The 100-year family

Longer lives, fewer children

Finance and wealth
Welfare
Culture and society
Financial planning
Education
Social care
Pensions

About the ILC

The International Longevity Centre UK (ILC) is the UK's specialist think tank on the impact of longevity on society. The ILC was established in 1997, as one of the founder members of the International Longevity Centre Global Alliance, an international network on longevity.

We have unrivalled expertise in demographic change, ageing and longevity. We use this expertise to highlight the impact of ageing on society, working with experts, policy makers and practitioners to provoke conversations and pioneer solutions for a society where everyone can thrive, regardless of age.

Authors: Les Mayhew and David Smith

Professor Les Mayhew and Dr. David Smith work in the Faculty of Actuarial Science and Insurance at Cass Business School, City, University of London.

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Contents

Summary

This paper investigates how the role and resilience of the family in the UK has changed over time, and explores how it is coming under increasing pressure from external demographic and economic forces.

We investigate these effects using a novel approach based on survivorship. We also propose a new way to define 'family,' using a framework flexible enough to model a range of family structures and situations: by centring analysis on the 'focal woman.'

Survivorship is the probability of living to a given age (see section 2 for more detail); we take this data from the UK Office for National Statistics (ONS) life tables for England and Wales. These are constructed using mortality data and are available from the mid-nineteenth century onwards.

We construct the joint survivorship of typical families based on the number of births. We also employ novel 'family accounting' methods to quantify and analyse the potential overlapping of care responsibilities that face today's families.

Our work is informed by the effects of two broad transitions, widely recognised by demographers as occurring across many societies:

- The first is the progression from high to low mortality and from high to low fertility rates (the average number of children born to each woman in any given population). These two changes combine to produce a surge in population and economic growth, accompanied by rapid increases in life expectancy. In the UK this period lasted from around the mid-nineteenth to the mid-twentieth century or later.
- The nature of the second transition is not universally accepted among demographers, but broadly it refers to the in any given population societal changes that have taken place since the 1970s; these include changes in family structures, and a shift towards women choosing to have fewer children, later in life. The arrival of protected rights and wider access to education for women during this period have been key factors in driving these shifts.

For the purposes of this paper, the term **'fertility rate'** is used to mean the average number of children born per each woman of childbearing age in the UK. We are interested not only in the number of children born to each woman but the timing and spacing of their births.

We posit that the economic benefits of the first transition are in danger of being reversed by the second, and that our social, political and economic structures are not aligned to support the families in which we now live. We explore this possibility through analysis of family structures in a context of increasingly stretched welfare systems, widening inequalities and ageing populations. This context raises questions:

- Whether our population can continue to replace itself given that families are having fewer children, later in life: our analysis indicates that, at a family level, our increased longevity does not offset the decline in fertility rates.
- How to address the additional strain on our underfunded social, health and state pension systems, with more older people living alone, and a greater need within today's smaller, older families for external support.
- How to address the likelihood that the tendency towards older families leads to each family's main carer being responsible for multiple generations at once.
- Whether the additional burdens of juggling work and caring responsibilities will have a further stagnating effect on the wider economy.
- How to address the inheritance gap that delays the passing of wealth to the next generation as we all live longer.

We believe that society must adjust itself to the new reality, by taking steps to move into a third transition. This will require action to enable more of us spend our additional years in good health and in decent housing, with the capacity to undertake paid work, to care for our families, or to do both. We suggest that as part of this transition there may be a need for:

- Reformulated personal financial services to address the current gaps in provision at the family level
- A new approach to social protection that focuses on families as well as individuals

Our analysis shows that the changes occurring during the second transition have put society on a demographic escalator to economic stagnation, and that matters can only get worse. We believe it will take conscious action by the UK's decision-makers to make a third transition reality and step off the escalator.

1. Introduction

Why study families?

Many of our societal structures are based on the premise that the family is the bedrock of society. We assume that family members will provide mutual support and share resources over the life course.

Amid ageing populations and falling fertility rates, families are less resilient than they once were. These factors have potential implications for society as a whole, but often go unnoticed until there is a crisis. One reason for this is that national data collection systems tend to focus on populations and individuals, rather than families; as a result children and older people are often ignored by analysts such as demographers and actuaries (Schoen, 2018).

Policymakers do sometimes consider family units; we see this in policies that concern planning for families and safeguarding children, as well as the tax and benefit systems. We also see assumptions made about the role of families in the construction of our care system. We see less evidence of this when it comes to financial services.

Esping Andersen (2009) identifies population ageing and falling fertility rates as key drivers of change in our society, along with the changing role of women in societies, and the shift towards a knowledge economy. In this paper we focus on the role of women in families as a way to examine how families have shifted over time and what this could mean for society.

Our aim in writing this paper is to use a fresh analytic perspective to provide a practical study of how demographic changes are affecting families. We do not intend it to be a theoretical treatise on individuals or their descendants or on families as an economic construct. A rich literature on these topics already exists (e.g. Jagers and Sagitov, 2008; Becker, 1991; Esping-Andersen and Billari, 2015).

Using life tables, we show how increased longevity, as well as forming partnerships, and having children later in life, may affect the size and other characteristics of the typical UK family. We show how these variables can have a significant impact on the lives of individuals; for example they influence who provides care for whom and when, as well as inheritance issues.

Defining the family

The definition of 'family' is a key element of this paper. Families have changed significantly since the nineteenth century, mainly due to lower fertility and higher survival rates. Another driver of change in families is economic necessity, as families are increasingly geographically separated when members move away to seek work.

In this paper we define family through kinship to a 'focal woman.' This concept, which we introduce in this paper, looks at one woman's links up and down the generations, including her parents, grandparents, siblings, partner(s), children, and grandchildren.

Our research combines actuarial methods with representative family demographics, to find that families affected by increases in life expectancy are smaller and on average older. In particular, we consider how long-term demographic trends in society are reflected at the family level.

Women over the last 100 years

Our research centres on a 100-year period for two reasons. The first is to allow us to cover the entire life course of our hypothetical 'focal woman,' reinforcing the pivotal role of women in inter-generational analysis.

The second is that this period echoes the influential book by Gratton and Scott (2016), *The 100-year life – living and working in an age of longevity,* which focuses on the life courses and work choices of individuals rather than on families.

Compared with a century ago, women are more likely to work through their adult lives, increasingly juggling childcare with earning a living (Charlton and Murphy, 1997). Women now tend to have their first child some ten years later than they did 100 years ago (Bernhardt et al, 2015).

We examine how this affects the 'average' family in the UK.

Fewer births, longer lives

An extreme example of external factors affecting familial support ratios comes from China. The one-child policy, introduced in 1969, created a generation of families with one child, two parents and four grandparents, the so-called '4-2-1' family. This generation despairingly talks about the need to become 'rich' before they are 'old,' to counteract the wealth imbalance with other generations.

Many other countries have also seen birth rates fall to below the level where the current population can be replaced ('replacement levels') without the imposition of artificial limits. Although surveys show that six out of ten women think two children is the ideal (Sobotka, T. and É. Beaujouan 2014), it is not the rule; even if it were, that rate would not reach replacement level.

If we wind the clock back to the eighteenth century, we find that the parents of the celebrated Austrian composer Franz Schubert, Franz Theodor Schubert and Elisabeth Vietz, had 15 children between 1783 and 1801. Of these, seven died in their first year of life, and three before their 6th birthday. The composer himself died aged 31 and his mother at age 56. His father died aged 67, having fathered another five children with his second wife! Although common at the time, this would hardly occur today, thanks to wide access to birth control and improved healthcare.

With regional variations, up to 20% of infants died before their first birthday in England and Wales over the period of 1851 to 1911.[1] This greatly affected life expectancy at birth: in 1841, life expectancy was over seven years higher at age eight (50.9 years) than it was at birth (43.8 years); now life expectancy at birth is the highest. Someone born in 2020 can expect to live until they are 94.

We also now see less variation in the age of death, reflecting long-term increases in life expectancy, health and wellbeing. This may indicate a trend towards 'fixed life spans' or at least much less variability compared with the past.

Lower birth and mortality rates also reduce the number of living grandchildren and increase the number of living grandparents. Murphy (2011) describes the emergence of the 'beanpole' family, with more vertical links through the generations and fewer horizontal links through siblings, with fewer aunts, uncles, nieces and nephews.

As the average age of the family rises, there is a higher incidence of the chronic health conditions that are more prevalent in older people. This stretches the resources of smaller families with fewer potential carers, and triggers a need for support from external agencies. In addition, the costs of providing care to family members across the life course can be financially crippling and eat into legacies.

[1] *Populations past - An interactive atlas of Victorian and Edwardian population:* http://www.populationspast.org/

1.1 The context for changes in the family

Demographers discuss two typical demographic transitions which most countries undergo at some point in their history: the first delivering a 'bulge' of working-age people and rapid increases in economic growth, while the second is characterised by fluctuating birth rates below the levels required for population replacement.

The first transition involves a reduction in mortality rates, followed by a drop in birth rates. The period between these two changes produces a bulge of people of working age, often referred to as a 'demographic dividend.' In the UK this transition unfolded over roughly a 150-year period. Mortality rates were in decline from the end of the eighteenth century, but birth rates did not start to fall until the 1880s, with the transition finishing in around 1950.

Before the first transition formal systems of social protection were rudimentary, with very low coverage. Poor sanitary conditions meant that infectious diseases were rife until the end of the nineteenth century, with families largely fending for themselves. Life expectancy at birth was very low and it was common for there to be more children per family living in overcrowded homes.

Other countries in Europe experienced a similar transition around the same time as the UK; there are also many accelerated examples from the post-war period, including Japan, Singapore, South Korea, and more recently China and India (Bloom et al, 2003).

