ESSAYS ON ANTICOMPETITIVE EFFECTS OF VERTICAL CONTRACTS

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Abstract

The first half of this thesis focuses on the effect of conditional rebates by an incumbent when the firm's product is a must stock for the buyer. It is inspired by the Intel case. The first essay analyses the baseline model and shows that although the incumbent can exclude the entrant by conditional rebate, the incumbent does not use the rebate to exclude the entrant, because just selling the must stock units with a high price is more profitable. Moreover, the model shows the limitation of the model in the sense that the incumbent has an ability to set incredibly high price to capture all the surplus of the market.

Developing the main insight of the baseline model in the first essay, the second essay analyses four extended models to analyse the anticompetitive effects of conditional rebate and other pricing behaviours. The first extended model considers the case where the incumbent's good is not literally 'must stock', but just has superiority over the competitors' products. The results suggest that we can overestimate the exclusionary effect of conditional rebates when we erroneously assume a quite strong must stock property. The second, third and fourth extended models show that the incumbent can have an incentive to exclude a more efficient competitor in the following situations; 1) there is uncertainty about the must stock proportion, 2) there is uncertainty about demand size or 3) the must stock property is vulnerable in the sense that it becomes weaker if entry happens.

The second half of this thesis develops a framework to analyse the competition through platforms that help consumers to get awareness of the sellers. It is motivated by the hotel booking platform cases. An informative advertising model is adapted to model platform investment. The third essay analyses the case where there are a monopoly platform and a monopoly seller and the effect of a narrow price parity clause on competition. The result shows that the introduction of the price parity clause increases the platform's investment, which has a positive effect on the consumer surplus through giving more opportunity for consumers to become aware of the product. On the other hand, it also has a negative effect on consumer surplus by raising the direct sales price. In some situations, the net effect on consumer welfare can be negative.

The fourth essay considers the case where there are duopoly platforms and duopoly sellers and analyses the effect of wide price parity clauses on competition. The results show that wide price parity clauses have a more straightforward effect on competition in the sense that the clauses make it possible for the platforms to avoid competition through undercutting commissions and so set their commissions at the collusive level. The analysis in this chapter shows that without wide price parity clauses the platforms try to undercut each other's commissions, but the clauses eliminate the possibility of undercutting by the other platform. The result also shows that the introduction of the price parity clause increases the platform's investment.

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Introduction

Most countries adopt competition laws to stimulate their economies and increase welfare by protecting effective competition and prohibiting conduct that may harm competition. Competition authorities that enforce such laws have regulated cartels and bid riggings, along with vertical restraints of dominant firms to exclude competitors. For example, predatory pricing, exclusive dealing, and conditional rebates are typical examples of unilateral conduct.

While the anticompetitive effects of horizontal agreement such as cartels and bid riggings are relatively clear and, hence, deemed per se illegal in many jurisdictions, vertical contract is difficult to differentiate from legal and pro-competitive conduct. This is because these contracts used widely and many of those contracts are considered to be pro-competitive.

In light of the increasing importance of considering a variety of factors and effects on competition to evaluate whether a vertical contract should be prohibited by competition laws, it is extremely important to clarify in which situation such conduct is likely to harm competition, and to set the criteria to be used to judge whether such conduct violates competition law, in order to minimise both the chilling effects on enterprises' pro-competitive actions by over-enforcement and the adverse effect on competition by under-enforcement. To establish such criteria, economic theory can be fairly informative. Therefore, the aim of this thesis is to examine the anti-competitive and pro-competitive effects of vertical contract.

Based on this motivation, this thesis adopts a microeconomic theoretical approach, particularly using industrial organisation and game theory. Specifically, this thesis will set up oligopoly models where a dominant firm is allowed to adopt a vertical contract and analyse the effects on competition such as market foreclosure effect. This thesis will also examine whether competition laws should prohibit such conduct by a dominant firm and, if so, what criteria should be introduced to judge the illegality of such activities.

This thesis consists of five chapters. The first three chapters focus on one type of vertical contract, conditional rebate inspired by the Intel case handled by the European Commission. The last two chapters are concerned about the behaviours of consumers, sellers and platformers in platform industry (especially, online hotel booking) and the effects on competition and consumer welfare of another type of vertical contract, price parity clauses or most-favoured-nation clauses, inspired by the hotel booking platform cases handled by national competition authorities in Europe.

The first three chapter cover conditional rebates, which are behaviours such that a firm offers a certain reward to its customers when the customers meet a certain condition, such as transaction volume and the share of the purchasing volume from the firm on total purchasing volume proposed by the firm.

Chapter 1 is the introduction of the three chapters. It introduces the background of the regulatory issues about conditional rebates, especially the Intel case, which inspired the first half of this thesis. This chapter also covers the basic mechanism of harm on competition by conditional rebates and the relevant researches so far.

Chapter 2 analyses a baseline model about conditional rebate under simple assumptions. This chapter considers a situation where two suppliers, an incumbent firm and an entrant, compete with each other to sell a good to one representative buyer. This model is close to Aghion and Bolton (1987), which analyse exclusive dealing and find that this can exclude the more efficient competitor if there is uncertainty about the entrant's cost. In Chapter 2, we consider the case where there is no uncertainty. This model also assumes that the competition between the incumbent and the entrant is Bertrand competition. Like Ide et. al. (2016) and Chone and Linnemer (2016), it is assumed that the incumbent has a competitive advantage such that the incumbent's good is a "must-stock" item. Our model differs from their models in the sense that we do not restrain the price of the incumbent firm and we do not assume a Nash bargaining process between the incumbent firm and the buyer.

The results of analysis show that the optimal behaviour for the incumbent is one of two types. The first pricing behaviour is setting the maximum price such that the incumbent can exclude the entrant. In the linear pricing case, the optimal price is to set the price just below the entrant's cost (competitive linear pricing). This type of pricing is preferred by the incumbent when the entrant's cost is higher than a certain point. The incumbent's profit from this pricing is equal to the difference between the entrant's cost and the incumbent's cost. On the other hand, in the two-part tariff case and the conditional rebate case, the incumbent can exclude the entrant regardless of the entrant's cost through setting a high fixed price in the two-part tariff case (exclusive two-part tariff) and setting the pre-discount price very high in the conditional rebate case (exclusive conditional rebate). This gives the incumbent a monopoly profit without constrained by the entrant.

The second type of the optimal behaviour for the incumbent is giving up competing with the entrant and exploiting the profit from the "must stock" part of the market by setting the highest possible price to let the entrant enter the market. Then, the incumbent can extract the full of the buyer's surplus (exploitative pricing). The incumbent prefers such pricing when the entrant's cost is lower than a certain point. The threshold point is equal to the incumbent's cost in the two-part tariff case and the conditional rebate case. The threshold in the linear pricing case is higher than the incumbent's cost. The lower the entrant's cost, the higher the profit the incumbent obtains, because the lower entrant's cost means the larger room that the incumbent can exploit.

The results of the analysis also show that although an incumbent can exclude an entrant by exclusive two-part tariff or conditional rebate, the incumbent prefers the exploitative pricing behaviour over exclusive two-part tariff or conditional rebate when the entrant is more efficient. This is because exploitative pricing behaviour is more profitable. The result is that an incumbent will not use exclusive two-part tariff or conditional rebate to foreclose the more efficient entrant from the market because the incumbent does not have an incentive to do so. This result is similar to the view of the Chicago School that there is no foreclosure of a more efficient entrant through exclusive dealing, but it is importantly different from the Chicago School's view in that consumers can be made worse off.

While illustrating the important mechanism, this simple model predicts that the incumbent may set incredibly high price to capture all the surplus of the market. Chapter 3 analyses four extended models with more realistic assumptions.

The first extended model considers the case where the incumbent's good is not literally 'must stock', but merely be the option strongly preferred the competitors' products is analysed. The result shows that the profitability of exploitative pricing and exclusive conditional rebate / two-part tariff is constrained by the possibility that the incumbent might be excluded by the entrant. This predicts that the incumbent's price can be not so high. Because this possibility both constrains the profitability of exploitative pricing and exclusive conditional rebate / two-part tariff, the result in the baseline model that the incumbent does not introduce exclusive conditional rebate / two-part tariff to exclude the entrant who is more efficient than the incumbent still holds in this model.

The second and third extended models in Chapter 3 introduces uncertainty. The second model assumes uncertainty about must-stock proportion and the third model assumes uncertainty about the size of the contestable demand. Both models show that the incumbent can have an incentive to exclude the entrant by conditional rebate. This is because the exclusive conditional rebate can adapt to such uncertainty better than exploitative pricing. The third model also shows that a market-share-based rebate is better for the incumbent than a quantity-based rebate and two-part tariff.

The fourth extended model about conditional rebate analyses the case where the game is played more than once and the must-stock property of the incumbent's good is eroded when the buyer becomes familiar with the entrant's product. The analysis shows that exclusion of the more efficient entrant can happen and, hence, maintaining the must stock property of its product can be the motivation to introduce exclusive conditional rebate for the incumbent.

Chapter 4 and Chapter 5 covers the competition through platform and the effect of price parity clauses on such competition, inspired by the hotel booking platform cases in Europe. In recent years, online platformer grows rapidly. Such companies offer a variety of services such as e-commerce, travel booking and brings huge benefits to consumers and small and small/medium sized enterprises by making the transactions between them easier. On the other hand, it is pointed out that most favoured nation

(MFN) clauses or platform parity clauses, which prohibit companies using platforms from selling at more preferable condition (e.g. price) on the other platforms or their own website, can have negative impacts on competition.

Platform parity clauses or MFNs are divided into two types depending on the scope they cover. If the restriction applies to other platforms, such clauses are called "wide" MFNs or parity clauses. On the other hand, if the restriction applies only to direct sales from the party and not to other platforms, such clauses are called "narrow" MFNs or parity clauses. There is no consensus across competition authorities with respect to the regulatory approach toward narrow and wide price parity clauses. For example, while French, Italy and Swedish authorities concluded that narrow-platform parity clauses can be justified in the hotel booking platform cases, the German Federal Cartel Office published the decision that not only wide-platform parity clauses but also narrow-platform parity clauses should be prohibited.

From those backgrounds, Chapter 4 and Chapter 5 analyse economic models considering this aspect by adapting the informative advertising model introduced in Grossman and Shapiro (1984), which is the linear Hotelling model assuming that only a part of consumers know about the existence of products. This is because this type of model can take into account the important motivation for enterprises to use such platformers, which is consumer awareness. This model allow us to consider a platform's incentive to make investment for increasing consumer awareness.

Chapter 4 covers the case where there are one seller and one platform and the seller can sell its product to the consumers directly. This chapter analyses the effect of narrow price parity clause. The result shows that the introduction of the price parity clause increases the platform's investment, which has a positive effect on the consumer surplus through giving more opportunities to be aware of the product. On the other hand, it also has the negative effect on the consumer surplus by the rise of the direct sales price and in some situations the net effect on the consumer welfare can be negative. The introduction of price parity clauses is likely to have negative effect on consumer welfare when the number of active consumers is small or the seller is less known by the consumers.

Chapter 5 covers the case where there are two sellers and two platforms but the seller cannot sell directly. This chapter analyses the effect of wide price parity clause on competition. The result shows that wide price parity clauses enables platforms to set higher commissions compared to when the clauses are not allowed. Consequently, this causes the increase of sellers' prices and the increase of the investment by platforms. While the former has a negative impact on consumer welfare, the latter has a positive impact on consumer welfare.

However, the improvement of consumer welfare through the introduction of wide price parity clauses is unlikely to occur when there is a fierce competition between the two sellers. Moreover, the wide price parity clauses have a more straightforward effect on competition in the sense that the clauses make it possible for the platforms to avoid the competition through undercutting commissions and to set their commissions to the level where they can set where they collude with each other.

On the other hand, the results in Chapter 4 show that the possible price rise by the narrow parity clause is limited to the direct sales price. In addition, the motivation of introducing a narrow price parity clause, preventing a free riding is more plausible reason than that of wide price parity clauses, avoiding the competition through undercutting commissions. Those results imply that wide price parity clauses could be more strictly regulated than narrow price parity clauses.

Chapter 1: Introduction about Conditional Rebate and its Issues from the Viewpoints of Competition Policy and Economics

1.1. Motivation

Most countries adopt competition laws to stimulate the economy and increase welfare by protecting effective competition and prohibiting conduct that may harm competition. Competition authorities enforcing such laws have regulated cartels and bid riggings, along with unilateral conduct (or vertical restraints) of dominant firms to exclude competitors. Such conduct can include predatory pricing, exclusive dealing, and conditional rebates. Such exclusionary conduct can bring about negative effects such as raising prices above the competitive level, decreasing the efficiency of the economy, and diminishing consumer welfare.

Although the anticompetitive effects of cartels and bid riggings are relatively clear and although such conduct is in many jurisdictions deemed to be *per se* illegal, unilateral conduct has a feature that makes it difficult to differentiate from legal and pro-competitive conduct. For example, selling goods at discount prices is supposed to be desirable from the perspective of competition policy in general. However, discounting by a dominant firm to exclude its competitors or to enforce predatory pricing can violate competition law. Reflecting this difficulty, most jurisdictions judge the legality of such conduct by considering various related factors, including its pro-competitive effects, such as efficiency. For example, American courts have assessed unilateral conduct based on the rule of reason, an approach that decides whether an action is unlawful by weighing its pro-competitive effects against its anticompetitive effects. Moreover, the European Commission adopts an "effects-based approach" against exclusionary conduct, which focuses on the effects of the alleged conduct on consumers along with efficiency gains from the conduct.

In this way, recognising the difficulty of distinguishing between exclusionary conduct and pro-competitive conduct, competition authorities and courts have taken a cautious attitude against exclusionary conduct. In addition, the business industry has required clarification of the criteria for the illegality of such conduct because the ambiguous border of the illegality would cause a chilling effect and prevent pro-competitive action. On the other hand, in more than a few cases, massive multinational corporations have excluded their competitors and negatively affected their competition by abusing their dominant positions. For example, certain competition authorities, including the European Commission, the Federal Trade Commission, and Japan Fair Trade

Commission, have investigated a large semiconductor company, Intel, which was allegedly preventing its rivals from selling their central processing units (CPUs) to their customers by abusing its dominant position. In one instance, it introduced rebates to customers on the condition that almost all of the CPUs incorporated into their computers must be those made by Intel. The European Commission ultimately imposed a large monetary penalty on Intel in 2009.

In light of these situations, it is extremely important to set the criteria for unilateral conduct such that the constraint on pro-competitive actions by over-enforcement and the adverse effect on competition by under-enforcement are minimised in a sufficiently clear manner to prevent any chilling effects. In establishing such criteria, economic theory can be fairly informative. Therefore, this the following chapters examine the anticompetitive and pro-competitive effects of exclusionary rebates and the appropriate criteria to regulate such conduct.

1.2. Characteristics of Conditional Rebates

1.2.1 General Definition

Conditional rebates or loyalty rebates refers to discounts such that customers can obtain when the customers meet certain conditions. The conditions of such rebates vary by case, and normally the conditions regard purchase volume of customers or the share of the firm providing such rebates. For example, in the case of Intel, the company's rebates to personal computer manufacturers were conditional upon purchasing 95% or 100% of CPUs used in their personal computers, a condition deemed by the European Commission to be an abuse of a dominant market position.

According to International Competition Network (2009), conditional or loyalty rebates can also be classified by a method to calculate the amount of discounts. If the discounts are given only to the volume exceeding the threshold in a certain period, such rebates are called "incremental rebates". If the amount depends on the total purchased volume in a specific period, such rebates are categorised as "retroactive rebates". For instance, assume a company offers a 10% discount rebate whose threshold is £1,000. A customer purchases £1,200. If the rebate is incremental, the customer obtains a £20 refund: (£1,200 – £1,000) × 10% = £20. On the other hand, if the rebate is retroactive, the value is £120: £1,200 × 10% = £120. Hence, if the same discount rate is applied, retroactive rebates provide larger payments than do incremental payments because the former takes into account threshold trade volume.

Conditional rebates are categorised as one type of non-linear pricing. The revenue functions of retroactive rebates and incremental rebates are exemplified in Figures 1.1 and 1.2, respectively. Another type of non-linear pricing is a two-part tariff. When a customer wants to trade with a company offering a two-part tariff, the customer has to pay a certain amount in fixed costs. Aside from the fixed cost, the customer is also required to pay variable costs depending on the quantity purchased. The revenue function is exemplified in Figure 1.3. As Figures 1.1 and 1.2 demonstrate, while the revenue function of a two-part tariff is smooth and continuous, the revenue functions of conditional rebates can be discontinuous or kinked. In addition, a three-part type is also categorised as non-linear pricing. Like a two-part tariff, a three-part tariff consists of variable costs and fixed costs. However, when a customer is offered a three-part tariff, certain allowances are granted to the customer. In other words, as long as the purchasing volume is below a certain threshold, the costs that the customer has to pay include the fixed costs only. The revenue function of a three-part tariff is exemplified in Figure 1.4, which shows that whilst the revenue functions of a three-part tariff are kinked and continuous, the first segment of the revenue functions is flat, as is the case for incremental rebates.

Figure 1.1 Revenue function of retroactive rebates

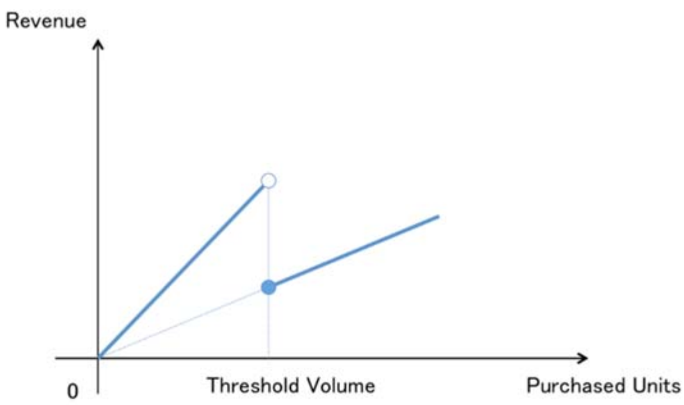


Figure 1.2 Revenue function of incremental rebates

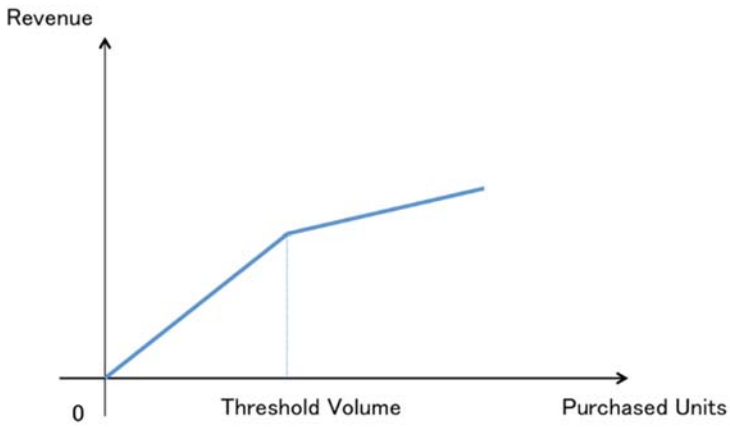


Figure 1.3 Two-part tariff

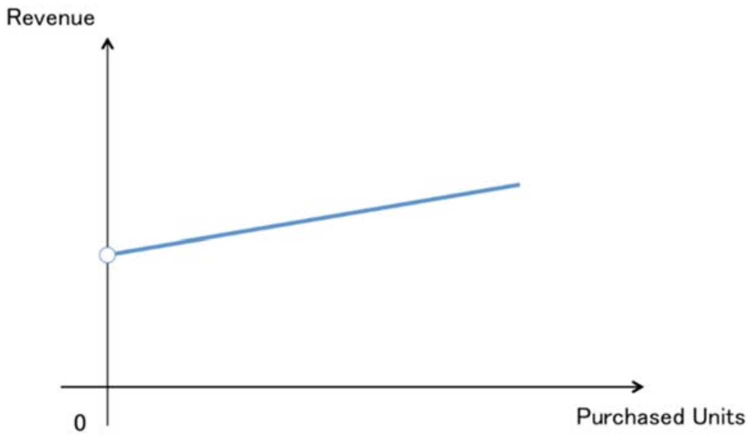
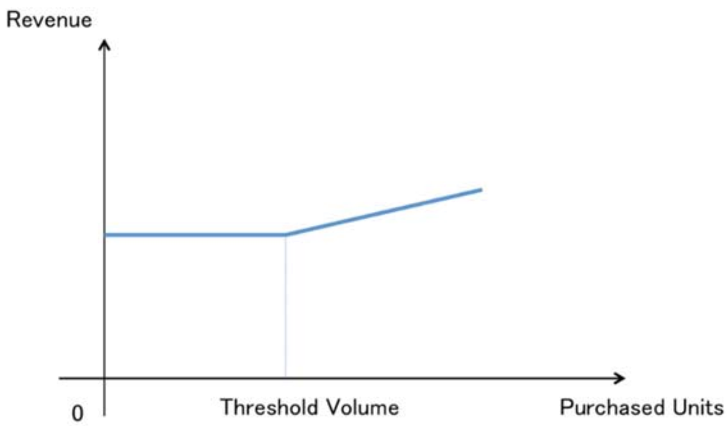


Figure 1.4 Three-part tariff



As is the case for other types of exclusionary conduct, conditional rebates are deemed to be normal business activities and beneficial for consumers in many situations because they decrease prices and increase the quantity of production. On the other hand, when a company with dominant power introduces such rebates, they may harm competition, as discussed in the following chapters of this thesis. For example, Maier-Rigaud (2006) and Bishop and Walker (2010) discuss that when buyers must purchase a certain amount of goods from a company at the same time as the company and its competitors compete for the remaining demand, the company can exclude its competitors (including those who are equally efficient or more efficient than the company) by introducing rebate schemes. They explain that such exclusion occurs because when a company introduces rebate programmes, the price for the remaining demand can be negative or smaller than the marginal cost of the company itself. The price for the remaining demands called an “effective price”. If the effective price is negative or smaller than the company’s marginal cost, its equal or more efficient competitor cannot compete with the company without incurring a loss. Maier-Rigaud (2006) calls this effect as “suction effect”.

As Bishop and Walker (2010) point out, such exclusionary effects are similar to those of another type of exclusionary unilateral conduct, “tying”. Tying is conduct such that a company offers one product or service on the condition that a buyer must purchase another only from the company. A dominant company’s use of tying is also regarded as a measure that may exclude its competitors by leveraging the dominance of the company in one market to influence another competitive market.

1.2.2 Regulatory Approach towards Conditional Rebates

As explained earlier, competition authorities and courts in general take the rule of reason approach against exclusionary unilateral conduct. The other options are *per se* legal and *per se* illegal approach. Compered to *per se* approaches, it is certain that a Rule of Reason approach is difficult to carry out in real cases. This is because many factors have to be considered to examine whether the alleged conduct is illegal. However, Motta (2009) points out that taking into account that unilateral conduct has both pro-competitive and anticompetitive effects, the only measure consistent with economic analysis is supposed to be the rule of reason approach.

Because the rule of reason remains ambiguous, however, certain economic tests have been used to judge whether exclusionary conduct is illegal. From the viewpoint of economic theory, whether alleged conduct reduces consumer (or total) welfare or not is a desirable benchmark. However, such a welfare test is extremely difficult to implement in actual cases due to the difficulty of measuring welfare. For this reason, other standards have been used in practice. As Vickers (2008) explains, the other examples of economic tests are the profit-sacrifice test and the as-efficient competitor test. The profit sacrifice test is also known as the “no-economic-sense test” or the “but-for test”. This type of test assesses whether a company suffers a loss because of its alleged conduct. The idea behind this test is that a rational firm does not carry out such behaviour unless it gains profit by excluding its competitors. The below-cost test, adopted for investigating predatory pricing in many jurisdictions, is a typical example of this kind of test.

The as-efficient competitor test evaluates whether such conduct makes it difficult for competitors, who are as efficient as the firm engaging in such conduct, to compete effectively with the firm. The logic of this test is as follows: the principle of competition law is not to protect “competitors” but to protect “competition”. Hence, on this view, a company that excludes its rivals does not necessarily need to be sanctioned. However, if a company’s behaviour excludes a competitor who is as efficient as the company with the exclusionary behaviour, such behaviour should still be prohibited, according to this test, because such behaviours distort competition.

Some competition authorities publish guidance papers to clarify their enforcement policy against exclusionary conduct. Conditional rebates are often supposed to share similarities with exclusive dealing, a type of unilateral conduct that makes it difficult for other companies to trade with their customers. Hence, their illegality should be judged by similar standards. In the guidance on enforcement policy against exclusionary conduct, the European Commission and the Japan Fair Trade Commission mention that they treat exclusive dealing similarly to how they treat conditional rebates. In addition, the European Commission mentions in its guidance paper that it will examine whether loyalty rebates by a dominant firm would foreclose a competitor as efficient as the company introducing the rebates.

While the rule of reason approach has gained popularity, especially with economists, the case laws in Europe and the United States seem to take different approaches. Until recently, the European courts and the European Commission were known by their hostile attitudes towards conditional rebates. Such attitudes have been often criticised as too formalistic. One of the representative cases is the Michelin II case, in which the European Commission classed Michelin's rebate programme, which included quantity rebates with multiple thresholds, as an abuse of a dominant position in 2001.[1] The European Court of First Instance upheld the Commission's Decision in 2003. Motta (2009) evaluates that this judgement substantially considers that conditional rebates by a dominant company are *per se* illegal. Motta (2009) then concludes that the European Commission and the Court of First Instance should have considered the pro-competitive effects of such rebates and shown how Michelin's rebate program had market foreclosure effects.

Then, the European Commission found that Intel's market share rebate fell under an abuse of dominant position in 2009.[2] This case was the first case of conditional rebates after the Commission clarified that it would adopt an effect-based approach on abuse of dominance cases. In the decision, the Commission focuses on the fact that Intel's CPUs were "must-stock" items for the computer manufacturers and the effective price of Intel's CPU was below its cost. The Commission further shows that the rebate programme "enables Intel to use the inelastic or "non-contestable" share of demand of each customer, …, as leverage to decrease the price for the elastic or 'contestable' share of demand" and can prevent as efficient competitors from expanding or entering the market.

In the Intel case, the European Commission carried out an "as-efficient-competitor (AEC) test" to quantitatively evaluate the effect of conditional rebates on competition in practice. The AEC test compares the actual contestable share with the share that competitors who are as efficient as the dominant firm need to compete with a dominant firm offering a rebate scheme based on the following formula:

[1] See the European Commission's decision against Michelin in COMP/E-2/36.041/PO (2001).
[2] See the European Commission's decision against Intel in COMP/C-3/37.990 (2009).

$$s \equiv \frac{R}{(ASP-AAC)/V},$$ [3]

where s denotes the required share.

If s is smaller than the required share, the rebate scheme is regarded as exclusive, because even the as-efficient-competitor cannot compete with the dominant company by setting its price equal to its cost given this rebate scheme. The amount of rebate given to the customer per unit is represented by R, while ASP is a sales price, AAC is the average avoidable cost of the dominant firm, and V denotes the number of the units that the customer requires to obtain the discount. The test is based on the logic that when a dominant firm's product is must-stock for the buyers and that firm introduces a rebate scheme, the effective price of the dominant firm's product for the contestable demand is lower than the dominant firm's nominal price. When the effective price is lower than the dominant firm's unit cost, the competitor who is as efficient as the dominant firm will be excluded from the market even if that competitor sets its price as low as possible (i.e., the dominant firm's cost).

The buyers' internal documents, in particular the quantities that the buyers think they must-stock are often used as a proxy for the share of the must-stock demand.[4] When such documents are not available, the share of Intel is sometimes alternatively used as the proxy.[5]

In 2014, the General Court upheld the European Commission's decision.[6] In its judgment, the court mentioned that the analysis of the anticompetitive effect including the AEC test is not necessary to establish the illegality of such rebate, because we should regard as illegal all exclusive rebates such that discounts are given on condition that customers purchase all or most of their requirements from a dominant company.

However, in 2017, the European Court of Justice annulled the General Court's judgement and remanded the case to the court on the ground that the court had failed to evaluate the Intel's argument to challenge the application of the AEC test by the European Commission.[7] Hence, this case is ongoing as of June 2019.

The application of the AEC test is not limited to the case of Intel. The test has also been used in other European cases. For example, in a decision concerning Google, the European Commission used this test. In this case, Google provides financial incentives to the device manufacturers "on condition that they exclusively pre-installed Google Search across their entire portfolio of Android devices". The European Commission

[3] para 1196 in COMP/C-3/37.990.
[4] For example, paras 1202-1213 in COMP/C-3/37.990.
[5] paras 1551-1558 in COMP/C-3/37.990.
[6] See the General Court's judgement in Case T□286/09 (2014).
[7] See the European Court of Justice's judgement in Case C□413/14P (2017).

proved that a rival search engine could not compensate device manufacturers for the loss of the payment from Google.[8]

In contrast to Europe's case law, in US case law, as Kobayashi (2005) introduces, the US courts charge a high burden to establish that loyalty discounts are anticompetitive on plaintiffs. Specifically, in the case of Barry Right Corporation about volume discounts for a single product, the First Circuit Court of Appeals clarified the negative attitude towards the illegality of volume discounts where the price is above cost in 1983. Kobayashi (2005) points out that the Court's approach is consistent with the approach presented in the Supreme Court's judgement in the 1993 Brooke Group case, famous for stating a regulatory standard of predatory pricing. In the case, the court held that to establish the illegality of predatory pricing, a claim must satisfy two conditions: (1) "that the prices complained of are below an appropriate measure of its rival's costs"; and (2) "a dangerous probability, of recouping its investment in below-cost prices". With regard to rebate cases related to multi-markets, it seems that there is room to establish illegality when the prices are above cost. In the case of Concord Boat Corporation about market share discounts, the Eighth Circuit Court of Appeals held in 1983 that "[b]ecause only one product, stern drive engines, is at issue here and there are no allegations of tying or bundling with another product, we do not find these cases persuasive".

Moreover, conditional rebates in the market in some cases entail that one party's product is must-stock in the market. For example, in the pulse oximetry product case (Masimo v. Tyco and Allied Orthopedic v. Tyco), the illegality of the rebate scheme by Tyco was debated in court.[9] The pulse oximetry products consist of durable goods, monitors, and consumable goods, sensors. The defendant, Tyco, produce both monitors and sensors. The plaintiffs, who are competitors in the sensor market, claimed that Tyco's product is must-stock in the sensor market, because some customers (i.e., hospitals) need to purchase sensors compatible with Tyco's monitors, and competitors cannot produce such sensors until Tyco's patents expired. However, the US courts have dealt with rebate cases as one type of discount and have not seriously treated the argument about the suction effect based on must-stock property.

Although the US case laws treat conditional rebates as tying or predatory pricing, some academics in the US claim that a specific approach is needed to evaluate illegality of conditional rebates like in the EU. Scott Morton and Abrahamson (2016) suggest that loyalty discounts should be judged based on a metric called "effective entrant burden (EEB)", which is the product of the threshold of a rebate and its discount divided by a contestable share. This metric summarises a financial penalty on buyers that purchase

[8] See the European Commission's press release on 18 July 2018, http://europa.eu/rapid/press-release_IP-18-4581_en.htm.
[9] For example, see Masimo v. Tyco Health Care Group, CV 02-4770 MRP (C.D. Cal. Mar. 22, 2006) and Allied Orthopedic v. Tyco Health Care Group, 592 F.3d 991 (9th Cir. 2010).

from competitors under a rebate scheme. The higher EEB means that the rebate places the higher financial burden on the competitors to compensate the loss of the discount to the buyers. As they recognise, this approach is consistent with the AEC test used in the EU. On the other hand, they emphasise that it is better to compare EEB with long run average incremental cost (LRAIC) rather than marginal cost or average cost, because fixed costs such as R&D costs to enter the market is important to evaluate the anticompetitive effect. They also point out that this approach is especially suitable to evaluate product-line cases in which competitor do not sell all types of the products such as the Tyco pulse oximetry product case.

1.3 Review of Theoretical Literature

Until recent years, economic studies on conditional rebates were limited. On the other hand, exclusive dealing has often been dealt with as an economic research topic. Similar to other types of exclusionary unilateral conduct, the economic discussion on exclusive dealing started with the Chicago School's criticism of intervening in such cases, and post-Chicago counterarguments followed.

According to Whinston (2006), the Chicago School, represented by Posner (1976) and Bork (1980), argued that if a firm wants to foreclose its competitor from the market by making a payment to the customer, the amount of the payment necessary to persuade the customer to agree to the contract is so large that the firm incurs a loss; hence, it does not make sense for the firm to make such a payment. Whinston (2006) also indicates that, since the 1980s, based on developments in economics such as game theory, it has been suggested that extending the Chicago School's models makes possible the explanation that rational firms can engage in exclusive dealing in order to exclude their rivals in certain situations.

The post-Chicago studies on exclusive dealing can be categorised into two types. The first type is the "rent-shifting" model conceived by Aghion and Bolton (1987), which shows that exclusive dealing can be used in order to exclude rivals when there exists uncertainty about rivals' costs. Aghion and Bolton (1987) demonstrate that the contract can be exclusionary for a sales contract between a dominant seller and a buyer such that if the seller purchases not from the dominant company but from other new entrants, the buyer must pay a certain amount of money for compensation. This is because this contract enables a dominant firm to exploit new entrants' profits and can prohibit their entry into the market in certain situations. The second type is the "naked exclusion" model represented by Rasmussen et. al. (1991). This model shows that a dominant firm can exclude its rivals by signing exclusive contracts with buyers when there are scale economies, respectively. Most research on exclusive dealing develops either model. For example, the model by Fumagalli and Motta (2006) is based on the exclusion model, but it consider the special case where the buyers compete with each other in the finial consumer market by using the dominant firm or the rivals' goods as inputs.

Based on the development of economic discussion on exclusive dealing and the accumulation of the antitrust cases of conditional rebates, conditional rebate is one of the areas of antitrust economics that is developing rapidly. For instance, Maier-Rigaud (2006) and Bishop and Walker (2010) explain how conditional rebates function as a tool to exclude competitors. They discuss that when buyers must purchase a certain amount from a company, and the company and its competitors compete for the remaining demand, the company can exclude a competitor who is equal to or more efficient than the company because the effective price of the company's products for the remaining demand can be lower than those of the equally efficient competitor. These models help to explain why the incumbent can exclude its competitors through rebate schemes when an incumbent's product is a must-stock item. Moreover, studies have analysed economic models regarding conditional rebates with an assumption that an incumbent's product is must-stock. Ide et. al. (2016) examines the effects of rebates where an incumbent's product is must-stock by applying the rent-shifting model and the naked exclusion model introduced in Aghion and Bolton (1987) and Rasmussen et. al. (1991), respectively. They conclude that under the rent-shifting assumption of cost uncertainty, the incumbent firm does not have an incentive to exclude the competitor by a rebate whose effective price is lower than its cost, since just focusing on selling the must-stock portion is more profitable. They also show that in the naked exclusion setup, while the foreclosure by the exclusive contract is profitable for the incumbent, foreclosure by rebate is not profitable because the incumbent cannot completely commit the buyers to comply with exclusivity through rebates.

Chone and Linnemer (2016) analyse the optimal price schedule of non-linear pricing, including rebates inspired by the rent-shifting model in the presence of the incumbent's must-stock property. However, they take different approach from Aghion and Bolton (1987) and Ide et. al. (2016) in the sense that they assume that a Nash bargaining process takes place after observing the incumbent's offer, and they agree on the point at which the surplus is shared depending on the parameter of the competitor's bargaining power against the buyer. One other difference is that they assume not only uncertainty about the competitor's information but also uncertainty about the share of the contestable market.

Chao et. al. (2018) introduce examines the foreclose effects of retroactive rebate in the case where a rival faces a capacity constraint. They set up the sequential model where there are three players: a dominant firm, a rival firm who is equally efficient as the dominant firm but subject to a capacity constraint, and a buyer. They also assume that the buyer's demand declines on the prices the buyer faces, in contrast to the horizontal demand that is adopted in Ide et. al. (2016). They show that the dominant firm can use the all unit discounts to partially exclude an equally efficient competitor that has a capacity constraint by offering a price schedule such that the quantity threshold to obtain a discount is high enough to guarantee the following: The buyer prefers not to purchase the competitor's good at an amount equal to the competitor's maximum capacity, which the buyer does prefer in the case in which the dominant firm offers linear pricing.

In addition, although the economic studies on conditional rebates shown above assume one-shot games, Ordover and Shaffer (2013) consider a two-period game where the buyer's demand consists of a captive unit and a contestable unit. It is assumed that the contestable unit can be supplied by either an incumbent or an entrant and the buyer will be locked-in to the seller supplying the contestable unit in the second period. They demonstrate that if the buyer incurs switching costs, if it switched supplier in the second period, the exclusion of the equally efficient entrant can happen in the equilibrium.

Some other economic studies on conditional rebates consider the case without the assumption that the goods of a company are must-stock items. Karlinger and Motta (2012) extend the naked exclusion model to conditional rebate cases. Chen and Shaffer (2014) also covers the naked exclusion model where there is uncertainty about the fixed cost that an entrant need to pay to analyse the effects of market share-based conditional rebate. They show that the optimal threshold percentage of conditional rebate can be less than 100 per cent of the market share, because in such situation the incumbent does not necessary have to fully compensate each buyer for the loss if it is sure that the entrant is excluded. Calzolari and Denicolo (2013) analyse a model in which there is incomplete information about demand and find that market share discounts can harm competition while exclusive contract is pro-competitive.

