As a result I consider knowledge of progression across the primary phase as an essential part of the content of teachers' knowledge. However, more than this, my research has renewed my belief in empowering teachers to reason mathematically and pedagogically, and bravely.

Bibliography

Adler, J., Hossain, S., Stevenson, M., Grantham, B., Clarke, J. and Archer, R., (2009) 'Interpretations of, and orientations to, 'understanding mathematics in depth': students in MEC programmes across institutions', Proceedings of the British Society for Research into Learning Mathematics, 29(3), pp. 1-6.

Adler, J. and Pillay, V., (2007) 'An Investigation into Mathematics for Teaching: Insights from a case', African Journal of Research in Mathematics, Science and Technology Education, 11(2), pp. 87-108.

Advisory Committee on Mathematics Education (ACME) (2008) Mathematics in Primary Years: A discussion paper for the Rose Review of the primary curriculum. London: The Royal Society.

Advisory Committee on Mathematics Education (ACME) (2013) Empowering teachers: success for learners. London: The Royal Society.

Advisory Committee on Mathematics Education (ACME) (2012) Raising the bar: developing able young mathematicians London: The Royal Society.

Ainley, J. and Luntley, M. (2005) 'What teachers know: the knowledge bases of classroom practice', M. Bosch (Ed.), Proceedings of CERME4, the Fourth Congress of the European Society for Research in Mathematics Education, pp. 1410-1419. Barcelona, Spain: FUNDEMI IQS, Universitat Ramon Llull. (Compact Disk)

Ainsworth, S., (2006) 'A Conceptual Framework for learning with Multiple Representations', Learning and Instruction, 16(3), pp. 183-198.

Al Zahrani, Y. and Jones, K. 'Coverage of topics during a mathematics pedagogy module for undergraduate pre-service primary teachers', Proceedings of the British Society for Research into Learning Mathematics, 32(3), pp. 7 - 12

Alexander, R., Rose, J. and Woodhead, C. (1992) Curriculum Organisation and Classroom Practice in primary Schools. London: DES

Alexander, R. (ed.) (2010) Children, their World, their Education: Final Report and Recommendations of the Cambridge Primary Review. London: Routledge.

Askew, M., Hodgen, J., Hossain, S. and Bretscher, N. (2010) Values and variables: Mathematics education in high-performing countries. London: Nuffield Foundation.

Askew, M., Brown, M., Johnson, D., Rhodes, V. and William, D. (1997) Effective Teachers of Numeracy. London: King's College London.

Aubrey, C. (1997) *Mathematics Teaching in the Early Years: An Investigation of Teachers'* Subject Knowledge. London: Falmer Press.

Ball, D. (1988) Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education. Unpublished PhD Thesis. Michigan State University.

Ball, D. and Bass, H. (2000) 'Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics', in Boaler, J. (ed) (2000) Multiple perspectives on the teaching and learning of mathematics, West Port CT: Ablex. pp. 83-104.

Ball, D., Thames, M. and Phelps, G. (2008) 'Content Knowledge for Teaching What Makes It Special?', Journal of teacher education, 59(5), pp. 389-407.

Ball, D. (1990) 'The Mathematical Understandings That Prospective Teachers Bring to Teacher Education', The Elementary School Journal, 90(4), pp. 449-466.

Banks, F., Leach, J. and Moon, B. (2005) 'New understandings of teachers' pedagogic knowledge ', Curriculum Journal, 16(3), pp. 293-329.

Barmby, P., Bilsborough, L., Harries, T. and Higgins, S. (2009) Primary Mathematics: Teaching for Understanding. Maidenhead: OUP McGraw Hill.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M. and Tsai, Y. (2010) 'Teachers' Mathematical Knowledge: Cognitive Activation in the classroom, and Student progress', American Educational Research Journal, 47(1), pp. 133-180.

Bednarz, N. and Proulx, J., (2009) 'Knowing and Using Mathematics in Teaching: Conceptual and Epistemological Clarifications', For the Learning of Mathematics, 29(3), pp. 11-17.

Brophy, J. (ed.) (1991) *Advances in research on teaching: Teachers' subject matter knowledge* and classroom instruction. Greenwich: JAI Press.

Brown, L., (2013) 'Learning as a Mathematics Teacher Educator through Narrative Interviewing ', Proceedings of the British Society for Research into Learning Mathematics, 33(1), pp. 1-6.

Brown, T., Mcnamara, O., Jones, L. and Hanley, U., (1999) 'Primary Student Teachers' Understanding of Mathematics and its Teaching', British Educational Research Journal, 25(3), pp. 299 – 302.

Bruner, J. (1996) The Culture of Education. Harvard University Press: London.

Bruner, J. (1978) 'The role of dialogue in language acquisition', in Sinclair, A., Jarvella, R. and Levelt, W. (ed.) *The Child's Conception of Language.* New York: Springer, pp. 241-256.

Bruner, J. (1966) Toward a Theory of Instruction. Cambridge, Mass.: Belkapp Press.

Bruner, J. (1977) The Process of Education. Cambridge Mass.: Harvard University Press.

Buber, M. (1958) I and thou. Trans. R. G. Smith. 2nd. New York: Charles Scribner's Sons.

Burgess H, and Mayes, A. (2008) 'Using e-learning to support primary trainee teachers' development of mathematical subject knowledge: An analysis of learning and the impact on confidence', *Teacher Development: An International Journal of teachers' professional* development, 12(1), pp. 37-55.

Burghes, D. (2012) Primary problems: A First Curriculum for Mathematics. London: Politeia.

Charmaz, K. (2005) 'Grounded Theory in the 21st Century: Applications for Advancing Social Justice Studies', in Denzin, N. and Lincoln, Y. (ed.) The Sage Handbook for Qualitative Research Third Edition. California: Sage, pp. 507-536.

Chase, S. (2005) 'Narrative Inquiry: Multiple Lenses, Approaches, Voices', in Denzin, N. and Lincoln, Y. (ed.) The Sage Handbook of Qualitative Research Third Edition. California: Sage, pp. 651-680.

Christians, C. (2005) 'Ethics and Politics in Qualitative Research', in Denzin, N. and Lincoln Y. (ed.) The Sage Handbook of Qualitative Research Third Edition. California: Sage, pp. 139-164.

Coffey, A. and Atkinson, P. (1996) Making Sense of Qualitative Data. California: Sage.

Compton, I. and Edwards, J. 'Relationships between the influences on primary teachers' mathematics knowledge', Proceedings of the British Society for Research into Learning Mathematics, 31(3), pp. 13-18.

Davis, B. and Renert, M. (2009) 'Mathematics-for-Teaching as Shared Dynamic Participation', For the Learning of Mathematics, 29(3), pp. 37-43.

Davis, B. and Renert, M. (2012) 'Profound understanding of emergent mathematics: broadening the construct of teachers' disciplinary knowledge', Educational Studies in Mathematics, 82(2), pp. 245-265.

Davis, B. and Simmt, E. (2006) 'Mathematics-for-Teaching: an Ongoing Investigation of the Mathematics that Teachers (Need to) Know', Educational Studies in Mathematics, 61(3), pp. 292-319.

Davis, B. and Simmt, E. (2003) 'Understanding Learning Systems: Mathematics Education and Complexity Science', Journal for Research in Mathematics Education, 34(2), pp. 137-167.

Davis, Z., Adler, J. and Parker, D., (2007) 'Identification with images of the teacher and teaching in formalised in service mathematics teacher education and the constitution of mathematics for teaching', Journal of Education, 42, pp. 33-60.

DCSF (2007) The Children's Plan: Building Brighter Futures. London; DCSF.

Denzin N. and Lincoln, Y. (2005) 'Introduction: The Discipline and Practice of Qualitative Research', in Denzin N. and Lincoln Y. (ed.) The Sage Handbook of Qualitative Research Third Edition. California: Sage, pp. 1-32.

DES (1988) Mathematics in the National Curriculum. London: HMSO.

DES (2002) Qualifying to Teach. London: DES / TTA.

Deutscher, I., Pestello, R. and Pestello, H. (1993) Sentiments and acts. New York: Aldine de Gruyter.

Dey, I. (1993) Qualitative Data Analysis. A User-Friendly Guide for Social Scientists. Routledge: London.

DFE (2011b) The Framework for the National Curriculum. A report by the Expert Panel for the National Curriculum Review. London: DFE.

DFE (2011a) *Teachers' Standards.* London: DFE.

DFE. Professional skills test. Available at: http://www.education.gov.uk/schools/careers/traininganddevelopment/professional/b00212154/introduction (Accessed: 20/11/13).

DFE (2011c) Training our next generation of outstanding teachers: Implementation plan. London: DFE.

DFE (2011a) Support and Aspiration: A new approach to Special Educational Need and Disability. Tell us what you think. Easy Read Version. London: DFE.

DFE (2012) Statutory Framework for the Early Years Foundation Stage. Cheshire: DFE.

DFE (2013) Mathematics Programmes of Study: Key Stages 1 and 2 National Curriculum in England. Cheshire: DFE.

DFE (2010) The Importance of Teaching: The Schools' White Paper London: DFE.

DFE (2012a) Teachers' Standards. London: DFE.

DFEE (1999) The National Numeracy Strategy. Suffolk: DFEE.

DFEE (1998) Teaching: High Status, High Standards. London: DFEE.

DFEE and QCA (1999) The National Curriculum. London: QCA.

DFES (2006 onwards) The Primary National Strategy. Nottingham: DfES.

DFES (2007) The Early Years Foundation Stage. Nottingham: DfES.

Ely, M., Anzul, M., Friedman, T., Garner, D. and McCormack Steinmetz, A. (1991) Doing Qualitative Research: Circles within circles. New York: Routledge Falmer.

Ernest P. (1989) 'The Impact of Beliefs on the Teaching of Mathematics', in Ernest, P. (ed.) Mathematics Teaching: the State of the Art. London: Falmer Press, pp. 249-254.

Fontana, A. and Frey, J. (2005) 'The Interview: From Neutral Stance to Political Involvement', in Denzin N. and Lincoln, Y. (ed.) The Sage Handbook of Qualitative Research Third Edition. California: Sage, pp. 695-728.

Freudenthal, H. (1973) Mathematics as an Educational Task. New York: Springer.

Freudenthal, H. (1991) Revisiting Mathematics Education: China Lectures. Dordrecht: Kluwer Academic Publishers.

Gates, P., (2006) 'Going beyond belief systems: exploring a model for the social influence on mathematics teacher beliefs', Educational Studies in Mathematics, 63(2), pp. 347-369.

Ginsburg, H. and Amit, M., (2008) ' What is teaching mathematics to young children? A theoretical perspective and case study', Journal of Applied Developmental Psychology, 29, pp. 274-285.

Goodson, I., (1995) 'The story so far: Personal knowledge and the political', International Journal of Qualitative Studies in Education, pp. 89-98.

Goulding, M., Rowland, T. and Barber, P. (2002) 'Does it matter? Primary Teacher Trainees' Subject Knowledge in Mathematics', British Educational Research Journal, 28(5), pp. 689-704.

Gray, E. and Tall, D. (2007) 'Abstraction as a natural process of mental compression', Mathematics Education Research Journal, 19(2), pp. 23-40.

Guba, E. and Lincoln Y. (2005) 'Paradigmatic Controversies, Contradictions, and Emerging Confluences', in Denzin, N. and Lincoln, Y. (ed.) The Sage Handbook of Qualitative Research Third Edition. California: Sage, pp. 191-216.

Hargreaves, A. (1981) 'Contrastive rhetoric and extremist talk: Teachers, hegemony and the educationist context', in Barton, L. and Walker, S. (ed.) Schools, teachers and teaching. Lewes: Falmer, pp. 303-329.

Hargreaves, A. (1984) 'Contrastive rhetoric and extremist talk', in Hargreaves, A. and Woods, P. (ed.) Classrooms and Staffrooms: The sociology of teachers and teaching. Milton Keynes: Open University Press, pp. 215-231.

Hiebert, J. and Carpenter, P. (1992) ' Learning and Teaching with Understanding ', in Grouws, D. (ed.) Handbook of Research on Mathematics Teaching and Learning. New York: Macmillan, pp. 65-97.

Hill, H., Blunk, M., Charalambous, C., Lewis, J., Phelps, G., Sleep, L. and Ball, D. (2008b) 'Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study ', Cognition and Instruction, 26(4), pp. 430-511.

Hill, H., Umland, K., Litke, E. and Kapitula, L., (2012) 'Teacher Quality and Quality Teaching: Examining the Relationship of a Teacher Assessment to Practice', American Journal of Education, 118(4), pp. 489-519.

Hill, H., Ball, D. and Schilling, S. (2008a) 'Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students', Journal for Research in Mathematics Education, 39(4), pp. 372-400.

Hill, H., Rowan, B. and Ball, D. (2005) 'Effects of teachers' mathematical knowledge for teaching on student achievement', American Educational Research Journal, 42(2), pp. 371-406.

Hill, H., Schilling, S. and Ball, D. (2004) 'Developing measures of teachers' mathematics knowledge for teaching', The Elementary School Journal, 105(1), pp. 11-30.

Kristendottir, J. (March 2013) 'Primary School Teachers Develop their Mathematics Teaching', Smith, C. (Ed.) Proceedings of the British Society for Research into Learning Mathematics, 33(1), pp. 37-42.

Kvale, S. (1996) Inter Views. California: Sage.

Leinhardt, G. and Smith, D. (1985) 'Expertise in Mathematics Instruction: Subject Matter Knowledge', Journal of Educational Psychology, 77, pp. 247-271.

Lincoln, Y. and Guba, E. (1985) Naturalistic Inquiry. London: Sage.

Ma, L. (1999) Knowing and Teaching Elementary Mathematics. Mahweh: Lawrence Erlbaum Associates Inc.

McNamara, D. (1991) 'Subject Knowledge and its Application: Problems and possibilities for teacher educators', British Educational Research Journal, 28(5), pp. 113-128.

Meredith, A., (1995) 'Terry's Learning: some limitations of Shulman's pedagogical content knowledge ', Cambridge Journal of Education, 25(2), pp. 175-187.

Miles, M. and Huberman, A. M. (1994) An Expanded Source book: Qualitative Data Analysis. Second edition. California: Sage.

Milik, A. and Boylan, M., (2013) 'Valuing Choice as an Alternative to Fixed Ability Thinking and Teaching in Primary Mathematics', Forum, 55(1), pp. 161-171.

Mullis, I., Martin, M., and Foy, P. (with Olson, J., Preuschoff, C., Erberber, E., Arora, A., and Galia, J.). (2008) TIMSS 2007 International Mathematics Report: Findings from *IEA's Trends* in International Mathematics and Science Study at the Fourth and Eighth Grades. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Murphy, C. (2006) '"Why do we have to do this?" Primary trainee teachers' views of a subject knowledge audit in mathematics', British Educational Research Journal, 32(2), pp. 227 – 250.

NCETM (2013) Mathematics Education Strategic Hubs.
Available on
https://www.ncetm.org.uk/resources/43530
(Accessed on 21/01/13).

Newell, R. (March 2011) 'The extent to which a primary maths teacher's success in the classroom is dependent on subject knowledge', Proceedings of the British Society for Research into Learning Mathematics, 31(1), pp. 103-108.

OECD (2013) OECD Skills Outlook 2013: First Results from the Survey of Adult Skills, London: OECD Publishing.

Ofsted (2012, 2013) Mathematics Survey Visits: Generic grade descriptors and supplementary subject-specific guidance for inspectors on making judgments during visits to schools. London: Ofsted.

Ofsted (2011) Good Practice in Primary Education: Evidence from twenty Successful Schools. London: Ofsted.

Ofsted (2008) Mathematics: Understanding the Score. London: Ofsted.

Ofsted (2012) Mathematics: Made to Measure. London: Ofsted.

Oslund, J. (2009) 'Mathematics for Teaching: an ethno poetic perspective,' in Swars, S., Stinson, D. and Lemons – Smith, S. (eds) Proceedings of the 31st Annual Meeting of the North American Chapter the International Group for the Psychology of Mathematics Education. Atlanta GA: Georgia State University.

Petrou, M. and Goulding, M. (2011) 'Conceptualising Teachers' Mathematical Knowledge in Teaching', in Rowland, T. and Ruthven, K. (ed.) Mathematical Knowledge in Teaching. London: Springer, pp. 9-26.

Poulson, L. (2001) 'Paradigm Lost? Subject Knowledge, Primary Teachers and Education Policy', British Journal of Educational Studies, 49(1), pp. 45-55.

