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Evidence of Power-law Behavior in Cognitive IoT Applications

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Abstract The rapidly evolving functionalities of the Internet of things (IoT) has led to the concurrence of a more intricate and interconnected network of devices. These devices specifically relate to a generic set of devices like the

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sensory devices, smart appliances, smart hubs, and other associated objects which permissively provide a ubiquitous connectivity among the devices. This is considered to be one of the most crucial prerequisites of the IoT technologies as several devices need to be included in the convolution for the acquisition, extraction, storage, processing, and other operations. The involvement of an extensively large number of devices adds to the complexity in the connectivity and communication between the devices. Thus in order to understand the scalability and the heterogeneity of these interconnected devices, the power-law exponent is an important measure. In this paper, we provide an analytical framework to illustrate the ubiquitous power-law behavior in the devices connected to the internet. It also provides the mathematical insight to characterize the dynamic behavior of the devices connected to the IoT. It emphasizes on the traits of the wireless sensor networks (WSNs), which are the elementary constituent of the IoT, and its future development and scopes in prospect of contemporary technologies.

Keywords Internet of Things · Wireless Sensor Networks · Power-law · Scalability · Interconnectivity

1 Introduction

In recent years, the internet of things (IoT) is experiencing a revolutionary upthrust in confronting the requirements of huge number of consumers globally by providing solutions for connecting a multitude of physical world devices serving enormous functionalities. The expansion of the IoT and its associated services have provided enormous solutions for the healthcare, academia, and business industries to name a few [1,2,3,4]. Attributing to the cutting edge growth in the performance of the communication technology and the scope of the sensing devices, the proficiency of the IoT has scaled up considerably. However, in order to achieve an entirely functional model for the IoT ecosystem, several crucial performance benchmarks are to be considered such as heterogeneity of the devices and interconnection network, topological constraints, efficient management of resources, interoperability among the devices, and optimizing the power consumption [9,10,11,12,13]. These factors are extensively dependent on the density of the network and the diverse collection of devices involved in the network.

In order to accomplish the transformations brought about by the expansion of the IoT, various reference models are to be taken into consideration. Several leading standards development organizations like the ITU-T, have been very effective throughout the principal development phase of several technologies and have made vital recommendations to enhance the capabilities of these technologies [28,29,30,31]. Apart from all these developments, there are some generic consensus which are yet to be solved on some of the most demanding technical potentiality of IoT. These demands are aimed at providing heterogeneity in the interconnection networks, handling multiplicity of the devices, minimizing the network complexities, improving the energy efficiency, and managing the numerous resources. All these necessities are predominantly influenced by the capabilities of the IoT system sustained by a massive collection of connected devices.

In the real world, many complex topological structures envelope a wide collection of heterogeneous systems, or devices, with specialized technological and computational capabilities. Initially, the application of complex networks was studied in connection to graph theory. In graph theory the fundamental focus was on regular graphs and their properties. In the 1950s the conception of large scale networks came into existence, but there were no specific design principles defined for random graphs. This lack of explicit design principles made the characterization of the anatomy of random networks more inevitable in order to understand the underlying dynamics of the complex networks. In [23,24,25,26,27], a more acceptable conceptualization of the random graph theory and the complex networks was presented, which subsequently lead to the advancements in substantial areas of network science.

At present several networks (like complex networks) are portraying an extensive degree of heterogeneity in nodes. Some popular example where these patterns can be distinctively observed include the World Wide Web (WWW), the internet, social networks, and the networks observed in aircrafts. There have been several studies on complex networks which reveal that in the course of progression in the topological support of network, certain types of emanating networks, referred to as the scale-free networks have emerged. These networks encompass several peculiarity in their coherent traits and parameters by displaying intrinsic heterogeneity in their behavior. This heterogeneous behaviour is observed due to the preferential attachment, which enables the nodes to establish several connections with the set of nodes initially present in the network [23,32,8,33]. The degree of the nodes evidently exhibit heterogeneity in their degree distribution, such that the network's degree follows the power-law distribution. The nodes involved in these networks have relatively short distance of inter-connectivity, which complies to the small-world property. These systems are very robust towards node failures, due to the densely interconnected network of nodes.

In this paper, we present a brief summary of some of the evolving dynamic networks technologies. We then provide an insight to the basic terminologies relevant to our study like the random graph theory concepts, the small-world concept, and the scale-free networks. We then switch to the convergence of these technologies with IoT, and conceptualize the scale-free traits of IoT. We then present the proposed IoT based framework for handling dynamic networks. These networks are observed to provide more befitting solutions as compared to the conventional theories of achieving robustness in communication networks. We then formalize the proposed scheme with several well-known mathematical models and establish the basis for our proposed scheme. We the provide the simulation results for the proposed model and provide an analysis of the theoretical results and the experimental results. We finally present the conclusive remarks on our proposed scheme with some suitable future directions.