Kolay et. al. (2004) consider the effect of quantity-based rebate when there are a dominant upstream manufacturer and a dominant retailer. They find that in the case where the demand size of the consumer is both known to the manufacturer and the retailer, conditional rebate can eliminate double marginalisation, because the manufacturer can choose the target quantity such that the retailer is induced to set the price such that the joint profit of the manufacture and the retailer is maximised. In this situation, the two firms enjoy higher profit and the consumers are better off due to the lower price compared to when the manufacture offers a linear pricing. They also show that in the case where the demand size is the private information that the manufacturer does not know, the manufacturer can obtain a higher profit by conditional rebate than when offering a two-part tariff, the rebate induce the retailer to reveal the true demand size. However, the effect on the welfare is ambiguous in this case.

Majumdar and Shaffer (2009) extend Kolay et. al.'s model by adding an assumption that there is a competitive fringe, which enables them to analyse the effect of market share-based conditional rebate. They analyse the case where there is uncertainty about the demand size (high or low) for the dominant manufacturer. They shows that the effect of such rebate on welfare is ambiguous and, hence, market share-based rebate may not be harmful. They point out that by such rebate the manufacturer can achieve the outcome that would happen if there is no demand uncertainty. In this outcome, the retailer cannot obtain the information rent in the high demand case and there is no distortion in output in the low demand case in the sense that the retailer has a smaller incentive to misreport the demand size. They conclude that their result support the circumspect approach against conditional rebate as taken in the US.

The next two chapters analyse the exclusionary effects and the welfare-related effects of linear pricing, two-part tariffs, and conditional rebates where an incumbent firm's goods include a must-stock item, as in the case of Intel, discussing in what situations those kinds of pricing behaviours should be prohibited.

Chapter 2: Anticompetitive Effects of Conditional Rebate Where an Incumbent’s Product Is Must-Stock Item

2.1. Introduction

As introduced in Chapter 1, whether conditional rebates have anticompetitive effects, especially when a dominant firm’s product is must-stock for buyers, is a hot topic in competition policy, but the debate still offers no clear conclusion, as evidenced by the fact that Intel case is still in dispute in European courts. Against this background, this chapter and the next chapter examine whether a company whose product is must-stock for buyers introduces conditional rebates to exclude its competitors through its suction effect, as summarised in Maier-Rigaud (2006), and whether such behaviour harms the consumer welfare or not through setting up and analysing a theoretical model.

In particular, this chapter considers a situation in which two suppliers, an incumbent firm and an entrant, compete with each other to sell a good to one buyer based on the economic models that analyse exclusive dealing, such as Aghion and Bolton (1987). This model also assumes the sequential game such that the entrant is a first mover and assuming Bertrand competition between the incumbent and the entrant, based on their model. As in the Intel case and in previous studies, it is assumed that the incumbent has a competitive advantage such that the incumbent’s good is a must-stock item. Hence, the model analysed in this chapter is similar to Ede et. al.’s (2016) model, which applies Aghion and Bolton’s model to a case in which an incumbent firm whose product is a must-stock item introduces a rebate. In addition, the assumption that the buyers demand is inelastic as long as the price is below a certain threshold, adopted in Ede et. al. (2016), is also used in this chapter.

On the other hand, this study adopts certain differences from Aghion and Bolton (1987) and Ede et. al. (2016). First, the model analysed in this chapter does not assume uncertainty, since this chapter aims to examine and confirm the Chicago studies on exclusive dealing represented by Posner (1976) and Bork (1980). In this case, it is not rational for an incumbent firm to foreclose its competitor by using exclusive contracts. What changes will happen when uncertainty is assumed are discussed in Chapter 3.

Another important difference from Ede et. al. (2016) is that the upper-bound price assumed in their model is relaxed. Specifically, their model assumes that the incumbent cannot set a higher price than the buyer’s reservation price for 1 unit. On the other hand, the model analysed in this chapter assumes that the buyer purchases the product as long as the total cost to procure the units the buyer demands does not exceed the buyer's reservation price. For example, in this model, even if the incumbent's price is higher than the buyer's reservation price for 1 unit, the buyer may purchase the must-stock goods from the incumbent when the buyer could purchase from the entrant at a lower price for the contestable demand. As discussed in the next chapter, this difference causes a difference in the results in the sense that the constraint for the incumbent to set a high price limit the profitability of a conditional rebate such that the competitor is excluded.

Moreover, this chapter and the next chapter also aim to deepen the understanding of must-stock property. While exclusive rebates with the existence of must-stock property of dominant firms' product are aggressively discussed in legal cases, policy debates and economic literatures, what must-stock means, exactly, is has not been clarified so far. From this perspective, the model set up in this chapter literally defines the must-stock property. It is assumed that if the buyer's purchase volume falls below a certain point, the buyer's willingness to pay for the product or service becomes zero. What happens when this assumption is relaxed is discussed in the next chapter.

This chapter is organised as follows. Section 2 explains the assumptions of the model and the process of the game. Section 3 then analyses the optimal behaviours of the incumbent, the entrant and the buyer when the incumbent offers linear pricing, a two-part tariff and a conditional rebate individually. Section 4 analyses the welfare in the three cases, and Section 5 analyses which pricings the incumbent prefers and offers concluding remarks.

2.2. Basic Assumptions

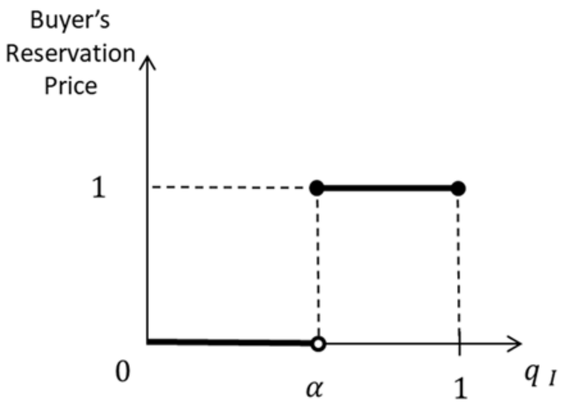
In order to analyse pricing behaviours of a dominant company, consider a situation in which an incumbent firm (I) and an entrant (E) compete with each other to sell a good to a buyer (B), as in the economic models of Aghion and Bolton (1987) and Rasmussen et. al. (1991). Incumbent firm I incurs constant marginal costs, $C_I = 0.5$, and E incurs a constant marginal cost, $C_E \in (0,1]$, to produce a unit of the good. Based on the Intel case, it is assumed that I has the advantage that, for a distributional reason, if the proportion of I's good on B's total purchase volume falls below a certain point, α, the benefit for B to purchase the good is significantly harmed, where $\alpha \in (0,1)$.[10]

For simplicity, in the following sections, a simple demand function is used. The demand size of B is assumed to be always 1, and B's willingness to pay for a unit of the good is 1 if the buyer purchases equal to or more than α units. It is assumed that if B fails to procure at least α units of the good from I, the value of 1 unit of the good for B is

[10] α must be strictly positive because some of the optimal prices obtained in this model are such prices that the denominator is α, the amount of the must-stock demand. This implies that if α is equal to zero, the price has a value that is divided by zero, entailing that the price becomes infinite.

decreased to zero. Hence, α units of I's product are literally must-stock for B in the sense that 1 unit of the good is useless for B if B cannot procure the must-stock item. It follows that if the minimum B's total cost to purchase 1 unit of the good is equal to or less than its willingness to pay, B decides to buy. Otherwise, B will not purchase anything. The case where I's good is not literally must-stock, but merely has superiority over E's good, is analysed in the Chapter 3. The relationship between B's reservation price for 1 unit of the product and the volume that B purchases from I can be summarised in Figure 2.1.

Figure 2.1 Buyer's reservation value for 1 unit of the product



Incumbent firm I can offer three types of pricing schemes: (1) linear pricing, (2) two-part tariff or (3) conditional rebate. In the first pricing scheme, I offers a fixed price per unit, P_I^L, where $P_I^L > 0$. In the second scheme, I offers a price schedule,(P_I^F, P_I^V), where $P_I^F > 0$ is the fixed cost that B has to pay when B purchases any amount from I, and $P_I^V \geq 0$ is the variable cost per unit. In the third scheme, I offers a price schedule, (P_I^R, β, γ), where $P_I^R \geq 0$ is a fixed price per unit but if the purchase volume from I to total purchase volume is equal to or more than a threshold volume, $\gamma \in (\alpha, 1]$, the unit price for I's good is discounted by $\beta \in (0, P_I^R]$, which means that the rebate is retroactive in the sense that the discount is applied to all the units B purchased. Entrant E offers only a linear pricing scheme, $P_E \geq 0$. The rebate scheme analysed in this section is a quantity-based rebate. Note that when the demand size is always fixed to 1, a quantity-based rebate and a market-share-based rebate—in which a discount is given on the condition that the share of purchase volume from I to total purchase volume is equal to or more than a threshold—have equivalent effects. These assumptions are quite similar to Ide et. al. (2016) that extend Aghion and Bolton (1987) to a conditional rebate case, although the model analysed in this and the next chapters does not assume uncertainty about the entrant's cost. However, unlike their model, I can offer the price larger than B's willingness to pay for 1 unit (i.e. P_I^L, P_I^F, P_I^V or P_I^R can be larger than 1).

The process of the game is assumed to be as follows:

1) C_E is realised and known to *I* and *E*.

2) *I* offers a pricing scheme to *B*.

3) After observing the pricing scheme offered by *I*, *E* offers its price to *B*.

4) *B* determines whether it trades with *I* and *E* or not and determines the quantities that *B* purchases from *I* and *E*. If the total cost for *B* is the same, *B* prefers to purchase *E*'s good as much as possible.

2.3. Analysis

This section analyses economic models based on the assumptions discussed in Section 2, and this chapter focuses on the case where *I*'s good is literally must-stock in order to analyse in particular, investigating whether exclusion of the more efficient entrant ($C_E < 0.5$) *b*y *I*'s two-part tariff or conditional rebate can happen or not. Sections 3.1, 3.2 and 3.3 examine the optimal price of *I* when *I* offers (1) linear pricing, (2) two-part tariff, and (3) conditional rebate individually.

2.3.1 Optimal Linear Pricing

In this section, the game in which both *I* and *E* offer linear pricing to *B* is analysed. In this game, *I*, *E* and *B* make their decisions in turn.

Analysis of this model is done by the backward induction approach (i.e., analysis on consumer behaviour, seller behaviour and platform behaviour in turn), which provides the below proposition, to be proved in subsequent sections.

Proposition 2.1. *If the incumbent offers a linear price to the buyer, the set of subgame perfect Nash equilibrium is given by the following:*

$$(P_I^{L^*}, P_E{}^*, q_I{}^*, q_E{}^*) = \begin{Bmatrix} (C_E - \varepsilon, +\infty, 1, 0)\ if\, C_E > \widetilde{C_E} \\ (\frac{1-C_E(1-\alpha)}{\alpha}, C_E, \alpha, 1-\alpha)\ if\, C_E \leq \widetilde{C_E} \end{Bmatrix},$$

where $\widetilde{C_E} = \frac{3-\alpha}{4-2\alpha}$, ε *is a small positive amount and* $P_E = +\infty$ *denotes that the entrant decides not to enter.*

The results summarised in Proposition 2.1 show that *I*'s optimal behaviours can be divided into two types. The first is when *I* sets a competitive price to capture the entire market (hereafter referred as *competitive linear pricing*). When *I* adopts this option, setting its price just below *E*'s marginal cost maximizes *I*'s profit ($P_I^L = C_E - \varepsilon$). This strategy gives I a high market share (more precisely, it completely dominates the market), but with a relatively low margin.

The second type is for I to give up competing with E and exploit the profit from the must-stock part of the market (hereafter referred as *exploitative linear pricing*).

The proposition also shows that the profits of competitive and of exploitative pricing are the same if C_E is equal to $\widetilde{C_E} = (3-\alpha)/(4-2\alpha)$. If $C_E > \widetilde{C_E}$, competitive linear pricing yields higher profit than exploitative linear pricing.

Finally, because the first derivative of $(3-\alpha)/(4-2\alpha)$ is positive, $\widetilde{C_E}$ is strictly increasing in α. When α is close to 0, $\widetilde{C_E}$ is close to 0.75, which is halfway between I's cost and the maximum cost of E. When α approaches 1, competitive linear pricing is almost never chosen in the equilibrium because $\widetilde{C_E}$ is close to 1. In this sense, the larger the must-have share of the market, the more likely it is that I will choose exploitative linear pricing; and even if α is close to 0, exploitative linear pricing will still be chosen by I if E has only a moderate cost disadvantage (i.e., if $C_E < 0.75$).

In order to prove Proposition 2.1, first analyse B's optimal behaviour. The optimal response of B given I and E's offers is shown by Lemma 2.1 below.

Lemma 2.1. *Given P_I^L and P_E, the buyer's optimal purchasing volume is*

$$(q_I{}^*, q_E{}^*) = \left\{ \begin{matrix} (1,0)\ if\ P_I^L < P_E\ and\ P_I^L\ \le 1 \\ (\alpha, 1-\alpha)\ if\ P_I^L \ge P_E\ and\ P_I^L\alpha + P_E(1-\alpha) \le 1 \\ (0,0)\ otherwise \end{matrix} \right\}.$$

Proof: See Appendix A

Lemma 2.1 shows that B purchases only I's good if I's price is lower than E's price as long as I's price does not exceed 1. If E's price is lower than I's price, B purchases both I's good and E's good as long as the total cost does not exceed B's willingness to pay for 1 unit. In this situation, all the contestable demand is covered by E's good.

Lemma 2.1 also implies that the expected profit for E can be written as

$$\Pi_E = \left\{ \begin{matrix} (P_E - C_E)(1-\alpha)\ if\ P_I^L \ge P_E\ and\ P_I^L\alpha + P_E(1-\alpha) \le 1 \\ 0\ if\ otherwise \end{matrix} \right\}.$$

This equation implies that E is subject to the following constraints to trade with B without incurring a loss:

$a)$ [Entrance condition] *E*'s price must be no higher than *I*'s price ($P_E \leq P_I^L$).

$b)$ [*B*'s willingness to pay] *E*'s price must be low enough to guarantee that the total cost does not exceed B's willingness to pay to make the buyer's demand strictly positive ($P_I^L\alpha + P_E(1-\alpha) \leq 1$).

$c)$ [Non-negative profit] *E*'s unit price must cover its cost ($P_E \geq C_E$).

Given *B*'s inelastic demand, *E* charges the highest possible price that meets the three conditions above as long as there exist such P_E. Otherwise, *E* decides not to enter the market. Entrant *E*'s optimal pricing given *I*'s offer to *B* can be written as Lemma 2.2 below.

Lemma 2.2. *Given* P_I^L*, the entrant's optimal pricing is*

$$P_E{}^* = \begin{Bmatrix} P_I^L \; if \; P_I^L \leq 1 \; and \; P_I^L \geq C_E \\ \frac{1-P_I^L\alpha}{1-\alpha} \; if \; P_I^L > 1 \; and \; \frac{1-P_I^L\alpha}{1-\alpha} \geq C_E \end{Bmatrix}.$$

If $P_I^L \leq 1 \; and \; P_I^L < C_E \; or \; if \; P_I^L > 1 \; and \; (1- P_I^L\alpha)/(1-\alpha) < C_E$ *, E decides not to enter the market.*

Proof: See Appendix A

Lemma 2.2 implies that while *E* offers the same price as *I*'s price as long as such pricing gives non-negative profit to *E* when *I*'s price does not exceed *B*'s willingness to pay for 1 unit, when *I*'s price is larger than *B*'s willingness to pay for 1 unit, *E* must set the price lower than *I*'s price because *B*'s willingness to pay is now binding. Note that it is quite feasible for *I* to set its price higher than 1 because when *I*'s good is a must-have good, *E*'s supply can make it worthwhile for *B* to purchase a combination of the expensive but must-have good of *I* and the cheaper good of *E*. If *I*'s price exceeds *B*'s willingness to pay, *E*'s profit is given by $1 - P_I^L\alpha - C_E(1-\alpha)$, which decreases as P_I^L increases. This decrease in *E*'s profit provides the opportunity for *I* to exploit *E*'s surplus. Incumbent firm *I* chooses its optimal behaviour taking *B* and *E*'s expected responses into account.

Lemmata 2.1 and 2.2 imply that the expected profit for *I* can be written as

$$\Pi_I = \begin{Bmatrix} P_I - 0.5 \; if \; P_I^L < C_E \\ (P_I - 0.5)\alpha \; if \; a) \; P_I^L > 1 \; and \; \dfrac{1-P_I^L\alpha}{1-\alpha} \geq C_E \; or \; b) \; P_I^L \leq 1 \; and \; P_I^L \geq C_E \\ 0 \; if \; otherwise \end{Bmatrix}.$$

Solving this profit-maximisation problem gives I's optimal behaviour, which is summarised in Lemma 2.3.

Lemma 2.3. *The incumbent's optimal pricing is*

$$P_I^{L^*} = \begin{cases} C_E - \varepsilon \; if \, C_E > \widetilde{C_E} \\ \frac{1-C_E(1-\alpha)}{\alpha} \; if \, C_E \leq \widetilde{C_E} \end{cases}, \text{ where } \widetilde{C_E} = \frac{3-\alpha}{4-2\alpha} \text{ and } \varepsilon \text{ is a small positive amount.}$$

The entrant enters the market only if $C_E \leq \widetilde{C_E}$.

Proof: See Appendix A

Lemma 2.3 shows that there are two types of equilibrium pricing for I: setting slightly lower than E's price (competitive linear pricing) or exploiting the profit from the must-stock part of the market (exploitative linear pricing). Note that while I's profit from the competitive linear pricing, which is given by $C_E - 0.5 - \varepsilon$, increases as C_E becomes larger, and I's profit from the exploitative linear pricing, which is given by $(1 - \alpha)(1 - C_E) + 0.5\alpha$, increases as C_E becomes smaller, because the lower cost for E implies that there is more room for I to exploit. The two profits have the same value if E's cost is $\widetilde{C_E}$.

When I chooses exploitative linear pricing, the price is set such that the total cost for B is equal to its reservation price. On the other hand, when I chooses competitive linear pricing, I's price is lower than B's reservation price for 1 unit, because E's entry works as a constraint. It follows that B's welfare is positive if $\widetilde{C_E} < C_E < 1$ and that if C_E is in this range, B's welfare decreases as C_E becomes larger.

Proposition 2.1 follows directly from Lemmata 2.1–2.3.

2.3.2 Optimal Two-Part Tariff

This section analyses the game with the following conditions: While E offers a linear pricing to B,[11] I offers a two-part tariff, (P_I^F, P_I^V), where $P_I^F > 0$ is the fixed cost that B has to pay when purchasing any amount from I, and $P_I^V \geq 0$ is the variable cost per unit.

[11] If it is assumed that E can offer a two-part tariff, the result does not differ from when E only offers linear pricing because E's product is not "must-stock"; hence, E cannot leverage the "must-stock" portion of the market to attract the contestable demand.

As in the linear pricing model analysed in Section 3.1, in the case at hand, *I*, *E* and *B* make their decisions in turn.

Analysis of this model provides the following proposition, which will be proved in the subsequent sections.

Proposition 2.2. *If the incumbent offers a two-part tariff to the buyer, there exist perfect Nash equilibria. In the equilibria, the incumbent offers a two-part tariff such that*

$$({P_I^F}^*, {P_I^V}^*) = \begin{Bmatrix} \text{any pair of } (P_I^F, P_I^V) \text{ that satisfies both } P_I^F = 1 - P_I^V \\ \text{and } 0 \le P_I^V < C_E \text{ if } C_E \ge 0.5 \\ \text{any pair of } (P_I^F, P_I^V) \text{ that satisfies both } P_I^F = 1 - P_I^V \alpha - C_E(1-\alpha) \\ \text{and } C_E < P_I^V < \frac{1 - C_E(1-\alpha)}{\alpha} \text{ if } C_E < 0.5 \end{Bmatrix}.$$

The set of the entrant's price and the buyer's purchasing volume is

$$({P_E}^*, {q_I}^*, {q_E}^*) = \begin{Bmatrix} (+\infty, 1, 0) \text{ if } C_E \ge 0.5 \\ (C_E, \alpha, 1-\alpha) \text{ if } C_E < 0.5 \end{Bmatrix},$$

where $P_E = +\infty$ *denotes that the entrant decides not to enter.*

The results summarised in Proposition 2.2 show that *I*'s optimal behaviours can be divided into two types, as with linear pricing. The first one sets a low variable price to exclude *E* (*exclusive two-part tariff*). When *I* adopts this option, *I* can exclude *E* by setting the variable price lower than *E*'s price. However, in contrast to linear pricing case, *I* can obtain a large profit by setting a high fixed price. In such a case, the most profitable exclusive two-part tariff is such that both the variable price is lower than *E*'s price and the total price for 1 unit is equal to *B*'s willingness to pay, and *I* can obtain a profit of 0.5 regardless of how efficient *E* is. Hence, this strategy not only enables *I* to dominate the market, but also gives *I* considerable profit.

The second type is giving up capturing the entire market by setting the variable price lower than *E*'s price and exploiting the profit from both the must-stock part of the market and the fixed price (*exploitative two-part tariff*). This strategy is equivalent to exploitative linear pricing in the linear pricing case.

This proposition also implies that the profits of exclusive two-part tariffs and exploitative two-part tariffs are the same if E is equally efficient to I ($C_E = 0.5$). If $C_E > 0.5$, an exclusive two-part tariff yields higher profit than does an exploitative two-part tariff. The result also implies that while *I* is able to exclude *E* by introducing an exclusive two-part tariff regardless of the value of *E*'s marginal cost, *I* has an incentive to do so only if *E* is equally or less efficient compared than *I* ($C_E \ge 0.5$) because if $C_E < 0.5$, an exploitative conditional rebate is more profitable.

Proposition 2.2 also shows that the incumbent's profit is strictly larger than *I*'s profit if *I* offers a linear pricing if $C_E > 0.5$.

In order to prove Proposition 2.2, first analyse *B*'s optimal behaviour. The optimal response of *B* given *I* and *E*'s offers is shown by Lemma 2.4 below.

Lemma 2.4. *Given* (P_I^F, P_I^V) *and* P_E*, the buyer's optimal purchasing volume is*

$$(q_I{}^*, q_E{}^*) = \begin{Bmatrix} (1,0)\ if\ P_I^V < P_E\ and\ P_I^F + P_I^V \le 1 \\ (\alpha,\ 1-\alpha)\ if\ P_I^V \ge P_E\ and\ P_I^F + P_I^V\alpha + P_E(1-\alpha) \le 1 \\ (0,0)\ otherwise \end{Bmatrix}.$$

Proof: See Appendix A

Lemma 2.4 shows that *B* purchases only *I'*s good if *I*'s variable price is lower than *E*'s price, but if *E*'s price is lower than *I*'s variable price, all the contestable demand is covered by *E*'s good as long as the total cost does not exceed *B*'s willingness to pay for 1 unit.

Lemma 2.4 also implies that the expected profit for *E* can be written as

$$\Pi_E = \begin{Bmatrix} (P_E - C_E)(1-\alpha)\ if\ P_I^V \ge P_E\ and\ P_I^F + P_I^V\alpha + P_E(1-\alpha) \le 1 \\ 0\ if\ otherwise \end{Bmatrix}.$$

Lemma 2.4 implies, in turn, that *E* is subject to the following constraints to trade with *B* without incurring a loss:

1) [Entrance condition] *E*'s price must be no higher than *I*'s variable price ($P_E \le P_I^V$).

2) [B's willingness to pay] *E*'s price must be low enough to make the buyer's demand strictly positive ($P_I^F + P_I^V\alpha + P_E(1-\alpha) \le 1$).

3) [Non-negative Profit] *E*'s unit price covers its cost ($P_E \ge C_E$).

Given *B*'s inelastic demand, *E* charges the highest possible price subject to the three conditions above as long as there exist such P_E. Otherwise, *E* decides not to enter the market. Entrant *E*'s optimal pricing given can be written as Lemma 2.5 below.

Lemma 2.5. *Given* (P_I^F, P_I^V)*, the entrant's optimal pricing is*

$$P_E{}^* = \begin{Bmatrix} P_I^V\ if\ P_I^F + P_I^V \le 1\ and\ P_I^V \ge C_E \\ \frac{1-P_I^F-P_I^V\alpha}{1-\alpha}\ if\ P_I^F + P_I^V > 1\ and\ \frac{1-P_I^F-P_I^V\alpha}{1-\alpha} \ge C_E \end{Bmatrix}.$$

If $P_I^F + P_I^V \leq 1$ *and* $P_I^V < C_E$ *or if* $P_I^F + P_I^V > 1$ *and* $(1 - P_I^F - P_I^V\alpha)/(1 - \alpha) < C_E$*, the entrant decides not to enter the market.*

Proof: See Appendix A

This lemma implies that, as in the case of linear pricing, *E* offers the same price as *I*'s variable price as long as such pricing gives non-negative profit to *E* when the total cost for 1 unit purchased only from *I* does not exceed *B*'s willingness to pay for 1 unit. If the total cost exceeds *B*'s willingness to pay, *E* must set its price lower than *I*'s variable price, because *B*'s willingness to pay is now binding.

Lemmata 2.4 and 2.5 imply that the expected profit for *I* can be written as

$$\Pi_I = \left\{ \begin{array}{c} P_I^F + P_I^V - 0.5 \; if \; P_I^F + P_I^V \leq 1 \; and \; P_I^V < C_E \\ P_I^F + (P_I^V - 0.5)\alpha \; if \; a) \; P_I^F + P_I^V > 1 \; and \; \dfrac{1 - P_I^F - P_I^V\alpha}{1-\alpha} \geq C_E \; or \\ b) \; P_I^F + P_I^V \leq 1 \; and \; P_I^V \geq C_E \\ 0 \; if \; otherwise \end{array} \right\}.$$

Solving this profit-maximisation problem gives *I*'s optimal behaviour, which is summarised in Lemma 2.6 below.

Lemma 2.6. *The incumbent's optimal pricing is*

$$({P_I^F}^*, {P_I^V}^*) = \left\{ \begin{array}{c} any \; pair \; of \; (P_I^F, P_I^V) \; that \; satisfies \; both \; P_I^F = 1 - P_I^V \\ and \; 0 \leq P_I^V < C_E \; if C_E \geq 0.5 \\ any \; pair \; of \; (P_I^F, P_I^V) \; that \; satisfies \; both \; P_I^F = 1 - P_I^V\alpha - C_E(1-\alpha) \\ and \; C_E < P_I^V < \frac{1 - C_E(1-\alpha)}{\alpha} \; if C_E < 0.5 \end{array} \right\}.$$

Proof: See Appendix A

Proposition 2.2 follows directly from Lemmata 2.4–2.6.

The result in the two-part tariff case shows that in contrast to linear pricing case, *B* does not benefit from competition even when *E*'s cost is high because *I* blocks entry by exclusive two-part tariff. Another key difference from the linear pricing case is that an exploitative two-part tariff is chosen only when *E* is less efficient than *I* (i.e. $C_E < 0.5$), which implies that inefficient production never occurs in this case.

2.3.3 Optimal Conditional Rebate

This section considers a game in which while *E* offers linear pricing to *B*,[12] *I* offers a conditional rebate scheme, (P_I^R, β, γ), where $P_I^R \geq 0$ is a pre-discount price per unit and where $\beta \in (0, P_I^R]$ is the discount per unit that *B* can obtain if the purchase volume from *I* is equal to or more than a threshold, $\gamma \in (\alpha, 1]$. As with the linear pricing model analysed in Section 3.1, in the present case *I*, *E* and *B* make their decisions in turn.

Analysis of this model provides the following proposition, which will be proved in subsequent sections.

Proposition 2.3. *If the incumbent offers a conditional rebate to the buyer, there exist perfect Nash equilibria. In the equilibria, the incumbent offers a conditional rebate such that*

$$(P_I^{R*}, \beta^*, \gamma^*) = \begin{Bmatrix} \text{any triple of } (P_I^R, \beta, \gamma) \text{ that satisfies } P_I^R - \beta = 1, \gamma = 1 \text{ and } P_I^R > \dfrac{1 - C_E(1-\alpha)}{\alpha} \text{ if } C_E \geq 0.5 \\ \text{any triple of } (P_I^R, \beta, \gamma) \text{ that satisfies } P_I^R = \dfrac{1 - C_E(1-\alpha)}{\alpha} \text{ and } P_I^R - \beta > \dfrac{1 - C_E(1-\gamma)}{\gamma} \text{ if } C_E < 0.5 \end{Bmatrix}.$$

The set of the entrant's price and the buyer's purchasing volume is

$$({P_E}^*, {q_I}^*, {q_E}^*) = \begin{Bmatrix} (+\infty, 1, 0) \text{ if } C_E \geq 0.5 \\ (C_E, \alpha, 1-\alpha) \text{ if } C_E < 0.5 \end{Bmatrix},$$

where $P_E = +\infty$ denotes that the entrant decides not to enter.

The results summarised in Proposition 2.3 show that like the two-part tariff case, *I*'s optimal behaviours can be divided into two types. The first type is setting a high enough pre-discount price that giving up the discount by purchasing the cheaper good of *E* is unreasonable for *B* (*exclusive conditional rebate*). When *I* adopts this option, *I* can exclude *E* by offering a rebate scheme such that the effective price for the contestable demand of *B*, who must buy α units, is lower than *E*'s marginal cost as long as the total cost does not exceed *B*'s willingness to pay for 1 unit. However, in contrast to the case

[12] If it is assumed that *E* can offer conditional rebate, the result does not differ from when *E* only offers linear pricing, as in the two-part tariff case, because *E*'s product is not "must-stock"; hence, *E* cannot leverage the "must-stock" portion of the market to attract the contestable demand.

of linear pricing, I can obtain a large profit by setting a high discounted unit price ($P_I^R - \beta$). In such a case, the most profitable conditional rebate satisfies the following three conditions: the discount is given if B purchases only from I ($\gamma = 1$); the effective price is lower than E's marginal cost ($P_I^R - \beta/(1-\alpha) < C_E$); and the discounted unit price is equal to B's willingness to pay for 1 unit ($P_I^R - \beta = 1$). In this situation, I can obtain a profit of 0.5 regardless of how efficient E is. Hence, this strategy not only enables I to dominate the market, but also gives I large profit.

The second type is giving up capturing the entire market and exploiting the profit from both the must-stock part of the market and the fixed price (*exploitative conditional rebate*). This strategy is equivalent to exploitative linear pricing in the linear pricing case.

Proposition 2.3 also shows that the profit is the same as the profit when I offers a two-part tariff. It follows that Proposition 2.2 also shows that I's profit is strictly larger than the profit attained if I offers linear pricing, given that $C_E > 0.5$. Note that Chapter 3 covers the extended model where a market-share-based rebate has a different effect than a two-part tariff and quantity-based rebate.

In order to prove Proposition 2.3, first analyse B's optimal behaviour. The optimal response of B given I and E's offers is shown in Lemma 2.7 below.

Lemma 2.7. *Given* (P_I^R, β, γ) *and* P_E, *the buyer's optimal purchasing volume is*

$$(q_I{}^*, q_E{}^*) = \left\{ \begin{array}{c} (1,0)\ if\ P_E > P_I^R - \beta\ and\ P_I^R - \beta\ \leq 1 \\ (\gamma,\ 1-\gamma)\ if\ P_I^R - \frac{\beta\gamma}{\gamma-\alpha} < P_E \leq P_I^R - \beta\ and\ (P_I^R - \beta)\gamma + P_E(1-\gamma) \leq 1 \\ (\alpha, 1-\alpha)\ if\ P_E \leq P_I^R - \frac{\beta\gamma}{\gamma-\alpha}\ and\ P_I^R\alpha + P_E(1-\alpha) \leq 1 \\ (0,0)\ otherwise \end{array} \right\}.$$

Proof: See Appendix A

Lemma 2.7 shows that the expected profit for E can be written as

$$\Pi_E = \left\{ \begin{array}{c} (P_E - C_E)(1-\gamma)\ if\ P_I^R - \dfrac{\beta\gamma}{\gamma-\alpha} < P_E \leq P_I^R - \beta\ and\ (P_I^R - \beta)\gamma + P_E(1-\gamma) \leq 1 \\ (P_E - C_E)(1-\alpha)\ if\ P_E \leq P_I^R - \dfrac{\beta\gamma}{\gamma-\alpha}\ and\ P_I^R\alpha + P_E(1-\alpha) \leq 1 \\ 0\ if\ otherwise \end{array} \right\}.$$

This lemma implies that E has two options to trade with B: (1) capturing the whole of the contestable demand by offering a price lower than the effective price of I's good for the contestable demand or (2) offering a price low enough to make B purchase only the threshold volume to obtain the discount from I and purchase all other units from E. Note that if the threshold is equal to 1, E can enter only if it offers its price lower than the effective price. Hereafter, this section focuses only on the case in which the incumbent introduces the rebate requiring the buyer not to trade with the entrant at all ($\gamma = 1$), since as shown in the appendix, the imperfect exclusive rebate scheme ($\gamma < 1$) is never chosen in the equilibrium.

If the threshold to obtain the discount is equal to B's demand, E is subject to the following constraints to trade with B without incurring a loss:

1) [Entrance condition] E's price must be no higher than I's effective price ($P_E \leq P_I^R - \beta/(1-\alpha)$).

2) [B's willingness to pay] E's price must be low enough to make the buyer's demand strictly positive ($P_I^R\alpha + P_E(1-\alpha) \leq 1$).

3) [Non-negative profit] E's unit price covers its cost ($P_E \geq C_E$).

Given B's inelastic demand, E charges the highest possible price while meeting the three conditions above as long as there exists such P_E. Otherwise, E decides not to enter the market. Entrant E's optimal pricing can be written as shown in Lemma 2.8.

Lemma 2.8. *Where $\gamma = 1$, given (P_I^R, β), the entrant's optimal pricing is*

$$P_E{}^* = \begin{Bmatrix} P_I^R - \frac{\beta}{1-\alpha} \; if \, P_I^R - \beta \leq 1 \; and \; P_I^R - \frac{\beta}{1-\alpha} \geq C_E \\ \frac{1-P_I^R\alpha}{1-\alpha} \; if \, P_I^R - \beta > 1 \; and \; \frac{1-P_I^R\alpha}{1-\alpha} \geq C_E \end{Bmatrix}.$$

If $P_I^R - \beta \leq 1$ and $P_I^R - \beta/(1-\alpha) < C_E$ or if $P_I^R - \beta > 1$ and $(1 - P_I^R\alpha)/(1-\alpha) < C_E$, the entrant decides not to enter the market.

Proof: See Appendix A

Lemma 2.8 implies that if $\gamma = 1$, E has to set its price strictly lower than the effective price of I's good for the contestable demand in order to capture whole the contestable demand. This requirement applies because if B purchases any unit from E, B loses an opportunity to obtain the discount from I, so in order to trade with B, E must compensate B's loss of the discount that B could have obtained.

This lemma also implies that if *I*'s discounted price is larger than 1, *E* must set its price lower than the effective price, because *B*'s willingness to pay is now binding.

Lemmata 2.7 and 2.8 imply that where $\gamma = 1$, the expected profit for *E* can be written as

$$\Pi_I = \begin{Bmatrix} P_I^R - \beta\ - 0.5\ if\ P_I^R - \beta \leq 1\ and\ P_I^R - \dfrac{\beta}{1-\alpha} < C_E \\ (P_I^R - 0.5)\alpha\ if\ a)\ P_I^R - \beta > 1\ and\ \dfrac{1 - P_I^R\alpha}{1-\alpha} \geq C_E \\ or\ b)\ P_I^R - \beta \leq 1\ and\ P_I^R - \dfrac{\beta}{1-\alpha} \geq C_E \\ 0\ if\ otherwise \end{Bmatrix}.$$

Solving this profit-maximisation problem gives *I*'s optimal behaviour, which is summarised in Lemma 2.9 below.

Lemma 2.9. *The incumbent's optimal pricing is*

$$\left(P_I^{R^*}, \beta^*, \gamma^*\right) = \begin{Bmatrix} any\ triple\ of\ (P_I^R, \beta, \gamma)\ that\ satisfies\ P_I^R - \beta = 1, \gamma = 1\ and\ P_I^R > \dfrac{1 - C_E(1-\alpha)}{\alpha}\ if C_E \geq\ 0.5 \\ any\ triple\ of\ (P_I^R, \beta, \gamma)\ that\ satisfies\ P_I^R = \dfrac{1 - C_E(1-\alpha)}{\alpha}\ and\ P_I^R - \beta > \dfrac{1 - C_E(1-\gamma)}{\gamma}\ if C_E <\ 0.5 \end{Bmatrix}.$$

Proof: See Appendix A

Proposition 2.3 follows directly from Lemmata 2.7–2.9.

The result in the conditional rebate case is the same as that of the two-part tariff case in the sense that the exploitative conditional rebate is chosen only when *E* is less efficient than *I*, and inefficient production, which happens when $0.5 < C_E < \widetilde{C_E}$ in the case of linear pricing, never occurs in this case.

2.3.4 Welfare Analysis

This section analyses welfare when *I* offers a linear pricing, a two-part tariff and a conditional rebate, respectively. The results are summarised in Table 2.1, whose details are discussed in the following sections.