There is less consensus over the nature of the second transition, which may be a direct consequence of the first. This transition is characterised by a tendency for people to live together rather than marry, with high divorce rates and fluctuating sub-replacement birth rates (e.g. Bumpass et al, 1991; Bumpass et al, 2000). Sub-replacement birth rates can lead to higher immigration, as seen in Germany, or population decline, as seen in Japan.

Although it is not universally accepted whether the second transition is a permanent change or may still be reversed, it certainly describes a situation faced by many countries: traditional familial support systems are under strain, while social protection systems such as pensions, health and social care face financial pressure (Goldscheider et al, 2015; Cherlin, 2016: Zaidi and Morgan, 2017).

One thing is clear: as the population ages, the proportion who are of working age compared to older people falls. The comparison between these two groups, or its inverse, is referred to as the 'old age dependency ratio.' This has obvious implications for pensions, health care and social care.

International practice defines old age as beginning at age 65, which is a useful benchmark. In the UK the ratio of the population aged 20 - 64 compared to those aged 65+ has fallen from around 4:1 in 2000 to 3:1. ONS population projections predict the ratio will fall to 2:1 by 2040 (ONS population Projections[2]).

Will the second transition ever end? Some researchers talk about the 'longevity dividend' (e.g. Olshansky et al, 2007). Here this is defined as taking advantage of a longer life expectancy to work and be productive for longer, and to take up other valuable but unpaid activities, such as volunteering and care giving (Mayhew, 2018).

However, this dividend depends on our longer lives being spent in good health, and on us being financially independent, living in a decent home within a supportive family structure (Mayhew, 2019). To reap that dividend, we argue that it will be necessary to improve healthy life expectancy and reduce inequalities in society.[3]

A more cynical view of the longevity dividend welcomes the business opportunities it creates in providing services to older people, including people with chronic illnesses and disabilities. But there are also huge opportunities for businesses in activities for healthier older consumers, ranging from tourism and leisure to building retirement housing.

1.2 Structure of the paper

This paper builds on various illustrations that show why families are as important in studies as individuals and populations.

In section two, we describe our analytical approach, based on the concepts we have adopted and the sources of data used. We use a stylised model to explain how survivorship has evolved since the first demographic transition, and the trends going forward. We then

[2]Population projections can be found at www.ons.gov.uk/peoplepopulationandcommunity/populationandmigration/populationprojections

[3]As a step in this direction, in 2018 the UK Government's Secretary of State for Health and Social Care set a very ambitious target to increase healthy life expectancy by at least five years by 2035 for England, whilst also reducing the gap in life expectancy between the richest and poorest (Marteau et al 2019).

switch our focus to show how trends in survival and fertility affect individual families.

This requires a definition of the 'family.' For this, we introduce our concept of the 'focal woman.' The stylised analysis shows how each generation is affected by changes in fertility rates and life expectancy throughout the focal woman's life, producing distinctive family structures and demographic patterns.

In section three, we provide illustrations using life table data. Our focus is on a 100-year time window anchored to the year of birth of the focal woman. This shows the changes that are caused when we apply different scenarios throughout her life course. We derive new metrics to describe within-family changes, such average size and age and the person-years lived by all family members.

Section four covers the provision of care for different family members during the focal woman's life and the dependencies that arise. We introduce the concept of 'family accounting,' a tool used in this instance for evaluating periods of overlapping caring responsibilities as arising from births, deaths, illnesses and disabilities.

A final section concludes and suggests ways forward.

2. Modelling approach

Our modelling approach is heavily stylised to communicate complex processes more simply. For our illustrations we use life tables to measure survivorship, based on ONS life table data from 1841 to 2066, a period which spans the two demographic transitions under discussion, and also peers into the future.

Life tables are based on the probability of any individual surviving to their next birthday. From this one can determine survivorship, i.e. the proportion of the population that will survive to a given age, as well as life expectancy and other demographic measures such as the mortality rates used by actuaries and demographers.

There are two types of life tables: cohort and period. Cohort life tables track the individuals born in a given year and complete once that cohort has died out. Period life tables are based on the mortality rates experienced in the population today.

Although both can be used in our model, we consider a cohort-based approach more useful as it can encompass mortality rate improvements over time. Since users of these methods, such as demographers or actuaries, will tend to look forward to predict what the future families might look like, the results should be more appropriate.

The model can be used in two ways:

- **Prospectively:** as a probabilistic model estimating birth years and birth rates, then deriving survival probabilities from cohort life tables to determine the size and survival of families over time
- **Retrospectively**: as a deterministic model that examines what has already happened rather than speculating on what might, using known birth and death dates, as might be the case when used by a historian or genealogist

A deterministic model is typically used for studying single families with complete demographic facts such as years of birth, death and union. A probability-based model is applied to populations of families, using average fertility and survival rates based on the year of birth and gender of individual family members.

We can also use the model to combine data for deceased family members with speculative details for those who are still alive. Such applications could include analyses of the hypothetical effects of untimely deaths, illness, divorce or inheritance.

There are several limitations to our model. For example, we assume that the survivorship of one family member does not affect the survivorship of other members.

Although our illustrations are based on national averages for survival rates across the whole population, it is possible to use survival rates for more limited groups, such as sufferers of diseases like cancer. This would allow us to analyse the potential effect of such a disease on families. In this case we would be applying specific survival rates to the circumstances of a particular family. Such applications are not considered further here but they are an obvious progression.

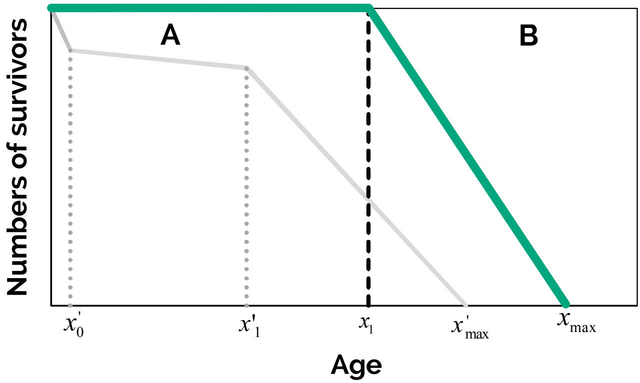
2.1 Conceptualising survivorship

Mathematically the survival curve, $S(x)$, denotes the probability of a person surviving to age x. With no loss of generality, we describe a simple, stylised model from which we derive the relationship between survival, life expectancy and the distribution of ages at death which has been used in previous published research (e.g. Mayhew, 2001; Mayhew and Smith 2020).

Imagine a stationary population in which there are a constant number of births and deaths, there is no migration, and which experiences the same mortality regime each year. Figure 1 shows the survival curves for two points in time.

The vertical axis shows the number of survivors $l(x)$ and the horizontal axis shows age, x. At age zero the probability of being alive is one, but this declines with age until death, when the probability is zero. The first example (A), shown in grey, uses rates from before the first demographic transition, while (B) is from a time after the transition.

Figure 1: Two stylised survival curves: A and B



Key:

A Including infant and childhood mortality:

x_0 =age of subsidence of infant and childhood mortality

x_1 = age of onset of adult mortality

x_{max} = maximum age to which anyone lives

B Excluding infant and childhood mortality:

x_1 =age of onset of adult mortality

x_{max} = maximum age to which anyone lives.

A reflects this period's high mortality levels and shorter life spans. The first years of life are marked by high infant and childhood mortality levels, up to age x'_0, after which levels subside slightly. We define x'_1 as the age at which adult deaths begin, and x'_{max} as the maximum age to which anyone lives.

All three of these ages are somewhat fuzzy quantities in the real world, i.e. we cannot pin an exact figure to them. However, our purpose is to use them as devices to anchor and compare distributions and mortality processes, rather than to determine them empirically.

In B, the survival curve is much more rectangular in shape, reflecting a huge decrease in childhood and adult mortality. Age x_1 is defined as when adult deaths begin, while the cohort dies out by age x_{max}, the maximum age to which anyone lives. In developed countries, x_1 starts from about age 60 onwards and x_{max} is about 100 years or slightly more. For simplicity, we assume there are no deaths before x_1, i.e. the probability of being alive is one.

During the transition between A and B, the maximum age to which anyone lives increases from x'_{max} to x'_{max}. The actual data indicate three phases: moderate increases in life expectancy from 1850 to 1900 (phase 1), reduced infant and childhood mortality with much more rapid increases between 1900 and 1950 (phase 2), then more moderate increases thereafter (phase 3).

The significant increase in life expectancy between A and B is a combination of falling mortality levels in young and early adulthood, increases in the age of onset of mortality, and an increase in the maximum age to which anyone lives (Mayhew and Smith, 2020). Today, the largest increases in life expectancy reflect more individuals in their 70s reaching 80, more in their 80s reaching 90, and so on (Mayhew and Smith, 2015).

The practical implications of this are easily illustrated by example. For instance, a woman born in 1850 had only a 50% chance of surviving to age 50; however, if she were born in 1950 she would have a 50% chance of living to 87, and if born in 2000, to 93. This inevitably leads to an increase in the older population.

A key question for policy makers is to ask what opportunities might result from these changes, if we can achieve the third demographic transition and capture the longevity dividend. The answer is necessarily speculative, but the survival distribution would become more rectangular in shape, tending towards lives of a fixed length.

In this case, the gap between x_1 and x_{max} would eventually narrow to nothing. We can verify whether this is currently happening by comparing median life expectancy with the maximum ages lived. We would expect these to converge over time (Mayhew and Smith, 2015). However, it seems the evidence is not very strong as yet.[4]

The analysis we have looked at so far has been based on individuals. In the next two sections, we use our stylised model of survivorship to investigate how these transitions affect families before, during and after the first transition.