Pring, R. (2000) Philosophy of Educational Research. London: Continuum.

QCA and DFEE (2000) Curriculum Guidance for the Foundation Stage London: Qualifications and Curriculum Authority.

Resnick, L., Bill, V. and Lesgold, S. (1992) ' Developing thinking abilities in arithmetic class ', in Demetrieu, A., Shayer, M. and Efklides, A. (ed.) Neo Piagetian theories of cognitive development. London: Routledge.

Robson, C. (1993) Real World Research. Oxford: Blackwell.

Rose, J. (2009) The Independent Review of the Primary Curriculum. Nottingham: DCSF.

Rowland, T. and Turner, F. (2008) 'How shall we talk about ‘subject knowledge’ for mathematics teaching?' Proceedings of the British Society for Research into Learning Mathematics, 28(2), pp. 91 – 96.

Rowland, T., Turner, F., Thwaites, A. and Huckstep, P. (2009) Developing Primary Mathematics Teaching. London: Sage.

Rowland, T. and Ruthven, K. (ed.) (2011) Mathematical Knowledge in Teaching. London: Springer.

Rowland, T., Huckstep, P. and Thwaites, A., (2003) 'The Knowledge Quartet', Proceedings of the British Society for Research into Learning Mathematics, 23(3), pp. 97 – 102.

Rowland, T., Huckstep, P. and Thwaites, A. (2005) ‘Elementary teachers’ mathematics subject knowledge: the knowledge quartet and the case of Naomi’. Journal of Mathematics Teacher Education 8(3) pp. 255 – 28.

Ruthven, K. (2011) 'Conceptualising Mathematical Knowledge in Teaching', in Rowland, T. and Ruthven, K. (ed.) Mathematical Knowledge in Teaching. London: Springer, pp. 83-98.

Ryan, J. and Williams, J. (2011) 'Teachers’ Stories of Mathematical Subject Knowledge: Accounting for the Unexpected', in Rowland, T. and Ruthven, K. (ed.) Mathematical Knowledge in Teaching. London: Springer, pp. 251-271.

Satre, J. (1981) The Family Idiot: Gustave Flaubert, 1821 - 1857 (Vol 1). Chicago: University of Chicago Press.

Schostak, J. (2002) Understanding, Designing and Conducting Qualitative Research in Education. Buckingham: OUP.

Schwab, J. (1978) Selected Essays in Westbury, I. and Wilkof, N. (ed.) Science, Curriculum and Liberal Education. Chicago: University of Chicago Press.

Schwandt, T. (1989) 'Recapturing moral discourse in evaluation', Educational Researcher, 18(8), pp. 11-16.

Shulman, L. (1987) 'Knowledge and Teaching: Foundations of the New Reform', Harvard Educational Review, 57(1), pp. 1-21.

Shulman, L. (1986) 'Those Who Understand: Knowledge Growth in Teaching', Educational Researcher, 15(2), pp. 4-14.

Simon, M. (2006) 'Key Developmental Understandings in Mathematics: A Direction for Investigating and Establishing Learning Goals', Mathematical Thinking and Learning, 8(4), pp. 359-371.

Simon, M. (1995) 'Reconstructing Mathematics Pedagogy from a Constructivist Perspective', Journal for Research in Mathematics Education, 26(2), pp. 114-145.

Skemp, R. (1989) Mathematics in the Primary School. London: Routledge.

Slavin, R. Lake, C. and Groff, C. (2009) What works in Teaching Maths? York: The University of York.

Smithers, A. (2013) *Confusion in the ranks: How good are England's Schools?* London: Sutton Trust.

Stake, R. (2005) 'Qualitative Case Studies', in Denzin, N. and Lincoln, Y. (ed.) The Sage Handbook of Qualitative Research Third Edition. California: Sage, pp. 443-466.

Strauss, A. (1987) Qualitative Analysis for Social Scientists. Cambridge: Cambridge University Press.

Strauss, A. and Corbin, J. (1998) Basics of Qualitative Research. 2nd edition. London: Sage.

Stronach, I. and Maclure, M. (1997) Educational Research Undone: The Postmodern Embrace. Buckingham: OUP.

Sturman, L., Burge, B., Cook, R. and Weaving, H. (2012) TIMSS 2011: Mathematics and Science Achievement in England. Slough: NFER.

Teaching and Development Agency (2007) Professional Standards for Teachers. London: TDA.

The Primary National Strategy. Finding All Possibilities
Available at:
file:///Z:/Primary/Primary%20Maths/NCETM%202008%20CD/Professional%20Development.html#e (Accessed: 20/11/13).

The Primary National Strategy. Primary National Strategy Professional Development Material.
Available at:

file:///Z:/Primary/Primary%20Maths/NCETM%202008%20CD/Professional%20Development.html#e (Accessed: 20/11/13).

The Quality Assurance Agency for Higher Education (2011) The UK Quality Code for Higher Education. Gloucester: The Quality Assurance Agency for Higher Education.

Thomas, G. (2013) How to do your research project. 2nd Ed. London: Sage.

Thompson, I. and Bramald, R. (2002) An investigation of the relationship between young *children's understanding of the concept of place value and their competence at mental addition*. Newcastle: University of Newcastle.

Thwaites, A., Rowland, T. and Huckstep, P. (2005) The Knowledge Quartet: a framework for developing mathematics teachers' content knowledge. Paper presented at the European Conference on Educational Research, University College Dublin, 7-10 September 2005

Turner - Bisset, R. (2006) 'The Knowledge Bases of the Expert Teacher', British Educational Research Journal, 25(1), pp. 39-55.

Turner, F. (2012) 'Using the Knowledge Quartet to develop mathematics content knowledge: the role of reflection on professional development', Research in Mathematics Education, 14(3), pp. 253-272.

Turner, F. and Rowland, T. (2011) 'The Knowledge Quartet as an Organising Framework for Developing and Deepening Teachers' Mathematical Knowledge', in Rowland, T. and Ruthven, K. (ed.) Mathematical Knowledge in Teaching. London: Springer, pp. 195-212.

Twiselton, S. (2000) 'Seeing the Wood for the Trees: the National Literacy Strategy and Initial Teacher Education; pedagogical content knowledge and the structure of subjects', The Cambridge Journal of Education, 30(3), pp. 391-403.

Vorderman, C., Porkess, R., Budd, C., Dunne, R. and Rahman - Hart, P. (2011) A world class mathematics education for all our young people. London: DFE.

Walford, G. (2001) Doing Qualitative Educational Research. London: Continuum.

Walker, M., Straw, S., Jeffes, J., Sainsbury, M., Clarke, C. and Thom, G. (2013) Evaluation of the Mathematics Specialist Teacher (MaST) programme. London: NFER.

Warburton, R., (2012) 'Continuous and discrete knowledge: analysing trainee teachers' mathematical content knowledge change through 'knowledge maps'', Proceedings of the British Society for Research into Learning Mathematics, 32(1), pp. 53-58.

Watson, A. and Barton, B. (2011) 'Teaching mathematics as the contextual application of modes of mathematical enquiry', in Rowland, T. and Ruthven, K. (ed.) Mathematical Knowledge in Teaching. London: Springer, pp. 65-82.

Watson, A. & Mason, J. (2002). Extending Example Spaces as a Learning /Teaching Strategy in Mathematics, in A. Cockburn & E. Nardi (Eds.) Proceedings of PME 26, University of East Anglia, 4, pp. 377-385.

Williams, P. (2008) The Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools. London: DCSF.

Wilson, S., Shulman, L. and Richert, A. (1987) ' 150 Different ways of knowing: Representations of Knowledge in Teaching,' ', in Calderhead, J. (ed.) Exploring Teachers' Thinking. London: Cassell, pp. 104-124.

Yin, R. K. (1989) Case study research: Design and methods. London: Sage.

Appendices

Appendix i	Initial email to MaSTs informing them of the research
Appendix ii	Details of participating MaSTs
Appendix iii	Expectations for teachers participating in the MaST programme
Appendix iv	Interview schedule
Appendix v	Pen portraits
Appendix vi	Coding for question 2
Appendix vii	Percentages of responses not coded
Appendix viii	Final validating email
Appendix ix	Consent form and information sheet
Appendix x	Outline of the MaST programme
Appendix xi	Incidences of codes across the responses of MaSTs in both the first and second interview to question 2
Appendix xii	Extracts from exemplar transcripts

Appendix i Initial email to MaSTs informing them of the research

Dear MaST Teachers

As you may know, I am working on finishing a Doctoral degree. As part of my studies, I hope to investigate the knowledge developed by teachers taking the MaST role.

I am very interested in finding some of you who would not mind being part of my study. This would involve meeting me for an interview at the beginning and at the end of the programme. The interview would take about an hour, and would take place somewhere close to or in your school.

If you are happy to participate, I will ensure that you are anonymous. The interviews will be taped and transcribed, replacing your name with a code. You will be asked to agree transcripts before they are used in any of my work, and then all interview tapes will be destroyed. You will be free to withdraw at any time.

This study is completely separate from the programme itself.

Please let me know if you are happy to participate.

Gina

Appendix ii Details of participating MaSTs

MaST	Teaching experience at the beginning of the research	Age Range taught so far	Type of school	Other roles held
A	12 years	Years 3 to 6	Primary	Mathematics lead
B	7 years	Years 1, 2, 3 and 5	Primary	Mathematics lead, geography lead, Head of Key Stage 1 and Early Years, acting deputy head teacher
C	4 years	Years 2, 5 and 6	Primary	Mathematics lead
D	13 years	Years 1 to 6	Special school	Mathematics lead
E	2 years	Years 4 and 5	Primary school	
F	12 years	Years 4, 5 and 6	Two primary schools during the	Mathematics lead, leader of learning

			two year period	
G	6 years	Reception and nursery	Two primary schools with nurseries during the two year period	Mathematics lead
H	10 years	Years 4, 5 and 6	Primary with nursery	Mathematics lead, Well Being lead, Modern Foreign Languages lead, acting deputy head teacher
I	13 years	Years 1 to GCSE but mostly 4 to 6	Junior	Mathematics lead, Head of year 4, basic skills coordinator, acting deputy head teacher
J	11 years	Years 1 to 6	Primary	Mathematics lead
K	13 years	Years 2, 3, 4, 5 and 6	Primary	Mathematics lead

Appendix iii Expectations for teachers participating in the MaST programme

The teachers who are selected will be expected to:

- Have taught mathematics to primary school children in two or more year groups
- Meet at least the minimum mathematics qualifications required, be a full-time qualified teacher in a primary school and be able to identify strengths and gaps in their knowledge of mathematics within the primary curriculum
- Have worked alongside teaching colleagues as part of a professional development activity, or engaged in some aspect of in-service provision or support to staff
- Be enthusiastic about and committed to developing the knowledge, skills and understanding needed to become a Mathematics Specialist teacher
- Be well placed to lead improvements in the teaching and learning of mathematics in their school and have a good working relationship with the school's leadership team
- Be prepared to undertake personal, extended professional learning activities, maintain a professional learning log and undertake professional development tasks that will involve them carrying out in-school support to colleagues.

DCSF Schedule outlining the contract for the MaST programme p.18

Appendix iv Interview schedule (Interview 2)

In my study, I hope to find out about the knowledge you have developed as a MaST, and explore what you feel is deep subject knowledge of primary mathematics

Show information sheet. (Appendix x)

Is there anything else you'd like to know before we start?

Are you happy to sign the permission form and continue the interview?

I'd like us to talk about two things: the area of maths you chose last time we met and your views on deep subject knowledge

So that we can talk about your knowledge and how it might have changed during the programme, I'd like you to talk about the area of the mathematics curriculum which we discussed last time.

Which year group are you currently teaching?

Can you talk about how you would approach this area of maths with your current class?

Let's look at what you said last time.

Do you think your views have changed?

Consideration of transcript of last interview

Thank you

Follow up probes:

Can you tell me more about…?

What do you mean by….

Can you give me an example of…?

I'm sorry, I am not sure exactly what you meant by…

Would you agree than that you'd identify … as important?

Now our final question

Williams said that MaSTs should have deep subject knowledge of primary mathematics, what do you think this is?

Let's look at what we discussed last time, have your views changed?

Consideration of transcript

Follow up probes:

Can you tell me more about…?

What do you mean by….

Can you give me an example of…?

I'm sorry, I am not sure exactly what you meant by…

Would you agree than that you'd identify … as important?

How does your knowledge compare to a class teacher with QTS?

How does your knowledge compare to a teacher who has taught every year group in the primary school?

How do you develop knowledge of areas we didn’t cover in the programme?

Concluding with:

Is there anything else you’d like to add in your analysis of deep subject knowledge?

Anything else?

Anything else?

Thank you. That was very useful and interesting.

Is there anything you’d like to add to anything we have discussed today?

Any questions you’d like to ask?

I very much appreciate the help you have given me today. Your views are very valuable to my study. The transcript will be emailed for you to check its accuracy. It will not be used until you give me permission.

Thank you very much for your time.

Appendix v Pen portraits

MaST K

MaST K qualified as a teacher in 1990, and had taught for 13 years before taking the role of MaST, mostly in Years 4 to 6 and Year 2. As she began the programme she was working with a Year 2 class. She took the lead for mathematics in her primary school. Her first degree was in Education and Mathematics. She had completed a Masters degree in Education which included an element of mathematics education. This was her last mathematics qualification. Her professional development in mathematics had taken the form of subject leader courses led by her Local Authority. In her first two years as MaST she taught Year 2, and then moved to Year 3.

In both interviews MaST K chose to talk about her teaching of place value. In the second interview she identified that the role of MaST had highlighted to her research undertaken by Thompson and Bramald (2002), about the teaching of place value. This had led her to question her teaching of column value of number early on, and to trial the teaching of quantity value in year 2, her current class at the time, and progress to column value later on. She reflected on her use of language of place value, identifying the key ideas of place value. She also spoke about how she could support other teachers in considering this approach to the progression in the teaching of place value across her school. She identified the research findings as key to both changes in her own practice and that of her colleagues. She had given her assignment, which was also on place value, to a colleague to read. She identified her academic reading, especially of research findings, as key to her new knowledge, and to the way she would continue to develop knowledge.

A further key point about her knowledge related to MaST K's experience of moving from the top of Key Stage 2 to a Year 2 class. This has coincided with the beginning of the programme. She talks about how she had assumed that she could adapt her knowledge of Key Stage 2 to teach Key Stage 1.

'Where I hadn't really thought about doing it a different way, … I have never had any training for key stage 1, and hadn't known a lot about the maths down there and I guess I thought I could drag everything down .. rather than start at the other end and building up .. because I had no other experience apart from *my own children and what they knew… I don't think I would ever have thought about that if I hadn't done the MaST, I would never have looked at it from that other angle'* K2

MaST K's new knowledge of the research on place value and the curriculum for Key stage 1 worked together:

'Yes, being aware that you can't be ready for the concepts in key stage 2 in key stage 1, so you have got to introduce things using the concepts they understand, and try not to add the complex parts, so we could juggle with big numbers in key stage 1 quite happily as long as we talk about *their value, like 20 and 4, but we can't juggle big numbers when we are talking about 2 tens and* 4 units, because that adds another layer, and there are probably other areas of maths where that is al*so the case and I just haven't met them yet' K2*

She sums this up by saying *'I think looking at progression and having the opportunity to look at up to date research has been really useful for me' K2*

MaST K used figurative language to identify how these changes in knowledge impacted on her ‘It knocked me flying in one way because I thought I was really good with my maths, I thought I *understood it all’ K2*

MaST K also reflected on new knowledge relating to using and applying mathematics. In completing her second assignment she had identified an issue relating to achievement of girls in this area of mathematics and had implemented and evaluated a change in her approach, building on these girls’ skills in literacy.

She identified how important it was that the programme engaged her intellectually, *‘I think the* MaST pulls apart what you think you know, and it does take you places where you think oh I *don’t know any more and you have to construct your understanding again, and I think that is a* really healthy thing t*o do, I don’t think you get opportunities in school, in your training, in any* of the local training I have done to pull something apart at quite an intellectual level before you *can then put it back together’*

K2

Coded in second interview question 1	Coverage	Number of references
Law or structure, basic mathematical idea	6.44 %	1
Progression tracking backwards, often from error	14.37 %	6
Progression tracking forwards	4.93 %	2
Reference to theory, literature, research findings, assignments	22.34 %	9
Use of resources and images	0.72 %	1
Vocabulary	7.76 %	4
Using and Applying mathematics, open ended investigations	5.71%	2

MaST J

MaST J was a Key Stage 1 teacher in a primary school. She had taught for 11 years as she took the role of MaST, after qualifying in 1998. She had taught Years 1 to 6 and as the programme began she was working with a year 1 - 2 class. The subject of her degree had been education and mathematics, and this was her last mathematics qualification. She has received and provided Local Authority professional development in mathematics prior to the programme. She was the maths subject leader in her school. During the first two years of the role she had taught a mixed Year 1 and 2 class, teaching mathematics to a group of Year 2 children in the second year. She then moved on to teach Year 2.