Table 2.1 Summary of the welfare analysis

i) $C_E \leq 0.5$

	I's Surplus	E's Surplus	B's Surplus	Total Welfare
Linear Pricing	$(1-\alpha)(1-C_E)+0.5\alpha$	0	0	$[(1-\alpha)(1-C_E)+0.5\alpha]\omega$
Two-Part Tariff	$(1-\alpha)(1-C_E)+0.5\alpha$	0	0	$[(1-\alpha)(1-C_E)+0.5\alpha]\omega$
Conditional Rebate	$(1-\alpha)(1-C_E)+0.5\alpha$	0	0	$[(1-\alpha)(1-C_E)+0.5\alpha]\omega$
Maximum Attainable Total Welfare ($\omega = 0.5$)	n. a.	n. a.	n. a.	$[(1-\alpha)(1-C_E)+0.5\alpha]0.5$
Maximum Attainable Total Welfare ($\omega < 0.5$)	n. a.	n. a.	n. a.	$[(1-\alpha)(1-C_E)+0.5\alpha](1-\omega)$

ii) $C_E > 0.5$

	I's Surplus	E's Surplus	B's Surplus	Total Welfare
Linear Pricing ($0.5 < C_E \leq \widetilde{C_E}$)	$(1-\alpha)(1-C_E)+0.5\alpha$	0	0	$[(1-\alpha)(1-C_E)+0.5\alpha]\omega$
Linear Pricing ($C_E > \widetilde{C_E}$)	$C_E - 0.5 - \varepsilon$	0	$1 - C_E + \varepsilon$	$(2\omega - 1)C_E + 1.5 - 2\omega$
Two-Part Tariff	0.5	0	0	0.5ω
Conditional Rebate	0.5	0	0	0.5ω
Maximum Attainable Total Welfare ($\omega = 0.5$)	n. a.	n. a.	n. a.	0.25
Maximum Attainable Total Welfare ($\omega < 0.5$)	n. a.	n. a.	n. a.	$0.5(1-\omega)$

2.3.4.1 Welfare of Each Player under the Linear Pricing Case

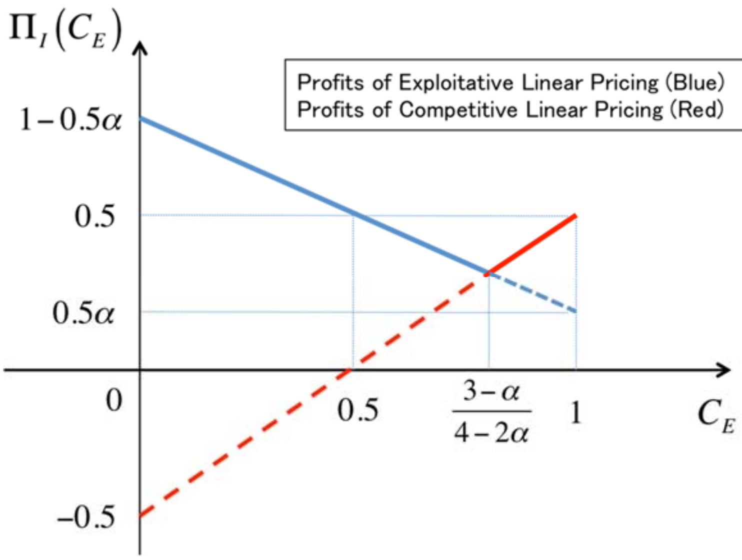
As shown in Section 3.1, in the game where both I and E offer linear pricing to B, I will choose either to set a competitive price to capture the entire market (competitive linear pricing) or to exploit the profit from the must-stock part of the market (exploitative linear pricing). Incumbent firm I's profit of the competitive linear pricing is $C_E - 0.5 - \varepsilon$ and that of the exploitative linear pricing is $(1-\alpha)(1-C_E)+0.5\alpha$.

If $C_E = 1$, I prefers competitive linear pricing because the profit of such pricing is $0.5 - \varepsilon$, while the profit of the exploitative linear pricing is 0.5α. However, the gap between the two levels of profit shrinks because the profits of the competitive linear pricing decrease as C_E becomes smaller, while the profits of the exploitative linear pricing increase as C_E becomes smaller. As shown in Section 3.1, the profits from exploitative linear pricing exceed that of the competitive linear pricing if

(2.1) $\quad C_E \leq \widetilde{C_E}.$

The relationship between I's surplus from its optimal linear pricing and E's cost is shown in Figure 2.2 below.

Figure 2.2 Incumbent's surplus when Incumbent offers a linear pricing

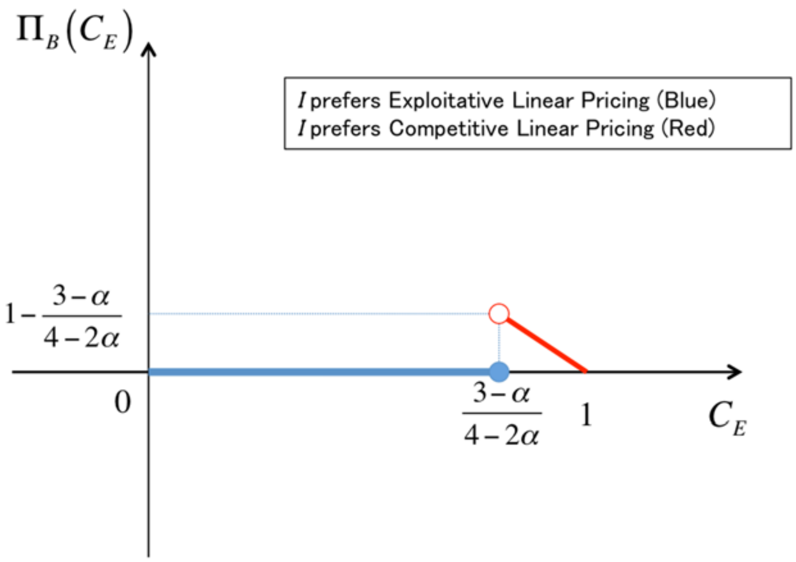


In contrast to I's surplus, E's surplus is always zero. The results show that if Inequality (2.1) does not hold, I prefers exploitative linear pricing. In this situation, E can enter the market and sell $1 - \alpha$ units to B. However, E's profit remains zero because I sets the highest possible price to let E enter the market and let B decide to purchase from both I and E. Hence, only if E sets the lowest possible price (C_E) can E enter the market, and E does set such pricing. If E set its price higher than C_E, B would decide to purchase from neither I nor E since the total cost exceeds B's willingness to pay, though P_E may be cheaper than P_I^L. In other words, in this game, a more efficient E implies more exploitation of E's potential profits by I. If Inequality (2.1) holds, I prefers to set its price just below E's cost to exclude E. Hence, in this situation, E is unable to enter the market. This inability implies that the producer's surplus is equal to I's surplus.

With regard to B's welfare, if Inequality (2.1) does not hold, I will block the entry by setting its price just below E's cost ($P_I^L = C_E - \varepsilon$). In this case, B purchases 1 unit from I and does not trade with E at all, meaning that B's surplus ($1 - C_E + \varepsilon$) is just above zero (ε) if $C_E = 1$, and it increases as C_E becomes smaller, because P_I^L becomes lower.

However, when E's cost falls below a certain point, B's surplus becomes zero, since I will set its price at the maximum price that B finds acceptable so as to maximises profit from the must-stock portion. This sequence of events happens when Inequality (2.1) does not hold. The relationship between B's surplus from its optimal linear pricing and E's cost is shown in Figure 2.3 below.

Figure 2.3 Buyer's surplus when Incumbent offers a linear pricing



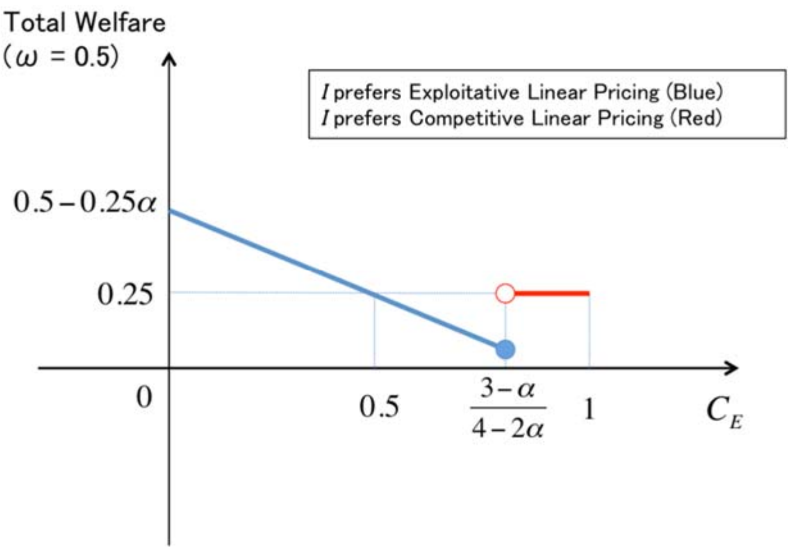
In order to analyse the total welfare of the economy, the weighted total welfare can be defined as $\omega[(I's\,profit)+(E's\,profit)]+(1-\omega)(B's\,surplus)$, where ω denotes the weight of the producer's surplus in the total welfare. Because most competition authorities put more emphasis on consumer welfare, only the cases where ω satisfies $0 \leq \omega \leq 0.5$ are analysed in this chapter. If $\omega = 0.5$, the producer's surplus and the consumer's surplus are equally weighted. If $0 < \omega < 0.5$, the consumer's surplus is more heavily emphasised than is the producer's surplus. If $\omega = 0$, the producer's surplus is not regarded at all when considering the total welfare. Since B's surplus is always zero, the total welfare is the weighted average of I's profit and B's surplus. When Inequality (2.1) does not hold, the weighted total welfare is $\omega(C_E - 0.5 - \varepsilon) + (1-\omega)(1 - C_E + \varepsilon) \approx (2\omega - 1)C_E + 1 - 1.5\omega$. It follows that when C_E decreases from 1, the weighted total welfare increases if $0 < \omega < 0.5$ and is constant if $\omega = 0.5$, as long as Inequality (2.1) does not hold. This difference between the case in which $\omega = 0.5$ and that in which $\omega < 0.5$ where Inequality (2.1) does not hold can be explained as follows: If Inequality (2.1) is not satisfied, B always purchases 1 unit from I regardless of C_E. This implies that the sum of the producer's surplus and the buyer's surplus is fixed at 0.5 when Inequality (2.1) does not hold. Though C_E does not have any influence on the quantities B purchases from I and E, it affects the condition of the transaction between I and B. What happens in this situation is a transfer of profits between I and B. If $C_E = 1$, I's profits is just below 0.5 and B's profit is just above 0.

As C_E decreases from 1, more profits are transferred from I to B as long as Inequality (2.1) is not satisfied.

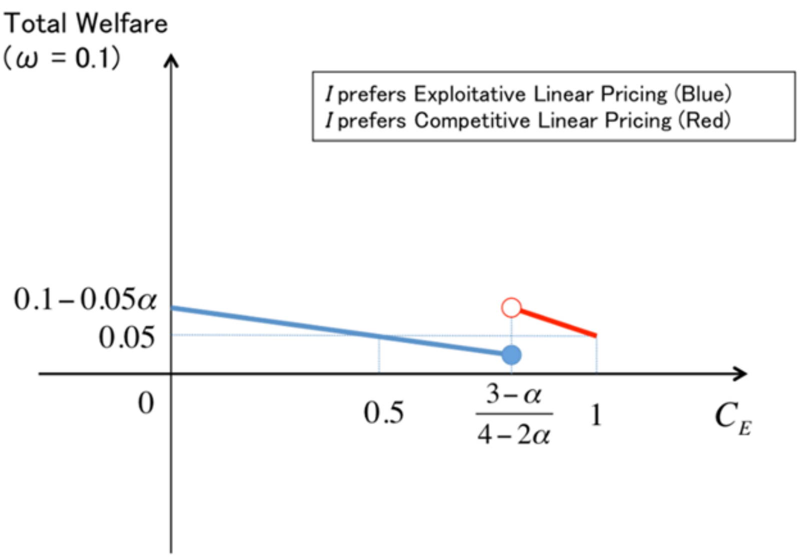
When Inequality (2.1) is satisfied, the weighted total welfare is equal to I's surplus multiplied by ω, which implies that the weighted total welfare increases as C_E approaches to zero if $0 < \omega < 0.5$ and is always zero if $\omega = 0$. Figure 2.4 below shows the weighted total welfare when the producer's surplus and B's surplus are equally weighted ($\omega = 0.5$), when the consumer surplus is the more appreciated ($\omega = 0.1$) and when the consumer surplus is only appreciated ($\omega = 0$).

Figure 2.4 Total welfare when Incumbent offers a linear pricing

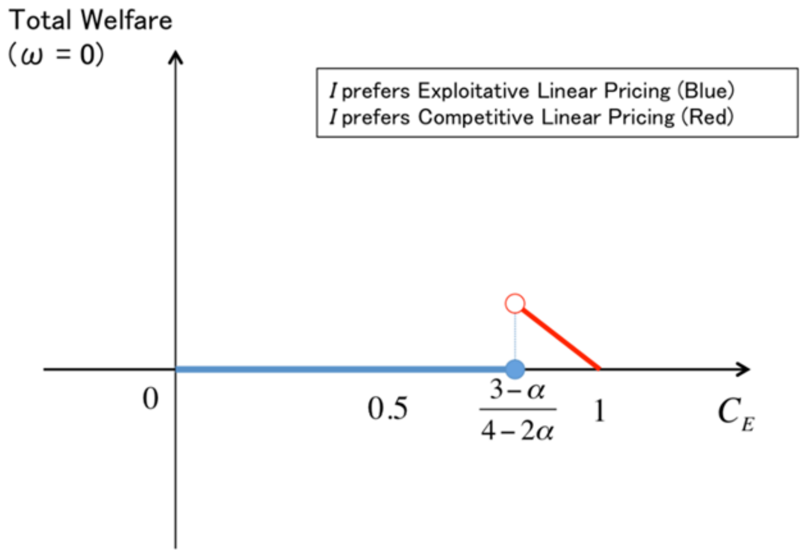
i) $\omega = 0.5$



ii) $\omega = 0.1$



iii) $\omega = 0$



The result above show that in this case, the weighted total welfare is $(2\omega - 1)C_E + 1 - 1.5\omega$ if Inequality (2.1) holds and $\omega[(1-\alpha)(1-C_E) + 0.5\alpha]$ if Inequality (2.1) does not hold. Comparison between the actual weighted total welfare and the maximum attainable weighted total welfare is carried out next. In this chapter, the maximum attainable weighted total welfare is defined as the weighted total welfare that is maximised on the condition that the profits of each player must be non-negative.

When $\omega = 0.5$, the maximum attainable welfare is achieved if efficient production is achieved. Entrant E is more efficient than I if $C_E < 0.5$ and less efficient than I if $C_E > 0.5$. It follows that if $C_E < 0.5$, efficient production is achieved when I sells α units and E sells $1 - \alpha$ units to B. If $C_E > 0.5$, efficient production is achieved when I sells 1 unit to B. If $C_E = 0.5$, efficient production is achieved as long as B purchases 1 unit. However, the result above shows that if $0.5 < C_E \leq \widetilde{C_E}$, I prefers to sell α units to B by adopting exploitative linear pricing though I, which is more efficient than E, and hence, in this segment inefficient production arises. The efficient production happens if $0 \leq C_E \leq 0.5$ or $C_E > \widetilde{C_E}$.

When $0 \leq \omega < 0.5$, the maximum attainable welfare can be achieved only if the total cost for B is as low as possible, because B's surplus is emphasised more. The lowest possible cost for B is 0.5 if $C_E \geq 0.5$ and C_E if $C_E < 0.5$. However, the realised total cost for B is 1 if Inequality (2.1) holds and $C_E - \varepsilon$ if Inequality (2.1) does not hold. It follows that in this situation, the maximum attainable welfare always fails to be achieved.

Therefore, the maximum attainable welfare is achieved only if both of the following conditions are satisfied: ω $= 0.5$ and 2) $0 \leq C_E \leq 0.5$ or $C_E > \widetilde{C_E}$.

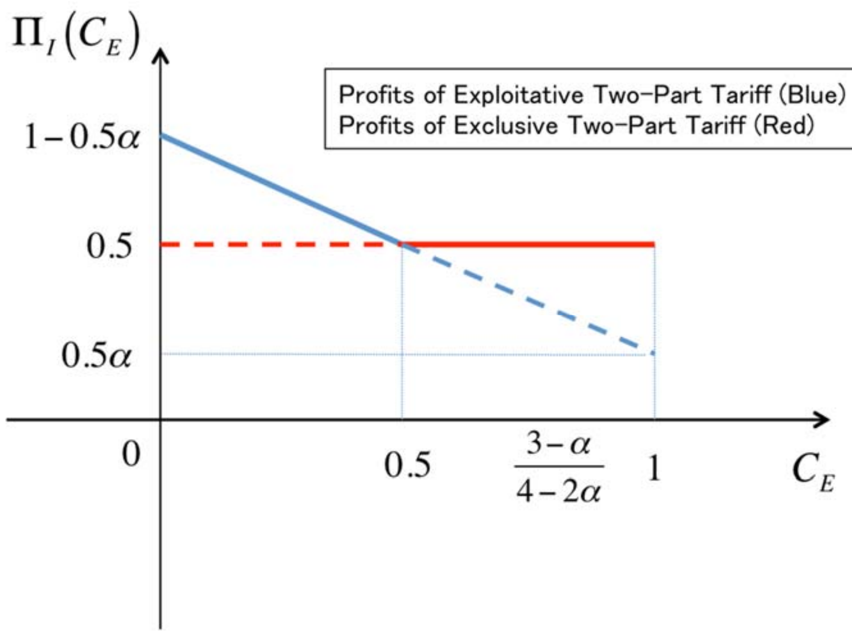
Another interesting result is that B's benefit from the competition between I and E arises only if $C_E > \widetilde{C_E}$. Otherwise, B's total cost is equal to its willingness to pay because I prefers the exploitative linear pricing.

2.3.4.2 Welfare of Each Player under the Two-Part Tariff Case

As shown in Section 3.2, in the game that I offers a two-part tariff to B, I will choose either setting a low variable price and a high fixed price to exclude E (exclusive two-part tariff) or will exploit the profit from the must-stock part of the market by setting (P_I^F, P_I^V) such that both $P_I^F = 1 - P_I^V\alpha - C_E(1-\alpha)$ and $P_I^V \geq C_E$ are satisfied (exploitative two-part tariff). The profit of the exclusive two-part tariff is always 0.5. The profit of the exploitative two-part tariff is $(1-\alpha)(1-C_E) + 0.5\alpha$, which is equal to the profit of exploitative linear pricing. The two match because when I adopts these two pricing behaviours, I aims to sell α units to B at the highest possible price that B can accept, which satisfies $P_I^F + P_I^V\alpha = 1 - C_E(1-\alpha)$.

In the linear pricing case, I prefers the alternative option, namely competitive linear pricing only when C_E is high (e.g., $C_E > \widetilde{C_E}$). On the other hand, in the two-part tariff case, I prefers the exclusive two-part tariff when $C_E \geq 0.5$, because the profit of the exclusive two-part tariff is 0.5 regardless of the value of C_E. The relationship between I's surplus from its optimal two-part tariff and E's cost is shown in Figure 2.5 below.

Figure 2.5 Incumbent's surplus



On the other hand, E's surplus is also always zero since the entry is blocked by I's two-part tariff when I adopts the exclusive two-part tariff (e.g., $0.5 \leq C_E \leq 1$), and I sets such high price that the only price E can set is equal to E's marginal cost when I adopts the exploitative two-part tariff (e.g., $0 \leq C_E < 0.5$). In addition, B's surplus is always zero since the total cost for B is always 1. Since E's welfare and B's welfare are always zero, the weighted total welfare is equal to I's surplus multiplied by ω. Hence, the weighted total welfare is $[(1-\alpha)(1-C_E)+0.5\alpha]\omega$ if $0 \leq C_E < 0.5$, and it is 0.5ω if $0.5 \leq C_E \leq 1$.

The above result shows that if $\omega = 0.5$, the weighted total welfare when I introduces a two-part tariff is equal to the maximum attainable weighted total welfare. The two are equal because efficient production is always achieved. In this situation, I sells 1 unit to B if I is more efficient, and I sells only α units and E sells $1-\alpha$ units to B if otherwise. Hence, compared to the linear pricing case analysed in Section 3.1, the weighted total welfare when $0.5 < C_E \leq \widetilde{C_E}$ is improved.

If $\omega < 0.5$, the weighted total welfare when I offers a two-part tariff is always below the maximum attainable weighted total welfare, as in the case of linear pricing. This is because while the lowest possible cost for B is 0.5 if $C_E \geq 0.5$ and C_E if $C_E < 0.5$, the realised total cost for B is always 1. This implies that compared to the linear pricing case, weighted total welfare declines in the case of a two-part tariff.

Therefore, compared to the linear pricing case analysed in Section 3.1, the weighted total welfare is improved if $\omega = 0.5$ and $0.5 < C_E \leq \widetilde{C_E}$, stays constant if $\omega = 0.5$ and $0 \leq C_E \leq 0.5$ or $C_E > \widetilde{C_E}$ and worsens if $\omega < 0.5$ and $C_E \geq 0.5$.

Moreover, in contrast to the case of linear pricing, B's benefit from the competition between I and E, which happens when Inequality (2.1) does not hold in the linear pricing case, does not arise because B's total cost is always equal to its willingness to pay.

2.3.4.3 Welfare of Each Player under the Conditional Rebate Case

As shown in Section 3.3, in the game that I offers a conditional rebate to B, I will choose either offering a rebate scheme such that E cannot compete with it (exclusive conditional rebate) or will give up excluding E by a conditional rebate and instead exploit the profit from the must-stock part of the market (exploitative conditional rebate). Incumbent firm I's profit from the exclusive conditional rebate is always 0.5. The profit of the exploitative conditional rebate is $(1-\alpha)(1-C_E)+0.5\alpha$. As such, the profits of the two options are the same as the two-part tariff case. Note that Chapter 3 will cover the extended model where market-share-based rebate has a different effect from two-part tariff and quantity-based rebate.

Hence, I chooses the exploitative conditional rebate if $C_E < 0.5$ and chooses the exclusive conditional rebate if $C_E \geq 0.5$. Therefore, the relationship between I's surplus from its optimal conditional rebate and E's cost is the same as that in the two-part tariff, as shown in Figure 2.5. As with the two-part tariff case, E's surplus and B's surplus are always zero. In addition, the weighted total welfare is $[(1-\alpha)(1-C_E)+0.5\alpha]\omega$ if $0 \leq C_E \leq 0.5$, and it is 0.5ω if $0.5 \leq C_E \leq 1$.

The above result shows that like the two-part tariff case, if $\omega = 0.5$, the weighted total welfare when *I* introduces a conditional rebate is equal to the maximum attainable weighted total welfare and if $\omega < 0.5$; the weighted total welfare when *I* offers a conditional rebate is always below the maximum attainable weighted total welfare. In addition, as in the two-part tariff case, efficient production is always achieved at *B*'s benefit from the competition between *I* and *E* not rising.

2.3.5 Endogenous Pricing Type

From the results regarding the welfare of each player under the two pricing types, linear pricing and conditional rebate[13] (as summarised in Table 2.1), Proposition 2.4 can be derived.

Proposition 2.4. *When the incumbent can introduce either linear pricing or conditional rebate,*

i) the incumbent's optimal behaviour is exclusive conditional rebate if $C_E \geq 0.5$ *and exploitative linear pricing or conditional rebate if* $C_E < 0.5$; *and*

ii) the surplus of the entrant and the buyer is zero regardless of the value of the entrant's cost, which means that the buyer is worse off and the incumbent is better off compared to the case where conditional rebate is not allowed if $\widetilde{C_E} < C_E < 1$. *If* $\omega = 0.5$. *The maximum attainable weighted total welfare is always achieved, which means the welfare is improved when* $0.5 < C_E \leq \widetilde{C_E}$ *since the entry of the less efficient entrant that occurs in the case where conditional rebate is not allowed does not happen. On the other hand, if* $\omega < 0.5$, *the maximum attainable weighted total welfare is never achieved, and where* $C_E \geq 0.5$ *holds, the welfare is worse than the case where only the linear pricing is allowed.*

The results show that in those three cases, *I* will choose from either one of two types of optimal pricing behaviours. The first is the pricing behaviour that excludes *E*. When *I* offers linear pricing, it does so by setting its price just below *E*'s marginal cost (competitive linear pricing). When *I* offers a rebate scheme, *I* can exclude *E* by offering a rebate scheme such that *E* cannot compete with it (exclusive conditional rebate). Note that if *I* can choose a two-part tariff, *I* is indifferent to whether *I* chooses a two-part tariff or conditional rebate, because the two types have equivalent effects. The profit of the competitive linear pricing is $C_E - 0.5 - \varepsilon$, and the profit of the exclusive two-part

[13] For simplicity, two-part tariff is not taken into account. However, even if two-part tariff is also considered, the result will not substantially change, because two-part tariff and conditional rebate have equivalent effects.

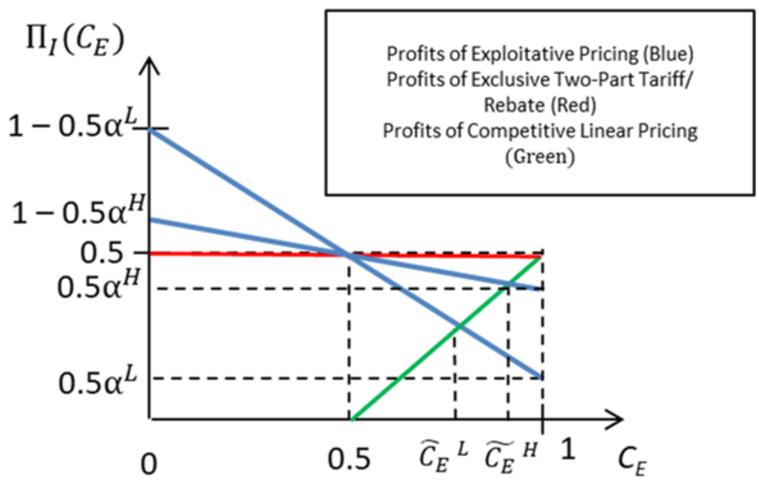
tariff and the exclusive conditional rebate is 0.5. It follows that if these three types of pricing behaviours are available to I, I always prefers the exclusive two-part tariff and the conditional rebate to the competitive linear pricing since $C_E \leq 1$.

The other option is to give up excluding E and focus instead on exploiting profit from the must-stock part of the market. As shown in Section 3.3, this option is also available when I offers a conditional rebate (exploitative conditional rebate). However, the exploitative conditional rebate is substantially equal to the linear pricing to exploit the profit from the must-stock part of the market (exploitative linear pricing). The profit of the exploitative linear pricing is given by $(1-\alpha)(1-C_E)+0.5\alpha$.

Hence, if the two types of pricing behaviours are available to I, the optimal pricing for I is either the exploitative pricing or the exclusive conditional rebate. Therefore, if the two types of pricing behaviours are available to I, I prefers exclusive linear pricing if $C_E < 0.5$ and prefers exclusive conditional rebate if $C_E \geq 0.5$.

With regard to the comparative statics about the proportion of the must-stock part of the market, α, the size of α does not have any influence on I's surplus of competitive linear pricing and exclusive conditional rebate and E's and B's surplus. On the other hand, it does influence I's surplus of exploitative pricing, since the larger α means that there is the more room for I to exploit E's efficiency. Hence, if $C_E < 0.5$, I's surplus of exploitative pricing increases as α becomes larger, and if $C_E < 0.5$, I's surplus of exploitative pricing decreases as α becomes larger. This effect also implies that the value of C_E such that competitive linear pricing and exploitative pricing are equivalent for I, $\widetilde{C_E}$, increases as α becomes larger. These relationships are summarised in Figure 2.6.

Figure 2.6 Incumbent's surplus when α is high and low



The results of the analysis in this chapter also show that where I's product is a must-stock item, a conditional rebate can work as a powerful tool to exclude E, even though the rivals are more efficient than the incumbent. However, this model shows that I will not use a rebate scheme (and two-part tariff) to exclude the more efficient entrant. If this model is accurate, the exclusion by a dominant firm's exclusionary pricing behaviour may not usually occur. Hence, determining in what situations a dominant firm uses a two-part tariff or a rebate scheme to exclude the entrant may be important. Chapter 3 will show that if the game is repeated, exclusion of the more efficient competitor can happen.

The other interesting result is that exploiting the profit from the must-stock part of the market is an attractive option for I. When E is more efficient than I, I prefers the exploitative option, although the exclusive two-part tariff and the exclusive conditional rebate gives I a profit of 0.5, which is equal to the profit that I can obtain when there is no rival in the market. This result implies that if E is the more efficient, I may set its price much higher than B's willingness to pay. For example, if $C_E = 0.4$ and $\alpha = 0.5$, the optimal pricing for I is setting its price at 1.6. If $C_E = 0.4$ and $\alpha = 0.2$, the optimal price is 3.4. Note that the minimum pre-discount price (P_I^R) necessary to exclude E by conditional rebate increases as α becomes smaller, since the condition to exclude E is given by

(2.2) $\quad P_I^R > [1 - C_E(1-\alpha)]/\alpha = (1 - C_E)/\alpha + C_E.$

However, setting such a high pre-discount price satisfying Inequality (2.2) seems not to happen normally. Furthermore, the results imply that if I uses the exclusive conditional rebate, the price offered to B can greatly exceed B's willingness to pay. For example, assume that I will set the lowest possible P_I^R when it prefers exclusive conditional rebate and that $\varepsilon \geq 0.001$ must be satisfied. In this circumstances, if $C_E = 0.5$ and $\alpha = 0.8$, the incumbent will exclude E and obtain the profit of 0.5 by setting P_I^R at 1.126 and β at 0.126. If $C_E = 0.5$ and $\alpha = 0.1$, the incumbent will exclude E by setting P_I^R at 5.501 and β at 4.501. If $C_E = 0.1$ and $\alpha = 0.1$, the incumbent will exclude E by setting P_I^R at 9.101 and β at 8.101. However, such pricing also seems unrealistic.

One possible explanation for these unrealistic results is that the assumption that I's good is literally a must-stock item is too strong. In reality, the must-stock item may not be literally must-stock, but merely be the option strongly preferred over the competitors' products, as will be discussed in the Chapter 3.

2.4. Conclusion

The results of this chapter's analysis show that in each of the three pricing behaviours, the optimal behaviour for the incumbent is one of two types. The first pricing behaviour is setting the maximum price such that the incumbent can exclude the entrant. This type of pricing is preferred by the incumbent when the entrant's cost is higher than a certain point. In the case of linear pricing, the optimal price is to set just below the entrant's cost (competitive linear pricing). The incumbent's profit from this pricing is equal to the difference between the entrant's cost and the incumbent's cost. On the other hand, in the two-part tariff case and the conditional rebate case, the incumbent can exclude the entrant by obtaining a monopoly profit regardless of how efficient the entrant is through setting a high fixed price in the two-part tariff case (exclusive two-part tariff) and setting the pre-discount price very high in the conditional rebate case (exclusive conditional rebate). If the incumbent chooses this type of pricing, the entrant's profit is zero. The buyer's surplus is positive in the linear pricing case, but the surplus is zero in the two-part tariff case and the conditional rebate case.

The second type of optimal behaviour for the incumbent is to give up competing with the entrant and instead exploit the profit from the must-stock part of the market by setting the highest possible price to let the entrant enter the market. The total cost for the buyer is equal to its willingness to pay (exploitative pricing). This type of optimal behaviour is common to the three cases, and the incumbent prefers such pricing when the entrant's cost is lower than a certain point. The lower the entrant's cost, the higher the profit the incumbent obtains, because the lower cost for the entrant means more room that the incumbent can exploit. If the incumbent chooses this type of pricing, the entrant's profit and the buyer's profit are zero.

The results of the analysis also show that although an incumbent can exclude an entrant by exclusive two-part tariff or conditional rebate even if the entrant is more efficient than the incumbent, the incumbent prefers exploitative pricing behaviour over an exclusive two-part tariff or conditional rebate because exploitative pricing behaviour is more profitable. The result is that an incumbent will not use exclusive two-part tariffs or conditional rebates to foreclose the more efficient entrant from the market because the incumbent does not have an incentive to do so. This result is similar to the view of the Chicago School that there is no foreclosure of a more efficient entrant through exclusive dealing, but it is importantly different from the Chicago School's view in that consumers can be made worse off.

Note that the result that the incumbent does not introduce a rebate to exclude the more efficient entrant is similar to Ide et. al. (2016), although the logic is different. In their model, the incumbent does not introduce exclusive conditional rebate because the profitability is limited under the assumption prohibiting setting a high list price. On the other hand, this chapter shows that if this assumption is relaxed, the incumbent can obtain a monopoly profit by using conditional rebate such that the list price is higher than the buyer's willingness pay and the unit price after the discount is applied is equal to the buyer's willingness pay. However, the incumbent does not use such rebate, because it can obtain a larger profit through exploitative pricing, which is also prohibited in Ide et. al. (2016).

The welfare analysis shows that when the incumbent offers an exclusive conditional rebate in the equilibrium, the surplus of the entrant and the buyer is zero. However, the improvement of the buyer's surplus by prohibiting the conditional rebate and two-part tariff happens only when the entrant's cost is very high (i.e., $\widetilde{C_E} < C_E \leq 1$). In addition, it is shown that prohibiting the two-part tariff and conditional rebate decreases total welfare when the entrant's cost is a little less efficient than that of the incumbent (i.e., $0.5 < C_E \leq \widetilde{C_E}$), because the two-part tariff and conditional rebate prevent inefficient production in the sense that *I* chooses to dominate the market rather than only producing the good for the must-stock demand.

The aim of this chapter has been to identify a mechanism that comes into force with exclusive conditional rebates. Its discussion has been highly simplified. At least one prediction does not seem to happen normally in the sense that the incumbent sets an extremely high pre-discount price when it excludes the entry by conditional rebate or extremely high price when it exploits the entrant's efficiency. We relax some assumptions in the next chapter in order to explore this observation. One of the possible explanations of those unrealistic results is that the assumption that *I*'s good is literally a must-stock item is too strong. Section 3 of the next chapter will analyse the case in which the must-stock item may not be literally must-stock, but is superior to competitors' products. Chapter 3 also covers the extensions to the baseline model analysed in this chapter in order to explore the situations in which the exclusion of the more efficient entrant by conditional rebate and two-part tariff happens and in which the exclusion is best achieved by market-share-based rebate rather than a two-part tariff and quantity-based rebate.

2.5. Appendix A

2.5.1 Proof of Lemma 2.1

In this situation, only linear pricing schemes are available to *B*, and *B* must purchase at least α units from *I* unless *B* decides not to buy the good. Furthermore, since both *I* and *E* offer only linear pricing, *B* prefers a cheaper good for its contestable demand, whose size is $1-\alpha$. Those imply that the purchase volume of *B* from *I* and *E*, (q_I, q_E), must be either $(1,0)$, $(\alpha, 1-\alpha)$ or $(0,0)$. When (q_I, q_E) is equal to $(1,0)$, *B*'s total cost can be written as P_I^L. When (q_I, q_E) is equal to $(\alpha, 1-\alpha)$, *B*'s total cost can be written as $P_I^L\alpha + P_E(1-\alpha)$. Hence, the condition that *B* prefers purchasing $(1,0)$ to purchasing $(\alpha, 1-\alpha)$ is $P_I^L < P_I^L\alpha + P_E(1-\alpha)$. This equation can be modified to $P_I^L < P_E$. On the other hand, if *B*'s total cost to purchase 1 unit is more than 1, *B* decides not to purchase at all, because the cost exceeds its willingness to pay. ■

2.5.2 Proof of Lemma 2.2

The conditions $a)$ and $b)$ can be modified as follows:

$a')$ $P_E \leq P_I^L$ and $b')$ $P_E \leq \frac{1-P_I^L\alpha}{(1-\alpha)}$.

These modifications imply that while the upper bound of P_E in the condition $a')$, P_I^L, is increasing in P_I^L, the upper bound in the condition $b')$, $(1-\ P_I^L\alpha)/(1-\alpha)$, is decreasing in P_I^L. The upper bounds in the conditions $a')$ and $b')$ coincide with each other at a single point, where $P_I^L = 1$ holds. It follows that which constraint is binding depends on whether P_I^L is larger than 1. If $P_I^L < 1$, only $a')$ is binding. In this situation, *E*'s optimal behaviour is to set the highest possible price satisfying $a')$, which is equal to P_I^L as long as the condition $c)$ is met (i.e., *E*'s unit price covers its cost ($P_I^L \geq C_E$)).

If $P_I^L > 1$, only $b')$ is binding. In this situation, the highest possible price satisfying $b')$ is

(A2.1) $P_E = (1-\ P_I^L\alpha)/(1-\alpha)$.

Offering the price satisfying Equation (A2.1) is *E*'s optimal behaviour as long as the condition $c)$ is satisfied (i.e., $(1-\ P_I^L\alpha)/(1-\alpha) \geq C_E$)). ■

2.5.3 Proof of Lemma 2.3

The proof of Lemma 2.3 is divided into two steps. In *Step 1* we find optimal linear pricing such that both $P_I^L \leq 1$ and $P_I^L < C_E$ (optimal competitive linear pricing). In

Step 2, we find the optimal linear pricing such that $P_I^L > 1$ and $C_E \leq (1 - P_I^L\alpha)/(1 - \alpha)$ hold or both $P_I^L \leq 1$ and $P_I^L \geq C_E$ hold (optimal exploitative linear pricing); we then analyse which type of linear pricing is more profitable.