[4]If we take the 95th percentile as a proxy for maximum age, we find a maximum age of 100 in 1950, compared with 105 in 2000: a five year difference. This compares with a difference of six years based on changes in median life expectancy. This suggests that convergence is proceeding very slowly.

2.2 Defining the family

If we use a definition of the family that encompasses all people linked by blood ties or marriage, the resulting families would be extremely large, with little real connection between some members. They may be disparate geographically and not even in social contact with each other.

Surveys such as the UK Government Census measure household composition by recording people living at the same address; they do not define kinship links between relatives who live elsewhere.[5]

If we take households as a proxy for families, the definition is useful in describing how a family is organised financially and physically, but it ignores strong bonds such as those between parents, siblings and grandchildren living elsewhere. (Willekins, 2010).

Box 1 refers to some of the definitions found in the literature on families, households etc. (for example Schoen 2018, Chapter 12). Because we think close biological links more accurately define family bonds, our definition of the family focuses on the concept of the 'focal woman.'

We use this individual as a point of reference for analysing family structures that include members born before and after her (see Box 1). Apart from her partner, all family members are biologically related, either as parents or grandparents, siblings, children, grandchildren or great-grandchildren.

This is an example of a 'kinship' approach, in which the family is traced through the female rather than male lineage. The number of members included is a matter of choice: we exclude cousins, nieces and nephews from our examples. But the gender, birth year and family relationship of each member is required. The constituent members of the family, along with their biological relationship to the focal woman, are referred to as the family pool.

The time windows in our illustrations are based on each focal woman's year of birth. This allows us to consider changes that take place over several generations, and to align our analysis with the reproductive cycle to consider issues such as the impact of fewer children in a family.

[5]Past and present census forms can be found at https://census.ukdataservice.ac.uk/use-data/censuses/forms.aspx

Box 1: Defining the family

Definitions

A **household** is a group of people under one roof who share some resources

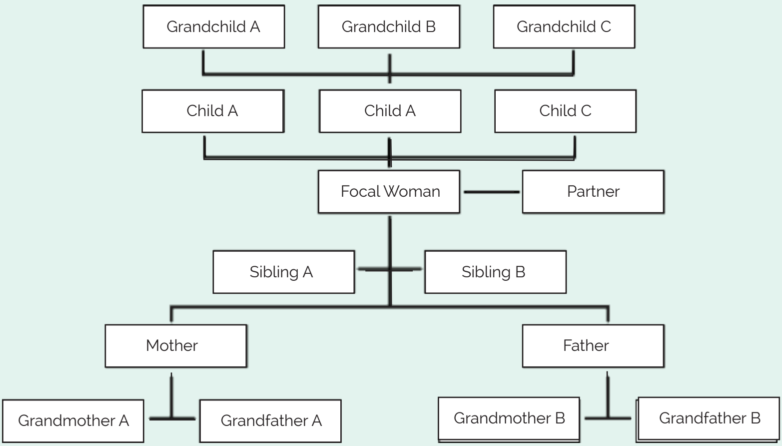
A **family** consists of households of related individuals

The **family pool** defines the related individuals to be included in the analysis, which may include one or more households (e.g. grandparents, parents, children and grandchildren)

The **focal woman** is the family member whose date of birth marks the starting point of the 100-year time frame that forms the analytical object of this study

A **nuclear family** is a household consisting of two parents and children

A **family tree** is a diagram of family members in the form of a generational hierarchy

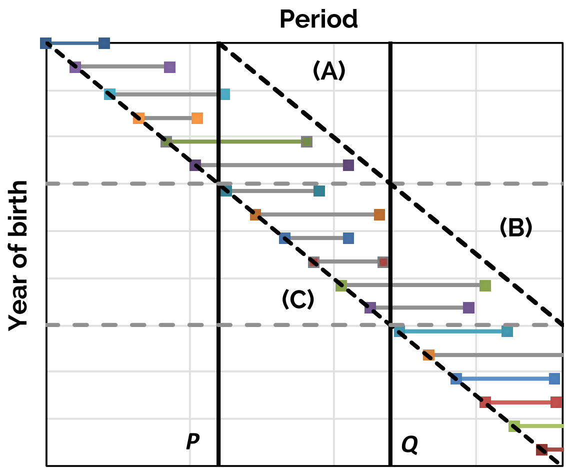


Schematic of a family tree with five generations showing grandparents, parents, the focal woman, her siblings and partner, her own children and her grandchildren.

2.3 The cyclical nature of families

We labour these definitions because it is necessary to show how families change over time. Figure 2 is a schematic representation of this.

Figure 2: A simple birth and deaths process



Key:
The horizontal axis shows calendar years and the vertical axis shows the birth years of individual family members. Each horizontal line represents a family member, with the length representing how long that member lives. Hatched lines show the start and end point of each cohort.

The family size at any point in time is the number of horizontal lines crossed by the appropriate vertical line, such as lines P and Q. P in this context could represent the year of birth of our focal woman, and Q could be 100 years later.

Section A includes family members born before time P, who die after P but before Q. Section B includes those born between times P and Q who die after Q. Section C includes those born after P but who die before Q.

At time P the family has three living members; at time Q it has four. This is found by counting the number of horizontal lines starting after P and crossing Q. There are two family members who are born after P but die before Q.

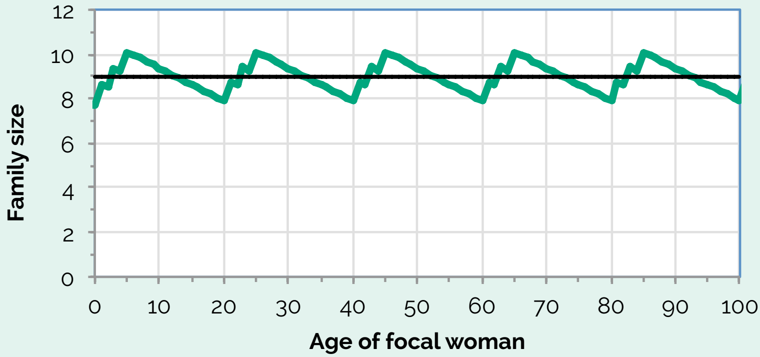
The person years (the total years lived by family members during a defined time period) lived by a family unit are a measure of its size and longevity. This figure is found by summing the years spent by each living member inside the time window defined by P and Q. This highly simplified stylised approach contrasts with the real world, in which the state of being alive is governed by probability distributions – in our case, life tables.

To show this we splice together the survival curves activated each time a new person enters the family. We then total the number of family members alive in each year to obtain family size at any point in time.

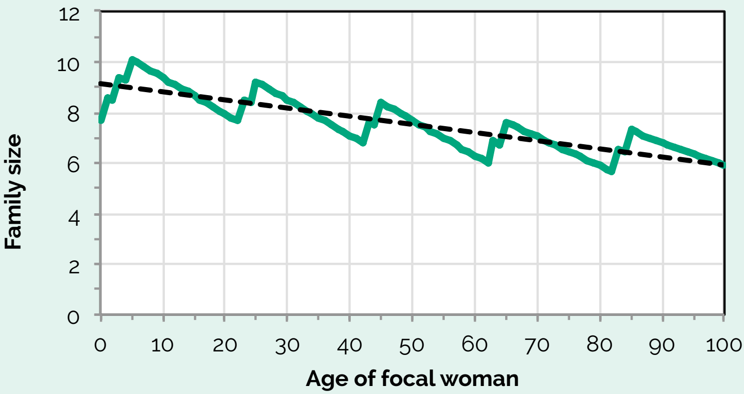
Examples of the patterns obtained are given in Box 2 which are based on the focal woman's age. The survival function corresponds to survival curve B in Figure 1 (i.e. it is based on low infant mortality and adult mortality up to a maximum age, characteristic of the post-transition phase).

Box 2: Schematic examples

A. Baseline stable state based on three children



B. Children born later in life, with reduction from three to two children plus improved life expectancy



The first case (A) assumes the focal woman has three children born in quick succession each year, starting from age 20. The pattern repeats with her daughter having three children, and so on. This is an example of a 'stable state,' which produces five cycles, with a trough-to-peak length of ten years, and trough-to-trough length of 20 years.

Case A produces 15 births in 5 reproductive cycles (3 births per cycle). It assumes a relatively low life expectancy of 60 years, with an average family age of 30. Family size varies between 8 and 10, with an average of 9 for the 100-year time window. This pattern will be sustained indefinitely if nothing changes.

The rising part of the cycle represents a birth and the declining part represents a death. The average age is counter-cyclical, i.e. lowest at the peaks and highest at the troughs. Depending on how long she lives, the focal woman in theory becomes a grandmother at 40, a great grandmother at 60 and a great-great grand mother at 80.

The second example (B) is based on fewer children, born later in the mother's life, making it more characteristic of the second transition phase. We assume that life expectancy at birth increases from 60 to 70 years during the focal woman's life, reflecting wider improvements in the general population. Births are now spaced at 21-year intervals instead of 20, yielding four and a bit cycles instead of five as in case A.

Family size is 8 initially, as in case A, but it declines to 6 at the end of the 100-year period. This decline occurs despite the increase in life expectancy over the period, meaning that the increased life expectancy cannot compensate for the fall in birth rate. Assuming this pattern persists, this implies that families will continue to get smaller over time.

2.4 Fixed-length life-spans

We saw earlier that the benefits of improved survivorship may include a tendency for life spans to become fixed in length (as evidenced by the increased clustering of deaths at a certain ages). This could lessen inequalities in health, and potentially give people and their families more certainty about life choices.

