Ontology in the Game of Life

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ABSTRACT: The game of life is an excellent framework for metaphysical modelling. It can be used to study ontological categories like space, time, causality, persistence, substance, emergence, and supervenience. It is often said that there are many levels of existence in the game of life. Objects like the glider are said to exist on higher levels. Our goal here is to work out a precise formalization of the thesis that there are various levels of existence in the game of life. To formalize this thesis, we develop a set-theoretic construction of the glider. The method of this construction generalizes to other patterns in the game of life. And it can be extended to more realistic physical systems. The result is a highly general method for the set-theoretical construction of substances.

KEYWORDS: substance; construction; levels; game of life; glider.

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1. Introduction

According to Quine, a *Democritean World* is a distribution of binary values over the points of space-time (Quine, 1969: 147-155). About the same time that Quine was thinking of Democritean Worlds, the mathematician John Conway was developing the *game of life* (Gardner, 1970). The game of life provides many opportunities for serious ontological study. It is simple enough that many ontological problems appear there with striking clarity. And it is complex enough that the solutions to these problems have at least some reasonable chance of scaling up to our universe. Within the game of life, you can study problems of space, time, causality, persistence, and substance.

The game of life has also been used to study problems associated with supervenience and emergence (Bedau, 1997; Beer, 2004; Hovda, 2008). Closely associated with these concepts is the thesis is that there are various *levels of existence*. Each level has its own ontology and is described by its own language. For instance, Bedau (1997) distinguishes between a *micro-level* and a *macro-level*. Dennett (1991) uses the game of life to illustrate his distinction between the physical, design, and intentional levels. The thesis that there are different levels of existence is especially intriguing. On the one hand, it links up with the classical idea of the great chain of being (Lovejoy, 1936). On the other hand, it links up with more recent technical work involving mereology and set theory, as well as recent work in object-oriented programming and the study of software ontologies.

Our goal here is to work out a precise formalization of the thesis that there are various levels of existence in the game of life. We will use a single game of life question to develop this thesis. The question is: *what is a glider?* To answer this question, we develop a *method* of set-theoretic construction that is highly general. As we apply this method, we will produce a series of constructions. These constructions are *set-theoretic* *representations* of real ontological categories (e.g. space-time points, processes, substances). It should be clear that we are only doing *metaphysical modelling* (see Meixner, 2010). This modelling can help us precisely understand the relations among real ontological categories.

2. The Nature of the Glider

We assume familiarity with the game of life (for an introduction, see Poundstone, 1985). For our purposes, space is infinite in all directions. Time is one-way infinite from an initial state. Time in the game of life is understood classically (that is, it is absolute). Every point in space-time has a field value either 0 or 1. Thus space-time supports a binary scalar field. This field evolves according to the causal law of life.

A point is *active* if its field value is 1. Activity is analogous to the presence of stuff (matter) in the game of life. On this interpretation, the evolution of the activity in the field is the evolution of its material content. Figure 1 shows an intriguing distribution of activity. When the game of life is animated by running it on a computer and displaying the states of space on the screen, this distribution appears to move. The apparent motion of the distribution is extremely vivid. The apparent motion is produced by well-known aspects of the human visual system.

For many people, the apparent motion of the distribution is so powerful that it produces the strong belief that there really does exist an entity that is moving. This production raises the venerable philosophical question about the relation of appearance to reality: under what conditions do appearances justify the existence of some associated real entity? For the distribution in Figure 1, many writers do accept the inference from appearance to reality: the apparent motion justifies the existence of an entity that is moving. This entity is *the glider*. Friends of glider-existence say that gliders *supervene* on the activities of the points or even that they *emerge* from those activities. If gliders do exist, then they are not space-time points. Hence they must exist on some *higher level* of existence.

We are not interested in either attacking or defending the existence of the glider. We are interested in trying to make precise formal sense out of what it means to say that gliders exist and that there are many levels of existence in the game of life. Our goal is to formalize the concept of the glider by giving an exact set-theoretic construction of the glider. The techniques used in this construction can be directly applied to other diachronic regularities in the game of life (e.g. blinkers, spaceships, logic gates, Turing machines, etc.). And these techniques can be extended to more realistic physical systems.