Step 1: Optimal competitive linear pricing

Lemma 2.2 implies that if *I* chooses competitive linear pricing, *I* has to offer P_I^L such that both $P_I^L \leq 1$ and $P_I^L < C_E$. In this situation, setting P_I^L just below C_E maximizes *I*'s profit. In other words, the most profitable price is

(A2.2) $P_I^L = C_E - \varepsilon.$

The profit of *I* when *I* chooses competitive linear pricing and offers a price satisfying Equation (A2.2), Π_I^{CL}, can be written as

(A2.3) $\Pi_I^{CL} = C_E - 0.5 - \varepsilon.$

Step 2: Optimal exploitative linear pricing

Lemma 2.2 also implies that if *I* chooses exploitative linear pricing, *I* has to offer P_I^L such that either both $P_I^L > 1$ and $C_E \leq (1 - P_I^L\alpha)/(1 - \alpha)$ hold or both $P_I^L \leq 1$ and $P_I^L \geq C_E$ hold. These conditions can be combined as follows:

(A2.4) $C_E \leq P_I^L \leq \frac{1-C_E(1-\alpha)}{\alpha}.$

Because *I*'s profit is given by $P_I^L\alpha$, the most profitable price satisfying Inequality (A2.4) is

(A2.5) $P_I^L = [1 - C_E(1 - \alpha)]/\alpha.$

The of *I* when *I* chooses exploitative linear pricing and offers the price satisfying Equation (A2.5), Π_I^{EPL}, can be written as

(A2.6) $\Pi_I^{EPL} = (1 - \alpha)(1 - C_E) + 0.5\alpha.$

From Equation (A2.3) and (A2.6), the condition *I* prefers to offer competitive linear pricing can be written as

$C_E - 0.5 > (1 - \alpha)(1 - C_E) + 0.5\alpha.$

The rearrangement yield the following:

$C_E > \frac{3-\alpha}{4-2\alpha}.$ ■

2.5.4 Proof of Lemma 2.4

As Lemma 2.1 shows, the purchase volume of *B* from *I* and *E*, (q_I, q_E), must be either $(1, 0)$, $(\alpha, 1-\alpha)$ or $(0, 0)$. As in the linear pricing case, the condition that B prefers purchasing only from *I* to purchasing just $1-\alpha$ units from *I* is the variable cost of *I*'s product is lower than that of *E*'s product. The difference from the linear pricing case is the total cost for B when $(q_I, q_E) = (1, 0)$ is $P_I^F + P_I^V$, and the total cost when $(q_I, q_E) = (\alpha, 1-\alpha)$ is $P_I^F + P_I^V\alpha + P_E(1-\alpha)$.■

2.5.5 Proof of Lemma 2.5

The conditions $a)$ and $b)$ can be modified as follows:

$a')$ $P_E \leq P_I^V$ and $b')$ $P_E \leq \frac{1-P_I^F-P_I^V\alpha}{1-\alpha}$.

This modification implies that while the upper bound of P_E in the condtion $a')$, P_I^V, is increasing in P_I^V, the upper bound in the condition $b')$, $(1-\ P_I^F-\ P_I^V\alpha)/(1-\alpha)$, is decreasing in P_I^V. The upper bounds in the conditions $a')$ and $b')$ coincide with each other at a single point, where $P_I^F + P_I^V = 1$ holds. It follows that which constraint is binding depends on whether $P_I^F + P_I^V$ is larger than 1 or not. If $P_I^F + P_I^V < 1$, only $a')$ is binding. Hence, *E*'s optimal behaviour is to set the highest possible price satisfying $a')$, which is equal to P_I^V as long as the condition $c)$ is met ($P_I^V \geq C_E$).

If $P_I^F + P_I^V > 1$, only $b')$ is binding. In this situation, the highest possible price satisfying $b')$ is

(A2.7) $P_E = (1-\ P_I^F-\ P_I^V\alpha)/(1-\alpha)$.

Offering the price satisfying Equation (2.7) is *E*'s optimal behaviour as long as condition $c)$ is satisfied (i.e., $1-\ P_I^F-\ P_I^V\alpha)/(1-\alpha) \geq C_E$). ■

2.5.6 Proof of Lemma 2.6

The proof of Lemma 2.6 is divided into two steps. In *Step 1* we find the optimal exclusive two-part tariff. In *Step 2*, we find the optimal exploitative two-part tariff and analyse which type of two-part tariff is more profitable.

Step 1: Optimal exclusive two-part tariff

Lemma 2.5 implies that if *I* chooses an exclusive two-part tariff, *I* has to offer P_I^L such that both $P_I^F + P_I^V \leq 1$ and

(A2.8) $P_I^V < C_E$.

Note that in this situation, since *I*'s profit is given by $P_I^F + P_I^V - 0.5$, *I* can obtain a large profit by setting high fixed cost while lowering its variable price to exclude *E*. Hence, the most profitable (P_I^F, P_I^V) satisfies;

(A2.9) $P_I^F + P_I^V = 1$.

Incumbent firm I's profit when I chooses exclusive two-part tariff and offers (P_I^F, P_I^V) satisfying Inequality (A2.8) and Equation (A2.9), Π_I^{EC2T}, can be written as

(A2.10) $\Pi_I^{EC2T} = 0.5$.

Step 2: Optimal exploitative two-part tariff

Lemma 2.5 also implies that if I chooses exploitative two-part tariff, I have to offer (P_I^F, P_I^V) such that both $P_I^F + P_I^V > 1$ and $C_E \leq (1 - P_I^F - P_I^V\alpha)/(1-\alpha)$ are satisfied. The condition can be modified as

$P_I^F + P_I^V\alpha \leq 1 - C_E(1-\alpha)$.

Because I's profit is given by $P_I^F + (P_I^V - 0.5)\alpha$, the most profitable (P_I^F, P_I^V) satisfies

(A2.11) $P_I^F + P_I^V\alpha = 1 - C_E(1-\alpha)$.

I's profit when I chooses exploitative two-part tariff and offers the price satisfying Equation (A11), Π_I^{EP2T},can be written as

(A2.12) $\Pi_I^{EP2T} = (1-\alpha)(1-C_E) + 0.5\alpha$.

Because $P_I^F + P_I^V > 1$, $P_I^F > 0$ and $C_E \leq (1 - P_I^F - P_I^V\alpha)/(1-\alpha)$ must be satisfied, such pricing must be subject to

(A2.13) $C_E < P_I^V < [1 - C_E(1-\alpha)]/\alpha$.

From Equation (A2.10) and Inequality (A2.13), the condition that I prefers to offer exclusive two-part tariff can be written as follows:

$0.5 \geq (1-\alpha)(1-C_E) + 0.5\alpha$.

Rearrangement of this equation yields

$C_E \geq 0.5$. ■

2.5.7 Proof of Lemma 2.7

As Lemma 2.1 shows, the possible pairs of the purchase volume of B from I and E, (q_I, q_E) are $(1,0)$, $(\alpha, 1-\alpha)$ or $(0,0)$. However, in this situation, while the unit price of E's good is constant, the unit price of I's good differs depending on whether the purchase volume of B from I achieves the threshold volume or not. While the unit price of I's good is P_I^R if $q_I < \gamma$, the unit price of I's good is $P_I^R - \beta$ if $q_I \geq \gamma$. This implies that $(\gamma, 1-\gamma)$ is the other possible pair of (q_I, q_E).

When (q_I, q_E) is equal to $(1, 0)$, B's total cost can be written as $P_I^R - \beta$. When (q_I, q_E) is equal to $(\gamma, 1-\gamma)$, B's total cost can be written as $(P_I^R - \beta)\gamma + P_E(1-\gamma)$. It follows that the condition that B prefers purchasing $(1, 0)$ to purchasing $(\gamma, 1-\gamma)$ is $P_I^R - \beta < (P_I^R - \beta)\gamma + P_E(1-\gamma)$. This equation can be modified to

$$P_E > P_I^R - \beta.$$

When (q_I, q_E) is equal to $(\alpha, 1-\alpha)$, B's total cost can be written as $P_I^R\alpha + P_E(1-\alpha)$. Hence, the condition that B prefers purchasing $(\gamma, 1-\gamma)$ to purchasing $(\alpha, 1-\alpha)$ is $(P_I^R - \beta)\gamma + P_E(1-\gamma) < P_I^R\alpha + P_E(1-\alpha)$. This equation can be modified to

$$P_E > P_I^R - \frac{\beta\gamma}{\gamma-\alpha}.$$

On the other hand, if B's total cost to purchase 1 unit is more than 1, B decides not to purchase at all, because the cost exceeds its willingness to pay. ■

2.5.8 Proof of Lemma 2.8

If $\gamma = 1$, the conditions a) and b) can be modified as follows:

a') $P_E \leq P_I^R - \frac{\beta}{1-\alpha}$ and

b') $P_E \leq \frac{1-P_I^R\alpha}{(1-\alpha)}$.

This modification implies that while the upper bound of P_E in the condtion a'), $P_I^R - \beta/(1-\alpha)$, is increasing in P_I^R, the upper bound in the condition b'), $(1 - P_I^R\alpha)/(1-\alpha)$, is decreasing in P_I^R. The upper bounds in the conditions a') and b') coincide with each other at a single point, where $P_I^R - \beta = 1$ holds. It follows that which constraint is binding depends on whether $P_I^R - \beta$ is larger than 1. If $P_I^R - \beta < 1$, only a') is binding. Hence, E's optimal behaviour is to set the highest possible price satisfying a'), which is equal to $P_I^R - \beta/(1-\alpha)$ as long as the condition c) is met ($P_I^R - \beta/(1-\alpha) \geq C_E$).

If $P_I^R - \beta > 1$, only b') is binding. In this situation, the highest possible price satisfying b') is equal to $(1 - P_I^R\alpha)/(1-\alpha)$, the same as for Equation (A2.1). Offering the price satisfying Equation (A2.1) is E's optimal behaviour as long as condition c) is satisfied (i.e., $(1 - P_I^R\alpha)/(1-\alpha) \geq C_E$).■

2.5.9 Proof of Lemma 2.9

The proof of Lemma 2.9 is divided into three steps. In *Step 1*, we find the optimal exclusive conditional rebate when I decides to offer the optimal rebate scheme such that $\gamma = 1$ holds. In *Step 2*, we show that any exclusive conditional rebate scheme such that $\gamma < 1$ cannot be in equilibrium because such a rebate is less profitable than the exclusive conditional rebate satisfying $\gamma = 1$. In *Step 3*, we find the optimal exploitative conditional rebate and analyse which type of conditional rebate is more profitable.

Step 1: Optimal exclusive conditional rebate satisfying $\gamma = 1$

Lemma 2.9 implies that if *I* chooses exclusive conditional rebate satisfying $\gamma = 1$, *I* has to offer (P_I^R, β) such that both $P_I^R - \beta \leq 1$ and

(A2.14) $P_I^R < C_E + \beta/(1-\alpha)$.

Note that since *I*'s profit is given by $P_I^R - \beta - 0.5$ in this situation, *I* can obtain a large profit by setting a high fixed cost while setting a pre-discount fixed price so high that giving up the discount by purchasing a cheaper good from *E* is unreasonable for *B*. Hence, *I*'s profit is maximised when (P_I^R, β) satisfies the following condition:

(A2.15) $P_I^R - \beta = 1$.

By substituting Equation (A2.15) into Inequality (A2.14), the condition of P_I^R for profit-maximising exclusive conditional rebate can be rewritten as

(A2.16) $P_I^R > [1 - C_E(1-\alpha)]/\alpha$.

Incumbent firm *I*'s profit when *I* chooses exclusive conditional rebate and offers (P_I^R, β) satisfying Equation (A2.15) and Inequality (A2.16), Π_I^{ECCR}, can be written as

(A2.17) $\Pi_I^{ECCR} = 0.5$.

Step 2: Optimal exclusive conditional rebate satisfying $\gamma < 1$

Lemma 2.8 also implies that *I* is also able to exclude *E* by offering a rebate scheme such that $\gamma < 1$. The profit *I* can obtain by selling γ units to *B* can be written as $(P_I^R - \beta - 0.5)\gamma$. That profit must satisfy both the constraint of *B*'s willingness to pay $((P_I^R - \beta)\gamma + P_E(1-\gamma) \leq 1)$ and the condition for *E* to enter the market $(P_E \geq C_E)$. These conditions can be combined as follows:

$(P_I^R - \beta - 0.5)\gamma \leq 1 - C_E(1-\gamma) - 0.5\gamma$.

However, $1 - C_E(1-\gamma) - 0.5\gamma$ is equal to or smaller than the profit from exclusive conditional rebates (i.e., 0.5) if $C_E \geq 0.5$, and it is smaller than the profit from exploitative conditional rebate (i.e., $1 - C_E(1-\alpha) - 0.5\alpha$) if $C_E < 0.5$. Hence, a rebate scheme such that $\gamma < 1$ cannot represent optimal pricing for *I*.

Step 3: Optimal exploitative conditional rebate

Lemma 2.8 also implies that if *I* chooses an exploitative two-part tariff, *I* has to offer (P_I^R, β) such that both $P_I^R - \beta > 1$ and $C_E \leq (1 - P_I^R \alpha)/(1 - \alpha)$ are satisfied. The condition can be modified as

$$P_I^R \alpha \leq 1 - C_E(1 - \alpha).$$

Because *I*'s profit is given by $P_I^R \alpha$, the most profitable price (P_I^R, β) satisfies the following condition:

(A2.18) $P_I^R = \frac{1 - C_E(1-\alpha)}{\alpha}$.

Incumbent firm *I*'s profit when *I* chooses an exploitative conditional rebate and offers a price satisfying Equation (A2.18), Π_I^{EPCR}, can be written as

(A2.19) $\Pi_I^{EPCR} = (1 - \alpha)(1 - C_E) + 0.5\alpha$.

Note that in order to make such exploitation possible, *I* needs to exclude the possibility that *B* purchases γ units from *I* and $1 - \gamma$ units from *E*. The condition that *I* excludes the possibility that *B* purchases γ units from *I* and $1 - \gamma$ units from *E* can be written as

$$(P_I^R - \beta)\gamma + C_E(1 - \gamma) > 1.$$

This condition can be rewritten as

$$P_I^R - \beta > \frac{1 - C_E(1-\gamma)}{\gamma}.$$

Note that $[1 - C_E(1 - \gamma)]/\gamma > 1$ holds.

From Equations (A2.17) and (A2.19), the condition that *I* prefers to offer exclusive conditional rebate can be written as follows:

$$0.5 \geq (1 - \alpha)(1 - C_E) + 0.5\alpha.$$

This equation can be modifies in this way:

$C_E \geq 0.5$. ■

Chapter 3: Extension Models Related to Anticompetitive Effects of Conditional Rebate Where an Incumbent's Product Is a Must-Stock Item

3.1. Introduction

In Chapter 2, the anticompetitive effects of linear pricing, two-part tariffs and conditional rebates where an incumbent firm's product is a must-stock item have been analysed. The results show that an incumbent can exclude an entrant by exclusive two-part tariff or conditional rebate even if the entrant is more efficient than is the incumbent. However, when the entrant is more efficient, the incumbent prefers exploitative pricing behaviour over exclusive two-part tariffs or conditional rebates because exploitative pricing behaviour is more profitable. The result implies that an incumbent will not use exclusive two-part tariffs or conditional rebates to foreclose the more efficient entrant from the market because the incumbent does not have an incentive to do so; this conclusion is similar to the view of the Chicago School on exclusive dealing. In addition, the results of the baseline model analysed in Chapter 2 show the limitations of the model in the sense that the incumbent has an ability to set an incredibly high price to capture all the surplus of the market.

This chapter expands the basic theoretical framework to analyse the anticompetitive effects of conditional rebates and other pricing behaviours raised in Chapter 2. In Section 2 of this chapter, we introduce the basic assumptions common to the extended models analysed in this chapter. Sections 3–6 cover four extension models. Section 3, considering the limitation of the baseline model such that the must-stock assumption may be too strong, analyses the case where the incumbent's good is not literally must-stock, but is only superior to competitors' products.

The next three sections show that by adding certain assumptions, the incumbent can have an incentive to exclude the more efficient entrant by conditional rebate. Section 4 analyses the case of uncertainty about must-stock proportions. Section 5 covers the case where uncertainty is assumed regarding the size of the contestable demand. In this extended model, a market-share-based rebate is a better scheme to exclude the entrant than is a two-part tariff or quantity-based rebate in the sense that the market-share-based rebate adapts to the uncertainty better than do other pricing scheme. Section 6 analyses the model such that the game is played more than once and that the must-stock property of the incumbent's good is not fixed over time, in order to show that the incumbent can have an incentive to exclude the more efficient entrant to maintain its advantage of must-stock property. Finally, Section 7 gives concluding remarks and policy implications.

3.2. Basic Assumptions

The basic assumptions in the extended models analysed in this chapter are common to those in Chapter 2, unless otherwise specified. It is assumed that there are three types of players; an incumbent firm (*I*) and an entrant (*E*) compete with each other to sell a good to a buyer (*B*). Incumbent firm *I* incurs a constant marginal costs, $C_I = 0.5$, and *E* incurs a constant marginal cost, $C_E \in (0,1]$, to produce a unit of the good. Based on the case of Intel, *I* is assumed to have the advantage that from a distributional perspective, if the proportion of *I*'s good on *B*'s total purchase volume falls below a certain point, α, the benefit for *B* to purchase the good is significantly harmed, where $\alpha \in (0,1)$.[14]

The demand size of *B* is assumed always to be 1. Buyer *B*'s willingness to pay for a unit of the good is 1 if *B* purchases equal to or more than α units. It is assumed that if *B* fails to procure at least α units of the good from *I*, the value of 1 unit of the good for *B* is decreased to zero. Hence, α units of *I*'s product is literally must-stock for *B* in the sense that 1 unit of the good is useless for *B* if *B* cannot procure the must-stock item. These conditions entail that if *B*'s minimum total cost to purchase 1 unit of the good is equal to or less than its willingness to pay, *B* decides to buy. Otherwise, *B* will not purchase at all. The case where *I*'s good is not literally 'must-stock' but merely superior to *E*'s good ($0 < \lambda < 1$) is analysed in Section 3.

Incumbent firm *I* can offer three types of pricing schemes; (1) linear pricing, (2) two-part tariff or (3) conditional rebate. In the first pricing scheme, *I* offers a fixed price per unit, P_I^L where $P_I^L > 0$. In the second scheme, *I* offers a price schedule, (P_I^F, P_I^V), where $P_I^F > 0$ is the fixed cost that *B* has to pay when *B* purchases any amount from *I* and where $P_I^V \geq 0$ is the variable cost per unit. In the third scheme, *I* offers a price schedule, (P_I^R, β, γ), where $P_I^R \geq 0$ is a fixed price per unit, but if the purchase volume from *I* to total purchase volume is equal to or more than a threshold volume, $\gamma \in (\alpha, 1]$, the unit price for *I*'s good is discounted by $\beta \in (0, P_I^R]$. The rebate is therefore a retroactive rebate in the sense that the discount is applied to all units *B* purchases. Entrant *E* offers only a linear pricing scheme, $P_E \geq 0$. Since our interest is the conditional rebate such that the entry is blocked ($\gamma = 1$) and since such a rebate is more profitable than the exclusive rebate, such that $\gamma < 1$ holds in the baseline model analysed in Chapter 2, we assume γ is equal to 1, for simplicity. In other words, we consider the rebate only such that the discount is given on the condition that *B* does not purchase from *E* at all.

The rebate scheme analysed in this section is quantity-based unless otherwise specified. Note that when the demand size is always fixed to 1, a quantity-based rebate and a market-share-based rebate—in which a discount is given on the condition that the share of purchase volume from *I* to total purchase volume is equal to or more than a threshold—have equivalent effects. As discussed in Section 5, if there is uncertainty about the size of the contestable demand, a quantity-based rebate and a market-share-

[14] α must be strictly positive because some of the optimal prices obtained in this model are such prices that the denominator is α, the amount of the must-stock demand. This implies that if α is equal to zero, the price has a value that is divided by zero, meaning that the price becomes infinite.

based rebate have different effects. Hence, only in this case both a quantity-based rebate and a market-share-based rebate are analysed.

The process of the game is assumed to be as follows:

1) C_E is realised and known to I and E.

2) I offers a pricing scheme to B.

3) After observing the pricing scheme offered by I, E offers its price to B.

4) B determines whether it trades with I and E or not, and the quantities that B purchases from I and E. If the total cost for B is the same, B prefers to purchase E's good as much as possible.

3.3. Optimal Pricing Where the Must-Stock Nature of Incumbent's Product Is Limited

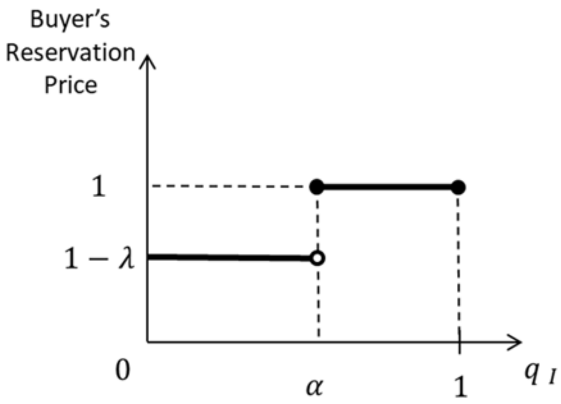
Chapter 2 analyses the baseline case where I's good is literally must-stock, and this analysis reveals the limitation that derives from the assumption of such strong "must-stock" nature. Taking that limitation into account, this section analyses the case where I's good is not literally must-stock, but is simply superior to competitors' products. Like the baseline model analysed in Chapter 2, assuming that the game is played only once, this section analyses an optimal behaviour of I when I offers (1) linear pricing, (2) a two-part tariff, and (3) a conditional rebate individually. Specifically, this section considers whether the extreme results persist such that the optimal pricing for I is set quite high so as to maximise profit from the must-stock part of the market. In the analysis of the cases where I offers (2) two-part tariff and (3) conditional rebate, only the equilibrium such that E is excluded is analysed. This is because there may exist another type of equilibrium, exploitation by I, which is equivalent to the exploitative equilibrium in a linear pricing case, as discussed in Chapter 2.

This section is organised as follows. Section 3.1 explains the assumptions of the model and the process of the game. Then, Section 3.2 analyses the optimal behaviours of the incumbent, the entrant and the buyer when the incumbent offers linear pricing, an exclusive two-part tariff and an exclusive conditional rebate individually. That section also analyses which pricings the incumbent prefers, and then in Section 3.3 concluding remarks are offered.

3.3.1 Assumptions

This section relaxes the assumption of such strong "must-stock" nature. To relax this assumption, we introduces a valuable, $\lambda \in (0,1)$, to measure the extent to which the must-stock property of I's product is strong. Specifically, it is assumed that if B fails to procure at least α units of the good from I, the value of 1 unit of the good for B is decreased by λ. Hence, if that is a case, B pays at most $1-\lambda$ to obtain 1 unit of the good. As such, in the case where α units of I's product are literally must-stock for B, as is analysed in Chapter 2, λ is equal to one. Hence, if the minimum total cost for B to purchase 1 unit of the good is equal to or less than its willingness to pay, B decides to buy. Otherwise, B will not purchase at all. The relationship between B's reservation price for 1 unit of the product and the volume that B purchases from I can be summarised in Figure 3.1 below.

Figure 3.1 Buyer's reservation value for 1 unit of the product



3.3.2 Analysis

This section analyses the game in which I can offer (1) linear pricing, (2) two-part tariffs or (3) conditional rebates. Analysis of this model by the backward induction approach provides the following proposition, which will be proved in the following sections.

Proposition 3.1. *Suppose the incumbent can introduce either linear pricing, a two-part tariff or conditional rebates.*

i) The incumbent is indifferent between exclusive two-part tariffs and exclusive conditional rebates.

ii) If either $a)$ $\lambda \geq 0.5$ *and* $C_E \geq 0.5$ *or* $b)$ $\lambda \leq 0.5$ *and* $C_E \geq min\left[\overline{C_E}, 0.5+\frac{\lambda}{\alpha}\right]$ *hold, there exist perfect Nash equilibria where the incumbent offers an exclusive conditional rebate, where* $\overline{C_E} = \frac{0.5(1-\alpha^2)+\alpha-\alpha\lambda}{1+\alpha-\alpha^2}$*. In the equilibria, the incumbent offers a conditional rebate such that*

$$(P_I^{R^*}, \beta^*) = \begin{Bmatrix} any\ pair\ of\ (P_I^R, \beta)\ that\ satisfies\ P_I^R - \beta = 1\ and\ P_I^R > \frac{1-C_E(1-\alpha)}{\alpha}\ if\ C_E \geq 1-\lambda \\ any\ pair\ of\ (P_I^R, \beta)\ that\ satisfies\ P_I^R - \beta = C_E + \lambda - \varepsilon\ and\ P_I^R > \frac{1-C_E(1-\alpha)}{\alpha}\ if\ C_E < 1-\lambda \end{Bmatrix},$$

where ε *is a small positive amount.*

Equivalently, the incumbent chooses the exclusive two-part tariff.

In this equilibrium, the entrant decides not to enter and the set of the buyer's purchasing volume is $(q_I^*, q_E^*) = (1, 0)$.

If neither $a)$ $\lambda \geq 0.5$ *and* $C_E \geq 0.5$ *nor* $b)$ $\lambda \leq 0.5$ *and* $C_E \geq min\left[\overline{C_E}, 0.5 + \frac{\lambda}{\alpha}\right]$ *hold, there exists a unique perfect Nash equilibria where the incumbent offers an exploitative pricing. The set of the prices and the buyer's purchasing volume in the equilibrium is given by*

$$(P_I^{L^*}, P_E^*, q_I^*, q_E^*) = \left\{\begin{array}{c} \left(\frac{1-C_E(1-\alpha)}{\alpha}, C_E, \alpha, 1-\alpha\right) \; if 1-\lambda < C_E < 0.5 \\ \left(\frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon, \frac{1-\lambda-C_E\alpha^2}{1-\alpha^2} + \varepsilon, \alpha, 1-\alpha\right) if \; 1 - \frac{\lambda}{\alpha^2} < C_E \leq 1-\lambda \\ \left(C_E + \frac{\lambda}{\alpha^2} - \varepsilon, C_E + \frac{(1-\alpha)\lambda}{\alpha^2} - \varepsilon, \alpha, 1-\alpha\right) if \; 0.5 - \frac{\lambda}{\alpha^2} < C_E \leq 1 - \frac{\lambda}{\alpha^2} \\ (+\infty, 1-\lambda, 0, 1) \; if \; C_E \leq 0.5 - \frac{\lambda}{\alpha^2} \end{array}\right\},$$

where $P_I^L = +\infty$ *denotes that the incumbent leaves the market and* ε *is a small positive amount.*

iii) If both $\lambda \leq 0.5$ *and* $min\left[\overline{C_E}, 0.5 + \frac{\lambda}{\alpha}\right] \leq C_E < 1 - \lambda$ *hold, the buyer's surplus is positive even though the incumbent chooses an exclusive conditional rebate. If either both* $\lambda \geq 0.5\frac{\alpha^2}{1+\alpha}$ *and* $0.5 - \frac{\lambda}{\alpha^2} < C_E < 1 - \frac{\lambda}{\alpha^2}$ *or* $\lambda < 0.5\frac{\alpha^2}{1+\alpha}$ *and* $0.5 - \frac{\lambda}{\alpha^2} < C_E < 0.5 + \frac{\lambda}{\alpha}$ *hold, the buyer's surplus is positive even though the incumbent chooses exploitative pricing. Otherwise, the buyer's surplus is zero.*

iv) If $C_E < min\left[1 - \lambda, \overline{C_E}, 0.5 + \frac{\lambda}{\alpha}\right]$ *hold, the entrant's profit is positive. Otherwise, the entrant's profit is zero.*

v) The improvement of the profit of the entrant due to a policy that prohibits two-part tariff and conditional rebate occurs only if

$a)$ *both* $0.5\frac{\alpha^2}{1+\alpha} \leq \lambda < 0.5$ *and* $\overline{C_E} < C_E < \frac{0.5(1-\alpha^2)+\alpha+\lambda}{1+\alpha-\alpha^2}$ *hold or*

$b)$ *both* $\lambda < 0.5\frac{\alpha^2}{1+\alpha}$ *and* $0.5 + \frac{\lambda}{\alpha} < C_E < 1 - \lambda$ *hold.*

The improvement of the surplus of the buyer by prohibiting a two-part tariff and conditional rebate happens only if

$a)$ *both* $\lambda \geq 0.5$ *and* $\widetilde{C_E} < C_E \leq 1$ *hold,*

$b)$ *both* $0.5\frac{\alpha^2}{1+\alpha} \leq \lambda < 0.5$ *and* $\frac{0.5(1-\alpha^2)+\alpha+\lambda}{1+\alpha-\alpha^2} < C_E \leq 1$ *hold or*

$c)$ *both* $\lambda < 0.5\frac{\alpha^2}{1+\alpha}$ *and* $\widetilde{C_E} < C_E \leq 1$ *hold, where* $\widetilde{C_E} = \frac{3-\alpha}{4-2\alpha}$.

If $d)$ *both* $0.5\frac{\alpha^2}{1+\alpha} \leq \lambda < 0.5$ *and* $\overline{C_E} \leq C_E \leq \frac{0.5(1-\alpha^2)+\alpha+\lambda}{1+\alpha-\alpha^2}$ *hold or*

$e)$ *both* $\lambda < 0.5\frac{\alpha^2}{1+\alpha}$ *and* $0.5 + \frac{\lambda}{\alpha} \leq C_E < 1 - \lambda$ *hold, the buyer becomes worse off by prohibiting the two-part tariff and conditional rebate.*

Proposition 3.1 *i)* shows that conditional rebates and two-part tariffs have an equivalent effect, as was found in the baseline model of Chapter 2.

Proposition 3.1 *ii)* shows that like in the baseline model, exclusive two-part tariffs or exclusive conditional rebates are chosen by *I* in the equilibrium when C_E is higher than a certain value. Proposition 3.1 *ii)* also implies that the profitability of exploitative pricing, exclusive two-part tariff, and exclusive conditional rebate is limited if $C_E < 1 - \lambda$, influenced by the possibility that *I* is excluded by *E*. It follows that if λ is small (i.e., the difference from the baseline model is large), *I*'s behaviour is more likely to be influenced by such possibility.

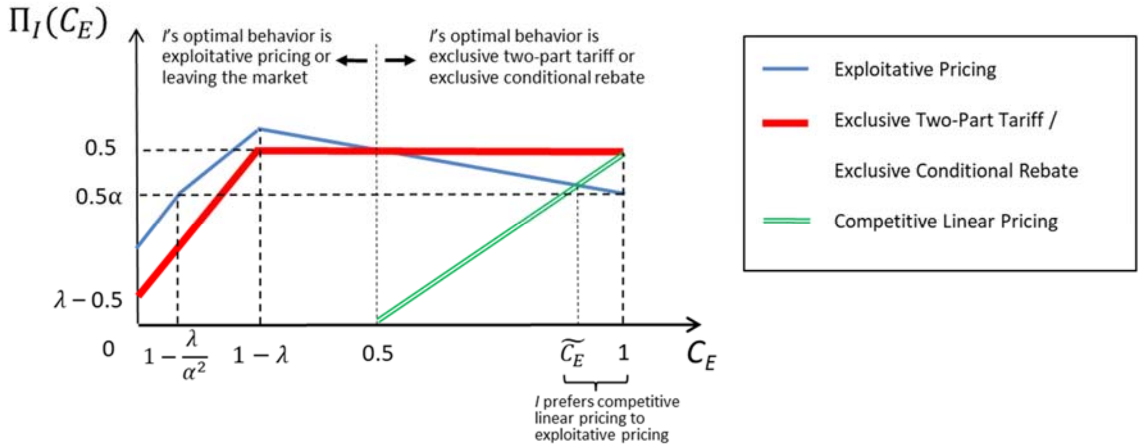
Moreover, Proposition 3.1 *ii)* shows that if *E* is extremely efficient ($C_E < 0.5 - \lambda/\alpha^2$), *I* leaves the market even in the equilibrium, which does not happen in the baseline model. This situation occurs because *I* anticipates that *E* will undercut its price to exclude *I* even when *I* sets the best exploitative pricing.

It is also shown that, as in the baseline model, *I* still does not have an incentive to exclude the more efficient *E* by a two-part tariff or conditional rebate. This incentive is lacking because if $C_E < 1 - \lambda$ holds while the possibility that *I* is excluded by *E* constrains the profitability of exploitative pricing, the possibility even more severely constrains the profitability of exclusive conditional rebates.

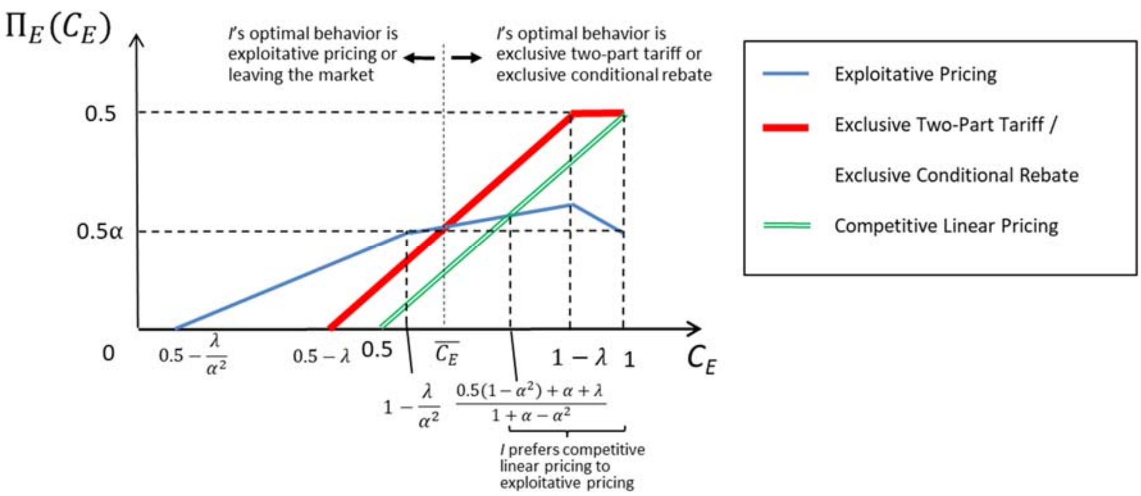
The profit of *I* for each type of equilibrium pricing and *E*'s profit and *B*'s surplus depending on *I*'s behaviours can be summarised as in Figures 3.2, 3.3 and 3.4. The details of each type of equilibrium pricing are discussed in subsequent sections.