In both interviews MaST J chose to discuss her teaching of fractions. She reflected that her approach had changed in that she used problem solving at the beginning of her teaching of fractions, rather than at the end of the teaching. She gave several examples of how she taught mathematics in a context not necessarily thought of as mathematical and which spread across the curriculum. '*I take it and put in context straight away' J2*. There are also examples of non-mathematical contexts where she is able to identify opportunities for mathematical learning.

In her second interview she reported having a very able child in her class and needing to extend this child's learning. Her knowledge of the curriculum higher than that she was currently teaching was helping her to meet this child's needs. She referred to this knowledge as being connected, '*it is perhaps seeing all those connections, I didn't see so much before and how it is all linked, interlinked… so I would say my knowledge before, it wasn't sort of the spokes on a wheel it was more of a spider's web with the connections in betwee*n but now it has almost become like a piece of fabric with all the connections within the connections, so yes I can see that whole mosaic of knowledge and how it does all connect together so I can then just say Ok we are doing this but actually I can take *you in this way, so yes it is exciting for me to have her,'* J2. She tracks forward through the curriculum to ensure progress for this particular child.

MaST J also uses the image of piece of fabric when she discusses how different her knowledge as MaST is when compared to a teacher who has taught every year group. *'I think there is a* difference, why I think there is a difference is that you would have, again like perhaps it is like a quilt and that you can sometimes when you teach a year group you have your square of quilt, a *colour of quilt and you might just touch the edges… but you don't think of it as the whole quilt..* whereas the course has just kind of encouraged us to think of it as the whole quilt and think about where you are and how the pieces get together and I think.. as sort as a teacher in general it would be good to know all the different pieces of the quilt and teach the different year *groups, but I don't think you would be able to' J2.*

MaST J reflects on the way in which her new knowledge has changed the way she plans. She describes how she is able to interpret a statement from the National Curriculum, 'Yes it has, it has changed the way I plan, because I now I plan using the national curriculum, so I go from *that document… with my knowledge* I can break that down to what the statement means, so if we are looking at number sequences we are looking at multiples and we are looking at odd and even numbers and we are looking at hundred squares and the patterns within the hundred squares and we are looking at, putting multiples and what patterns could they see and *discussing what that looks like…' J2.*

'The ladder built both ways.. what they need to do to get to this point, and then some of those of the bits of the ladder missing, so then you think OK what rung is missing, is it because they *really don't understand what 17 is or they are just muddling up 71 and 17…' J2.* This is another image she chooses, that her knowledge is like a ladder. There seems to be focus on progression, tracking forwards and backwards.

MaST J reflected that she had developed this sort of knowledge in areas not covered by the programme. She suggests that this is part of her reasoning, and her ability to transfer the knowledge after developing it in some areas. 'But then it *is the process isn't it, it is perhaps thinking a bit more as a teacher…. And thinking Ok how do we link things… that you just then use the skills from that to apply to other…' J2*

Her reflections include an identification of a key structure of number. She discusses difficulties children encounter with learning about the teen numbers and the importance of their experience of larger numbers which follow a clear structure. In this she discusses how children's learning could be limited by teachers, *'so some pe*ople might say Ok they are muddling up those teen *numbers so they just put a cap on twenty and they don't look at the higher numbers until they* have got this but it is thinking well if they understand the structure then they understand those connections, s*o I think it was not having that fear as well..'J2* . She uses her knowledge of the curriculum beyond that of her current year group.

J2

Coded in second interview question 1	Coverage	Number of references
Connecting similar mathematical ideas, horizontal connections	5.41 %	3
Context for mathematics	9.75 %	4
Law or structure, basic mathematical idea	2.66 %	2
Mathematics in a non-mathematical context	7.30 %	2
Progression tracking backwards, often from error	7.69 %	3
Progression tracking forwards	8.79 %	6
Use of resources and images	3.41 %	3

MaST I

MaST I was a Key Stage 2 teacher and mathematics subject leader in a junior school. She began the role of MaST in her 13th year of teaching. She had a wide range of teaching, from year 1 to GCSE, but had most experience in Years 4 and 6. She had completed an Education degree, including a mathematics specialism which was her last mathematics qualification. Her most recent mathematics professional development had been in the form of Local Authority led maths courses, attending leading teacher and subject leader courses. She was the subject leader for mathematics in the school, as well as head of Year 4, acting deputy in the first year of the programme and Basic Skills coordinator. She taught Year 4 throughout the programme. At the end of the two year period the staff took the decision to change from ability grouping for mathematics across year groups to mixed ability grouping. This was a decision MaST I was pleased about.

MaST I chose to discuss her teaching of fractions, decimals, percentages, ratio and proportion. She felt that her knowledge as MaST had resulted in greater confidence to teach mixed ability groups and to differentiate her questioning and use of resources. This knowledge was both mathematical and pedagogical, and she discussed her understanding of children's misconceptions as an example of it.

A further example of the impact of her new knowledge was in the way she began a topic of work. Using multiplication as an example, she explained how instead of beginning her teaching with what she assumed would be an appropriate next step even though she had fully recognised the need for assessment for learning; she would now begin with a series of calculations, ranging from quite simple to more complex. This would allow her to assess where she started her teaching, rather than beginning with the usual objectives set for that year group. 'I know exactly which children I need to focus, which children are ready to go on to the grid method and *beyond and which children I need to do more practical activities with, use the arrays or… whatever methods I decide to use.. so that is definitely something which has come up' I2*. Here she draws on her knowledge of learning below the year group to match her teachings to individual needs. '*it is different, I haven't done that before, and I think it is because.. partly the* confidence and the sessions that we had, and the mixed ability, realising that I have got to approach it differently, because it is going to benefit all the children because actually in my set *one there would have been some children that weren't ready to go ahead, they might have been* brilliant at adding or subtracting or something but not secure with multiplication so I would *assume that they are and ready to move on to the next stage.. it wasn't an assumption I should have made then' I2*. She explains how she analyses this learning and plans for progression forwards.

In her reflections MaST I identified the distributive law and talks about how she now introduces it to support children's understanding. 'Where they have to understand the distributive law and *I've told them, I use that language with them…and they like… I say to them you don't know* have to use that word but this is what it is, but I modelled it with that group by showing then 7 x 5 is the same as 5 x 5 and 2 x 5 and we drew it out and I showed them pictures of 5 x 5 and 2 x 5 and recombined it and going right bac*k to understanding why the grid method works…and I have never done that before, I thought… I have never actually done it.. I've done it by showing* 24 as two lots of ten and 4 but I have never done it other than partitioning by place value, I have never partitioned with the 5 and the 2 and I am sure that is something come from MaST.. *so that is something that I have used…' I2*

When considering how she developed knowledge in areas of mathematics not covered by the programme MaST I talks about how the academic side of the programme affected knowledge she had already acquired. *' I think may be it did because it made me kind of evaluate my subject* knowledge, so I may have had it somewhere lurking in the depths but it has made me really think about what I was doing*, but also the reading I did for the course, for the assignments' I2.*

MaST I

Coded in second interview question 1	Coverage	Number of references
Change in differentiation	1.76 %	1
Law or structure, basic mathematical idea	3.48 %	1
Meeting children's individual needs, not progression	3.09 %	3
Progression tracking backwards, often from error	13.64 %	4
Progression tracking forwards	3.08 %	2
Reference to theory, literature, research findings, assignments	4.41 %	2
Use of resources and images	1.76 %	1

MaST H

MaST H was acting deputy head of a primary school in the first year of the programme, as well as leading Well Being and Modern Foreign Languages across the school. She had taught for ten years as she took the role of MaST, mostly in Key Stage 2 teaching Year 4, mixed Years 4 and 5, and Year 6. In the first two years of the role she taught year 6 and then moved to years 4 – 5. She had never taught below Year 4. Her degree was related to mathematics. She had undertaken work as a Primary National Strategy consultant for mathematics before taking the role of MaST, and this had been her main source of gaining professional development in mathematics, and delivering it. She was the mathematics subject leader for her current school which was a primary school, including a nursery.

In both interviews, MaST H chose to discuss her teaching of calculation. Her second interview covered the following key points. She discussed at length about how she felt the knowledge gained as MaST had altered the way she sequenced her teaching. For example when teaching a formal calculation she would analyse the smaller skills which were part of the calculation and ensure these were fully secure first. 'Since doing the MaST, as I said I think my subject knowledge has definitely deepened now, where I thought I had subject knowledge when we spoke at the beginning of the course, I thought I had, it was knowing what addition is and knowing how to do it yourself and then being able to teach it to children, I thought that was your subject knowledge, but now I realise actually it is really deeper than that, it is knowing the *progression of it, knowing the skills you need before you teach that… Skills that the children* need to have before they can go.. so for example if you are teaching multiplication you know that the children need to be able to add the numbers back together, they need to be able to *partition the numbers if you are using the grid method, so I didn't see that so much as the* subject knowledge whereas now I really see that actually yes, deeper subject knowledge is knowing what skills they have to have to be able to do those calculations, knowing the next step, *if they are able to do that, knowing where they have come from,…' H2*

MaST H also describes staff meetings where she has discussed counting, calculating and problem solving with her staff. She had identified a key calculation strategy which did not support progression in learning across her school and wanted to address this. 'so we are looking at different methods and what to encourage children to use and not use and getting them to use their own methods, it has made me think about that lot more as well, and that it is not just one *set method, and what methods… fit in to the progression well and which methods are standing alone and don't really support… we have just starting to look at mental calculation,.. and with* addition, there are two mental ways of adding when they partition the tens and partition the *units and add it back together but we were thinking that doesn't s*upport when they do mental subtraction so we are encouraging more when they take the whole number and we encourage that earlier on in the school, and we realise and I emphasise to the staff that they need to be *able to partition we can't not because that* comes later and they still need to do that but with mental strategies then we need to work with the whole number, and then that opened up actually we are going to look at the whole calculation policy again and look at what methods are more supportive for children and what ones can be a little bit more of an issue to children and not *help them quite so much…so I think my approach has changed quite a lot' H2.*

MaST H identified a change in the way she approached calculation, drawing on her understanding of steps of progression. '*When I do the work sometimes I don't specifically* allocate work to children, I give them the steps for the children and then they choose where they feel they are, and then if they are a little bit less confident they might go to the step below, do a *few, check those and then move on… if they are not getting it obviously they get help with it* because sometimes I find it difficult because we set children in groups but I have found that with calculations the children are so out of their groups and they are so individual and I felt it is really important to get each child where they are, rather than giving the child you can do TU – *U actually they might be able…so I think that gives children the opportunities a little bit more to* choose where they are and they are pretty good, they have a good knowledge of where they are, *I haven't found they have chosen that really easy group because they are easy I can get those* done, sometimes they do choose a little but too hard but then we look and say shall we move a *step back…I have it up on the board and then they select the group they are doing' H2.*
She attributes her ability to do this to being a MaST*, 'I mean the difference with MaST, I knew* the progression around my areas whereas I feel more confi*dent' H2.*

MaST H also reflect that this knowledge of progression enables her to support her colleagues, *'when we were looking at progression in counting we were showing progression from* foundation through and I think that made people realise that my knowledge has widened, *perhaps it made me realise that before key stage 1 people didn't come to me so often because they thought I wouldn't be able to help them in that area, and now they know I have gone from* foundation right the way through I think it has instilled confidence in that area which is good, I was probably giving out *the image that I didn't know' H2*

MaST H identifies an area where she would like to research further which also relates to the sequencing of her teaching and progression in learning. '*I don't know whether the philosophy is* better to go in hard and then work a little bit more at it and then know what all the pit falls are *or whether to start off at a lower level and build up, that is one thing I don't know I haven't* done enough reading *it on it…. I would be really interested in doing a little bit of research… a* teacher came back to me after teaching subtraction and she said oh I did it really easy to begin with, no crossing boundaries to begin with and she said oh they have all got it well and then the next day she did start doing with boundaries and obviously they did find it hard but she said I *think they are getting it but when they had zeros in, they had no chance.. and I don't really go for the let's teach it easy ,because I think th*ey are getting into a false sense of security just being *able to do the 6 take away the 4….the 80 take away the 60… and because.. I just feel that going* with it and let them know all the pitfalls, yes they find it harder to begin with, because you can say *that is the worse it gets, it won't get any harder but I don't know which is the best approach..' H2*

MaST H

Coded in second interview question 1	Coverage	Number of references
Context for mathematics	2.28 %	1
Progression tracking backwards, often from error	17.55 %	7
Progression tracking forwards	19.98 %	5
Reference to theory, literature, research findings, assignments	8.82 %	5

MaST G

MaST G was an Early Years teacher and mathematics subject leader in a primary school. As she took the role of MaST she was in her 6^{th} year of teaching after qualifying in 2005 and completing a degree in Philosophy. Her last mathematics qualification had been her GCSE and she had attended Local Authority mathematics and early years courses as her latest forms of mathematics professional development. During the programme she changed schools but continued to teach Reception and nursery children throughout the two years.

MaST G chose to discuss her teaching of number in both interviews. In the second interview she identifies that her practice has changed in that she is aware that she must not ‘limit’ the children’s learning. *‘there are things that I’ve, I think more carefully about now, rather than limiting…little examples like rather than limiting, if I am doing a game where they have numbe*r cards and they have to choose the right number, I might put out white boards and pens so they can write the numbers as well, and just trying to think of ideas to expand their options and definitely ideas that are not limiting their thinking by the activities that I do.. I did a number line activity that I might have done two years ago with zero to ten or zero to twenty, and I would have had the zero to twenty cards and they would take it in turns to pick one up and order them, but now I give them blank paper, ask them to write a number that they know and then to order *those numbers on a number line and it doesn’t matter if they have got the same ones or if they* have got a million or if they have got 3, it is them knowing those numbers and taking ownership, so definitely think I have changed that view,.. just thinking a bit more deeply about how I give *them the opportunity to show what they know…’ G2*

‘I will think of it as unlimited’ G2.

MaST G’s knowledge of later learning changes the way she teaches number. For example, she reflects on the structure of the number system and the importance of avoiding later misconceptions. ‘I am really conscious of things like zero is not the last number, there is *numbers below that… I know it is little things, the kids m*ight not pick up on but to me it is quite *important that we don’t limit… , because the question might come up one day and you have taught them all your life that it starts at zero and it goes upward… so I think about that in terms* of my teaching practice about how I have got to be conscious about what I might be setting them up for, and just making sure they have got a really firm understanding of the basic *principles, otherwise they are going to struggle further up the school' G2*

Her new knowledge appears to allow her to identify and respond to unexpected learning, 'I think mathematically, like if there was someone, a child who was to go off on a tangent with something I would rather see if we could incorporate it rather than pulling them straight back to *what I had envisaged for the activity' G2*

MaST G's knowledge as MaST appears to relate to progression, *'I have a knowledge of, having* done the course, what comes before and what comes after.. just thinking what would be the next *progression for them' G2.* This knowledge allows her to remove limits and allow children to progress more quickly than she might have done before. This relates to her more able children but she also reflects on her knowledge of early mark making, gathered during her writing of an assignment, and her ability to identify more basic learning. As she teaches the younger children, she seems to focus on new knowledge of ideas later on the curriculum, tracking the curriculum forwards more than she does backwards.

MaST G

Coded in second interview question 1	Coverage	Number of references
Early mark making	3.46 %	1
Mathematics in a non-mathematical context	0.56 %	1
Progression tracking backwards, often from error	4.06 %	1
Progression tracking forwards	24.84 %	8
Reference to theory, literature, research findings, assignments	1.05 %	2
Use of resources and images	6.28 %	2
Key idea and those on the periphery	6.18%	1

MaST F

MaST F was a Key Stage 2 teacher and maths subject leader in a primary school. She took the role of MaST in her 12th year of teaching after completing a Natural Sciences degree which included some mathematical learning such as engineering. This was her last mathematics qualification. She had undertaken Local Authority courses as her last mathematics professional development. She was the leader of learning as well as mathematics subject leader. During the two years MaST F moved to another primary school where she took the role of mathematics subject leader. MaST F had taught predominately from Years 4 to 6. In the two years of the programme she taught Year 4 and then moved to year 6.