2 Background

Most of the physical world objects, irrespective of their capabilities, can be efficiently converged into a single functional unit by utilizing the networking abilities of IoT. In 1960, Erdös and Rênyi [6], proposed the random graph model for large-scale networks, which was designed by combining the conventional approaches of the graph-theoretic concepts and the notion of statistical physics. In [7], Watts and Strogatz studied the importance of the small-world networks for some dynamical systems. They extended the regular graph model by introducing some irregularities to its structure, to better study the versatility of dynamical systems. Their model showed properties analogous to the random graphs, in terms of path lengths.

Barábasi and Albert [8] studied the scale-free properties of large-scale networks. They put forward two generic rules : (i) The new vertices associated with a network followed preferential attachment; and (ii) The network topology displayed continuous expansion with the inclusion of new vertices. These features gave rise to the concept of the scale-free distributions in large networks. This model illustrates the self-organizing properties of the vertices involved in a network, which resembles the fundamental properties of real WSNs [50,51,52,53]. The applications of several probabilistic models can be observed in the studies [41,42,44,43]. These systems worked well under the influence of growing uncertainties in the system. Barrat et al. [49], studied a network model which considered the edge weights of each connected node. This weight parameters enables the addition of several essential functionalities associated with a network. Some of them include the variations in the connection strength, factors affecting the connection, intensity of the connection, and so on. This type of network is most suitable for evaluating the performance of the WSNs involved in the IoT.

3 Basic Concepts

In this section, some of the primary concepts associated with the evolution of the scale-free networks is discussed. A brief account on the evolution and applicabilities of the random graphs is first given, which resulted in the invention of various evolving technologies. We start with the elementary concepts of the random graphs and then quickly upsurge to the unfolding traits of the scale-free networks.

3.1 Random Networks

The evolution of the random graphs has been an elemental revelation to many developing technologies. Over the years several patterns in nature as well as complex systems have evolved around this theory. In [15], Erdös and Rényi, were the first to bring into focus the concept of random graphs. Random graphs have evolved as a consequence of the irregularities observed in the topological arrangements of the regular graphs [32,15,16,17]. The emergence of random graphs have induced a rich set of concepts associated with the degree distribution of the graphs and have favored the growth of several associated perceptions. The complexity of arrangements observed specifically in the random networks and the uncertainty of their organization have made their study prominent while dealing with complex networks. The random graph theory has several applications, to mention a few, it can be used for determining the connectedness of the network, it can be used for handling attacks in random networks, along with application sin exploratory tree structures [40].

3.2 Small-World Networks

The small-world systems are mostly observed to be highly clustered, which is quite similar to the behavior portrayed by regular lattices [8]. This phenomena can be observed in several research applications like the spread of contagious diseases, social networks, and the internet [20,19,21]. A peculiar examples of the small-world networks can be observed in the electricity grid distribution lines, as the distribution lines need to be highly resilient towards the occurrence of any failures in the transmission grid. The small-world networks show an increasingly growing amount of disorder in the network topology which makes it suitable for modelling the behaviour of dynamical systems [18].

3.3 Scale-Free Networks

In 1999, Barábasi and Albert [8,22], initially studied the evolution of scale-free networks which were based on the behavior of real world networks. A large number of real world networks display the scale-free property. The scale-free networks initially constitute of a fixed number of nodes in the network, which shows continuous growth with the addition of randomly incoming nodes. These newly added nodes require to be connected to the existing network, without interfering with the existing topology of the nodes.

Fig. 1, shows the topology of the scale-free network, where the newly added nodes are connected to the central node. The large scale-free networks may constitute of several central nodes. These central nodes usually have a higher number of connections as compared to the other nodes present in the network.

There are two important feature of the scale-free network:

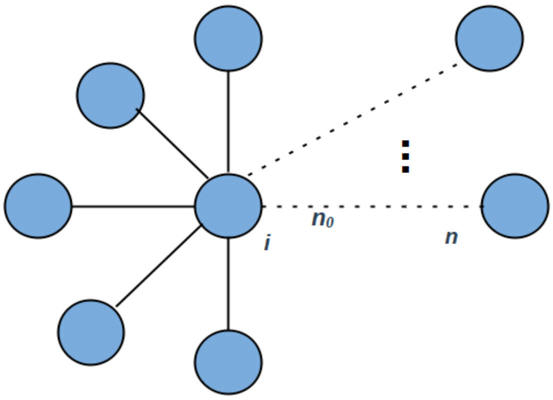


Fig. 1: A topological representation of the scale-free network with the newly added nodes.

- Topological growth: As the new incoming nodes are continuously added to the network, the network scales up. A system constituting of an initial number of nodes (say n_0) is considered. If we consider a random set of incoming nodes (say R_n), such that the new node n (for $n \in R_n$) with e_n edges, such that $e_n \leq n_0$.
- Preferential attachment of nodes: While selecting from the existing set of nodes with which the newly added nodes can be connected , a probability based attachment principle is used [8,32] which enables the new node to connect to the existing set of nodes (say N) depending upon the degree (say k_i) of the node N nodes.