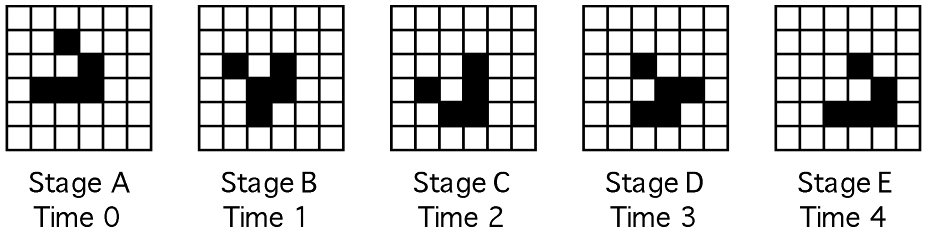

Figure 1. The apparent motion of a glider.

3. Formalizing the Nature of the Glider

3.1 Ordinals

Ordinals. Ordinal numbers were originally defined by von Neumann (1923). The initial ordinal is denoted 0. It the empty set {}. So 0 = {}. For every ordinal n, the next ordinal is n+1. The ordinal n+1 is {0, . . n}. So 1 = {0} = {{}}. And 2 = {0, 1} = {{}, {{}}}. And 3 = {0, 1, 2}. Figure 2 depicts the membership diagram of the ordinal 3. A arrow directed from dot x to dot y in Figure 2 indicates x is a member of y.

Ordinals are ordered in two ways. First, they are ordered with respect to one another. The order relation is identical with the membership relation. For any two ordinals m and n, m is less than n iff m is a member of n. Second, every ordinal is *internally ordered* by the membership relation. For any ordinal n, the diagram of its internal membership relation is also the diagram of the less than relation. So, an ordinal is a set that is *intrinsically* well-ordered by the membership relation. The less than relation is not extrinsic to the ordinal. It is part of the definition of the ordinal. Hence there is a sense in which the ordinal *encodes* that relation. An ordinal is a set that encodes its essential nature in its structure.

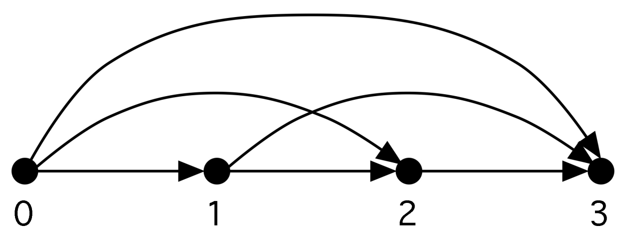

Figure 2. The membership diagram of the ordinal 3.

3.2 Space and Time

Space-Time. Space-time is (Z × Z) × N. The third coordinate is the timeline. Time runs from 0 through all the finite ordinal numbers. A *region* is any set of points. Say region x is a subregion of region y iff x is a subset of y.

Moments. A moment is an instantaneous slice of space-time. It is the set of all space-time points at a single time coordinate. For each t in N, the moment M(t) is the set of all points (x, y, t) such that x and y are in Z. Thus the moment $M(t) = (Z \times Z) \times \{t\}$.

Spatial Regions. A spatial region is any set of simultaneous points. The points in a spatial region all have the same time coordinate; they are all in the same moment. For every moment M(t), the set of spatial regions REG(t) is the power set of M(t).

Our first proposed definition for the glider is that it is a spatial region with a certain shape. To define this shape, we start with the fact that every glider is centered on a point. This point is (x, y, t). We define the shape of the glider around the center. For any point (x, y, t), the glider-shaped region around that point is the set {(x, y+1, t), (x+1, y, t), (x-1, y-1, t), (x, y-1, t), (x+1, y-1, t)}. Our first proposal says that every glider-shaped region is a glider. Of course, this proposal faces an immediate fatal objection: a glider is a region in which all the points are *active*. Beer says "the label 'glider' refers to the distinctive pattern of five ON cells"(2004: 312). Any definition of the glider must include this activity.

3.3 Material Content

Qualified Points. A qualified point has some material content – it is a point along with the value of the binary scalar field at that point. More formally, a qualified point is a pair of the form (point, field-value). A qualified point is active if its field-value is 1; it is inactive if its field-value is 0. Since points are coordinate triples, a qualified point has the form ((x, y, t), v). The set of qualified points in the game of life is $(Z \times Z \times N) \times \{0, 1\}$. An active point is a pair of the form (point, field value 1). An inactive point is a pair of the form (point, field value 0).