In both interviews MaST F discussed her teaching of division. In her second interview she identifies new knowledge in terms of the curriculum and learning expected to take place before that of her current class. This knowledge impacted on her teaching. *'I have basically, I think a* lot of what I said in here holds true, what I taught year 4 is still the way I approach it.. I think what is different is I have a better appreciation of where the children have come from, or should *come from.. I remember saying this time that it seemed like a real struggle in year 4 and I don't* think I really knew what they were supposed to have done before they got to me, I mean I kind of did because I have seen the framework and I knew, and I was subject leader, but I think having been on the course I have a better idea of the kinds of experiences they should have had and *what kinds of understanding … and what that would mean if they don't, and the kind of gaps… I know I talked about gaps there… because I felt there were gaps but I think I know now more… why, if that makes sense… because if they had only ever done sharing…no wonder they struggle…' F2.*

'I think what's changed is my understanding…why children struggle ' F2.

In her reflection she identifies the two structures of division, and how this knowledge impacts on her teaching.

She identifies the need for visual and practical experiences and guides her staff towards this, differentiating between recording on white boards and practical experiences. Her position in Year 6 by the time of the second interview enabled her to consider progression throughout the school *'I think the confidence of being on the course and realising why children struggle has helped me intervene in other year groups' F2.* She reflects that this knowledge is not just knowing what other year groups should do, but understanding alternatives. *'I don't think it is* just the practical things for teaching each particular year group, yes you see more and you *would know the areas of weakness but you wouldn't necessarily know what to do about them or* how to approach them differently or how it could be tau*ght differently… the whole idea of teaching through problems and not just presenting children with sets of drills, you wouldn't* come to that on your own would you? Unless it was the culture of your school and you were lucky through your own training, but I think that actually doing the readings for the course, and *the training days, they give you a totally different perspective than just through teaching' F2.* In reflection on the teaching of chunking as a written form of division, she suggests that children should have access to a range of methods and their understanding was of fundamental importance. *'it is kind of controversial but I have come to think that is the actual method is less important than the children's understanding, and the mental is much mor*e important, so if they understand how to tackle it, then basically they will be able to do chunking, .. we gave them an old year 6 SATs paper recently and there was a long division question on it, it was like *Anghileri's research really, and the children* who understood had a range of different methods to get there, some used short division with big numbers, some used their own form of recording, *basically chunking, keeping track of, which they were able to do, but it wasn't the chunking they* were taught, w*e don't really do that here, and I think that is what is important, it is important* that they know what they are doing, and keep track of it somehow, not which formal method you do, so I can see us as a school not teaching chunking as a formal method but making sure they *understand division well enough so they can keep track and find an answer' F2.*

MaST F

Coded in second interview question 1	Coverage	Number of references
Context for mathematics	4.42 %	2
Games	0.25 %	1
Law or structure, basic mathematical idea	1.62 %	2
Progression tracking backwards, often from error	10.83 %	3
Rapid recall, basic skills	0.65 %	1
Using and applying mathematics: open ended investigations	0.85 %	1
Use of resources and images	12.66 %	5

MaST E

MaST E was a Key Stage 2 teacher in a primary school. She began the role of MaST in her third year of teaching, qualifying in 2007, three years previously, after completing a degree in Natural Sciences with Biology. This degree included a module relating to mathematics which was her last mathematics qualification and mathematics professional development. In the first interview, she was not the mathematics coordinator in her present school but was shortly to change schools where she would take this position. Before the programme she had taught years 4 and 5 and undertaken work as a supply teacher across the primary phase. During the programme she taught years 4 to 6, with most time in Years 4 and 5.

Interview 2 took place in MaST E's new school, in a staff room during the school day. I noticed that other members of staff entered and walked through this room during the interview, at which point MaST E hesitated and paused until the room was empty again. The interview was held when MaST E was relatively new in the school, being there one full term. I considered that she might be hesitant about others hearing the views she shared with me in the interview.

In both interviews MaST E chose to discuss the teaching of calculations. In the second interview she implied that she had undertaken the MaST training early in her career and that her views and knowledge had changed due to experience as well as the role of MaST. Changes were identified as relating to children's errors, *'I was very conscious of children making m*istakes and wanting them desperately to get everything right and actually now I am more inclined to let *them make mistakes and that is where the learning comes in'* E2. She talked about feeling 'relaxed' and 'freer,' and less 'traditional' in her approach.

MaST E identified specific knowledge from the MaST programme relating to the use of the array in the teaching of multiplication.

In her discussion she refers to her understanding of progression in each area of mathematics and how this structures her teaching. 'what I try to do early on in the year is go on to looking at arrays and we have perhaps an investigation at using arrays and getting the children to re familiarise themselves with them and from that I try to keep it more grown up as they are older c*hildren and I don't know want them to think oh I am going back to doing this again, so I try to* keep it more investigating using arrays and then actually take them the whole progression very quickly over the course of about a week, say Monday we will do arrays and then Tuesday then we will start adding some number and then Wednesday may be just number and then actually take them through the process, depending on the group obviously, but I find that helps them to *just to revisit what they have done before…'* E2. She reflects that this was new practice, *'No I'd* go straight in with this is what they are supposed to be doing, I will teach them what the curriculum says for this year group, and then coming unstuck and discovering the gaps in *children's knowledge an*d then having to go back to try and fill those gaps, whereas I find that by doing a recap, show me what you know, helps me understand where they are and helps them *to review what they know,' E2.*

*'I have adopted the approach of show me what you know first a*nd then we will take it from *there,… rather than this is what you should know so that is what I am teaching,' E2.*

'.. It was almost as if we had children peeling off, if you like, they got to a certain stage and then it became apparent, that was their ceiling, that was their limit, I am comfortable at this stage *but I am not sure what…you could tell when you had gone too far…and that was quite useful* because they peeled off and at the end of the week it was the more able ones that we were moving on

…. It was but it was longer term, it was vey, useful… and the more able children by the end of the week they were able to see where they were going next, I think the procedure and knowing

where you were going next was really important…there is an awful lot of .. vagueness… I know from when I started teaching I was quite vague in some aspects. If they know where they are going and they can see where they are going it makes a huge difference and with that we sort of introduced it as this is ... we are starting with multiplication, I know that a lot of you know what *you are doing, but let's just go back to the beginning and have a look at it and unpick it and* presented it to them in that way, it made it less, oh we are doing this again, we have done this, I *didn't hav*e any of that, it was a case of just go back and have a look and move it on, and this is where we are going to end up but we are going to take it from there to there and this is our *route and we are going to have this journey together… E2*

This knowledge changes how she starts a topic, 'We all do a show me lesson at the start of a *unit which is very much putting the ball in the children's court, show me what you know, and* the planning is based on that and we have been trying to make the children more responsible for *their learning….' E2.*

MaST E reflects that this knowledge has arisen as MaST, and would not have been acquired from teaching other year groups. *'I think the difference would be, for me anyway.. the specific* focus on different areas, it is actually made me unpick exactly what I am doing, but if someone *had taught year 1, year 2 they wouldn't necessarily have gone through the process of unpicking what they are doing, they just pick up the curriculum and off we go' E2*

Her teaching approach drew on a variety of contexts for mathematics, 'fences and drain covers *and things like that' E2.* In some cases this is where she has a particular learning objective in mind such as the teaching of the array and in other cases she uses examples chosen by the children for unplanned mathematical learning 'We had photographs of different things around the school and different things around the area and we found these wonderful photos of... fences and drain covers and things like that.. and we said to the children, what is it, what can you see, *where would you find it and left it very open for them to say what they can see and …that went* quite well, and we also did a find me exercise when we were out in the playground, we said find me something that has got an array on it, ..find me something with parallel lines, find me *something.. and they absolutely loved that…*' E2.

MaST E

Coded in second interview question 1	Coverage	Number of references
Context for mathematics	5.63 %	3
Mathematics in a non-mathematical context	2.75 %	1
Meeting children's individual needs, not progression	2.65 %	1
Progression tracking backwards, often from error	8.98 %	5
Progression tracking forwards	8.53 %	5
Rapid recall, basic skills	3.48 %	1
Reference to theory, literature, research findings, assignments	5.35 %	2
Talk	0.92 %	1
Using and applying mathematics: open ended investigations	0.23 %	1
Unique to my school	3.72 %	1
Use of ICT resource	1.66 %	1
Use of resources and images	7.01 %	4

MaST D

MaST D was a teacher at a special school who had taught for 13 years before taking the role as MaST, qualifying in 1997. She had completed a mathematics degree which was her last mathematics qualification. She had led mathematics professional development for a group of schools and had attended special school subject leader courses. She was the mathematics coordinator in the primary phase of her school. During the programme she had taught years 1 and 2, and then took a mixed years 3, 4, 5 and 6 class of children with severe needs relating to ASD.

MaST D chose to discuss using and applying mathematics. In the second interview she discussed how she was supporting staff in teaching using and applying mathematics and data handling together. She reported that staff did not seem to think there was enough time spent on teaching data handling but that she interpreted data handling in a different way 'everything we *do, using symbols and having signs round our classroom is data handling…I tried to show them* all that they are covering it, they c*an evidence it, it doesn't have to be on a lesson plan' D2*. These are recurring themes in MaST D's second interview. Firstly she is able to link areas of mathematics together. She attributes her ability to do this as stemming from knowledge and understanding of mathematics, *'if you don't understand maths as a subject you can't see the links between the different areas' D2.*

A second theme is her ability to identify mathematics in a range of contexts. *'We are* introducing topics now so I have taken two topics and done a plan encompassing data handling *and using and applying for, I think it is animals and transport' D2.*
'They couldn't see the links with everyday classroom stuff and data handling and using and applying' D2.
This involves her with not only finding a context for a particular mathematical idea but also being able to identify mathematical learning opportunities in a context, 'I think it is being able to unpick the maths in something and look at every little aspect of what they are doing there and b*e able to find something..' D2.*

Another theme is her ability to identify the key mathematical ideas which she wants children to learn, and distinguish them from ideas on the periphery. She does this in her discussion of counting, data handling and time.
'Just simple things like one to one correspondence… trying to explain to my TAs what one to one correspondence actually means, so I say to them for children it is really quite hard because they have to touch one thing each time, so straight away you are b*ringing a number into it…* and they need to slow down, it is more than doing that is that, and that is that and that is that, if *we spend the whole time going count count count they don't understand why… and things like* changing the order and making it a wo*bbly line and a circle and randomly across the table…'* D2.

'I had another one, another person who wanted to, she was doing a pictogram, and she wanted.. they couldn't associate a cat with a picture of a cat, so why don't you use animal counters, and do a 3D graph, and when you want to record it, either photograph it or *photograph the counters, and then use those, but don't get them to take them all off because they will think they are doing again, just get them to swap,… it is just little simple things lik*e *that, she hadn't looked at their development level and worked out.. and what I tend to say if* someone is P4, I say if you think about that in normal child development that child would be about 2 and a half, so if you think about your child at 2 and a half, could he have done a pictogram, no, but he could have told you how many animals there were, and if you asked him to put the cats in a line he would have done, and if you had asked him to put the dogs in a line he would have done and he would have said oh look that is higher than that, but you are *introducing all those grid lines and he doesn't need it, if you read a pictogram it is to read information from, it doesn't matter if it is not, the lines aren't all straight, as long as the* animals are at the s*ame level and the same height, it didn't matter, and it is that sort of thing, to have the confidence to break away from the pictogram has lines and numbers down the side…* The most important thing is reading off the information, it is not even about the making the pictograms, that is just a nice task to do, you want them to be able to tell you how many more, cats than dogs, it is information finding, not just a pretty picture, it is information finding,.. I think it is not having the understanding to know.. *maybe they didn't know why they were doing it either,.. it is just one of those things you do, you teach pictograms, you teach whatever…' D2.* *'I had someone come and ask me because she's got time to do next term and she's got P4 and it doesn't say anything about hours until P8 but I said no but you've got a world of language there… you can do all the nice stuff, you can still do the nice making of clocks and stuff because they have still got to know that a clock is a time piece… so I gave her the whole o*f the using and *applying part of that curriculum… and said just look through and see if you can find anything* which is broadly related to time and I said if you think about it even using the word next is time.. like they all thought that all data handling is graphs and pictograms everyone thinks time *is clocks but it is not, is it?' D2.* This knowledge changes the way she structures activities*, 'I*

wanted them to do their own recording because we tend to structure their recording all the *time… we give them* things to stick on and whatever... it takes away.. they have to worry about *glue…and those who don't like glue on their fingers… so all I did was get some coloured* counters, and say sort them into colours and tell me how many and then draw them on your pap*er to say how many are the same… and my TAs said why are we doing that… and I said they* are doing so much in that, they are sorting, they are counting, they are recording, they are counting, they have got one to one correspondence, they can tell you how ma*ny they have got… there is so much in there… but you could have given them say yellow circles to stick on, which takes them longer…takes away from the maths.. and they did that may be three or four times in ten minutes… it just simplified it..' D2.*

Her language of progression suggests a ladder *'I do seem to be able unpick it and go back down and back up…' D2.*

She reflects that the role and training of MaST has provided knowledge of the curriculum above that which she usually teaches and this has helped her to accelerate learning, *'what really helped* me in the MaST course was doing the assignment and having to really think about something a *lot.. and it kind of focuses your mind a bit and it was…Oh I didn't think about that,… and for* me, the afternoons and *the days were good but they were always a bit above what I needed…* but it was good because it keeps you in track, you lose track of normality of what a normal eleven year old does and from that I think I pushed on a couple of my kids more than I would have *done because I wasn't scared to, because I knew I could step back.. and if they had gaps in* their learning and something was missing, I knew I could step back and work out what it was *they hadn't learn how to do' D2*. However the knowledge she uses most is of how to address gaps and slow down learning, 'and you tend to start breaking things down more and more in *your head.. I've noticed a child in my class, we are trying to get him to learn his number bonds* to ten, and he is so ingrained with one two three *four…., and I've noticed my TAs are trying to* rush him on to the next step too quick and said no, we need to get that secure, and even things like knowing to turn the numicon tile over to make it fit, and no we to go slowly with him, it is a whole new con*cept for him, and if he gets the pictures in his head, he'll start to work out,, he can count to ten but I don't think he knows what 10 is..' D2.*

MaST D differentiates between needs in her school, *'if I do a P5child, and I have got P5 in my* room but my chi*ldren have got ASD and their minds work very differently… usually I try to* think of a tactile idea and a non tactile idea.. for different needs some children need the *systematic work. Some need the multi sensory stuff more' D2. 'Like a lot of ASD children* are not very good at orally telling you what they are doing, but they can do it, and the children with *cerebral palsy aren't very good at counting on their fingers but they can say on the counting stick' D2.*

MaST D

Coded in second interview question 1	Coverage	Number of references
Connecting similar mathematical ideas, horizontal connections	4.27 %	6
Context for mathematics	3.37 %	3
Key ideas and those on periphery	17.44 %	3
Law or structure, basic mathematical idea	6.24 %	3
Mathematics in a non-mathematical context	3.32 %	3
Meeting children's individual needs, not progression	4.48 %	3
Progression tracking backwards, often from error	6.68 %	5
Progression tracking forwards	5.13 %	4
Reference to theory, literature, research findings, assignments	2.41 %	2
Use of resources and images	19.14 %	5

MaST C

MaST C was Mathematics coordinator and Key Stage 2 teacher in a primary school. She had four years' teaching experience as she took the role of MaST, qualifying in 2006 with a BA Ed degree in primary education and geography with QTS. Her last mathematics qualification had been her A level. She had attended professional development courses such as the Local

Authority subject leader courses. During the programme she taught Years 5 and 2 and then moved to Year 6 where she taught top set for mathematics.

