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Research Article

Passivity and Synchronization of Coupled Different Dimensional Delayed Reaction-Diffusion Neural Networks with Dirichlet Boundary Conditions

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Two types of coupled different dimensional delayed reaction-diffusion neural network (CDDDRDNN) models without and with parametric uncertainties are analyzed in this paper. On the one hand, passivity and synchronization of the raised network model with certain parameters are studied through exploiting some inequality techniques and Lyapunov stability theory, and some adequate conditions are established. On the other hand, the problems of robust passivity and robust synchronization of CDDDRDNNs with parameter uncertainties are solved. Finally, two numerical examples are given to testify the effectiveness of the derived passivity and synchronization conditions.

1. Introduction

In recent years, complex networks (CNs) have attracted much attention since they are ubiquitous under the circumstance of our daily life, for instance, communication networks, food webs, and social networks. Coupled neural networks (CNNs), as a particular type of CNs, have been put into use successfully in various fields, e.g., pattern recognition, chaos generators design, and brain science [1–3]. Strictly speaking, these applications in a large extent are depending on some properties of dynamics in CNNs (e.g., synchronization). Thus, the problem of synchronization for CNNs has attracted comprehensive attention and been developed into a hot research topic. So far, many important and interesting results have been derived on this topic recently [4–10]. Through exploiting the properties of random variables, Yang et al. [4] acquired some synchronization conditions for randomly delayed CNNs. In [7], several synchronization criteria were gained for delayed CNNs in accordance with the Lyapunov functional strategy.

Nevertheless, most of the synchronization results in aforesaid literatures [4–10] neglected reaction-diffusion phenomena. In fact, the reaction-diffusion phenomena are unavoidable for CNs such as neural networks and cellular networks when they are implemented by means of electric circuits in practical situations [11]. Hence, it is necessary to research coupled reaction-diffusion neural networks (CRDNNs). Recently, some significant synchronization results for CRDNNs have been acquired [12–17]. In [12], the authors researched synchronization of CRDNNs and presented some synchronization criteria. By designing appropriate pinning controllers, several synchronization criteria were established for CRDNNs in [17]. Furthermore, owing to the existence of external interferences, the noises of environment, and equipment restrictions, it is very difficult to ensure network models containing the certain parameter values in some practice situations. Consequently, some authors studied parametric uncertainties of neural networks [18–22], and a few interesting results have been derived regarding robust synchronization of CNNs with uncertain parameters [23–25]. In [24], the scholars presented the memristive CNNs model with parametric uncertainties and obtained some adequate conditions for guaranteeing the robust synchronization for the considered network. By utilizing impulsive functional strategy combined with the stability theory, Li et al. [25] presented some robust synchronization criteria for the CNNs with uncertainties.

Actually, passivity is also one of the most important behaviors of dynamics for CNs which can guarantee a system's internal stabilization in system theory. Due to the potential applications in plenty of fields, e.g., fuzzy control and sliding mode control, the passivity problem of CNNs has been investigated extensively in the past years [26–33]. In [27], the authors proposed several passivity criteria for delayed CNNs. Ren et al. [28] analyzed the model of CRDNNs and established some passivity and pinning passivity conditions for the considered network. Unfortunately, the passivity results in aforementioned studies [26–33] are based on the case that the dimension of input is identical with output. As far as we know, only few scholars have addressed the problem of passivity for the network with nonidentical dimensional output and input [34–36]. The authors in [34] established some conditions for ensuring that the CRDNNs with the input and output in different dimensions achieve passivity. Ren et al. [36] discussed the (pinning) passivity problems of CNNs with nonidentical dimensional output and input and acquired some corresponding passivity criteria.

Note that the networks are composed of identical nodes in the aforementioned works [4–36]. Unfortunately, this case is very rare in the real-world networks. Consequently, the CNs consisting of nonidentical nodes in the same dimension [37–41] have been discussed firstly by researchers. Zhao et al. [39] dealt with the synchronization problem of CNs with nonidentical nodes. As a matter of fact, the networks constructed by nonidentical nodes of different dimensions can reflect more real networks in many circumstances. Note that the networks constructed by nonidentical nodes in different dimensions can exhibit different and even more sophisticated dynamical behaviors, which makes the passivity and synchronization methods for the networks with the same dimensional nonidentical nodes or identical nodes in the above-mentioned works [4–41] invalid. Hence, it is necessary and meaningful to develop some new stabilization and synchronization strategies for the network with nonidentical nodes of different dimensions [42–45]. In [43], by designing appropriate decentralized controllers, the authors devoted to establishing stabilization and synchronization criteria for CNs consisting of nonidentical nodes. Up to now, only a few researchers considered CNNs constructed by the nonidentical nodes of different dimensions [46–48]. In [47], the authors investigated generalized synchronization of delayed CNNs with different dimensional nodes by making use of the Lyapunov functional method. To the best of knowledge, the problems of synchronization and passivity for CDDDRDNNs have not yet been investigated. Consequently, it is essential to put forth some efforts to study passivity and synchronization of CDDDRDNNs.

In terms of the above introduction, the main aim in this paper is to address passivity and synchronization of CDDDRDNNs without and with parametric uncertainties. On the one hand, we discuss the CDDDRDNNs without parametric uncertainties, and several conditions are derived to guarantee the considered network to achieve passivity and synchronization. On the other hand, the problems of robust synchronization and robust passivity for CDDDRDNNs with parametric uncertainties are also studied.

2. Preliminaries

Let the matrix $G \in \mathbb{R}^{n\times n}$, the notation $G<0$ $(G>0, G\leq 0, G\geq 0)$ signifies G is symmetric and negative (positive, seminegative, and semipositive) definite. $\lambda_m(\cdot)$ $(\lambda_M(\cdot))$ represents the minimum (maximum) eigenvalue of the corresponding matrix. An open-bounded domain in $\mathbb{R}^{\varrho}$ with smooth boundary $\partial\Omega$ is defined by $\Omega = \{m = (m_1, m_2, \cdots, m_{\varrho})^T \| |m_\sigma| < \zeta_\sigma, \sigma = 1, 2, \cdots, \varrho\}$. For any $z(m,t) = (z_1(m,t), z_2(m,t), \cdots, z_n(m,t))^T \in \mathbb{R}^n$, we have

$$\|z(\cdot,t)\|_2 = \left(\int_\Omega \sum_{i=1}^n z_i^2(m,t)\mathrm{d}m\right)^{1/2}. \tag{1}$$

Definition 1 (see [49]). Let $u(m,t) \in \mathbb{R}^p$ and $y(m,t) \in \mathbb{R}^q$ denote the input and output of a system. Assume that there exists a storage function $S: [0,+\infty) \longrightarrow [0,+\infty)$ which satisfies

$$\int_{t_0}^{t_l} \Pi(u,y)\mathrm{d}t \geq S(t_l) - S(t_0), \tag{2}$$

for any $t_0, t_l \in [0,+\infty)$ and $t_0 \leq t_l$, then the system with supply rate $\Pi(u,y)$ is dissipative. Moreover, a system is passive if the system is dissipative with

$$\Pi(u,y) = \int_\Omega y^T(m,t)Qu(m,t)\mathrm{d}m, \tag{3}$$

where the matrix $Q \in \mathbb{R}^{q\times p}$. Furthermore, assume that a system is dissipative with

$$\begin{aligned}\Pi(u,y) = &\int_\Omega y^T(m,t)Qu(m,t)\mathrm{d}m \\ &- \int_\Omega y^T(m,t)M_1 y(m,t)\mathrm{d}m \\ &- \int_\Omega u^T(m,t)M_2 u(m,t)\mathrm{d}m,\end{aligned} \tag{4}$$

in which $M_1 \in \mathbb{R}^{q\times q} \geqslant 0, M_2 \in \mathbb{R}^{p\times p} \geqslant 0$, $\lambda_m(M_1) + \lambda_m(M_2) > 0$, and $Q \in \mathbb{R}^{q\times p}$, then the system is strictly passive. Especially, if $M_1 > 0$, then the system is called to be output-strictly passive; if $M_2 > 0$, then the system is called to be input-strictly passive.

Lemma 2.1 (see [50]). *Let Ω be a cube $|m_\sigma| < \zeta_\sigma$ $(\sigma = 1, 2, \cdots, \varrho)$ and real-valued function $w(m) \in C^1(\Omega)$ satisfy $w(m)|_{\partial\Omega} = 0$. Then,*

$$\int_\Omega w^2(m)dm \leqslant \zeta_\sigma^2 \int_\Omega \left(\frac{\partial w(m)}{\partial m_\sigma}\right)^2 \mathrm{d}m, \tag{5}$$

where $m = (m_1, m_2, \cdots, m_\varrho)^T$.

3. Passivity and Synchronization of CDDDRDNNs

3.1. Network Model. The CDDDRDNNs considered in this section is stated as follows:

$$\begin{aligned}\frac{\partial w_i(m,t)}{\partial t} &= D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 w_i(m,t)}{\partial m_\sigma^2} - B_i w_i(m,t) \\ &\quad + A_i f_i(w_i(m,t)) + J_i + G_i u_i(m,t) \\ &\quad + Z_i \varphi_i(w_i(m,t-\tau_i(t))) + \sum_{j=1}^{N} c_{ij} H_{ij} w_j(m,t),\end{aligned} \tag{6}$$

in which $i = 1, 2, \cdots, N$, $w_i(m,t) = (w_1^{(i)}(m,t), w_2^{(i)}(m,t), \cdots, w_{\xi_i}^{(i)}(m,t))^T \in \mathbb{R}^{\xi_i}$ denotes the state vector of ith neuron; $B_i \in \mathbb{R}^{\xi_i \times \xi_i} = \mathrm{diag}(b_1^{(i)}, b_2^{(i)}, \cdots, b_{\xi_i}^{(i)}) > 0$; $A_i = (a_{gh}^{(i)})_{\xi_i \times \xi_i}$, $Z_i = (z_{gh}^{(i)})_{\xi_i \times \xi_i}$; $f_i(w_i(m,t)) = (f_1^{(i)}(w_1^{(i)}(m,t)), f_2^{(i)}(w_2^{(i)}(m,t)), \cdots, f_{\xi_i}^{(i)}(w_{\xi_i}^{(i)}(m,t)))^T$; $D_i = \mathrm{diag}(d_1^{(i)}, d_2^{(i)}, \cdots, d_{\xi_i}^{(i)}) > 0$; $\varphi_i(w_i(m,t-\tau_i(t))) = (\varphi_1^{(i)}(w_1^{(i)}(m,t-\tau_i(t))), \varphi_2^{(i)}(w_2^{(i)}(m,t-\tau_i(t))), \cdots, \varphi_{\xi_i}^{(i)}(w_{\xi_i}^{(i)}(m,t-\tau_i(t))))^T$, $f_l^{(i)}(\cdot)$, and $\varphi_l^{(i)}(\cdot)\,(l = 1, 2, \cdots, \xi_i)$ denote the activation functions for the lth neuron in neural network i; $J_i = (J_1^{(i)}, J_2^{(i)}, \cdots, J_{\xi_i}^{(i)})^T$; $u_i(m,t) \in \mathbb{R}^{p_i}$ denotes the control input; the inner coupling matrix is defined by $H_{ij} \in \mathbb{R}^{\xi_i \times \xi_j}$; $G_i \in \mathbb{R}^{\xi_i \times p_i}$ is a known matrix; $C = (c_{ij})_{N \times N}$ is the coupling configuration matrix denoting coupling weight, which satisfies $c_{ij} \neq 0\,(i \neq j)$ if node i and node j are connected, or else $c_{ij} = 0$. In addition, the time-varying delay $\tau_i(t)$ satisfies $0 \leqslant \tau_i(t) \leqslant \tau_i$ and $\dot{\tau}_i(t) \leqslant \delta_i < 1$.

Remark 1. As a special type of CNs, CNNs have attracted much attention due to their extensive applications on plenty of fields. So far, a large number of scholars have acquired some interesting research results about synchronization and passivity of CNNs [4–36]. Nevertheless, the considered CNs in the aforementioned literature are made up of identical nodes. As a matter of fact, it is utterly impractical that the networks have totally identical nodes in many practical situations. For instance, due to the differences of the parameters, it is impossible that the neurons in the nervous system of neural networks are entirely the same as each other. Consequently, it is significant to study CNNs composed of nonidentical nodes. To our knowledge, a few investigators have discussed CNs consisting of the same dimensional nonidentical nodes in recent years [37–41]. However, the networks with nonidentical nodes of different dimensions can describe more practical networks. In addition, the considered networks in these existing works did not take the reaction-diffusion terms into account. Therefore, we pay our attention on the CRDNNs with different dimensional nodes. As far as we know, this is our first step toward addressing the passivity and synchronization problems of CDDDRDNNs.

For network (6), the initial value condition and boundary value condition are described by

$$\begin{aligned} w_i(m,t) &= \vartheta_i(m,t), \\ (m,t) &\in \Omega \times [-\rho, 0], \end{aligned} \tag{7}$$

$$\begin{aligned} w_i(m,t) &= 0, \\ (m,t) &\in \partial\Omega \times [-\rho, +\infty), \end{aligned} \tag{8}$$

where $i = 1, 2, \cdots, N$, $\vartheta_i(m,t)$ is bounded and continuous on $\Omega \times [-\rho, 0]$ and $\rho = \max_{i=1,2,\cdots,N}\{\tau_i\}$.

Suppose that the functions $f_l^{(i)}(\cdot)$ and $\varphi_l^{(i)}(\cdot)$ satisfy the following global Lipschitz condition; there exist positive constants ψ_{il} and $\widehat{\psi}_{il}$ such that

$$\begin{aligned} \left| f_l^{(i)}(v_1) - f_l^{(i)}(v_2) \right| &\leqslant \psi_{il} |v_1 - v_2|, \\ \left| \varphi_l^{(i)}(v_1) - \varphi_l^{(i)}(v_2) \right| &\leqslant \widehat{\psi}_{il} |v_1 - v_2|, \end{aligned} \tag{9}$$

hold for all $v_1, v_2 \in \mathbb{R}$, $l = 1, 2, \cdots, \xi_i$, $i = 1, 2, \cdots, N$.

Let the constant vector $\widehat{O} = (\widehat{o}_1^T, \widehat{o}_2^T, \cdots, \widehat{o}_N^T)^T$ be an equilibrium point of an isolated node of network (6). Then,

$$\begin{aligned} &D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 \widehat{o}_i}{\partial m_\sigma^2} - B_i \widehat{o}_i + A_i f_i(\widehat{o}_i) + J_i + Z_i \varphi_i(\widehat{o}_i) \\ &\quad + \sum_{j=1}^{N} c_{ij} H_{ij} \widehat{o}_j = 0, \end{aligned} \tag{10}$$

where $\widehat{o}_i = (\widehat{o}_1^{(i)}, \widehat{o}_2^{(i)}, \cdots, \widehat{o}_{\xi_i}^{(i)})^T \in \mathbb{R}^{\xi_i}$, $i = 1, 2, \cdots, N$.