This can create some interesting effects, especially if birth intervals are an exact multiple of lifespan. Instead of fluctuating as per the examples in Box 2, family size would be represented as straight line, staying constant throughout the 100-year time window. Average age would still vary though, depending on birth intervals.

Annex B explains this further with examples.

Clearly, such developments are not something that can be designed or engineered, but are the result of a long trend. However, they have the capacity to transform financial products designed to mitigate for the uncertainty of when death will occur.

3. Selected illustrations

In this section we use ONS life tables populated with past, current and future survival data. They are designed to show the effects of varying fertility rates and longevity levels on family size and age. We refer to this as a 'family survivorship model,' and we apply it in different contexts in the following sections.

We have designed the illustrations to show the effects of changes in survivorship over time, and to reflect features of the first and second demographic transitions. The technical detail of how we have applied the life tables and the family metrics used within the model is given in Annex A.

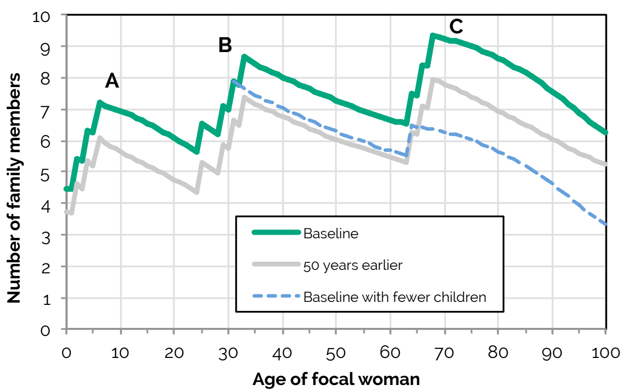
We start by looking at changes in family size over our 100-year time period. We would expect family size to equal the survival probabilities of all other family members, expressed as a function of the focal woman's age, x.

We can make the following variations in our model:

- Each family member's birth year
- The number of siblings
- The year our focal woman forms a relationship with her partner
- How many children she has
- How many grandchildren she has

The cohort life table based on each member's year of birth predicts their survival. These tables are derived from figures for England and Wales but are a good proxy for the whole UK; i.e. we would not expect the results to vary by much.

Figure 3: Comparing family sizes for three related scenarios



Key:
A = siblings
B = children
C = grandchildren

Figure 3 represents three scenarios for three different family pools, the parameters of which are given in Table 1. It compares the family size in each scenario. The pattern in each case generally follows the stylised examples in section 2 but now the survivorship figures are real.

Table 1: Birth years and gender for each scenario

Family member	Scenario 1	Scenario 2	Scenario 3	Gender
Focal woman	1956	1906	1956	F
Partner	1950	1900	1950	M
Father	1920	1870	1920	M
Mother	1920	1870	1920	F
Grandmother A	1900	1850	1900	F
Grandfather A	1900	1850	1900	M
Grandmother B	1900	1850	1900	F
Grandfather B	1900	1850	1900	M
Sibling 1	1958	1908	1958	M
Sibling 2	1960	1910	1960	F
Sibling 3	1962	1910	1960	F
1st child	1985	1935	1985	M
2nd child	1987	1937	1987	F
3rd child	1989	1939	n/a	F
1st grandchild	2020	1970	2020	M
2nd grandchild	2022	1972	n/a	F
3rd grandchild	2024	1974	n/a	M

The baseline (first) scenario has a focal woman born in 1956, with four grandparents, two parents, three younger siblings, an older partner, three children and three grandchildren. In the second scenario she was born 50 years earlier in 1906. In the third, her birth date remains 1956 but she has only two children and one grandchild.

In scenario 1, her parents were born in 1920 and her partner in 1950. She has three younger siblings born at two-year intervals starting in 1958. Her grandparents were all born in 1900. We assign a probability of one for her mother and father being alive in 1956 because her birth requires that the mother be alive at the time of birth. It is also very likely in the father's case, although this assumption can be varied.

We assign a probability of 0.64 to the grandmothers being alive at the time of her birth, and 0.58 to the grandfathers, based on their years of birth and the year in which the focal woman was born, according to the relevant ONS life tables. Summing the probabilities of all parents and grandparents gives us an estimate of 4.5 family members alive at the time of her birth. We repeat the calculation for the expected number of family members alive for each year of her life until she notionally reaches the age of 100.

The expected family pool increases to seven by the time she is nine years old (point A). The family then reduces in size with the demise of her grandparents. She marries her partner at 25 and has three children in quick succession from age 29 to 33, increasing the expected family size to 8.5 (point B).

The first of her three grandchildren is born when she is 64, increasing the family size to just over nine when she is 69 (point C). The troughs between these peaks represent first her grandparents, then her parents dying.

The larger family size in scenario 1, compared with scenario 2, is due to general increases in life expectancy between 1906 and 1956. However the underlying birth cycle is similar: peaks commence slightly sooner as mortality is higher, leading to a smaller difference between trough and peak.

In scenario 3, the effect of having fewer children and grandchildren is shown as a hatched line. From around the age of 70, the family size dips below that of the second scenario. Table 2 summarises the differences between scenarios.

Table 2: Family metrics by scenario

Metrics	Scenario 1	Scenario 2	Scenario 3
Maximum family size	9.3	8.0	7.9
Minimum family size	4.4	3.7	3.3
Average family size	7.3	6.0	6.0
Years lived/100 year life	740	608	606

This table shows the maximum, minimum and average family size. Summing over age gives the total number of person years lived by family members in the century following the birth of the focal woman. Scenario 1 produces 740 person years, 132 more years than scenario 2. Scenario 3 has a slightly smaller total than scenario 2, despite the advantage of improved longevity.

In scenarios 1 and 2, family size is at maximum when the focal woman is 68. In scenario 3 this happens when she is 31, as there is only one grandchild. We also see that family size reduces at the end of each 100-year cycle regardless of scenario.

In scenario 3, family size would be at its smallest should the focal woman reach 100 years of age (in contrast with scenarios 1 and 2, where this happens when she is a child). This means that lower birth rates and increased longevity make it less likely that our focal woman will have support from her family at the end of her life.

3.1 Effects of increased life expectancy on average family age

Our analysis also allows us to draw out other effects. We continue with our baseline case A in a series of new illustrations. The first of these concerns the variation in the average family age as a function of the focal woman's age: x .

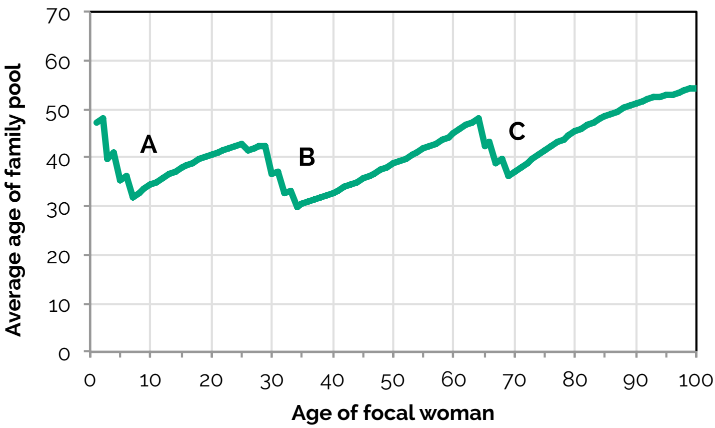
We calculate this average family age as the weighted sum of the probabilities of family members being alive, where x ranges from 0 to 100 (see Annex A.4). This variation is represented in Figure 4; it is a mirror image of the pattern in Figure 3.

The average age rises and falls with each new generation and is therefore counter-cyclical compared with family size. This will occur as long as the number and spacing of births is unchanged.

The difference between peak to trough ranges from around 12 to 18 years in the baseline case. Note also that the overall trend is upwards, due to the increased life expectancy of individual members.

All other things being equal, family size and average age are a function of the chosen time window: i.e. the more recent the time window, the greater the average age.

Figure 4: Average age of the family over time



Key:
A = the effect of siblings
B = the effect of her children
C = the effect of her grandchildren

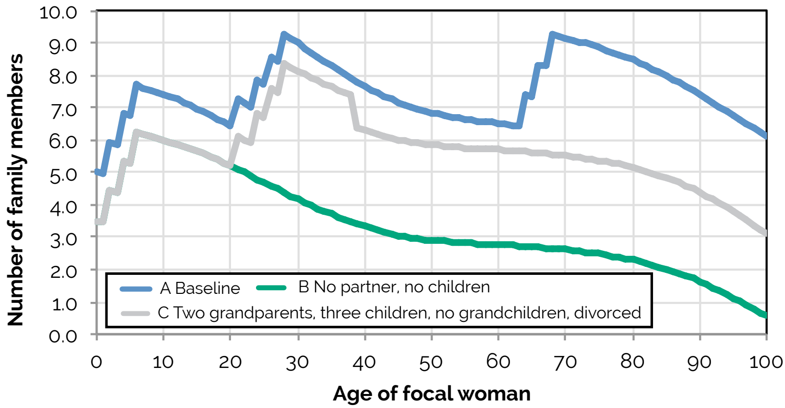
3.2 The effects of discontinuities on families

Any changes to the reproductive cycle give rise to different outcomes. If the focal woman does not have children or grandchildren, if she experiences the death of a partner, or if she divorces, any of these cases produces a smaller family size at every age, with a lower number of person years lived.

The illustration in Figure 5 represents a small sample of possible outcomes in three cases. A is the baseline case from Figure 3; scenario B supposes no partner and no children; and scenario C supposes a divorce or separation leading to two grandparents, three children and no grandchildren.

We can create many variations with different numbers and timing of children, multiple partners and step families. More complicated scenarios obviously require careful consideration of who should be included in the family pool.

