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A Procedure for the Calculation of the Perceived Loudness of Sonic Booms

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A Procedure for the Calculation of the Perceived Loudness of Sonic Booms

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Implementing a method to calculate the human ear's perceived loudness of a sonic boom requires consulting scattered literature with varying amounts of detail. This work describes a comprehensive implementation of Stevens' Mark VII in Python, called PyLdB. References to literary works are included in enough detail so that the reader could use this work as a guide to implement the Mark VII algorithm. The details behind the mathematics of the Mark VII algorithm are included and PyLdB is used to calculate the perceived loudness of an example pressure signature. PyLdB is benchmarked against a widely used and validated code by NASA called the Loudness Code for Asymmetric Sonic Booms that also implements the Mark VII methodology. PyLdB can be applied in conjunction with other tools to calculate the perceived loudness of sonic booms and facilitate the optimization of aircraft to reduce loudness levels.

Nomenclature

A	=	amplitude
B	=	equivalent loudness at 80 Hz
E	=	band energy level
E_T	=	total energy in a signal
F	=	loudness summation factor
f_C	=	one-third octave band central frequency
FFT	=	fast Fourier transform
f_l	=	one-third octave band lower frequency
f_s	=	sampling frequency
f_u	=	one-third octave band upper frequency
$LCASB$	=	Loudness Code for Asymmetric Sonic Booms
L_{eq}	=	equivalent loudness
L_ℓ	=	lower loudness limit
L_p	=	sound pressure level
L_u	=	upper loudness limit
N	=	total number of data points
N_B	=	frequency band number
P	=	perceived loudness (PLdB)
p	=	pressure
$\hat{p}$	=	Fourier transform of pressure
p_a	=	ambient pressure
p_o	=	reference pressure ($20\,\mu Pa$)

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$RMSE$	=	root mean squared error
S	=	loudness in sones
SBJ	=	supersonic business jet design
SSC	=	supersonic canard design
S_m	=	maximum loudness in sones
S_t	=	total loudness in sones
S_{xx}	=	power spectrum
t_o	=	start of time stamp
X	=	equivalent loudness transformation constant
Δf	=	frequency resolution in FFT
Δt	=	time step
ω	=	frequency (rad/s)
ϕ	=	phase shift (rad)

I. Introduction

Quantifying the perceived loudness of a sound such as a sonic boom has been the subject of significant research over the years and has applications in the optimization of supersonic aircraft design. The object of perceived loudness research is to relate sounds to a scale that accurately represents human perception. Such a scale allows the loudness of a sound to be measured and subsequently reduced to prevent annoyance and damage to the human ear. By calculating a perceived loudness in decibels (PLdB) for the pressure signature generated by a shock wave, tools can be developed for the optimization of supersonic aircraft with reduced loudness. The purpose of this paper is to summarize, outline, and implement the traditional algorithm for calculating perceived loudness as reported by Stevens to obtain perceived loudness results for representative supersonic aircraft geometries [1]. These results are compared to results obtained using NASA's Loudness Code for Asymmetric Sonic Booms (LCASB).

A sonic boom refers to the sound created when an aircraft flies in a supersonic flow regime. Supersonic flow is strongly influenced by the presence of shock waves, which are regions over which flow properties such as pressure, temperature, velocity, and density change rapidly [2]. The central parameter used in the calculation of perceived loudness is pressure. The changes in pressure caused by a shock wave at altitude can propagate down to the ground level, interacting with the human ear. An example of a propagated pressure signature measured at the ground is shown in Fig. 1, where the pressure metric represents a change from ambient pressure, $p - p_a$ or Δp.

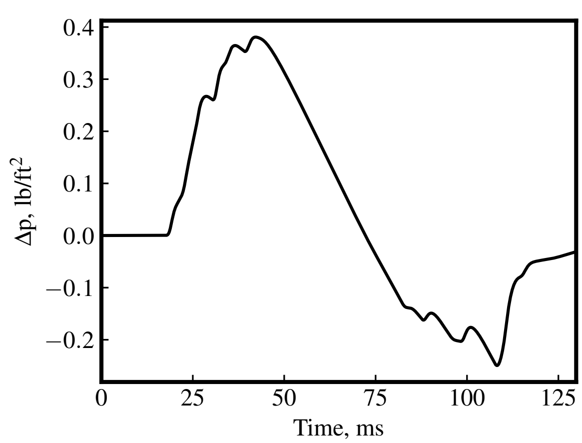

Fig. 1 Example ground pressure signature from a sonic boom.

There are a variety of methods, such as the A, B, C, and D-weightings, that attempt to estimate loudness values using different frequency-based weighting scales. The A, B, and C-weighting scales were introduced by Fletcher and Munson [3] using 40, 70, and 100 decibel equal-loudness contours respectively. Each scale uses a reference frequency of 1000 Hz to scale the sound pressure level of a given noise. The D-weighting scale was created to improve upon the A-weighting scale for measuring the noisiness of common sounds but has fallen into disuse due to a lack of perceived benefit over the A-weighting scale [4–6]. The A-weighting scale was quickly adopted as the standard for measuring noise due to its emphasis on the 1 kHz to 4 kHz frequency range, which early studies showed was the range where humans were more sensitive to hearing loss [7]. Figure 2 shows the A, B, C, and D-weighting scales plotted by frequency with the threshold equal-loudness contour, also called the absolute threshold of hearing, plotted for reference [8].

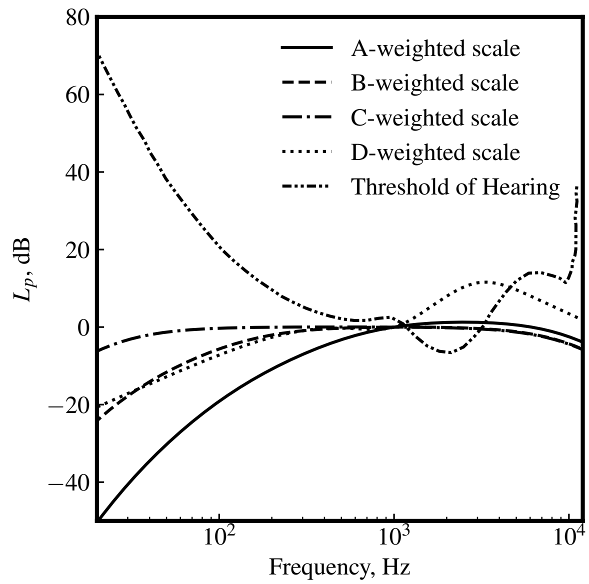

Fig. 2 A, B, C, and D-weighting scales compared to the threshold of hearing contour [8].

A study by Pierre [7] showed that there are significant issues with using the A-weighted frequency scale to determine the loudness of sonic booms. The A-weighting scale has been shown to underestimate the loudness of sounds over 60 decibels, which a sonic boom can easily surpass. This occurs because the A-weighting scale weights the contribution of low frequencies (where much of the sonic energy is located) much lower than the mid-to-high frequencies as can be seen in Fig. 2 [9].