For the error vector $z_i(m,t) = (z_1^{(i)}(m,t), z_2^{(i)}(m,t), \cdots, z_{\xi_i}^{(i)}(m,t))^T = w_i(m,t) - \widehat{o}_i$, we have

$$\begin{aligned}\frac{\partial z_i(m,t)}{\partial t} &= D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} - B_i z_i(m,t) + A_i \widehat{f}_i(z_i(m,t)) \\ &\quad + G_i u_i(m,t) + Z_i \widehat{\varphi}_i(z_i(m,t-\tau_i(t))) \\ &\quad + \sum_{j=1}^{N} c_{ij} H_{ij} z_j(m,t),\end{aligned} \tag{11}$$

where $i = 1, 2, \cdots, N$, $\widehat{f}_i(z_i(m,t)) = f_i(w_i(m,t)) - f_i(\widehat{o}_i) = (f_1^{(i)}(w_1^{(i)}(m,t)) - f_1^{(i)}(\widehat{o}_1^{(i)}), f_2^{(i)}(w_2^{(i)}(m,t)) - f_2^{(i)}(\widehat{o}_2^{(i)}), \cdots, f_{\xi_i}^{(i)}(w_{\xi_i}^{(i)}(m,t)) - f_{\xi_i}^{(i)}(\widehat{o}_{\xi_i}^{(i)}))^T$ and $\widehat{\varphi}_i(z_i(m,t-\tau_i(t))) = \varphi_i(w_i(m,t-\tau_i(t))) - \varphi_i(\widehat{o}_i) = (\varphi_1^{(i)}(w_1^{(i)}(m,t)) - \varphi_1^{(i)}(\widehat{o}_1^{(i)}), \varphi_2^{(i)}(w_2^{(i)}(m,t)) - \varphi_2^{(i)}(\widehat{o}_2^{(i)}), \cdots, \varphi_{\xi_i}^{(i)}(w_{\xi_i}^{(i)}(m,t)) - \varphi_{\xi_i}^{(i)}(\widehat{o}_{\xi_i}^{(i)}))^T$.

The output vector $y_i(m,t) \in \mathbb{R}^{q_i}$ of the network (11) is given as follows:

$$y_i(m,t) = K_1^{(i)} z_i(m,t) + K_2^{(i)} u_i(m,t), \tag{12}$$

where $K_1^{(i)} \in \mathbb{R}^{q_i \times \xi_i}$ and $K_2^{(i)} \in \mathbb{R}^{q_i \times p_i}$ are the known matrices.

In the whole paper, we denote

$$
\begin{aligned}
& z(m,t)=\left(z_1^T(m,t), z_2^T(m,t), \cdots, z_N^T(m,t)\right)^T, u(m,t)=\left(u_1^T(m,t), u_2^T(m,t), \cdots, u_N^T(m,t)\right)^T, \\
& y(m,t)=\left(y_1^T(m,t), y_2^T(m,t), \cdots, y_N^T(m,t)\right)^T, L=\left(\frac{1}{1-\delta_1} I_{\xi_1}, \frac{1}{1-\delta_2} I_{\xi_2}, \cdots, \frac{1}{1-\delta_N} I_{\xi_N}\right), \\
& p=p_1+p_2+\cdots+p_N, q=q_1+q_2+\cdots+q_N, \\
& \xi=\xi_1+\xi_2+\cdots+\xi_N, \\
& D=\operatorname{diag}(D_1, D_2, \cdots, D_N), \\
& Z=\operatorname{diag}(Z_1, Z_2, \cdots, Z_N), \\
& G=\operatorname{diag}(G_1, G_2, \cdots, G_N), \\
& B=\operatorname{diag}(B_1, B_2, \cdots, B_N), \\
& A=\operatorname{diag}(A_1, A_2, \cdots, A_N), \\
& \Psi_i=\operatorname{diag}\left(\psi_{i1}^2, \psi_{i2}^2, \cdots, \psi_{i\xi_i}^2\right), \\
& \Psi=\operatorname{diag}(\Psi_1, \Psi_2, \cdots, \Psi_N), \\
& \widehat{\Psi}_i=\operatorname{diag}\left(\widehat{\psi}_{i1}^2, \widehat{\psi}_{i2}^2, \cdots, \widehat{\psi}_{i\xi_i}^2\right), \\
& \widehat{\Psi}=\operatorname{diag}\left(\widehat{\Psi}_1, \widehat{\Psi}_2, \cdots, \widehat{\Psi}_N\right), \\
& \widehat{f}(z(m,t))=\left(\widehat{f}_1(z_1(m,t)), \widehat{f}_2(z_2(m,t)), \cdots, \widehat{f}_N(z_N(m,t))\right)^T, \\
& \widehat{\varphi}(z(m,t-\tau(t)))=\widehat{\varphi}_1(z_1(m,t-\tau_1(t))), \widehat{\varphi}_2(z_2(m,t-\tau_2(t))), \cdots, \\
& \widehat{\varphi}_N(z_N(m,t-\tau_N(t)))^T, \widehat{H}=\left(\widehat{H}_{ij}\right)_{N\times N}, \text{ where } \widehat{H}_{ij}=c_{ij}H_{ij}, \widehat{K}_1=\operatorname{diag}\left(K_1^{(1)}, K_1^{(2)}, \cdots, K_1^{(N)}\right), \\
& \widehat{K}_2=\operatorname{diag}\left(K_2^{(1)}, K_2^{(2)}, \cdots, K_2^{(N)}\right).
\end{aligned} \tag{13}
$$

3.2. Passivity Analysis. It follows from (11) and (12) that the following system with output $y_i(m,t) \in \mathbb{R}^{q_i}$ and input $u_i(m,t) \in \mathbb{R}^{p_i}$ can be obtained as follows:

$$
\begin{cases}
\dfrac{\partial z_i(m,t)}{\partial t}=D_i \displaystyle\sum_{\sigma=1}^{\varrho} \dfrac{\partial^2 z_i(m,t)}{\partial m_\sigma^2}-B_i z_i(m,t)+A_i \widehat{f}_i(z_i(m,t))+G_i u_i(m,t)+Z_i \widehat{\varphi}_i(z_i(m,t-\tau_i(t)))+\displaystyle\sum_{j=1}^{N} \widehat{H}_{ij} z_j(m,t), \\
y_i(m,t)=K_1^{(i)} z_i(m,t)+K_2^{(i)} u_i(m,t),
\end{cases} \tag{14}
$$

where $\widehat{H}_{ij}=c_{ij}H_{ij}$ and $i=1,2,\cdots,N$.

Theorem 1. *System (14) reaches output-strict passivity if there are matrices $Q \in \mathbb{R}^{q\times p}$, $P=\operatorname{diag}(P_1, P_2, \cdots, P_N)>0$ ($P_i \in \mathbb{R}^{\xi_i\times\xi_i}$), and $0<M_1 \in \mathbb{R}^{q\times q}$ satisfying*

$$
PD+DP \geqslant 0, \tag{15}
$$

$$
\begin{pmatrix} \Phi_1 & \Lambda_1 \\ \Lambda_1^T & \Lambda_2 \end{pmatrix} \leqslant 0, \tag{16}
$$

where $\Phi_1=-\sum_{\sigma=1}^{\varrho}(1/\zeta_\sigma^2)(PD+DP)-PB-BP+PAA^TP+PZZ^TP+\Psi+\widehat{\Psi}L+P\widehat{H}+\widehat{H}^TP+\widehat{K}_1^TM_1\widehat{K}_1, \Lambda_1=PG+\widehat{K}_1^T M_1\widehat{K}_2-\widehat{K}_1^TQ, \Lambda_2=\widehat{K}_2^TM_1\widehat{K}_2-\widehat{K}_2^TQ-Q^T\widehat{K}_2$.

Proof. Construct the Lyapunov functional for system (14) as follows:

$$
\begin{aligned}
V(t)= & \sum_{i=1}^{N} \int_{\Omega} z_i^T(m,t) P_i z_i(m,t) \mathrm{d}m \\
& +\sum_{i=1}^{N} \sum_{j=1}^{\xi_i} \frac{\widehat{\psi}_{ij}^2}{1-\delta_i} \int_{t-\tau_i(t)}^{t} \int_{\Omega}\left(z_j^{(i)}(m,h)\right)^2 \mathrm{d}m\, \mathrm{d}h.
\end{aligned} \tag{17}
$$

Then,

$$\begin{aligned}
\dot{V}(t) &= 2\sum_{i=1}^{N}\int_{\Omega} z_i^T(m,t)P_i\Bigg(D_i\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} - B_i z_i(m,t) + A_i\widehat{f}_i(z_i(m,t)) + G_i u_i(m,t) + Z_i\widehat{\varphi}_i(z_i(m,t-\tau_i(t)))\\
&\quad + \sum_{j=1}^{N}\widehat{H}_{ij}z_j(m,t)\Bigg)\mathrm{d}m - \sum_{i=1}^{N}\sum_{j=1}^{\xi_i}\frac{\widehat{\psi}_{ij}^2(1-\dot{\tau}_i(t))}{1-\delta_i}\int_{\Omega}\left(z_j^{(i)}(m,t-\tau_i(t))\right)^2\mathrm{d}m + \sum_{i=1}^{N}\sum_{j=1}^{\xi_i}\frac{\widehat{\psi}_{ij}^2}{1-\delta_i}\int_{\Omega}\left(z_j^{(i)}(m,t)\right)^2\mathrm{d}m\\
&\le 2\sum_{i=1}^{N}\int_{\Omega} z_i^T(m,t)P_i\Bigg(D_i\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} - B_i z_i(m,t) + A_i\widehat{f}_i(z_i(m,t)) + Z_i\widehat{\varphi}_i(z_i(m,t-\tau_i(t))) + \sum_{j=1}^{N}\widehat{H}_{ij}z_j(m,t)\\
&\quad + G_i u_i(m,t)\Bigg)\mathrm{d}m - \sum_{i=1}^{N}\int_{\Omega} z_i^T(m,t-\tau_i(t))\widehat{\Psi}_i z_i(m,t-\tau_i(t))\mathrm{d}m + \sum_{i=1}^{N}\int_{\Omega} z_i^T(m,t)\frac{\widehat{\Psi}_i}{1-\delta_i}z_i(m,t)\mathrm{d}m\\
&= 2\int_{\Omega} z^T(m,t)P\Bigg(D\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z(m,t)}{\partial m_\sigma^2} - Bz(m,t) + A\widehat{f}(z(m,t)) + Gu(m,t) + Z\widehat{\varphi}(z(m,t-\tau(t))) + \widehat{H}z(m,t)\Bigg)\mathrm{d}m\\
&\quad + \int_{\Omega} z^T(m,t)\widehat{\Psi}Lz(m,t)\mathrm{d}m - \int_{\Omega} z^T(m,t-\tau(t))\widehat{\Psi}z(m,t-\tau(t))\mathrm{d}m.
\end{aligned} \tag{18}$$

By means of the boundary condition and Green's formula, one can obtain the following:

$$\int_{\Omega} z_g^{(i)}(m,t)\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z_h^{(i)}(m,t)}{\partial m_\sigma^2}\mathrm{d}m = -\sum_{\sigma=1}^{\varrho}\int_{\Omega}\frac{\partial z_g^{(i)}(m,t)}{\partial m_\sigma}\frac{\partial z_h^{(i)}(m,t)}{\partial m_\sigma}\mathrm{d}m, \tag{19}$$

where $h = 1,2,\cdots,\xi_i$, $i = 1,2,\cdots,N$, and $g = 1,2,\cdots,\xi_i$. Let $P_i = (p_{gh}^{(i)})_{\xi_i\times\xi_i}$, one has

$$\begin{aligned}
\int_{\Omega} z^T(m,t)PD\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z(m,t)}{\partial m_\sigma^2}\mathrm{d}m &= \sum_{i=1}^{N}\sum_{g=1}^{\xi_i}\sum_{h=1}^{\xi_i}p_{gh}^{(i)}d_h^{(i)}\int_{\Omega} z_g^{(i)}(m,t)\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z_h^{(i)}(m,t)}{\partial m_\sigma^2}\mathrm{d}m\\
&= -\sum_{i=1}^{N}\sum_{\sigma=1}^{\varrho}\sum_{g=1}^{\xi_i}\sum_{h=1}^{\xi_i}p_{gh}^{(i)}d_h^{(i)}\int_{\Omega}\frac{\partial z_g^{(i)}(m,t)}{\partial m_\sigma}\frac{\partial z_h^{(i)}(m,t)}{\partial m_\sigma}\mathrm{d}m\\
&= -\sum_{\sigma=1}^{\varrho}\int_{\Omega}\left(\frac{\partial z(m,t)}{\partial m_\sigma}\right)^T PD\frac{\partial z(m,t)}{\partial m_\sigma}\mathrm{d}m.
\end{aligned} \tag{20}$$

Then, we obtain

$$\begin{aligned}
2\int_{\Omega} z^T(m,t)PD\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z(m,t)}{\partial m_\sigma^2}\mathrm{d}m &= -\sum_{\sigma=1}^{\varrho}\int_{\Omega}\left(\frac{\partial z(m,t)}{\partial m_\sigma}\right)^T\\
&\quad\cdot(PD+DP)\frac{\partial z(m,t)}{\partial m_\sigma}\mathrm{d}m.
\end{aligned} \tag{21}$$

Obviously, there is a real matrix $\Gamma \in \mathbb{R}^{\xi\times\xi}$ satisfying

$$PD + DP = \Gamma^T\Gamma. \tag{22}$$

Thus,

$$\begin{aligned}
\left(\frac{\partial z(m,t)}{\partial m_\sigma}\right)^T(PD+DP)\frac{\partial z(m,t)}{\partial m_\sigma} &= \left(\frac{\partial z(m,t)}{\partial m_\sigma}\right)^T\Gamma^T\Gamma\frac{\partial z(m,t)}{\partial m_\sigma}\\
&= \left(\frac{\partial(\Gamma z(m,t))}{\partial m_\sigma}\right)^T\frac{\partial(\Gamma z(m,t))}{\partial m_\sigma}.
\end{aligned} \tag{23}$$