Figure 5: Comparing average family sizes based on three related scenarios



3.3 Dividing families into 'young' and 'old' members

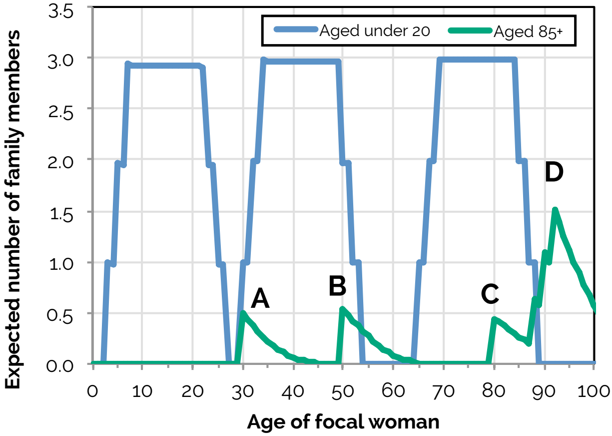
We can also look at younger and older family members separately. Depending on how we define the cut-off ages that separate 'young' and 'old' individuals, these divisions can be helpful in describing whether a family is big or small, young or old, and changing over time.

The results may be useful for drawing out the possible implications of changes in family structures. Depending on the purpose of the analysis, it could inform investigation into the need for financial support throughout our lives and for financial transfers from older to younger generations, such as for paying for education or long-term care.

For illustration, we will assume two age cut-offs: a lower limit, L, below which individuals are classed as children or young adults in full-time education; and an upper age limit, U, above which individuals cease to be fully independent. Annex A.5 explains how cuts-offs are represented in the model.

The model is the same as the baseline case A in Figure 5, but family members are now categorised according to their age. Figure 6 shows the results. In this illustration L is defined as the age at which formal education ceases, which we take to be 20, and we have assumed a value of 85+ for U.

Figure 6: Expected number of living family members aged under 20 or 85+



Key:

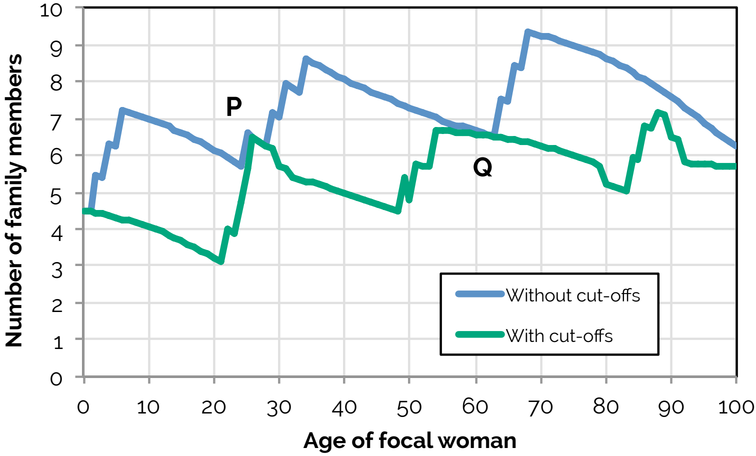
A = grandparents
B = parents
C = partner
D = siblings

Three generations of children are represented (siblings, the focal woman's children and grandchildren). The number of family members aged 85+ peaks in the middle of her life (grandparents and parents) and at the end (partner and siblings).

The placement and heights of these spikes can be considered to be partly the result of improving longevity.

One way to show the effect family ageing has on the availability of carers is to distinguish those family members that are available as carers from those needing care. Applying the same age cut-offs as before, we get the chart in Figure 7.

Figure 7: Variations family size showing the availability of potential carers



The series touch at two points, P and Q. At these points there are no family members under the age L, or above age U, and so care is not an issue. In this example the focal woman is 27 at P (just before she has children and before her grandparents reach U) and 63 at Q (when her parents have died but before grandchildren are born).

We can conceive of many other cases, such as the availability of grandparents to care for their grandchildren, or in which the focal woman cares for a sick partner, young children and frail older parents simultaneously. We return to this issue in section 4 when we discuss sandwich generations.

3.4 Within-family dependency ratios

The ratio of the number of older people to working age individuals in a population is generally referred to as the 'old age dependency ratio.' Earlier we drew attention to the fact that by 2040 there might be only two adults aged 20 to 64 for every individual aged 65+, where 65 is the assumed pensionable age. This is half the number of working-age adults for every pensionable-age adult seen in 2000.

If this is happening at a population level, we ask how it would be reflected at a family level. What are the effects in terms of the financial support ratios within the family and how is the ratio affected if children (as non-earners) and older people are included?

As before, let there be two age cut-offs: a lower limit, L, below which an individual is classed as a child or young adult in full-time education; and an upper limit, U, above which they are assumed to be of pensionable age but not yet physically dependent.

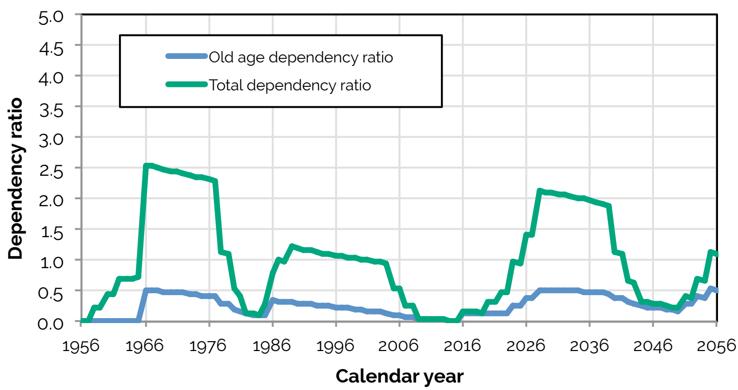
In the following illustration, we use two definitions:

1. **The old age dependency ratio**: this is the proportion of the population above a given age cut-off U (in which we assume a pensionable age of 65), compared to working-age adults between *L* and *U*, where *L* is assumed to be 20. A ratio of one means one older person for each working-age adult.
2. **The total dependency ratio**: this is the ratio of family members between ages L and U to the number of members of age U and older, plus those of L and younger, i.e. it includes both young and old family members alike. A ratio of 2 say, would imply that there are two young or older family members for each working age adult.

The higher the value of the ratio, the greater would be the degree of dependency in the family. We are interested in how it varies over the life course of the focal woman. As before, we base our illustration on baseline scenario A.

Figure 8 shows that both ratios tend to rise and fall together, as the arrival of new children tends to correspond with the ageing of older family members. However, the total dependency ratio is always because it includes both older people and children. The old age ratio usually stays below 0.5, i.e. there are two adults for each pensionable family member.

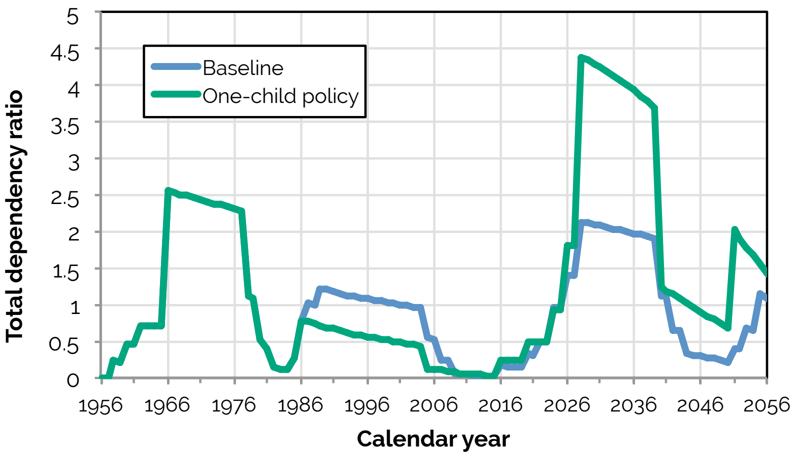
Figure 8: Dependency ratios for case A with a focal woman born in 1956



To offer an extreme example, we have created an illustration where we imagine China's defunct one-child policy being applied to our focal woman. The one-child policy was in force in China from 1979 to 2016. It was intended to stem the rapid growth of the population and create better conditions for economic growth and advancement. Given the real-world timings in China, the policy would not have affected her parents but would apply to her and her child. We can now show, using the model, why this policy was a failure.

We represent the imagined effect of this policy on family size and on total dependency ratio in Figure 9. The family becomes top heavy, in terms of the number of older people, in the 2020s and 2030s, with a total dependency ratio of nearly 4.5 compared with 2 in the baseline scenario.

Figure 9: The total dependency ratio due to a one-child-policy compared with the baseline scenario



4. Further uses for the basic model

There are a number of related applications for the basic model as illustrated in the previous sections. For example, we saw that there will be peaks and troughs in the need for, and provision of, care across any family according to the age and health of family members (and a range of other factors).

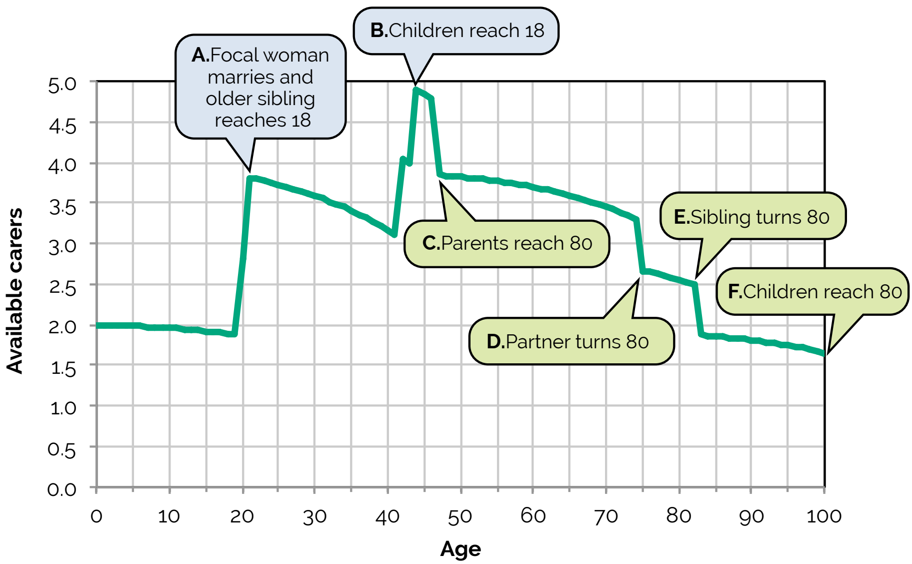
The model could be used deterministically, using concrete data stating when limitations on caring come in to play, based on real families. However, for the purposes of this section we have based our calculations on a set of informed assumptions.