Figure 3.2 Incumbent's profit for each type of pricing

$a)\ \lambda \geq 0.5$



$b)\ 0.5\frac{\alpha^2}{1+\alpha} \leq \lambda < 0.5$



$c)\ \lambda < 0.5\frac{\alpha^2}{1+\alpha}$

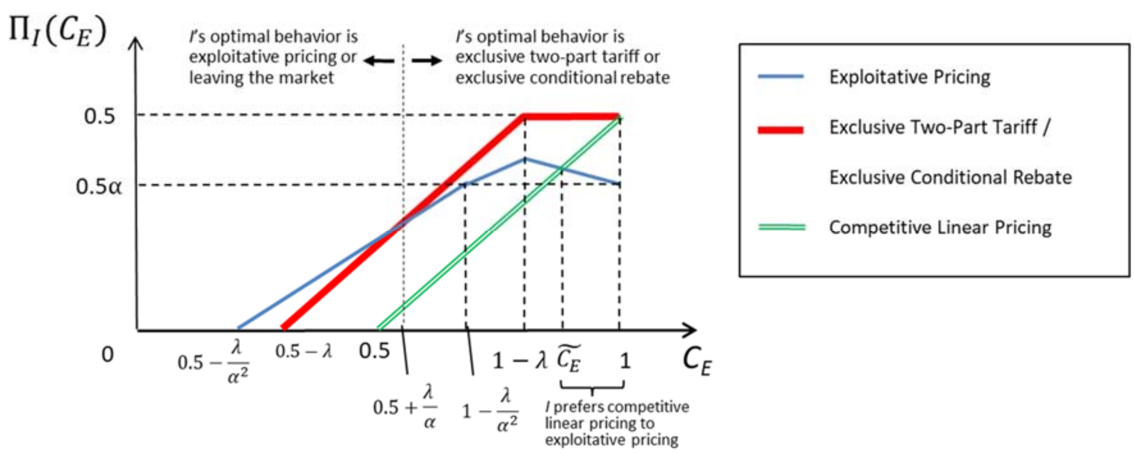
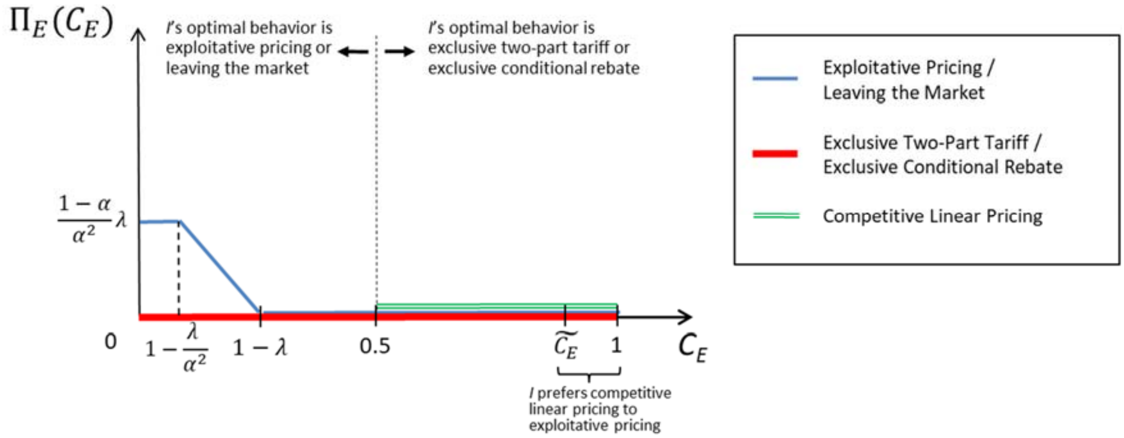
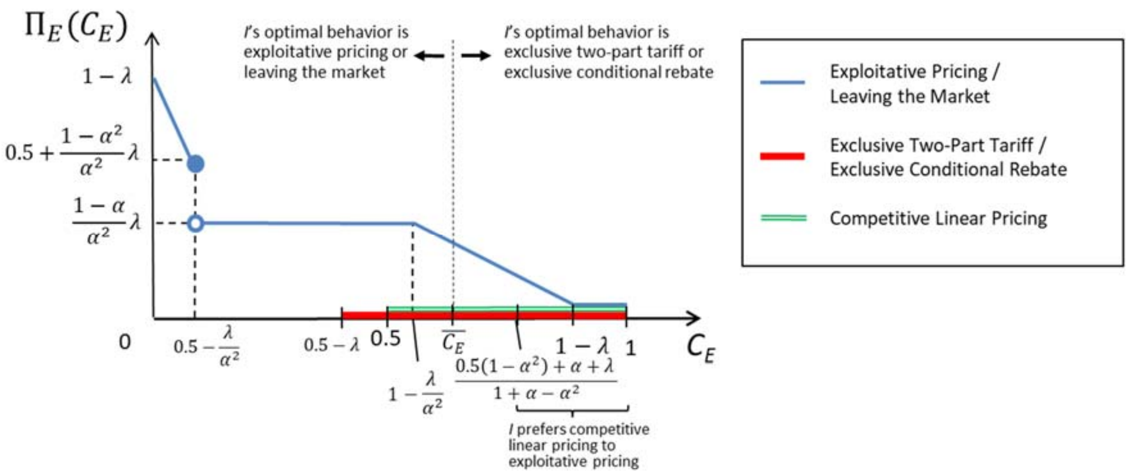


Figure 3.3 Entrant's profit when Incumbent offers competitive linear pricing, exploitative pricing and exclusive two-part tariffs or conditional rebates

$a)\ \lambda \geq 0.5$



$b)\ 0.5\frac{\alpha^2}{1+\alpha} \leq \lambda < 0.5$



$c)\ \lambda < 0.5\frac{\alpha^2}{1+\alpha}$

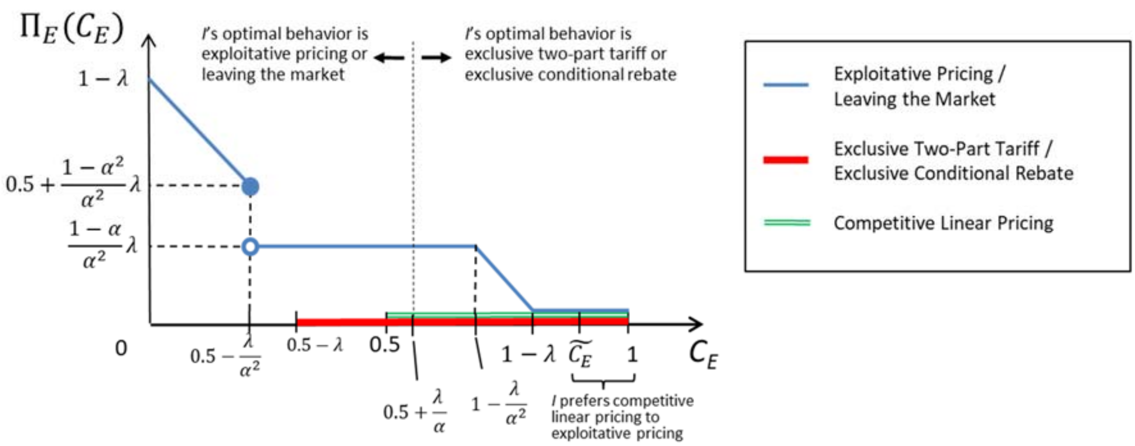
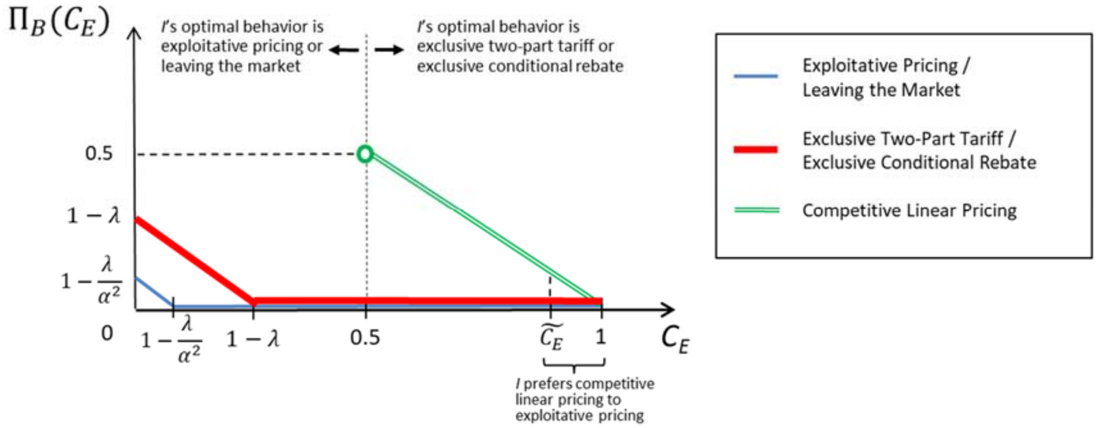
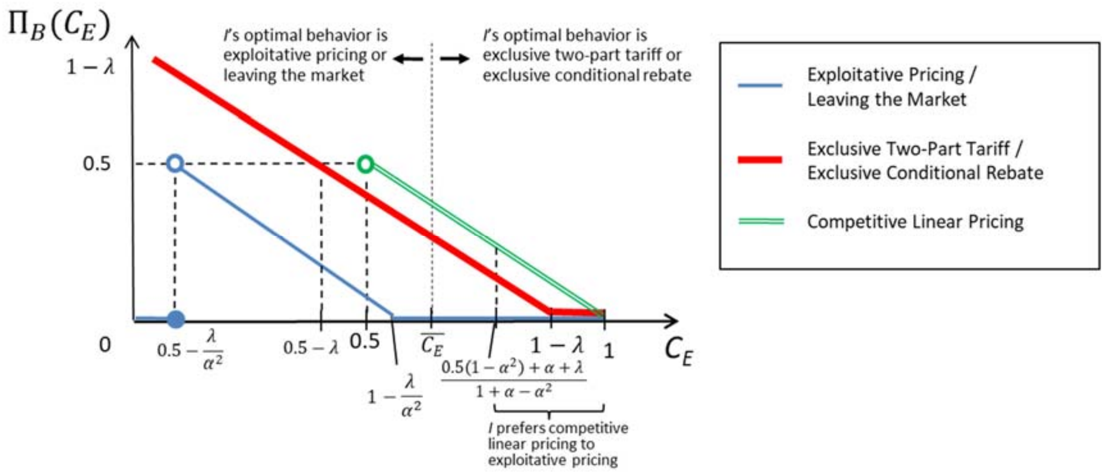


Figure 3.4 Buyer's surplus when Incumbent offers competitive linear pricing, exploitative pricing and exclusive two-part tariffs or conditional rebates

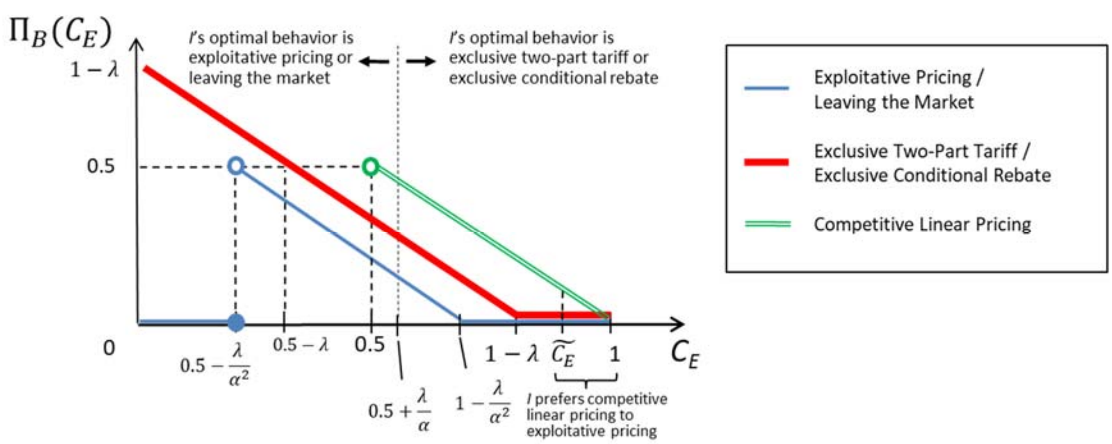
$a)\ \lambda \geq 0.5$



$b)\ 0.5\frac{\alpha^2}{1+\alpha} \leq \lambda < 0.5$



$c)\ \lambda < 0.5\frac{\alpha^2}{1+\alpha}$



The results summarised in Figure 3.2 show that if λ is large ($\lambda \geq 0.5$), the result is similar to the baseline model analysed in Chapter 2 in the sense that *I*'s optimal behaviour is exclusive conditional rebate or two-part tariff if $C_E \geq 0.5$ and exploitative pricing if $C_E < 0.5$. The result differs from the baseline model only when the entrant's cost is close to zero (i.e., $C_E < 1 - \lambda$). In this situation, the profitability of the exploitative pricing is limited by the possibility that *I* is excluded by *E*.

On the other hand, if λ is small ($\lambda < 0.5$), the situation becomes more complicated because *I*'s pricing is influenced by the possibility that *I* is excluded by *E* unless C_E is high. In this situation, exploitative pricing can be optimal even when the entrant is less efficient. In this situation, *I* even leaves the market in a state of equilibrium when C_E is close to zero ($C_E < 0.5 - \lambda/\alpha^2$), because the expected profit from exploitative pricing is negative. Note that *I* never leaves the market if $\lambda \geq 0.5$ holds.

With respect to *B*'s surplus, as summarised in Proposition 3.1 *iii),* if *I* chooses exploitative linear pricing in the equilibrium, *B* can have a positive surplus when $0.5 - \lambda/\alpha^2 < C_E < 1 - \lambda/\alpha^2$ holds. If *I* chooses the exclusive two-part tariff or conditional rebate in the equilibrium, *B* can have a positive surplus when $C_E < 1 - \lambda$ holds. Note that as long as C_E is in the range where those two types of pricing are optimal for *I*, *B*'s surplus increases as C_E becomes smaller. However, as in the baseline model, when *I* chooses exploitative linear pricing or exclusive conditional rebate in the equilibrium but the possibility that *E* dominates the market does not effectively constrain *I*'s behaviour, which can happen when $C_E \geq 1 - \lambda$ holds, *B*'s surplus is zero. Moreover, *B*'s surplus is also zero when $C_E < 0.5 - \lambda/\alpha^2$ holds, because in this situation *E* becomes a monopolist after *I* leaves the market.

With respect to *E*'s profit in the equilibrium, as summarised in Proposition 3.1 *iv),* the profit is zero when *I* chooses exclusive two-part tariffs or conditional rebates in the equilibrium, because the market entry is blocked. When $C_E < 1 - \lambda$ holds and *I* chooses exploitative pricing, while *I*'s profitability of exploitative pricing is limited by the possibility that *E* excludes *I*, *E* can obtain a positive profit instead. In this situation, as long as C_E is a little less than $1 - \lambda$, *E*'s surplus increases as C_E becomes smaller. However, if C_E decreases to $1 - \lambda/\alpha^2$, *E*'s surplus is constant with C_E because in this situation *B* enjoys the improvement of *E*'s efficiency.

Proposition 3.1 *v)* summarises the effect of prohibiting two-part tariffs and conditional rebates. In this case, the improvement of *E*'s profit and *B*'s surplus can occur. However, such improvement can happen only if *E* is less efficient than *I*. Moreover, if $\lambda < 0.5$, *B* can be worse off by prohibiting those price schemes when *E* is less efficient than the *I*. This is because in this situation, exclusive conditional rebate and two-part tariffs are more beneficial for the buyer than is exploitative pricing when $C_E < 1 - \lambda$ holds.

3.3.2.1 Optimal Linear Pricing

In this section, the game where both I and E offers a linear pricing to B is analysed. In this game I, E and B make their decisions in turn. The only difference between this case and the model analysed in the linear pricing case in the baseline model analysed in Chapter 2 is that in this case, B may purchase only from E when this option is the most attractive for B despite the decrease of B's willingness to pay by λ. It follows that the purchase volume of B from I and E, (q_I, q_E), must be either $(1,0)$, $(\alpha, 1-\alpha)$, $(0,0)$ or $(0,1)$. Hence, the optimal behaviour of I in this case can be obtained by analysing the situation in which B prefers to purchase only from E.

Analysis of this model by the backward induction approach provides the following proposition, which will be proved in subsequent sections.

Proposition 3.2. [Equilibrium linear pricing] *If the incumbent offers a linear price to the buyer, the set of the subgame perfect Nash equilibrium can be written as follows:*

a) If $\widetilde{C_E} > 1-\lambda$,

$$(P_I^{L^*}, {P_E}^*, {q_I}^*, {q_E}^*) = \left\{\begin{array}{c} (C_E-\varepsilon, +\infty, 1, 0) if\ C_E > \widetilde{C_E} \\ \left(\frac{1-C_E(1-\alpha)}{\alpha}, C_E, \alpha, 1-\alpha\right)\ if 1-\lambda < C_E \le \widetilde{C_E} \\ \left(\frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon, \frac{1-\lambda-C_E\alpha^2}{1-\alpha^2} + \varepsilon, \alpha, 1-\alpha\right) if\ 1-\frac{\lambda}{\alpha^2} < C_E \le 1-\lambda \\ \left(C_E + \frac{\lambda}{\alpha^2} - \varepsilon, C_E + \frac{(1-\alpha)\lambda}{\alpha^2} - \varepsilon, \alpha, 1-\alpha\right) if\ 0.5 - \frac{\lambda}{\alpha^2} < C_E \le 1-\frac{\lambda}{\alpha^2} \\ (+\infty, 1-\lambda, 0, 1)\ if\ C_E \le 0.5 - \frac{\lambda}{\alpha^2} \end{array}\right\},$$

where $\widetilde{C_E} = \frac{3-\alpha}{4-2\alpha}$, ε *is a small positive amount and* $P_{i\in I,E} = +\infty$ *denotes that i decides to leave or not to enter.*

b) If $1-\lambda \ge \widetilde{C_E}$,

$$(P_I^{L^*}, {P_E}^*, {q_I}^*, {q_E}^*) = \left\{\begin{array}{c} (C_E-\varepsilon, +\infty, 1, 0)\ if\ C_E > \overline{C_E} \\ \left(\frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon, \frac{1-\lambda-C_E\alpha^2}{1-\alpha^2} + \varepsilon, \alpha, 1-\alpha\right) if\ 1-\frac{\lambda}{\alpha^2} < C_E \le \overline{C_E} \\ \left(C_E + \frac{\lambda}{\alpha^2} - \varepsilon, C_E + \frac{(1-\alpha)\lambda}{\alpha^2} - \varepsilon, \alpha, 1-\alpha\right) if\ 0.5 - \frac{\lambda}{\alpha^2} < C_E \le 1-\frac{\lambda}{\alpha^2} \\ (+\infty, 1-\lambda, 0, 1)\ if\ C_E \le 0.5 - \frac{\lambda}{\alpha^2} \end{array}\right\},$$

where $\overline{C_E} = \frac{0.5(1-\alpha^2)+\alpha+\lambda}{1+\alpha-\alpha^2}$, ε *is a small positive amount and* $P_{i\in I,E} = +\infty$ *denotes that i decides to leave or not to enter.*

As shown in Proposition 3.2, five types of equilibria exist, corresponding to the five types of pricing behaviours for I: competitive linear pricing, unconstrained exploitative linear pricing, first-type constrained exploitative linear pricing, second-type constrained exploitative linear pricing and leaving the market. Based on these behaviours, the outcome of the game can be classified into the following five outcomes; while the first two outcomes are equivalent to the possible outcomes in the baseline model analysed in Chapter 2, the last three outcomes do not arise in the baseline model:

1) Competitive exclusion by *I* (*I*'s optimal behaviour: *competitive linear pricing*)

This outcome arises when C_E is quite high ($C_E > \min\left[\widetilde{C_E}, \overline{C_E}\right]$). When C_E has such a high value, *I* prefers to undercut its price low enough to exclude the possibility of the entrance of the market by *E*, which is less efficient than *I*. In this case, *B* enjoys a non-negative profit derived from the competition (i.e., threat of entry), although *E* gets nothing. Hence, in this case, the surplus of the economy is shared by *I* and *B*.

2) Unconstrained exploitation by *I* (*I*'s optimal behaviour: *unconstrained exploitative linear pricing*)

This outcome occurs when C_E is low enough that exploitative linear pricing is more profitable than competitive linear pricing, but so high that competitive pressure is not applied by the possibility that *E* dominates the market ($C_E > 1 - \lambda$). In this situation, *I* can fully exploit all the surplus of the market. Hence, although *E* enters the market, the surpluses of *E* and *B* are zero, respectively.

3) Joint full exploitation by *I* and *E* (*I*'s optimal behaviour: *first-type constrained exploitative linear pricing*)

This outcome occurs when C_E decreases to the point where *I* can no longer extract all the surplus due to the threat that *E* could attract all *B*'s demand, including the must-stock part of the market if *I*'s price is too high. This threat constrains *I*'s ability to extract all the benefit of *E*'s efficiency from the must-stock part of the market. In this case, *I* can obtain the positive profit, but *I* can no longer dominate the entire surplus, while *E* can obtain a non-negative profit, which is equal to the profit *I* loses. Buyer *B*'s surplus is always zero, which implies that the benefits of the economy are shared by *I* and *E*.

4) Joint partial exploitation by *I* and *E* with non-negative profit for *B* (*I*'s optimal behaviour: *second-type constrained exploitative linear pricing*)

This outcome occurs when C_E is further decreased to the point where first-type constrained exploitative linear pricing is no longer optimal for *I* due to the threat that *E* could attract all *B*'s demand, but *I* can still obtain non-negative profit from exploitative linear pricing. In this case, *I* and *E* can obtain non-negative profits, but the joint profit of the two is not equal to the total surplus. Instead, *B* obtains non-negative profit owing to competitive pressure. Hence, in this situation, *I*, *E* and *B* share the surplus of the economy. Note that one of the conditions for the fourth segment to exist is $\lambda < \alpha^2$.

5) Unconstrained exploitation by *E* (*I*'s optimal behaviour: leaving the market)

This outcome happens when C_E is so low that *I* leaves the market because *I* cannot compete with such an efficient entrant. In this case, *E* dominates the market. It follows that E obtains the entire surplus and that I and B get nothing. Note that one of the conditions by which this outcome can occur is $\lambda < 0.5\alpha^2$.

The last three types of equilibria do not happen in the baseline model analysed in Chapter 2. Their absence shows that when C_E is low, the likelihood increases that E excludes I or constrains I's ability to extract all the benefit of E's efficiency, from which follows the various outcomes, limited exploitation and E's dominance.

Proposition 3.2 also shows that the result differs slightly depending on whether the threshold point of C_E —where I's profits from competitive linear pricing and unconstrained exploitative linear pricing are equal, $\widetilde{C_E}$—is larger than $1-\lambda$, the threshold point where the optimal pricing changes from unconstrained exploitative linear pricing to first-type constrained exploitative linear pricing.

In order to prove Proposition 3.2, first analyse B's optimal behaviour. The optimal response to B given I and E's offers is shown by Lemma 3.1 below.

Lemma 3.1. *Given P_I^L and P_E, the buyer's optimal purchasing volume is*

$$(q_I{}^*, q_E{}^*) = \begin{cases} (1,0)\ if\ P_I^L < P_E\ and\ P_I^L \le 1 \\ (\alpha,\ 1-\alpha)\ if\ P_I^L - \frac{\lambda}{\alpha} < P_E \le P_I^L\ and\ P_I^L\alpha + P_E(1-\alpha) \le 1 \\ (0,1)\ if\ P_E \le P_I^L - \frac{\lambda}{\alpha}\ and\ P_E \le 1-\lambda \\ (0,0)\ otherwise \end{cases}.$$

Proof: See Appendix B

Lemma 3.1 implies that the minimum price that E needs to set in order to sell 1 unit to B is equal to I's price subtracted by the decrease of B's willingness to pay for 1 unit when B's purchases from only E are divided by the demand size of the must-stock item (i.e., λ/α).

Entrant E chooses its optimal behaviour accounting for B's expected response, as summarised in Lemma 3.1. In order to analyse E's best response given I's price, consider the maximum profits when E sells $1-\alpha$ units and 1 unit individually, and compare them. Entrant E's optimal behaviour and profit when it decides to sell $1-\alpha$ units is analysed in Chapter 2 as summarised in Lemma 2.2. Then, consider E's optimal behaviour when it decides to sell 1 unit. From Lemma 3.1 and the definitions of the model, the following conditions need to be met for E to sell 1 unit to B without incurring a loss:

1) E's price must be no higher than I's price subtracted by the decrease of B's willingness to pay divided by α ($P_E \le P_I^L - \lambda/\alpha$) as shown in Equation (2.3).

2) E's price must be low enough to make the buyer's demand strictly positive. The condition is $P_E \leq 1 - \lambda$

3) E's unit price covers its cost ($P_E \geq C_E$)

Given B's inelastic demand, E charges the highest possible price that meets the three conditions above as long as there exist such P_E. Otherwise, E decides not to enter the market.

E's optimal pricing given can be written as Lemma 3.2 below.

Lemma 3.2. *Given P_I^L, the entrant's optimal pricing is*

$$P_E{}^* = \left\{ \begin{array}{c} P_I^L \ if \ C_E \leq P_I^L \leq min\left[C_E + \frac{\lambda}{\alpha^2} - \varepsilon, 1\right] \\ \frac{1-P_I^L\alpha}{1-\alpha} \ if \ 1 < P_I^L \leq min\left[\frac{1-(1-\alpha)C_E}{\alpha}, \frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon\right] \\ P_I^L - \frac{\lambda}{\alpha} \ if \ min\left[C_E + \frac{\lambda}{\alpha^2}, \frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right)\right] \leq \ P_I^L \leq 1 + \frac{1-\alpha}{\alpha}\lambda \\ 1 - \lambda \ if P_I^L > 1 + \frac{1-\alpha}{\alpha}\lambda \ and \ C_E \leq 1 - \lambda \end{array} \right\},$$

where ε is a small positive amount.

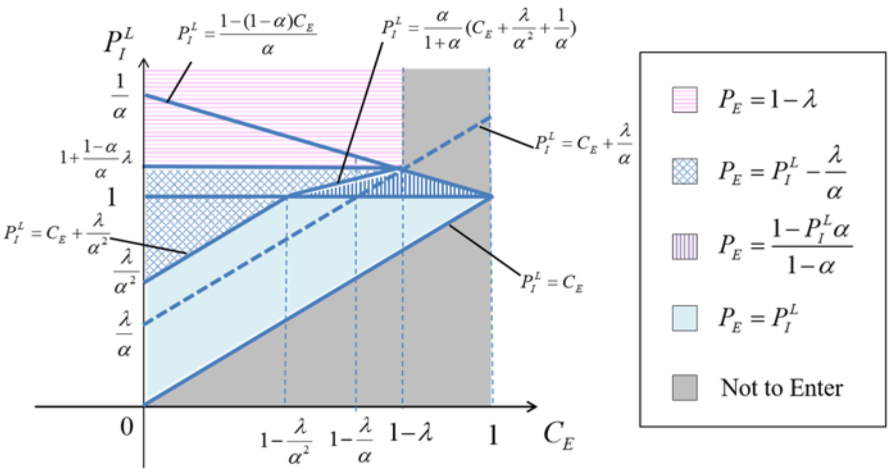
If $P_I^L < C_E$ or if $P_I^L > \frac{1-(1-\alpha)C_E}{\alpha}$ and $C_E > 1 - \lambda$, E decides not to enter the market.

Proof: See Appendix B

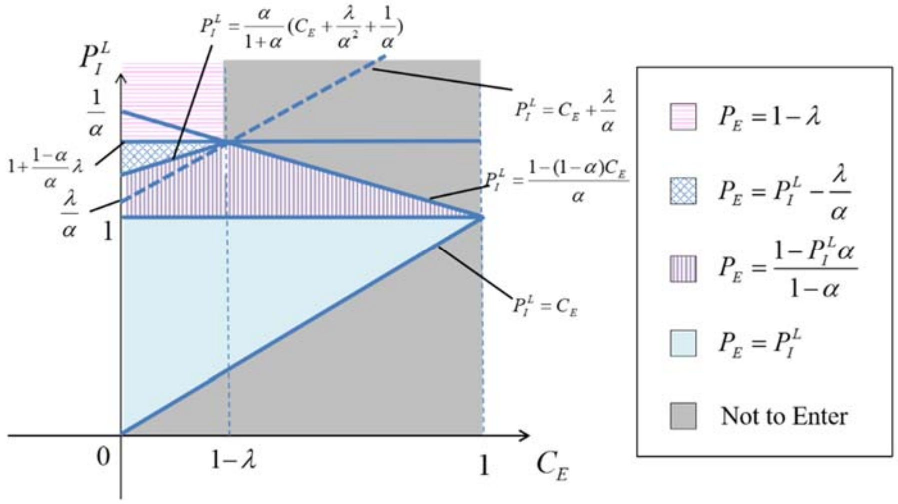
The optimal E's behaviour in Lemma 3.2 can be summarised in Figure 3.5 below.

Figure 3.5 Entrant's optimal response given Incumbent's price

i) if $\lambda < \alpha$



ii) if $\lambda \geq \alpha$



Lemma 3.2 and Figure 3.5 show that E dominates the market if E's marginal cost is lower than B's willingness to pay for 1 unit when B purchases only from E (i.e., $C_E \leq 1 - \lambda$) and I's price is high enough that selling 1 unit by setting at a lower price is more profitable than selling α units (i.e., $P_I^L \geq \min[C_E + \lambda/\alpha^2, [\alpha/(1+\alpha)][C_E + \lambda/\alpha^2 + 1/\alpha]]$).

In the linear pricing model analysed in Chapter 2, I's optimal behaviours can be divided into two types: competitive linear pricing or exploitative linear pricing. When I introduces competitive linear pricing ($P_I^L = C_E - \varepsilon$), there is no room for E to sell any unit to B. On the other hand, Lemma 2.2 implies exploitative linear pricing may be constrained by competitive pressure such that E dominates the market. In Chapter 2, when I uses exploitative pricing, I fully exploits E's efficiency by setting a price such that E will enter the market by setting its lowest possible price, C_E and B's total cost is equal to its willingness to pay (*unconstrained exploitative linear pricing*). However, in this model, unconstrained exploitative linear pricing may not be possible. If that is the case, I must set its price low enough to guarantee that E prefers not to dominate the market.

Incumbent firm I's optimal behaviour taking such constraint into account is summarised in Lemma 3.3.

Lemma 3.3. *The incumbent's optimal pricing is*

a) if $1 - \lambda > \widetilde{C_E}$,

$$P_I^{L^*} = \left\{ \begin{array}{c} C_E - \varepsilon \; if \, C_E > \widetilde{C_E} \\ \frac{1-C_E(1-\alpha)}{\alpha} \; if \, 1-\lambda < C_E \leq \widetilde{C_E} \\ \frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon \; if \; 1 - \frac{\lambda}{\alpha^2} < C_E \leq 1 - \lambda \\ C_E + \frac{\lambda}{\alpha^2} - \varepsilon \; if \; 0.5 - \frac{\lambda}{\alpha^2} < C_E \leq 1 - \frac{\lambda}{\alpha^2} \end{array} \right\}, \text{ where } \widetilde{C_E} = \frac{3-\alpha}{4-2\alpha}; \text{ or}$$

b) if $1 - \lambda < \widetilde{C_E}$,

$$P_I^{L^*} = \begin{Bmatrix} C_E - \varepsilon \; if \, C_E > \frac{0.5(1-\alpha^2)+\alpha+\lambda}{1+\alpha-\alpha^2} \\ \frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon \; if \; 1 - \frac{\lambda}{\alpha^2} < C_E \le \frac{0.5(1-\alpha^2)+\alpha+\lambda}{1+\alpha-\alpha^2} \\ C_E + \frac{\lambda}{\alpha^2} - \varepsilon \; if \; 0.5 - \frac{\lambda}{\alpha^2} < C_E \le 1 - \frac{\lambda}{\alpha^2} \end{Bmatrix}.$$

If $C_E \le 0.5 - \frac{\lambda}{\alpha^2}$, I *decides to leave the market. The entrant enters the market only if* $C_E \le \frac{0.5(1-\alpha^2)+\alpha+\lambda}{1+\alpha-\alpha^2}$.

Proof: See Appendix B

Lemma 3.3 shows that similar to the baseline model in Chapter 2, *I* will choose either to set a competitive price to capture the entire market (competitive linear pricing) or to exploit the profit from the must-stock part of the market (exploitative linear pricing). The difference from the baseline model in Chapter 2 is that the possibility that *E* can dominate the market does constrain *I*'s ability to extract all the benefits of *E*'s efficiency and, hence, such unconstrained exploitative linear pricing may not be optimal behaviour for *I*.

This situation happens occurs if $C_E \le 1 - \lambda$. If C_E is slightly lower than $1 - \lambda$, *I*'s optimal exploitative pricing is to set ${P_I}^L$ slightly lower than $[\alpha/(1+\alpha)][C_E + \lambda/\alpha^2 + 1/\alpha]$ ("first-type constrained exploitative linear pricing"), because *I* needs to set a price low enough to guarantee that *E* sets a price such that *E* sells only α units rather than setting a price low enough to exclude *I*. If C_E is lower than a certain point, $1 - \lambda/\alpha^2$, *I*'s optimal exploitative pricing changes. In this situation, *I* sets ${P_I}^L$ slightly lower than $C_E + \lambda/\alpha^2$ ("second-type constrained exploitative linear pricing").

If C_E is further down to $0.5 - \lambda/\alpha^2$, *I* decides to leave the market, because in this situation, *I* can obtain non-negative profits by neither exploitative linear pricing nor competitive linear pricing.

Proposition 3.2 follows directly from Lemmata 3.1–3.3.

3.3.2.2 Optimal Two-Part Tariff

This section analyses the game that while *E* offers linear pricing to *B, I* offers a two-part tariff, (P_I^F, P_I^V), where $P_I^F > 0$ is the fixed cost that *B* has to pay when purchasing any amount from *I* and $P_I^V \ge 0$ is the variable cost per unit. The only difference between this case and the two-part tariff case analysed in Chapter 2 is that in this case, *B* may purchase only from *E* when this option is the most attractive for *B*. As well as the linear pricing case analysed in the previous section, the purchase volume of *B* from *I* and *E*, (q_I, q_E), must be either (1,0), $(\alpha, 1-\alpha)$, (0,0) or (0,1). Hence, the optimal behaviour of I in this case can be obtained by analysing in what situation B prefers to purchase only from E.

Analysis of this model through backward induction provides the following proposition, which will be proved in subsequent sections.

Proposition 3.3. [Optimal exclusive two-part tariff] *The incumbent can exclude the entrant by offering the two-part tariff such that both* $P_I^F + P_I^V = min[1, C_E + \lambda - \varepsilon]$ *and* $P_I^V < C_E$ *hold and can obtain a profit of* $min[0.5, C_E + \lambda - 0.5 - \varepsilon]$ *as long as* $C_E \geq 0.5 - \lambda$*, where* ε *is a small non-negative amount.*

Proof: See Appendix B

Proposition 3.3 shows there are two types of equilibria in which E is excluded from the market, and those equilibria correspond to the following two types of exclusive pricing behaviour for I; unconstrained exclusive two-part tariffs and constrained exclusive two-part tariffs. Based on this proposition, the outcome of the game can be classified into the following two outcomes. While the first outcome is equivalent to the possible outcome in the baseline model analysed in Chapter 2, the second outcome does not occur in the baseline model.

Note that I does choose the exclusive two-part tariff only if the profit is higher than that of exploitative pricing. The condition is shown in Proposition 3.1.

1') Unconstrained exclusion by I (I's optimal behaviour: *unconstrained exclusive two-part tariff*)

This outcome arises when C_E is high ($C_E > \min[0.5, 1 - \lambda]$). In this case, I can capture the entire surplus, although E and B get nothing. This result is maintained as long as C_E does not fall below a certain point, 0.5.

2') Constrained exclusion by I (I's optimal behaviour: *constrained exclusive two-part tariff*)

This outcome happens when C_E is decrease to the point where the competitive pressure by the possibility that E dominates the market ($C_E \leq 1 - \lambda$) works, but is still high enough that an exclusive two-part tariff is more profitable than exploitative pricing ($C_E > 0.5$). In this case, though I can exclude E, I no longer captures the entire surplus of the economy. The rest of the surplus is shared by B.

Note that if $C_E < 0.5 - \lambda$ holds, I prefers not to introduce an exclusive two-part tariff because the profit will be negative.

This result shows that when C_E is low, the possibility that E excludes I constrains I's ability to obtain profit from the two-part tariff, from which follows that the divergent outcomes, with limited profitability from the exclusive two-part tariff, may arise.

3.3.2.3 Optimal Conditional Rebate

In this section, the game considered is that while E offers a linear pricing to B, I offers a conditional rebate scheme, (P_I^R, β), where $P_I^R \geq 0$ is a pre-discount price per unit, and $\beta \in (0, P_I^R]$ is the discount per unit B can obtain if B purchases 1 unit form I. The only difference between this case and the two-part tariff case analysed in Chapter 1 is that in this case, B may purchase only from E when this option is the most attractive for B. As well as the linear pricing case analysed in the previous section, the purchase volume of B from I and E, (q_I, q_E), must be either $(1,0)$, $(\alpha, 1-\alpha)$, $(0,0)$ or $(0,1)$. Hence, the optimal behaviour of I in this case can be obtained by analysing in what situations B prefers to purchase only from E.

Analysis on this model by the backward induction approach provides the following proposition, to be proved in the subsequent sections.

Proposition 3.4. [Optimal exclusive conditional rebate] *The incumbent can exclude the entrant by offering the conditional rebate such that both* $P_I^R - \beta = min[1, C_E + \lambda - \varepsilon]$ *and* $P_I^R > [1 - C_E(1-\alpha)]/\alpha$ *hold and can obtain a profit of* $min[0.5, C_E + \lambda - 0.5 - \varepsilon]$, *where* ε *is a small non-negative amount.*

Proposition 3.4 shows that a conditional rebate has the same effects as a two-part as in the baseline model in Chapter 2. In specific, there are two types of equilibria in which E is excluded from the market, corresponding to the following two types of exclusive pricing behaviours from I; unconstrained exclusive conditional rebates and constrained exclusive conditional rebates. Based on this result, the outcome of the game can be classified into the following two outcomes. While the first outcome is equivalent to the possible outcome in the baseline model analysed in Chapter 2, the second outcome does not occur in the baseline model.

Note that I does choose the exclusive conditional rebate only if the profit is higher than that of exploitative pricing. The condition is shown in Proposition 3.1.

1') Unconstrained exclusion by I (I's optimal behaviour: *unconstrained exclusive conditional rebate*)

This outcome arises when C_E is high ($C_E > \min[0.5, 1-\lambda]$). In this case, I can capture the entire surplus, though E and B get nothing. This result is maintained as long as C_E does not fall below a certain point, 0.5.

2') Constrained exclusion by I (I's optimal behaviour: *constrained exclusive conditional rebate*)

This outcome happens when C_E decreases to the point where the competitive pressure from the possibility that E dominates the market ($C_E \leq 1-\lambda$) works, but C_E remains high enough that exclusive conditional rebates are more profitable than exploitative pricing ($C_E > 0.5$). In this case, although *I* can exclude *E*, *I* no longer captures the entire surplus of the economy. The rest of the surplus is shared by *B*.

This shows that when C_E is low, the possibility that *E* excludes *I* constrains *I*'s ability to obtain large profit from the exclusive conditional rebate, from which follows that the divergent outcomes, with limited profitability from the exclusive conditional rebate, may arise.

Note that if $C_E < 0.5 - \lambda$ holds, *I* prefers not to introduce exclusive two-part tariff because the profit will be negative.