In contrast to these weighting scales, the procedure used by Stevens has been proven to be effective at measuring the perceived loudness of sonic booms and is generally used as a quantitative measure for sonic boom loudness [10, 11]. Stevens' Mark VII algorithm was designed to effectively calculate loudness for the entire human auditory range, including the range at which a sonic boom is found [1]. To ensure that the PLdB weighting correlated well to the loudness of a sound based on the response of the human ear, the Mark VII was based on a reference frequency near the ear's most sensitive region and was benchmarked against multiple weighting systems, including the A-weighting scale [1]. Figure 3 shows the equal-sone contours used by the Mark VII to weight the band loudness values and calculate the perceived loudness.

The Mark VII has been used extensively to predict the loudness of sonic booms [4, 11, 12]. Jackson and Leventhall [13] showed that the Mark VII procedure developed by Stevens could be easily expressed mathematically using Fig. 3 and tabulated conversions. This allows the procedure to be applied to a computer program efficiently regardless of the composition of the ground signature. Shepherd and Sullivan [14] outlined and applied the Mark VII algorithm to better understand the effect of overpressure and rise time in the loudness of sonic booms. Due to the ease with which the Mark VII is implemented into a computer program, it can be used to calculate and optimize the perceived loudness of supersonic aircraft.

II. Traditional Procedure for Loudness Calculation

The Mark VII perceived loudness calculation can be described in three steps. The first step in the calculation requires a determination of the one-third octave bands of the ground signature and their respective sound pressure levels. Next, the sound pressure levels, measured in units of decibels, are converted to 3150 Hz equivalent loudness levels using the equal-loudness contours shown in Fig. 3. Finally, the equivalent loudness values are converted to a perceived loudness, measured in PLdB, using tabulated conversions and a power law.

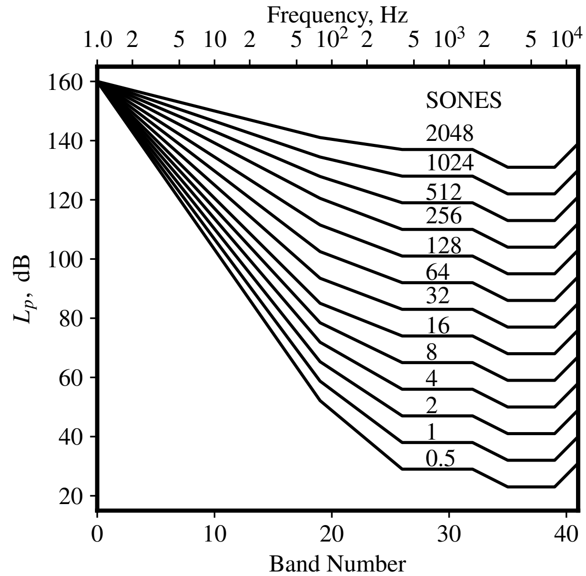

Fig. 3 Contours of equal perceived magnitude in sones used by Stevens' Mark VII algorithm.

The frequency spectrum of any signal can be separated into bands. Octave or one-third octave frequency bands are most commonly used, where each band is defined with a central frequency, an upper frequency limit, and a lower frequency limit. The upper limit of any band must be equivalent to the lower limit of the adjacent frequency band. One-third octave bands will be used in the algorithm outlined in this paper, though mention will be made of any changes necessary to make the computations with octave bands.

The nominal and calculated one-third octave band central frequencies, upper band limits, and lower band limits for bands from 1.0 Hz to 12.5 kHz are shown in Table 1. These can be calculated by defining a lower and upper band central frequency, as well as a reference band about which to begin the calculations. As defined in Hayes' work [15], the ratio of the center frequency of any one-third octave band i to the next one-third octave band is given by

$$f_{C_{i+1}} / f_{C_i} = 2^{1/3} \tag{1}$$

while the corresponding lower band limit is

$$f_l = f_C / 2^{1/6} \tag{2}$$

and the upper band limit is

$$f_u = 2^{1/6} f_C \tag{3}$$

The nominal values will be used throughout the present work and the calculated values were found by using Eq. (1) with a reference value of 3150 Hz.

A pressure signature, such as the one shown in Fig. 1, has energy content in many, if not all, of the one-third octave bands shown in Table 1. Performing a Fourier transform on the pressure signature allows the energy content in each band to be analyzed. Since the pressure signature is generally a set of discretized points, a Fast Fourier Transform (FFT) is performed to analyze the energy content.

The FFT considers a discrete dataset as a periodic function and therefore any dataset that does not start and end at the same value can experience spectral leakage. Spectral leakage refers to the creation of new frequency components in the frequency spectrum that don't exist in the signature, but rather have leaked from neighboring frequencies. To reduce the effect of spectral leakage in the output of the FFT, a Hanning window [17] is applied to the front and rear of the pressure signature. This ensures that the leading and trailing edges of the pressure signature are brought back to zero smoothly so that the periodic extension of the data and its derivatives are continuous.

Since sonic booms contain more energy on the lower ranges of the frequency spectrum, it is important that the signature has enough data points to provide proper resolution to the FFT. As seen in Fig. 1, pressure signatures generated

Table 1 Nominal and calculated one-third octave bands [16].

Band number	Nominal Center Frequency (Hz)	Calculated Center Frequency (Hz)	Frequency Limits (Hz)
0	1.0	0.97	0.89 - 1.12
1	1.25	1.22	1.12 - 1.41
2	1.6	1.54	1.41 - 1.78
3	2	1.94	1.78 - 2.24
4	2.5	2.44	2.24 - 2.82
5	3.15	3.08	2.82 - 3.55
6	4	3.88	3.55 - 4.47
7	5	4.88	4.47 - 5.62
8	6.3	6.15	5.62 - 7.08
9	8	7.75	7.08 - 8.91
10	10	9.77	8.91 - 11.2
11	12.5	12.3	11.2 - 14.1
12	16	15.5	14.1 - 17.8
13	20	19.53	17.8 - 22.4
14	25	24.61	22.4 - 28.2
15	31.5	31.01	28.2 - 35.5
16	40	39.06	35.5 - 44.7
17	50	49.22	44.7 - 56.2
18	63	62.01	56.2 - 70.8
19	80	78.13	70.8 - 89.1
20	100	98.44	89.1 - 112
21	125	124	112 - 141
22	160	156	141 - 178
23	200	197	178 - 224
24	250	248	224 - 282
25	315	313	282 - 355
26	400	394	355 - 447
27	500	496	447 - 562
28	630	625	562 - 708
29	800	788	708 - 891
30	1000	992	891 - 1120
31	1250	1250	1120 - 1410
32	1600	1575	1410 - 1780
33	2000	1984	1780 - 2240
34	2500	2500	2240 - 2820
35	3150	3150	2820 - 3550
36	4000	3969	3550 - 4470
37	5000	5000	4470 - 5620
38	6300	6300	5620 - 7080
39	8000	7938	7080 - 8910
40	10000	10000	8910 - 11200
41	12500	12600	11200 - 14100