4.1 Case study 1: A lack of carers

For the first extension to the basic model, we consider scenarios in which the focal woman may find herself without family members able to provide care (as they are themselves either too young or too old), especially if she lives to 100.

For our case study, we will posit a family where those aged under 18 and over 80 are unavailable as carers. Figure 10 shows how key life events trigger changes to the number of family members available to provide care.

Figure 10: Hypothetical availability of family carers for the focal woman during her life



As a child, the focal woman lives with her parents and one younger sibling. We assume that she forms a partnership and moves out at 21, to a partner who is six years older. According to our assumptions, she has just under four potential carers at this point in her life (point A).

We assume she has two children at 24 and 26. They reach 18 when she is in her 40s, increasing the number of potential carers to just under five, the maximum in her life time (point B). At point C her parents turn 80; based on our assumptions they are now no longer available to care for her, reducing the number of potential carers to under four.

At 74, our focal woman's partner turns 80, making them unavailable to care; at age 82 her sibling turns age 80, reducing the number of potential carers to two (point E). Assuming she lives to 100, her children will shortly become too old to provide care (point F).

In summary, the analysis suggests she could she could run out of carers especially if she is pre-deceased by her children. Life tables suggest that one or two children today is usually enough to ensure that at least one will be alive when the focal woman reaches 80, say, but not if she lives to 100 years.

Our assumptions about the primary carers in this example exclude the possibility of grandchildren taking over caring duties. Conventionally grandparents provide care for grandchildren, but the reverse is unusual (though some research from the US that suggests this may be changing).[6] In general we still know very little about care exchanges between grandchildren and grandparents, or how care works in more complicated family structures, making this an area for further research.

4.2 Case study 2: Sandwich generations

As more women have children later in life, it becomes more likely that our focal woman would be faced with caring for her children and her parents at the same time. The literature uses the terms 'pivot' or 'sandwich' carers to describe people who, in certain phases of their lives, are called upon to simultaneously care for their children, older parents and/or possibly a partner.

[6] E.g. see https://www.seniorly.com/resources/articles/a-new-age-of-caregiving-grandchildren-caring-for-grandparents

As people have children later, they are responsible for looking after them at a time when their parents are also older and more likely to be affected by age-related illnesses.

Our illustration for this case study reverses the situation in section 4.1; it considers how many family members our focal woman might be asked to provide care for at each point of her life. We have removed the probabilistic element of the model and assumed that we know the birth and death dates of all family members.

Our aim is to calculate how many years our focal woman will spend as a carer throughout her life, given the circumstances of her family. We also estimate the extent to which she will care for several family members simultaneously, and how many other members will be able to share the burden of care.

Clearly each individual's requirements for care are not dependent solely on their age but a range of other factors, including their health, any disabilities, and the extent to which other sources of care are available.

We find that the extent of any 'sandwich' phases depends on the birth year of the oldest parent and the birth year of the youngest child. This period is most likely to start in our focal woman's middle age (around 45) and to potentially last for many years.

We analyse two examples, each with three care phases: raising children, caring for older parents, and caring for an older partner. We do not include looking after grandchildren. We call the tabulated outputs 'family accounts,' which enumerate the care phases over the life course, including any overlaps.

We start by creating a schedule of events, including the dates of birth, death and marriage for all family members. We centre our analysis on the birth year of the focal woman and immediate family members. In both cases we limit the family size to seven: the focal woman, and her mother and father, partner and three children.

Example 1: No sandwich phases

Table 3a lists the events that affect the focal woman and Table 3b lists those that affect the relationships between her and the other six family members.

Table 3b indicates that for children, care starts at birth and ends at age 18; for members already over 18, care is arbitrarily assumed to begin at 80.

Tables 3a and b: Family chronography of events in Example 1

(a) **Focal woman**

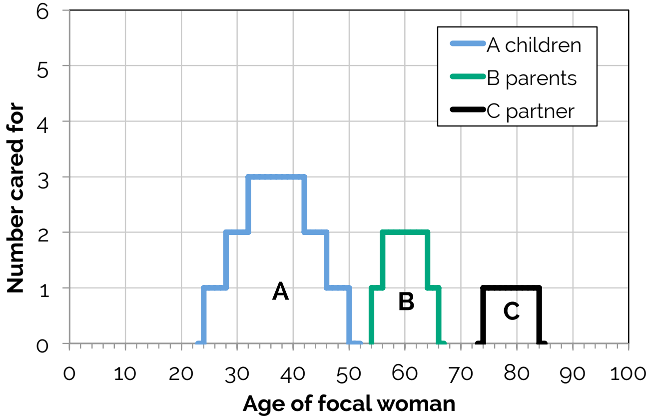
Birth	Death	Marriage	Age at death	Age at marriage	Length of marriage	Father's age at birth	Mother's age at birth	Partner's age at marriage
1956	2046	1984	90	28	56	26	24	34

(b) **Family care accounts**

Family member	Year of birth	Age difference	Year of death	Focal woman's age at death	Adulthood threshold age	Focal woman's age at threshold	Age at which care required	Age of focal woman when care required
Father	1930	−26	2020	64	n/a	n/a	80	54
Mother	1932	−24	2022	66	n/a	n/a	80	56
Partner	1950	−6	2040	84	n/a	n/a	80	74
Child 1	1980	24	2070	n/a	18	42	80	n/a
Child 2	1984	28	2074	n/a	18	46	80	n/a
Child 3	1988	32	2078	n/a	18	50	80	n/a

The pattern that arises from these assumptions is shown in Figure 11. We observe three separate phases for the focal woman: A, B and C. A represents caring for her children, B caring for her parents and C caring for her partner.

Figure 11: Care phases by age of focal woman and family size (example 1)



The start and end years, and duration, of each phase of caring are shown in Table 4. In this case periods of care do not overlap, so there are no sandwich years.

Table 4: Start, end and duration of care per family member (Example 1)

Family member	Carer's age at start of care	Carer's age at end of care	Duration of care (years)
Father	54	64	10
Mother	56	66	10
Partner	74	84	10
Child 1	24	42	18
Child 2	28	46	18
Child 3	32	50	18

Example 2: Multiple sandwich years

This example deals with a more complex case in which care phases overlap. Table 5a lists the events that affect the focal woman and Table 5b lists those that affect the relationships between her and the other six family members.

Table 5a and b: Family chronography of events in Example 2

(a) **Focal woman**

Birth	Death	Marriage	Age at death	Age at marriage	Length of marriage	Father's age at birth	Mother's age at birth	Partner's age at marriage
1956	2046	1980	90	24	40	36	34	40

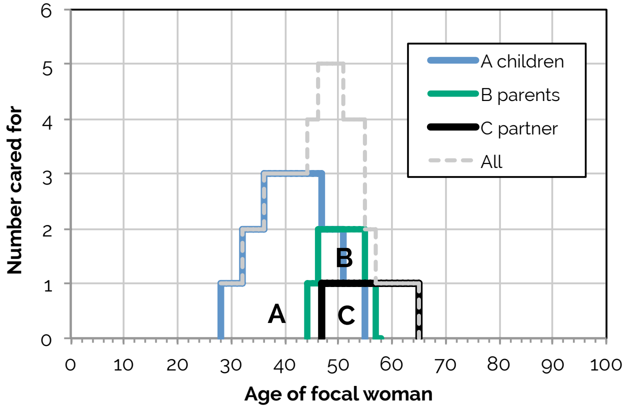
(b) **Family details**

Family member	Year of birth	Age difference	Year of death	Focal woman's age at death	Adulthood threshold age	Focal woman's age at threshold	Age at which care required	Focal woman's age when care required
Father	1920	–36	2010	54	n/a	n/a	80	44
Mother	1922	–34	2012	56	n/a	n/a	80	46
Partner	1940	–16	2020	64	n/a	n/a	63	47
Child 1	1984	28	2070	n/a	18	46	80	n/a
Child 2	1988	32	2074	n/a	18	50	80	n/a
Child 3	1992	36	2078	n/a	18	54	80	n/a

The focal woman's details are the same as in Example 1. However, we now assume that her parents and partner are all ten years older than in Example 1. This scenario also builds in a need to provide care for her partner from the age of 63.

The pattern that arises from these assumptions is shown in Figure 12. The care phases A, B and C still represent caring for her children, caring for her parents and caring for her partner respectively, but they are now overlapping.

Figure 12: Care phases by age of focal woman and number cared for (example 2)



The start and end years, and duration, of each phase are shown in Table 6a.

There are four groups of sandwich years in Table 6b: caring for children and parents (12 years from 44 to 46), caring for parents and partner (9 years from 47 to 56), caring for partner and children (7 years from 47 to 54), and caring for parents, partner and children (7 years from 47 to 54). This amounts to 12 consecutive years of sandwich caring from age 44 to age 56 and 36 years of caring altogether from age 28 to 64.