Distributions. A distribution is an active spatial region. It is some set of active points at some instant in time (in the same moment). For each time t in N, the set of distributions at t is P(t). To define P(t), recall that M(t) is the set of all space-time points at the single time t. For each moment M(t), the total set of active points over that moment is $M(t) \times \{1\}$. Now, a distribution is just any set of active points at some moment. Each subset of $(M(t) \times \{1\})$ is a distribution. So P(t) is the power set of $(M(t) \times \{1\})$. The set of distributions over space-time is P. P is the union of the P(t) for all t in N.

Our second proposal is that a glider is a distribution with a certain shape. For any point (x, y, t), the glider centered on that point is the distribution {((x, y+1, t), 1), ((x+1, y, t), 1), ((x-1, y-1, t), 1), ((x, y-1, t), 1), ((x+1, y-1, t), 1)}. Unfortunately, this second proposal fails. Distributions are static. However, gliders persist – they are temporally extended. As Beer also points out, "a glider is a coherent localized pattern of spatiotemporal activity in the life universe that continuously reconstitutes itself" (2004: 311). Any definition of the glider must include the fact that its nature is diachronic.

3.4 Sequences

Sequences. Since ordinals are intrinsically ordered, we can use them to order other sets. Given some set S whose size is n, we define a 1-1 correspondence *f* between the ordinal n and S. For example, let S be {A, B, C, D, E}. The ordinal 5 is the set {0, 1, 2, 3, 4}. It's easy to associate 5 with S. One association looks like this: 0 → A; 1 → B; 2 → C; 3 → D; 4 → E. Formally, each arrow is an ordered pair. Thus 0 → A denotes the ordered pair (0, A). The association from 5 to S is the function *f*. Since the ordinal 5 is ordered, the function *f* from 5 to S induces a parallel order on S. The order is A < B < C < D < E. A sequence is a 1-1 correspondence from some ordinal to a set. For our example, the sequence is the function *f* = {(0, A), (1, B), (2, C), (3, D), (4, E)}.

Our third proposal is that a glider is a sequence of distributions. For example, let the objects A through E be the distributions in Figure 1. The sequence of these distributions is the function *f* = {(0, A), (1, B), (2, C), (3, D), (4, E)}. Accordingly, our third proposal identifies the glider with that function. This proposal captures the diachronic nature of the glider. However, this diachronic nature is *extrinsic* to the glider. It is the result of a function from something external – an ordinal – onto the distributions. The stages of the glider are ordered by their links to ordinals – not to one another. But the diachronic nature of the glider is intrinsic. Any metaphysically adequate definition of the glider should include this intrinsicness. We want gliders (or any persistent things) to be more like ordinals. Just as the ordinal 3 encodes its own order within its structure, so any persistent thing should encode its diachronic order within its structure.

3.5 Chains

Chain. A pair (u, v) is linked to a pair (v, w). Those two pairs are linked by their common member v. A chain is a series of pairs in which every pair but the last is linked to exactly one next pair. Chains are either finite or infinite.

Finite Chain. For any set of pairs X, say X is a finite chain iff there exists some ordinal n+1 and there exists some function *f* from n+1 onto X such that *f* is 1-1 and for every k in n, the last item in *f*(k) is identical with the first item in *f*(k+1). For example, consider the set of pairs X = { (A, B), (B, C), (C, D), (D, E)}. There is a function *f* that maps the ordinal 4 onto this X as follows: 0 → (A, B), 1 → (B, C), 2 → (C, D), 3 → (D, E). For k varying from 0 through 2, the last item of *f*(k) is the first item of *f*(k+1). Thus *f*(2) is (C, D) and *f*(3) is (D, E). These two pairs are linked at D.

Infinite Chain. For any set of pairs X, say X is an infinite (or endless) chain iff there exists some function *f* from the set of natural numbers N onto X such that *f* is 1-1 and for every k in N, the last item in *f*(k) is identical with the first item in *f*(k+1).

Chains of Distributions. A chain of distributions is any set x such that every member of x is a pair of distributions and x is a chain. The set of these chains is K. Note that chains need not be spatially, temporally, or causally connected. Thus {(X, Y), (Y, Z)} is a chain *no matter where or when* the distributions X, Y, and Z appear.

