Nietzsche's Philosophy of Mathematics

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ABSTRACT: Nietzsche has a surprisingly significant and strikingly positive assessment of mathematics. I discuss Nietzsche's theory of the origin of mathematical practice in the division of the continuum of force, his theory of numbers, his conception of the finite and the infinite, and the relations between Nietzschean mathematics and formalism and intuitionism. I talk about the relations between math, illusion, life, and the will to truth. I distinguish life and world affirming mathematical practice from its ascetic perversion. For Nietzsche, math is an artistic and moral activity that has an essential role to play in the joyful wisdom.

1. Introduction

At first sight, it would appear that Nietzsche has nothing to say about mathematics, or at least nothing positive to say. Both of those initial impressions are wrong. A careful reading of Nietzsche's own writings with mathematical concerns in mind reveals that he neither ignores nor hates mathematics.[1] In fact, Nietzsche's attitude towards mathematics is surprisingly positive, and his explicit remarks on mathematical topics, though not by themselves extensive, form an elaborate and coherent whole when situated in the larger context of his writings on epistemological and metaphysical themes.

Nietzsche's philosophy of mathematics is best summarized by aphorism 246 of the *Gay Science:* "Let us introduce the refinement and rigor of mathematics into all sciences as far as this is at all possible, not in the faith that this will lead us to know things but in order to *determine* our human relation to things. Mathematics is merely the means for general and ultimate knowledge of man."[2] In the context of Nietzsche's perspectivism, the claim that mathematics is the means for any *ultimate* knowledge is striking. Equally striking is his claim that math is a means towards human *self-knowledge*.

To understand Nietzsche's philosophy of mathematics, we need to look briefly at the nature of mathematics itself. On the one hand, there are at least two ways to characterize mathematics negatively: mathematics is neither empirical science nor arithmetic. It is *not* concerned with predictions or explanations, and it is *not* concerned with numerical calculations. Nietzsche correctly draws both those distinctions. On the other hand, there are at least three ways to characterize mathematics positively:[3] firstly as logic (logicism); secondly as the study of formal systems (formalism); and thirdly as the

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[1]In this respect Nietzsche's philosophy of mathematics should be contrasted with that of Heidegger.
[2]Trans. W. Kaufmann (New York: Random House, 1974).
[3]Stephan Korner, *The Philosophy of Mathematics: An Introductory Essay* (New York: Dover, 1968).

activity of intuitive constructions (intuitionism). It will perhaps come as no surprise that Nietzsche is not a logicist. On the contrary, his thoughts on mathematics lie very close to intuitionism, though at some places Nietzsche adopts formalist positions.

In what follows we offer a preliminary discussion of Nietzsche's philosophy of mathematics. Much work remains to be done in this area. Here we discuss the origin of mathematics in the division of the continuum, the restriction of Nietzschean mathematics to finitary methods, the relations between Nietzschean mathematics and formalism and intuitionism, math as artistic and moral activity, and finally the role of mathematics in the gay science. We hope our work here will at least dispell the *meconnaisance* according to which Nietzsche has little significant or positive to say about mathematics.

2. The Origin of Mathematics in the Division of the Continuum

The will to power is a continuous flux (HH II:11; GS 109, 112; WP 512, 715) that human cognition necessarily divides. So divided, the will to power is a plurality of *identical cases* (WP 512, 521, 532, 544, 551, 552, 568, 569), and as such initially becomes amenable to human comprehension. Nietzsche explicitly argues that this division of the continuum is foundational for both logic (WP 512) and also for arithmetic, in as much as it is the basis for the very possibility of calculation (WP 521, 551, 568). This division of the continuum is not merely of mathematical relevance; indeed, it is the initial and *most fundamental* of all those errors with biological utility (BGE 4; GS 109, 110), namely, the commission of the error of equality (GS 111). Making this mistake is necessary for all human *cognitive* activity, but it is particularly relevant with regards to mathematics:

> The laws of numbers were invented on the basis of the initially prevailing error that there are various identical things (but actually there is nothing identical) or at least that there are things (but there is no "thing"). The assumption of multiplicity always presumes that there is something, which occurs repeatedly. But this is just where error rules; even here, we invent entities, unities, that do not exist. . . . To a world that is *not* our idea, the laws of numbers are completely inapplicable: they are valid only in the human world. (HH I:19)[4]