MaST C chose to discuss her teaching of division. Over the two years she had reflected on how to ensure children in her school understood division as both grouping and sharing and how she could successfully support progression to understanding and proficiency in the written methods. She articulated the different structures of division and how they linked to the written method. She had worked with colleagues in modelling the chunking procedure practically and recording thinking on a number line, refining her whole school calculation guidance over the two years. She demonstrated application of her knowledge of progression in her teaching of her own class, 'I have now got top set year 6 so my target audience that I am working with is completely different to what I had before, so with my years 6s we have talked about division that if it is dividing by tens, TU and HTU divided by units then they are more than happy with bus stop *method and with our calculation policy that is the final stage if they are confident and I've got* 12 levels 5s and I am hoping to put in 2 to the level 6 paper, so they are really confident with the understanding, and then obviously when it comes to division by TU they are using chunking *because they don't know their 24 times tables, but even with the confident top year 6 set they* still *didn't understand chunking at the start of the year…it is really interesting that they understood the quick fix bus stop method but they didn't understand the chunking so I had to* work a lot on division facts so that if I know that 20 divided by 2 is 10 then 200 divided by 2 is *100 so I had to work on a lot on that then I had to go on to decimals so if it was I don't know 3.6* divided by 6 is 0.6 that actually they could just do 36 divided by 6 and then manipulate the answer and so that is the level I am a*t with my years 6s' C2.*

As she reflects on her teaching of division there are numerous examples of her knowledge of connections and links, 'manipulating what 10 times, 20 times and 30 times and 40 times is, so I have had to do a lot of work on times tables and linking it to if we know that number fact, so if *we know that, what other number facts…' C2. 'in the mental maths test when it says write 7* *tenths as a decimal they couldn't do that and I said that is really easy it is just like writing 7 units and I had to break it down what all the columns stood for and then link that to their division' C2.* Most of these links are hierarchical, in that they move between increasingly complex mathematical ideas or procedures.

She sums this approach up as 'I think it is kind of unpicking where they were and then *narrowing the gaps' C2.* Her aim is understanding *'I think the problems with that is with division, they didn't fully understand why they were dividing, they knew they had to take away 10 lots of something but they didn't really understand why and if they did understand why they got confused with the subtracting….'C2.*

She identifies that knowledge of progression has come from both the MaST training and her experience of different year groups. *'But I think before the maths specialist course I am not sure I really understood, because to be honest I started teaching in year 5, didn't fully understand where the children had come from, I knew I had to teach them this chunking method, and if it killed me I was going to teach them, take away ten times, take away ten times and I think going down to year 2 and then going back to year 6 has made me more aware of the stages of learning… so in order to help, to be able to help those year 3 teachers I can only do that because I taught it in year 2, now I know where they are going in year 6 so I can see the many stepping stones but, maybe when I was in year 5 having not taught lower down, I didn't may be fully appreciate why the children had those gaps and why there was misunderstanding' C2.*

'I think may be just from the MaST course really unpicked where the transition, like I've never really thought before' C2.

'I'd like to think a maths specialist would be able to … fully understand what happens from the moment they enter school to when they go to secondary school.. so to be able to identify , look at a group of children or a class and identify where their gaps are and how to move them forward, by understanding the progression, but not having necessarily taught all those year groups …'C2.

MaST C provides several examples of her ability to take an example of learning and apply her knowledge of how this learning might progress. She uses this knowledge to support her colleagues.

*'We talked about this child, I said what about in your water are*a could you give them some *containers that you couldn't fill from one to the other and then she started to say well the other day there was this child who was trying to fill up the watering can from the tap but he couldn't* get the watering can to fit underneath the tap and he struggled and struggled and I watched him and in the end he flooded the whole area and I told him off! So I said could you suggest or say to him to go and get another container which could fit under the tap which that would have been t*o do with shape and space.. that would have been perfect… but she just got really cross and he just kept turning the tap further up and she said she didn't have the patience, and I think* that is what is hard in foundation, how do you get them to use that initiative without telling them *or getting cross…I know what in year 1 and 2 what they are going to need to know, so I know* the skills they are going need to have and she thinks that is just flooding the classroom but actually when they are in year 1 and 2 when they are looking at capacity if he had gone to get another container which was smaller, he would have had some grasp of smaller and larger *instead of being told to turn off the tap' C2.*

There are also several examples of MaST C's application of her knowledge of progression where she identifies a problem or gap and tracks back to curriculum and learning which should have been secure previously, *'and now, even with my year 6s the top set,.. like in fractions I* mentioned fractions and they all said oh n*o we don't want to do it and I said right we are going* to do a quick ten minute practice and I said right, in year 2 this is what you would have done and in year 3 this is what you would have done and along the line I managed to pick them all up and now they can do representing the quotient as a fraction or a decimal so somewhere along *the line I clicked where they were and where they had some misconceptions,… and maybe where you have got teachers… have weaknesses in year 3 and 4 they don't know what they cover in year 1 and 2 and they probably don't have an inclination to find out and they are too* *scared to find about year 5and 6 and they are just in their little bubble and they don't understand the full picture' C2.*

*'I think in year 6 constantly I am con*stantly doing test based questions and constantly saying, for example we did a sats paper before Christmas and none of my children could do the shape with the coordinates and the parallelogram and it had two points and they had to identify what the missing one was and none of my group could do that so I had to go.. this week I have built up.. so the lady in my class today could do that question, so I had to go right back to Ok what are the properties of quadrilaterals, we had to a lesson on parallelograms, looking at acute and *obtuse angles, then we had to a lesson on, then I built on well, actually they didn't know how to* use a protractor so we had to do that when we was there, and then we did a thing actually talking about what is an angle, turns, they di*dn't know that, then we talked about parallel and* perpendicular lines and how we could test what a parallel and perpendicular lines look like and then investigate them round the classroom and then we had to do coordinates to build up to today so they could recognise that is a parallelogram and those angles are the same so therefore those lengths must be the same and so that is where the point is, so that is a whole *week's worth of work to build up to do that one question..*

…. It took quite a long time to plan actually, because I was thinking well they can't do that, so they can't do that, and they didn't really know what a quadrilateral was so they didn't know what a parallelogram was .. and it took me a week, well obviously they came up with other things.. *they couldn't estimate angles,.. they didn't know that in that parallelogram there was* going to be two acutes and two obtuses, and yet they were level 4 but there were so many gaps in their understanding for them to be able to answer one question because of what had gone *before,.. and I think in year 6 teaching like that… me and my partner, we have to do that all the* time in order to achieve one thing, those little mini steps have got to be in place

*… And I don't think I would have done that if I hadn't d*one the maths specialist course, I think I would have gone, oh come on guys, that line is the same as that line,

…Well I would never have understood, they didn't even know what a parallelogram was, if I didn't understand all of that shape and space topic, I would have just have said, well they don't know coordinates'C2.

This quote includes her ability to trace progression back to what I argue is a basic idea, that angle is a measurement of turn, although in this case it could be seen to be a static example of angle.

MaST C's knowledge of progression leads her to question the appropriateness of expecting children's learning to be at the required level for their year groups. *'If had we only taught in year 4 and 5, …I didn't really care where the children ha*d come from, I just knew what I had to *get them to,' C2.*

'and maybe where you have got teachers…have weaknesses in year 3 and 4 they don't know what they cover in year 1 and 2 and they probably don't have an inclination to find out and they are too scared *to find about year 5and 6 and they are just in their little bubble and they don't understand the full picture' C2*

MaST C reflects on the need to explore mathematical ideas in real contexts in her teaching. She also discusses how she supports her reception teacher in identifying mathematical learning as it happens in contexts which are unplanned and not primarily mathematical: 'she laid out all her maths and I could say actually that child is doing maths but you might not realise that they are I can help he*r to identify what she is already doing' C2*. The example of her reflections on the water activity is another example of MaST C's awareness of mathematical learning in a context.

MaST C

Coded in second interview question 1	Coverage	Number of references
Connecting similar mathematical ideas, horizontal connections	3.06 %	3
Context for mathematics	5.08 %	1

Law or structure, basic mathematical idea	3.01 %	3
Mathematics in a non-mathematical context	4.80 %	2
Mathematics in play	5.39 %	2
Progression tracking backwards, often from error	27.72 %	9
Progression tracking forwards	14.04 %	8
Rapid recall, basic skills	0.84 %	2
Reference to theory, literature, research findings, assignments	3.45 %	3
Using and Applying mathematics: open ended investigations	2.35 %	1
Use of resources and images	7.70 %	4
Vocabulary	2.73 %	1
Key idea and those on the periphery	0.84%	1

MaST B

MaST B was a Key Stage 1 primary school teacher who had been teaching for seven years as she took the role of MaST. She had experience of teaching years 3, 5, 1 and 2 after qualifying in 2003. She had gained a degree in geography and tourism. Her last qualification in mathematics was her GCSE. She had attended local authority subject leader courses and mathematics courses. She took the role of mathematics and geography coordinator, head of Key Stage 1 and NQT mentor. By the second interview MaST B was acting deputy head of the school, as well as Early Years and Key Stage 1 leader. She was by then teaching Year 2 and had taught in Key Stage 1 throughout the two year programme.

In both interviews MaST B discussed her teaching of division.

In the second interview she identifies new knowledge in terms of progression. 'My approach has changed in knowing where they have come from and where they are going to, so probably *in that respect' B2.* This knowledge of progression and the learning usually taking place above and below her class changes her teaching of her class '*I suppose because I've looked at progression from R to year 7… I've got children in my class this year for example who are* working at foundation stage level and so when we did division it was particularly important that they had those hands on experiences, practical, in those early stages of sharing, and may be the bottoms could be looking at where they have come from in year 1, so they have probably progressed to sharing, which is the first thing, I am aware they do a lot of sharing in year 1 and it moves on to grouping in year 2 and by the end of year 2 I have got some children doing chunking already which is what we get them to think about before they move on to the bus stop *method which they move on to at the end of it' B2.*

She uses this knowledge to support other staff in year groups higher than hers, 'sometimes the problems which children have.. can be tracked back to possibly gaps in knowledge which they *might not have picked up further back in their learning,' B2.*

'I know we had a discussion with a year 5 teacher last week and she has some children who are finding the very early concept of counting on quite tricky so we talked about simple things she could do, even simple things like games of snakes and ladders and the concept of counting on *and counting back and she said I haven't even thought about those sort* of things for a maths *lesson.. for her it hadn't even occurred to her that was maths learning but just a free time playing and some children need to have experience of physically counting and recognising.. '*

B2

In particular this new knowledge helps her to extend the more able children, 'I think sometimes I might have possibly shied away from vocabulary which I think is too advanced for them and *actually I am underestimating them..' B2.*

'I've been more brave to get opportunities for maths in other areas,'B2.

'I think I am braver in my teaching because I am more confident, I do know it, I do know where they are going so if it is appropriate than it is OK to do it, before I was more tentative, maybe I should make sure they are really secure but not something else, but that is not really good teaching in that respect because if they are ready to do it than they should move on and do it, I am probably braver and more creative in my teaching of division, I am not basing on it on paper and pencil methods which I think a lot of teachers, when they think division, they go *straight to paper and pencil.. we try to do in real life scenarios and situations'B2*

The word ‘brave’ is indicative of emotive language used to describe changes in her approach. She also talks about how ‘*I’m not so scared now to do problems solving with division as well because it doesn’t have to be’ B2*.

MaST B also uses figurative language relating to travelling and cementing *‘if we have talked* about the vocabulary enough and they have got an understanding and they can use it in the *right context there is no reason why we shouldn’t use the correct vocabulary, because otherwise* another word might pop up and they might not have an understanding of it and if you can get it sort of cemented now, it sor*t of travels with them as they go through the school’ B2.*
‘we would have done it the day before but we do it in geography so it cements it, putting it in meaningful contexts and it has had a big impact.’B2.

She emphasises the importance in her teaching of using practical activities and relating mathematical ideas to real contexts *‘we probably try to do more real life experiences as oppose to going straight to doing pencil and paper ways’ B2.*
*‘We are doing remainders, to try to make understanding for the ch*ildren I gave each table a packet of smarties, they have got to make sure everybody has the same amount and it was really interesting to watch them give them out and there were some left on the table and there was a discussion, what shall we do with them, is it fair to have them what do you do with them.. the discussion was they had to remain away from everyone else because they have not been shared equally, divided between the group of people and that was really interesting and it was quite a good model, an image for them, to understand the actual concept of a remainder because it can *be …abstract in many ways’ B2*

MaST B talks of how new knowledge as MaST has enabled her to provide more open ended tasks for her class, ‘.. *it can be quite open ended and I’m* trying to get them to think about *division… there was a question or a challenge I’ve given when you’ve got a certain amount and* you can find different ways to share equally, like the smarties but it was more open ended and they have to find all the differ*ent .. and before I wouldn't have tried it with year 2 because I didn't think their understanding was there but they seem to be better because they've had more hands on experiences, to move it off to the more abstract, I don't know if that's…'B2*

MaST B distinguishes between the grouping and sharing structures of division in interview 2, but not interview 1, and identifies this as new knowledge for the MaST training 'we do lots of *grouping and sharing and halving….maybe I've got more of an understanding, be*cause I wrote my essay on it, the grouping and sharing and the difference between them, and how it is *important that you distinguish the difference between them to the children,'B2.*

She also discusses the connections she makes in her teaching of division, implying this is also new knowledge *'when I am teaching division I do more relate it to multiplication as well, the understanding that you do need to have… an understanding of multiplication to be able to use* division and vice versa because they are connected, and that has helped with our big maths because we do something called fact families where the children have one fact and they have to manipulate the numbers round in different ways, and that the children have to have an understanding that those two *are quite closely linked,' B2.*

MaST B also talks about an increased focus on mental calculation and rapid recall. This seems to have increased her expectations of her children, '*again I don't think we would have done that if I hadn't studied it in detail* first of all and had the opportunity of doing big maths and thinking *that is appropriate for year 2 and I might have said actually I don't think it is' B2.*

MaST B

Coded in second interview question 1	Coverage	Number of references
Change in differentiation	0.34 %	1

Connecting similar mathematical ideas, horizontal connections	3.16 %	2
Context for mathematics	6.33 %	3
Law or structure, basic mathematical idea	3.90 %	2
Mathematics in play	2.29 %	1
Progression tracking backwards, often from error	10.46 %	5
Progression tracking forwards	16.23 %	8
Rapid recall, basic skills	1.66 %	2
Reference to theory, literature, research findings, assignments	12.14 %	7
Using and applying mathematics: open ended investigations	6.76 %	4
Unique to my school	1.10 %	1
Use of resources and images	10.51 %	7
Vocabulary	5.62 %	3

MaST A

MaST A was a Key Stage 2 teacher in a primary school. She had been teaching for 12 years when she took the role of MaST, after qualifying in 1997. She had completed a B Ed with a specialism in Physical Education. Her last mathematics qualification was her GCSE. She had attended courses relating to her role as subject leader but these tended to cover management rather than mathematics. She was the school's mathematics coordinator. During the programme she taught mathematics to Years 5 and 6 in sets.

MaST A chose to discuss the teaching of multiplication in both interviews. She reflected on a whole school approach which she had implemented. This involved whole school assemblies and championships with the reciting of table facts. She had implemented a whole school daily counting session. She also had provided written guidance for staff, 'we had to tighten up our calculation policy it was a bit loose and there were issues with some teachers not using correct *strategies' A2.*

'I think we've slowed it down, there were year 4 teachers and they were going to teach year 4 objectives and they didn't have the basic skills, so we had to slow it down, until they can do this they are not *going to do the next' A2.*

MaST A approached multiplication as connected to counting. She articulated progression from extended counting to the written method for multiplication 'lots of counting stick activities, like *if you've done the sevens, you do the* seventies, and the 0.7s and then using that to work the *grid… we're using grid and then just at year 6 if they are up for it then they use the shorter..'A2.*

'We're looking at the next level at using it for division facts and bigger numbers, mental partiti*oning, if you can do 7 you can do 17..' A2*

She identifies that she has acquired new knowledge particularly in the earlier curriculum, 'I *would say the MaST has helped enormously with that because… Probably wrongly the* perception of junior teachers is that *it is counting and colouring in and playing with stuff… but* having done the course now and spent time with early year foundation specialists and key stage 1 teachers, you see actually what goes on at that level, and the impact it has coming up through the *school.. I think from year 1 up I would have been confident before but now I think if I don't know the answers I am better equipped to find them' A2.*

MaST A had adopted a whole school approach based largely on errors she had identified in her teaching of the oldest children. This had caused her to consider progression in her school. '*Counting has been highlighted as an issue, counting backward particularly, when we've done simple subtractions, children are just flummoxed, questions like… every week we do a* basic *skills test and the level 3 one which I'm doing with my year 5s every week it starts with a* question like what is 4 less than 401 or what is 5 less than 302, it is something close to the hundred, and the strategies the children were using or trying to use to solve that and how far wrong they were getting it sort of highlighted to me that there was a lack of counting, so having done a bit more looking into it, the current year 5s are the last group who have missed that good teaching but the counting stuff which has been happening in the last few years seems to be *having an impact, the basic skills it seems…because my second assignment was on subtraction and just simple things, counting backwards or counting on to find the difference… and we were* finding from year 5 it was a struggle but coming through was a real strength, so the impact of *the counting focus is working, or has worked…' A2.*

MaST A discusses her approach to key mathematical ideas of the commutative law and identified ICT as a useful model for this, '*we've got ICT packages its multiplying monkeys* when you put them into sets and you look at 3 lots of 4 and 4 lots of 3 and you can look at the *relationships' A2*.