Our fourth proposal is that the glider is a chain of distributions. Using the distributions from Figure 1, the glider is the chain {(A, B), (B, C), (C, D), (D, E)}. Figure 3 is a rough (not exact) sketch of the membership structure of the chain. This fourth proposal captures all our previous reasoning about the nature of the glider. Given our earlier definitions, the chain encodes the fact that the glider in Figure 1 has five stages. Every stage encodes information about shape and point activity. The stages are intrinsically linked within the chain. But there is a problem. Although the chain that we gave as an example has various orders and connectivities, none of those orders or connectivities are implied by (or encoded in) the proposed definition of the glider as a chain of distributions. Such chains need not have any physical orders or connectivities. For any type of order or connectivity that gliders do have, we must explicitly build that type into the definition of the glider.

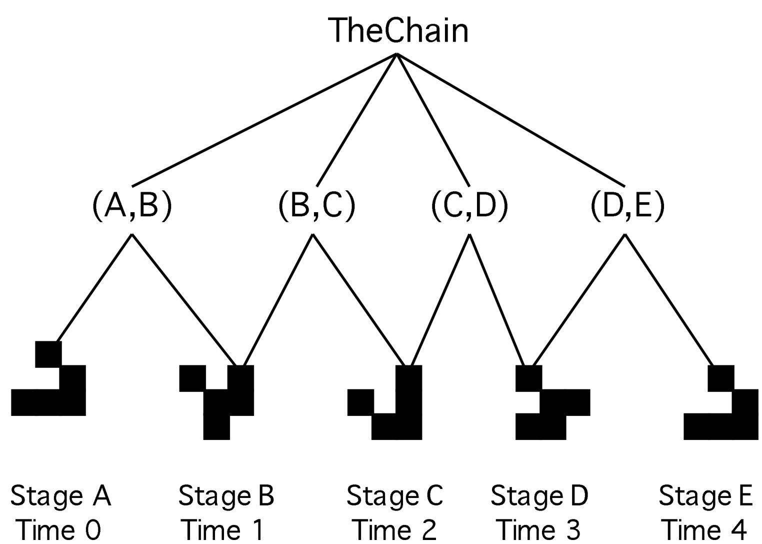

Figure 3. The glider as a chain of distributions.

3.6 Substances

Substance. A substance is some *substratum* along with some *attributes*. We interpret this set-theoretically as an ordered tuple in which the substratum comes first. Thus a substance is a tuple of the form $(s, p_1, \ldots p_n)$. The first item s is the substratum. We think of any substratum as a set. The terms p_1 through p_n are the attributes of the substratum.

Attributes. Each attribute is some type of property. Properties are defined extensionally as sets of instances. And to instantiate the property is to be a member of the property. So each p_i is some set. The fact that the substratum s has the property p_i is expressed by requiring that s is a member of p_i. Hence for any substance, the first item of the tuple is a member of every other item in the tuple. There are at least three types of attributes. First, if s has some property p, then s is a member of p. So the attribute of s is p. Second, if every member of s has some property p, then s is a member of the power set of p. So the attribute is pow p. And third, if every member of s is in some relation R, then s is a member of the power set of R. So the attribute is pow R.

Material Thing. A material thing is a substance. Any material thing (in the game of life or otherwise) has to contain some stuff – it has to have some material content. Since distributions contain active points, they provide the material content of a material thing. They provide the matter. Although a material thing has to contain some stuff, it does not have to be formally, spatially, temporally, or causally connected. But we want to be able to add these connectivities as we define various types of material things. Consequently, the substratum x of a material thing is a set of ordered pairs. This substratum has exactly one attribute: every pair in this substratum is a pair of distributions. This means that the substratum x is a subset of $P \times P$. Hence x is a member of pow $P \times P$. Putting this all together, a material thing is a tuple (x, pow $P \times P$).

Refinement. Substances are refined by adding attributes. When a substance of one type is refined, the result is a subtype. The subtype relation determines a *genus-species hierarchy*. The concept of refinement for substances is much like the concept of refinement in *object-oriented programming*. This similarity reveals intriguing connections between object-oriented programming and set-theoretic construction. We will refine material things into concatenations; concatenations into processes; and so on.

Concatenation. A concatenation is a species of material thing. It is derived by adding the attribute of formal connectivity. Chains are formally connected. This means that, for any concatenation, the substratum x is a member of the set of chains K. Thus a concatenation has the form (x, pow $P \times P$, K). Although a concatenation is a chain, this connectivity is merely formal. It is neither spatial, nor temporal, nor causal.