from sonic booms have a very short duration, which requires a high sampling frequency to properly describe the signature. The number of discrete frequencies resolved in an FFT is equal to half of the total number of data points, and the FFT has a maximum frequency value equal to half of the sampling frequency. The spacing between resolved frequencies in an FFT can be described by

$$\Delta f = f_s / N \tag{4}$$

which is controlled directly by the duration of the signal and is generally predetermined. In signal processing, zero-padding is a method often used to provide increased resolution to the frequency bins in an FFT. By adding banks of zeros to both sides of the signal, the number of resolved frequencies in the FFT is increased by increasing N while keeping f_s constant. This allows the energy content in the lower frequency bands to be measured more accurately.

The output of the FFT gives a summation of the Fourier coefficients in both the real and imaginary space. Often this information is viewed in terms of a power spectral density, which shows the relative power of each discretized frequency in the signal. The FFT is transformed to the power spectral density through

$$S_{xx} = |FFT^2|\,\Delta t^2 \tag{5}$$

where Δt is the time step in the pressure signature. Figure 4 shows the power spectral density as a function of frequency for the ground signature in Fig. 1. The spectral power of a pressure signature is divided between positive and negative frequencies by the FFT. By multiplying the power of the nonzero, positive frequencies by 2, a one-sided power spectrum is obtained, which can be used in conjunction with the frequency bands to calculate the energy in the pressure signature.

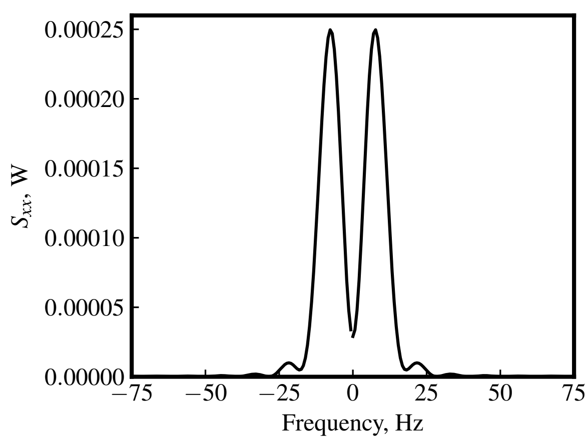

Fig. 4 Power spectral density as a function of frequency.

Parseval's theorem [18] allows for the energy content to be calculated using the power spectral density [19]. Parseval's theorem in the context of this problem says that for any given waveform, $p(t)$, starting at $t_o = 0$, the energy of the wave is proportional to

$$E_T = \int_0^\infty |p(t)|^2 \, \mathrm{d}t \tag{6}$$

which is equal to the integral of the Fourier transform over the positive frequency domain,

$$E_T = \int_0^\infty |\hat{p}(f)|^2 \, \mathrm{d}f \tag{7}$$

Since we are concerned with the energy content contained within each one-third octave frequency band, Eq. (7) can be separated by the additive properties of integration, and the energy in each band can be calculated as

$$E = \int_{f_{l_i}}^{f_{u_i}} |\hat{p}(f)|^2 \, \mathrm{d}f \tag{8}$$

Equation (8) can be solved for the discrete pressure signature by using a numerical integration method. To account for all of the energy within each one-third octave band, a linear interpolation is performed to produce an estimate for the spectral power at each of the one-third octave band frequency limits, f_l and f_u. This ensures that there is a value of S_{xx} that can be used in the integration for each of the band limits.

The sound pressure level for a given frequency band can be found using

$$L_p = 10 \log_{10} \left(E/p_o^2\right) - 3 \tag{9}$$

where the reference pressure p_o is defined to be the lower threshold of human hearing with a value of 20μPa. As there is a discrepancy between the units in Eqs. (7) and (9), it is necessary to divide the band energy by some reference time. According to Johnson and Sullivan, this reference time is 0.07 s. This is defined as the critical time of the human auditory system, and represents the time for the auditory system to fully respond to acoustic stimuli based on experimental data [12].

Two assumptions are made when subtracting the 3 decibels shown in Eq. (9). It is assumed that the majority of the energy in the signature is contained in the two pulses at the front and rear of the waveform and that the time separating these pulses is greater than the critical time of the auditory system. Making these assumptions means that the human ear will hear two distinct events of equal loudness over the duration of this pressure signature. Since the definition of the total energy in Eq. (6) is taken over the entire waveform, not just a single pulse, that energy must be divided equally into the two events. In this context, the sound pressure level can be described using

$$L_p = 10 \log_{10} \left(E/(2\, p_o^2)\right) = 10 \log_{10} \left(E/p_o^2\right) - 10 \log_{10} (2) \tag{10}$$

which is very nearly equivalent to Eq. (9).

Using the values of L_p for each of the frequency bands, Stevens' Mark VII method can be implemented to find the perceived loudness value for the pressure signature. The L_p values (in decibels) for each of the frequency bands are first transformed to an equivalent loudness value with reference to the 3150 Hz frequency band. The 3150 Hz band is chosen as the reference band for the purpose of better approximating the perception of sound by the human ear [1]. The transformations from sound pressure level, L_p, to an equivalent loudness, L_{eq}, for each of the one-third octave frequency bands are given in Jackson's work [13], which is reproduced in Table 2. The formulation of the equations referenced in this table can be found in Appendix A.

After calculating the L_{eq} for each frequency band using the transformations in Table 2, a second transformation is performed from loudness in decibels to loudness in sones. A sone is defined by Stevens as the perceived magnitude of the standard 3150 Hz sound with an L_p of 32 decibels [1]. Each of the equivalent loudness values are transformed from decibels to sones using a table provided by Jackson [13], which is included for convenience in Table B.1 of Appendix B. Values of L_{eq} may be linearly interpolated to find a value in sones.

After the value in sones for each of the frequency bands is found, the total loudness is found using

$$S_t = S_m + F\left(\sum_{i=1}^{N} S_i - S_m\right) \tag{11}$$

where S_i is the equivalent loudness in sones of each frequency band and S_m is the maximum sone value in all of the frequency bands. Stevens calls F the summation factor, and explains that it is related to the masking and mutual inhibition of each band with its adjacent bands [1]. The values for F can be found using S_m in Table B.2 of Appendix B. It should be noted that if an octave band analysis has been used up to this point, 4.9 dB should be subtracted from the loudness of the band with the highest sone value and a new sone value calculated for that band. The value of F should then be doubled in Eq. (11) [13].