Let $\phi(m,t)=\Gamma z(m,t)$, for $(m,t)\in\partial\Omega\times[-\rho,+\infty)$, it follows from boundary condition (8) that $\phi(m,t)=\Gamma z(m,t)=0$. In terms of Lemma 2.1, we can get

$$\sum_{\sigma=1}^{\varrho}\int_{\Omega}\left(\frac{\partial\phi(m,t)}{\partial m_\sigma}\right)^T\frac{\partial\phi(m,t)}{\partial m_\sigma}\mathrm{d}m\geq\sum_{\sigma=1}^{\varrho}\frac{1}{\zeta_\sigma^2}\int_{\Omega}\phi^T(m,t)\phi(m,t)\mathrm{d}m=\sum_{\sigma=1}^{\varrho}\frac{1}{\zeta_\sigma^2}\int_{\Omega}z^T(m,t)(PD+DP)z(m,t)\mathrm{d}m. \tag{24}$$

Therefore,

$$2\int_{\Omega}z^T(m,t)PD\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z(m,t)}{\partial m_\sigma^2}\mathrm{d}m\leq-\sum_{\sigma=1}^{\varrho}\frac{1}{\zeta_\sigma^2}\int_{\Omega}z^T(m,t)\cdot(PD+DP)z(m,t)\mathrm{d}m. \tag{25}$$

In addition, we can derive that

$$\begin{aligned}
2\int_{\Omega}z^T(m,t)PA\widehat{f}(z(m,t))\mathrm{d}m&=2\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t)P_iA_i\widehat{f}_i(z_i(m,t))\mathrm{d}m\\
&\leq\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t)P_iA_iA_i^TP_iz_i(m,t)\mathrm{d}m+\sum_{i=1}^{N}\int_{\Omega}\widehat{f}_i^T(w_i(m,t))\widehat{f}_i(w_i(m,t))\mathrm{d}m\\
&=\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t)P_iA_iA_i^TP_iz_i(m,t)\mathrm{d}m+\sum_{i=1}^{N}\sum_{l=1}^{\xi_i}\int_{\Omega}\left[f_l^{(i)}\left(w_l^{(i)}(m,t)\right)-f_l^{(i)}\left(\widehat{o}_l^{(i)}\right)\right]^2\mathrm{d}m\\
&\leq\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t)P_iA_iA_i^TP_iz_i(m,t)\mathrm{d}m+\sum_{i=1}^{N}\sum_{l=1}^{\xi_i}\int_{\Omega}\psi_{il}^2\left(z_l^{(i)}(m,t)\right)^2\mathrm{d}m\\
&=\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t)P_iA_iA_i^TP_iz_i(m,t)\mathrm{d}m+\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t)\Psi_iz_i(m,t)\mathrm{d}m\\
&=\int_{\Omega}z^T(m,t)\left(PAA^TP+\Psi\right)z(m,t)\mathrm{d}m.
\end{aligned} \tag{26}$$

Similarly, we have

$$\begin{aligned}
&2\int_{\Omega}z^T(m,t-\tau(t))PZ\widehat{\varphi}(z(m,t-\tau(t)))\mathrm{d}m\\
&=2\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t-\tau_i(t))P_iZ_i\widehat{\varphi}_i(z_i(m,t-\tau_i(t)))\mathrm{d}m\\
&\leq\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t)P_iZ_iZ_i^TP_iz_i(m,t)\mathrm{d}m+\sum_{i=1}^{N}\int_{\Omega}z_i^T(m,t-\tau_i(t))\widehat{\Psi}_iz_i(m,t-\tau_i(t))\mathrm{d}m\\
&=\int_{\Omega}z^T(m,t)PZZ^TPz(m,t)\mathrm{d}m+\int_{\Omega}z^T(m,t-\tau(t))\widehat{\Psi}z(m,t-\tau(t))\mathrm{d}m.
\end{aligned} \tag{27}$$

By (25)–(27), one has

$$\begin{aligned}
\dot{V}(t)\leq&\int_{\Omega}z^T(m,t)\left[-\sum_{\sigma=1}^{\varrho}\frac{1}{\zeta_\sigma^2}(PD+DP)-PB-BP+PAA^TP\right.\\
&\left.+PZZ^TP+\Psi+\widehat{\Psi}L+P\widehat{H}+\widehat{H}^TP\right]z(m,t)\mathrm{d}m\\
&+2\int_{\Omega}z^T(m,t)PGu(m,t)\mathrm{d}m.
\end{aligned} \tag{28}$$

Thus,

$$\begin{aligned}
&\dot{V}(t) - 2\int_{\Omega} y^T(m,t)Qu(m,t)\mathrm{d}m + \int_{\Omega} y^T(m,t)M_1 y(m,t)\mathrm{d}m \\
&\leq \int_{\Omega} z^T(m,t)\left[-\sum_{\sigma=1}^{\varrho}\frac{1}{\zeta_\sigma^2}(PD+DP) - PB - BP + PAA^TP + PZZ^TP + \Psi + \widehat{\Psi}L + P\widehat{H} + \widehat{H}^TP\right]z(m,t)\mathrm{d}m \\
&\quad + \int_{\Omega} z^T(m,t)\widehat{K}_1^T M_1 \widehat{K}_1 z(m,t)\mathrm{d}m + \int_{\Omega} z^T(m,t)\widehat{K}_1^T M_1 \widehat{K}_2 u(m,t)\mathrm{d}m + \int_{\Omega} u^T(m,t)\widehat{K}_2^T M_1 \widehat{K}_1 z(m,t)\mathrm{d}m \\
&\quad + \int_{\Omega} u^T(m,t)\widehat{K}_2^T M_1 \widehat{K}_2 u(m,t)\mathrm{d}m - 2\int_{\Omega} z^T(m,t)\widehat{K}_1^T Q u(m,t)\mathrm{d}m - 2\int_{\Omega} u^T(m,t)\widehat{K}_2^T Q u(m,t)\mathrm{d}m \\
&\quad + 2\int_{\Omega} z^T(m,t)PGu(m,t)\mathrm{d}m \\
&= \int_{\Omega} z^T(m,t)\left[-\sum_{\sigma=1}^{\varrho}\frac{1}{\zeta_\sigma^2}(PD+DP) - PB - BP + PAA^TP + PZZ^TP + \Psi + \widehat{\Psi}L + P\widehat{H} + \widehat{H}^TP + \widehat{K}_1^T M_1 \widehat{K}_1\right]z(m,t)dm \\
&\quad + 2\int_{\Omega} z^T(m,t)\left(PG + \widehat{K}_1^T M_1 \widehat{K}_2 - \widehat{K}_1^T Q\right)u(m,t)\mathrm{d}m + \int_{\Omega} u^T(m,t)\left(\widehat{K}_2^T M_1 \widehat{K}_2 - \widehat{K}_2^T Q - Q^T\widehat{K}_2\right)u(m,t)\mathrm{d}m \\
&= \int_{\Omega} \varpi^T(m,t)\begin{pmatrix}\Phi_1 & \Lambda_1 \\ \Lambda_1^T & \Lambda_2\end{pmatrix}\varpi(m,t)\mathrm{d}m,
\end{aligned} \tag{29}$$

where $\varpi(m,t) = (z^T(m,t), u^T(m,t))^T$. On the basis of (16), one gets

$$2\int_{\Omega} y^T(m,t)Qu(m,t)\mathrm{d}m - \int_{\Omega} y^T(m,t)M_1 y(m,t)\mathrm{d}m \geq \dot{V}(t). \tag{30}$$

Then,

$$\begin{aligned}
&2\int_{t_0}^{t_l}\int_{\Omega} y^T(m,t)Qu(m,t)\mathrm{d}m\,\mathrm{d}t \\
&\quad - \int_{t_0}^{t_l}\int_{\Omega} y^T(m,t)M_1 y(m,t)\mathrm{d}m\,\mathrm{d}t \geq V(t_l) - V(t_0),
\end{aligned} \tag{31}$$

for any $t_0, t_l \in [0,+\infty)$ and $t_l \geqslant t_0$. In other words,

$$\begin{aligned}
&\int_{t_0}^{t_l}\int_{\Omega}\left(y^T(m,t)Qu(m,t) - y^T(m,t)\frac{M_1}{2}y(m,t)\right)\mathrm{d}m\,\mathrm{d}t \\
&\quad \geq S(t_l) - S(t_0),
\end{aligned} \tag{32}$$

where $S(t) = V(t)/2$.

The following results can be deduced by using the similar method. □

Corollary 1. *System (14) realizes passivity if there are matrices $P = \mathrm{diag}(P_1, P_2, \cdots, P_N) > 0$ ($P_i \in \mathbb{R}^{\xi_i\times\xi_i}$) and $Q \in \mathbb{R}^{q\times p}$ satisfying*

$$PD + DP \geq 0, \tag{33}$$

$$\begin{pmatrix}\Phi_2 & PG - \widehat{K}_1^T Q \\ G^TP - Q^T\widehat{K}_1 & -\widehat{K}_2^T Q - Q^T\widehat{K}_2\end{pmatrix} \leq 0, \tag{34}$$

where $\Phi_2 = -\sum_{\varrho=1}^{\varrho}(1/\zeta_\varrho^2)(PD+DP) - PB - BP + PAA^TP + PZZ^TP + \Psi + \widehat{\Psi}L + P\widehat{H} + \widehat{H}^TP$.

Corollary 2. *System (14) realizes input-strict passivity if there exist matrices $P = \mathrm{diag}(P_1, P_2, \cdots, P_N) > 0$ ($P_i \in \mathbb{R}^{\xi_i\times\xi_i}$), $Q \in \mathbb{R}^{q\times p}$, and $0 < M_2 \in \mathbb{R}^{p\times p}$ satisfying*

$$PD + DP \geq 0, \tag{35}$$

$$\begin{pmatrix}\Phi_2 & PG - \widehat{K}_1^T Q \\ G^TP - Q^T\widehat{K}_1 & M_2 - \widehat{K}_2^T Q - Q^T\widehat{K}_2\end{pmatrix} \leq 0, \tag{36}$$

where $\Phi_2 = -\sum_{\sigma=1}^{\varrho}(1/\zeta_\sigma^2)(PD+DP) - PB - BP + PAA^TP + PZZ^TP + \Psi + \widehat{\Psi}L + P\widehat{H} + \widehat{H}^TP$.

Remark 2. To the best of our knowledge, the concept of passivity is proposed for the first time in circuit analysis and has discovered comprehensive potential applications in lots of areas after that. Over the past few decades, some scholars have investigated the passivity of CNNs and CRDNNs and derived many meaningful results [26–33]. Note that the passivity problem is solved in the abovementioned works based on the case that the input has identical dimension as the output. As far as we know, only a few passivity results have been obtained for the CRDNNs with different dimensional input and output until now [34–36]. However, the networks considered in these works are coupled by identical nodes. Therefore, we devote ourselves to studying the passivity of CRDNNs consisting of different dimensional nodes (i.e., CDDDRDNNs), in which the input has different dimensions as output. Hence, the acquired passivity results are more general and less conservative in this paper.

3.3. Synchronization Analysis. Definition 2. Network (6) achieves synchronization if for all $i = 1, 2, \cdots, N$,

$$\lim_{t \longrightarrow +\infty} \left\| w_i(\cdot, t) - \hat{o}_i \right\|_2 = 0, \tag{37}$$

under the condition $u_i(m, t) = 0$.

Theorem 2. *Network (6) is synchronized if there are matrices $P = \operatorname{diag}(P_1, P_2, \cdots, P_N) > 0$ $(P_i \in \mathbb{R}^{\xi_i \times \xi_i})$ satisfying*

$$PD + DP \geq 0, \tag{38}$$

$$W_1 + P\widehat{H} + \widehat{H}^T P < 0, \tag{39}$$

where $W_1 = -\sum_{\sigma=1}^{\varrho} 1/\zeta_\sigma^2 (PD + DP) - PB - BP + PAA^T P + P ZZ^T P + \Psi + \widehat{\Psi} L$.

Proof. Choose the same Lyapunov functional as (17) for system (11), then,

$$\begin{aligned}
\dot{V}(t) = {} & 2\sum_{i=1}^{N} \int_{\Omega} z_i^T(m,t) P_i \left(D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} + A_i \widehat{f}_i(z_i(m,t)) + Z_i \widehat{\varphi}_i(z_i(m, t-\tau_i(t))) - B_i z_i(m,t) + \sum_{j=1}^{N} \widehat{H}_{ij} z_j(m,t) \right) \mathrm{d}m \\
& + \sum_{i=1}^{N} \sum_{j=1}^{\xi_i} \frac{\widehat{\psi}_{ij}^2}{1-\delta_i} \int_{\Omega} \left(z_j^{(i)}(m,t) \right)^2 \mathrm{d}m - \sum_{i=1}^{N} \sum_{j=1}^{\xi_i} \frac{\widehat{\psi}_{ij}^2 (1-\dot{\tau}_i(t))}{1-\delta_i} \int_{\Omega} \left(z_j^{(i)}(m, t-\tau_i(t)) \right)^2 \mathrm{d}m \\
\leq {} & 2\sum_{i=1}^{N} \int_{\Omega} z_i^T(m,t) P_i \left(D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} + A_i \widehat{f}_i(z_i(m,t)) + Z_i \widehat{\varphi}_i(z_i(m, t-\tau_i(t))) - B_i z_i(m,t) + \sum_{j=1}^{N} \widehat{H}_{ij} z_j(m,t) \right) \mathrm{d}m \\
& + \sum_{i=1}^{N} \int_{\Omega} z_i^T(m,t) \frac{\widehat{\Psi}_i}{1-\delta_i} z_i(m,t) \mathrm{d}m - \sum_{i=1}^{N} \int_{\Omega} z_i^T(m, t-\tau_i(t)) \widehat{\Psi}_i z_i(m, t-\tau_i(t)) \mathrm{d}m \\
\leq {} & \int_{\Omega} z^T(m,t) \left[-\sum_{\sigma=1}^{\varrho} \frac{1}{\zeta_\sigma^2} (PD + DP) - PB - BP + PAA^T P + PZZ^T P + \Psi + \widehat{\Psi} L + P\widehat{H} + \widehat{H}^T P \right] z(m,t) \mathrm{d}m \\
\leq {} & \Upsilon_1 \|z(\cdot, t)\|_2^2,
\end{aligned} \tag{40}$$

where $\Upsilon_1 = \lambda_M(-\sum_{\sigma=1}^{\varrho} (1/\zeta_\sigma^2)(PD + DP) - PB - BP + PA A^T P + PZZ^T P + \Psi + \widehat{\Psi} L + P\widehat{H} + \widehat{H}^T P) < 0$.