4 Convergence with IoT

In this section, we conceptualize the existence of the power-law behaviour observed in the topological arrangement of dynamic WSNs, and its relevance for characterizing the increased amount of uncertainty observed in the degree distributions of large heterogeneously spread IoT networks. These networks are exclusively tolerant towards the changes caused by the addition of new incoming sensor nodes in the network, and are also more reliable in-terms of the occurrence of node failures.

4.1 Conceptualizing the Small-World Scenario for WSNs

Wireless sensor networks (WSNs), have enormously developed in their potentiality and configurations over the years. In order to perceive the continually changing events a densely deployed WSN is required for obtaining the factual and cumulative data of the environment. This involves the association of a huge number of coupled sensor nodes over a large area for sensing the data coming from heterogeneous sources. The WSNs usually are portrayed as self-organizing systems which efficiently customize the network topology of the sensor nodes, for the dynamic management of the network activities. These traits distinguish the WSNs from most static, wired, and constrained systems, by making them more adaptable to faults, distance of communication, and power constraints. Due to it's capabilities of being versatile, and ubiquitous clustered, the WSNs display high degree of disorder in their organization.

In recent times, the interest in the study of the small-world properties of different scenarios is expanding hugely. Several researchers have explored alternating fields of the real world and have observed intriguing outcomes, which obey the small-world property. Unlike the properties of the regular lattices, the networks which obey the small-world property have relatively higher connectivity among the nodes, they show considerable variations in their degree distributions, and have reduced path lengths between the node. In [24,34,35, 36], the authors have extensively addressed the issues in genomic sequencing in-terms of the small-world network. Further the studies in [37,38,39], have revealed that the functionalities of the human brain network follows the small-world property in response to external stimuli.

The topology of the WSNs is said to follow the small-world property characterized by the densely clustered sensor nodes possessing diverse sensing capabilities. Previously several issues in the WSNs prevented their extensive use in a multitude of fields. Some of the drawbacks in the static network topology of the WSNs, are as follows:

- Inaccuracy in the Signal: Due to the increase in the distance between the two communicating nodes, the propagated signal usually lacks accuracy at the receiving end. These inaccuracies are induced from several conflicting situations like presence of huge obstacles in the ray of sight (ROS) of the signal, delays caused due to excessive path lengths.

- Node Failure: The sensors nodes and microcontrollers involved in a WSN have usually limited power resources. While accomplishing the exchange of information between far off nodes, these nodes usually loose their efficiency due to the constrained power supply. Hence the network may suffer from node failures, their by minimizing the reliability of the WSN.

- Energy Wastage: In static WSNs, there is no source for regulating the activation and the deactivation of the sensor nodes. All the nodes in the network work uniformly at the same level. This sometimes results in the wastage of energy resources of the sensor nodes as in a network of sensor nodes not all the sensors need to be employed concurrently for accomplishing a certain task.

– Network Topology: The topology of the static WSNs is similar to that of the regular lattice. It follows an order degree distribution. This topology may not satisfying for continuously changing requirements. Thus making it unsuitable for largely growing networks.

Hence it can be said that the regular WSNs may fail to efficiently handle the increase in the number of sensor nodes, and the growing interconnection network topology. Thus bringing about the need for a more robust WSN topology, which can adaptively handle the increase in the amount of uncertainty in the network. This brings in the approach for a large network constituted of heterogeneous sensor nodes with distinct sensing capabilities. These nodes need to be organized in a highly coupled interconnection network such that, they provide an accurate exchange of data. Further this network organization is much desired as, it reduces the delay in transmission caused due to lengthy communication paths, and works well against node failures, in an integrative way. These systems are ideal especially in heathcare sectors, and the defense, as they facilitate an uninterrupted flow of data throughout the system, without causing any bifurcations in the perceived data. Thus the notion of the small-world properties of the WSN emerges, which can be appropriately advantaged as a working model for providing an integrated characterization of the dynamic WSNs. By conceptualizing the small-world properties of the WSNs, we can achieve befitting results in terms of the robustness of the network, functionality, reduced propagation-delays, and improved signal quality.

4.2 Conceptualizing the Scale-Free Networks for IoT

In this section, we extend our study towards the application of the scale-free properties of IoT. From the studies made in [48,49], it is evident that the scale-free networks provide a better understanding of the frequently changing networks and for networks which have varying flow of information throughout the system. The most fundamental functional modules of the IoT constitute of the sensor nodes which are used for the inception of multiple events. In order to infer the data from a wider area, a huge WSN is desired with immensely high performance constraints. Large WSNs usually require to be dynamic in order to efficiently utilize the resources involved. The WSNs with dynamic properties show a high rate of clustering with the cluster head (or, the central node) being connected to a larger number of sensor nodes. Thus the cluster heads are rich in the number of links they establish with each sensor node in the cluster, and are therefore said to have a higher degree distribution.