3. The Finitude of the World of Force and the Denial of Actual Infinities in Mathematics

Much work in the foundations of mathematics was stimulated by problems associated with infinity. No one denies that potential infinity is a valid mathematical concept (e.g. one can always add one to a number *again*, or divide a number by two *again*). The concept of potential infinity justifies the principle of mathematical induction. What remains controversial is the notion of *actual infinity* (e.g. the definition of a real

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[4]Trans. M. Faber & S. Lehmann (Lincoln, NE: University of Nebraska Press, 1984).

number as the limit of an infinite sequence, or the use of the notion of the set of all subsets of an infinite set). Insofar as they do not accept realist interpretations of actual infinities in mathematics, formalists and intuitionists are finitists.

Nietzsche is likewise a finitist, repeatedly asserting that the world is finite (Z "Of the Three Evil Things" 1; WP 595, 1062, 1066, 1067). In arguing for the eternal return of the same, Nietzsche argues against the notion of actual infinity with respect to force (WP 1062) though he permits the potential infinity of the world's temporal sequence (WP 1066). He criticizes the notion of an unlimited force as *unthinkable*: "the world, as force, may not be thought of as unlimited, for it *cannot* be so thought of; we forbid ourselves the concept of an infinite force as incompatible with the concept 'force'" (WP 1062).[5] That a force cannot be thought of as infinite, and that Zarathustra's wisdom mocks all "infinite worlds" (Z "Of the Three Evil Things" 1), suggest that actual infinities are themselves inconceivable. When Nietzsche declares that "the world has become 'infinite' for us again" (GS 374), the infinity to which he refers is clearly potential.[6] The "horizon of the infinite" to which the adventurous free spirit may set sail (GS 124) is potential: the horizon is never reached but one may always progress towards it. Indeed, if the eternal return is true then the world has a recursive hence finitary construction; since the world is the maximal object that can be conceived, Nietzsche is entirely committed to finitary methods.

4. From the Division of the Continuum to Mathematics as the Science of Formal Systems

In his positivist moments, Nietzsche's approach to mathematics basically agrees with the formalist program of Hilbert: "on Hilbert's showing, classical mathematics has as its hard core a perceivable, or at least in principle perceptually constructible subject-matter, to which fictitious, imperceivable and perceptually non-constructible objects, in particular various infinite totalities, are adjoined."[7] Hilbert's formalist program is perfectly coherent with Nietzsche's positivistic assertion that

> We possess scientific knowledge today to precisely the extent that we have decided to *accept* the evidence of the senses . . . *Or* science of formulae, sign-systems: such logic and that applied logic, mathematics. In these reality does not appear at all, not even as a problem; just as little as does the question what value a system of conventional signs such as constitutes logic can possibly possess. (TI "'Reason' in Philosophy" 3)[8]

[5]Trans. W. Kaufmann & R. J. Hollingdale (New York: Random House, 1968).
[6]Importantly, the perspectives mentioned in GS 374 are *possibilities*; and Nietzsche rejects any totalizing conception of them as "the Unknown One".
[7]Korner, *The Philosophy of Mathematics*, p. 82.
[8]Trans. R. J. Hollingdale (New York: Penguin Books, 1968).

Just as Nietzsche distinguishes between science founded on the senses and science of sign-systems, so the formalist distinguishes between *finitary* and *transfinitary* mathematics.

Finitary math includes those structures generatable by finite methods: the integers and the potential infinities generated by mathematical induction. Finite methods in mathematics begin with the production of recurrent identical cases, which Hilbert symbolizes by strokes "|". Elementary number theory is concerned with the study of series of such strokes formed from an initial stroke by the adjunction of another stroke -- the production of a recurrent identical case. Such series include |, ||, |||, and so on, which can be named "1", "2", "3", and so on. The equality operator = is a method for answering whether two stroke-series have an identical form; the < operator is a method for telling whether one stroke-series is longer than another. On these simple foundations it is possible to build all the ordinary concepts of rational arithmetic (addition, subtraction, multiplication, and division) to demonstrate the laws (e.g. associative, commutative, distributive) of these operations. Insofar as a longer stroke-series can always be constructed from a given one, it is possible to ground the principle of mathematical induction. With the principle of induction it is possible to define a variety of notions (e.g. prime numbers) and to permit recursive definitions (e.g. the definition of factorials). The production of an initial stroke and the recurrent production of identical cases (yielding a stroke-series) are all that is required to secure finite mathematics.