From (40), we have

$$\|z(\cdot, t)\|_2^2 \leqslant \frac{\dot{V}(t)}{\Upsilon_1}. \tag{41}$$

From (17) and (40), one knows that all terms of $V(t)$ are bounded and $V(t)$ is nonincreasing. Hence, $V(t)$ converges to a finite nonnegative real number. According to (41), we can infer that $\lim_{t \longrightarrow +\infty} \int_0^t \|z(\cdot, t)\|_2^2 \mathrm{d}t$ exists and is finite. Furthermore, since $\tau_i(t)$ is bounded, it is easily derived that

$$\lim_{t \longrightarrow +\infty} \sum_{i=1}^{N} \sum_{j=1}^{\xi_i} \frac{\widehat{\psi}_{ij}^2}{1-\delta_i} \int_{t-\tau_i(t)}^{t} \int_{\Omega} \left(z_j^{(i)}(m,h) \right)^2 \mathrm{d}m\, \mathrm{d}h = 0. \tag{42}$$

Hence, $\lim_{t \longrightarrow +\infty} \int_{\Omega} z^T(m,t) P z(m,t) \mathrm{d}m$ is not only existing but also is a nonnegative real number. Next, we will prove that $\lim_{t \longrightarrow +\infty} \int_{\Omega} z^T(m,t) P z(m,t) \mathrm{d}m = 0$. If not, one can get

$$\lim_{t \longrightarrow +\infty} \int_{\Omega} z^T(m,t) P z(m,t) \mathrm{d}m = \varsigma > 0. \tag{43}$$

Obviously, there is a real constant $\theta > 0$ such that

$$\int_{\Omega} z^T(m,t) P z(m,t) \mathrm{d}m > \frac{\varsigma}{2}, \quad t \geq \theta. \tag{44}$$

Hence,

$$\|z(\cdot, t)\|_2^2 > \frac{\varsigma}{2\kappa}, \quad t \geq \theta, \tag{45}$$

where $\kappa = \lambda_M(P)$. According to (40) and (45), we obtain

$$\dot{V}(t) < \frac{\Upsilon_1 \varsigma}{2\kappa}, \quad t \geq \theta. \tag{46}$$

It follows from (46) that

$$-V(\theta) \leq V(+\infty) - V(\theta) = \int_{\theta}^{+\infty} \dot{V}(t)\mathrm{d}t < \int_{\theta}^{+\infty} \frac{\Upsilon_1 \varsigma}{2\kappa}\mathrm{d}t = -\infty. \tag{47}$$

This is unreasonable. Consequently,

$$\lim_{t \longrightarrow +\infty} \int_{\Omega} z^T(m,t) P z(m,t) \mathrm{d}m = 0. \tag{48}$$

Then, we have $\lim_{t\longrightarrow+\infty}\|z(\cdot,t)\|_2 = 0$. Hence, network (6) is synchronized. □

4. Robust Passivity and Robust Synchronization of CDDDRDNNs

4.1. Network Model. Taking the parameter uncertainties into account, a CDDDRDNN with uncertain parameters is stated as follows:

$$\begin{aligned}\frac{\partial w_i(m,t)}{\partial t} &= D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 w_i(m,t)}{\partial m_\sigma^2} - B_i w_i(m,t) + A_i f_i(w_i(m,t)) \\ &\quad + J_i + G_i u_i(m,t) + Z_i \varphi_i(w_i(m,t-\tau_i(t))) \\ &\quad + \sum_{j=1}^{N} c_{ij} H_{ij} w_j(m,t),\end{aligned} \tag{49}$$

in which $i = 1,2,\cdots,N, w_i(m,t), J_i, G_i, u_i(m,t), f_i(w_i(m,t)), \varphi_i(w_i(m,t-\tau_i(t))), H_{ij}$, and c_{ij} represent the same senses as in model (6). The parameters D_i, B_i, A_i, and Z_i can be changed within a certain parameter range of precisions as follows:

$$\begin{aligned}D^{(P)} &:= \left\{D_i = \mathrm{diag}\left(d_s^{(i)}\right) : \underline{D}_i \leq D_i \leq \overline{D}_i, 0 < \underline{d}_s^{(i)} \leq d_s^{(i)} \leq \overline{d}_s^{(i)}, \quad i = 1,2,\cdots,N, s = 1,2,\cdots,\xi_i, \forall D_i \in D^{(P)}\right\}; \\ B^{(P)} &:= \left\{B_i = \mathrm{diag}\left(b_s^{(i)}\right) : \underline{B}_i \leq B_i \leq \overline{B}_i, 0 < \underline{b}_s^{(i)} \leq b_s^{(i)} \leq \overline{b}_s^{(i)}, \quad i = 1,2,\cdots,N, s = 1,2,\cdots,\xi_i, \forall B_i \in B^{(P)}\right\}; \\ A^{(P)} &:= \left\{A_i = \left(a_{gh}^{(i)}\right)_{\xi_i\times\xi_i} : \underline{a}_{gh}^{(i)} \leq a_{gh}^{(i)} \leq \overline{a}_{gh}^{(i)}, \quad i = 1,2,\cdots,N, g = 1,2,\cdots,\xi_i, h = 1,2,\cdots,\xi_i, \forall A_i \in A^{(P)}\right\}; \\ Z^{(P)} &:= \left\{Z_i = \left(z_{gh}^{(i)}\right)_{\xi_i\times\xi_i} : \underline{z}_{gh}^{(i)} \leq z_{gh}^{(i)} \leq \overline{z}_{gh}^{(i)}, \quad i = 1,2,\cdots,N, g = 1,2,\cdots,\xi_i, h = 1,2,\cdots,\xi_i, \forall Z_i \in Z^{(P)}\right\}.\end{aligned} \tag{50}$$

In addition, for convenience, we denote

$$\begin{aligned}\tilde{a}_{gh}^{(i)} &= \max\left\{\left|\underline{a}_{gh}^{(i)}\right|, \left|\overline{a}_{gh}^{(i)}\right|\right\}, \\ \tilde{z}_{gh}^{(i)} &= \max\left\{\left|\underline{z}_{gh}^{(i)}\right|, \left|\overline{z}_{gh}^{(i)}\right|\right\}, \\ \vartheta_{i_A} &= \sum_{g=1}^{\xi_i}\sum_{h=1}^{\xi_i}\left(\tilde{a}_{gh}^{(i)}\right)^2, \\ \vartheta_{i_Z} &= \sum_{g=1}^{\xi_i}\sum_{h=1}^{\xi_i}\left(\tilde{z}_{gh}^{(i)}\right)^2, \\ \vartheta_A &= \mathrm{diag}\left(\vartheta_{1_A} I_{\xi_1}, \vartheta_{2_A} I_{\xi_2}, \cdots, \vartheta_{N_A} I_{\xi_N}\right) \in \mathbb{R}^{\xi\times\xi}, \\ \vartheta_Z &= \mathrm{diag}\left(\vartheta_{1_Z} I_{\xi_1}, \vartheta_{2_Z} I_{\xi_2}, \cdots, \vartheta_{N_Z} I_{\xi_N}\right) \in \mathbb{R}^{\xi\times\xi}, \\ \underline{D} &= \mathrm{diag}(\underline{D}_1, \underline{D}_2, \cdots, \underline{D}_N), \\ \underline{B} &= \mathrm{diag}(\underline{B}_1, \underline{B}_2, \cdots, \underline{B}_N),\end{aligned} \tag{51}$$

where $g = 1,2,\cdots,\xi_i$ and $h = 1,2,\cdots,\xi_i, i = 1,2,\cdots,N$.

According to (10), we can get the error system $z_i(m,t)$ of network (49) as follows:

$$\begin{aligned}\frac{\partial z_i(m,t)}{\partial t} &= D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} - B_i z_i(m,t) + A_i \widehat{f}_i(z_i(m,t)) \\ &\quad + G_i u_i(m,t) + Z_i \widehat{\varphi}_i(z_i(m,t-\tau_i(t))) \\ &\quad + \sum_{j=1}^{N} c_{ij} H_{ij} z_j(m,t),\end{aligned} \tag{52}$$

where the ranges of D_i, B_i, A_i, and Z_i are described by using (50).

4.2. Robust Passivity Analysis. The output vector $y_i(m,t)$ of system (52) is defined similarly as (12). In terms of (12) and (52), the error system with output $y_i(m,t) \in \mathbb{R}^{q_i}$ and input $u_i(m,t) \in \mathbb{R}^{p_i}$ can be described by

$$\begin{cases}\dfrac{\partial z_i(m,t)}{\partial t} = D_i \displaystyle\sum_{\sigma=1}^{\varrho} \frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} - B_i z_i(m,t) + A_i \widehat{f}_i(z_i(m,t)) \\ \quad + G_i u_i(m,t) + Z_i \widehat{\varphi}_i(z_i(m,t-\tau_i(t))) + \displaystyle\sum_{j=1}^{N} \widehat{H}_{ij} z_j(m,t), \\ y_i(m,t) = K_1^{(i)} z_i(m,t) + K_2^{(i)} u_i(m,t),\end{cases} \tag{53}$$

where $\widehat{H}_{ij} = c_{ij}H_{ij}$ and $i = 1, 2, \cdots, N$.

Theorem 3. *If there are matrices $P = diag(P_1, P_2, \cdots, P_N) > 0$ $(P_i = diag(p_1^{(i)}, p_2^{(i)}, \cdots, p_{\xi_i}^{(i)}) \in \mathbb{R}^{\xi_i \times \xi_i})$, $Q \in \mathbb{R}^{q\times p}$, and $0 < M_1 \in \mathbb{R}^{q\times q}$ satisfying*

$$\begin{pmatrix} \Phi_3 & \Lambda_1 \\ \Lambda_1^T & \Lambda_2 \end{pmatrix} \leq 0, \tag{54}$$

where $\Phi_3 = -\sum_{\sigma=1}^{\varrho} (2/\zeta_\sigma^2) P\, \underline{D} - 2P\, \underline{B} + (\vartheta_A + \vartheta_Z)P^2 + \Psi + \widehat{\Psi}L + P\widehat{H} + \widehat{H}^T P + \widehat{K}_1^T M_1 \widehat{K}_1$, $\Lambda_1 = PG + \widehat{K}_1^T M_1 \widehat{K}_2 - \widehat{K}_1^T Q$, $\Lambda_2 = \widehat{K}_2^T M_1 \widehat{K}_2 - \widehat{K}_2^T Q - Q^T \widehat{K}_2$, then, system (53) under the given ranges of parameters (50) is output-robustly passive.

Proof. Choose the same Lyapunov functional as (17) for system (52). Then,

$$\begin{aligned}
\dot{V}(t) = {} & 2\sum_{i=1}^{N} \int_\Omega z_i^T(m,t) P_i \left(D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2} - B_i z_i(m,t) + A_i \widehat{f}_i(z_i(m,t)) + G_i u_i(m,t) + Z_i \widehat{\varphi}_i(z_i(m,t-\tau_i(t))) + \sum_{j=1}^{N} \widehat{H}_{ij} z_j(m,t) \right) dm \\
& - \sum_{i=1}^{N}\sum_{j=1}^{\xi_i} \frac{\widehat{\psi}_{ij}^2 (1 - \dot{\tau}_i(t))}{1-\delta_i} \int_\Omega \left(z_j^{(i)}(m,t-\tau_i(t))\right)^2 dm + \sum_{i=1}^{N}\sum_{j=1}^{\xi_i} \frac{\widehat{\psi}_{ij}^2}{1-\delta_i} \int_\Omega \left(z_j^{(i)}(m,t)\right)^2 dm \\
\leqslant {} & 2\int_\Omega z^T(m,t) P \left(D \sum_{\sigma=1}^{\varrho} \frac{\partial^2 z(m,t)}{\partial m_\sigma^2} - Bz(m,t) + A\widehat{f}(z(m,t)) + Gu(m,t) + Z\widehat{\varphi}(z(m,t-\tau(t))) + \widehat{H}z(m,t) \right) dm \\
& + \int_\Omega z^T(m,t) \widehat{\Psi} L z(m,t) dm - \int_\Omega z^T(m,t-\tau(t)) \widehat{\Psi} z(m,t-\tau(t)) dm.
\end{aligned} \tag{55}$$

Obviously,

$$\begin{aligned}
-2\int_\Omega z^T(m,t) PBz(m,t) dm &= -2\sum_{i=1}^{N}\sum_{h=1}^{\xi_i} p_h^{(i)} b_h^{(i)} \int_\Omega \left(z_h^{(i)}(m,t)\right)^2 dm \leq -2\sum_{i=1}^{N}\sum_{h=1}^{\xi_i} p_h^{(i)} \underline{b}_h^{(i)} \int_\Omega \left(z_h^{(i)}(m,t)\right)^2 dm \\
&= -2\int_\Omega z^T(m,t) P\, \underline{B}\, z(m,t) dm.
\end{aligned} \tag{56}$$

Similar to the deduction of (25), one can get

$$\begin{aligned}
2\int_\Omega z^T(m,t) P D \sum_{\sigma=1}^{\varrho} \frac{\partial^2 z(m,t)}{\partial m_\sigma^2} dm &\leq -\sum_{\sigma=1}^{\varrho} \frac{2}{\zeta_\sigma^2} \int_\Omega z^T(m,t) PDz(m,t) dm \\
&= -\sum_{i=1}^{N}\sum_{h=1}^{\xi_i}\sum_{\sigma=1}^{\varrho} \frac{2}{\zeta_\sigma^2} p_h^{(i)} d_h^{(i)} \int_\Omega \left(z_h^{(i)}(m,t)\right)^2 dm \\
&\leq -\sum_{i=1}^{N}\sum_{h=1}^{\xi_i}\sum_{\sigma=1}^{\varrho} \frac{2}{\zeta_\sigma^2} p_h^{(i)} \underline{d}_h^{(i)} \int_\Omega \left(z_h^{(i)}(m,t)\right)^2 dm \\
&= -\sum_{\sigma=1}^{\varrho} \frac{2}{\zeta_\sigma^2} \int_\Omega z^T(m,t) P\underline{D}\, z(m,t) dm.
\end{aligned} \tag{57}$$