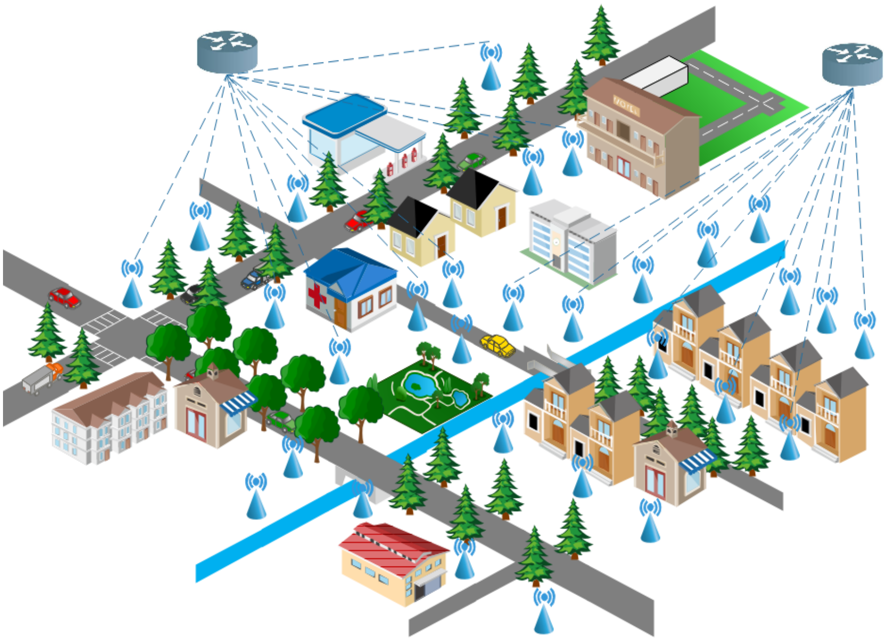
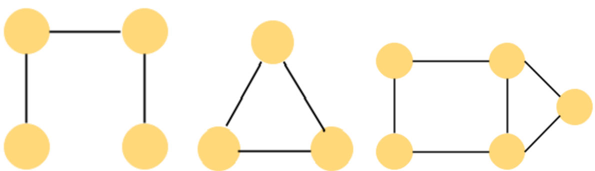


Fig. 2: Deployment scenario for various IoT devices.



In densely clustered networks like the one desired for modeling the IoT with a heterogeneous set of sensor nodes [22,8,48], a huge number of links between the sensor nodes exists. This makes the network more resilient towards link failures and can therefore be used for dynamic systems. When the number of sensor nodes in the IoT architecture scale up continually, the system becomes more vulnerable to failures. Thus, the dynamic capabilities of the scale-free networks makes it suitable to model the IoT with huge number of nodes.

Fig. 2, shows a scenario for the deployment of several sensor nodes in a city with each central node connected to the sink or, the gateways. This particular case is quite similar to the study in [48], with the gateways acting as the central nodes for several other central nodes (cluster heads) present in the cluster of the sensor nodes. These gateways have a higher degree of connection as observed in [48,49].

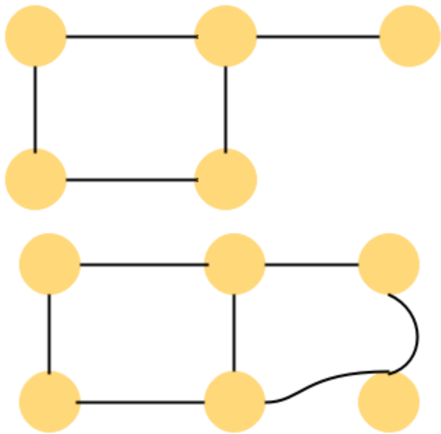


Fig. 3: An example of a graph for addition of a new node in a scale-free network.

Fig.3, shows the basic graph examples with varying number of nodes and different topological arrangements.

4.3 Generation of scale-free WSN

In this section, we provide the simulations in MATLAB for generating the scale-free networks [8,45]. We have considered three specific cases for which we generate the scale-free sensor networks. The first case which we have considered is constituted of 100 nodes ($N = 100$), with the average degree distribution of the nodes as $\langle k \rangle = 4$, the overall links (or edges) among the sensor nodes in the network is $E = 400$, and the interconnection probability of the sensor nodes is $p = 0.5$, such that $0 < p < 1$. In [32], it is said that with the increase in the probability of the interconnection link p, the amount of uncertainty of the network increases, till it reaches 1. Fig. 4, shows the simulated sensor network with the above parameters. In the second case, we consider a WSN with number of sensor nodes as $N = 50$, the total number of links as $E = 200$, the average degree distribution given by $\langle k \rangle = 4$, and the interconnection probability as $p = 0.6$. The network architecture can be observed in Fig. 5. In the third case, we consider a WSN with 20 nodes ($N = 20$), the degree distribution of the nodes is taken as $\langle k \rangle = 4$, with 120 links ($E = 120$), and an interconnection probability of 0.4 ($p = 0.4$). These networks favor the dynamic behaviour of the IoT, by augmenting the performance of the system in-terms of reliability, ubiquity, and robustness. Table 1, provides a comprehensive account of the specific simulation parameters used for generating the three scale-free WSNs.