Finally, a perceived loudness, P, in PLdB can be calculated using

$$S_t = 2^{(P-32)/9} \tag{12}$$

which can be rearranged to yield

$$P = 32 + 9 \log_2(S_t) \tag{13}$$

This procedure can be used in an algorithm to calculate the perceived loudness of any pressure signature. The implementation of this algorithm will be covered in the following section on a specific pressure signature, followed by a validation of the results.

Table 2 Equivalent loudness transformations for a 3150 Hz reference frequency [13]. All equations are derived in Appendix A.

Band number N_B	Frequency f_C (Hz)	Using band number, N_B and corresponding loudness, L_p	Using band frequency, f_C and corresponding loudness, L_p
0 to 18	1 to 63	Find B from Eq. (A.12) $L_p = B$ and $N = 19$ $L_\ell = 86.5$ dB and $L_u = 131.5$ dB $L_{eq} = \begin{cases} \text{Eq.(A.6)}, & B < L_\ell \\ \text{Eq.(A.8)}, & L_\ell \leq B \leq L_u \\ \text{Eq.(A.10)}, & B > L_u \end{cases}$	Find B from Eq. (A.11) $L_p = B$, $f = 80$, and $X = 10.5$ $L_\ell = 86.5$ dB and $L_u = 131.5$ dB $L_{eq} = \begin{cases} \text{Eq.(A.5)}, & B < L_\ell \\ \text{Eq.(A.7)}, & L_\ell \leq B \leq L_u \\ \text{Eq.(A.9)}, & B > L_u \end{cases}$
19	80	$L_\ell = 86.5$ dB and $L_u = 131.5$ dB $L_{eq} = \begin{cases} \text{Eq.(A.6)}, & L_p < L_\ell \\ \text{Eq.(A.8)}, & L_\ell \leq L_p \leq L_u \\ \text{Eq.(A.10)}, & L_p > L_u \end{cases}$	$L_\ell = 86.5$ dB and $L_u = 131.5$ dB $L_{eq} = \begin{cases} \text{Eq.(A.5)}, & L_p < L_\ell \\ \text{Eq.(A.7)}, & L_\ell \leq L_p \leq L_u \\ \text{Eq.(A.9)}, & L_p > L_u \end{cases}$
20 to 25	100 to 315	Same as previous, with L_l and L_u from Table A.1	Same as previous, with L_l and L_u and X given by: f(Hz) 100 125 160 200 250 315 X(dB) 9.0 7.5 6.0 4.5 3.0 1.5
26 to 31	400 to 1250	Use Eq. (A.1) with $X = 8$	Use Eq. (A.1) with $X = 8$
32 to 34	1600 to 2500	Use Eq. (A.3)	Use Eq. (A.2) with X given by: f(kHz) 1.6 2.0 2.5 X(dB) 6.0 4.0 2.0
35 to 39	3150 to 8000	$L_{eq} = L_p$ dB	$L_{eq} = L_p$ dB
Above 39	Above 8000	Use Eq. (A.2)	Use Eq. (A.1) with: $X = 4$ dB for $f = 10$ kHz and $X = 8$ dB for $f = 12.5$ kHz

III. An Example Perceived Loudness Calculation

An example calculation of perceived loudness will be performed with the pressure signature shown in Fig. 5. One-third octave bands were defined with central frequencies from 1.0 Hz to 12.5 kHz. A Hanning function was used to window to the first and last 800 points of the signature to bring the trailing edge back to zero smoothly. It should be noted that the number of points used in the Hanning window has a significant effect on the PLdB calculation, and should be varied with caution.

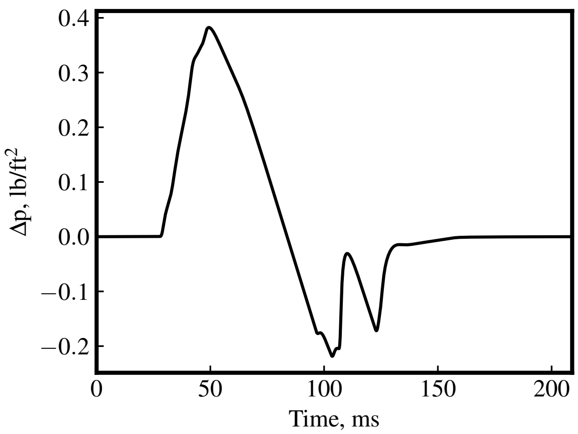

Fig. 5 Ground signature used in example calculations.

The 36,000-point pressure signature was zero-padded with 144,000 points on each end of the signal to produce a frequency resolution of Δf = 0.53 Hz in the FFT. The one-sided power spectrum shown in Fig. 6 was found using the FFT performed on the zero-padded, windowed signal. Values of S_{xx} were found at each of the frequency band boundaries using linear interpolation to estimate the total power in the pressure signature. The energy content of each frequency band was then found using Simpson's rule for the integration [20] in Eq. (7).

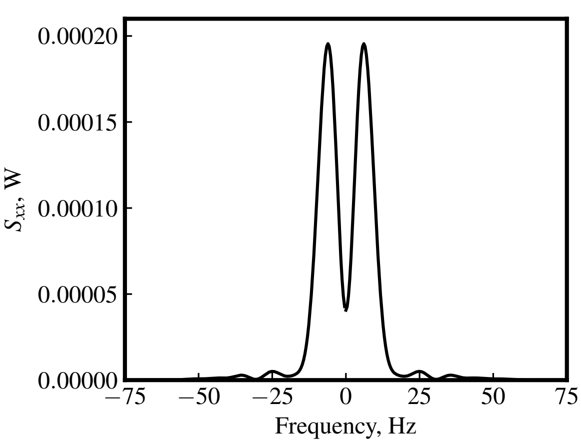

Fig. 6 Power spectral density for the example signature in Fig. 5.

At this point, a check of Parseval's theorem, described in Eq. (6) and Eq. (7), can be made to ensure that all of the energy in the signature was accounted for properly. The value of the integral in Eq. (6) is equal to 0.004123274, while the value of the energy integral, Eq. (7) yields 0.004123261 for this pressure signature. The 0.0003 % difference between these two values can be accounted for by recognizing estimation made at the frequency boundaries. In addition, the numerical approximation being made by the integration could yield this kind of error. Due to these considerations, Parseval's theorem dictates that the energy of the wave before and after the Fourier transform remains the same, and that all the energy in the wave has been accounted for in the loudness calculations.

The sound pressure levels for each of the one-third octave bands is found by using the energy in each band in Eq. (9). The resulting loudness in decibels is plotted in Fig. 7. Using the steps contained in Table 2, 3150 Hz equivalent loudness levels can be found, as shown in Fig. 8, and converted to sones using Table B.1. The loudness of each band in sones is shown in Fig. 9. The maximum value in sones, 14.75 for this signature, is then used in conjunction with Table B.2 to produce a value for F of 0.2. F is then used in Eq. (11) to find the total loudness, S_t, which is 42.35 sones. Finally, the perceived loudness is calculated to be 80.64 PLdB using Eq. (13).