In addition,

$$\begin{aligned}
2\int_{\Omega} z^{T}(m,t)PA\widehat{f}(z(m,t))\mathrm{d}m &= 2\sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t)P_iA_i\widehat{f}_i(z_i(m,t))\mathrm{d}m \\
&\leq \sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t)P_iA_iA_i^{T}P_iz_i(m,t)\mathrm{d}m + \sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t)\Psi_iz_i(m,t)\mathrm{d}m \\
&\leq \sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t)\vartheta_{i_A}P_i^{2}z_i(m,t)\mathrm{d}m + \sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t)\Psi_iz_i(m,t)\mathrm{d}m \\
&= \int_{\Omega} z^{T}(m,t)\left(\vartheta_AP^{2}+\Psi\right)z(m,t)\mathrm{d}m,
\end{aligned} \tag{58}$$

$$\begin{aligned}
2\int_{\Omega} z^{T}(m,t)PZ\widehat{\varphi}(z(m,t-\tau(t)))\mathrm{d}m &= 2\sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t)P_iZ_i\widehat{\varphi}_i(z_i(m,t-\tau_i(t)))\mathrm{d}m \\
&\leqslant \sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t)P_iZ_iZ_i^{T}P_iz_i(m,t)\mathrm{d}m + \sum_{i=1}^{N}\int_{\Omega} z_i^{T}(m,t-\tau_i(t))\widehat{\Psi}_iz_i(m,t-\tau_i(t))\mathrm{d}m \\
&= \int_{\Omega} z^{T}(m,t)\vartheta_ZP^{2}z(m,t)\mathrm{d}m + \int_{\Omega} z^{T}(m,t-\tau(t))\widehat{\Psi}z(m,t-\tau(t))\mathrm{d}m.
\end{aligned} \tag{59}$$

According to (56)–(59), one gets

$$\begin{aligned}
\dot{V}(t) \leq &\int_{\Omega} z^{T}(m,t)\left[-\sum_{\sigma=1}^{\varrho}\frac{2}{\zeta_\sigma^{2}}P\underline{D}-2P\underline{B}+(\vartheta_A+\vartheta_Z)P^{2}\right. \\
&\left.+\Psi+\widehat{\Psi}L+P\widehat{H}+\widehat{H}^{T}P\right]z(m,t)dm \\
&+2\int_{\Omega} z^{T}(m,t)PGu(m,t)\mathrm{d}m.
\end{aligned} \tag{60}$$

Hence,

$$\begin{aligned}
&\dot{V}(t)-2\int_{\Omega} y^{T}(m,t)Qu(m,t)\mathrm{d}m+\int_{\Omega} y^{T}(m,t)M_1y(m,t)\mathrm{d}m \\
&\leq \int_{\Omega} z^{T}(m,t)\left[-\sum_{\sigma=1}^{\varrho}\frac{2}{\zeta_\sigma^{2}}P\underline{D}-2P\underline{B}+(\vartheta_A+\vartheta_Z)P^{2}+\Psi+\widehat{\Psi}L\right. \\
&\left.+P\widehat{H}+\widehat{H}^{T}P+\widehat{K}_1^{T}M_1\widehat{K}_1\right]z(m,t)\mathrm{d}m+2\int_{\Omega} z^{T}(m,t) \\
&\cdot\left(PG+\widehat{K}_1^{T}M_1\widehat{K}_2-\widehat{K}_1^{T}Q\right)u(m,t)\mathrm{d}m \\
&+\int_{\Omega} u^{T}(m,t)\left(\widehat{K}_2^{T}M_1\widehat{K}_2-\widehat{K}_2^{T}Q-Q^{T}\widehat{K}_2\right)u(m,t)\mathrm{d}m \\
&= \int_{\Omega} \varpi^{T}(m,t)\begin{pmatrix}\Phi_3 & \Lambda_1\\ \Lambda_1^{T} & \Lambda_2\end{pmatrix}\varpi(m,t)\mathrm{d}m.
\end{aligned} \tag{61}$$

From (54), one can get

$$2\int_{\Omega} y^{T}(m,t)Qu(m,t)\mathrm{d}m-\int_{\Omega} y^{T}(m,t)M_1y(m,t)\mathrm{d}m \geq \dot{V}(t). \tag{62}$$

Then,

$$\begin{aligned}
&2\int_{t_0}^{t_l}\int_{\Omega} y^{T}(m,t)Qu(m,t)\mathrm{d}m\,\mathrm{d}t-\int_{t_0}^{t_l}\int_{\Omega} y^{T}(m,t)M_1y \\
&(m,t)\mathrm{d}m\,\mathrm{d}t \geq V(t_l)-V(t_0),
\end{aligned} \tag{63}$$

for any $t_0, t_l \in [0,+\infty)$ and $t_l \geq t_0$. In other words,

$$\begin{aligned}
&\int_{t_0}^{t_l}\int_{\Omega}\left(y^{T}(m,t)Qu(m,t)-y^{T}(m,t)\frac{M_1}{2}y(m,t)\right)\mathrm{d}m\,\mathrm{d}t \\
&\geq S(t_l)-S(t_0),
\end{aligned} \tag{64}$$

in which $S(t)=V(t)/2$.

The following results can be deduced by using the similar method. □

Corollary 3. *System (53) under the given ranges of parameters (50) is robustly passive if there are matrices $P=\mathrm{diag}(P_1, P_2, \cdots, P_N)>0$ ($P_i=\mathrm{diag}(p_1^{(i)}, p_2^{(i)}, \cdots, p_{\xi_i}^{(i)}) \in \mathbb{R}^{\xi_i\times\xi_i}$) and $Q\in\mathbb{R}^{q\times p}$ satisfying*

$$\begin{pmatrix}\Phi_4 & PG-\widehat{K}_1^{T}Q\\ G^{T}P-Q^{T}\widehat{K}_1 & -\widehat{K}_2^{T}Q-Q^{T}\widehat{K}_2\end{pmatrix}\leq 0, \tag{65}$$

where $\Phi_4=-\sum_{\sigma=1}^{\varrho}(2/\zeta_\sigma^{2})P\underline{D}-2P\underline{B}+(\vartheta_A+\vartheta_Z)P^{2}+\Psi+\widehat{\Psi}L+P\widehat{H}+\widehat{H}^{T}P$.

Corollary 4. *System (53) under the given ranges of parameters (50) is input-robustly passive if there are matrices $P=\mathrm{diag}(P_1,P_2,\cdots,P_N)>0$ ($P_i=\mathrm{diag}(p_1^{(i)},p_2^{(i)},\cdots,p_{\xi_i}^{(i)})\in\mathbb{R}^{\xi_i\times\xi_i}$), $Q\in\mathbb{R}^{q\times p}$, and $0<M_2\in\mathbb{R}^{p\times p}$ satisfying*

$$\begin{pmatrix}\Phi_4 & PG-\widehat{K}_1^{T}Q\\ G^{T}P-Q^{T}\widehat{K}_1 & M_2-\widehat{K}_2^{T}Q-Q^{T}\widehat{K}_2\end{pmatrix}\leq 0, \tag{66}$$

where $\Phi_4 = -\sum_{\sigma=1}^{\varrho}(2/\zeta_\sigma^2)P\underline{D} - 2P\underline{B} + (\vartheta_A + \vartheta_Z)P^2 + \Psi + \widehat{\Psi}L + P\widehat{H} + \widehat{H}^T P$.

4.3. Robust Synchronization Analysis. Definition 3. Network (49) under the given ranges of parameter (50) realizes robust synchronization if for all $D_i \in D^{(P)}, B_i \in B^{(P)}, A_i \in A^{(P)}$, and $Z_i \in Z^{(P)}$, $i = 1, 2, \cdots, N$,

$$\lim_{t\longrightarrow+\infty}\left\|w_i(\cdot,t) - \widehat{o}_i\right\|_2 = 0, \tag{67}$$

under the condition $u_i(m,t) = 0$.

Theorem 4. *Network (49) under the given ranges of parameters (50) achieves robust synchronization if there are matrices* $P = \mathrm{diag}(P_1, P_2, \cdots, P_N) > 0$ ($P_i = \mathrm{diag}(p_1^{(i)}, p_2^{(i)}, \cdots, p_{\xi_i}^{(i)}) \in \mathbb{R}^{\xi_i\times\xi_i}$) *satisfying*

$$W_2 + P\widehat{H} + \widehat{H}^T P < 0, \tag{68}$$

where $W_2 = -\sum_{\sigma=1}^{\varrho}(2/\zeta_\sigma^2)P\underline{D} - 2P\underline{B} + (\vartheta_A + \vartheta_Z)P^2 + \Psi + \widehat{\Psi}L$.

Proof. Select the same Lyapunov functional as (17) for system (52); then,

$$\begin{aligned}
\dot{V}(t) \le{}& 2\sum_{i=1}^{N}\int_\Omega z_i^T(m,t)P_i\Bigg(D_i\sum_{\sigma=1}^{\varrho}\frac{\partial^2 z_i(m,t)}{\partial m_\sigma^2}\\
&- B_i z_i(m,t) + Z_i\widehat{\varphi}_i(z_i(m,t-\tau_i(t)))\\
&+ A_i\widehat{f}_i(z_i(m,t)) + \sum_{j=1}^{N}\widehat{H}_{ij}z_j(m,t)\Bigg)\mathrm{d}m\\
&+ \sum_{i=1}^{N}\int_\Omega z_i^T(m,t)\frac{\widehat{\Psi}_i}{1-\delta_i}z_i(m,t)\mathrm{d}m\\
&- \sum_{i=1}^{N}\int_\Omega z_i^T(m,t-\tau_i(t))\widehat{\Psi}_i z_i(m,t-\tau_i(t))\mathrm{d}m\\
\le{}& \int_\Omega z^T(m,t)\Bigg[-\sum_{\sigma=1}^{\varrho}\frac{2}{\zeta_\sigma^2}P\underline{D} - 2P\underline{B} + (\vartheta_A+\vartheta_Z)P^2\\
&+ \Psi + \widehat{\Psi}L + P\widehat{H} + \widehat{H}^T P\Bigg]z(m,t)\mathrm{d}m\\
\le{}& \Upsilon_2\|z(\cdot,t)\|_2^2,
\end{aligned} \tag{69}$$

where $\Upsilon_2 = \lambda_M(-\sum_{\sigma=1}^{\varrho}(2/\zeta_\sigma^2)P\underline{D} - 2P\underline{B} + (\vartheta_A + \vartheta_Z)P^2 + \widehat{\Psi}L + \Psi + P\widehat{H} + \widehat{H}^T P) < 0$.

Then, $\lim_{t\longrightarrow+\infty}\|z(\cdot,t)\|_2 = 0$ can be proved similarly as in Theorem 2. Consequently, network (49) under the given ranges of parameters (50) realizes robust synchronization. □

Remark 3. In Section 3, we deal with the passivity and synchronization problems of CDDDRDNNs and establish some adequate conditions to achieve the passivity and synchronization of network (6). Note that the parameters in matrices D_i, B_i, A_i, and Z_i of network model (6) are fixed. However, it is utterly unrealistic for the networks with some certain parameters due to the noises of environment and equipment restrictions in some practical situations [18–25]. Thus, it is necessary to consider the case that the parameters in matrices D_i, B_i, A_i, and Z_i of network model (6) belong to some given ranges and investigate robust dynamical properties of the considered network. As far as we know, the robust synchronization and robust passivity of CDDDRDNNs with parametric uncertainties have never been studied. In this section, we present several robust synchronization and robust passivity criteria of CDDDRDNNs with uncertain parameters in Theorems 3 and 4 and Corollaries 3 and 4, respectively.

5. Numerical Examples

Example 1. Consider the following CDDDRDNNs:

$$\begin{aligned}
\frac{\partial w_i(m,t)}{\partial t} ={}& D_i\sum_{\sigma=1}^{\varrho}\frac{\partial^2 w_i(m,t)}{\partial m_\sigma^2} - B_i w_i(m,t) + A_i f_i(w_i(m,t))\\
&+ J_i + G_i u_i(m,t) + Z_i\varphi_i(w_i(m,t-\tau_i(t)))\\
&+ \sum_{j=1}^{3}c_{ij}H_{ij}w_j(m,t),
\end{aligned} \tag{70}$$

where $i = 1,2,3, \xi_1 = 3, \xi_2 = \xi_3 = 2, f_l^{(i)}(v) = \varphi_l^{(i)}(v) = (|v+1| - |v-1|/8), l = 1,2,3, \Omega = \{m| -0.5 < m < 0.5\}$, $D_1 = \mathrm{diag}(0.6, 0.8, 0.9)$, $D_2 = \mathrm{diag}(0.5, 0.7)$, $D_3 = \mathrm{diag}(0.8, 0.9)$, $B_1 = \mathrm{diag}(0.8, 0.9, 1.2)$, $B_2 = \mathrm{diag}(0.7, 0.9)$, $B_3 = \mathrm{diag}(0.6, 0.9)$, $\tau_i(t) = 0.04i - 0.02ie^{-t}$, $\tau_1 = 0.04$, $\delta_1 = 0.02$, $\tau_2 = 0.08$, $\delta_2 = 0.04$, $\tau_3 = 0.12$, $\delta_3 = 0.06$ and $J_1 = (0,0,0)^T$, $J_2 = J_3 = (0,0)^T$, $u_1(m,t) = (8\sqrt{t}\cos(\pi m), 5\sqrt{t}\cos(\pi m), 10\sqrt{t}\cos(\pi m))^T$, $u_2(m,t) = (16\sqrt{t}\cos(\pi m), 10\sqrt{t}\cos(\pi m))^T$, $u_3(m,t) = (24\sqrt{t}\cos(\pi m), 15\sqrt{t}\cos(\pi m))^T$. The matrices $A_i = (a_{gh}^{(i)})_{\xi_i\times\xi_i}$, $Z_i = (z_{gh}^{(i)})_{\xi_i\times\xi_i}$, $C = (c_{ij})_{3\times3}$, $G_i, H_{ij}, K_1^{(i)}$, and $K_2^{(i)}$ are chosen as follows:

$$\begin{aligned}
A_1 &= \begin{pmatrix} 0.5 & 0.6 & 0.7 \\ 0.6 & 0.4 & 0.5 \\ 0.3 & 0.2 & 0.8 \end{pmatrix}, \\
A_2 &= \begin{pmatrix} 0.4 & 0.6 \\ 0.3 & 0.4 \end{pmatrix}, \\
A_3 &= \begin{pmatrix} 0.3 & 0.6 \\ 0.5 & 0.4 \end{pmatrix}, \\
Z_1 &= \begin{pmatrix} 0.4 & 0.6 & 0.7 \\ 0.6 & 0.3 & 0.5 \\ 0.5 & 0.2 & 0.6 \end{pmatrix}, \\
Z_2 &= \begin{pmatrix} 0.3 & 0.6 \\ 0.5 & 0.4 \end{pmatrix}, \\
Z_3 &= \begin{pmatrix} 0.5 & 0.6 \\ 0.6 & 0.4 \end{pmatrix}, \\
G_1 &= \begin{pmatrix} 0.2 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 \end{pmatrix}, \\
G_2 &= \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.1 \end{pmatrix}, \\
G_3 &= \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{pmatrix}, \\
H_{11} &= \begin{pmatrix} 0.2 & 0.1 & 0.3 \\ 0.1 & 0.3 & 0.2 \\ 0.3 & 0.2 & 0.4 \end{pmatrix}, \\
H_{12} &= \begin{pmatrix} 0.3 & 0.2 \\ 0.3 & 0.5 \\ 0.2 & 0.4 \end{pmatrix}, \\
H_{13} &= \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \\ 0.1 & 0.2 \end{pmatrix}, \\
H_{21} &= \begin{pmatrix} 0.2 & 0.3 & 0.1 \\ 0.1 & 0.4 & 0.2 \end{pmatrix}, \\
H_{22} &= \begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.3 \end{pmatrix}, \\
H_{23} &= \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.4 \end{pmatrix}, \\
H_{31} &= \begin{pmatrix} 0.1 & 0.2 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}, \\
H_{32} &= \begin{pmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{pmatrix}, \\
H_{33} &= \begin{pmatrix} 0.4 & 0.3 \\ 0.2 & 0.4 \end{pmatrix}, \\
K_1^1 &= \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.2 & 0.4 & 0.1 \\ 0.3 & 0.2 & 0.1 \end{pmatrix}, \\
K_1^2 &= \begin{pmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \\ 0.2 & 0.1 \end{pmatrix}, \\
K_1^3 &= \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.4 \\ 0.2 & 0.1 \end{pmatrix}, \\
K_2^1 &= \begin{pmatrix} 0.2 & 0.2 & 0.3 \\ 0.1 & 0.3 & 0.2 \\ 0.2 & 0.1 & 0.3 \end{pmatrix}, \\
K_2^2 &= \begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \\ 0.2 & 0.4 \end{pmatrix}, \\
K_2^3 &= \begin{pmatrix} 0.1 & 0.2 \\ 0.1 & 0.3 \\ 0.3 & 0.2 \end{pmatrix}, \\
C &= \begin{pmatrix} -0.1 & 0.3 & 0.2 \\ 0.2 & -0.4 & 0.2 \\ 0.3 & 0.2 & -0.8 \end{pmatrix}.
\end{aligned} \tag{71}$$