Algorithm 1, shows a procedure for generating a random collection of sensor nodes which is inline with [32]. Here we consider a dynamic WSN, such that N represents the set of nodes involved in the system. Initially the system is said to have n_0 nodes, along with a degree distribution of the i^{th} sensor nodes represented as k_i. If we consider n to be the set of existing nodes, such that $n \in N$, then the n_i randomly incoming sensor nodes can be incrementally added to the WSN. The algorithm generates a random set of sensor nodes based on the probability of the degree distribution of the sensor nodes given as,

$$\prod(k_i) = \frac{k_i}{\sum_j k_j}. \tag{1}$$

Thus from Eq. (1), we obtain the optimum value for the degree distribution of the sensor nodes.

Algorithm 1: An algorithm for random node generation for scale-free WSNs

Input : n_0, k_i, n
Output: k_i
$N \leftarrow$ set of all the nodes present in the system.
$n_0 \leftarrow$ set of nodes initially present in the system such that $n_0 \in N$.
$k_i \leftarrow$ degree distribution of all the nodes in the system.
$n \leftarrow$ set of existing nodes.
$n_i \leftarrow$ set of newly added nodes.
$k_i \leftarrow$ degree distribution of newly added nodes.
for $n_i! = 0$ *and* $n_i \in N$ **do**
    $n = n + n_i$
    Compute:
    $\prod(k_i) = \frac{k_i}{\sum_j k_j}$
end

5 The IoT Framework

In this section, we present two holistic frameworks based on the conventional small-world phenomena and the scale-free networks [32,8,49]. The applicability of these two scenarios have been previously discussed in Section-4. It was

Table 1: Parameters for generation of random nodes for the scale-free IoT framework.

Nodes (N)	Edges (E)	Probability (p)	Node degree (k)
100	400	0.4	4
50	200	0.6	4
20	120	0.4	6

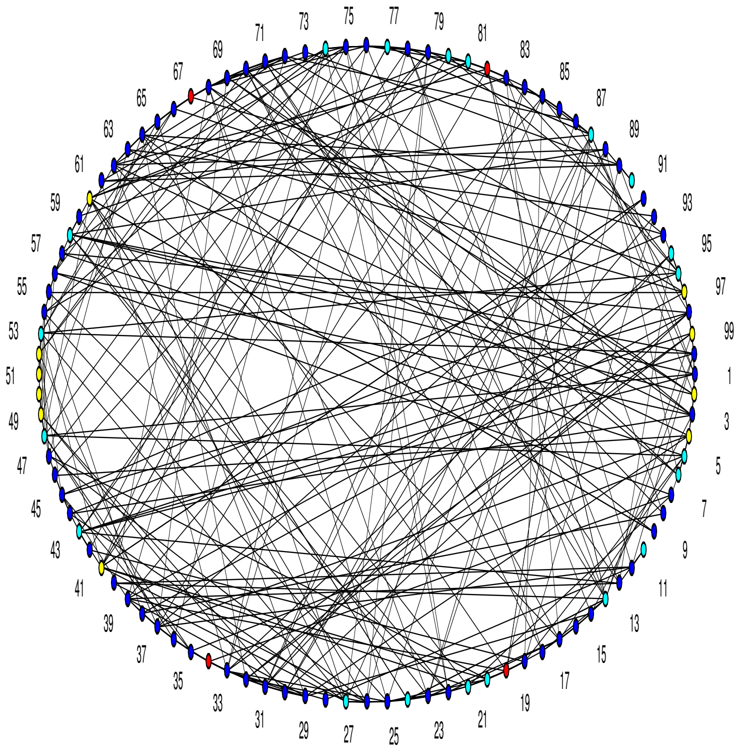


Fig. 4: Representation of the scale-free sensor nodes for $N = 100$, $E = 400$, $p = 0.5$, $\langle k \rangle = 4$.

observed that large network structures like the IoT, which are characteristically constituted of a relatively heterogeneous set of sensor nodes, converge with the dynamical systems with temporally changing configurations. Thus by accomplishing a scale-free model for the IoT, the resources can be better utilized (like power supply and memory constraints), which consequently provide a system with enhanced capabilities. It is observed that the degree distribution of the WSNs, under the influence of uncertainties in the dynamic network, follows a power-law distribution. Some other applications of the power-law distribution can be found in [47,46].

5.1 The Small World Scenario

If we consider a WSN to have node set, $v = \{v_1, v_2, \ldots, v_n\}$ where $n = |v|$ denotes the total number of nodes in the WSN.