Proof: See Appendix B

Note also that Propositions 3.3 and 3.4 show that the profit is the same as the profit from the exclusive two-part tariff.

3.3.2.4 Endogenous Pricing Type

According to the results shown in Propositions 3.2, 3.3 and 3.4, the surplus of *I*, *E* and *B* when *I* introduces competitive linear pricing, exploitative linear pricing , exclusive two-part tariffs and exclusive conditional rebates can be summarised in Table 3.1 and 3.2 below.

Table 3.1 Surplus of each player when Incumbent offers linear pricing

$a)\ 1 - \lambda < \widetilde{C_E}$

C_E	Optimal *I*'s Behaviour	*I*'s profit	*E*'s profit	*B*'s surplus
$\widetilde{C_E} < C_E \leq 1$	1) Competitive linear pricing	$C_E - 0.5 - \varepsilon$	0	$1 - C_E + \varepsilon$
$1 - \lambda < C_E \leq \widetilde{C_E}$	2) Unconstrained exploitative linear pricing	$(1-\alpha)(1-C_E) + 0.5\alpha$	0	0
$1 - \frac{\lambda}{\alpha^2} < C_E \leq 1 - \lambda$	3) 1st type constrained exploitative linear pricing	$\frac{\alpha^2 C_E + \lambda + 0.5\alpha(1-\alpha)}{1+\alpha} - \varepsilon$	$\frac{1-\lambda-C_E}{1+\alpha} - \varepsilon$	0
$0.5 - \frac{\lambda}{\alpha^2} < C_E \leq 1 - \frac{\lambda}{\alpha^2}$	4) 2nd type constrained exploitative linear pricing	$\alpha C_E + \frac{\lambda}{\alpha} - 0.5\alpha - \varepsilon$	$\frac{(1-\alpha)\lambda}{\alpha^2} - \varepsilon$	$1 - \frac{\lambda}{\alpha^2} - C_E + \varepsilon$
$0 < C_E \leq 0.5 - \frac{\lambda}{\alpha^2}$	5) Leaving the market	0	$1 - \lambda - C_E$	0

$b)\ 1-\lambda \geq \widetilde{C_E}$

C_E	Optimal I's Behaviour	I's profit	E's profit	B's surplus
$\acute{C_E} < C_E \leq 1$	1) Competitive linear pricing	$C_E - 0.5 - \varepsilon$	0	$1 - C_E + \varepsilon$
$1 - \frac{\lambda}{\alpha^2} < C_E \leq \acute{C_E}$	3) 1st type constrained exploitative linear pricing	$\frac{\alpha^2 C_E + \lambda + 0.5\alpha(1-\alpha)}{1+\alpha} - \varepsilon$	$\frac{1-\lambda-C_E}{1+\alpha} - \varepsilon$	0
$0.5 - \frac{\lambda}{\alpha^2} < C_E \leq 1 - \frac{\lambda}{\alpha^2}$	4) 2nd type constrained exploitative linear pricing	$\alpha C_E + \frac{\lambda}{\alpha} - 0.5\alpha - \varepsilon$	$\frac{(1-\alpha)\lambda}{\alpha^2} - \varepsilon$	$1 - \frac{\lambda}{\alpha^2} - C_E + \varepsilon$
$0 < C_E \leq 0.5 - \frac{\lambda}{\alpha^2}$	5) Leaving the market	0	$1 - \lambda - C_E$	0

Table 3.2 Surplus of each player when Incumbent offers exclusive two-part tariff or conditional rebate

C_E	Optimal I's Behaviour	I's surplus	E's profit	B's surplus
$1 - \lambda < C_E \leq 1$	1') Unconstrained exclusive two-part tariff or conditional rebate	0.5	0	0
$0.5 - \lambda < C_E \leq 1 - \lambda$	2') Constrained exclusive two-part tariff or conditional rebate	$C_E + \lambda - 0.5$	0	$1 - \lambda - C_E$
$0 < C_E \leq 0.5 - \lambda$	Not using exclusive two-part tariff or conditional rebate	0	$1 - \lambda - C_E$	0

By comparing I's optimal profit when I introduces linear pricing and exclusive conditional rebates as summarised in Tables 3.1 and 3.2 and Figures 3.2, 3.3 and 3.4, Proposition 3.1 can be proven.

3.3.3 Effect on Incumbent's Ability to Set Extremely High Prices

The baseline model in Chapter 2 makes a prediction that seems unlikely to happen in the sense that the incumbent sets an extremely high pre-discount price when it excludes E's entry by conditional rebates or extremely high prices, exploiting the entrant's efficiency. This section examines to what extent the possibility that *I* is excluded by *E* can limit the ability to set extremely high prices.

The results in the model analysed in this chapter show that the profitability of such behaviour is constrained by the competitive pressure that *E* dominates the market undermining the profitability of exploitative pricing when C_E is smaller than $1 - \lambda$, in contrast to the baseline model analysed in Chapter 2. For instance, if $C_E = 0.4$, $\alpha = 0.5$ and $\lambda = 0.3$, the optimal linear pricing for *I* is to set its price at 1.2, while the optimal exploitative price in the baseline model is 1.6. If $C_E = 0.4$ and $\alpha = 0.2$ and $\lambda = 0.3$, the optimal exploitative linear pricing for *I* is to set its price at 2.15, while that the optimal exploitative price in the baseline model is 3.4.

Moreover, such competitive pressure also has an influence on the profitability of exclusive conditional rebates when C_E is smaller than $1 - \lambda$. For example, assume that *I* will set the lowest possible P_I^R when it prefers exclusive conditional rebates and that $\varepsilon \geq 0.001$ must be satisfied. In these circumstances, if $C_E = 0.5$, $\alpha = 0.8$ and $\lambda = 0.3$, the incumbent will exclude *E* and obtain the profit of 0.5 by exclusive conditional rebates, such that $P_I^R = 0.875$ and $\beta = 0.076$, which are 1.126 and 0.126 in the baseline model. If $C_E = 0.5$, $\alpha = 0.1$ and $\lambda = 0.3$, the incumbent will exclude E by setting P_I^R at 3.5 and β at 2.701, which are 5.501 and 4.501 in the baseline model. While exclusive conditional rebate is optimal for *I* when $C_E = 0.5$ in the baseline model, in this case, exclusive conditional rebate is not optimal when $C_E = 0.5$ and $\lambda < 0.5$. If $C_E = 0.1$, $\alpha = 0.1$ and $\lambda = 0.3$, the incumbent will exclude E by setting P_I^R at 3.1 and β at 2.701, which are 9.101 and 8.101 in the baseline model, although such pricing gives *I* negative profit. The relationship between the optimal nominal price, P_I^R, and the optimal discounted price, $P_I^R - \beta$, with C_E is shown by Figure 3.4 below.

When the difference from the baseline model is small but the entrant is efficient enough to enter the market (i.e., $C_E < 1 - \lambda < 0.5$), the competitive pressure still works. For example, if $C_E = 0.1$, $\alpha = 0.8$ and $\lambda = 0.8$, the incumbent's optimal pricing is exploitative pricing such that the unit price is approximately equal to 1.15, corresponding to 1.225 in the baseline model. If $C_E = 0.1$, $\alpha = 0.1$ and $\lambda = 0.8$, the incumbent's optimal pricing is exploitative pricing such that the unit price is approximately equal to 8.19, which is 9.1 in the baseline model. If $C_E < 1 - \lambda < 0.5$ holds, exclusive conditional rebates are not compatible with equilibrium, because exploitative pricing is always more profitable.

On the other hand, if C_E is weakly larger than $1 - \lambda$, the incumbent's optimal behaviours are the same as the baseline model, because the possibility that the entrant dominates the market does not effectively constrain the incumbent's behaviour. It follows that when the difference from the baseline model is small (i.e., λ is close to 1), the incumbent is more likely to set an extremely high price.

Therefore, under this relaxed assumption about the must-stock property, the incumbent's optimal pricing can have a more realistic values, although such a result occurs only when $C_E \leq 1 - \lambda$. Moreover, as with the baseline model, the optimal price becomes lower when α is high. These results imply that the equilibrium price is more likely to have a realistic value when λ is low or α is high.

3.4. Optimal Pricing Where There Is Uncertainty about Must-stock Proportion

Chapter 2 analyses the baseline case where there is no uncertainty, and the result shows that *I* does not have an incentive to exclude the more efficient entrant by two-part tariffs or conditional rebates, because exploitative pricing is more profitable. This section analyses the case in which uncertainty exists about must-stock proportion, α, and shows that *I* may have an incentive to exclude the more efficient entrant with two-part tariffs and conditional rebates.

This section analyses an optimal behaviour of *I* when *I* offers (1) linear pricing and (2) conditional rebates individually. Specifically, it considers whether the extreme results persist, such that the optimal pricing for *I* is to set quite high so as to maximise profit from the must-stock part of the market. In the analysis of the cases where *I* offers conditional rebate, only the equilibrium such that *E* is excluded is analysed. Because conditional rebates and two-part tariffs have an equivalent effect, this section does not cover two-part tariffs. The next section covers the case in which market-share-based rebates have different effect from two-part tariffs and quantity-based rebates.

This section is organised as follows. Section 4.1 explains the assumptions specific to this section. Section 4.2 then analyses the optimal behaviours of the incumbent, the entrant and the buyer when the incumbent offers linear pricing and an exclusive conditional rebate individually. That section also analyses the pricings the incumbent prefers and offers concluding remarks.

3.4.1 Assumptions

The assumption specific to this section is uncertainty about must-stock proportion, α. Specifically, it is assumed that $\alpha_i = \alpha_H$ or α_L on a 50-50 basis where $0 < \alpha_L < \alpha_H < 1$. It is also assumed that α_i is realised after *I* offers a pricing scheme to *B* and that *I* knows only the distribution of α_i (50-50). On the other hand, *E* and *B* make their decisions after observing that α is realised.

This scenario implies that the process of the game can be written as follows:

1) C_E is realised and known to *I* and *E*.

2) *I* offers a pricing scheme to *B*.

3) α_i is realised and known to *E*

4) After observing the pricing scheme offered by *I*, *E* offers its price to *B*.

5) B determines whether it trades with I and E or not, along with the quantities that B purchases from I and E. If the total cost for B is the same, B prefers to purchase E's good as often as possible.

The difference from the baseline model is that the third stage, the realisation of α_i, is added.

3.4.2 Analysis

This section analyses the game in which I can offer (1) linear pricing or (2) conditional rebates. Analysis of this model by the backward induction approach provides the following proposition, which will be proved in the following sections.

Proposition 3.5. *Suppose the incumbent can introduce either linear pricing or conditional rebate.*

i) If $C_E \geq 0.5 - \frac{\frac{\alpha_H-\alpha_L}{4\alpha_H}}{1-\frac{\alpha_H-\alpha_L}{2\alpha_H}-\bar{\alpha}}$*, the incumbent's optimal behaviour is exclusive conditional rebate such that both* $P_I^R - \beta = 1$ and $P_I^R > \frac{1-C_E(1-\alpha_L)}{\alpha_L}$ *hold. In the equilibrium, the entrant decides not to enter and the set of the buyer's purchasing volume is* $({q_I}^*, {q_E}^*) = (1, 0)$.

If $C_E < 0.5 - \frac{\frac{\alpha_H-\alpha_L}{4\alpha_H}}{1-\frac{\alpha_H-\alpha_L}{2\alpha_H}-\bar{\alpha}}$*, the incumbent's optimal behaviour is and exploitative linear pricing such that* $P_I^{L*} = \frac{1-C_E(1-\alpha_H)}{\alpha_H}$*. The set of the entrant's price and the buyer's purchasing volume in the equilibrium is given by*

$$({P_E}^*, {q_I}^*, {q_E}^*) = \left\{ \begin{array}{c} \left(\frac{1-\frac{\alpha_L}{\alpha_H}[1-C_E(1-\alpha_H)]}{1-\alpha_L}, \alpha_L, 1-\alpha_L \right) \; if \, \alpha_i = \alpha_L \\ (C_E, \alpha_H, 1-\alpha_H) \; if \, \alpha_i = \alpha_H \end{array} \right\}.$$

ii) The entrant obtains a profit of $(\alpha_H - \alpha_L)(1 - C_E)/\alpha_H(1 - \alpha_L)$ *if the incumbent introduces exploitative pricing in the equilibrium and* $\alpha_i = \alpha_L$*. Otherwise, the entrant's profit is zero. The buyer's surplus is always zero.*

iii) If $\acute{C}_E < C_E \leq 1$*, prohibition of the conditional rebate increases the buyer's profit, where*

$$\acute{C}_E = \frac{3-\bar{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}{4-2\bar{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}} \text{ and } \bar{\alpha} = \frac{\alpha_H+\alpha_L}{2}.$$

If $0.5 - \frac{\frac{\alpha_H - \alpha_L}{4\alpha_H}}{1 - \frac{\alpha_H - \alpha_L}{2\alpha_H} - \bar{\alpha}} \leq C_E \leq \acute{C}_E$, *prohibition of the conditional rebate increases the entrant's profit.*

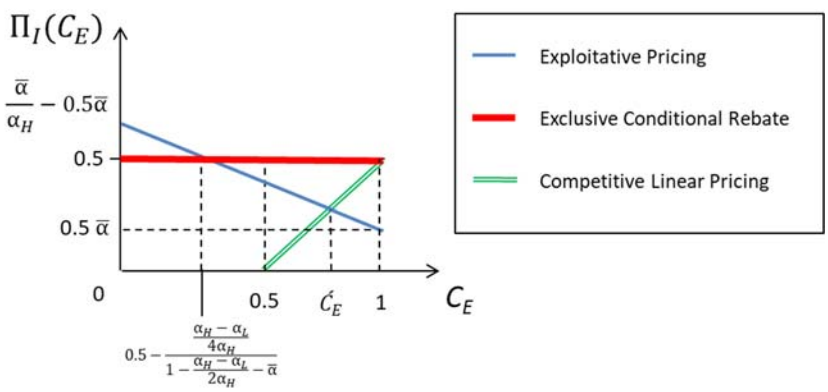
If $C_E < 0.5 - \frac{\frac{\alpha_H - \alpha_L}{4\alpha_H}}{1 - \frac{\alpha_H - \alpha_L}{2\alpha_H} - \bar{\alpha}}$, *prohibition of the conditional rebate changes neither the entrant's profit nor the buyer's surplus.*

Proposition 3.4 *i)* shows that, as in the baseline model, the exclusive conditional rebate is chosen by *I* in the equilibrium when C_E is higher than a certain point, and *I* chooses exploitative pricing when C_E is lower than the threshold. However, in this model, the threshold point is lower than 0.5. This point implies that in this case *I* may exclude the more efficient entrant by conditional rebate in the equilibrium, which never happens in the baseline model analysed in Chapter 2. This situation occurs because the existence of the uncertainty about the must-stock proportion only limits the profitability of exploitative pricing and does not affect the profitability of the exclusive conditional rebate. Note that *I* can exclude *E* and obtain monopoly profit by offering a two-part tariff, with an equivalent effect in this model if the two-part tariff is available because, as in the baseline model, conditional rebates and two-part tariffs have an equivalent effect in this model.

The profitability of exploitative pricing is limited by the uncertainty because under such uncertainty, *I* cannot set a price such that *I* can completely exploit the profit in both the high α case and the low α case. Taking this constraint into account, *I* sets a price such that *I* can exploit the profit completely only when $\alpha_i = \alpha_H$ in the equilibrium. In exchange, *E* can obtain a positive profit when $\alpha_i = \alpha_L$ and *I* chooses exploitative pricing in the equilibrium, as shown in Proposition 3.4 *ii)*. Note that the profit becomes larger as C_E becomes smaller. On the other hand, the buyer's surplus is always zero.

The expected profit of *I* for each type of equilibrium pricing can be summarised in Figures 3.6. The details of each type of equilibrium pricing will be discussed in the following sections.

Figure 3.6 Incumbent's expected profit for each type of pricing



Proposition 3.5 *iii)* summarises the effect of prohibiting conditional rebates. The results show that if two-part tariffs and conditional rebates are prohibited, the improvement of *E*'s profit or *B*'s surplus can occur. If C_E is extremely high (i.e., $\acute{C_E} < C_E \leq 1$), the prohibition of conditional rebates improves *B*'s surplus, because in this situation, *I* introduces competitive linear pricing as its second best option, meaning that *E*'s profit remains zero. If C_E is less than the threshold but is high enough to prefer exclusive conditional rebate to exploitative pricing, the prohibition on conditional rebates improves *E*'s profit, because in this situation *I* introduces exploitative pricing as its second best option, and *B*'s surplus is still zero.

3.4.2.1 Optimal Linear Pricing

In this section, the game where both *I* and *E* offers a linear pricing to *B* is analysed. In this game *I*, *E* and *B* make their decisions in turn. Let ${P_E}^{H^*}$ and ${P_E}^{L^*}$ denote *E*'s optimal response given *I*'s pricing when α_i is α_H and α_L, respectively.

Analysis on this model by the backward induction approach provides Proposition 3.6, proven in the subsequent sections.

Proposition 3.6. *If the incumbent offers a linear price to the buyer, this game has a unique subgame perfect Nash equilibrium:*

$$({P_I^{L^*}}, {P_E}^{H^*}, {P_E}^{L^*}, {q_I}^*, {q_E}^*) = \begin{Bmatrix} (C_E - \varepsilon, +\infty, +\infty, 1, 0) \; if \, C_E > \acute{C_E} \\ (\frac{1-C_E(1-\alpha_H)}{\alpha_H}, C_E, \frac{1-\frac{\alpha_L}{\alpha_H}[1-C_E(1-\alpha_H)]}{1-\alpha_L}, \alpha_i, 1-\alpha_i) \; if \, C_E \leq \acute{C_E} \end{Bmatrix},$$

where $\acute{C_E} = \frac{3-\bar{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}{4-2\bar{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}$, $\bar{\alpha} = \frac{\alpha_H+\alpha_L}{2}$ *and* $P_E = +\infty$ *denotes that E decides not to enter.*

The results summarised in Proposition 3.6 show that as in the baseline model, competitive linear pricing is chosen by *I* in the equilibrium when C_E is higher than a certain point, and *I* chooses exploitative pricing when C_E is lower than the threshold. However, Proposition 3.6 shows that the equilibrium for exploitative linear pricing is the price at which *I* can exploit the profit completely only when $\alpha_i = \alpha_H$. It follows that the profitability of exploitative linear pricing is limited. On the other hand, the existence of the uncertainty about the must-stock proportion does not limit the profitability of competitive linear pricing.

In order to prove Proposition 3.6, we first analyse *B* and *E*'s optimal behaviours.

The following lemmata about the optimal behaviours of *B* and *E* can be obtained by substituting α_i into α in Lemmata 2.1 and 2.2.

Lemma 3.4. *Given P_I^L and P_E, the buyer's optimal purchasing volume is*

$$(q_I{}^*, q_E{}^*) = \left\{\begin{matrix} (1,0)\ if\ P_I^L < P_E\ and\ P_I^L\ \leq 1 \\ (\alpha_i, 1-\alpha_i)\ if\ P_I^L \geq P_E\ and\ P_I^L\alpha_i + P_E(1-\alpha_i) \leq 1 \\ (0,0)\ otherwise \end{matrix}\right\}.$$

Lemma 3.5. *Given P_I^L, the entrant's optimal pricing is*

$$P_E{}^* = \left\{\begin{matrix} P_I^L\ if\ P_I^L \leq 1\ and\ P_I^L \geq C_E \\ \frac{1-P_I^L\alpha_i}{1-\alpha_i}\ if\ P_I^L > 1\ and\ \frac{1-P_I^L\alpha_i}{1-\alpha_i} \geq C_E \end{matrix}\right\}.$$

If $P_I^L \leq 1$ and $P_I^L < C_E$ or if $P_I^L > 1$ and $(1-\ P_I^L\alpha_i)/(1-\alpha_i) < C_E$, E decides not to enter the market.

In the baseline model, *I*'s optimal behaviours can be divided into two types. The first type is when *I* sets a competitive price to capture the entire market (*competitive linear pricing*). When *I* adopts this option, setting P_I^L just below C_E maximizes *I*'s profit ($P_I^L = C_E - \varepsilon$). In this case, such pricing also may be optimal, since the profit does not depend on α_i.

The second type is for *I* to give up competing with *E* and exploit the profit from the must-stock part of the market by setting the price higher than 1 ("exploitative linear pricing"). This type of pricing is affected by uncertainty about α. If *I* adopts this type of pricing, the resulting profit can be calculated as follows:

$$(3.1)\ \ \Pi_I = \left\{\begin{matrix} 0.5(P_I - 0.5)\alpha_H + 0.5(P_I - 0.5)\alpha_L\ if\ \ 1 < P_I^L \leq \frac{1-C_E(1-\alpha_H)}{\alpha_H} \\ 0.5(P_I - 0.5)\alpha_L\ if\ \frac{1-C_E(1-\alpha_H)}{\alpha_H} < P_I^L \leq \frac{1-C_E(1-\alpha_L)}{\alpha_L} \\ 0\ if\ P_I^L > \frac{1-C_E(1-\alpha_L)}{\alpha_L} \end{matrix}\right\}.$$

Solving this profit-maximisation problem gives *I*'s optimal behaviour, which is summarised in Lemma 3.6 below.

Lemma 3.6. *The incumbent's optimal pricing is*

$$P_I^{L*} = \left\{\begin{matrix} C_E - \varepsilon\ if C_E > \overline{C_E} \\ \frac{1-C_E(1-\alpha_H)}{\alpha_H}\ if C_E \leq \overline{C_E} \end{matrix}\right\},\ where\ \overline{C_E} = \frac{3-\bar{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}{4-2\bar{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}},\ \bar{\alpha} = \frac{\alpha_H+\alpha_L}{2}\ and\ \varepsilon\ is\ a\ small$$

positive amount.

The entrant enters the market only if $C_E \leq \overline{C_E}$.

Proof: See Appendix B

Proposition 3.6 follows directly from Lemmata 3.4–3.6.

3.4.2.2 Optimal Conditional Rebate

Analysis of this model by the backward induction approach provides the following proposition.

Proposition 3.7. [Optimal exclusive conditional rebate] *The incumbent can exclude the entrant by offering the conditional rebate such that* both $P_I^R - \beta = 1$ and $P_I^R > [1 - C_E(1 - \alpha_L)]/\alpha_L$ *hold, and the expected profit is* 0.5.

Proof: See Appendix B

Proposition 3.7 shows that, as in the baseline model, *I* can obtain a monopoly profit by exclusive conditional rebate regardless of the values of C_E and α_i. This result implies that the existence of the uncertainty about the must-stock proportion does not limit the profitability of exclusive conditional rebates in contrast to exploitative pricing. Such an asymmetric effect of the uncertainty causes the exclusion of the more efficient entrant by conditional rebate, which never happens in the baseline model.

Note that like in the baseline model, conditional rebates and two-part tariffs have an equivalent effect in this model. Incumbent firm *I* can exclude *E* and obtain a profit of 0.5 by offering two-part tariff (P_I^F, P_I^V) such that both $P_I^F = 1 - P_I^V$ and $0 \leq P_I^V < C_E$ hold.

Note that *I* does choose the exclusive conditional rebate only if the profit is higher than that of exploitative pricing. The condition is shown in Proposition 3.5.

3.4.2.3 Endogenous Pricing Type

According to the results shown in Propositions 3.6 and 3.7, the surplus of *I*, *E* and *B* when *I* introduces competitive linear pricing, exploitative linear pricing and exclusive conditional rebates can be summarised in Table 3.3.

Table 3.3 Surplus of each player for each type of Incumbent's pricing

a) Surplus of each player if $\alpha_i = \alpha_H$

I's Behaviour	*I*'s profit	*E*'s profit	*B*'s surplus
Competitive linear pricing	$C_E - 0.5 - \varepsilon$	0	$1 - C_E + \varepsilon$
Exploitative linear pricing	$(1 - \alpha_H)(1 - C_E) + 0.5\alpha_H$	0	0
Exclusive conditional rebate	0.5	0	0

b) Surplus of each player if $\alpha_i = \alpha_L$

I's Behaviour	*I*'s profit	*E*'s profit	*B*'s surplus
Competitive linear pricing	$C_E - 0.5 - \varepsilon$	0	$1 - C_E + \varepsilon$
Exploitative linear pricing	$\frac{\alpha_L}{\alpha_H}[1 - (1 - \alpha_H)C_E] - 0.5\alpha_L$	$\frac{(\alpha_H - \alpha_L)(1 - C_E)}{\alpha_H(1 - \alpha_L)}$	0
Exclusive conditional rebate	0.5	0	0

c) Expected surplus of each player

I's Behaviour	*I*'s profit	*E*'s profit	*B*'s surplus
Competitive linear pricing	$C_E - 0.5 - \varepsilon$	0	$1 - C_E + \varepsilon$
Exploitative linear pricing	$\frac{\bar{\alpha}}{\alpha_H}[1 - (1 - \alpha_H)C_E] - 0.5\,\bar{\alpha}$	$0.5\frac{(\alpha_H - \alpha_L)(1 - C_E)}{\alpha_H(1 - \alpha_L)}$	0
Exclusive conditional rebate	0.5	0	0

* $\bar{\alpha} = 0.5\alpha_H + 0.5\alpha_L$.

With a comparison of *I*'s optimal profit when *I* introduces linear pricing and exclusive conditional rebates (as summarised in Table 3.3 and Figure 3.6), Proposition 3.5 can be obtained.

3.5. Optimal Pricing Where There Is Uncertainty about Size of Contestable Demand

Section 4 analyses the case in which uncertainty exists about must-stock proportion and shows that I may have an incentive to exclude the more efficient entrant with two-part tariffs and conditional rebates. This section analyses the case in which there is uncertainty about the size of the contestable demand. The baseline model in Chapter 2 covers only a quantity-based rebate, which is a rebate scheme such that B obtains a discount of β when the purchase volume from I exceeds the threshold determined by I. In the baseline model, a quantity-based rebate and a market-share-based rebate (i.e., a rebate scheme such that B obtains a discount when the share of purchase volume from I to total purchase volume exceeds a threshold) have an equivalent effect, because the demand size is fixed to 1. However, as discussed below, under the assumption of the uncertainty about the size of the contestable demand, I may have an incentive to exclude the more efficient entrant, and the market-share-based conditional rebate works better than the quantity-based conditional rebate and two-part tariff.

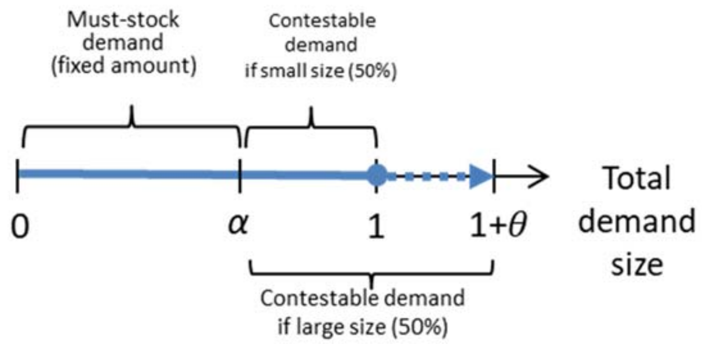
As with Chapter 2, this section analyses the optimal behaviour of I when I offers (1) linear pricing, (2) two-part tariffs, and (3) conditional rebates individually—specifically, analyses whether the extreme results of optimal pricing for I being to set at a high price so as to maximise the profit from the must-stock part of the market persist. In the analysis of the cases where I offers two-part tariffs and conditional rebates, only the equilibrium such that E is excluded is analysed.

This section is organised as follows. Section 5.1 explains the assumptions specific to this section. Section 5.2 then analyses the optimal behaviours of the incumbent, the entrant and the buyer when the incumbent offers linear pricing, an exclusive two-part tariff and an exclusive conditional rebate individually. Section 5.2 also analyses which pricings the incumbent prefers and presents concluding remarks.

3.5.1 Assumptions

The assumption specific to this section is uncertainty about the size of the contestable demand. In particular, while it is assumed that the size of the must-stock demand is fixed to $\alpha \in (0,1)$, the size of the contestable demand is low or high on a 50-50 basis. When the size of the contestable demand is low, the amount is $1-\alpha$. When the size of the contestable demand is high, the amount is $1-\alpha+\theta$, where $\theta > 0$. For simplicity, it is also assumed θ is small enough to guarantee that in the equilibrium, the incumbent will not choose the options such that its profit becomes zero with a 50% probability. The buyer's demand size under the uncertainty about the size of contestable demand can be summarised in Figure 3.7.

Figure 3.7 Buyer's demand size under uncertainty about the size of contestable demand



Moreover, it is also assumed that the size of the contestable demand is realised after I offers a pricing scheme to B and I only knows the distribution of α_i (50-50). On the other hand, E and B make their decisions after observing that the size of the contestable demand is realised.

This sequence of events implies that the process of the game can be written as follows:

1) C_E is realised and known to I and E.

2) I offers a pricing scheme to B.

3) The size of the contestable demand is realised and known to I and E.

4) After observing the pricing scheme offered by I, E offers its price to B.

5) B determines whether it trades with I and E or not, along with the quantities that B purchases from I and E. If the total cost for B is the same, B prefers to purchase E's good as much as possible.

The difference from the baseline model is that the third stage, namely the realisation of the size of the contestable demand, is added.

In addition, this section assumes that I can offer not only quantity-based conditional rebates (P_I^R, β, γ), as with the baseline model, but also market-share-based conditional rebates. For simplicity, this section considers a market-share-based rebate scheme (P_I^{MR}, β^M) such that the discount of $\beta^M \in (0, P_I^{MR}]$ is given only if I has a 100% share of B's purchase volume. Since in this case, the total demand size can be larger than 1, this section does not assume that the threshold volume of quantity-based rebate, γ, is equal to 1.

3.5.2 Analysis

This section analyses the game where I can offer (1) linear pricing, (2) two-part tariffs, and (3) conditional rebates. Analysis of this model by the backward induction approach provides the following proposition, which will be proved in the following sections.

Proposition 3.8. *Suppose the incumbent can introduce either linear pricing, two-part tariffs or conditional rebates.*

i) If $C_E \geq 0.5 - 0.5\theta/(1-\alpha)$, *the incumbent's optimal behaviour is exclusive market share-based rebate such that both* $P_I^{MR} - \beta^M = 1$ *and* $P_I^{MR} > [1-(1-\alpha+\theta)C_E]/(\alpha-\theta)$ *hold. In the equilibrium, the entrant decides not to enter and the set of the buyer's purchasing volume is*

$$({q_I}^*, {q_E}^*) = \begin{Bmatrix} (1,0) \; if the\ contestable\ demand\ is\ small \\ (1+\theta,0) \; if\ the\ contestable\ demand\ is\ large \end{Bmatrix}.$$

If $C_E < 0.5 - 0.5\theta/(1-\alpha)$, *the incumbent's optimal behaviour is the exploitative pricing such that the unit price is equal to* $[1 - C_E(1-\alpha)]/\alpha$. *The set of the entrant's price and the buyer's purchasing volume in the equilibrium is given by*

$$({P_E}^*, {q_I}^*, {q_E}^*) = $$
$$\begin{Bmatrix} (C_E, \alpha, 1-\alpha) \; if the\ contestable\ demand\ is\ small \\ \left(\frac{\theta + C_E(1-\alpha)}{1-\alpha+\theta}, \alpha, 1-\alpha+\theta\right) \; if\ the\ contestable\ demand\ is\ large \end{Bmatrix}.$$

ii) The entrant obtains a profit of $0.5\theta(1-C_E)$ *if the incumbent introduces exploitative pricing in the equilibrium and the size of the contestable demand is large. Otherwise, the entrant's profit is zero. The buyer's surplus is always zero.*

iii) If $\widehat{C_E} < C_E \leq 1$, *prohibition of two-part tariff and conditional rebate increases the buyer's profit, where*

$$\widehat{C_E} = \frac{3+\theta-\alpha}{4+2\theta-2\alpha}.$$

If $0.5 - \frac{0.5\theta}{1-\alpha} \leq C_E \leq \widehat{C_E}$, *prohibition of two-part tariff and conditional rebate increases the entrant's profit.*

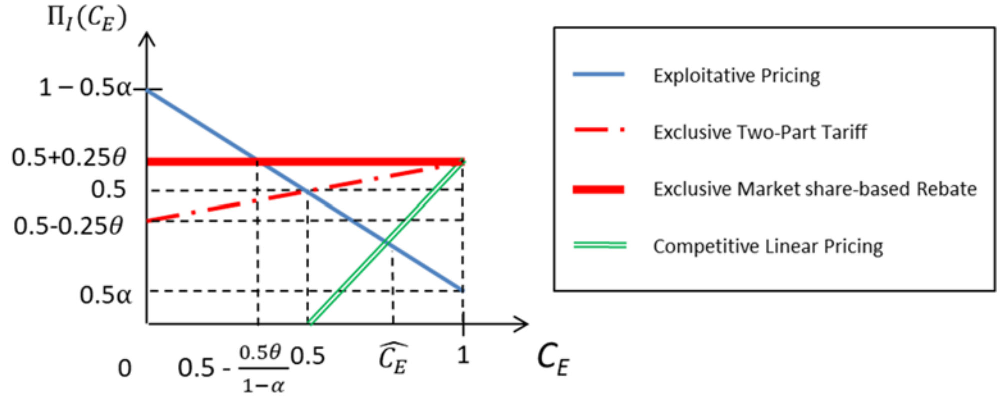
If $C_E < 0.5 - \frac{0.5\theta}{1-\alpha}$, *prohibition of two-part tariff and conditional rebate does not change the entrant's profit or the buyer's surplus.*

Proposition 3.8 *i)* shows that, as in the baseline model, exclusive conditional rebate is chosen by *I* in the equilibrium when C_E is higher than a certain point, and *I* chooses exploitative pricing when C_E is lower than the threshold. However, in this model, *I* strictly prefers exclusive market-share-based rebates to exclusive two-part tariffs and exclusive quantity-based rebates, although in the baseline model these three types of price schedule have an equivalent effect. Moreover, in this model, the threshold point is lower than 0.5. This result implies that in this case, *I* may exclude the entrant by market-share-based rebate in the equilibrium, which never happens in the baseline model, because the existence of the uncertainty about the size of the contestable demand limits the profitability of exploitative pricing, exclusive two-part tariffs and exclusive conditional rebates and does not have an effect on the profitability of exclusive market-share-based rebates.

The profitability of exploitative pricing is limited by the uncertainty because under such uncertainty, I cannot set a price such that I can completely exploit the profit in both the large contestable demand case and the small contestable demand case. Taking this constraint into account, I sets a price such that in the equilibrium I can exploit the profit completely only when the size of the contestable demand is small. In exchange, E can obtain a positive profit when the size of the contestable demand is small, and I chooses the exploitative pricing in the equilibrium, as shown in Proposition 3.4 *ii)*. Note that the profit becomes larger as C_E becomes smaller. On the other hand, the buyer's surplus is always zero.

The expected profit of I for each type of equilibrium pricing can be summarised in Figures 3.2, 3.3 and 3.6 below. The details of each type of equilibrium pricing will be discussed in the following sections.

Figure 3.8 Incumbent's expected profit for each type of pricing



Proposition 3.8 *iii)* summarises the effect of prohibiting conditional rebates. The results show that if two-part tariffs and conditional rebates are prohibited, the improvement of E's profit or B's surplus can occur. If C_E is extremely high (i.e.,$\widehat{C_E} < C_E \leq 1$), the prohibition of conditional rebates improves B's surplus, because in this situation, I introduces competitive linear pricing as its second best option, which entails that E's profit remains zero. If C_E is less than the threshold but is high enough to prefer an exclusive market-share-based rebate to exploitative pricing, the prohibition of two-part tariffs and conditional rebates improves E's profit, because in this situation I introduces exploitative pricing as its second-best option, so B's surplus remains zero.

The result summarised in Proposition 3.8 differs from the models analysed so far in the sense that I strictly prefers exclusive market-share-based rebates to exclusive two-part tariffs or quantity-based rebates. This preference exists because while I's ability to obtain monopoly profit by exclusive market-share-based rebates is not undermined by uncertainty about the size of the contestable demand, such uncertainty limits the profitability of exclusive quantity-based rebates as well as exclusive two-part tariffs and exploitative pricing. Such asymmetric effects of uncertainty on exclusive market-share-based rebates and other exclusive pricing causes the exclusion of the more efficient entrants through market-share-based rebates, which does not occur in the baseline model. This result suggests that a market-share-based rebate is more harmful to the consumer than is a two-part tariff or a quantity-based rebate.

3.5.2.1 Optimal Linear Pricing

In this section, the game where both *I* and *E* offer linear pricing to *B* is analysed. In this game *I*, *E* and *B* make their decisions in turn. Let ${P_E}^{S^*}$ and ${P_E}^{B^*}$ denote *E*'s optimal response, given *I*'s pricing when the determined contestable demand size is small and big, respectively, and ${q_k}^{S^*}$ and ${q_k}^{B^*}$ denote *B*'s optimal purchasing volume from $k \in I, E$ when the determined contestable demand size is small and big, respectively.

Analysis of this model by the backward induction approach provides Proposition 3.9, to be proved in the following sections.