The package also used the array as a model and MaST A had considered this ' Yes, that was my *first assignment, the array…we looked at that at our level, but we didn't sort of translate that into practice in the school' A2.*

MaST A refers to specific authors she had read which had supported her thinking and the findings from research studies on setting.

She also reflects on the individual needs of her school and the way she has supported staff to meet them, '*With the children here, I know this isn't typical,.. our children, their brain* pathways I see like a jungle, if you break them down regularly the path remains open, if you *don't do it for a term it is forgotten and you are back to square one, so we have sort of evolved our curriculum to a place where every week we'll do one day on the four operations and we'll* perhaps just plate spin for the three of them and then look at something new in division say and then come back to it the next week and I think too long not doing things is too easily forgotten in *our school….it is trying to keep all those paths open and everything ticking at the same time'* A2.

MaST A

Coded in second interview question 1	Coverage	Number of references
Connecting similar mathematical ideas, horizontal connections	2.04 %	3

Law or structure, basic mathematical idea	1.84 %	2
Learning from other MaSTs during the programme and beyond	2.14 %	1
Progression tracking backwards, often from error	7.12 %	1
Progression tracking forwards	6.86 %	3
Rapid recall, basic skills	4.17 %	2
Reference to theory, literature, research findings, assignments	6.47 %	3
Use of ICT resource	2.24 %	2
Use of resources and images	3.10 %	2
Whole school approach, unique to my school	2.28 %	2

Appendix vi Coding for question 2

The following table shows the codes resulting from my analysis of responses to question 2 and explains how I decided the content would be categorised.

Name of code	Explanatory notes and examples
Understanding	This code was used when MaSTs referred to either the importance of their own or children's full understanding of mathematical ideas. One MaST referred to the definition of mathematical understanding by Skemp (1989). Some MaSTs make general comments about good mathematical understanding being a feature of deep subject knowledge or about children's secure understanding being a consequence of deep subject knowledge. The MaSTs also provide examples where they feel that the role

	has increased their understanding. This is often in relation to their increased understanding of other year groups, which is also coded under progression. MaST K states in the second interview, *'I have got an understanding of the big sweep of it.'*
Progression	This code was used when the MaSTs made reference to their deep subject knowledge as their understanding of progression of mathematics and mathematical learning across the primary phase. The MaSTs often referred to their increased understanding of previously unfamiliar areas of the primary phase, and the ability to trace progression throughout. '*I'd like to think a maths specialist would be able to … fully* understand what happens from the moment they enter school to when they go to secondary school.. so to be able to identify , look at a group of children or a class and identify where their gaps are and how to move them forward, by understanding the progression, but not having necessarily *taught all those year groups.'* C2 The code also included examples of MaSTs discussing their knowledge of the sequence or timing of teaching, *'the progression, really understanding* the whole range of that subject.. if you are talking about calculation really understanding how it starts off and builds up and leads onto, and I think that is having a deep *subject knowledge of maths, the whole range of it....'* H2 This code was also used when MaSTs linked basic ideas to more complex ones which would occur later in children's learning. For example MaST I discusses the significance of her ability to link pattern to later learning of algebra. This feature of deep subject knowledge included MaSTs' comments on their ability to consider the use of resources eg I2 'the number counting

	stick isn't just one to a hundred it is one or whatever number you want it to be, *and the bead strings aren't necessarily one to a hundred it could be zero to one, or whatever you want it to be'*
Identifying gaps, misconceptions	This code was used for responses which referred to deep subject knowledge as enabling MaSTs to identify gaps and misconceptions in children's learning, and also in their colleagues' knowledge. *'I think with deep subject* knowledge it is knowing where there might be possible barriers to learning along the way, because if you really understand what the subject is about, *then you'd probably know where misconceptions can come and issues can come up'* H2
Using and applying mathematics: open ended investigations	This code included any reference MaSTs made to open ended investigation
Assessment for learning, start with the children	This code was used when MaSTs referred to deep subject knowledge as drawing on assessment for learning strategies. 'I think I have taken the ceilings out more, I think and I always had the groups, but I did think what is this group doing next, and this group doing next, not necessarily, and I *don't know if some of those children could have gone further, and I am* really listening to them and not being afraid to move the groups around, *they are used to me saying, actually we won't…., because the*y do go in fits *and starts' J2*
Unique to my school	In some cases MaSTs referred to their deep subject knowledge as including understanding of some aspect which was unique to their school.
Knowing more advanced maths	Although related to the progression code, this code was used for examples when MaSTs referred to their deep subject knowledge as including specific

eg Key Stage 3	areas of the Key Stage 3 curriculum.
Theory and research	Several MaSTs referred to deep subject knowledge as knowledge of theoretical ideas and research findings and they often named these specifically. Some discussed international comparisons and one MaST summed up by saying 'because in our MaST role we have to tell staff or ask staff or explain things to staff I am able to *explain it … a lot more* confidently because I am able to give some reasoning and theory and *background to it whereas before I wouldn't have been able to stand up in a* staff meeting and say this is what we are doing, and the theory and understanding behind *it maybe… in that respect maybe my subject knowledge is deeper because am able to justify …some approaches, maybe'* B2
Knowing why	This code was used when MaSTs explained that their deep subject knowledge was concerned with knowing why for example errors occur, or why we can calculate the nine times table on our fingers. MaSTs explained that this was important in their role in supporting other colleagues' practice.
Explain in different ways, different approaches	MaSTs often referred to deep subject knowledge as a repertoire of ways to explain or approaches to take to areas of mathematics. '*I think, OK let's take this apart and let's try to put it is your speak so you can access it'* J2
Use of resources	This code was used to examples where MaSTs discussed their deep subject knowledge as relating to the use of resources, a point made most about older children
Pedagogy	When MaSTs discussed pedagogy by name this code was used. Although other codes discuss area which are part of pedagogy, eg use of resources and identification of misconceptions, this code allowed me to see where the MaSTs were formally distinguishing between subject and pedagogical

	knowledge and recognising that their knowledge was not purely mathematical. *'I think it was as much the pedagogi*cal knowledge, the way, how children learn maths and how best to present it to them, I think that is *what I most value'* F2
Mathematics in play	One MaST acknowledged knowledge of how mathematics is learned in play as part of deep subject knowledge.
Key idea or periphery	One MaST identified examples of the need to be able to pin point the key mathematical idea of a lesson and those ideas which were peripheral as part of deep subject knowledge. This was the MaST from the special school. For example 'she was *doing a pictogram, and she wanted.. they couldn't associate a cat with a picture of a cat, so why don't you use animal* counters, and do a 3D graph, and when you want to record it, either photograph it or photograph the counters, and then use those, but don*'t get* them to take them all off because they will think they are doing again, just *get them to swap,… it is just little simple things like that, she hadn't looked* at their development level and worked out.. and what I tend to say if someone is P4, I say if you think about that in normal child development that child would be about 2 and a half, so if you think about your child at 2 and a half, could he have done a pictogram, no, but he could have told you how many animals there were, and if you asked him to put the cats in a line he would have done, and if you had asked him to put the dogs in a line he would have done and he would have said oh look that is higher than that, *but you are introducing all those grid lines and he doesn't need it, if you* read a pi*ctogram it is to read information from, it doesn't matter if it is not, the lines aren't all straight, as long as the animals are at the same level and the same height, it didn't matter, and it is that sort of thing, to have the* confidence to break away from the pictogram has lines and numbers down

	the side' D2
Making connections between similar mathematical ideas, horizontal connections	MaSTs identified the ability to make links between mathematical ideas as part of deep subject knowledge. '*if you don't understand maths as a subject you can't see the links between the different areas…To me deep subject knowledge is knowing the links* between all the different areas without having to think about it too hard.. so knowing the links between using and applying *and data handling… or* counting and understanding, or shape and space and using and data *handling'* D2
Identify mathematics in a non-mathematical context	This code was used when the MaSTs referred to the ability to look at any context and identify the opportunity for specific mathematical learning.
Identify context for maths	This code was used when MaSTs referred to deep subject knowledge as including their knowledge of how to use a different area of the curriculum or real life context to explore a particular mathematics idea.
Discussion of structures and laws	One MaST made a reference to the way her knowledge of the different structures of subtraction allowed her to support colleagues in the teaching of the operation.
Extend able, support less able	This code was used when MaSTs referred to deep subject knowledge as supporting them in their understanding of the teaching of less and more able children.
Knowing how and when to access support	Several MaST identified the knowledge of how to find out information as significant
Flexibility	One MaST identified flexibility as part of deep subject knowledge

Challenge others	One MaST identified the ability to challenge colleagues as part of deep subject knowledge
Patterns	This code was used when MaSTs identified the significance of patterns in mathematics and the ability to recognise this as part of deep subject knowledge.
Passion	This code was used when MaSTs recognised a passion for mathematics as part of deep subject knowledge.

Appendix vii Percentages of responses not coded

I considered it important to examine the parts of the interviews which were not coded. For example, I considered how much of the responses to question 1 were not coded, as a percentage of the whole of the second interview. The percentage included the un coded words of the MaST, the whole interview included my words. These varied from 0.04% of the interview to 19.46. N Vivo allowed me to see easily the phrases which had not been coded and allowed me to check that there was nothing significant I had missed. The un coded sections related to discussions of other areas not related to the question, the teaching of other subjects or school issues which were either confidential or not connected to mathematics.

MaST	Percentage of the whole interview un coded
A	8.53%
B	1.18%
C	3.04%
D	13.5%
E	9.73%
F	8.39%
G	1.52%
H	3.78%
I	19.46%
J	5.6%
K	0.04%

Appendix viii Final validating email

Dear …..

You very kindly offered to help me with my doctoral research, which I have now almost completed. If you remember I was seeking to explore the knowledge you developed in your role as MaST, and the meaning of deep subject knowledge. I interviewed you twice and asked you to discuss your approaches to teaching one area of the maths curriculum, and to help me to understand what deep subject knowledge really means to you.

I have collated and analysed all the interviews, twenty two in total, and have drawn the following conclusions. I promised that I would share these with you.

I would be very grateful if you could let me know if these conclusions make sense to you and if you have any comments? As usual, your views are incredibly important to me in this research. Just a brief email would be very much appreciated.

Conclusions:

When you all discussed your teaching of your chosen area of mathematics, there were no uniform approaches. This suggests that is no 'MaST way of teaching'. However there were some common themes.

The most common was to do with progression. It seems that you all were concerned about progression in the first interview, and this was sometimes voiced in terms of, for example, progression through National Curriculum levels. During the programme you seemed to increase your understanding of the whole primary phase including year groups you had never taught before. By the second interview you responded differently to progression, talking more about new knowledge to address progression across your schools based on analysis of children's learning and of the mathematical ideas in the primary curriculum.

You appeared to be using this knowledge to advise other members of staff in terms of progression. You seemed to be more confident in supporting teachers and Teaching Assistants who were teaching year groups where you had little or no experience. Sometimes you used your experience with your own classes to help you to do this. For example when you had taught younger children you might have promoted practices with older children based on what had been effective in your classroom. When you taught older children and had repeatedly found a difficulty or gap in their understanding you promoted changes across your school to ensure secure learning in this area with younger children.

I also found that you used this knowledge of progression across the primary phase to change your own teaching in your classroom. There were examples of changes in classroom practice, often for less able children, where you would draw on your understanding of learning which usually takes place before your year group. I found examples of changes in how you plan and how you begin topics. Several of you appear to begin topics by covering a lot of the learning which you might have expected to be secure before your year group, instead of perhaps assuming this was in place. There were numerous examples of how when you find your children have gaps in learning or make errors, you can track back through the curriculum to find the source of the difficulty. Sometimes you track right back to the underlying mathematical principles related to the difficulty, even if they might have been expected to be covered in the early years. You also gave examples of how you might remove limits for more able children.

The most striking common theme of all interviews is that you seem to call on this knowledge of the whole primary phase to give you renewed confidence to teach objectives outside your year group, usually below it. Many of you also talked about supporting other staff in this. (This might become more significant in the future given that the new curriculum is arranged in year group objectives.)

Progression was the most common theme in your discussion of your teaching approach, discussed in some form or other by all of you. Most of you also discussed how you use models and resources, promote rapid recall of facts and try to connect mathematical ideas. Many of you discussed teaching the underlying ideas of the area you had chosen, much more in the second interview than the first, for example the commutative or distributive law, the structure of the number system or grouping and sharing for division.

When asked to define deep subject knowledge in my second question, you all again mentioned the need for knowledge of progression. Most of you also discussed how you felt you needed to understand mathematics as a MaST. When I asked you to talk about this in more detail you defined understanding for example as knowledge of gaps, errors and misconceptions, being able to explain why, and being to explain in different ways. Some of you also characterised deep subject knowledge as connected knowledge which enables you to make connections between ideas. Some of you identified aspects of using and applying mathematics as a feature of deep subject knowledge.

You said that deep subject knowledge is different to that gained from experience. When asked if you could have gained the same knowledge from teaching every year group across the primary phase, you all felt that this would have given you a different type of knowledge. You seemed to suggest that your knowledge as MaST is different from that of any qualified teacher, even an experienced teacher. You were confident to support teachers across your schools in areas of the curriculum not covered by the programme.

I found that some of you saw deep subject knowledge as demonstrating some aspects of Masters level study. You referred to literature you had read as part of the course and what you had learned from assignments. Some of you suggested that you had this knowledge already but that it had been forgotten or not used, and was triggered by the academic part of the programme.

Please let me know, just by a brief email, if you think I have missed the point of what you were saying or if this seems to resonate with what you now feel.

I am very grateful for your help. I know you are busy people and I could never have done this without you.

Gina

Appendix ix Consent form and information sheet

Thank you for considering being part of my research project.

The Williams Review of Mathematics Teaching in Early Years Settings and Primary Schools (2008) recommends that every primary school should have one specialist mathematics teacher who has deep subject and pedagogical knowledge.

In this project, I am hoping to explore the knowledge that MaSTs use and recognise as deep.

I hope to invite you, as a MaST participating in the Mathematics Specialist Teacher Programme, to agree to two interviews at the beginning and at the end of the programme. In the interviews I will ask you to talk about your views on the nature of deep subject knowledge, and to discuss one area of the mathematics curriculum, which you will choose. I hope that this will help both you and I to consider how your knowledge develops.

The interviews will be analysed in order to identify any common views. I will ensure that all participants are anonymous and prevent them from being identified as much as is possible. Interviews will be taped and transcribed, using a code for each participant. You will be asked to agree transcripts and then all interview tapes will be destroyed.

Paper notes will be kept in a locked filing cabinet until destroyed. Electronic notes will be kept in the secure University computer system.

You will be offered a summary of my findings.

You will be able to contact me on:

gina.donaldson@canterbury.ac.uk

01227 782176

If you are happy to participate, please sign the attached form and give it to me, and keep this information sheet.

Permission form

I ____ agree to participate in the project Investigating the knowledge of the Mathematics Specialist Teachers.

I am aware of the aims of the project.

I am aware of the data which will be collected from me during the interviews. I know how my identity will be protected.

I am aware that I can choose not to participate in the project.

When transcripts of my interviews have been agreed by me, I am happy for them to be reported in such a way as to minimise my identification. The transcript will not be used until I agree it.

Signed ____

Date ____________________

Appendix x Outline of the MaST programme

The Primary Mathematics Specialist Teacher Post Graduate Certificate

The Programme Learning Outcomes are:

1. To demonstrate systematic understanding of the mathematical ideas and essential processes underpinning the curriculum for Early Years, Key Stages 1, 2 and 3.

National programme expectations: To demonstrate successful completion of the first year teachers would be expected to:	Module learning outcomes

2. To demonstrate conceptual understanding of the theoretical frameworks of the pedagogy of mathematics, and use these systematically to critically evaluate evidence of the impact of pedagogies on children's learning.
3. To demonstrate critical understanding of techniques of collaborative professional development in primary mathematics and evaluate critically the effectiveness of the outcomes of professional development.