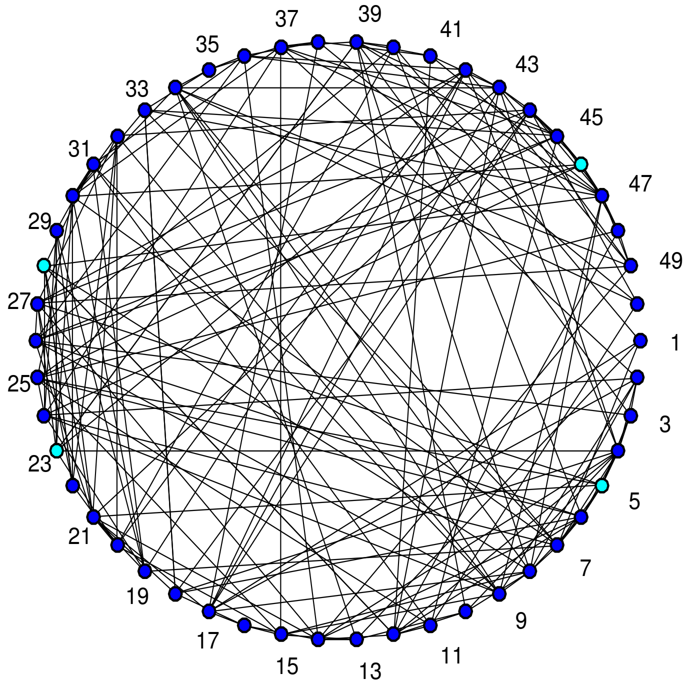


Fig. 5: Representation of the scale-free sensor nodes for $N = 50$, $E = 200$, $p = 0.6$, $\langle k \rangle = 4$.

In the random or small-world WSN topology. the connection probability p with i node having k_i degrees follow binomial distribution with parameters n and p i.e.,

$$k_i = B\left(n, p\right). \tag{2}$$

So,

$$F\left(k_i = k\right) = C_k^{n-1} p^k {(1-p)}^{n-1-k}. \tag{3}$$

The probability of i^{th} node connected k existing nodes is p^k, similarly, probability of the number of edges connected to the i^{th} node is ${(1-p)}^{n-1-k}$.

Let X_k be any random variable, representing the number of nodes in the WSN with degree k. Hence the expectation value for the number of nodes with degree k can be obtained from Eq.(3) as follows,

$$E\left(X_k\right) = nF\left(k_i = k\right) = \alpha_k, \tag{4}$$

where,

$$\alpha_k = nC_k^{n-1} p^k {(1-p)}^{n-1-k}. \tag{5}$$

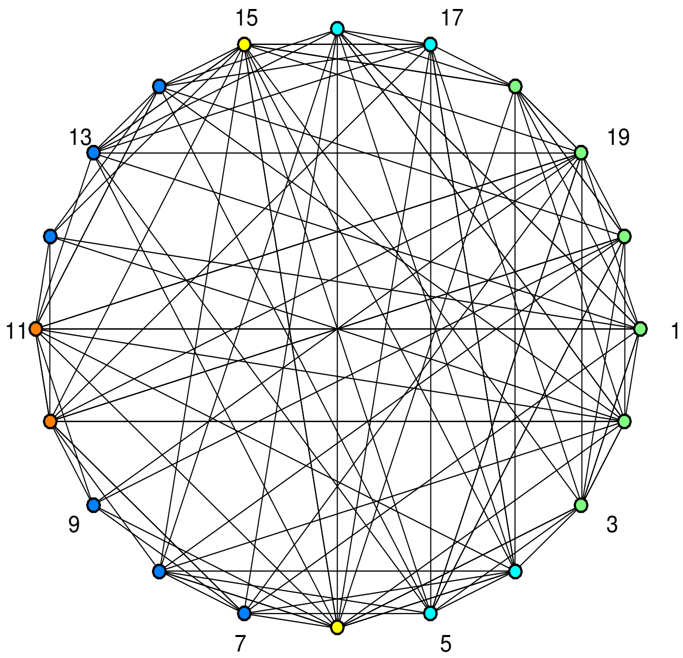


Fig. 6: Representation of the scale-free sensor nodes for $N = 20$, $E = 120$, $p = 0.4$, $\langle k \rangle = 6$.

When, $n >> 0$, and p is very small, the Eq.(3) can be rewritten as,

$$p(k) = \lim_{n\to\infty} \frac{(n-1)(n-2)\cdots(n-2-k)(n-1-k)!}{k!(n-1-k)!} \times \left(\frac{\alpha_k}{n}\right)^k \left(1-\frac{\alpha_k}{n}\right)^{n-1-k} \tag{6}$$

$$p(k) = \lim_{n\to\infty} \frac{n-1}{n-1}.\frac{n-2}{n-1}.\frac{n-3}{n-1}. \ldots \frac{n-1-(1+k)}{n-1} \left(\frac{{\alpha_k}^k}{k!}\right)\left(1-\frac{\alpha_k}{n}\right)^{n-1}\left(1-\frac{\alpha_k}{n}\right)^{-k} \tag{7}$$

$$\lim_{n\to\infty}\left(1-\frac{\alpha_k}{n}\right)^{n-1} = e^{-\alpha_k} \tag{8}$$

$$F(X_k = k) = \frac{e^{-\alpha_k}\alpha_k^k}{k!} = \frac{e^{-\langle k\rangle}\langle k\rangle^k}{k!} \tag{9}$$

where $\langle k \rangle$ is the average degree of the random networks or small world WSN topologies. Therefore from Eq.(9), it is evident that for large number of nodes the degree distribution of the random, or small-world WSN topology follows Poisson distribution.