Case 1. Obviously, the equilibrium point of an isolated node of network (70) is $\widehat{O} = (0,0,0,0,0,0,0)^T$ and $\psi_{il} = \widehat{\psi}_{il} = 0.25$. It is easy to calculate the following matrices P, Q, and M_1 satisfying (15) and (16):

$$M_1 = \begin{pmatrix} 4.9725 & -0.7644 & -0.7186 & -0.0353 & -0.0591 & -0.0363 & -0.0508 & -0.0707 & -0.0289 \\ -0.7644 & 4.3790 & -0.9676 & -0.0454 & -0.0765 & -0.0477 & -0.0609 & -0.0850 & -0.0352 \\ -0.7186 & -0.9676 & 4.5415 & -0.0363 & -0.0613 & -0.0386 & -0.0533 & -0.0742 & -0.0306 \\ -0.0353 & -0.0454 & -0.0363 & 5.3616 & 0.0154 & -0.0287 & -0.0215 & -0.0299 & -0.0122 \\ -0.0591 & -0.0765 & -0.0613 & 0.0154 & 5.3503 & -0.0393 & -0.0360 & -0.0501 & -0.0206 \\ -0.0363 & -0.0477 & -0.0386 & -0.0287 & -0.0393 & 5.3398 & -0.0222 & -0.0309 & -0.0128 \\ -0.0508 & -0.0609 & -0.0533 & -0.0215 & -0.0360 & -0.0222 & 5.5126 & 0.2342 & 0.0883 \\ -0.0707 & -0.0850 & -0.0742 & -0.0299 & -0.0501 & -0.0309 & 0.2342 & 5.6688 & 0.1327 \\ -0.0289 & -0.0352 & -0.0306 & -0.0122 & -0.0206 & -0.0128 & 0.0883 & 0.1327 & 5.4320 \end{pmatrix},$$
$$Q = \begin{pmatrix} 4.3345 & 5.8441 & 5.2880 & -0.0640 & 0.0404 & -0.0208 & 0.0671 \\ -3.7523 & 7.7721 & -3.5249 & -0.0102 & -0.0813 & -0.0252 & -0.1125 \\ 2.9109 & -10.7922 & 3.1831 & -0.0047 & -0.0718 & -0.0219 & -0.0890 \\ 0.0218 & 0.0085 & 0.0367 & 11.6387 & -10.2133 & 0.0546 & 0.0150 \\ -0.1185 & -0.3761 & -0.2903 & -2.9389 & 3.7925 & -0.0623 & -0.1872 \\ 0.0379 & 0.1798 & 0.1093 & -0.9002 & 11.2576 & 0.0067 & 0.0789 \\ -0.5730 & -0.9531 & -1.1102 & -0.1287 & -0.0881 & 22.6726 & -64.9202 \\ 0.2752 & 0.4871 & 0.5449 & 0.0814 & 0.0072 & -16.9261 & 50.4379 \\ 0.0993 & 0.1553 & 0.1884 & -0.0423 & -0.0047 & 8.0204 & 3.4236 \end{pmatrix}, \tag{72}$$

and $P = \mathrm{diag}(P_1, P_2, P_3)$, where

$$P_1 = \begin{pmatrix} 1.4434 & -0.4099 & -0.3575 \\ -0.4099 & 1.1014 & -0.1331 \\ -0.3575 & -0.1331 & 1.3827 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 1.8096 & -0.1693 \\ -0.1693 & 1.5437 \end{pmatrix}, \tag{73}$$
$$P_3 = \begin{pmatrix} 1.4530 & -0.0737 \\ -0.0737 & 1.2896 \end{pmatrix}.$$

On the basis of Theorem 1, the network (70) with the input $u(m,t) \in \mathbb{R}^7$ and output $y(m,t) \in \mathbb{R}^9$ as described in (12) realizes output-strictly passivity. Figure 1 displays the simulation results.

Case 2. $\psi_{il}, \widehat{\psi}_{il}$, and $\widehat{O}$ are similar as those in Case 1. By exploiting the MATLAB Toolbox, the matrix P satisfying (38) and (39) can be computed as follows:

$$P = \mathrm{diag}(P_1, P_2, P_3), \tag{74}$$

where

$$P_1 = \begin{pmatrix} 0.2165 & -0.0111 & -0.0091 \\ -0.0111 & 0.1750 & -0.0054 \\ -0.0091 & -0.0054 & 0.1493 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 0.2567 & -0.0117 \\ -0.0117 & 0.1929 \end{pmatrix}, \tag{75}$$
$$P_3 = \begin{pmatrix} 0.1780 & -0.0118 \\ -0.0118 & 0.1520 \end{pmatrix}.$$

According to Theorem 2, the network (70) realizes synchronization. Figures 2–4 show the simulation results.

Example 2. Consider the following CDDDRNNs:

$$\begin{aligned} \frac{\partial w_i(m,t)}{\partial t} &= D_i \sum_{\sigma=1}^{\varrho} \frac{\partial^2 w_i(m,t)}{\partial m_\sigma^2} - B_i w_i(m,t) + A_i f_i(w_i(m,t)) \\ &\quad + J_i + G_i u_i(m,t) + Z_i \varphi_i(w_i(m,t-\tau_i(t))) \\ &\quad + \sum_{j=1}^{3} c_{ij} H_{ij} w_j(m,t), \end{aligned} \tag{76}$$

in which $i = 1,2,3, \xi_1 = 3, \xi_2 = \xi_3 = 2, \Omega = \{m \mid -0.5 < m < 0.5\}$, $f_l^{(i)}(v) = \varphi_l^{(i)}(v) = (|v+1| - |v-1|)/4$, $l = 1,2,3$, $\tau_i(t) = 0.02i - 0.04ie^{-t}$, $\tau_1 = 0.02, \delta_1 = 0.04, \tau_2 = 0.04, \delta_2 =$

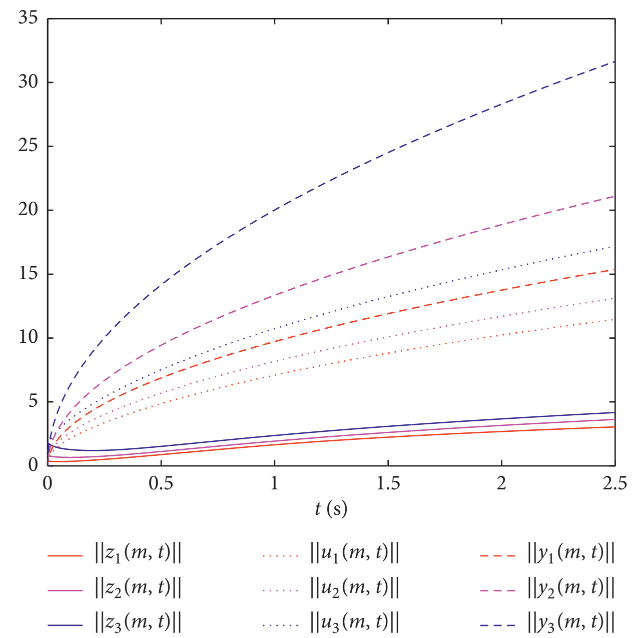

Figure 1: Change processes of $z_i(m,t)$, $u_i(m,t)$ and $y_i(m,t)$ in network (70).

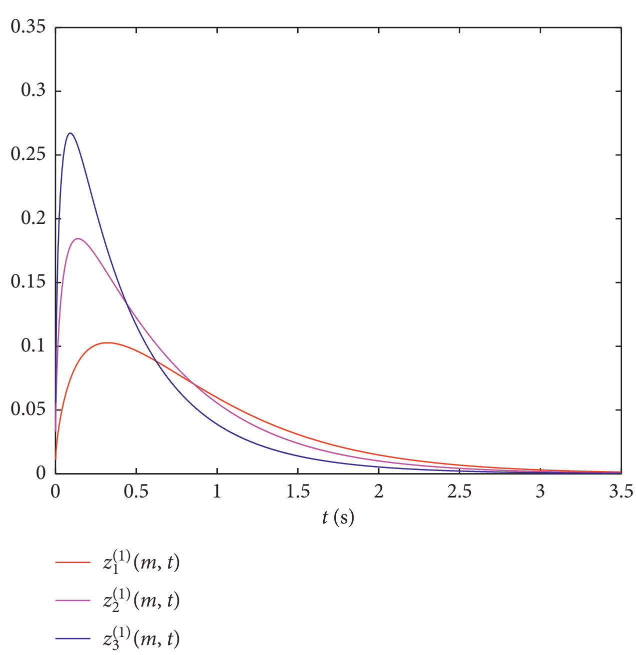

Figure 2: Change processes of $z_j^{(1)}(m,t)$ in network (70).

0.08, $\tau_3 = 0.06, \delta_3 = 0.12$, and $J_1 = (0,0,0)^T, J_2 = J_3 = (0,0)^T, u_1(m,t) = (3\sqrt{t}\cos(\pi m), 4\sqrt{t}\cos(\pi m), 6\sqrt{t}\cos(\pi m))^T, u_2(m,t) = (6\sqrt{t}\cos(\pi m), 8\sqrt{t}\cos(\pi m))^T, u_3(m,t) = (9\sqrt{t}\cos(\pi m), 12\sqrt{t}\cos(\pi m))^T$. The matrices $C = (c_{ij})_{3\times3}, G_i, H_{ij}, K_1^{(i)}$, and $K_2^{(i)}$ are chosen as follows:

$$
\begin{aligned}
G_1 &= \begin{pmatrix} 0.2 & 0.1 & 0.4 \\ 0.3 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \end{pmatrix}, \\
G_2 &= \begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{pmatrix}, \\
G_3 &= \begin{pmatrix} 0.3 & 0.4 \\ 0.2 & 0.3 \end{pmatrix}, \\
H_{11} &= \begin{pmatrix} 0.2 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0.3 \end{pmatrix}, \\
H_{12} &= \begin{pmatrix} 0.2 & 0.4 \\ 0.4 & 0.2 \\ 0.3 & 0.3 \end{pmatrix}, \\
H_{13} &= \begin{pmatrix} 0.3 & 0.6 \\ 0.3 & 0.4 \\ 0.2 & 0.5 \end{pmatrix}, \\
H_{21} &= \begin{pmatrix} 0.2 & 0.3 & 0.4 \\ 0.4 & 0.5 & 0.3 \end{pmatrix}, \\
H_{22} &= \begin{pmatrix} 0.3 & 0.4 \\ 0.5 & 0.6 \end{pmatrix}, \\
H_{23} &= \begin{pmatrix} 0.3 & 0.4 \\ 0.5 & 0.4 \end{pmatrix}, \\
H_{31} &= \begin{pmatrix} 0.3 & 0.2 & 0.3 \\ 0.4 & 0.3 & 0.2 \end{pmatrix}, \\
H_{32} &= \begin{pmatrix} 0.3 & 0.4 \\ 0.3 & 0.4 \end{pmatrix}, \\
H_{33} &= \begin{pmatrix} 0.3 & 0.4 \\ 0.4 & 0.2 \end{pmatrix}, \\
K_1^1 &= \begin{pmatrix} 0.2 & 0.1 & 0.2 \\ 0.2 & 0.3 & 0.1 \\ 0.2 & 0.5 & 0.3 \end{pmatrix}, \\
K_1^2 &= \begin{pmatrix} 0.3 & 0.4 \\ 0.4 & 0.5 \\ 0.1 & 0.3 \end{pmatrix}, \\
K_1^3 &= \begin{pmatrix} 0.4 & 0.2 \\ 0.2 & 0.3 \\ 0.3 & 0.2 \end{pmatrix}, \\
K_2^1 &= \begin{pmatrix} 0.4 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.2 \\ 0.3 & 0.4 & 0.1 \end{pmatrix}, \\
K_2^2 &= \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \\ 0.3 & 0.1 \end{pmatrix}, \\
K_2^3 &= \begin{pmatrix} 0.3 & 0.2 \\ 0.4 & 0.5 \\ 0.3 & 0.4 \end{pmatrix}, \\
C &= \begin{pmatrix} 0.2 & -0.8 & 0.5 \\ -0.7 & 0.1 & 0.2 \\ 0.4 & 0.3 & -0.6 \end{pmatrix}.
\end{aligned}
\tag{77}
$$

The parameters $D_i = \operatorname{diag}(d_1^{(i)}, d_2^{(i)}, \cdots, d_{\xi_i}^{(i)}), B_i = \operatorname{diag}(b_1^{(i)}, b_2^{(i)}, \cdots, b_{\xi_i}^{(i)}), A_i = (a_{gh}^{(i)})_{\xi_i\times\xi_i}$, and $Z_i = (z_{gh}^{(i)})_{\xi_i\times\xi_i} (i = 1, 2, 3)$ in network (76) can be changed in the precisions given as follows:

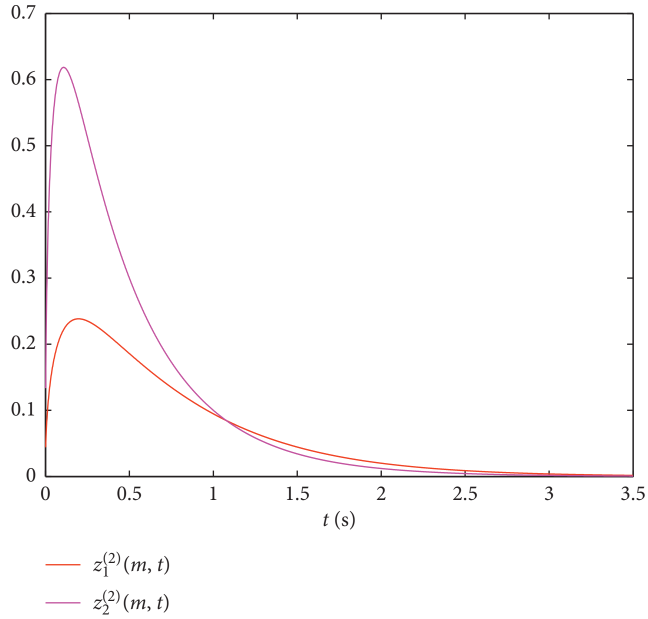

FIGURE 3: Change processes of $z_j^{(2)}(m,t)$ in network (70).