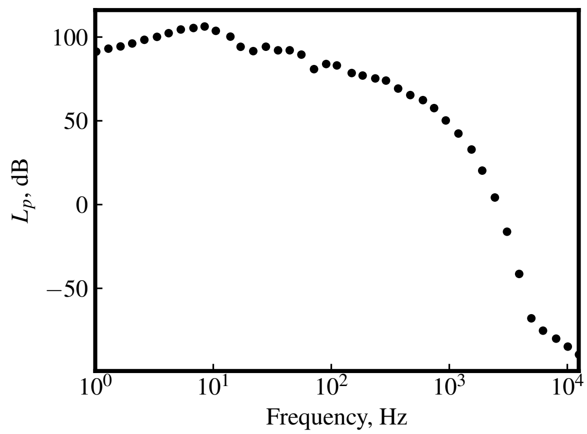

Fig. 7 L_p **values for each one-third octave band.**

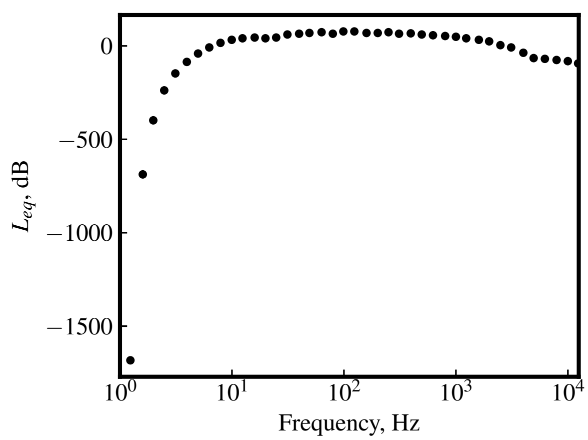

Fig. 8 L_{eq} **based on a 3150 Hz reference frequency for each one-third octave band.**

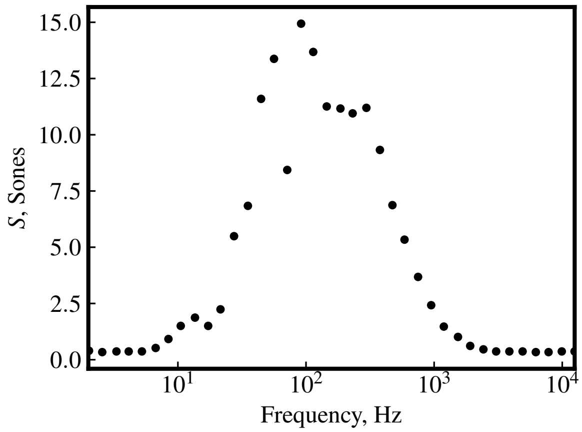

Fig. 9 Sone levels for each one-third octave band.

IV. Results and Validation

The specific implementation of the described method used in this work was written in the Python programming language and is called PyLdB*. To validate PyLdB, the calculated PLdB is compared with the results generated by NASA's LCASB program, another perceived loudness calculator using Stevens' methodology. Table 3 shows the PLdB values generated by PyLdB and LCASB for four ground signatures from reported supersonic geometries. The geometries include the NASA 25D concept geometry [21], the Lockheed Martin N+2 design [22], a supersonic business jet design (SBJ) [23], and a supersonic canard design (SSC) [24].

Table 3 PLdB values generated by PyLdB and LCASB for several ground signatures.

Ground Signature	PyLdB	LCASB	ΔPLdB	% Difference
NASA 25D	78.06	78.09	0.03	0.04%
Lockheed Martin N+2	80.08	80.14	0.06	0.08%
SBJ	97.08	97.10	0.02	0.02%
SSC	99.30	99.33	0.03	0.03%

Table 3 compares the resulting perceived loudness calculated by PyLdB and LCASB and shows that the loudness calculated by PyLdB fit very closely with those from LCASB. The magnitude of the percent difference between the two programs indicate that these errors are likely outside of the range to which the formulations in Appendix A are accurate. To validate PyLdB over a larger domain, a set of 10,000 N-wave-type signatures were generated with peak overpressures varying from 0.0 to 0.7 $\mathrm{lb/ft^2}$ and rise times from 0.0 to 0.2 s. The overpressure and rise time are defined in Fig. 10 on an N-wave pressure signature. The contour in Fig. 11 represents the error between the loudness calculated by PyLdB and the loudness calculated by LCASB for the domain covered by the 10,000 generated signatures. Note that the maximum error is approximately 0.20 PLdB.

Since common ground signatures are not perfect N-waves, a set of more realistic signatures were generated and tested in the same domain as the N-wave signatures. These signatures were created using an N-wave with a defined overpressure and rise time that was supplemented by five sine waves of the form

$$p(t) = A \sin(\omega t + \phi) \tag{14}$$

where A is the amplitude, ω is the frequency of oscillation and ϕ is the phase shift of the signal. This produced a ground signature like that shown in Fig. 12. The purpose of generating these signatures was not to create realistic ground signatures, but rather to subject PyLdB to a wide range of signatures and validate the results of PyLdB to those of LCASB. The five sine waves contained five evenly-spaced values for each of the parameters. The values of A ranged

*https://github.com/usuaero/PyLdB

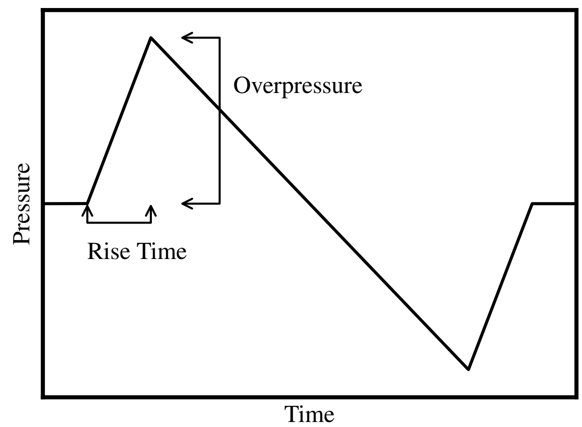

Fig. 10 Definition of overpressure and rise time in an N-wave.