Mapping of DCSF National Programme end of year expectations to Module Learning Outcomes, and Module Learning Outcomes to Post Graduate Skills:

undertake personalised self-supportive study as a result of carrying out an initial audit of their mathematics knowledge, skills and understanding with follow-up self-reviews of their progress extend their understanding of progression in some of the key concepts, language and notation that support the learning of mathematics within the EYFS and primary curriculum carry out sustained enquiry refining their own use of key mathematical processes involved in problem solving, reasoning and communication	Demonstrate systematic understanding of the mathematical ideas and essential processes underpinning the curriculum for Early Years, Key Stages 1, 2 and 3 covered in the module.
reflect on the pedagogy of teaching mathematics and on the teaching approaches they use most and least frequently and why demonstrate a broader repertoire of fit-for-purpose teaching approaches with emerging evidence of how they can be used to strengthen and deepen children's learning of mathematics, in particular children's decision making, language and mental skills	Reflect critically on the pedagogy of mathematics and the related literature and research findings, demonstrating a critical engagement with theoretical frameworks, using these to evaluate critically evidence of how teaching approaches can be used to strengthen and deepen children's learning of mathematics
	Module 1b (Year 1)
use peer coaching and mentoring strategies to work collaboratively with colleagues aimed at sharing, reviewing and deepening subject knowledge and understanding of progression in mathematics discuss with a senior colleague how mentoring and peer coaching can be used to strengthen the learning and teaching	Reflect on and evaluate critically peer coaching and mentoring strategies for working collaboratively with colleagues aimed at sharing, reviewing and deepening colleagues' subject

of mathematics in the school and involve a colleague in collaborative professional development activity to improve assessment practices and the learning and teaching of mathematics	knowledge and understanding of progression in mathematics
National programme expectations: To demonstrate successful completion of the second year teachers would be expected to:	Course 2 learning outcomes
	Module 2a (Year 2)
continue to undertake self-supportive study of their mathematics knowledge, skills and understanding and identify future continuing professional development needs and refreshment strategies acquire a detailed understanding of progression in mathematics from the EYFS into KS3 curriculum, and the role of language, notation and symbolism in supporting mathematical reasoning, and knowledge of how key concepts progress and inter-relate show confidence in selecting and using the essential processes to support learning in mathematics, in particular, problem solving, reasoning and sustaining enquiry	Demonstrate systematic understanding of the mathematical ideas and essential processes underpinning the curriculum for Early Years, Key stages 1, 2 and 3.
apply their understanding of the pedagogy of mathematics teaching to their own and school-wide practices and undertake in-school collaborative classroom-focused, lesson study activity focused on improving children's learning in mathematics demonstrate an extended repertoire of fit-for-purpose teaching approaches supported by good practice and	Demonstrate conceptual understanding of the theoretical frameworks of the pedagogy of mathematics, and use these systematically to critically evaluate evidence of the impact of pedagogies on children's learning

evidence of impact and research to develop colleagues' pedagogy in mathematics	
	Module 2b (Year 2)
undertake in-school collaborative professional development using a lesson study approach, to improve the learning and teaching of mathematics aligned to the school's priorities, and to undertake the evaluation of impact on provision and standards undertake the role of mentor or peer coach to build professional capacity in the school that will improve the learning of mathematics	Demonstrate critical understanding of techniques of collaborative professional development in primary mathematics and evaluate critically the effectiveness of the outcomes of professional development.

Assignments

Year 1

Part 1

Equivalent to 4000 words Indicative title: Supporting Learning in Primary Mathematics

Provide a literature review of the key theory and research findings concerning progression in children's learning in one area of mathematics from Early Years to Key Stage 3. Draw on your own mathematical investigations and an appendix where you trace the progression of key concepts, essential processes, language and symbolism from EY to KS3 in the area of mathematics.

Reflect on one aspect of pedagogy you have used to teach this area of mathematics. Critique the relevant literature, theory and research findings. Evaluate the children's learning by critically reviewing the evidence collected in your classroom.

Part 2

Equivalent to 2000 words

A critical reflection on an example of mentoring or coaching

Reflect critically on one incident of peer coaching or mentoring. Draw on literature to critically evaluate evidence of impact on colleagues' mathematical subject and pedagogical knowledge.

Year 2

Part 1

4000 words

Explore the key theory and research findings concerning the area or aspect of mathematics from Early Years to Key Stage 3 which you have identified as a need for your school. This is essentially a literature review of where you would like to move the school to. Then use this to analyse the current practice in your school. You might include as an appendix and refer to parts of your Professional Learning Log and children's work.

Part 2

2000 words

A critical reflection on collaborative professional development designed to impact on the subject and pedagogical knowledge of colleagues in this area or aspect, including a critical evaluation of outcomes of the professional development.

Appendix xi

Incidences of codes across the responses of MaSTs in both the first and second interview to question 2: What do you see as deep subject knowledge?

Codes repeated in each interview are underlined.

	Interview 1	Interview 2
A	Progression Understanding Identifying misconceptions Knowledge of National Curriculum levels Using and Applying (open ended investigations) Mathematics in a context	Progression Assessment for learning Unique to my school
B	Progression KS 3 mathematics	Understanding Knowing why Theory and research
C	Understanding Progression Knowing why Explain in different ways	Understanding Progression Knowing why Assessment for learning Theory and research Maths in play Pedagogy Use of resources

D	<u>Progression</u> <u>Key ideas and those on the periphery</u> <u>Connecting mathematical ideas: horizontal connections</u> Understanding Explain in different ways Patterns	<u>Progression</u> <u>Key ideas and those on the periphery</u> <u>Connecting mathematical ideas: horizontal connections</u> Using and Applying (open ended investigations) Identify mathematics in a context
E	<u>Understanding</u> <u>Progression</u> Assessment for learning Knowing why Explain in different ways Connecting mathematical ideas: horizontal connections	<u>Understanding</u> <u>Progression</u> Identifying misconceptions Using and Applying mathematics (open ended investigation)
F	<u>Progression</u> <u>Using and Applying (open ended investigations)</u> <u>Explaining in different ways</u> Identifying misconceptions Knowing why Connecting mathematical ideas: horizontal connections	<u>Progression</u> <u>Using and Applying (open ended investigations)</u> <u>Explaining in different ways</u> Understanding Pedagogy Structures and laws Identifying context for mathematics
G	<u>Using and Applying (open ended investigations)</u> <u>Identifying context for mathematics</u>	<u>Using and Applying (open ended investigations)</u> <u>Identifying context for</u>

	Understanding Less and more able children	mathematics Progression Identify misconceptions Explain in different ways Connecting mathematical ideas: horizontal connections Challenge others Flexibility Access support
H	Understanding Progression Identifying gaps and misconceptions	Understanding Progression Identifying gaps and misconceptions Theory and research Explain in different ways
I	Progression Connecting mathematical ideas: horizontal connections Patterns Understanding Identifying misconceptions KS3 mathematics Less and more able children	Progression Connecting mathematical ideas: horizontal connections Patterns Knowing why Explain in different ways Use of resources Identify context for mathematics
J	Understanding Passion for mathematics Theory and research Access support	Understanding Passion for mathematics Progression Assessment for learning

	Patterns Connecting mathematical ideas: horizontal connections	Key stage 3 mathematics Knowing why Explain in different ways Use of resources
K	Understanding Progression Using and Applying (open ended investigations) Connecting mathematical ideas: horizontal connections	Understanding Progression Using and Applying (open ended investigations) Connecting mathematical ideas: horizontal connections Assessment for learning Theory and research Pedagogy

Appendix xii Extracts from exemplar transcripts

T denotes teacher, I denotes interviewer

MaST D Interview 2

1. So what about using and applying with your class…
2. T Well I've been working on data handling this term as part of my role as maths coordinator, part of the school development plan, and we are introducing a new developmental curriculum, and I have looked at data handling and using and applying, together,.. it is a really good link.. to show people, because we only had two data handling weeks a year.. and using and applying every term and everyone was saying they couldn't do enough on data handling,.. this development curriculum has shown that everything we do, using symbols and having signs round our classroom is data handling…

   I tried to show them all that they are covering it, they can evidence it, it doesn't have to be on a lesson plan.. so,.. we are introducing topics now so I have taken two topics and done a plan encompassing data handling and using and applying for, I think it is animals and transport, and different performance indicators for the P levels more than anything… up to level 1,.. different ways of using those subjects..

   it was more that everyone was saying we're not doing enough.. and I said no, you are doing this and that and they couldn't see the links with everyday classroom stuff and data handling and using and applying
3. I Was it more that they saw data handling as tables and charts…
4. T Yes they saw it as pictograms.
5. I How have you got that knowledge then, because we have never done data handing on our course…

6. T Well I think you just pick it up,.. this developmental curriculum.. it makes perfect sense, I think it is being able to unpick the maths in something and look at every little aspect of what they are doing there and be able to find something.. but I had someone come and ask me because she's got time to do next term and she's got P4 and it doesn't say anything about hours until P8 but I said no but you've got a world of language there… you can do all the nice stuff, you can still do the nice making of clocks and stuff because they have still got to know that a clock is a time piece… so I gave her the whole of the using and applying part of that curriculum… and said just look through and see if you can find anything which is broadly related to time and I said if you think about it even using the word next is time.. like they all thought that all data handling is graphs and pictograms everyone thinks time is clocks but it is not, is it?
7. I What do you usually find?
8. T It is just going lower enough
9. I I am really interested in that
10. T I don't know.. I think there is experience and there is knowledge and that is the difference… I think a problem is our cohort has changed and it has gone a lot lower than what it was.. and people are in some ways still churning out what we used to do, so think, oh it is time and we do that on one day, that on one day that on one day… and then a lot of people when they do time do days of the week but we do days of the week every day in circle time, so they don't need to do days of the week
11. I So …
12. T Yes… and I have been showing them it is all the time..

    I think it is having the understanding of maths to be able to unpick everything, if you don't understand maths as a subject you can't see the links between the different areas…and... just simple things like one to one correspondence… trying to explain to my TAs what one to one correspondence actually means, so I say to them for children it is really quite hard because they have to touch one thing each time, so straight away you are bringing a number into it… and they need to slow down, it is more than doing that is that, and that is that and that is that, if we spend the whole time going count count count they don't understand why… and things like changing the order and making it a wobbly line and a circle and randomly across the table…

13. I How do you write about a topic for a class you are not teaching yourself?
14. T I unpick, a brain storm of the topic and then I use the other curriculum as a bit of inspiration, oh you could do that…. They will say things like… sorting activities, or pre sorting activities like one to one and you can do one to one with anything… any object…matching label to object, which is all data handling and using and applying, they can transfer it to anything…

    I tend to … if I do a P5child, and I have got P5 in my room but my children have got ASD and their minds work very differently… usually I try to think of a tactile idea and a non tactile idea.. for different needs some children need the systematic work. Some need the multi sensory stuff more

    I think it is experience… you can be in special needs and teach year 6 the whole time, so you haven't got that breadth, you could be in special needs and teach ASD children all the time.. and part of it I think is watching my own children develop.. seeing them.. like a bench mark. I do tend to compare with them
15. I.. So you have become a maths specialist and done the course and that has taken two years, but another way of doing it would be to have asked you to teach R to year 6, so if we had gone that way.. would you have the same sort of knowledge as you have got now?
16. T.. different I think, I think.. what really helped me in the mast course was doing the assignment and having to really think about something a lot.. and it kind of focuses your mind a bit and it was…Oh I didn't think about that,… and for me, the afternoons and the days were good but they were always a bit above what I needed… but it was good because it keeps you in track, you lose track of normality of what a normal eleven year old does and from that I think I pushed on a couple of my kids more than I would have done because I wasn't sacred to, because I knew I could step back.. and if they had gaps

in their learning and something was missing, I knew I could step back and work out what it was they hadn't learn how to do

…and you can see the links between one and the other,… and you tend to start breaking things down more and more in your head.. I've noticed a child in my class, we are trying to get him to learn his number bonds to ten, and he is so ingrained with one two three four…., and I've noticed my TAs are trying to rush him on to the next step too quick and said no, we need to get that secure, and even things like knowing to turn the numicon tile over to make it fit, and no we to go slowly with him, it is a whole new concept for him, and if he gets the pictures in his head, he'll start to work out,, he can count to ten but I don't think he knows what 10 is.. , there is just no point… it is a waste of time, I had a boy last year who had been moved on to tens and units in the vertical way.. and I said why are you doing that and he was.. I don't know.. and we went right back and I used the dienes apparatus, and he really understood that

17. I So what do you mean?
18. T very small steps, and all the time you've got to make sure they are solid before you move on to the next one otherwise you are just going to confuse them
19. I And do you feel you can do that from R to year 6, in those different needs
20. T Yes, year 6 here…I haven't taught level 5 and 6 for so long.. I look at… sometimes, not very often, not the Strategy because I find it quite confusing… I have got some text books,… but with my class this year we are starting times tables… not very... simple, two times table… and I am going to have a check-up because I haven't taught it before, so I need to refocus, on what arrays are … so I need to look for that
21. I So is that part of your knowledge, knowing where to look…
22. T Yes, I suppose it is… it is now, and checking, and I wouldn't have known about the new methods either if I hadn't done the MaST course because I haven't taught at such a high level for so long, and they have bought in things I hadn't heard of before.
23. I So if a teacher comes up and asks you a question about a child who is finding things difficult or an area of maths they are finding difficult….

24. T I’d probably have to look at their file, to see if there are things missing…because you can get a level with 80% but sometimes children don’t achieve because of whatever they have got, like a lot of ASD children are not very good at orally telling you what they are doing, but they can do it, and the children with cerebral palsy aren’t very good at counting on their fingers but they can say on the counting stick, so I would look back and see if there was anything missing…

25. I But would the class teacher not be able to see that though?

26. T I don’t think they can all the time… I think a lot of the teachers are so used to teaching maths at a lower level, that they don’t, they haven’t been on training courses for a long long time, and they haven’t learnt how to pick it apart yet, why he can’t do that. Like something really simple like, in writing, getting a white board and them transcribing a sentence and then they copy it and a lot of them write it at the top, and then they have got a long way to travel… to copy, if you write at the bottom and put them paper at the bottom…they can just do that… you are making it more difficult, just take out the middle man…

27. I Like the use of the ruler last time… is there anything else?

28. T Um… I would discuss with them and ask them what they’d tried… one says well I give him the dinosaur counters and he doesn’t want to count them he just wants to play with them, and I said well he would do wouldn’t he they are dinosaurs… so take away the dinosaurs for a bit and give him something else and then go back to the dinosaur counters, it is not that he doesn’t want to count, it is just that he has got an association with dinosaurs,.. and I had another one, another person who wanted to, she was doing a pictogram, and she wanted.. they couldn’t associate a cat with a picture of a cat, so why don’t you use animal counters, and do a 3D graph, and when you want to record it, either photograph it or photograph the counters, and then use those, but don’t get them to take them all off because they will think they are doing again, just get them to swap,… it is just little simple things like that, she hadn’t looked at their development level and worked out.. and what I tend to say if someone is P4, I say if you think about

that in normal child development that child would be about 2 and a half, so if you think about your child at 2 and a half, could he have done a pictogram, no, but he could have told you how many animals there were, and if you asked him to put the cats in a line he would have done, and if you had asked him to put the dogs in a line he would have done and he would have said oh look that is higher than that, but you are introducing all those grid lines and he doesn't need it, if you read a pictogram it is to read information from, it doesn't matter if it is not, the lines aren't all straight, as long as the animals are at the same level and the same height, it didn't matter, and it is that sort of thing, to have the confidence to break away from the pictogram has lines and numbers down the side

29. I Can you explain…?

30. T The most important thing is reading off the information, it is not even about the making the pictograms, that is just a nice task to do, you want them to be able to tell you how many more, cats than dogs, it is information finding, not just a pretty picture, it is information finding,.. I think it is not having the understanding to know.. maybe they didn't know why they were doing it either,.. it is just one of those things you do, you teach pictograms, you teach whatever…

    …I suppose because I am a mathematician I've got the confidence to say yes but they are doing their pictogram like this… they are giving me the information… which is what I've been doing.. with my level 1s or P 7 and P6…. I wanted them to do their own recording because we tend to structure their recording all the time… we give them things to stick on and whatever... it takes away.. they have to worry about glue…and those who don't like glue on their fingers… so all I did was get some coloured counters, and say sort them into colours and tell me how many and then draw them on your paper to say how many are the same… and my TAs said why are we doing that… and I said they are doing so much in that, they are sorting, they are counting, they are recording, they are counting, they have got one to one correspondence, they can tell you how many they have got… there is so much in there… but you could have given them say yellow circles to stick on, which takes them longer…takes away from the maths.. and they did that may be three or four times in ten minutes… it just simplified it..