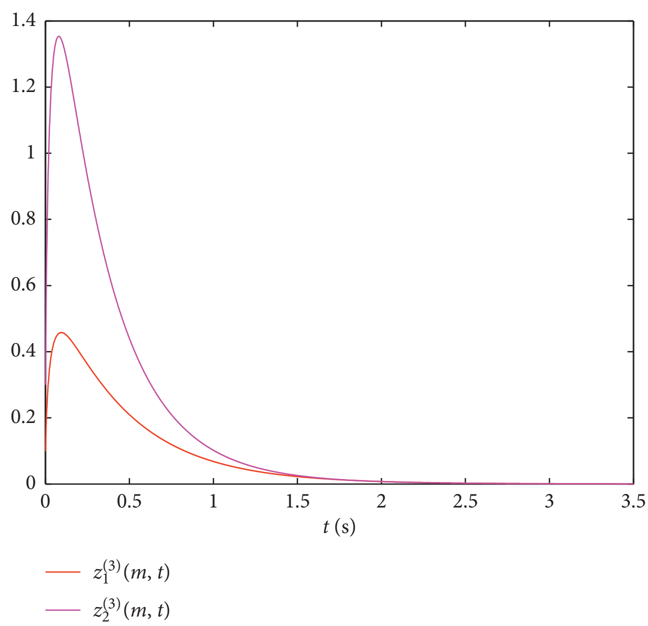

FIGURE 4: Change processes of $z_j^{(3)}(m,t)$ in network (70).

$$\begin{aligned}
D^{(P)} &:= \left\{D_i = \operatorname{diag}\left(d_s^{(i)}\right) : \underline{D}_i \le D_i \le \overline{D}_i, 0 < 0.4is \le d_s^{(i)} \le 0.6is, \quad i = 1, 2, \cdots, N, s = 1, 2, \cdots, \xi_i, \forall D_i \in D^{(P)}\right\}; \\
B^{(P)} &:= \left\{B_i = \operatorname{diag}\left(b_s^{(i)}\right) : \underline{B}_i \le B_i \le \overline{B}_i, 0 < 1.2is \le b_s^{(i)} \le 1.6is, \quad i = 1, 2, \cdots, N, s = 1, 2, \cdots, \xi_i, \forall B_i \in B^{(P)}\right\}; \\
A^{(P)} &:= \left\{A_i = \left(a_{gh}^{(i)}\right)_{\xi_i \times \xi_i} : \frac{1}{g+h} + 0.2i \le a_{gh}^{(i)} \le \frac{1}{g+h} + 0.3i, \quad i = 1, 2, \cdots, N, g = 1, 2, \cdots, \xi_i, h = 1, 2, \cdots, \xi_i, \forall A_i \in A^{(P)}\right\}; \\
Z^{(P)} &:= \left\{Z_i = \left(z_{gh}^{(i)}\right)_{\xi_i \times \xi_i} : \frac{1}{g+h} + 0.3i \le z_{gh}^{(i)} \le \frac{1}{g+h} + 0.4i, \quad i = 1, 2, \cdots, N, g = 1, 2, \cdots, \xi_i, h = 1, 2, \cdots, \xi_i, \forall Z_i \in Z^{(P)}\right\}.
\end{aligned} \tag{78}$$

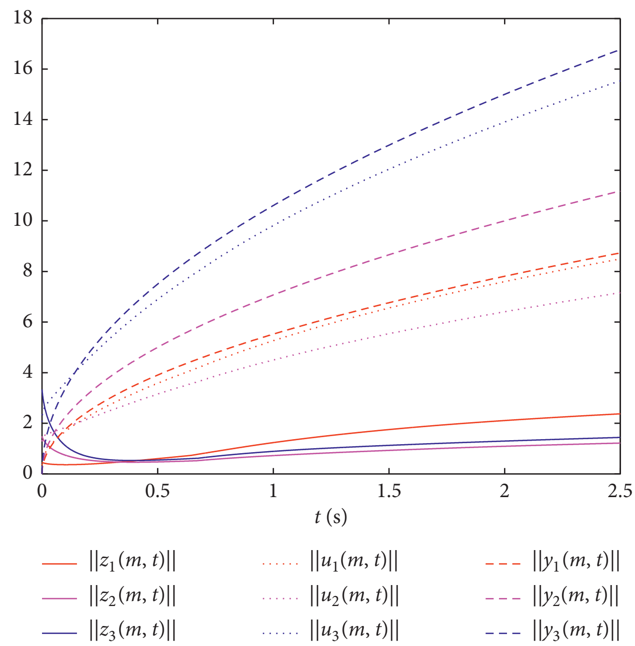

FIGURE 5: Change processes of $z_i(m,t)$, $u_i(m,t)$ and $y_i(m,t)$ in network (76).

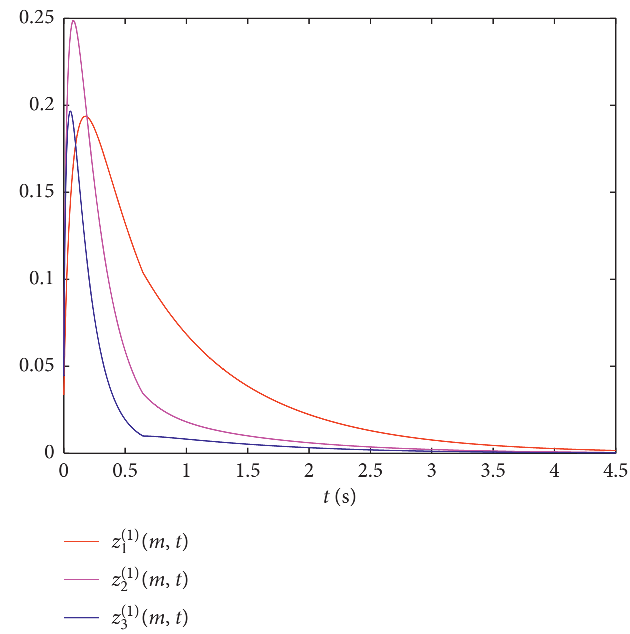

FIGURE 6: Change processes of $z_j^{(1)}(m,t)$ in network (76).

Case 1. Obviously, $\psi_{il} = \widehat{\psi}_{il} = 0.5$ and the equilibrium point of an isolated node of network (76) is $\widehat{O} = (0,0,0,0,0,0,0)^T$.

It is not difficult to calculate the matrices P, Q, and M_1 satisfying (54) as follows:

$$M_1 = \begin{pmatrix} 1.2297 & -0.0937 & -0.1183 & 0.0318 & 0.0413 & 0.0159 & -0.0142 & -0.0103 & -0.0115 \\ -0.0937 & 1.2081 & -0.1195 & 0.0410 & 0.0530 & 0.0217 & -0.0207 & -0.0166 & -0.0171 \\ -0.1183 & -0.1195 & 1.1776 & 0.0622 & 0.0803 & 0.0340 & -0.0330 & -0.0281 & -0.0277 \\ 0.0318 & 0.0410 & 0.0622 & 1.2876 & -0.0192 & 0.0102 & -0.0097 & -0.0074 & -0.0079 \\ 0.0413 & 0.0530 & 0.0803 & -0.0192 & 1.2710 & 0.0095 & -0.0124 & -0.0093 & -0.0101 \\ 0.0159 & 0.0217 & 0.0340 & 0.0102 & 0.0095 & 1.3233 & -0.0062 & -0.0049 & -0.0051 \\ -0.0142 & -0.0207 & -0.0330 & -0.0097 & -0.0124 & -0.0062 & 1.3534 & 0.0455 & 0.0449 \\ -0.0103 & -0.0166 & -0.0281 & -0.0074 & -0.0093 & -0.0049 & 0.0455 & 1.3585 & 0.0431 \\ -0.0115 & -0.0171 & -0.0277 & -0.0079 & -0.0101 & -0.0051 & 0.0449 & 0.0431 & 1.3386 \end{pmatrix},$$
$$Q = \begin{pmatrix} 0.9904 & 0.0197 & 0.4052 & -0.0358 & -0.0044 & 0.0347 & -0.0164 \\ -1.7563 & 1.4252 & 2.8033 & 0.0946 & 0.0507 & -0.2257 & -0.0020 \\ 1.3650 & 0.3042 & -1.7168 & 0.0407 & 0.0357 & 0.0309 & -0.0368 \\ -0.0357 & -0.1531 & -0.0033 & -1.4861 & 3.9352 & 0.0425 & -0.0718 \\ 0.0929 & 0.3681 & 0.0193 & 1.0186 & -2.3689 & -0.1092 & 0.0390 \\ -0.0214 & -0.2121 & 0.0412 & 2.5131 & 0.2759 & 0.0837 & -0.0048 \\ -0.0274 & -0.0498 & 0.0196 & -0.0190 & -0.0630 & 4.6554 & -0.9467 \\ -0.0133 & 0.1213 & 0.0760 & -0.0019 & -0.0240 & 3.5739 & -0.0527 \\ 0.0312 & -0.1258 & -0.1058 & 0.0045 & 0.0832 & -6.8413 & 2.9637 \end{pmatrix}, \tag{79}$$

and $P = \operatorname{diag}(P_1, P_2, P_3)$, where

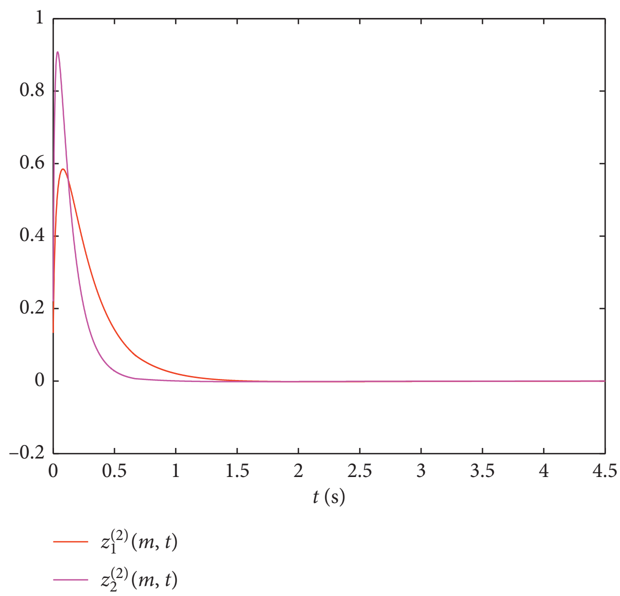

Figure 7: Change processes of $z_j^{(2)}(m,t)$ in network (76).

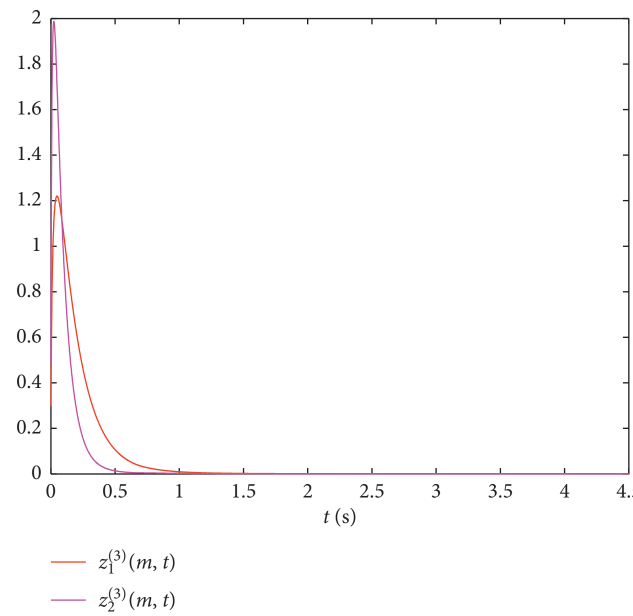

Figure 8: Change processes of $z_j^{(3)}(m,t)$ in network (76).

$$P_1 = \begin{pmatrix} 0.2842 & 0 & 0 \\ 0 & 0.2886 & 0 \\ 0 & 0 & 0.1766 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 0.2473 & 0 \\ 0 & 0.1490 \end{pmatrix}, \quad (80)$$
$$P_3 = \begin{pmatrix} 0.1724 & 0 \\ 0 & 0.0778 \end{pmatrix}.$$

In terms of Theorem 3, network (76) with the input $u(m,t) \in \mathbb{R}^7$ and output $y(m,t) \in \mathbb{R}^9$ as described in (12) under the given parameters defined in (78) realizes output-strictly passivity. Figure 5 displays the simulation results.

Case 2. ψ_{il}, $\widehat{\psi}_{il}$, and $\widehat{O}$ are the same as those in Case 1. By exploiting the MATLAB Toolbox, the matrices P satisfying (68) can be computed as follows:

$$P = \operatorname{diag}(P_1, P_2, P_3), \quad (81)$$

where

$$P_1 = \begin{pmatrix} 0.1876 & 0 & 0 \\ 0 & 0.1007 & 0 \\ 0 & 0 & 0.0677 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 0.1015 & 0 \\ 0 & 0.0512 \end{pmatrix}, \quad (82)$$
$$P_3 = \begin{pmatrix} 0.0672 & 0 \\ 0 & 0.0340 \end{pmatrix}.$$

According to Theorem 4, network (76) under the given ranges of parameters defined in (78) achieves robust synchronization. Figures 6–8 show the simulation results.