Changes. We can add temporal connectivity by defining changes. A change is a pair of distributions in which the first is immediately temporally prior to the second. For instance, in the glider in Figure 1, the stage A changes into the stage B. So (A, B) is a change. Recall that P(t) is the set of distributions at time t. For each time t in N, let T(t) be the set of all pairs of the form (x, y) where x is in P(t) and y is in P(t+1). Let T be the union of the T(t) for all t in N.

Process. A process is a species of concatenation. It is derived by adding the attribute of temporal connectivity. Since processes are concatenations, the pairs in the substratum of any process already form a formally connected chain. Thus all we need to do to gain temporal connectivity is to require each of these pairs to be a change. For any process, each pair in the substratum x is a member of the set of changes T. This means that x is a subset of T; so x is a member of pow T. Putting this all together, a process is a tuple (x, pow $P \times P$, K, pow T). Processes are formally and temporally connected.

Our fifth proposal is that the glider is a process. A glider is any set of the form (x, pow P $\times$ P, K, pow T). For instance, the glider in Figure 1 is defined as this process: ({(A, B), (B, C), (C, D), (D, E)}, pow P $\times$ P, K, pow T). As a process, the glider has material content. The matter is distributed through space and time. The distribution is formally and temporally connected. However, as a process, the distribution of the matter in the glider is not causally connected. Nor does it evolve in any regular way. It is still necessary to add these two attributes.

3.7 Space-Time Worms

Field Operator. Each distribution is all the active points in some instantaneous state of a binary scalar field. The game of life has a rule that says how this scalar field changes over time. This rule is expressible as an operator on the scalar field. The field operator ϕ maps distributions over any moment M(t) to those over the next moment M(t+1). At any time t, ϕ maps every distribution in any P(t) to those in P(t+1). So ϕ(t) is the set of all (p, q) such that p is in P(t) and q is in P(t+1) and the life rule transforms p into q. The field operator ϕ is the union of the ϕ(t) for all t in N.

Worms. A space-time worm is a species of process. It is a process in which the chain of changes is causally connected. The attribute of causal connectivity is added by saying that each pair in x is in the field operator ϕ. So x is a subset of ϕ. This means x is a member of the power set of ϕ. A worm has the form (x, pow P $\times$ P, K, pow T, pow ϕ). For the game of life, causal connectivity implies spatial connectivity.

Our sixth proposal is that the glider is a space-time worm. A glider is any set of the form (x, pow P $\times$ P, K, pow T, pow ϕ). For instance, the glider in Figure 1 is the worm ({(A, B), (B, C), (C, D), (D, E)}, pow P $\times$ P, K, pow T, pow ϕ). As a worm, the glider has material content. The matter is distributed through space and time. The distribution is formally, temporally, causally, and spatially connected. However, as a worm, the distribution of the material content of the glider does evolve in any regular way. There is no program that governs the changes of its shapes. But the most distinctive attribute of the glider is that it does evolve in a regular way! We need to talk about shapes and programs.

3.8 From Programs to Machines

Shape Equivalence. The active points in any distribution have some shape. Two distributions have the same shape iff there is some coordinate shift that preserves the positions of their active points. Formally, distribution x is the same shape as distribution y iff there is some translation F such that F(x) = y. The translation shifts coordinates on either the X or Y axis or both. Shape equivalence is strictly translational. It does not include rotations or reflections. Rotations and reflections are not allowed because (for instance) the horizontal and vertical bars of the blinker are regarded as two distinct shapes but they are rotationally equivalent. And shape equivalence does not include any other permutations. For any distributions x and y, say $x \equiv y$ iff x is the same shape as y.

Shapes. The shape equivalence relation partitions the set P of distributions into equivalence classes. Each equivalence class is a shape. The set of all shapes is S.

Field Operator on Shapes. The field operator can be lifted from distributions to shape-equivalence classes of distributions. The result is a more abstract field operator Φ that maps shapes onto shapes. For any shapes X and Y, the operator Φ maps X onto Y iff for every distribution x, x is in X iff $\phi(x)$ is in Y.

Cycle of Shapes. A cycle of shapes is a loop in the graph of Φ. It is a cyclical attractor in the dynamics of Φ. Say C is a cycle of shapes iff (1) C is a sequence of shapes $\langle C_1, C_2, \ldots C_n \rangle$; and (2) for every i less than n, Φ maps C_i onto the next shape C_{i+1}; and (3) Φ maps the last shape C_n onto the first shape C_1.