I do seem to be able unpick it and go back down and back up…I don't like to see people not progressing in an area and it might be my fault, because if I am making it hard for them…they all seem to do well in maths

31. I So is your knowledge different to someone who has taught all the year groups in your school, and different needs?
32. T Probably because we tend to straddle… abilities and we tend to straddle it… we don't have in year 3 you do this and year 4 you do this,…we are more fluid. And all the time start with the children…see where they are and go back… and sometimes,… because we have work boxes in the morning and it is something they can do independently,…and sometimes it will be a harder thing that they have learnt how to do and then they'll have a taught session on the easier thing that works up to what they can do, … and they don't realise that and it isn't until you get to the end of it that they go…Ahh…

    So if we've been doing some work on handling data it will be something on handling data that I know they can do or if it is weight and then their homework is heavy and light, if I know they can do heavy and light I will send a heavy and light activity… we do a lot of sorting stuff all day, a lot of using and applying,… where will that one go, put the big ones there… lots of reinforcement activities all the time…
33. .. so deep subject knowledge…
34. T …To me deep subject knowledge is knowing the links between all the different areas without having to think about it too hard.. so knowing the links between using and applying and data handling… or counting and understanding, or shape and space and using and data handling…
35. I How do you see those links…?
36. T I would say it is a very spidery web…. I would say it is very much… I went on a course recently about the early years foundation stage and about the national

curriculum, and the man had made these polygons things that you put together, it was all the different early learning goals and he said you can put them together in any way to get across from one to another and I think you can do that in maths as well… if you did it, recognising numbers and if you did shape you still need to know your numbers to count how many sides there are… I think it is the same in mainstream…say if you are talking about shape and space and how many sides has a square got. If the children haven't got the concept of 4 or 3 that hasn't got any relevance for them and especially here because people so desperately want to prove that children are progressing, they bombard them with all these questions rather than giving them lots of shapes to play with, they say look it has got three sides three corners so you can put a tick in the box to show progression…rather than letting them just explore them…

MaST F Interview 2

1. T: I … I have basically, I think a lot of what I said in here holds true, what I taught year 4 is still the way I approach it.. I think what is different is I have a better appreciation of where the children have come from, or should come from.. I remember saying this time that it seemed like a real struggle in year 4 and I don't think I really knew what they were supposed to have done before they got to me, I mean I kind of did because I have seen the framework and I knew, and I was subject leader, but I think having been on the course I have a better idea of the kinds of experiences they should have had and what kinds of understanding … and what that would mean if they don't, and the kind of gaps… I know I talked about gaps there… because I felt there were gaps but I think I know now more… why, if that makes sense… because if they had only ever done sharing…no wonder they struggle…
2. I: And has that changed …
3. T: um… I suppose, just being aware of , I think I focus more on mental division now than I did before…so we talked before about chunking up….so my own class, I do play

division games, and try to do worded problems in lots of different situations so they think about it… I think that is different….division maybe is one of the areas that has changed least, .. lots of areas have changed in my teaching since doing Mast, I think I was kind of on the right track with division anyway…

4. I: You were puzzled by division and you had already given it lots of thought…
5. T: Yes…I think what's changed is my understanding…why children struggle. Maybe I give more time to it, I make sure it is visual, when I have helped other year groups, I think the confidence of being on the course and realising why children struggle has helped me intervene in other year groups maybe, so I worked with year 3 last year, to help them use and apply… and subtraction, and making it visual, so they could have them physically doing things,… realising perhaps we don't do enough of that in the school, not just with white boards, but practical division with different situations and different objects, sharing and grouping, so it has maybe helped me intervene. When I worked with Year 3 it was helping them develop division using number lines and repeated subtraction….It has helped me know that when I look at infants' work, or, I haven't been able to watch lessons yet here, it has helped me to know what to look for
6. I: Can you explain how?
7. T: To get them to… I suppose so, but I don't think in any straight forward way, it is not like I am wanting them to do things too early…it is more having a fully rounded experience at that level, and especially practically,.. without that doing something, dividing things, putting them in groups, I think they don't get it…when I have had an intervention group, working on something, I will make sure that the TA does it practically, because they have been missing out on that, I plan that way for those children to have those experiences
8. I: So what sort of knowledge do you need to be a MaST?
9. T: .. all the problem solving we did as part of the course and I think that is deep subject knowledge, you can do the maths, you understand it, they weren't usually straight forward things were they, so I think that kind of confidence… mathematical

confidence…and enjoyment I think…I really enjoyed that part of the course, I miss it already…and I think the people I met who were probably best at their role really enjoyed maths

10. I: So it is not, … you could do the programme or you could teach every year group…
11. T:…Would you end up with the same?.. No I don't think you would, I think it is the readings, and I think it is the actual. I don't think it is just the practical things for teaching each particular year group, yes you see more and you would know the areas of weakness but you wouldn't necessarily know what to do about them or how to approach them differently or how it could be taught differently… the whole idea of teaching through problems and not just presenting children with sets of drills, you wouldn't come to that on your own would you? Unless it was the culture of your school and you were lucky through your own training, but I think that actually doing the readings for the course, and the training days, they give you a totally different perspective than just through teaching
12. I So deep subject knowledge that you are drawing on in your role here?
13. T: I think it was as much the pedagogical knowledge, the way, how children learn maths and how best to present it to them, I think that is what I most value
14. I: So it is ….
15. T: Yes, even the simple things like feeding back on planning, I have done a couple of year groups, planning scrutinises, what I can say now is really different, I can comment more on the progression, I've got a better understanding of progression in different areas, I can comment more on what the problem solving approaches they could use, could you make it cross curricular, have you thought about how to develop it through stories, it has really developed my ability to…
16. I: And, do you ever feel you are making mathematical comments?
17. T: Yes, so, progression, and so for instance I did a planning scrutiny recently and I was able to say really good sound progression in mental maths methods, and then in a

different year group, commenting on subtraction: make sure at some point you do finding the difference as well because here you are just taking away.

18. I….do you think you are drawing on new knowledge or…
19. T: Yes … I was doing it on my own, do you know what I mean, without any,…I picked things up as I taught, I always felt I was delivering OK but I feel much more confident, how to teach, how to help other people….

    What I have come to, through needing to revise the calculation policy, is all the things that happen in the infants are still so important to stay in there, and.. it is kind of controversial but I have come to think that is the actual method is less important than the children's understanding, and the mental is much more important, so if they understand how to tackle it, then basically they will be able to do chunking, .. we gave them an old year 6 SATs paper recently and there was a long division question on it, it was like Anghileri's research really, and the children who understood had a range of different methods to get there, some used short division with big numbers, some used their own form of recording, basically chunking, keeping track of, which they were able to do, but it wasn't the chunking they were taught, we don't really do that here, and I think that is what is important, it is important that they know what they are doing, and keep track of it somehow, not which formal method you do, so I can see us as a school not teaching chunking as a formal method but making sure they understand division well enough so they can keep track and find an answer
20. I That is great, thank you…

MaST K Interview 2

1. I: So here we are after two years and now you are the MaST, do you think the way you teach place value has changed at all?

2. T: Yes I think there has been a major shift in one aspect of place value in the language that I use, I still l agree with what I said before about all the practical equipment, and the importance of teaching place value, but through the MaST course I was really struck by the Thompson paper, and for me it was the biggest thing about MaST was it gave me access to research, and it made me look at things from a completely different view point and it is almost like you think the maths you are doing is common sense, and some one looks at it from a completely different angle which you would never have thought, so for me reading that paper and finding out, it was a very convincing paper, … that I shouldn't be talking about columns with the younger children, because they would find that more difficult to understand. So I was trying to refer to the values of the numbers, so rather than saying 2 tens and 4 units I would say 20 and 4. For me that was a huge change in my language and I kept making mistakes because I'd got used to saying it the other way…
3. I: Do you think it has had an impact on the children's learning?
4. T:… I don't know, I think I worried that if I didn't use columns they wouldn't move on as quickly, so we are always worried, we have got to get them moving on to another level, and I think I would first have thought well of course you have got to tell them about the columns, or you would hold them back, and it definitely hasn't held anyone back with their understanding and it hasn't, …how much positive value I think it is too early for me to tell. But I have definitely started to tell other teachers in the school about that research and it comes out of the blue for them too, they say really, why is that?...I guess for me that is quite hard, how can I explain to non-specialists? The reasons behind things, I don't think they work at that deep a level, I suppose that touches on deep subject knowledge… I think they teach the way they have always taught, they find it hard to think deeply about some of those concepts, about whether you talk about columns or not

   ….Last year I was in the interesting position where I worked with someone else all year, I was trying to explain to this other teacher what I had learnt on the course about this and she didn't get it with my explanation, then I gave her my MaST essay because I had written about place value and I said read that little bit and see what you think. And she has got the point where I guess she trusts me and she will use the other language but I don't think she really understands why, she trusts me on that, and I haven't spread that message further yet

5. I: But if Thompson and Bramald's research is right…
6. T: Yes, later, and I think it was being in year 2, because I don't usually teach in Key Stage 1, it was a new experience being with the younger children, and I thought I would just drag the higher stuff down, to that level
7. I: That is interesting
8. T: Where I hadn't really thought about doing it a different way, … I have never had any training for Key Stage 1, and hadn't known a lot about the maths down there and I guess I thought I could drag everything down .. rather than start at the other end and building up .. because I had no other experience apart from my own children and what they knew
9. I Can you explain what you mean?
10. T: I don't think I would ever have thought about that if I hadn't done the MaST, I would never have looked at it from that other angle,
11. I: And also that idea of dragging it down,
12. T: Yes
13. I: You were in year 2 before you started MaST, had you already come to the conclusion that you couldn't just drag down the curriculum for Key Stage 2 into Key Stage 1, or did that come as part of MaST?
14. T: I guess I was really experienced in what they would have to meet in Key Stage 2, and so I felt it was the best of my abilities to give them opportunities to get them to start meeting that, in that format, so I would introduce the columns because I knew that they would be using them in Key Stage 2, but now I am thinking where are they at, what the bridge was…

15. I: but you are still using what you know they need at Key Stage 2

16. T: Yes, being aware that you can't be ready for the concepts in Key Stage 2 in Key Stage 1, so you have got to introduce things using the concepts they understand, and try not to add the complex parts, so we could juggle with big numbers in Key Stage 1 quite happily as long as we talk about their value, like 20 and 4, but we can't juggle big numbers when we are talking about 2 tens and 4 units, because that adds another layer, and there are probably other areas of maths where that is also the case and I just haven't met them yet

    … It knocked me flying in one way because I thought I was really good with my maths, I thought I understood it all, and I guess I thought am I going to learn very much from this MaST? And I have

17. I: Have you learnt new knowledge?

18. T: I think the MaST pulls apart what you think you know, and it does take you places where you think oh I don't know any more and you have to construct your understanding again, and I think that is a really healthy thing to do, I don't think you get opportunities in school, in your training, in any of the local training I have done to pull something apart at quite an intellectual level before you can then put it back together

19. I: But you've done a masters already

20. T: Yes, but not all in maths… but it was done in a different ways, I think looking at progression and having the opportunity to look at up to date research has been really useful for me

21. I: Can you explain how?

22. T: I guess the language is important to think about the language which is used with children, so that was something I hadn't talked very much with place value with staff, but now I do more, I think talk about language in other areas of maths, … I think place value was the area where I learnt most, there were other areas where it just consolidated and there weren't so many surprises

I don’t know if this answers the question, but when I was doing my second assignment, and it was a whole school project, because I did an inset day for the staff on what we were doing and I gave them tasks to do, and we were all sort of on board and all of a sudden people began to speak to me about maths it was almost as if maths wasn’t talked about before, but because we had had this big input and I was pushing things forward, it would be just over coffee in the staff room someone would say, oh do you know what happened in maths today… and I think the teachers in my school are fairly confident in their teaching of maths, pretty competent as well, so it is not often that they are stuck, but they have been doing a lot more sharing of the enjoyment of a lesson, or these children are hung up at this point and we discuss it more, so I think somehow the mast raised the profile more, being able to come to me and talk about maths more, perhaps it made the teacher think about their own teaching as well, they were going to reflect more on their own teaching

And I think somehow, I don’t know why, in the staff room we don’t seem to be tending to talk about how your lessons go, may be you think you are showing off, or on the other hand if things haven’t gone so well you don’t want to share it…I guess no one thinks they are bad enough to ask for my help, they think she is busy, I won’t disturb her, it is only a little thing…

23. I: How do you help teachers in year groups you haven’t taught?
24. T: I’m much better in my understanding of the whole continuum…
25. I: So if someone asked you for help with Year 1, how do you work out…
26. T: I’ve got a really good knowledge of the National Curriculum levels, and I guess I use them as my guide, so I think right Year 1, where should they be, what sort of concepts should they be covering, and then I will be picking up ideas I have seen around to suggest activities
27. I: And what do you think of the ideas of not doing the MaST programme at all, but becoming a MaST because you have taught year R,1,2,3,4,5,6? Do you think that would give you the same sort of knowledge?

28. T: No… because I was starting to do a role like that before hand, but I wasn't getting the input for my ideas I suppose, yes I think I need more than just the classroom experience to be good at advising teachers, and saying it again, getting access to that research, when I have supported teachers before I would never have thought of going and looking at maths research, whereas now, that might be one of my first port of calls now, looking at that, I am enjoying subscribing to some of the maths journals now, where you get the ideas to.. otherwise I am just sort of making the most of my limited experience and not gaining from the experience of all these people who have done these big research projects because some things are not common sense
29. I: in what way?
30. T: You need some people with a bit of space, to think of questions, to think I wonder if… and to look into that… I really enjoy looking into that and being able to feed that information into school and that is my worry now I am a MaST, how do I keep up with that because it takes time, and we were forced to do it on the course and how do I keep that discipline up?

    Yes, see the value of… and you do get a lot from going into different classroom and see how different teachers approach things
31. I: So you start off your teaching career, so when you get QTS you have to have secure subject knowledge, you wouldn't have got QTS without secure subject knowledge, how is the deep subject knowledge different from the secure subject knowledge that you had?
32. T: My pedagogical knowledge has increased, the most probably, I probably had a lot of gaps in different year groups at the beginning, so an understanding of that progression has grown, I think it was quite good when I got to MaST, because I had spent a lot of time on different maths projects…
33. I: Yes…

34. T: I think I was running and ready for the deeper stuff, it slotted in, because I have a mathematics experience I have always been strong on the using and applying, investigating, justifying
35. I: It is interesting that you mention using and applying, has your approach to that changed?
36. T: There have been aspects of it, because it came into my second assignment, because I was trying to solve the problem of why our girls weren't making progress, and making maths look a bit more like English, because that is where the girls are happy, so I guess I have turned my using and applying a bit on its head in that I am starting to use more literature skills in maths, trying to get the children to write their thoughts down, trying to model writing in maths…so I guess that part of my using and applying has changed but I have, using and applying has always been important to me, to incorporate that through all my other lessons in maths, and also when I am training staff, it is always very high on my agenda, using and applying…
37. I: So deep subject knowledge…
38. T: I think I have a problem with deep subject knowledge because to say I have got deep subject knowledge is that I am there, and I feel so much that I am on the edge, I have only just started…. I have always been very very interested in how children learn and I guess that is what I am looking for in the research… you just want that magic formula, this one will work, and it is finding those little things that work, that click for children, a deeper level for…than going through the motions of teaching this concept and this concept, I want to really understand, what do children think, and how can I help them with their thinking, to move on to the next step
39. I: Is that mathematical?
40. T: I guess not completely, some parts of it, some parts of it are how different children learn… and I think it is all these links between all the different aspects of maths and keeping aware of what a big subject area it is, it is vast, the content we have to cover in each year group seems huge, when you compare it to other curriculum areas, I think that

is where you can feel swamped, and may be because there is so much of it, I am more confident because I have got an understanding of the big sweep of it, but other teachers struggle because there are so many different concepts.. when I am teaching a lesson those links are probably there all the time, whereas other people, they are, I am getting to this objective, and they never go sideways

41. I: Thanks you that is really useful, is there anything you would add
42. T: Only that I worry about the research, the gap between school and research is growing.. but that is all I think