The care requirement in this example is obviously not manageable by one person alone, or even with the support of other family members. External help would be needed in the form of babysitting, care sharing with other families, and privately purchased care. The merit of family care accounts is that they provide a schedule that allows care pressures to be anticipated to a degree. Such information might be useful for assessing risk and for designing family care insurance policies.

On a technical note, in order to make an estimate of the number of sandwich years in advance, we can assume three generations with the focal woman caring for the generation on either side.

If we name the age at which children attain independence L and the age at which care becomes necessary U, the condition for sandwich years to occur is $s - f > U - L$, where s is the youngest child's year of birth and f is the oldest parent's year of birth. The number of sandwich years is given by $(s - f) - (U - L) = n$.

In reality n is probabilistic, since it depends on U.

Tables 6a and b: Start, end and duration of care phases in example 2, plus sandwich years

(a) **Care years given to named family member**

Family member	Carer's age at start of care	Carer's age at end of care	Duration of care (years)
Father	44	54	10
Mother	46	56	10
Partner	47	64	17
Child 1	28	46	18
Child 2	32	50	18
Child 3	36	54	18

(b) **Sandwich years**

Sandwich years	Parents/ children	Parents/ partner	Partner/ children	Parents, partner and children	Any overlap
Start age (A)	44	47	47	47	44
End age (B)	54	56	54	54	56
Duration (B-A)	10	9	7	7	12

5. Conclusions

This paper has explored the role and resilience of the family in the UK over time, and how it has come under pressure from external and demographic forces. Families are difficult to analyse; they are hard to define and information about them is sparse – particularly if we wish to analyse demographic changes in families over time.

Information about individuals and households is more plentiful but obscures the wider role and influence of families. This makes it more difficult to formulate and evaluate policies at a family level which could make a positive contribution elsewhere in society. It probably explains the relative lack of focus on families.

We used a novel approach based on survivorship, proposing a new way to define 'family' using a framework flexible enough to model a range of family structures and situations. We chose to centre our analysis on the birth year of the 'focal woman' and follow her immediate family (grandparents, parents, siblings, children and grandchildren), but this could be extended.

We used life tables to construct the joint survivorship of typical families based on a 100-year window to represent her life course. Our methods showed how changes in the timing and number of births within each family, improvements in life expectancy and the onset of illness and disability could affect the ability of families to cope with raising children and looking after older family members.

Our scenarios were informed by two demographic transitions; one lasting 150 years that completed in the 1950s, and a second from the 1970s onwards.

There are many ways in which both transitions affect how we live today. Although the average family in the UK is economically better off, the second transition has arguably contributed indirectly to a widening in inequalities as measured by life expectancy, income and housing.

Families are economically less resilient as a result, with fewer family members to look after an increasingly ageing family unit, and more family members living by themselves and/or geographically separated from the unit. Although average incomes are higher, it is the difference between individuals, households and families that is at issue.

One piece of independent evidence for this is the increasing inefficiency of the housing market. Homes are becoming increasingly under-occupied, thanks to population ageing and a reduction in three-generational households. Projections show that around 2.4 million people aged over 85 will live alone by 2040, with huge implications for social care provision.

This is because people are living longer and tending to stay in their own homes for as long as possible, with all the risks this entails. Meanwhile, at the other end of the age range young families are struggling to get on the housing ladder. Building our way out of this problem may not be the solution.

We saw that the older our focal woman is when she has children, the greater the probability that she will simultaneously care for her children, her parents and possibly her partner.

Household and childcare duties tend to fall disproportionately on women, with pressure at maximum when they reach their late 40s and 50s. Since earning potential is highest in middle age this tends to affect the prosperity and wellbeing of the whole family.

There are many ways to address this issue, including sharing household duties, flexible employment policies, and greater access to paid for care services.

Another consequence of increased longevity is an increase in the age of inheritance. The net result is that wealth is likely to be passed through the generations more slowly and may even skip a generation. A concrete but typical example based on the table in Annex C helps to show how much it has changed.

Potential solutions

Despite the economic benefits that came with the first transition, it is clear that the second transition is reversing some of those benefits. Unless we take steps to address the economic and demographic implications of this we believe that society will be trapped on a demographic escalator that can only exacerbate these issues. There are potential solutions to these problems but it will require a third transition. The third transition will have a number of features:

Health

The cost of health and social care rises with age. An ageing population may consume an ever-greater share of national income and taxes. We are also seeing an increasing gap between healthy life expectancy and overall life expectancy, which increases dependency and cost. However, there is much we can do to prevent ill-health in later life, including work to address the social determinants of health and supporting people to maintain their own health as they age. Policy measures focusing on preventative care for conditions like obesity and smoking-related illness, and encouraging physical activity, would promote longer working and promote independence and reduce the risk of economic stagnation.

Review of family policies

This is probably among the more complicated areas to tackle because of hidden or unintended biases in different parts of the system. A review of these from a family perspective could help to identify the less effective policies, including how they affect employment, access to benefits and care, healthcare and housing for a more joined up, family-friendly approach. Currently policy responsibility is split between different government departments, which does not help.

More age-appropriate housing

The housing crisis has been building for over two decades, with ever-higher house prices and housing shortages. A re-focussing of policy to release under-occupied homes would make a difference, but there are shortages of good quality stock, with very little new retirement housing being built. Transaction costs are often prohibitive, discouraging downsizing. Families playing a more pivotal role could help, for example through home swaps, building 'granny flats' and so on.

Transferring wealth sooner

Passing wealth down the generations sooner can help children or grandchildren with their education and getting on the housing ladder. Since most personal wealth is in housing and pensions, both have a role to play. Median wealth in the critical 55 to 64 age group in the UK is around £500,000. Waiting until a person has died to transfer wealth is rarely the best option, especially with the issue of inheritance tax. However, there are obstacles to reforming this system; for example housing is an illiquid asset. Releasing equity, either by downsizing or borrowing against the asset (equity release) could help. On the other hand, pension wealth is outside the estate, making it transferrable.

A bigger role for financial services

Financial services play a big role in wealth management and estate planning especially in the areas of housing, pensions and savings. The insurance industry offers a great deal of individual protection, through health and life products, and in protecting assets such as contents insurance. However, family-oriented products are more limited in scope; they include family accident insurance or funeral plans. More creative products are needed, especially to help pay for care in later life, using not just income to pay premiums but also housing and pension wealth.

And finally

To sum up, we suggest that steps taken in these areas would help set the UK on course to a a third transition, where families are able to enjoy their longevity in good health, with financial independence. This would moderate increases in the cost of health and welfare due to the ageing population.

There are glimmers of progress and these should be celebrated. One example is the closing of the gap between male and female life expectancy which means couples will spend more time together, potentially leading to less isolation in later life. (Pickard et al, 2011; Mayhew and Smith, 2014).

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Annex A: Technical note

A.1 Family survival analysis

The notation we use is illustrated using two examples of actual survival distributions for women born 50 years apart. This may be visually compared with the stylised version shown in Figure 1. In cohort life tables, we calculate age-specific probabilities of death using mortality data from a group of individuals born in the same year, followed until all cohort members have died.

Our data contains a mixture of completed cohorts, partially completed cohorts and cohorts that have not yet begun. In other words, they are mixture of actual and projected mortality experience. In order to analyse survival in selected examples of families we need to adjust the mathematical notation used.

Let the probability of survival to age z of the i^{th} family member born in year t_i be

$$S^i(z, t_i)$$

The life expectancy of the i^{th} family member at birth is given by:

$$e_0^i(t) = \int_0^{\omega} S(x, t_i)\,dx$$

in which w is the upper age limit. Life expectancy at birth can be estimated from cohort life tables, using the l_x^i column by single year of age:

$$e_0 = \frac{1}{l_0^i} \sum_{x=1}^{x=\omega} l_x^i + 0.5$$

Now consider a person born in year t_0, whose age range is $0 \le S(x) \le 1$. She will be called the 'focal woman' or reference point. Let

$S(x, t_0) =$ probability of survival to age x, $0 \le S(x) \le 1$

If the age of the focal woman is x, we can redefine the age of the i^{th} family member as $z_i = x + \Delta x_i$, where $\Delta x_i = t_0 - t_i$

This quantity is positive for $t_0 > t_i$ and negative if $t_0 < t_i$ i.e. it depends on whether the i^{th} family member was born before or after the focal woman.

The probability of the i^{th} family member born in year t_i being alive when the focal woman's age is x is therefore:

$$S^i(z_i, t_i) = S^i(x + \Delta x_i, t_i)$$

Where $S^i(x + \Delta x, t_i)$ is a continuous function of age and period.

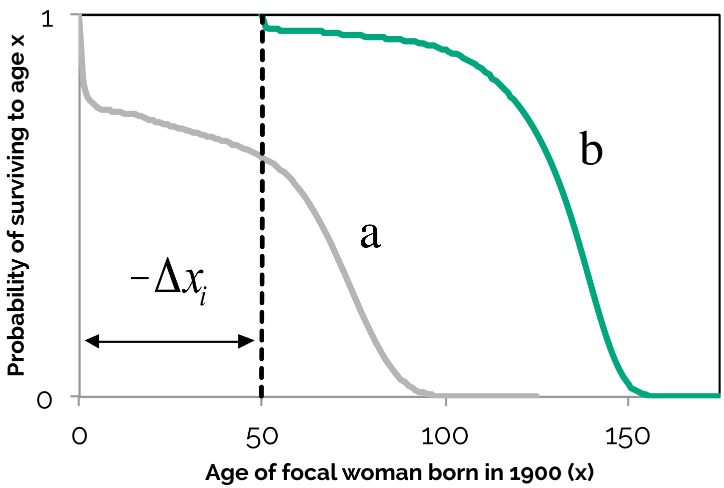
Computationally, we use the l_x^i column from cohort life tables to approximate the value of S at each age based on the focal woman's birth year (where l_0^i =100,000 for all *i*). This gives:

$$S^i(z_i, t_i) \approx \frac{1}{l_0^i} l^i_{x+\Delta x_i}$$

Figure A1 shows two examples of survival functions based on two women born in 1900 (a) and 1950 (b). The curves are shown as a function of the focal woman for a with the age displacement relative to b being $-\Delta x_i$ years, i.e. 50 years apart.