Cycles in the Game of Life. The set C*(1) is the set of all shape-cycles of length 1. Any cycle of length 1 is an identity cycle. Any stable shape in the game of life is in an identity cycle. For instance, the *block* in the game of life is a shape in an identity cycle. Shape-cycles of longer lengths are known as *oscillators*. The set C*(n) is the set of all shape-cycles of length n. For example, the *blinker* is an oscillator of cycle length 2. The glider is a mobile oscillator of cycle length 4. The set of all cycles of shapes in the game of life is C*. C* is the union of all C*(n) for all n in N.

Cyclical Equivalence. For any shapes X and Y, X is in the same cycle as Y iff there is some cycle C such that X is a shape in C and Y is a shape in C. This is an equivalence relation on shapes. Say X *is isocyclical with* Y iff X is in the same cycle as Y. Every cycle is *closed* under the field operator on shapes. This closure implies and is implied by cyclical equivalence. Closure means that the cycle is a closed attractor basin in the dynamics of the field operator.

Cyclical Program. A cyclical program M is a pair (C, Φ) where Φ is the field operator on shapes and C is a cycle of shapes. The shapes in C are the states in the state-space of the program M. The operator Φ is the transition operator of the program. Thus any program can be diagrammed as a state-transition network.

Cyclical Programs in the Game of Life. For every cycle C in C*, there is exactly one cyclical program (C, Φ). The set of all cyclical programs is M*. It is the set of all pairs of the form (C, Φ) such that C is a cycle in C*.

Conservation. Many processes in the game of life exhibit cyclical patterns. For instance, blocks (and all still lifes), blinkers (and all oscillators), and gliders (and all movers) exhibit cyclical dynamics. Processes with cyclical dynamics display a regularity that entails a kind of equivalence (their shapes are isocyclical). This equivalence is an invariant. They don't leave their attractor basin but remain stable within it. This regularity, invariance, or stability is one of the classical marks of thinghood. It is the foundation for persistence (and the illusion of an enduring substance). For any process that exhibits cyclical dynamics, the regularity is a conserved quality. It is a cyclical program. Given some conserved qualities, we build things. That is, things are constructed out of conserved qualities.

Extensions of Programs. A space-time worm w is an instance of a cyclical program (C, Φ) iff the initial distribution in the worm is a member of one of the shapes in C. The extension of a cyclical program (C, Φ) is the set of all worms that are instances of (C, Φ). The extension of (C, Φ) is denoted EXT(C, Φ).

Mechanical Things. A mechanical thing (aka a machine, aka a thing) is a substance. Since it is temporally connected, it is a process. Since it is causally connected, it is a space-time worm. It is an instance of some cyclical program (C, Φ). It therefore lies within the extension of the program. It is a member of EXT(C, Φ). We add this attribute to its bundle. Thus a mechanical thing is a tuple (x, pow $P \times P$, K, pow T, pow ϕ, EXT(C, Φ)).

Our seventh and final proposal is that a glider is a machine. It is a machine that lies in the extension of the glider-program. Let G be the sequence of glider shapes. Thus a glider is a tuple (x, pow $P \times P$, K, pow T, pow ϕ, EXT(G, Φ)). We think this fully captures the meaning of Beer's definition of the glider as "a coherent localized pattern of spatiotemporal activity in the life universe that continuously reconstitutes itself" (2004: 311). As a set, the glider exists many, many levels above the micro-level of the life grid – above the set of qualified space-time points. If levels of existence are the levels in the iterative hierarchy of pure sets, then there are many levels above the life grid. These levels are populated with objects that are in the game of life ontology.

4. Universes

4.1 Snapshots

Machines like the glider somehow exist in game of life universes. But what is a game of life universe? We formalize such universes as substances. We might try to treat a universe as a big machine. If a universe is a big machine, then the substratum of a universe is ultimately made of distributions. But there are two problems. The first problem is this: although machines cannot contain empty space, universes can contain empty space. Universes can contain inactive points – space-time points whose field values are 0. The second problem is this: although no machine can contain every point on the life grid, every universe must contain every point on the life grid. To deal with universes, we first need to build up some preliminary structures that are analogous to distributions. These analogous structures can contain empty space and must cover all points.