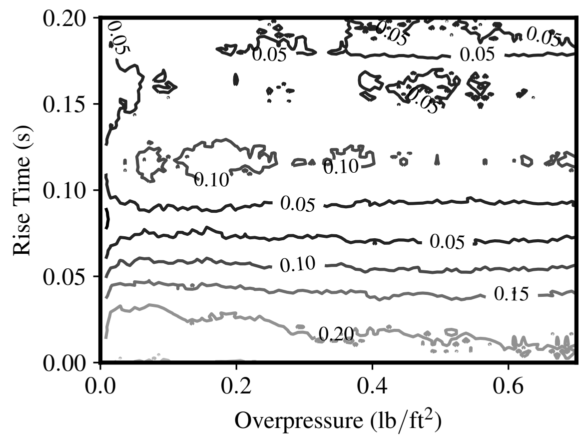

Fig. 11 Contour plot of the error between PyLdB and LCASB for N-waves.

from 0.01 to 0.03 $\mathrm{lb/ft^2}$, the values of ω ranged from 10 to 75 Hz, and the values of ϕ ranged from 0.01 to 0.1 rad. Figure 13 shows the error between PyLdB and LCASB of the sine-supplemented signatures. Note that the maximum error is approximately 0.7 PLdB.

Figures 11 and 13 show error values on the same order of magnitude as those reported in Table 3. This seems to indicate that there are discrepancies between the implementation of the Mark VII algorithm between the two programs, though the magnitude of the error is so low that there should be little cause for concern in the majority of applications.

To further investigate the error between PyLdB and LCASB, a series of tests were run to evaluate if the source of error could come from the approach taken by the two codes for windowing. The points used in the Hanning window were varied on both the N-waves and sine-supplemented N-waves. The overpressure was varied as before with the rise time set at a value of 100 ms. The error results are shown in Figs. 14 and 15 for the N-waves and sine-supplemented N-waves respectively.

Figures 14 and 15 show errors that are on the same order of magnitude as those shown in Figs. 11 and 13, indicating that the windowing is unlikely to be a significant cause of error between PyLdB and LCASB. The errors found in these studies fall within an acceptable range when considered alongside the precision inherent in the equations used. For this reason, the implementation discussed in this work can be considered an accurate model of perceived loudness calculation when compared to LCASB.

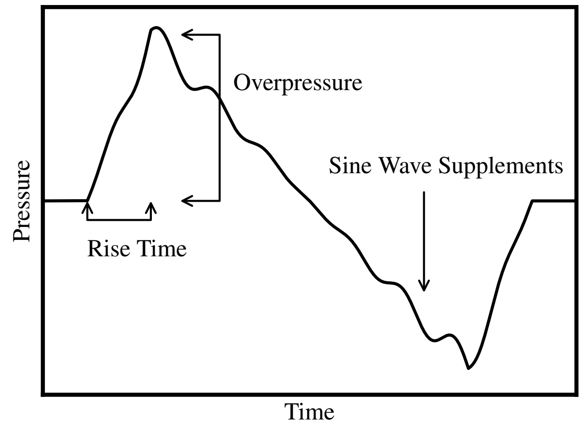

Fig. 12 Sine-wave-supplemented N-wave signature.

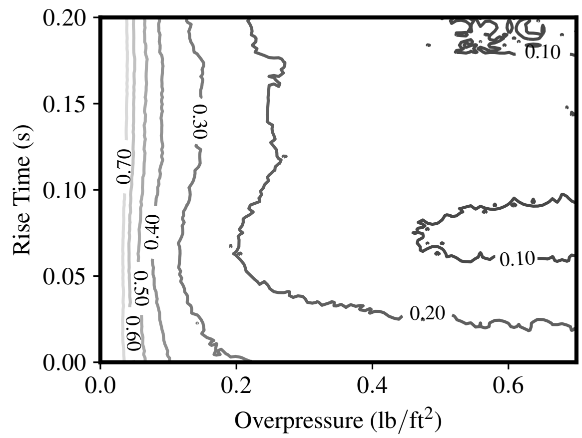

Fig. 13 Contour plot of the error between PyLdB and LCASB for sine-supplemented N-waves.

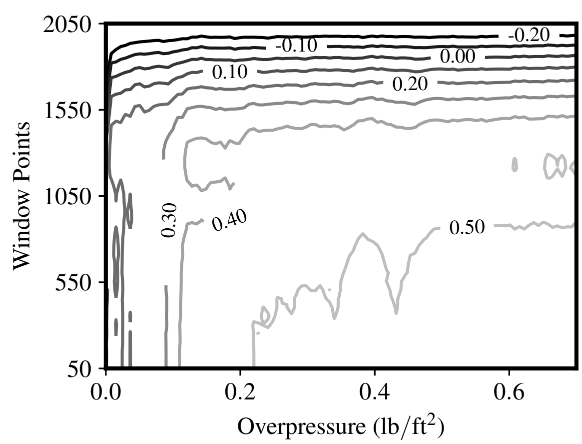

Fig. 14 Contour plot of the error between PyLdB and LCASB for windowed N-waves.

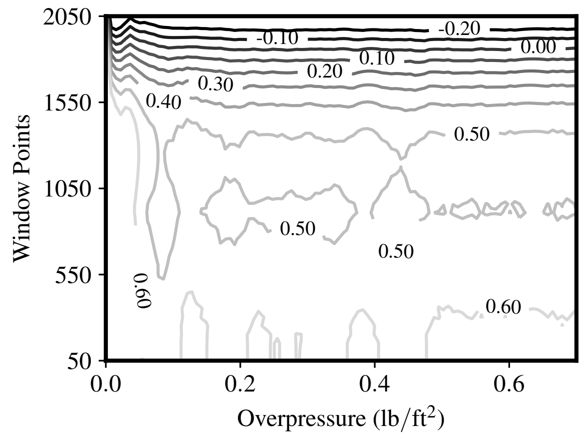

Fig. 15 Contour plot of the error between PyLdB and LCASB for windowed, sine-supplemented N-waves.

V. Conclusion

Stevens' Mark VII algorithm for the calculation of the perceived loudness from a pressure signature has been implemented and tested. Other methods, such as A, B, C, and D-weighting scales were discussed and shown to be less effective measuring devices for the loudness of sonic booms. Stevens' Mark VII procedure can be used in algorithmic form to calculate the PLdB of a sonic boom from a given pressure signature.

The Mark VII was implemented by first finding the nominal one-third octave band frequency limits. The pressure signature was then windowed using a Hanning window to reduce spectral leakage and zero-padded on the front and rear of the signature to increase the resolution of the FFT. The energy in each one-third octave band was calculated using Parseval's theorem, and a sound pressure level for each band was obtained. These sound pressure levels were converted into a perceived loudness using the perceived loudness calculations from Stevens' Mark VII algorithm.

An example was shown for a specific signature, which covered all of the steps in the Mark VII algorithm. The implementation of Stevens' algorithm used in this paper is called PyLdB. To validate the results obtained from PyLdB, the PLdB values were compared to those computed by the NASA LCASB program. Results were gathered using pressure signatures from four representative supersonic geometries. The difference in PLdB for all of the geometries was less than 0.1 %.

To investigate possible sources of error, N-wave signatures and sine-wave supplemented N-wave signatures were generated across a domain. Results showed that neither the shape of the N-wave or the way in which the signals were windowed had a marked effect on the error between PyLdB and LCASB. Future investigations on the effect of windowing and zero-padding could explain these results, though it is likely that the algorithm followed in the application of the Mark VII does not have the precision necessary to distinguish these errors.