Transfinitary mathematics is concerned with notions requiring actual infinities: for example, the real numbers (in which the actually infinitely divided continuum is presupposed), Cantor's transfinite arithmetic, the unrestricted use of the law of the excluded middle, and the axiom of choice. While concepts employed in finitary math correspond to the abstract features of empirical reality, those used in transfinitary math are at best merely consistent with the abstract features of empirical reality. Indeed, in order to be used they must be *proven* consistent with the system of finitary math. The study of a mathematical system including transfinite concepts is the study of a formal system of signs. To say that it is formal simply means that there are mechanical rules for manipulating the signs of the system without any consideration of their reference.

Nietzsche's finitism and his theory of recurrent identical cases clearly agree with formalist conception of finitary math; likewise, his division of scientific knowledge into empirical (here finitary math) and purely formal (here transfinitary math) accords with the formalist program. However, the formalist conception of math as the study of formal systems does not provide an adequate place for Nietzsche's treatments of math as an artistic and moral activity.

5. From the Division of the Continuum to Mathematics as the Activity of Intuitive Construction

The origin of mathematical activity is an aesthetic falsification: the division of the continuum into series of identical cases. Insofar as the identical cases are *constructed* (WP 544), and also insofar as they are constructed by means of a temporal division (i.e. division of a continuous *flux*), Nietzsche's thoughts about the foundations of logic and arithmetic are squarely in line with those of *intuitionism*. The intuitionist conception of math as a constructive activity accords very well with Nietzsche's view of math as *artistry*. Brouwer, one of the founders of intuitionism, asserts that

> intuitionist mathematics is an essentially languageless activity of the mind having its origin in the perception of *a move of time, i.e.* of the falling apart of a life moment into two distinct things, one of which gives way to the other, but is retained by memory. If the two-ity thus born is divested of all quality, there remains the *empty form of the common substratum of all two-ities. . . .*it is this empty form, which is the *basic intuition of mathematics*.[9]

Brouwer's basic intuition of mathematics (the *intuition* of an empty form) and Nietzsche's fundamental act of cognition are the same.[10]

The link between Nietzsche's philosophy of mathematics and intuitionism is significant insofar as the production of recurrent identical cases has a natural intuititionist interpretation. Suppose we limit ourselves to the single operation of making a stroke. Thus limited (i.e. thus dancing in chains), there is nothing possible besides repetition: making a recurrent identical case. As we experience (i.e. perceive) the next stroke-making action, we subsume it under the memory of the previous stroke (GS 114; BGE 230); we thus construct the next stroke-making operation with the aid of our memory of the previous stroke-making operation; the result is the construction of an *intuition of repetition*. Indeed, Nietzsche's descriptions of his insights into the eternal return of the same (GS 341) suggest that these are intuitions of repetition for which arguments using symbols are only given later (e.g. WP 1062). The restriction of intuitive constructions to those based on the intuition of repetition constrains intuition in such a manner as to avoid the speculative excesses which spring from its unfettered use in religion and morality (HH I:131).

Intuitive constructions are necessarily finitary. The intuitionist does not accept infinite totalities (actual infinities) into mathematics at all, not even as ideal fictions. Instead, intuitionism works entirely with potential infinities. Potential infinity suffices to construct the integers (via the principle of mathematical induction). Since the classical definition of the real numbers treats any segment of the the real number line as a structure whose infinite divisibility is actually given, the intuitionist rejects this classical definition in favor of one that constructs the real numbers as a system of infinitely proceeding sequences (which are always only potentially given).