Snapshot. A snapshot is an instantaneous state of the game of life field. Snapshots are analogous to distributions. However, distributions do not contain any empty space (that is, they don't contain any inactive points). Snapshots contain both active and inactive points. And any snapshot spans the entirety of space. A snapshot is a function from some moment to {0, 1}. It associates each point in the moment with either 0 or 1. For every time t in N, the set of snapshots at time t is S(t). S(t) is the set of all *f* such that *f* maps M(t) onto {0, 1}. The set of all snapshots S is the union of S(t) for all t in N.

Chains of Snapshots. A chain of snapshots is any set x such that every member of x is a pair of snapshots and x is a chain. The set of these chains is Q. We have a special interest in endless chains of snapshots. The set of all endless chains of snapshots is Q*.

Transition. A transition is a pair of snapshots in which the first is immediately temporally prior to the second. For each time t in N, let W(t) be the set of all pairs of the form (x, y) where x is in S(t) and y is in S(t+1). Let W be the union of the W(t) for all t in N.

Field Operator on Snapshots. Each snapshot is some instantaneous state of a binary scalar field. The game of life has a rule that says how this scalar field changes over time. We already defined a field operator for distributions; we now define it for snapshots. The field operator ψ maps snapshots over any moment M(t) to those over the next moment M(t+1). At any time t, ψ maps every snapshot in any S(t) to those in S(t+1). So ψ(t) is the set of all (p, q) such that p is in S(t) and q is in S(t+1) and the life rule transforms p into q. The field operator ψ is the union of the ψ(t) for all t in N.

4.2 From Movies to Universes

Movie. A movie is a substance. A movie is analogous to a process. Just as a process is defined over distributions, so a movie is defined over snapshots. Informally, it is a series of snapshots that is temporally connected and endless. It need not be spatially or causally connected. More formally, the substratum of a movie is a set x of pairs. The set x has several attributes. First, each pair in x is a pair of snapshots. Hence x is a member of pow S $\times$ S. Second, a movie is temporally connected. Since every pair contains temporally adjacent items, we can obtain temporal connectivity just by asserting that the pairs form a chain. We want this chain to be endless – movies go on forever. Thus x is a member of Q*. Third, each pair in x is a transition. This means that the items in any pair are temporally adjacent. Hence x is a subset of W; so x is a member of pow W. A movie is therefore a tuple (x, pow S $\times$ S, Q*, pow W). There is a set of all movies over the life grid.

Universe. A universe is a substance. A universe is analogous to a space-time worm. Just as a worm is defined over distributions, so a universe is defined over snapshots. The substratum of a universe is an endless chain of snapshots. Informally, a universe is an endless chain of snapshots that is temporally and causally connected. Since a movie is an endless chain of snapshots that is temporally connected, and universes just add causal connectivity, a universe is a type of movie. Thus any universe just adds the attribute of causal connectivity to any substance whose form is (x, pow $S \times S$, Q*, pow W). Causal connectivity is added by saying that each pair in x is in the field operator ψ. So x is a subset of ψ. This means x is a member of the power set of ψ. Putting this all together, a universe is a tuple of the form (x, pow $S \times S$, Q*, pow W, pow ψ).

Habitation. A substance s inhabits a universe U iff every active point in s is an active point in U. Every universe contains a maximal substance. Specifically, it contains a maximal space-time worm. All other worms are spatio-temporal parts of that worm. Some of the subworms of the maximal worm are machines. Thus for every universe, we can define the set of worms in that universe and the set of machines in that universe.

5. Further Work

According to Dennett (1991), there are three main levels of existence in the game of life. These are the physical level; the design level; and the intentional level. The design level falls apart into a hierarchy of sublevels. These sublevels are ranked by complexity. The glider is on the lowest sublevel (along with blinkers, eaters, and so on). Objects on lower design sublevels can be put together to make more complex objects on higher design sublevels. Higher design levels contain things like glider guns, puffer trains, and so on. And they contain logic gates (Rennard, 2001). The highest design level contains universal Turing machines (Rendell, 2001). And, beyond the design levels, Dennett says there is an intentional level. It would be nice to study these higher levels.

Further work can also take us beyond the game of life. Within the game of life, the glider is an example of a particle-like wave (Adamatzky, 1998). It is like a soliton. Solitons have been used to model particles in our universe – such as quarks interacting within a proton (Chen, 2006; Dauxois & Peyrard, 2006). So there is a path – presently very sketchy – from the physics of the glider to the physics of our universe. Following this path, we might try to produce set-theoretic constructions of particles in our universe.

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