The purpose of this work was to bring together all of the information necessary to implement Stevens' Mark VII procedure in an algorithmic format. This paper has outlined the implementation of the Mark VII algorithm from start to finish, with details behind the algorithm included. The implementation, PyLdB, has been validated using a LCASB, a benchmarked and widely accepted program, and can be used to determine the perceived loudness of a pressure signature.

Appendix A. Determining Equivalent Loudness Values

The method used to determine equivalent 3150 Hz loudness values in accordance with Table 2 is outlined here. The equations below relate directly to the equal-sone contours shown in Fig. 3. Additional details about the derivation of these equations can be found in Jackson and Leventhall's work [13].

For band frequencies above 8 kHz, the equivalent loudness can be found using

$$L_{eq} = (L_p - X)\ \text{dB} \tag{A.1}$$

with X taking values of 4 and 8 for the 10 and 12.5 kHz bands respectively. If working in band numbers, L_{eq} can be described using

$$L_{eq} = L_p + 4(39 - N_B)\ \text{dB} \tag{A.2}$$

where N_B is the band number.

Frequencies from 3.15 kHz to 8 kHz and band numbers from 30 to 35 have an equivalent 3150 Hz loudness equal to the loudness value found using Eq. (9).

For frequencies from 1.25 kHz to 3.15 kHz, Eq. (A.1) can be used, with X taking values of 6, 4, and 2 dB for the 1.6, 2, and 2.5 kHz bands respectively. Eq. (A.2) must be modified to accommodate band numbers 31 to 35, and can be written as

$$L_{eq} = L_p - 2(35 - N_B)\,\text{dB} \tag{A.3}$$

For frequencies from 400 Hz to 2.5 kHz, with corresponding band numbers of 26 to 31, Eq. (A.1) can be used with X set equal to 8.0.

The frequencies below 400 Hz in Fig. 3 converge logarithmically to a point that depends on the L_p found in the band. Frequencies from 80 Hz to 400 Hz converge to 115 dB at 1 Hz, and can be split into three ranges based on the value of L_p found in that band. The L_p limits corresponding to each band in this range can be seen in Table A.1.

Table A.1 Loudness limits for one-third octave bands from 80 Hz to 400 Hz used in calculating L_{eq}.

f_C (Hz)	L_ℓ (dB)	L_u (dB)
80	86.5	131.5
100	85.0	130.0
125	83.5	128.5
160	82.0	127.0
200	80.5	125.5
250	79.0	124.0
315	77.5	122.5
400	76.0	121.0

If the value of L_p for a given band in this range is less than the lower limit in Table A.1, then the equivalent loudness can be found using

$$L_{eq} = (A - 8)\,\text{dB} \tag{A.4}$$

where A is defined as

$$A = 115 - \frac{(115 - L_p)\log_{10}(400)}{\log_{10}(f_C)}\,\text{dB} \tag{A.5}$$

using central frequency values, and

$$A = 115 - 26\,\frac{(115 - L_p)}{N_B}\,\text{dB} \tag{A.6}$$

using band numbers. A represents the equivalent loudness at 400 Hz, which can then be converted to an equivalent loudness at 3150 Hz using Eq. (A.4).

If the value of L_p lies between the lower and upper limit described, the equal-sone contours have a constant slope approaching 1 Hz, and A is described using

$$A = (L_p - X)\,\text{dB} \tag{A.7}$$

with X taking values of 1.5, 3.0, 4.5, 6.0, 7.5, 9.0, and 10.5 for the 315, 250, 200, 160, 100, and 80 Hz bands respectively. In terms of band numbers, for this range, A is defined as

$$A = L_p - 1.5\,(26 - N_B)\,\text{dB} \tag{A.8}$$

For values of L_p greater than the upper limit, the equal-sone contours converge to 160 dB logarithmically, and A is calculated

$$A = 160 - \frac{(160 - L_p)\log_{10}(400)}{\log_{10}(f_C)}\,\text{dB} \tag{A.9}$$

using frequencies, and

$$A = 160 - 26\,\frac{(160 - L_p)}{N_B}\,\text{dB} \tag{A.10}$$

using band numbers.

For frequencies less than 80 Hz, an additional conversion factor, B must be utilized to find the equivlent 3150 Hz loudness values. B represents the equivalent loudness at 80 Hz and is calculated as

$$B = 160 - \frac{(160 - L_p)\log_{10}(80)}{\log_{10}(f_C)}\ \text{dB} \tag{A.11}$$

using frequencies, and

$$B = 160 - 19\,\frac{(160 - L_p)}{N_B}\ \text{dB} \tag{A.12}$$

using band numbers. Using the limits shown in Table A.1 for the 80 Hz band, and setting L_p equal to B and f equal to 80, a value of A can be found depending on the value of L_p. A can then be used with X equal to 10.5 to find the value of L_{eq}.

Appendix B. Sones Conversion Tables

Table B.1 Conversion from L_{eq} to loudness in sones.

L_{eq} (dB)	S (sones)	L_{eq} (dB)	S (sones)	L_{eq} (dB)	S (sones)	L_{eq} (dB)	S (sones)
1	0.079	36	1.36	71	20.2	106	299
2	0.087	37	1.47	72	21.8	107	323
3	0.097	38	1.59	73	23.5	108	348
4	0.107	39	1.71	74	25.4	109	376
5	0.118	40	1.85	75	27.4	110	406
6	0.129	41	2.00	76	29.6	111	439
7	0.141	42	2.16	77	32.0	112	474
8	0.153	43	2.33	78	34.6	113	512
9	0.166	44	2.52	79	37.3	114	553
10	0.181	45	2.72	80	40.3	115	597
11	0.196	46	2.94	81	43.5	116	645
12	0.212	47	3.18	82	47.0	117	697
13	0.230	48	3.43	83	50.8	118	752
14	0.248	49	3.70	84	54.9	119	813
15	0.269	50	4.00	85	59.3	120	878
16	0.290	51	4.32	86	64.0	121	948
17	0.314	52	4.67	87	69.1	122	1024
18	0.339	53	5.04	88	74.7	123	1106
19	0.367	54	5.44	89	80.6	124	1194
20	0.396	55	5.88	90	87.1	125	1290
21	0.428	56	6.35	91	94.1	126	1393
22	0.463	57	6.86	92	102	127	1505
23	0.500	58	7.41	93	110	128	1625
24	0.540	59	8.00	94	119	129	1756
25	0.583	60	8.64	95	128	130	1896
26	0.630	61	9.33	96	138	131	2048
27	0.680	62	10.1	97	149	132	2212
28	0.735	63	10.9	98	161	133	2389
29	0.794	64	11.8	99	174	134	2580
30	0.857	65	12.7	100	188	135	2787
31	0.926	66	13.7	101	203	136	3010
32	1.00	67	14.8	102	219	137	3251
33	1.08	68	16.0	103	237	138	3511
34	1.17	69	17.3	104	256	139	3792
35	1.26	70	18.7	105	276	140	4096

Table B.2 Conversion from S_m to a summation factor F.