Nietzsche's finitism and his theory of recurrent identical cases clearly agree with intuitionist conceptions mathematics. Moreover, intuitionism supports Nietzsche's treatments of math as an artistic and moral activity. The fact that the identical cases are *constructed* (WP 544) makes both logic and arithmetic much closer to art than to science.

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[9]Korner, *The Philosophy of Mathematics,* p. 122.

[10]It is worth noting an agreement between Nietzsche and the intuitionist on the relation between thought and language. For Nietzsche, thought and speech are distinct (GS 244, 298); for the intuitionist, math is languageless cognitive activity (Korner, p. 122). But since language is the totality of herd-signalling conventions (GS 354), the distinction between mathematical thought and herd consciousness marks mathematics (like music) as essentially free from herd perspectives. Math is *exceptional* thought.

Indeed, because the construction of identical cases via the division of the continuum is a deliberate falsification, Nietzsche explicitly asserts that "logic does *not* spring from will to truth" (WP 512). The intuition of repetition is itself a moral act (GS 114).

6. Mathematics as Artistic Activity

Mathematics, according to the intuitionists, is a constructive activity. In many respects this constructive activity is a kind of engineering comparable to architecture. The difference is that its products are abstract. Mathematicians construct proofs of theorems. As a kind of abstract architecture, math is much like mountaineering: just as the mountaineer constructs a *path* (which is abstract) from the base to to the top, so the mathematician constructs a *proof* from postulates to theorem. Mathematical thinking is artistry constrained by only one demand: *self-consistency.* This single constraint (GS 290) imposes the severest discipline (GS 293) on the mathematical imagination, resulting in a style that is rigorous and formal in terms of *manner*. But absolute submission to the single constraint of self-consistency leaves such thinking utterly free of *every other* constraint: consequently, the only other factors that can be brought to bear in mathematical work are aesthetic (e.g. elegance, simplicity (GS 226), clarity (GS 173), depth, revelation).

Mathematical activity exemplifies the kind of creation under constraint that Nietzsche calls *dancing in chains:* "making things dificult for oneself and then spreading over it the illusion of ease and facility" (HH II:140). The mathematician's constraints are the axioms and rules of inference or proof. The concept of dancing in chains shows what mathematicians have in common with some poets, particularly those of classical Greece. What Nietzsche says of Homer applies *mutatis mutandis* to any mathematical pioneer: "we can perceive an abundance of inherited formulae [i.e. axioms] and epic narrative rules [i.e. inference rules] within which he had to dance: and he himself created additional new conventions for those who came after him" (HH II:140). For instance, the intuitionists took up the methods of classical mathematics but prohibited the use of any actual infinities;[11] working within this additional constraint, the intuitionists developed an elegant and graceful construction of the real numbers quite distinct from their classical definition.[12]

Mathematics, like all other constructive practice, is ambiguous (GS 370; WP 846). It springs either from an *over-fullness of life* or from an *impoverishment of life*. Mathematics practiced from over-fullness of life aspires to the *grand style:* "To become master of the chaos one is; to compel one's chaos to become form: to become logical, simple, unambiguous, mathematics, law -- that is the grand ambition here" (WP 842). Again, "logical and geometrical simplification" is a sign of enhanced strength and so further enhances strength; the grand style is the high point of this self-intensification (WP 800). However, mathematics practiced from impoverishment of life is yet another

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[11]The intuitionists add other constraints, such as prohibitions against pure existence theorems, the use of the law of the excluded middle and axiom of choice (e.g. usages of the set of all subsets of an infinite set).

[12]Korner, *The Philosophy of Mathematics,* ch. 6., sec. 2.

intoxicant; those who are poor in life desire "logic and the conceptual understandability of existence -- for logic calms" (GS 370). Mathematics practiced from impoverishment produces "a 'world of truth' that can be mastered completely and forever with the aid of our square little reason" (GS 373). When applied (e.g. in science), this sort of mathematical practice is reductive, stripping existence of its rich ambiguity and degrading it to "a mere exercise for a calculator and an indoor diversion for mathematicians" (GS 373). Reductive formalization, applied to a piece of music (i.e. to the world-process, GS 109) grasps nothing of what is musical in it (GS 373; WP 624).