Proposition 3.9. *If the incumbent offers a linear price to the buyer, this game has a unique subgame perfect Nash equilibrium:*

$$({P_I^L}^*, {P_E}^{S^*}, {P_E}^{B^*}, {q_I}^{S^*}, {q_I}^{B^*}, {q_E}^{S^*}, {q_E}^{B^*}) = \begin{Bmatrix} (C_E - \varepsilon, +\infty, +\infty, 1, 1+\theta, 0, 0)\ if C_E > \widehat{C_E} \\ (\frac{1-C_E(1-\alpha)}{\alpha}, C_E, \frac{\theta + C_E(1-\alpha)}{1-\alpha+\theta}, \alpha, \alpha, 1-\alpha, 1-\alpha+\theta)\ if C_E \leq \widehat{C_E} \end{Bmatrix},$$

where $\widehat{C_E} = \frac{3+\theta-\alpha}{4+2\theta-2\alpha}$ *and* $P_E = +\infty$ *denotes that E decides not to enter.*

The results summarised in Proposition 3.9 show that there are two types of equilibrium. In the first type, *I* sets the price slightly lower than *E*'s marginal cost, which entails that *B* purchases only from *I* regardless of the realised value of the contestable demand size. This equilibrium is equivalent to competitive linear pricing in the baseline model.

In the second type of equilibrium, *I* sets the high price to exploit the profit from the must-stock part of the market. All contestable demand is covered by *E*'s product.

Note that the profit from competitive linear pricing in this model is $1 + 0.5\theta$ times larger than that in the baseline model, so *B*'s surplus is also $1 + 0.5\theta$ times larger than that in the baseline model.

On the other hand, the profit from exploitative linear pricing in this model is equal to that in the baseline model. The fact that the incumbent can exploit more when the determined contestable demand size is large implies that, in this case, the incumbent cannot fully exploit the profit through linear pricing. In this situation, *E* sets its price at $[\theta + C_E(1-\alpha)]/(1-\alpha+\theta)$ and obtains a profit of $(1-C_E)/(1-\alpha+\theta)$, meaning that *B*'s surplus is zero regardless of the determined demand size.

It follows that the incumbent is more likely to uses exploitative linear pricing in the sense that the threshold value of the entrant's cost to change the type of pricing, $\widehat{C_E}$, is lower than the threshold in the baseline model.

In order to prove Proposition 3.9, we first analyse *B* and *E*'s optimal behaviours.

The optimal behaviours if the determined contestable demand is $1-\alpha$ are same as in Lemmata 2.1 and 2.2. The optimal behaviours if the determined contestable demand is $1-\alpha+\theta$ can be obtained by substituting $1-\alpha+\theta$ into $1-\alpha$ in Lemmata 2.1 and 2.2 yielding the following lemmata.

Lemma 3.7. *Given P_I^L and P_E, the buyer's optimal purchasing volume is as follows:*

If the determined contestable demand is $1-\alpha$,

$$(q_I{}^*, q_E{}^*) = \begin{Bmatrix} (1,0)\ if\ P_I^L < P_E\ and\ P_I^L \leq 1 \\ (\alpha, 1-\alpha)\ if\ P_I^L \geq P_E\ and\ P_I^L\alpha + P_E(1-\alpha) \leq 1 \\ (0,0)\ otherwise \end{Bmatrix}.$$

If the determined contestable demand is $1-\alpha+\theta$,

$$(q_I{}^*, q_E{}^*) = \begin{Bmatrix} (1,0)\ if\ P_I^L < P_E\ and\ P_I^L \leq 1 \\ (\alpha, 1-\alpha+\theta)\ if\ P_I^L \geq P_E\ and\ P_I^L\alpha + P_E(1-\alpha+\theta) \leq 1+\theta \\ (0,0)\ otherwise \end{Bmatrix}.$$

Lemma 3.8. *Given P_I^L, the entrant's optimal pricing is as follows:*

If the determined contestable demand is $1-\alpha$,

$$P_E{}^* = \begin{Bmatrix} P_I^L\ if\ P_I^L \leq 1\ and\ P_I^L \geq C_E \\ \frac{1-P_I^L\alpha}{1-\alpha}\ if\ P_I^L > 1\ and\ \frac{1-P_I^L\alpha}{1-\alpha} \geq C_E \end{Bmatrix}.$$

If the determined contestable demand is $1-\alpha+\theta$,

$$P_E{}^* = \begin{Bmatrix} P_I^L\ if\ P_I^L \leq 1\ and\ P_I^L \geq C_E \\ \frac{1+\theta-P_I^L\alpha}{1-\alpha+\theta}\ if\ P_I^L > 1\ and\ \frac{1+\theta-P_I^L\alpha}{1-\alpha+\theta} \geq C_E \end{Bmatrix}.$$

We next consider the optimal behaviour of *I*. In this situation, two options are possible for the incumbent's optimal price. The first option is to set a competitive price to capture the entire market (equivalent to *competitive linear pricing* in the baseline model), regardless of the size of the contestable demand size. The second option is to give up competing with E and setting the maximum possible price to exploit the profit from the must-stock part of the market in both the large contestable demand case and the small contestable demand case (which is equivalent to *exploitative linear pricing* in the baseline model).

Although theoretically there exists an alternative option for setting the maximum possible price to exploit the profit from the must-stock part of the market in either the large contestable demand case or the small contestable demand case, the price is so high that B decides not to purchase at all in that case. However, this option is excluded by the assumption that θ is small enough to guarantee that in the equilibrium, with a 50% probability the incumbent will not choose the options such that its profit becomes zero.

Solving the profit-maximisation problem for I gives I's optimal behaviour, which is summarised in Lemma 3.9 below.

Lemma 3.9. *The incumbent's optimal pricing is*

$$P_I^{L^*} = \begin{Bmatrix} C_E - \varepsilon \; if \, C_E > \widehat{C_E} \\ \frac{1-C_E(1-\alpha)}{\alpha} \; if \, C_E \leq \widehat{C_E} \end{Bmatrix}, \text{ where } \widehat{C_E} = \frac{3+\theta-\alpha}{4+2\theta-2\alpha}.$$

The entrant enters the market only if $C_E \leq \widehat{C_E}$.

Proof: See Appendix B

Proposition 3.9 follows directly from Lemmata 3.7–3.9.

3.5.2.2 Optimal Two-Part Tariff

Analysis of this model by the backward induction approach provides Proposition 3.10.

Proposition 3.10. [Optimal exclusive two-part tariff] *The incumbent can exclude the entrant by offering the two-part tariff such that* $(P_I^{F^*}, P_I^{V^*}) = (1 - C_E + \varepsilon, C_E - \varepsilon)$ *and the expected profit is* $0.5 + 0.5\theta(C_E - 0.5) - \varepsilon$.

Proof: See Appendix B

Proposition 3.10 shows that, like in the baseline model, I can obtain monopoly profit through the optimal exclusive conditional rebate regardless of the values of C_E when the realised contestable demand size is small. However, when the realised contestable demand size is large, I cannot dominate the entire surplus of the market. On the other hand, B obtains a positive surplus.

It follows that the profitability of the exclusive two-part tariff is limited by the existence of uncertainty about the size of the contestable as well as exploitative pricing. It follows that the incumbent does not exclude the more efficient entrant through exclusive two-part tariffs if only linear pricing and conditional rebates are available for the incumbent, the same result as for the baseline model.

3.5.2.3 Optimal Conditional Rebate

Analysis of this model by the backward induction approach provides Proposition 3.11.

Proposition 3.11. [Optimal exclusive conditional rebate] *The incumbent can exclude the entrant by offering the market-share-based rebate such that both* $P_I^{MR} - \beta^M = 1$ *and* $P_I^{MR} > [1 - (1 - \alpha + \theta)C_E]/(\alpha - \theta)$ *hold, and the expected profit is* $0.5(1 + \theta)$. *The expected profit from the exclusive market-share-based rebate is always larger than the profit from the exclusive quantity-based rebate.*

Proof: See Appendix B

Proposition 3.11 shows that, as in the baseline model, I can obtain a monopoly profit by exclusive conditional rebates regardless of the values of C_E and the realised contestable demand size, and the expected profit from exclusive quantity-based rebate is strictly higher than that of exclusive market-share-based rebate.

Note that I does choose the exclusive market-share-based rebate only if the profit is higher than that of exploitative pricing. The condition is shown in Proposition 3.8.

3.5.2.4 Endogenous Pricing Type

According to the results shown in Propositions 3.9, 3.10 and 3.11, the surplus of I, E and B when I introduces competitive linear pricing, exploitative linear pricing and exclusive conditional rebates can be summarised as in Table 3.4.

Table 3.4 Surplus of each player for each type of Incumbent's pricing

a) Surplus of each player if the determined contestable demand is $1-\alpha$

I's Behaviour	I's profit	E's profit	B's surplus
Competitive linear pricing	$C_E - 0.5 - \varepsilon$	0	$1 - C_E + \varepsilon$
Exploitative linear pricing	$(1-\alpha)(1-C_E) + 0.5\alpha$	0	0
Exclusive two-part tariff	0.5	0	0
Exclusive market-share-based rebate	$0.5(1+\theta)$	0	0

b) Surplus of each player if the determined contestable demand is $1-\alpha+\theta$

I's Behaviour	I's profit	E's profit	B's surplus
Competitive linear pricing	$(1+\theta)(C_E-0.5)-\varepsilon$	0	$(1+\theta)(1-C_E)+\varepsilon$
Exploitative linear pricing	$(1-\alpha)(1-C_E)+0.5\alpha$	$0.5\theta(1-C_E)$	0
Exclusive two-part tariff	$0.5+\theta(C_E-0.5)-\varepsilon$	0	$\theta(1-C_E)+\varepsilon$
Exclusive market-share-based rebate	$0.5(1+\theta)$	0	0

c) Expected surplus of each player

I's Behaviour	I's profit	E's profit	B's surplus
Competitive linear pricing	$(1+0.5\theta)(C_E-0.5)-\varepsilon$	0	$(1+0.5\theta)(1-C_E)+\varepsilon$
Exploitative linear pricing	$(1-\alpha)(1-C_E)+0.5\alpha$	$0.5\theta(1-C_E)$	0
Exclusive two-part tariff	$0.5+0.5\theta(C_E-0.5)-\varepsilon$	0	$0.5\theta(1-C_E)+\varepsilon$
Exclusive market-share-based rebate	$0.5(1+0.5\theta)$	0	0

By comparing I's optimal profit when I introduces linear pricing and exclusive conditional rebates, as summarised in Table 3.4 and Figure 3.8, Proposition 3.8 can be proved.

3.6. Optimal Pricing Where Must-Stock Share, α, Is Not Fixed Over Time

Chapter 2 analyses the baseline case where the game played only once and the result shows that I does not have an incentive to exclude the more efficient entrant than itself by two-part tariff or conditional rebate, because exploitative pricing is more profitable. This section analyses the case where the game is played repeatedly. Moreover, in order to consider the case where the result of the previous game affect the value of α, it is assumed that the must stock property of the incumbent's product is eroded when the buyer becomes familiar with the entrant's product.

This section is organised as follows. Section 6.1 explains the assumptions specific to this section. Then, Section 6.2 analyses the case where the game is played twice. In Section 6.3, the number of times that the game is played is a finite but large number. and 6.4 consider the cases where the game is playes infinitely.

3.6.1 Assumptions

The assumption specific to this chapter is that this section assumes that the game is played repeatedly. In other words, the game is played "T" times. The basic assumptions of a game at period "t", where $t = 1, \dots, T$ and $T > 1$, are the same as that in the baseline model analysed in Chapter 2. For simplicity, in this game it is assumed that I offers linear pricing or conditional rebates because the results of the two-part tariff case and the conditional rebate case in Chapter 2 show that two-part tariffs and conditional rebates have equivalent effects. The variables regarding the price schedules of I and E at period t are written as P_{It},[15] β_t and P_{Et}, whose definitions are the same as those in Section 3. The expected profits in the next game are discounted by the discount factor, $\delta \in (0,1)$. For example, when the value of I's profit in period $t+1$, Π_{It+1}, is evaluated at period t, the expected value is described as $\delta \Pi_{It+1}$. This implies that when δ is close to 1, I places much more weight on the profits in the future periods than I does when δ is close to zero.

The must-have proportion of I's good at period t is written as α_t and $\alpha_1 \in (0,1)$. In order to consider the case in which the result of the previous game affect the value of

[15] The model analysed in this section does not differentiate P_I^L from P_I^R because such a distinction is not necessary. As the analysis in Chapter 2 shows, where the incumbent offers a exploitative conditional rebate, a conditional rebate (P_I^R, β, γ) is synonymous with a linear pricing (P_I^L); hence, it is possible to express all the possible sets of I's linear pricing and conditional rebate offers by parameters, $(P_{It}, \beta_t, \gamma_t)$.

α, it is assumed that where the entry happens, or where the entrant sells positive units to the buyer, α becomes 0 in the following games, for simplicity. In other words, the must-stock property of the incumbent's product is eroded when the buyer becomes familiar with the entrant's product. Specifically, this model assumes that $\alpha_t = \alpha_1$ if the entry has not happened in the previous periods ($q_{E1} = q_{E2} = \cdots = q_{Et-1} = 0$). On the other hand, $\alpha_t = 0$ if the entry has happened in the previous periods. Such the assumption is applicable in such a case that a user of a good can extract the true value of the good only if the user has enough experience to use the good.

For simplicity, it is also assumed that the order of the game is such that *I* moves first and then *E* moves, given *I*'s action is not changed regardless of the previous periods when $t \geq 2$ and that when *I* expects that *I* will be excluded from the market by *E*'s response *I* does not offer any price but chooses to leave the market.

Moreover, it is also assumed the goods are not storable. This assumption implies that *B* must consume what *B* purchases in the period.

As in previous models, *I* chooses the pricing behaviour that maximises its profit. The expected profits for *I* and *E* in the next game are discounted by the discount rate, $\delta \in (0,1)$. In addition, it is assumed that *I* and *E* are not allowed to borrow money from financial institutions. This assumption implies that the profits of the incumbent and the entrant in each game must not be negative. This assumption can be justified because a new entrant tends to have difficulty borrowing money from financial markets due to the lack of information on the likelihood that it will successfully enter the market, from the viewpoint of financial institutions.

This multi-period assumption is similar to Ordover and Shaffer (2012). However, there is a difference from their model over the assumption about the effect of the buyer's choice in one period on the next period. While they assume that the buyer's contestable demand will be locked-in to the seller supplying the contestable unit in the second period, the model analysed in this section assumes that the must-stock property of the incumbent's good is eroded when the buyer purchases from the entrant. Their assumption is more appropriate for a situation where buyers get locked-in, perhaps due to the necessity to learn how to use the specific product. On the other hand, the assumption in this paper can be more appropriate for a situation where a dominant incumbent with strong reputation such as the Intel case.

In order to analyse the optimal pricing behaviours that *I* and *E* set and the quantities that the buyer purchases from the incumbent and the entrant in each stage at the equilibrium by backward induction, it will be shown that the incumbent may block the entry if the entrant is more efficient than the incumbent.

3.6.2 Optimal Pricing When $T = 2$

This section analyses the case in which the game is repeated twice. In other words, this model consists of two periods. Analysis on this model by the backward induction approach provides Proposition 3.12, which will be proved in this section.

Proposition 3.12. *Where the game is played twice and α is assumed to be zero after the entry happens, the incumbent will choose to exclude the entrant by introducing exclusive conditional rebates for the two periods if* $0.5 \leq C_E \leq 1$ *and to introduce exclusive conditional rebates in the first period and exploitative pricing in the second period if* $0 < C_E < 0.5$ *and* $\delta \geq \tilde{\delta}$ *, where* $\tilde{\delta} = [0.5 - 0.5\alpha_1 - (1-\alpha_1)C_E]/[1 - 0.5\alpha_1 - (1-\alpha_1)C_E]$. *Otherwise, the incumbent will choose to adopt exploitative pricing in the first period and leave the market in the second period.*

Proposition 3.12 shows that under the assumption that the game is played twice and the must-stock proportion, α, is not fixed over time, the incumbent can use exclusive conditional rebates to exclude the more efficient entrant when the discount rate is large enough. Hence, in this model, the exclusion of the more efficient entrant by conditional rebate can happen, although it never happens in the baseline model. This difference arises because the long-term benefit from maintaining its advantage as the must-stock product can exceed the short-term profit from offering exploitative pricing in the first period.

In order to prove Proposition 3.12, we first analyse the optimal behaviours of each player in the second period. Then, the optimal behaviours of each player in the first period, anticipating the result that would happen in the second period, are analysed. Incumbent firm *I*'s optimal behaviour in the second period is shown in Lemma 3.10.

Lemma 3.10. *In the second period, the incumbent's optimal behaviour is to offer an exclusive conditional rebate if there is no entry in the first period and* $C_E \geq 0.5$, *exploitative pricing if there is no entry in the first period and* $C_E < 0.5$, *and competitive linear pricing if there is an entry in the first period and* $C_E \geq 0.5$; *the incumbent will leave the market if there is an entry in the first period and* $C_E < 0.5$.

Proof: See Appendix B

Lemma 3.10 shows that *I*'s optimal behaviour in the second period varies according to whether *I* is as or more efficient than *E* ($C_E \geq 0.5$).

Where $C_E \geq 0.5$ holds, according to Lemma 3.10, *I*'s optimal behaviour in the second period is to introduce exclusive conditional rebates, obtaining a profit of $\Pi_I^{ECCR} \equiv 0.5$ if $\alpha_2 = \alpha_1$, and to introduce competitive linear pricing, obtaining a profit of $\Pi_I^{CL} \equiv C_E - 0.5 - \varepsilon$ if $\alpha_2 = 0$.

We next prove the rest of Proposition 3.12. As analysed in Chapter 2, if *I* decides to offer a pricing scheme in the first period such that $\alpha_2 = \alpha_1$ is the result, the most profitable option is exclusive conditional rebates. Because in this situation, *I*'s optimal behaviour in the second period is to introduce exclusive conditional rebates again, *I*'s total profit can be written as

$$\Pi_I^{ECCR} + \delta\Pi_I^{ECCR} = (1+\delta)\Pi_I^{ECCR} = 0.5 + 0.5\delta.$$

To introduce exploitative conditional rebates in the first and second periods is the optimal behaviour for *I* regardless of the values of α_1 and δ where $C_E \geq 0.5$ holds. This behaviour is optimal because an exclusive conditional rebate is the most profitable option in the first period, and $\Pi_I^{ECCR} > \Pi_I^{CL}$ holds.

On the other hand, where $C_E < 0.5$ holds, according to Lemma 2-1, *I*'s optimal behaviour in the second period is to introduce exploitative pricing by obtaining a profit of $\Pi_I^{EP} \equiv (1-\alpha_1)(1-C_E) + 0.5\alpha_1$ if $\alpha_2 = \alpha_1$ and to leave the market if $\alpha_2 = 0$.

As analysed in Chapter 2, if *I* decides to offer a pricing scheme in the first period such that $\alpha_2 = \alpha_1$ is caused as a result, the most profitable option in the first period is an exclusive conditional rebate. Because in this situation, *I*'s optimal behaviour in the second period is to introduce exploitative pricing, *I*'s total profit can be written as

$$\Pi_I^{ECCR} + \delta\Pi_I^{EP}.$$

As analysed in Chapter 1, if *I* decides to offer a pricing scheme in the first period such that $\alpha_2 = 0$ is the result, the most profitable option in the first period is exclusive pricing. Because in this situation *I*'s optimal behaviour in the second period is to leave the market, *I*'s total profit is equal to Π_I^{EP}.

Therefore, the condition that *I* introduces exclusive conditional rebates in the first period where $C_E < 0.5$ holds can be written as

$$\Pi_I^{ECCR} + \delta\Pi_I^{EP} \geq \Pi_I^{EP}.$$

Because $\Pi_I^{EP} = (1-\alpha_1)(1-C_E) + 0.5\alpha_1 > 0$, this inequality can be modified to

(3.2) $\quad \delta \geq \frac{\Pi_I^{EP} - \Pi_I^{ECCR}}{\Pi_I^{EP}}.$

By substituting $\Pi_I^{EP} = (1-\alpha_1)(1-C_E) + 0.5\alpha_1$ and $\Pi_I^{ECCR} = 0.5$ into Inequality (3.2), the condition that *I* chooses the first option in the equilibrium can be written as

(3.3) $\quad \delta \geq \frac{0.5 - 0.5\alpha_1 - (1-\alpha_1)C_E}{1 - 0.5\alpha_1 - (1-\alpha_1)C_E}.$

Since $\Pi_I^{EP} > \Pi_I^{EP} - \Pi_I^{ECCR} > 0$, the threshold value $(\Pi_I^{EP} - \Pi_I^{ECCR})/\Pi_I^{EP}$ is somewhere between 0 and 1. Hence, in this model, depending on the value of δ, the exclusion of the more efficient entrant can occur. Let $\tilde{\delta}$ denote $[0.5 - 0.5\alpha_1 -$

$(1 - \alpha_1)C_E]/[1 - 0.5\alpha_1 - (1 - \alpha_1)C_E]$. From the above discussion, Proposition 3.12 can be proven directly.

3.6.3 Optimal Pricing When T is a Finite but Large Number

This section analyses the case in which the game is repeated by T times, where T is a large finite number. In this case, the results are similar to those analysed in Section 6.2. In other words, the following proposition holds;

Proposition 3.13. *Where the game is repeated* T *times and α is assumed to be zero after the entry happens, the incumbent will choose to exclude the entrant by introducing exclusive conditional rebate for the entire periods if* $0.5 \leq C_E \leq 1$ *and to keep introducing exclusive conditional rebate until period* $T - 1$ *and exploitative pricing at period* T *if* $0 < C_E < 0.5$ *and* $\delta \geq \tilde{\delta}$, *where* $\tilde{\delta} = [0.5 - 0.5\alpha_1 - (1 - \alpha_1)C_E]/[1 - 0.5\alpha_1 - (1 - \alpha_1)C_E]$. *Otherwise, the incumbent will choose to adopt exploitative pricing in the first period and to keep out of the market thereafter.*

Proof: See Appendix B

The results show that as in the two-period model analysed in Section 6.2, if the game is repeated many times, the exclusion of the more efficient entrant through conditional rebates may happen, depending of the values of α_1, C_E, and δ, although the pricing that maximises *I*'s profit in the first period is exploitative. By adopting exclusive conditional rebate, *I* can secure the advantage of its product so that *I*'s profit in the long-run can be larger than when *I* maximises its profit in the first period. This result implies that securing the advantage, the must-stock property of its product, can be an incentive for an incumbent supplier to exclude the more efficient entrant by introducing exclusive conditional rebates.

If the game is repeated twice and δ is larger than a certain threshold, *I* adopts exclusive conditional rebates in the first period and adopts exploitative pricing in the second period. If the game is repeated T times and δ is larger than a certain threshold, *I* will adopt exclusive conditional rebates not only in the first period but also in the following periods. This is the difference from the one-shot game analysed in the previous sections in which *I* does not have an incentive to exclude *E*, despite that the exclusion is possible.

The threshold values regarding whether the exclusion of the more efficient entrant happens or not when the game is played T times is same as the threshold value when the game is repeated twice: $[0.5 - 0.5\alpha_1 - (1 - \alpha_1)C_E]/[1 - 0.5\alpha_1 - (1 - \alpha_1)C_E]$. Incumbent firm *I*'s expected total profit when *I* adopts exploitative pricing in the first period is $(1 - \alpha_1)(1 - C_E) + 0.5\alpha_1$ regardless of the number of times the game is played, since the profits in the subsequent periods are zero due to the loss of *I*'s advantage. When I adopts exclusive conditional rebates, I's expected total profits in the two-period model differ from those in the T-period model. The profit in the second period in the two-period model is larger than that in the T-period model because I took exploitative pricing in the second period. On the other hand, in the T-period model, the profits from the third period are also counted in the expected total profit. As a result, the condition on whether the exclusion of the more efficient entrant happens or not in the two-period model is the same as that in the T-period model.

The threshold value, $[0.5 - 0.5\alpha_1 - (1 - \alpha_1)C_E]/[1 - 0.5\alpha_1 - (1 - \alpha_1)C_E]$, is close to zero if C_E is close to 0.5. As C_E decreases from 0.5, the threshold value becomes larger. This increase implies that when C_E is smaller than 0.5, the exclusion by I may not happen, and as C_E decreases, such exclusion becomes less likely.

3.6.4 Optimal Pricing When T Is Infinite

This section analyses the case in which the game is repeated infinitely with a constant probability that the game will terminate. To make the model realistic, it is assumed that after one period is finished, the game continues to the next period with a probability of $\theta \in (0,1)$. In this case, the results are similar to those analysed in Section 6.2. In other words, Proposition 3.14 holds.

Proposition 3.14. *Where the game is repeated infinitely with a constant probability that the game ends and α is assumed to be zero after the entry happens, the condition that I prefers to keep introducing exclusive conditional rebates is the same as the condition that I will choose to introduce exclusive conditional rebates in the first period shown in Proposition 3.12, except that δ is replaced by $\delta\theta$.*

Proof: See Appendix B

The result shows that if the game is repeated infinitely with a constant probability that the game will terminate, the threshold value regarding whether the exclusion of the more-efficient entrant happens or not is equal to the value when the game is repeated twice multiplied by $1/\theta$.

3.7. Policy Implications and Conclusions

3.7.1 Summary

This chapter has covered the four models expanding the baseline model analysed in Chapter 2. As with the baseline model, the results in all the four extended models show that the incumbent's optimal behaviour is either exploitative pricing, on the one hand, or exclusive conditional rebates or two-part tariffs, on the other. However, there are some differences from the baseline model.

Section 3 has considered the case where the incumbent's good is not literally must-stock, but is only superior to the competitors' products. If the entrant's cost is lower than the buyer's willingness to pay for 1 unit when the buyer purchases only from the entrant (i.e., $C_E < 1 - \lambda$), the profitability of exploitative pricing and exclusive conditional rebates or two-part tariffs is constrained by the possibility that the incumbent is excluded by the entrant. The equilibrium price is more likely to have a realistic value when λ is low or α is high. On the other hand, because this possibility both constrains the profitability of exploitative pricing and exclusive conditional rebates or two-part tariffs, the result persists in this model, from the baseline model, that the incumbent does not introduce exclusive conditional rebates or two-part tariffs to exclude the entrant who is more efficient than the incumbent.

Section 4 has examined the case assuming uncertainty about the must-stock proportion, α. The result shows that the incumbent can introduce exclusive conditional rebates or two-part tariffs to exclude the more efficient entrants. This ability is enabled by the existence of uncertainty about the must-stock proportion limiting only the profitability of exploitative pricing and not having an effect on the profitability of exclusive conditional rebates. The profitability of exploitative pricing is limited by the uncertainty because under such uncertainty, the incumbent cannot set a price such that it can completely exploit the profit under the uncertainty. Note that like in the baseline model, conditional rebates and two-part tariffs have an equivalent effect in this model.

The result that the incumbent can have an incentive to exclude the entrant by conditional rebates when there is the uncertainty about the must-stock proportion, is same as the conclusion of Chone and Linnemer (2016). However, Chone and Linnemer (2016) assume that the incumbent and the buyer jointly maximise their profit and the surplus is shared depending on the parameter of the competitor's bargaining power against the buyer. In contrast, Chapter 2 and this chapter analyse the models where the share of the surplus between the incumbent and the buyer are determined not exogenously but endogenously. The results in this chapter show that the incumbent does not need to collude with the buyer, because the incumbent can dominate the entire surplus of the market by exclusive rebates or exploitative pricing in the model where the incumbent, the entrant and the buyer move sequentially.

Moreover, it is also shown that relaxing the assumption that the incumbent cannot set the price larger than the buyer's willingness to pay for 1 unit, which is introduced in Ide et. al. (2016), makes the results differ from their model. As shown in Chapter 2, exclusive conditional rebate enables to obtain a monopoly profit by setting a high list price and a large discount to discourage the buyer to purchase from the entrant. While they conclude that the incumbent never uses a conditional rebate to exclude the entrant even when there is uncertainty about the entrant's cost or there exists a scale economy, this chapter shows that the incumbent may have an incentive to exclude the more efficient entrant when there is uncertainty about the must-stock proportion or the size of the contestable demand. The future topic could be the analysis of the situations where there is uncertainty about the entrant's cost and the incumbent is allowed to set a high list price.

Section 5 has scrutinized the case of uncertainty about the size of the contestable demand. It also shows that the incumbent can have an incentive to exclude the entrant by conditional rebate. However, the result in this model is different from the models analysed in the baseline model and in the previous sections in this chapter, in the sense that the incumbent strictly prefers exclusive market-share-based rebates to exclusive two-part tariffs or quantity-based rebates. This preference holds because such uncertainty limits the profitability of exclusive quantity-based rebates as well as exclusive two-part tariffs and exploitative pricing, while the incumbent's ability to obtain monopoly profit by exclusive market-share-based rebates is not undermined by uncertainty about the size of the contestable demand. Such an asymmetric effect of uncertainty concerning exclusive market-share-based rebates and other exclusive pricing causes the exclusion of the more-efficient entrant by market-share-based rebate, which never happens in the baseline model. This result suggests that a market-share-based rebate is more harmful to the consumer than is a two-part tariff or a quantity-based rebate.

Note that the results in Chapter 2 and this chapter show that the optimal threshold percentage of conditional rebates is always 100 per cent. However, in the real cases, the threshold can be less than 100 per cent. For example, in the Intel case, Intel offered 95 per cent as the threshold to some computer manufacturers. As shown in the theoretical analysis by Chen and Shaffer (2014), the optimal threshold percentage can be less than 100 per cent in some cases such as the situation where there are more than one buyers and scale economies. Taking such a situation into account is a challenge for the future study.

Section 6 has analysed the model such that the game is played more than once and the must-stock property of the incumbent's good is eroded when the buyer becomes familiar with the entrant's product. The analysis shows that exclusion of the more efficient entrant can happen. The results of the infinitely repeated game with a constant probability that the game ends also shows that the exclusion of the more efficient entrant is more likely to happen when the probability that the market disappears is high, for example where destructive innovation tends to occur. This result is similar to Ordover and Shaffer (2012) in the sense that the effect of the buyer's choice in one period on the next period gives the incumbent an incentive to exclude the entrant by conditional rebates, which implies that this model shows that the same result with their model can happen under the more flexible settings.

3.7.2 Policy Implications

The results in Chapter 2 and this chapter support that the desirable approach to judge the illegality of conditional rebates is a case by case basis approach based on the effect on competition, as currently taken in many jurisdictions, rather than a per illegal approach. As summarised in the previous section, conditional rebates may improve the welfare, while in some situations it harms the welfare through excluding a competitor. This finding is consistent with the previous studies showing that conditional rebate can improve welfare such as Calzolari and Denicolo (2013), Kolay et. al. (2004) and Majumdar and Shaffer (2009). Moreover, the previous works such as Chone and Linnemer (2016), Chao et. al. (2018) and Ordover and Shaffer (2013) present the situations where conditional rebates may harm competition. The results in these two chapters can add other possible situations to them and help to identify when the harm on competition is more likely by suggesting situations when incumbent have an incentive to exclude an entrant by exclusive conditional rebates.

With regard to methods to evaluate the illegality of conditional rebates, as introduced in Chapter 1, the AEC test is used to quantitatively evaluate the effect of conditional rebates on competition in practice. The AEC test compares the actual contestable share with the share that competitors who are as efficient as the dominant firm needs to compete with a dominant firm offering a rebate scheme based on the following formula;

$$s \equiv \frac{R}{(ASP - AAC)/V},$$

where s denotes the required share, and if s is smaller than the required share, the rebate scheme is regarded as exclusive, because even the as-efficient competitor cannot compete with the dominant company by setting its price as equal to its cost given this rebate scheme; where R is an amount of rebate given to the customer per unit; where ASP is a sales price and AAC is the average avoidable cost of the dominant firm; and where V denotes the number of the units that the customer requires to obtain the discount. The test is based on the rationale that when a dominant firm's product is must-stock for the buyers and it introduces a rebate scheme, the effective price of the dominant firm's product for the contestable demand is lower than the dominant firm's nominal price. When the effective price is lower than the dominant firm's unit cost, the competitor who is as efficient as the dominant firm will be excluded from the market even if the competitor sets its lowest possible price (i.e., the dominant firm's cost). Moreover, Scott Morton and Abrahamson (2016) suggest a similar approach to the AEC based on the metric called "effective entrant burden (EEB)".

The AEC test relies on the assumption that some part of the market is non-contestable (i.e., must-stock for buyers). This must-stock assumption is useful to consider the anticompetitive effect of conditional rebates. The existence of must-stock property can explain why a dominant firm can exclude its competitors by a conditional rebate and why competitors cannot react to the dominant company's pricing with same pricing behaviour.

However, the analysis in Chapter 2 and this chapter clarify the problems of AEC tests (and the EEB approach). First of all, the AEC tests treat the must-stock proportion as a fixed value, and it is assumed that the proportion does not depend on any variable, including the dominant firm's and the entrant's price schedules. The baseline model analysed in Chapter 2 shows that when it is assumed that the incumbent's product is literally must-stock, as in the AEC test, the incumbent does not use a conditional rebate to exclude the more efficient entrant, because covering only the must-stock market with high and exploitative pricing is more profitable. Furthermore, in such situation the incumbent may set an unrealistically high price, especially when the share of the must-stock demand is small. This possibility suggests that the AEC test assumes a too strong assumption about the must-stock property.

Such a strong assumption seems to hold in specific situations. Scott Morton and Abrahamson (2016) suggest that their EEB approach is especially suitable to evaluate product-line cases in which competitor do not sell all types of the products. The results in these two chapters confirm that their suggestion is reasonable. Those imply that the AEC test is likely to function properly when it is easy to differentiate between the products for the must-stock demand and the contestable demand. For example, such situation may happen when products consist of several parts and only an incumbent company can produce some part of the products by laws. The pulse oximetry product case (Masimo v. Tyco and Allied Orthopedic v. Tyco) can be a good example of such situation. In this case, the market consists of must-stock durable goods protected by the patents and contestable consumable goods and the rebate scheme applied to the purchase of both products. Another good example is found in Post Denmark II case.[16] In this case, Post Denmark has a statutory monopoly on the distribution of all letters, including direct advertisement mail, weighing up to 50 grams and the statutory monopoly covers 70 per cent of the bulk mail market. On the other hand, when it is difficult to differentiate between them as in the Intel case, the AEC test should be applied carefully.

In addition, the model analysed in Section 3 of this section shows that if there is an alternative option for the buyer such that it purchases all units from the entrant, such a possibility constrain the incumbent's behaviour. The result suggests that when the must-stock property is limited and when the entrant's price, compared to the incumbent' offer, is low enough to compensate the loss of not purchasing the incumbent's product, the buyer purchases only from the entrant, which means the share of the must-stock demand under this condition is zero. This result implies that the AEC test may overestimate the exclusivity of conditional rebates.[17]

In this way, one of the biggest problems of applying the AEC test is the difficulty in estimating the value of the must-stock share. In practice, the buyers' internal documents,

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[16] See the European Court of Justice's judgement in Case C□23/14 (2015).

[17] The problem of the non-constant must-stock proportion is considered to happen when the demand elasticities of the must-stock part of the demand and the contestable demand differ.

in particular the quantities that the buyers think are must-stock are often used as proxies of the size of the must-stock demand. Alternatively, the share of the incumbent is sometimes used as the proxy. However, such figures do not take into account what the firms will actually do if the competitor decreases the price to a level equal to the incumbent's cost. It follows that we must be careful when we apply the simple formula of the AEC test in practice.

On the other hand, we have shown that uncertainty about the must-stock proportion itself gives an incumbent an incentive to exclude the more-efficient competitor. This result suggests that we may not need to identify the exact figure of the must-stock proportion to quantitatively analyse the effect of conditional rebates.

Secondly, as shown in Chapter 2 and in this chapter, the fact that the incumbent can exclude the entrant by conditional rebate does not necessarily mean that the incumbent has an incentive to use a conditional rebate to exclude the entrant. The baseline model in Chapter 2 shows that if the incumbent's product is must-stock, the incumbent has no incentive to exclude the entrant through exclusive rebates, because exploitative pricing is more profitable. This discussion is applicable to the AEC test. For example, the positive result of the AEC test does not give an answer as to why the dominant firm chose the exclusive conditional rebate rather than exploitative pricing. It might be true that the strong must-stock assumption is wrong. With respect to the incentive to exclude the more efficient competitors, the factors considered in this chapter—such as uncertainty about the must-stock proportion and the demand size and the vulnerability of must-stock property such that it becomes weaker when entrance happens—can help to evaluate whether the exclusion of the more efficient competitors through conditional rebates makes sense for the dominant firm.

The results in these two chapters also imply that when we analyse the effect of conditional rebates or apply the AEC test, we must carefully consider what must-stock really means and to what extent a dominant firm's product is must-stock in a specific case. Those points seem to be not well focused so far, and hence, there is no consensus about the details of the definition of must-stock property. However, as discussed in these two chapters, the exclusivity of conditional rebates can be influenced by assumptions about must-stock property. This effect implies that misunderstanding of the must-stock property in a specific case can crucially undermine the accuracy of the AEC test, since the AEC test assumes a quite strong must-stock property and can overestimate the exclusivity of conditional rebates if the must-stock property is not as strong as assumed.

More precisely, the AEC test assumes that the product is homogeneous except that the buyers need to purchase the incumbents' products for a certain amount, and they do not purchase at all when it is not possible. However, in real cases, it may be true that the must-stock property is no more than one type of vertical or horizontal product differentiation where the dominant firm has an advantage over its competitors. If that is the case, we can apply the more generalised AEC test by calculating the must-stock proportion through certain demand-estimation methods for differentiated markets.