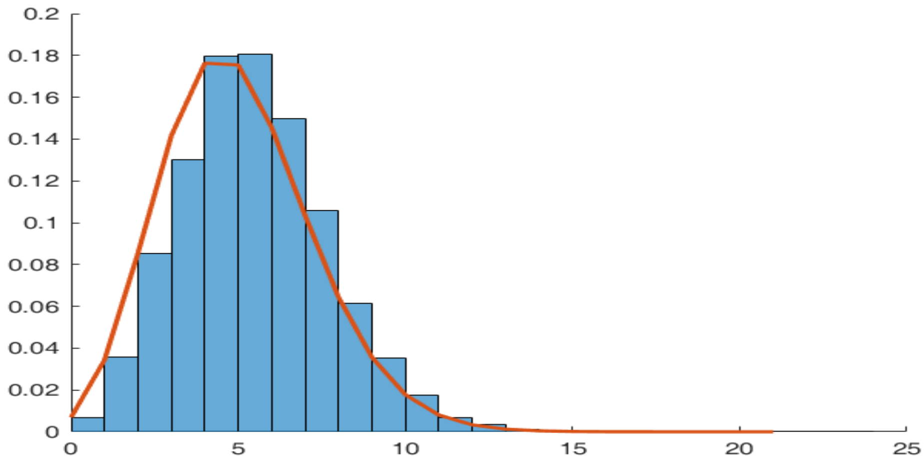


Fig. 7: Representation of the degree distribution with parameters N = 1000, and k_{avg} = 10.0368 for a random network.

In Fig. 7,we present the Poisson distribution in compliance with the degree distribution of the simulated data with 1000 nodes and an average degree distribution of $k_{avg} = 10.0368$, estimated for the particular simulated data using the most likelihood estimate (MLE).

5.2 The Scale-Free Scenario

The scaling up of the WSN in the support of IoT infrastructure falls in the category of random networks. The dynamical changes of the adhoc sensor nodes to the existing WSNs satisfies the scale-free network properties and the degree distribution of sensor nodes follows a Power-law behavior. A dynamic new sensor node is connected to node i with degree k_i and probability $\prod$ such that,

$$\prod(k_i) = \frac{k_i}{\sum_j k_j} \tag{10}$$

where $\sum_j k_j$ is the total number of edges in the existing WSN topology. Following Barabasi and Albert [32], the degree distribution can be modeled as,

$$\frac{\partial k_i}{\partial t} = n \prod(k_i) = \frac{n k_i}{\sum_{j=1}^{n} k_j} \tag{11}$$

After t time steps the WSN creates $n = t + n_0$ nodes and nt edges. Here, n_0 be the number of initial nodes in the WSN. At each time step a new node is added to the existing WSN and connected to n existing nodes, where $n \leq n_0$ The denominator sum is calculated as,

$$\sum_j k_j = 2nt - n \tag{12}$$

Using Eqs. (11) and (12), we get,

$$\frac{\partial k_i}{\partial t} = \frac{k_i}{2t} \tag{13}$$

The above equation can be solved under initial condition at node i, $k_i\left(t_i\right) = n$ is,

$$k_i\left(t\right) = n\sqrt{\frac{t}{t_i}} \tag{14}$$

The probability that a node having $k_i\left(t_i\right)$ degree less than k degree.

So,

$$F\left(k_i\left(t\right) < k\right) = F\left(t_i > \left(\frac{n}{k}\right)^2 t\right) \tag{15}$$

The probability density function (PDF) for the time step t_i is given by,

$$F\left(t_i\right) = \frac{1}{n_o + t} \tag{16}$$

$$F\left(t_i > \left(\frac{n}{k}\right)^2 t\right) = 1 - F\left(t_i \leq t\left(\frac{n}{k}\right)^2\right) \tag{17}$$

$$= 1 - t\left(\frac{n}{k}\right)^2 \frac{1}{n_0 + t} \tag{18}$$

The probability density function can be obtained,

$$p\left(k\right) = \frac{\partial P\left(k_i\left(t\right) < k\right)}{\partial k} = \frac{2n^2 t}{n_0 + t}\frac{1}{k^3} \tag{19}$$

For very large value of t i.e., $t >> 0$,

$$p\left(k\right) \sim 2n^2 k^{-3} \tag{20}$$

From Eq.(20), we obtain the degree distribution for a dynamic network of wireless sensor nodes. This behavior is typically portrayed by systems which are resilient to high degrees of disorder. Thus by providing a power-law degree distribution for the WSNs, we intend to add more reliability to the network. This can provide a futuristic framework for building the IoT applications.