S_m (sones)	F	S_m (sones)	F	S_m (sones)	F
0.181	0.100	2.16	0.308	25.4	0.190
0.196	0.122	2.33	0.304	27.4	0.190
0.212	0.140	2.52	0.300	29.6	0.190
0.230	0.158	2.72	0.296	32.0	0.190
0.248	0.174	2.94	0.292	34.6	0.190
0.269	0.187	3.18	0.288	37.3	0.190
0.290	0.200	3.43	0.284	40.3	0.191
0.314	0.212	3.70	0.279	43.5	0.191
0.339	0.222	4.00	0.275	47.0	0.192
0.367	0.232	4.32	0.270	50.8	0.193
0.396	0.241	4.67	0.266	54.9	0.194
0.428	0.250	5.04	0.262	59.3	0.195
0.463	0.259	5.44	0.258	64.0	0.197
0.500	0.267	5.88	0.253	69.1	O.199
0.540	0.274	6.35	0.248	74.7	0.201
0.583	0.281	6.86	0.244	80.6	0.203
0.630	0.287	7.41	0.240	87.1	0.205
0.680	0.293	8.00	0.235	94.1	0.208
0.735	0.298	8.64	0.230	102	0.210
0.794	0.303	9.33	0.226	110	0.212
0.857	0.308	10.1	0.222	119	0.215
0.926	0.312	10.9	0.217	128	0.217
1.00	0.316	11.8	0.212	138	0.219
1.08	0.319	12.7	0.208	149	0.221
1.17	0.320	13.7	0.204	161	0.223
1.26	0.322	14.8	0.200	174	0.224
1.36	0.322	16.0	0.197	188	0.225
1.47	0.320	17.3	0.195	203	0.226
1.59	0.319	18.7	0.194	219	0.227
1.72	0.317	20.2	0.193	237	0.227
1.85	0.314	21.8	0.192	256	0.227
2.00	0.311	23.5	0.191		

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References

[1] Stevens, S., "Perceived level of noise by Mark VII and decibels (E)," *The Journal of the Acoustical Society of America*, Vol. 51, No. 2B, 1972, pp. 575–601.

[2] Anderson, J., *Fundamentals of Aerodynamics*, McGraw-Hill series in aeronautical and aerospace engineering, McGraw-Hill Education, 2016.

[3] Fletcher, H., and Munson, W. A., "Loudness, its definition, measurement and calculation," *Bell Labs Technical Journal*, Vol. 12, No. 4, 1933, pp. 377–430.

[4] Young, R. W., and Peterson, A., "On estimating noisiness of aircraft sounds," *The Journal of the Acoustical Society of America*, Vol. 45, No. 4, 1969, pp. 834–838.

[5] Aarts, R. M., "A comparison of some loudness measures for loudspeaker listening tests," *Journal of the Audio Engineering Society*, Vol. 40, No. 3, 1992, pp. 142–146.

[6] Bennett, R. L., and Pearsons, K. S., "Handbook of aircraft noise metrics," 1981.

[7] Pierre Jr, R. L. S., Maguire, D. J., and Automotive, C. S., "The impact of A-weighting sound pressure level measurements during the evaluation of noise exposure," *Conference NOISE-CON*, 2004, pp. 12–14.

[8] ISO, B., "226: 2003: Acoustics–Normal equal-loudness-level contours," *International Organization for Standardization*, Vol. 63, 2003.

[9] McMinn, T., "A-weighting: Is it the metric you think it is?" *Acoustics 2013*, 2013, pp. 1–4.

[10] Loubeau, A., Naka, Y., Cook, B. G., Sparrow, V. W., and Morgenstern, J. M., "A new evaluation of noise metrics for sonic booms using existing data," *AIP Conference Proceedings*, Vol. 1685, AIP Publishing, 2015, p. 090015.

[11] Leatherwood, J. D., Sullivan, B. M., Shepherd, K. P., McCurdy, D. A., and Brown, S. A., "Summary of recent NASA studies of human response to sonic booms," *The Journal of the Acoustical Society of America*, Vol. 111, No. 1, 2002, pp. 586–598.

[12] Johnson, D., and Robinson, D., "Procedure for calculating the loudness of sonic bangs," *Acta Acustica united with Acustica*, Vol. 21, No. 6, 1969, pp. 307–318.

[13] Jackson, G., and Leventhall, H., "Calculation of the perceived level of noise (PLdB) using Stevens' method (Mark VII)," *Applied Acoustics*, Vol. 6, No. 1, 1973, pp. 23–34.

[14] Shepherd, K. P., and Sullivan, B. M., "A loudness calculation procedure applied to shaped sonic booms," 1991.

[15] Hayes, C. D., and Lamers, M. D., "Octave and One-third Octave Acoustic Noise Spectrum Analysis," 1967.

[16] ISO, "497: 1973: Guide to the choice of series of preferred numbers and of series containing more rounded values of preferred numbers," *International Organization for Standardization*, Vol. 5, 1973.

[17] Harris, F. J., "On the use of windows for harmonic analysis with the discrete Fourier transform," *Proceedings of the IEEE*, Vol. 66, No. 1, 1978, pp. 51–83.

[18] Parseval, M.-A., "Mémoire sur les séries et sur l'intégration complète d'une équation aux différences partielles linéaires du second ordre, à coefficients constants," *Mém. prés. par divers savants, Acad. des Sciences, Paris,(1)*, Vol. 1, 1806, pp. 638–648.

[19] Oppenheim, A. V., Willsky, A. S., and Nawab, S. H., *Signals &Amp; Systems (2Nd Ed.)*, Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1996.

[20] Atkinson, K. E., *An introduction to numerical analysis*, John Wiley & Sons, 2008.

[21] Ordaz, I., Geiselhart, K. A., and Fenbert, J. W., "Conceptual Design of Low-Boom Aircraft with Flight Trim Requirement," *Journal of Aircraft*, Vol. 52, No. 3, 2015, pp. 932–939.

[22] Morgenstern, J., Buonanno, M., Yao, J., Murugappan, M., Paliath, U., Cheung, L., Malcevic, I., Ramakrishnan, K., Pastouchenko, N., Wood, T., et al., "Advanced concept studies for supersonic commercial transports entering service in the 2018-2020 period phase 2," 2015.

[23] Mack, R. J., "A supersonic business-jet concept designed for low sonic boom," 2003.

[24] Chan, M. K., "Supersonic aircraft optimization for minimizing drag and sonic boom," PhD Dissertation, Stanford, California, 2003.