7. Mathematics as Moral Activity

The foundation of Nietzsche's philosophy of mathematics is the intuition of repetition via the construction of identical cases; this construction is a *moral action:* "As soon as we see a new image, we immediately construct it with the aid of all our previous experiences, depending on the degree of our honesty and justice. All experiences are moral experiences, even in the realm of sense perception" (GS 114). The degree of "honesty and justice" that one employs in this construction is the quality of a self-relation: it is one's honesty to oneself. Here the will to falsification operative in mathematics meets the will to truth, which has its origin in the imperative: "I will not deceive, not even myself" (GS 344); but with that, as Nietzsche says, "*we stand on moral ground*." The moral value and danger of math results from the imperative to avoid deception; insofar as this is originally an avoidance of *self-deception,* mathematics is useful as a means to self-knowledge (GS 246). Honesty in relation to oneself is the root of intellectual cleanliness (GS 2, 357), the lack of which has consequences far exceeding mathematical activity.

Training in the severe honesty (GS 293, 357) that is required for mathematical work is valuable because it teaches us to practice the strictest discipline in self-examination. This discipline in self-examination is precisely what is lacking in religious types (GS 319) who would rather rely on the proof of strength (GS 347; WP 171, 452; A 50) rather than on the rigorous kind of proof mathematics requires. Superstition (particularly religious superstition) springs from a lack of discipline in making inferences: "The greatest progress men have made lies in their learning to *draw correct conclusions*. . . . [This art] is learned late and still has not come to prevail. False conclusions are the rule in older times. And all people's mythologies, magic, superstition, religious worship, and law -- all are the inexhaustible sites of evidence for this thesis." (HH I:271). Rigorous methods of inquiry (HH I:633-5) and the rigorous conception of truth (GS 357) are fundamentally opposed to religious superstition.

The capacity of mathematical thought to dispell those errors which sustain reactive religious and political superstitions and intuitions (cf. GS 348) suggests the use of mathematical methods in the construction of higher human cultures. Mathematical honesty and artistry can be employed to construct a multiplicity of cultural systems (of which utilitarian calculi are degenerate examples). Nietzsche's final word on mathematics as a moral activity is thus expressed like this:

> Our knowledge has become scientific to the extent that it is able to employ number and measure. The attempt should be made to see whether a scientific order of values could be constructed simply on a numerical and mensural scale of force -- All other 'values' are prejudices, naiveties, misunderstandings. -- They are everywhere *reducible* to this numerical and mensural scale of force. The ascent on this scale represents every rise in value; the descent on this scale represents diminution of value. (WP 710)

8. Conclusion: The Role of Mathematics in the Gay Science

Nietzsche envisions a time when "artistic energies and the practical wisdom of life will join with scientific thinking to form a higher organic system" (GS 113). This higher organic system is the gay science. Mathematics plays a central role in the gay science.

The errors on which mathematics and logic are based are the most fundamental errors made by living things; they are *the falsest judgments, and consequently the most indisepensible* (BGE 4). It is precisely for this reason that mathematics is the common ground where art and science meet; as the most fundamental of all errors, the falsification of the world by numbers is where the scientific will to truth ends and where the artistic will to illusion begins. The great question of the modern age is whether human life can endure the incorporation of the scientific will to truth (GS 110, 343). The scientific will to truth renders the world meaningless and opens the door to nihilism and suicide (GS 107). This will to truth is dangerous; but living dangerously (GS 283) can be a sign of vitality: "It is a measure of the degree of strength of will to what extent one can do without meaning in things, to what extend one can endure to live in a meaningless world *because one organizes a small portion of it oneself*" (WP 585A). Mathematics *in the grand style* facilitates this organization by placing the scientific will to truth in the service of those errors without which we could not live. Math effects the transition from scientific nihilism to Dionysian affirmation (WP 1041); it is the artistic tonic needed to reconcile science with life.

A conception of mathematics as honest artistry whose will to power is a will to consistency reveals one way to resolve the apparent conflict between life and the will to truth: "the sublime consistency of and interrelatedness of all knowledge perhaps is and will be the highest means to *preserve* the universality of dreaming and the mutual comprehension of all dreamers and thus also *the continuation of the dream*" (GS 54).