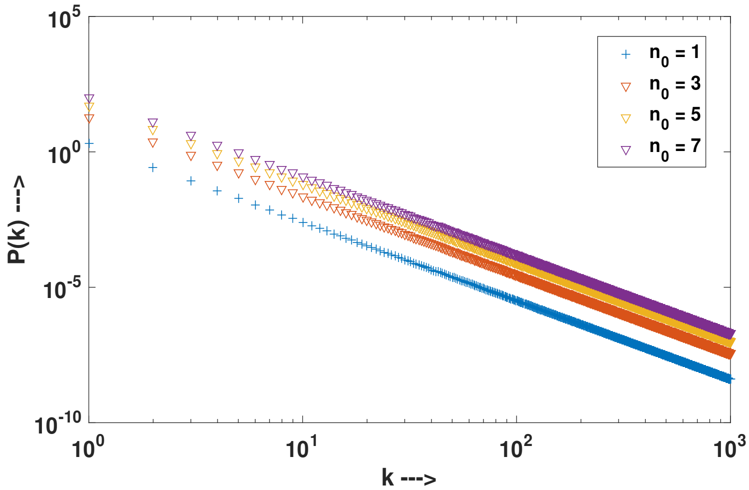


Fig. 8: Representation of the degree distribution for $n_0 = 1$, $n_0 = 3$, $n_0 = 5$, $n_0 = 7$, k^{-3}.

5.3 Simulation Results

In this section, we provide the simulation results obtained in compliance to the methods used in [45]. We build the simulation environment by initially considering $n = n_0 = 5$ heterogeneous sensor nodes with a degree distribution of $k = 3$ (here the heterogeneity of the sensor nodes is independent of the network topology and is only concerned with the coherent properties of the sensor nodes and the type of data perceived). The WSN network has a total of 2000 sensor nodes, by incrementally making addition of new sensor nodes to the network. At every step function a new node is added to the existing n sensor nodes in the WSN.

In Fig. 8, we provide an account of the theoretical distributions (obtained in Eq.(20)) observed for different values of n_0, considered in the logarithmic scale.

Fig. 9, gives an account of the power-law distribution for the simulated data. It is evident that with

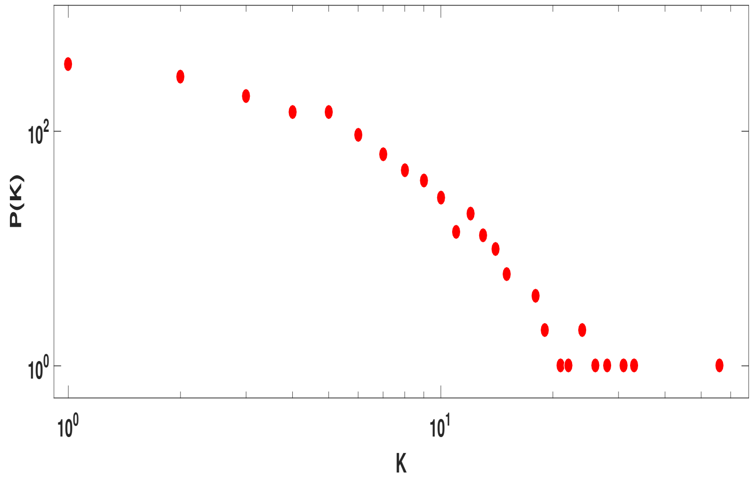


Fig. 9: Representation of the degree distribution of the empirical data with power-law distribution.

6 Conclusion and Future works

It can be concluded that with the rapid evolution of the Internet of things (IoT) and its functionalities, have led to the concurrence of a more intricate and interconnected network of devices. These devices specifically relate to a generic set of devices like the sensory devices, smart appliances, smart hubs, and other associated objects which permissively provide a ubiquitous connectivity among the devices. Thus it is inevitable for the IoT devices to obtain heterogeneity. This gives rise to certain amount of disorder in the network, which the conventional network topologies may not efficiently characterize. Thus a more dynamic alternative approach has been provided in this chapter to befit the requirements of the IoT ecosystem, by adding more reliability to the system, and thereby improving the overall performance of the system. The complexities involved due to the addition of extensively large number of devices to the static WSNs have been discussed. Thus in order to understand the scalability and the heterogeneity of these interconnected devices, the power-law exponent is an important measure. In this chapter, we have provided an analytical framework to illustrate the ubiquitous power-law behavior observed in the devices connected to the internet. We have also provided the mathematical insights to characterize the dynamic behavior of the devices connected to the IoT by emphasizing on the small-world traits of the wireless sensor networks (WSNs). Finally we have provided an analysis of the theoretical and simulation results obtained for the proposed model.

The study of scale-free networks and the small-world phenomena have lead to the evolution of several constituent theories whether it may be in the characterization of the human brain cell interactions, or the air-craft systems. Its applicability in new spheres of science and technology is still evolving. Thus a lot more factors are to be taken into consideration before formally implementing the scale-free network model for the IoT.

7 Conflict of Interest

There is no conflict of interest

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