The marked dip at early ages for the survival function (a) reflects infant mortality, which is partly responsible for the notable difference in life expectancy at birth: 50.3 years for woman a and 77.7.years for woman b.

Figure A1: The survival probabilities for two focal women, one born in 1900 (a) and one in 1950 (b), in which $-\Delta x_i$ is the age displacement relative to b



A.2 Conditional survivorship of parents and grandparents

While we can use these equations to calculate survivorship for people younger than the focal woman, we must use a different method for people who are older than her. For the woman to have been born, her mother must have been alive. For convenience we will also assume that her father was still alive as well. Similarly, we assume both pairs of grandparents have been alive when her father and her mother were born.

$$S^i(z_i, t_i) \approx \frac{1}{l^i_{a_i}} l^i_{x+\Delta x_i}$$

where a_i is the age at which we assume the i^{th} family member must be alive i.e. the age of the parent when the focal woman was born or the age of the grandparent when the relevant parent was born.

A.3 Expected family size

The expected number of family members for a focal woman aged x will be equal to the survival probabilities for other family members, expressed as a function of the focal woman's age.

$$N(x) = \frac{1}{l_0} \sum_i l^i_{x+\Delta x_i}$$

A.4 Average family age

The average age of family members at the time when the focal person is aged x, where x ranges from 0 to 100, is given by:

$$\bar{a} = \frac{\sum_{x=0}^{100} \sum_i x l_{x+\Delta x_i}}{\sum_i l_{x+\Delta x_i}}$$

A.5 Age cut-offs

Let there be two age cut-offs: a lower limit, L, below which a person is classed as a child or young adult in full-time education; and an upper limit, U, above which a person could be of pensionable age or at an age when they cease to be fully independent.

Using previous notation, the expected number of family members aged below L at x is:

$$N(x < L) = \sum_i S^i(z_i, t_i) \qquad z_i < L$$

And the expected number aged above U is:

$$N(x > U) = \sum_i S^i(z_i, t_i) \qquad z_i > U$$

A.6 Within-family dependency ratios

The old age dependency ratio is defined as:

$$D_U = \frac{P_{\geq U}}{P_{L<x<U}},$$

where L and U are the lower and upper cut-off ages and the total dependency ratio is:

$$D_{Total} = \frac{P_{x \geq U} + P_{x<L}}{P_{L<x<U}}$$

Annex B: Fixed-length lives

With the modal age of death becoming increasingly compressed, we can debate whether lives are slowly becoming more fixed in length and what implications this might have for family size. The following is a simple rule of thumb for imagining this relationship.

In survival terms, a fixed life span can be represented by the survival curve becoming more rectangular in shape (see Figure 1), which would signify everyone dying at around the same age.

We can compute examples for different fertility rates and life spans to determine family size as shown in Box B1. In practice, it will depend on the relationship between the birth and death cycles. Below are three illustrations of what might occur.

The first example (a) is based on a fixed lifespan of 80 years, with two births every 20 years. In this case, the average family size would be 8 in the steady state; it is given by 80/20 x 2 = 8.

The average age of family members will be 40 years (their life span divided by the birth interval or 80/2), but it would rise and fall as the birth cycle peaks and falls every 20 years, as shown by the hatched line in a.

Alternatively, if lifespan were 90 years with two births every 30 years, family size would be 9 (90/3 x 3 =9) and so on.

In the second example (b) we assume one birth every 20 years, with a much lower fixed lifespan of 55 years. In this case, family size fluctuates between 2 and 3 members. The period with three members lasts for 15 years and the period with two members lasts for five years.

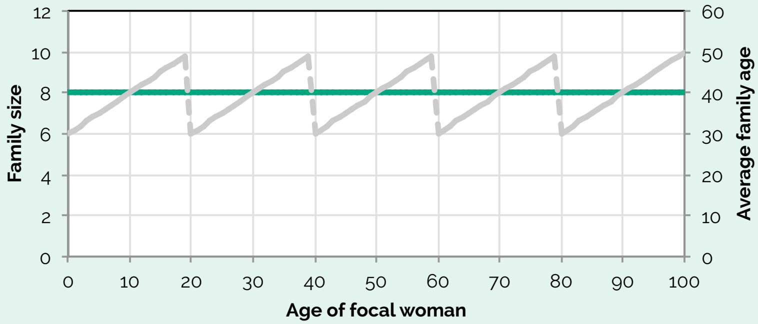
In the general case, these periods are found by subtracting the integer part of the average family size from the average itself and multiplying it by the birth interval.

In this case, we have (2.75 – 2.00) x 20=15 years at 3 family members and 5 years with 2 family members. The general rule is $\left[\frac{x}{b} - \operatorname{int}\frac{x}{b}\right] \times b$, where x is life expectancy and b is the birth interval. If there are multiple births at regular intervals, say two every 20 years, then family size doubles to between 4 and 6 members in this example.

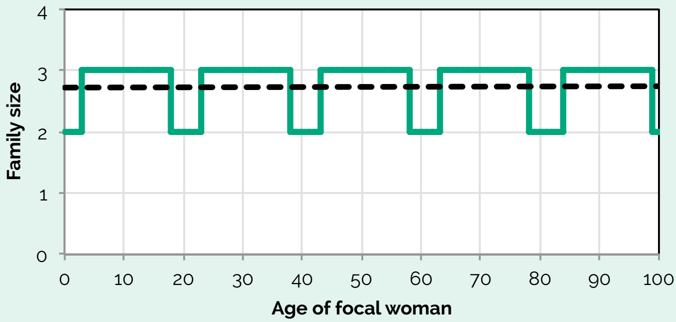
The third example (c) shows how patterns can become more complex. This case is based on a fixed lifespan of 55 years, with three children born one year apart every 20 years. Family size in this case fluctuates between 6 and 9.

Box B1: Examples of families based on fixed-length lives

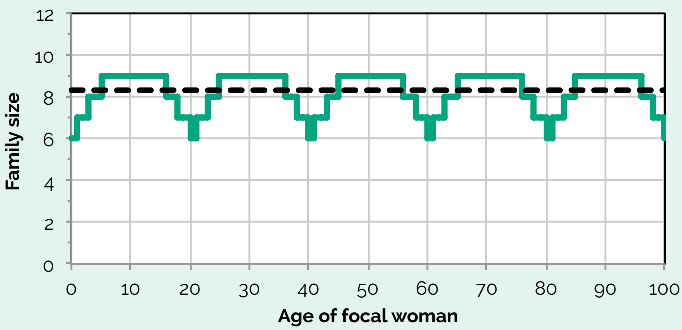
(a) Lifespan of 80 years, two births every 20 years (hatched line is average age)



(b) Lifespan 55 years, one birth every 20 years (hatched line is average size)



(c) Life span 55 years, 3 children born one year apart every 20 years



Annex C: Estimated age of inheritance

The focal woman's life expectancy and her age when she gives birth will affect the age a child can expect to receive any inheritance. Analysis shows that the age of inheritance has gradually increased over time thanks to improvements in life expectancy.

This is may be seen from table C1. Columns show the birth year for the focal woman, which range from 1841 to 2066. The rows show her age when her child was born and range from 18 to 50.

To give an example, the expected age of death of a woman born in 1920, having a child at age 26, is 79 years, putting her child at 53 when she inherits. If she was born in 1960 and had a child at 26, then her expected year of death would be 87 making her child's expected age of inheritance 61 – 8 years later. If she had been born in 2000, then it would be 12 years later, when she would be 65, and so on. Note that if she had a child at 30 rather than at 26 in 2000, the gap would be reduced back to 61.

Table C1: Estimated age of inheritance based on year of birth of focal woman and of her child

Age at birth of child	Year of birth											
	1840	1860	1880	1900	1920	1940	1960	1980	2000	2020	2040	2060
18	44	47	53	56	60	66	69	71	73	75	76	77
20	42	46	51	55	59	64	67	69	71	73	74	75
22	41	44	49	53	57	62	65	67	69	71	72	73
24	40	43	48	52	55	60	63	65	67	69	70	71
26	38	42	46	50	53	58	61	63	65	67	68	69
28	37	40	44	48	52	56	59	61	63	65	66	68
30	36	39	43	47	50	54	57	59	61	63	64	66
32	34	37	41	45	48	52	55	57	59	61	62	64
34	33	36	40	43	46	50	53	55	57	59	60	62
36	32	34	38	41	44	48	51	53	55	57	58	60
38	30	33	36	40	42	46	49	51	53	55	56	58
40	29	31	35	38	41	44	47	49	51	53	54	56
42	28	30	33	36	39	43	45	47	49	51	52	54
44	26	29	32	35	37	41	43	46	47	49	50	52
46	25	27	30	33	35	39	42	44	45	47	49	50
48	23	26	28	31	33	37	40	42	44	45	47	48
50	22	24	27	30	31	35	38	40	42	43	45	46

Key: Light blue under 50; dark blue 50-59; yellow 60 to 69; green 70 or over

International Longevity Centre UK

11 Tufton Street
London
SW1P 3QB
Tel : +44 (0) 20 7340 0440
www.ilcuk.org.uk