Remark 4. Section 3 is devoting to investigating the synchronization and passivity of CDDDRDNNs. First, the network model of CDDDRDNNs is presented in Section 3.1. In Section 3.2, the passivity of CDDDRDNNs with certain parameters is studied. Moreover, we establish some adequate conditions to ensure the network being output-strictly passive in Theorem 1, passive in Corollary 1, and input-strictly passive in Corollary 2, respectively. Then, in Theorem 2 of Section 3.3, a synchronization criterion is obtained for the considered network. Because the precise values of parameters are difficult to acquire because of noises of environment and equipment limitations, we address the problems of robust synchronization and passivity for CDDDRDNNs with parametric uncertainties in Section 4. The main distinction on two kinds of CDDDRDNNs in Section 3 and Section 4 is whether D_i, B_i, A_i, and Z_i in the network model of CDDDRDNNs is uncertain or not. More precisely, in Section 4.1, we give the CDDDRDNN model with parametric uncertainties firstly. After that, a robust output-strict passivity criterion is proposed for CDDDRDNNs with parameter uncertainties in Theorem 3 of Section 4.2. Meanwhile, we establish the related robust passivity condition in Corollary 3 and robust input-strict passivity condition in Corollary 4, respectively. Then, a robust synchronization criterion is obtained for the considered network in Theorem 4 of Section 4.3. In Section 5, we select the most intricate output-strict passivity condition for verifying the validity of the obtained passivity results. Actually, the related passivity or input-strict passivity condition can be illustrated similarly. In order to avoid repetition, we omit the simulation results for showing the effectiveness of the obtained passivity results in corollaries. Therefore, in Case 1 and Case 2 of Example 1, the correctness of the output-strict passivity and synchronization conditions in Theorems 1 and 2 for

CDDDRDNNs (70) with parametric certainties are demonstrated, respectively. Similarly, in Case 1 and Case 2 of Example 2, the robust output-strict passivity criterion and robust synchronization criterion in Theorem 3 and 4 for CDDDRDNNs (76) with uncertain parameters described by (78) are demonstrated, respectively.

6. Conclusion

The synchronization and passivity of CDDDRDNNs with and without parameter uncertainties have been investigated in this paper. First, several new criteria for CDDDRDNNs with parametric certainties have been derived to guarantee the passivity and synchronization by taking advantage of the Lyapunov functional method. Second, we have also studied the problems of robust synchronization and robust passivity for CDDDRDNNs with uncertain parameters. Third, two numerical examples have been provided to display the effectiveness of the obtained passivity and synchronization results. In our future work, it would be very interesting to study the pinning adaptive passivity and synchronization of CDDDRDNNs.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] B. A. Huberman and L. A. Adamic, "Internet: growth dynamics of the world wide web," *Nature*, vol. 401, no. 16, pp. 23–25, 1999.

[2] S. H. Strogatz and I. Stewart, "Coupled oscillators and biological synchronization," *Scientific American*, vol. 269, no. 6, pp. 102–109, 1993.

[3] Z. Zheng and L. Xie, "Finite-time path following control for a stratospheric airship with input saturation and error constraint," *International Journal of Control*, vol. 92, no. 2, pp. 368–393, 2019.

[4] X. Yang, J. Cao, and J. Lu, "Synchronization of randomly coupled neural networks with Markovian jumping and time-delay," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 2, pp. 363–376, 2013.

[5] J. Wang, H. Zhang, Z. Wang, and Q. Shan, "Local synchronization criteria of markovian nonlinearly coupled neural networks with uncertain and partially unknown transition rates," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 1953–1964, 2017.

[6] W. Wu and T. P. Chen, "Global synchronization criteria of linearly coupled neural network systems with time-varying coupling," *IEEE Transactions on Neural Networks*, vol. 19, no. 2, pp. 319–332, 2008.

[7] J. D. Cao, G. R. Chen, and P. Li, "Global synchronization in an array of delayed neural networks with hybrid coupling," *IEEE Transactions on Systems, Man, and Cybernetics: Cybernetics*, vol. 38, no. 2, pp. 488–498, 2008.

[8] Z. Tang, J. H. Park, T. H. Lee, and J. Feng, "Mean square exponential synchronization for impulsive coupled neural networks with time-varying delays and stochastic disturbances," *Complexity*, vol. 21, no. 5, pp. 190–202, 2016.

[9] H. Chen, P. Shi, and C.-C. Lim, "Exponential synchronization for markovian stochastic coupled neural networks of neutral-type via adaptive feedback control," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 7, pp. 1618–1632, 2017.

[10] H. Zhang, D. Gong, B. Chen, and Z. Liu, "Synchronization for coupled neural networks with interval delay: a novel augmented Lyapunov-Krasovskii functional method," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 1, pp. 58–70, 2013.

[11] A. Selivanov and E. Fridman, "Boundary observers for a reaction-diffusion system under time-delayed and sampled-data measurements," *IEEE Transactions on Automatic Control*, vol. 64, no. 8, pp. 3385–3390, 2019.

[12] J.-L. Wang and H.-N. Wu, "Synchronization and adaptive control of an array of linearly coupled reaction-diffusion neural networks with hybrid coupling," *IEEE Transactions on Cybernetics*, vol. 44, no. 8, pp. 1350–1361, 2014.

[13] J. L. Wang, H. N. Wu, and L. Guo, "Novel adaptive strategies for synchronization of linearly coupled neural networks with reaction-diffusion terms," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 2, pp. 429–440, 2014.

[14] J.-L. Wang, H.-N. Wu, T. Huang, and S.-Y. Ren, "Pinning control strategies for synchronization of linearly coupled neural networks with reaction-diffusion terms," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 27, no. 4, pp. 749–761, 2016.

[15] S. Dharani, R. Rakkiyappan, and J. H. Park, "Pinning sampled-data synchronization of coupled inertial neural networks with reaction-diffusion terms and time-varying delays," *Neurocomputing*, vol. 227, pp. 101–107, 2017.

[16] M. Xu, J.-L. Wang, and P.-C. Wei, "Synchronization for coupled reaction-diffusion neural networks with and without multiple time-varying delays via pinning control," *Neurocomputing*, vol. 227, pp. 82–91, 2017.

[17] S.-X. Wang, Y.-L. Huang, and B.-B. Xu, "Pinning synchronization of spatial diffusion coupled reaction-diffusion neural networks with and without multiple time-varying delays," *Neurocomputing*, vol. 227, pp. 92–100, 2017.

[18] Y. Du and R. Xu, "Robust synchronization of an array of neural networks with hybrid coupling and mixed time delays," *ISA Transactions*, vol. 53, no. 4, pp. 1015–1023, 2014.

[19] S. Li, X. Peng, Y. Tang, and Y. Shi, "Finite-time synchronization of time-delayed neural networks with unknown parameters via adaptive control," *Neurocomputing*, vol. 308, pp. 65–74, 2018.

[20] Z. Cai, X. Pan, L. Huang, and J. Huang, "Finite-time robust synchronization for discontinuous neural networks with mixed-delays and uncertain external perturbations," *Neurocomputing*, vol. 275, pp. 2624–2634, 2018.

[21] Y. Li, B. Luo, D. Liu, and Z. Yang, "Robust synchronization of memristive neural networks with strong mismatch characteristics via pinning control," *Neurocomputing*, vol. 289, pp. 144–154, 2018.

[22] X. Ding, J. Cao, A. Alsaedi, F. E. Alsaadi, and T. Hayat, "Robust fixed-time synchronization for uncertain complex-valued neural networks with discontinuous activation functions," *Neural Networks*, vol. 90, pp. 42–55, 2017.

[23] J. L. Liang, Z. D. Wang, Y. R. Liu, and X. H. Liu, "Robust synchronization of an array of coupled stochastic discrete-time delayed neural networks," *IEEE Transactions on Neural Networks*, vol. 19, no. 11, pp. 1910–1921, 2008.

[24] S. Yang, Z. Guo, and J. Wang, "Robust synchronization of multiple memristive neural networks with uncertain parameters via nonlinear coupling," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 7, pp. 1077–1086, 2015.

[25] P. Li, J. Cao, and Z. Wang, "Robust impulsive synchronization of coupled delayed neural networks with uncertainties," *Physica A: Statistical Mechanics and its Applications*, vol. 373, pp. 261–272, 2007.

[26] J.-L. Wang, M. Xu, H.-N. Wu, and T. Huang, "Finite-time passivity of coupled neural networks with multiple weights," *IEEE Transactions on Network Science and Engineering*, vol. 5, no. 3, pp. 184–197, 2018.

[27] N. Li and J. Cao, "Passivity and robust synchronisation of switched interval coupled neural networks with time delay," *International Journal of Systems Science*, vol. 47, no. 12, pp. 2827–2836, 2016.

[28] S.-Y. Ren, J. Wu, and P.-C. Wei, "Passivity and pinning passivity of coupled delayed reaction-diffusion neural networks with dirichlet boundary conditions," *Neural Processing Letters*, vol. 45, no. 3, pp. 869–885, 2016.

[29] B.-B. Xu, Y.-L. Huang, J.-L. Wang, P.-C. Wei, and S.-Y. Ren, "Passivity of linearly coupled neural networks with reaction-diffusion terms and switching topology," *Journal of the Franklin Institute*, vol. 353, no. 8, pp. 1882–1898, 2016.

[30] Y.-L. Huang, S.-Y. Ren, J. Wu, and B.-B. Xu, "Passivity and synchronization of switched coupled reaction-diffusion neural networks with non-delayed and delayed couplings," *International Journal of Computer Mathematics*, vol. 96, no. 9, pp. 1702–1722, 2019.

[31] Y.-L. Huang, B.-B. Xu, and S.-Y. Ren, "Analysis and pinning control for passivity of coupled reaction-diffusion neural networks with nonlinear coupling," *Neurocomputing*, vol. 272, pp. 334–342, 2018.

[32] Y.-L. Huang, S.-X. Wang, and S.-Y. Ren, "Pinning exponential synchronisation and passivity of coupled delayed reaction-diffusion neural networks with and without parametric uncertainties," *International Journal of Control*, vol. 92, no. 5, pp. 1167–1182, 2019.

[33] Y.-L. Huang, S.-H. Qiu, and S.-Y. Ren, "Finite-time synchronisation and passivity of coupled memristive neural networks," *International Journal of Control*, pp. 1–14, 2019.

[34] J.-L. Wang, H.-N. Wu, T. Huang, S.-Y. Ren, and J. Wu, "Passivity analysis of coupled reaction-diffusion neural networks with Dirichlet boundary conditions," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 2148–2159, 2017.

[35] J.-L. Wang, H.-N. Wu, T. Huang, S.-Y. Ren, and J. Wu, "Passivity of directed and undirected complex dynamical networks with adaptive coupling weights," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 8, pp. 1827–1839, 2017.

[36] S.-Y. Ren, J. Wu, S.-X. Wang, and Y.-L. Huang, "Passivity and pinning control of coupled neural networks with and without time-varying delay," *Transactions of the Institute of Measurement and Control*, vol. 40, no. 9, pp. 2708–2717, 2017.

[37] C. J. Xu, Y. Zheng, H. S. Su, and X. G. Liu, "Adaptive synchronization for nonlinear coupled complex network with nonidentical nodes," in *Proceedings of the 33rd Chinese Control Conference*, pp. 5750–5754, Nanjing, China, July 2014.

[38] S. Wang, H. Yao, S. Zheng, and Y. Xie, "A novel criterion for cluster synchronization of complex dynamical networks with coupling time-varying delays," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 7, pp. 2997–3004, 2012.

[39] J. Zhao, D. J. Hill, and T. Liu, "Synchronization of dynamical networks with nonidentical nodes: criteria and control," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 58, no. 3, pp. 584–594, 2011.

[40] X. Yang, Z. Wu, and J. Cao, "Finite-time synchronization of complex networks with nonidentical discontinuous nodes," *Nonlinear Dynamics*, vol. 73, no. 4, pp. 2313–2327, 2013.

[41] S. Lin, Y. Huang, and S. Ren, "Event-triggered passivity and synchronization of delayed multiple-weighted coupled reaction-diffusion neural networks with non-identical nodes," *Neural Networks*, vol. 121, pp. 259–275, 2020.

[42] M. Tan and W. Tian, "Finite-time stabilization and synchronization of complex dynamical networks with nonidentical nodes of different dimensions," *Nonlinear Dynamics*, vol. 79, no. 1, pp. 731–741, 2015.

[43] Y. Wang, Y. Fan, Q. Wang, and Y. Zhang, "Stabilization and synchronization of complex dynamical networks with different dynamics of nodes via decentralized controllers," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 59, no. 8, pp. 1786–1795, 2012.

[44] L. Zhang, Y. Wang, and Y. Huang, "Synchronization for non-dissipatively coupled time-varying complex dynamical networks with delayed coupling nodes," *Nonlinear Dynamics*, vol. 82, no. 3, pp. 1581–1593, 2015.

[45] L. Zhang, Y. Wang, Y. Huang, and X. Chen, "Delay-dependent synchronization for non-diffusively coupled time-varying complex dynamical networks," *Applied Mathematics and Computation*, vol. 259, pp. 510–522, 2015.

[46] M. Tan, "Stabilization of coupled time-delay neural networks with nodes of different dimensions," *Neural Processing Letters*, vol. 43, no. 1, pp. 255–268, 2016.

[47] Y. Huang, W. Chen, S. Ren, and Z. Zheng, "Analysis and pinning control for generalized synchronization of delayed coupled neural networks with different dimensional nodes," *Journal of the Franklin Institute*, vol. 355, no. 13, pp. 5968–5997, 2018.

[48] S. Lin, Y. Huang, and S. Ren, "Analysis and pinning control for passivity of coupled different dimensional neural networks," *Neurocomputing*, vol. 321, pp. 187–200, 2018.

[49] J.-L. Wang, Z. Qin, H.-N. Wu, and T. Huang, "Passivity and synchronization of coupled uncertain reaction-diffusion neural networks with multiple time delays," *IEEE*

Transactions on Neural Networks and Learning Systems, vol. 30, no. 8, pp. 2434–2448, 2019.

[50] J. G. Lu, “Global exponential stability and periodicity of reaction-diffusion delayed recurrent neural networks with Dirichlet boundary conditions,” *Chaos, Solitons & Fractals*, vol. 35, no. 1, pp. 116–125, 2008.