3.8. Appendix B

3.8.1 Proof of Lemma 3.1

Suppose B purchases only from E (i.e., $(q_I, q_E) = (0,1)$); B's willingness to pay for each unit decreases by λ, which entails that B's surplus is $1 - \lambda - P_E$. Next, suppose B purchases only from I, and B's surplus is $1 - P_I^L$. Finally, suppose B purchases α units from I and $1 - \alpha$ units from E, and B's surplus is $1 - P_I^L\alpha - P_E(1-\alpha)$. Lemma 1-1 shows that $1 - P_I^L \leq 1 - P_I^L\alpha - P_E(1-\alpha)$ holds if $P_I^L \geq P_E$. The condition that B prefers purchasing only from E to purchasing $1 - \alpha$ units from E can be written as

$$1 - \lambda - P_E \geq 1 - \alpha {P_I}^L - (1-\alpha)P_E.$$

This condition can be modified to

(A3.1) $P_E \leq {P_I}^L - \frac{\lambda}{\alpha}$.

Note that even if Equation (A3.1) holds, B will not trade with E when P_E exceeds B's willingness to pay, $1 - \lambda$. This resistance implies that in order for E to sell 1 unit to B, the condition that

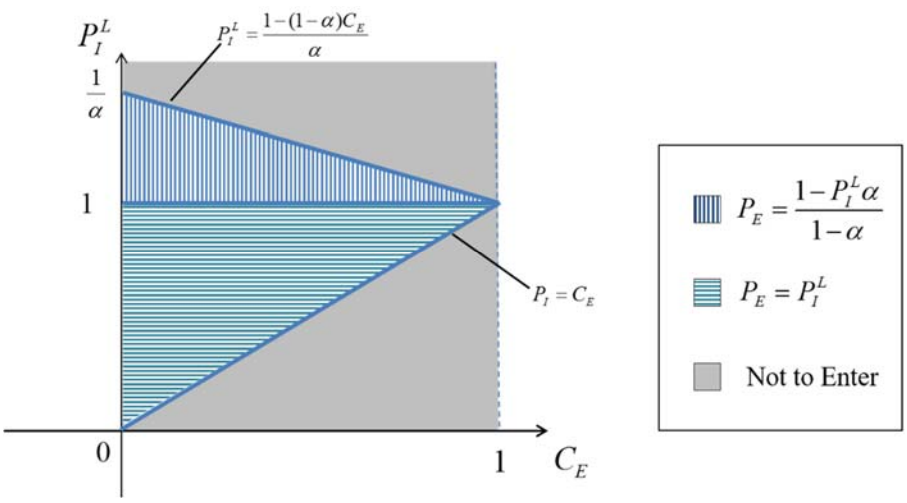
(A3.2) $P_E \leq 1 - \lambda$

must be satisfied. ■

3.8.2 Proof of Lemma 3.2

Firm E's optimal behaviour when it decides to sell $1 - \alpha$ units is equivalent to the optimal behaviour in the case analysed in Chapter 2, which is shown in Lemma 2.2, as can displayed in Figure 3.5 below.

Figure A3.1 E's most profitable pricing to sell $1 - \alpha$ units to B



The horizontal axis in Figure A3.1 represents *E*'s marginal cost, and the vertical axis represents *I*'s price (P_I^L). The upward-sloping line shows that the minimum P_I^L that *E* can sell $1-\alpha$ units to *B* without incurring a loss given. It follows that if (C_E, P_I^L) is located below this line (the bottom-right grey shaded area), *E* must give up entering the market because *E* cannot enter with a non-negative profit. The down-ward sloping line shows P_I^L such that *B*'s total cost to purchase α units from *I* and $1-\alpha$ units from *E* is equal to *B*'s willingness to pay when *E* sets its lowest possible price ($P_E = C_E$). This relationship implies if (C_E, P_I^L) is located above this line (the top-right grey shaded area), *E* must give up entering the market, because for *E* it is impossible to offer a price that is acceptable for *B* with a non-negative profit.

Hence, *E* can enter with a non-negative profit if (C_E, P_I^L) is in the triangle bounded by the upward-sloping line, the down-ward sloping line and the vertical axis of the graph. If (C_E, P_I^L) is in this area, as shown in Lemma 2.2, the optimal P_E depends on whether P_I^L is larger than 1 or not. If $P_I^L > 1$ (the top-left vertical-striped area), *E*'s optimal price is $(1-P_I^L\alpha)/(1-\alpha)$ because *B*'s willingness to pay is binding in this situation. If $P_I^L \leq 1$ (the bottom-left horizontal-striped area), *E*'s optimal price is P_I^L because *I*'s price is binding in this situation.

Then consider *E*'s optimal behaviour when it decides to sell 1 unit to *B*. According to conditions A) and B) above, the condition that *E* can sell 1 unit to *B* is

(A3.3) $P_I^L < C_E + \lambda/\alpha.$

In addition, according to the conditions B) and C) above, *E* cannot sell 1 unit if

(A3.4) $C_E < 1 - \lambda.$

This inability is the case because if Equation (A3.4) does not hold, *E* cannot offer its price equal to or lower than *B*'s willingness to pay without incurring a loss. Hence, *E* can sell 1 unit if both Equations (A3.3) and (A3.4) are satisfied. In this situation, *E* will set the highest price satisfying both Equations (A3.1) and (A3.2). These two equations are equalised if

$$P_I{}^L - \frac{\lambda}{\alpha} = 1 - \lambda \leftrightarrow P_I{}^L = 1 + \frac{1-\alpha}{\alpha}\lambda.$$

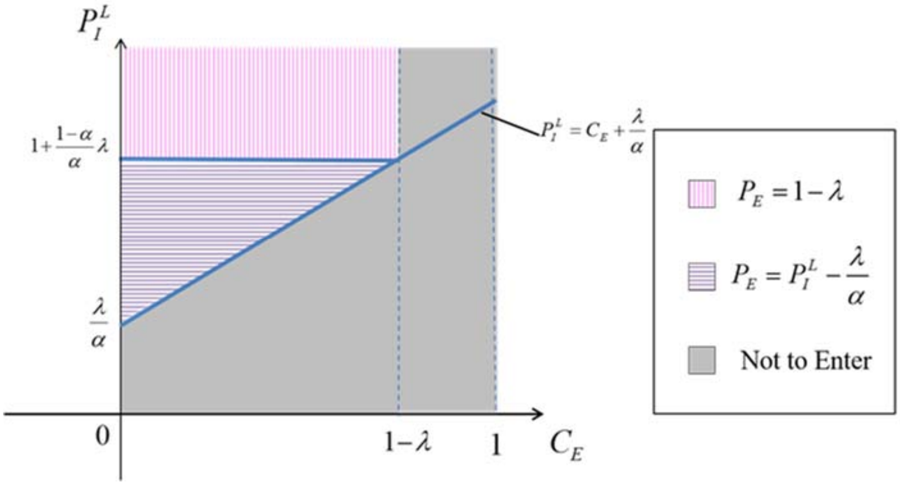
This condition implies that *E*'s most profitable price to sell 1 unit to *B* is $P_I^L - \lambda/\alpha$ if

(A3.5) $P_I^L \leq 1 + \frac{1-\alpha}{\alpha}\lambda,$

because Equation (A3.1) is binding in this situation. If Equation (A3.5) does not hold, *E*'s most profitable price to sell 1 unit to *B* is $1-\lambda$, because Equation (A3.2) is now binding.

From the discussion above, *E*'s optimal behaviour when it decides to sell 1 unit given P_I^L and C_E can be summarised as in Figure A3.2.

Figure A3.2 *E*'s most profitable pricing to sell 1 unit to *B*

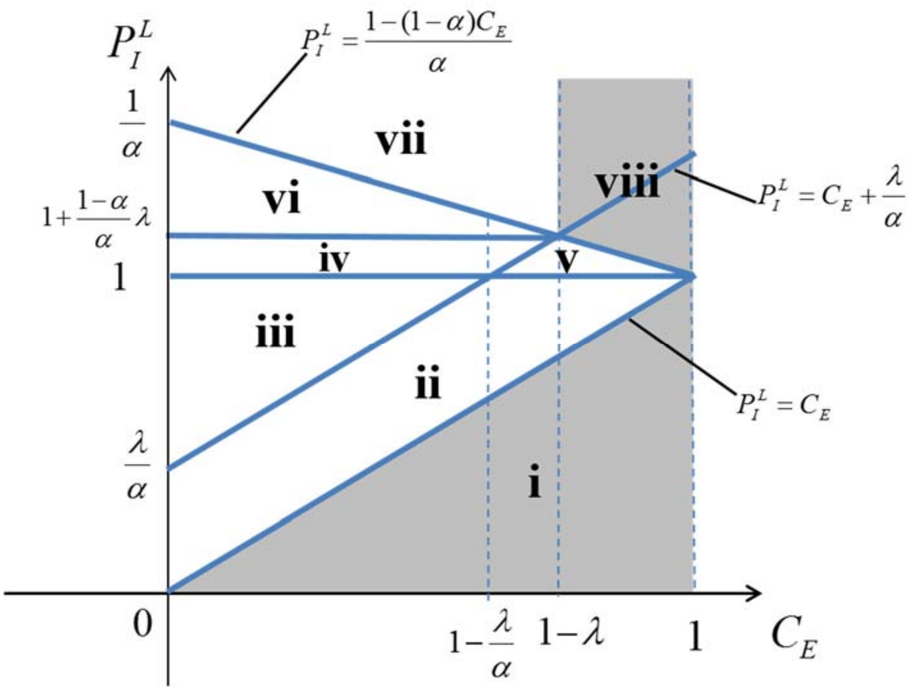


The horizontal axis in Figure A3.2 represents *E*'s marginal cost, and the vertical axis represents *I*'s price (P_I^L). The upward-sloping line shows that the minimum condition of P_I^L that *E* can sell 1 unit to *B* without incurring a loss, given as shown in Equation (A3.1). This minimum condition implies that if (C_E, P_I^L) is located below this line (the bottom grey shaded area), *E* must give up to sell 1 unit to *B* because *E* cannot do so with a non-negative profit.

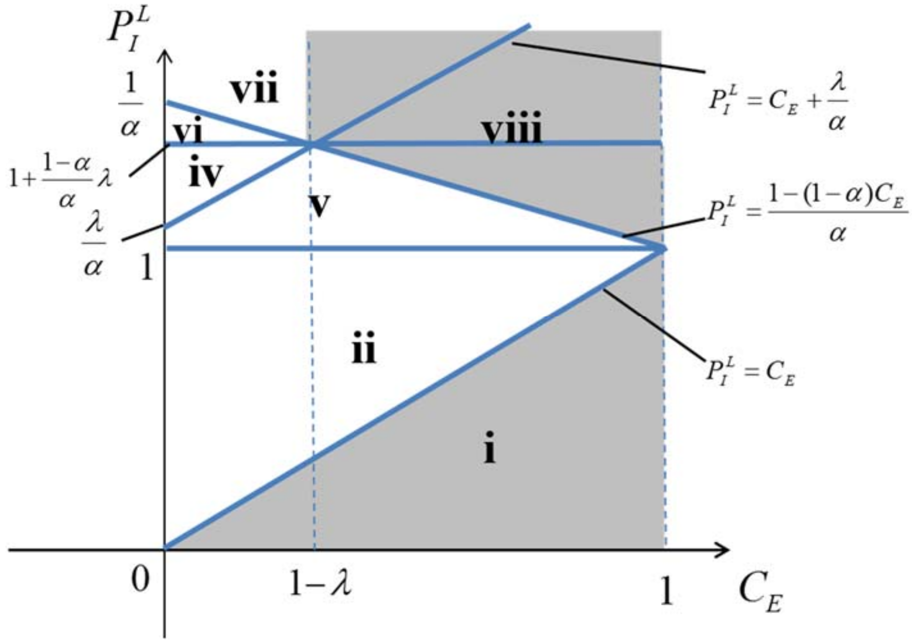
Figures A3.1 and A3.2 imply that this case can be divided into eight sub-cases, depending on the values of P_I^L and C_E, as summarised in Figure A3.3.

Figure A3.3 *E*'s most profitable pricing to sell 1 unit to *B*

i) if $\lambda < \alpha$



ii) if $\lambda \geq \alpha$



In each case, E's optimal response is presented below.

i) $P_I^L < C_E$ and viii) $P_I^L > \frac{1-(1-\alpha)C_E}{\alpha}$ *and* $C_E > 1-\lambda$

If (P_I^L, C_E) is in those areas, there is no way for E to sell any unit profitably. Hence, E has no choice but not to enter the market.

ii) $C_E \leq P_I^L < C_E + \frac{\lambda}{\alpha}$ and $P_I^L \leq 1$

If (P_I^L, C_E) is in this area, while E can sell $1-\alpha$ units by setting P_E at P_I^L, there is no way to sell 1 unit profitably. Hence, E's optimal pricing is to set P_E at P_I^L.

iii) $P_I^L \geq C_E + \frac{\lambda}{\alpha}$ and $P_I^L \leq 1$

If (P_I^L, C_E) is in this area, E can either sell $1-\alpha$ units by setting P_E at P_I^L or sell 1 unit by setting P_E at $P_I^L - \lambda/\alpha$. The condition that E prefers to sell $1-\alpha$ units can be written as

$$(P_I{}^L - C_E)(1-\alpha) > P_I{}^L - \frac{\lambda}{\alpha} - C_E.$$

The inequality can be modified to

(A3.6) $P_I{}^L < C_E + \frac{\lambda}{\alpha^2}$.

Hence, in this case, E's optimal pricing is to set P_E at P_I^L if (A3.6) holds and to set P_E at $P_I^L - \lambda/\alpha$ if (A3.6) does not hold.

iv) $P_I^L \geq C_E + \frac{\lambda}{\alpha}$ and $1 < P_I^L \leq 1 + \frac{1-\alpha}{\alpha}\lambda$

If (P_I^L, C_E) is in this area, *E* can either sell $1-\alpha$ units by setting P_E at $(1-P_I^L\alpha)/(1-\alpha)$ or sell 1 unit by setting P_E at $P_I^L - \lambda/\alpha$. The condition that *E* prefers to sell $1-\alpha$ units can be written as

$$(\frac{1-P_I{}^L\alpha}{1-\alpha} - C_E)(1-\alpha) > P_I{}^L - \frac{\lambda}{\alpha} - C_E.$$

The inequality can be modified to

(A3.7) $P_I{}^L < \frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right)$.

Hence, in this case, *E*'s optimal pricing is to set P_E at $(1-P_I^L\alpha)/(1-\alpha)$ if Equation (A3.7) holds and to set P_E at $P_I^L - \lambda/\alpha$ if (A3.7) does not hold. Note that $P_I{}^L = [\alpha/(1+\alpha)][C_E + \lambda/\alpha^2 + 1/\alpha]$ is an increasing function of C_E and passes the point that $(P_I^L, C_E) = (1-\lambda, 1+(1-\alpha)\lambda/\alpha)$.

v) $P_I^L < C_E + \frac{\lambda}{\alpha}$ and $P_I^L \leq \frac{1-(1-\alpha)C_E}{\alpha}$

If (P_I^L, C_E) is in this area, while *E* can sell $1-\alpha$ units by setting P_E at $(1-P_I^L\alpha)/(1-\alpha)$, there is no way to sell 1 unit profitably. Hence, *E*'s optimal pricing is to set P_E at $(1-P_I^L\alpha)/(1-\alpha)$.

vi) $P_I^L > 1 + \frac{1-\alpha}{\alpha}\lambda$ and $P_I^L \leq \frac{1-(1-\alpha)C_E}{\alpha}$

If (P_I^L, C_E) is in this area, *E* can either sell $1-\alpha$ units by setting P_E at $(1-P_I^L\alpha)/(1-\alpha)$ or sell 1 unit by setting P_E at $1-\lambda$. The condition that *E* prefers to sell $1-\alpha$ units can be written as

$$(\frac{1-P_I{}^L\alpha}{1-\alpha} - C_E)(1-\alpha) > 1-\lambda - C_E.$$

The inequality can be modified as $P_I^L < C_E + \lambda/\alpha$, which implies that this inequality does not hold in this case. This is because both $P_I^L < C_E + \lambda/\alpha$ and $P_I^L > 1 + (1-\alpha)\lambda/\alpha$ hold only if $C_E > 1-\lambda$, but there is no C_E such that both $P_I^L \leq [1-(1-\alpha)C_E]/\alpha$ and $P_I^L > 1+(1-\alpha)\lambda/\alpha$ hold. Hence, in this case, *E*'s optimal pricing is to set P_E at $1-\lambda$.

vii) $P_I^L \leq 1-\lambda$ and $P_I^L > \frac{1-(1-\alpha)C_E}{\alpha}$

If (P_I^L, C_E) is in this area, while E can sell 1 unit by setting P_E at $1-\lambda$, there is no way to sell $1-\alpha$ units profitably. Hence, *E*'s optimal pricing is to set P_E at $1-\lambda$. ■

3.8.3 Proof of Lemma 3.3

In the linear pricing model analysed in Chapter 2, *I*'s optimal behaviours can be divided into two types, competitive linear pricing or exploitative linear pricing. Competitive linear pricing (i.e., $P_I^L = C_E - \varepsilon$) is not constrained by the possibility that *E* dominates the market by setting a price much lower than *I*'s price, because *E* cannot set its price below *I*'s price when *I* chooses competitive linear pricing. Hence, this section focuses only on exploitative pricing, in particular how such pricing is constrained by the possibility that *E* dominates the market.

Exploitative linear pricing refers to a pricing behaviour that *I* gives up competing with *E* in the "contestable"[18] part of the market and exploiting the profit from the must-stock part of the market. It follows that in order to use exploitative pricing, *I* sets a price low enough to guarantee that *E* can enter the market and prefers sell only $1-\alpha$ units. Lemma 3.2 and Figure 3.5 show that the conditions are as follows:

(A3.7) ${P_I}^L < C_E + \frac{\lambda}{\alpha^2}$,

(A3.8) ${P_I}^L < \frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right)$ and

(A3.9) $P_I^L \leq \frac{1-(1-\alpha)C_E}{\alpha}$.

As shown in Figure 3.5, Equation (A3.7) is binding if $C_E \leq 1-\lambda/\alpha^2$, and Equation (A3.8) is binding if $1-\lambda/\alpha^2 < C_E \leq 1-\lambda$. Equation (A3.9) is binding if $C_E > 1-\lambda$.

In the exploitative pricing equilibrium, *I* sells only α units to *B*, and the profit is $(P_I^L - 0.5)\alpha$. Consequently, in such equilibrium, *I* sets the maximum price *I* can set. The maximum price that *I* can set to sell α units to *B* can be written as

$$
{P_I}^L = \left\{
\begin{array}{c}
\frac{[1-(1-\alpha)]C_E}{\alpha} \; if \; C_E > 1-\lambda \\
\frac{\alpha}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon \; if 1 - \frac{\lambda}{\alpha^2} < C_E \leq 1-\lambda \\
C_E + \frac{\lambda}{\alpha^2} - \varepsilon \; if \; C_E \leq 1 - \frac{\lambda}{\alpha^2}
\end{array}
\right\}.
$$

Incumbent firm *I* offers such a price as long as this price does not fall under its unit cost, 0.5. It follows that the condition that *I* prefers to leave the market rather than offering exploitative linear pricing can be written as

[18] Strictly speaking, under the assumption of this case, the entire of the market is contestable.

$\left(C_E + \frac{\lambda}{\alpha^2} - \varepsilon - 0.5\right)\alpha < 0.$

The inequality can be modified to

(A3.10) $C_E \leq 0.5 - \frac{\lambda}{\alpha^2}.$

Hence, the profit-maximising price when *I* decides to introduce exploitative linear pricing can be written as

$$P_I{}^L = \begin{Bmatrix} \frac{[1-(1-\alpha)]C_E}{\alpha}\ if\ C_E > 1-\lambda \\ \frac{\alpha^2}{1+\alpha}\left(C_E + \frac{\lambda}{\alpha^2} + \frac{1}{\alpha}\right) - \varepsilon\ if 1 - \frac{\lambda}{\alpha^2} < C_E \leq 1-\lambda \\ C_E + \frac{\lambda}{\alpha^2} - \varepsilon\ if\ 0.5 - \frac{\lambda}{\alpha^2} < C_E \leq 1 - \frac{\lambda}{\alpha^2} \\ +\infty\ if\ C_E \leq 0.5 - \frac{\lambda}{\alpha^2} \end{Bmatrix}$$, where $P_I = +\infty$ denotes that *I* decides to leave the market.

If such profit exceeds *I*'s profit from competitive linear pricing ($C_E - 0.5 - \varepsilon$), *I* prefers exploitative linear pricing to competitive linear pricing. This comparison leads directly to Lemma 3.3. ■

3.8.4 Proof of Proposition 3.2

Proposition 3.2 can be proved by showing that in what situations the possibility that *E* excludes *I* by setting low price constrains *I*'s ability to set the exclusive two-part tariff such that *I* not only dominates the market, but also profits much (i.e., unconstrained exclusive two-part tariff).

Lemma 2.5 shows that in the baseline model in Chapter 2, *I* can exclude *E* by offering a two-part tariff such that $P_I^V < C_E$. In this situation, such pricing is subject to the constraint of *B*'s willingness to pay (i.e., $P_I^F + P_I^V \leq 1$ must hold).

However, when there is a possibility that *E* excludes *I*, there is another condition to exclude *E* by a two-part tariff to eliminate that possibility. Entrant *E* can exclude *I* from offering the exclusive two-part tariff if *B*'s surplus when *B* purchases only from *I* is strictly larger than *B*'s surplus when *B* purchases only from *E*. This condition can be written as

$1 - P_I^F - P_I^V > 1 - \lambda - P_E.$

This condition can be modified to $P_E > P_I^F + P_I^V - \lambda.$

It follows that *I* can eliminate the possibility that *E* dominates the market by offering a two-part tariff such that $C_E > P_I^F + P_I^V - \lambda$, since $P_E \geq C_E$ must hold.

Hence, the optimal exclusive two-part tariff can be obtained by solving the following profit-maximisation problem;

$$\max_{P_I^F, P_I^V} P_I^F + P_I^V - 0.5$$

$$\text{s.t. } P_I^F + P_I^V \leq 1, P_I^F + P_I^V < C_E + \lambda \text{ and } P_I^V < C_E.$$

Solving this profit-maximisation problem gives Proposition 3.2. ■

3.8.5 Proof of Proposition 3.3

Proposition 3.3 can be proved by showing in what situation the possibility that *E* excludes *I* by setting a low price constrains *I*'s ability to set the exclusive conditional rebate such that *I* not only dominates the market, but also obtains large profit (i.e., unconstrained exclusive two-part tariff).

Lemma 2.8 shows that in the baseline model in Chapter 2, *I* can exclude *E* by offering a conditional rebate such that $P_I^R > [1 - C_E(1-\alpha)]/\alpha$. In this situation, such pricing is subject to the constraint of *B*'s willingness to pay (i.e., $P_I^R - \beta \leq 1$ must hold).

However, when there is a possibility that *E* excludes *I*, there is another condition to exclude *E* by a conditional rebate to eliminate the possibility. Entrant *E* can exclude *I*, offering the exclusive conditional rebate if *B*'s surplus when *B* purchases only from *I* is strictly larger than *B*'s surplus when *B* purchases only from *E*. This condition can be written as

$$1 - (P_I^R - \beta) > 1 - \lambda - P_E.$$

This condition can be modified to $P_E > P_I^R - \beta - \lambda$.

It follows that *I* can eliminate the possibility that *E* dominates the market by offering a conditional rebate such that $C_E > P_I^R - \beta - \lambda$ since $P_E \geq C_E$ must hold.

Hence, the optimal exclusive two-part tariff can be obtained by solving the following profit-maximisation problem;

$$\max_{P_I^R, \beta} P_I^R - \beta - 0.5$$

$$\text{s.t. } P_I^R - \beta \leq 1, P_I^R - \beta < C_E + \lambda \text{ and } P_I^R > [1 - C_E(1-\alpha)]/\alpha.$$

Solving this profit-maximisation problem yields Proposition 3.3. ■

3.8.6 Proof of Lemma 3.6

According to Equation (3.1), the price that maximises *I*'s profits when *I* decides to introduce exploitative linear pricing is $\frac{1-C_E(1-\alpha_H)}{\alpha_H}$ or $\frac{1-C_E(1-\alpha_L)}{\alpha_L}$.

The profit when setting $\frac{1-C_E(1-\alpha_H)}{\alpha_H}$ is

$$0.5\left(\frac{1-C_E(1-\alpha_H)}{\alpha_H}-0.5\right)\alpha_H+0.5\left(\frac{1-C_E(1-\alpha_H)}{\alpha_H}-0.5\right)\alpha_L.$$

The profit when setting $\frac{1-C_E(1-\alpha_L)}{\alpha_L}$ is $0.5\left(\frac{1-C_E(1-\alpha_L)}{\alpha_L}-0.5\right)\alpha_L$.

Since $0.5\left(\frac{1-C_E(1-\alpha_H)}{\alpha_H}-0.5\right)\alpha_H+0.5\left(\frac{1-C_E(1-\alpha_H)}{\alpha_H}-0.5\right)\alpha_L-0.5\left(\frac{1-C_E(1-\alpha_L)}{\alpha_L}-0.5\right)\alpha_L=0.5C_E\alpha_H+0.5\frac{1-C_E}{\alpha_H}\alpha_L>0$, the price that maximises *I*'s profits when *I* adopts this type of pricing is $\frac{1-C_E(1-\alpha_H)}{\alpha_H}$. The profit can be rewritten as $\Pi_I=1-\frac{\alpha_H-\alpha_L}{2\alpha_H}-0.5\overline{\alpha}-C_E(1-\frac{\alpha_H-\alpha_L}{2\alpha_H}-\overline{\alpha})$, where $\overline{\alpha}=\frac{\alpha_H+\alpha_L}{2}$.

The condition that *I* prefers exploitative linear pricing to competitive linear pricing is

$$1-\frac{\alpha_H-\alpha_L}{2\alpha_H}-0.5\overline{\alpha}-C_E\left(1-\frac{\alpha_H-\alpha_L}{2\alpha_H}-\overline{\alpha}\right)\geq C_E-0.5.$$

This condition can be rewritten as $C_E\leq\frac{3-\overline{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}{4-2\overline{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}$.

Note that $\frac{3-\overline{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}{4-2\overline{\alpha}-\frac{\alpha_H-\alpha_L}{\alpha_H}}<\frac{3-\overline{\alpha}}{4-2\overline{\alpha}}$. ■

3.8.7 Proof of Proposition 3.7

According to the baseline model analysed in Chapter 2, by offering a rebate scheme such that $P_I^R-\beta/(1-\alpha)<C_E$, $\gamma=1$ and $P_I^R-\beta=1$ hold, *I* can exclude *E* and obtain a profit of 0.5 (i.e., a monopoly profit) regardless of how efficient *E* is.

This finding implies that in the model analysed in this section, the condition that *I* can exclude *E* when the realised value of α is α_L is

$$P_I^R-\beta/(1-\alpha_L)<C_E\leftrightarrow P_I^R>\frac{1-C_E(1-\alpha_L)}{\alpha_L}.$$

Similarly, the condition that *I* can exclude *E* when the realised value of α is α_H can be written as

$$P_I^R>\frac{1-C_E(1-\alpha_H)}{\alpha_H}.$$

Note that

$$\frac{1-C_E(1-\alpha_L)}{\alpha_L}>\frac{1-C_E(1-\alpha_H)}{\alpha_H}.$$

It follows that by offering the rebate scheme such that both $P_I^R - \beta = 1$ and $P_I^R > [1 - C_E(1 - \alpha_L)]/\alpha_L$ hold, I can exclude E by obtaining the monopoly profit regardless of the realised value of α.■

3.8.8 Proof of Lemma 3.9

Since $\frac{1-P_I^L\alpha}{1-\alpha} < \frac{1+\theta-P_I^L\alpha}{1-\alpha+\theta}$ holds if $P_I^L > 1$,

Lemmata 3.7 and 3.8 imply that the quantities are given by

$$(q_I^S, q_I^B) = \left\{\begin{array}{ll} (1,1+\theta) & if\ P_I^L \leq 1\ and\ P_I^L < C_E \\ (\alpha,\alpha) & if\ P_I^L \leq 1\ and\ P_I^L \geq C_E\ or \\ & if\ P_I^L > 1\ and\ C_E \leq \frac{1-P_I^L\alpha}{1-\alpha} \\ (0,\alpha) & if\ \frac{1-P_I^L\alpha}{1-\alpha} < C_E \leq \frac{1+\theta-P_I^L\alpha}{1-\alpha+\theta} \\ (0,0) & if\ C_E > \frac{1+\theta-P_I^L\alpha}{1-\alpha+\theta} \end{array}\right\}.$$

According to the assumption concerning θ, $(q_I^S, q_I^B) = (0,\alpha)$ cannot happen in the equilibrium.

Hence, there are two possible options for the incumbent's optimal price. The first option is to set a price low enough to capture the entire market (which is equivalent to *competitive linear pricing* in the baseline model). The conditions of this type of pricing are $P_I^L \leq 1$ and $P_I^L < C_E$ and the profit is given by $\Pi_I = 0.5(P_I - 0.5) + 0.5(1 + \theta)(P_I - 0.5)$. Hence, setting P_I^L just below C_E maximizes I's profit ($P_I^L = C_E - \varepsilon$) when I decides to choose this option, and the expected profit is $(1 + 0.5\theta)(C_E - 0.5 - \varepsilon)$.

The second option is to give up competing with E and exploiting the profit from the must-stock part of the market by setting the price higher than 1 (which is equivalent to *exploitative linear pricing* in the baseline model). In order to implement this type of pricing, either $P_I^L \leq 1$ and $P_I^L \geq C_E$ or $P_I^L > 1$ and $C_E \leq (1 - P_I^L\alpha)/(1-\alpha)$ must hold. If I adopts this option, the profit is given by

$\Pi_I = 0.5\alpha(P_I - 0.5) + 0.5\alpha(P_I - 0.5)$. Hence, setting P_I^L at $[1 - C_E(1-\alpha)]/\alpha$ maximizes I's profit ($P_I^L = C_E - \varepsilon$) when I decides to choose this option, and the expected profit is $(1-\alpha)(1 - C_E) + 0.5\alpha$.

The condition that the incumbent prefers competitive linear pricing to exploitative linear pricing is

$(1 + 0.5\theta)(C_E - 0.5) > (1-\alpha)(1 - C_E) + 0.5\alpha.$

This equation can be modified to

$C_E > \frac{3+\theta-\alpha}{4+2\theta-2\alpha}$.■

3.8.9 Proof of Proposition 3.10

The proof of Proposition 3.10 is divided into two steps. In *Step 1* we find the optimal behaviours of *B* and *E* given the two-part tariff offered by *I*. In *Step 2*, we find the optimal exclusive two-part tariff anticipating the responses of *B* and *E* specified in *Step 1*.

Step 1: Optimal behaviours of B and E

The optimal behaviours if the determined contestable demand is $1-\alpha$ are same as in Lemmata 2.4 and 2.5. The optimal behaviours if the determined contestable demand is $1-\alpha+\theta$ can be obtained by substituting $1-\alpha+\theta$ into $1-\alpha$ in Lemmata 2.4 and 2.5. Hence, the optimal behaviours of *B* and *I* can be written as follows:

Given (P_I^F, P_I^V) and P_E, the buyer's optimal purchasing volume is as follows:

If the determined contestable demand is $1-\alpha$,

$$({q_I}^*, {q_E}^*) = \begin{Bmatrix} (1,0) \; if \; P_I^V < P_E \; and \; P_I^F + P_I^V \leq 1 \\ (\alpha, 1-\alpha) \; if \; P_I^V \geq P_E \; and \; P_I^F + P_I^V\alpha + P_E(1-\alpha) \leq 1 \\ (0,0) \; otherwise \end{Bmatrix}.$$

If the determined contestable demand is $1-\alpha+\theta$,

$$({q_I}^*, {q_E}^*) = \begin{Bmatrix} (1+\theta,0) \; if \; P_I^V < P_E \; and \; P_I^F + P_I^V(1+\theta) \leq 1+\theta \\ (\alpha, 1-\alpha+\theta) \; if \; P_I^V \geq P_E \; and \; P_I^F + P_I^V\alpha + P_E(1-\alpha+\theta) \leq 1+\theta \\ (0,0) \; otherwise \end{Bmatrix}.$$

Moreover, given (P_I^F, P_I^V), the entrant's optimal pricing anticipating the buyer's optimal response above is as follows:

If the determined contestable demand is $1-\alpha$,

$${P_E}^* = \begin{Bmatrix} P_I^V \; if \; P_I^F + P_I^V \leq 1 \; and \; P_I^V \geq C_E \\ \frac{1-P_I^F-P_I^V\alpha}{1-\alpha} \; if \; P_I^F + P_I^V > 1 \; and \; \frac{1-P_I^F-P_I^V\alpha}{1-\alpha} \geq C_E \\ +\infty \; if \; otherwise \end{Bmatrix}.$$

If the determined contestable demand is $1-\alpha+\theta$,

$${P_E}^* = \begin{Bmatrix} P_I^V \; if \; P_I^F + \frac{P_I^V}{1+\theta} \leq 1 \; and \; P_I^V \geq C_E \\ \frac{1+\theta-P_I^F-P_I^V\alpha}{1-\alpha+\theta} \; if \; P_I^F + P_I^V > 1 \; and \; \frac{1+\theta-P_I^F-P_I^V\alpha}{1-\alpha+\theta} \geq C_E \\ +\infty \; if \; otherwise \end{Bmatrix}.$$

Note that $P_E = +\infty$ denotes that *E* decides not to enter.

Step 2: Optimal exclusive two-part tariff

According to the optimal behaviours of *B* and *E* specified in *Step 1*, the conditions that *I* can exclude *E* through the two-part tariff regardless of the determined size of the contestable demand are $P_I^F + P_I^V \leq 1$, $P_I^F + P_I^V/(1+\theta) \leq 1$ and $P_I^V < C_E$. In this situation, *I*'s expected profit is given by

$\Pi_I = 0.5(P_I^F + P_I^V - 0.5) + 0.5[P_I^F + (1+\theta)(P_I^V - 0.5)] = P_I^F + (1 + 0.5\theta)(P_I^V - 0.5)$.

Hence, the optimal exclusive two-part tariff can be obtained by solving the following profit-maximisation problem:

$$\max_{P_I^F, P_I^V} P_I^F + (1+0.5\theta)(P_I^V - 0.5)$$

$$\text{s.t. } P_I^F + P_I^V \leq 1, P_I^F + P_I^V/(1+\theta) \leq 1 \text{ and } P_I^V < C_E.$$

Solving this profit-maximisation problem gives Proposition 3.10.■

3.8.10 Proof of Proposition 3.11

The proof of Proposition 3.11 is divided into three steps. In *Step 1* we find the optimal behaviours of *B* and *E* given the market-share-based rebate by *I*. In *Step 2*, we find the optimal exclusive market-share-based rebate anticipating the responses of *B* and *E* specified in *Step 1*. *Step 1* shows that the expected profit from the optimal exclusive market-share-based rebate is always larger than the profit from the exclusive quantity-based rebate.

Step 1: Optimal behaviours of B and E

The optimal behaviours if the determined contestable demand is $1-\alpha$ are the same as in Lemmata 2.7 and 2.8. The optimal behaviours if the determined contestable demand is $1-\alpha+\theta$ can be obtained by substituting $1-\alpha+\theta$ into $1-\alpha$ in Lemmata 2.7 and 2.8. Hence, the optimal behaviours of *B* and *I* can be written as below.

Given (P_I^{MR}, β^M), and P_E, the buyer's optimal purchasing volume is as follows:

If the determined contestable demand is $1-\alpha$,

$$(q_I^*, q_E^*) = \begin{Bmatrix} (1,0)\ if\ P_E > P_I^{MR} - \beta^M\ and\ P_I^{MR} - \beta^M \leq 1 \\ (\alpha, 1-\alpha)\ if\ P_E \leq P_I^{MR} - \frac{\beta^M}{1-\alpha}\ and\ P_I^{MR}\alpha + P_E(1-\alpha) \leq 1 \\ (0,0)\ otherwise \end{Bmatrix}.$$

If the determined contestable demand is $1-\alpha+\theta$,

$(q_I{}^*, q_E{}^*) =$

$$\begin{Bmatrix} (1+\theta, 0)\ if\ P_E > P_I^{MR} - \frac{\beta^M}{1-\alpha+\theta}\ and\ P_I^{MR} - \beta^M \le 1 \\ (\alpha, 1-\alpha+\theta)\ if\ P_E \le P_I^{MR} - \frac{\beta^M}{1-\alpha+\theta}\ and\ P_I^{MR}\alpha + P_E(1-\alpha+\theta) \le 1+\theta \\ (0,0)\ otherwise \end{Bmatrix}.$$

Moreover, given (P_I^{MR}, β^M), the entrant's optimal pricing anticipating the buyer's optimal response above is as follows:

If the determined contestable demand is $1-\alpha$,

$$P_E{}^* = \begin{Bmatrix} P_I^{MR} - \frac{\beta^M}{1-\alpha}\ if\ P_I^{MR} - \beta^M \le 1\ and\ P_I^{MR} - \frac{\beta^M}{1-\alpha} \ge C_E \\ \frac{1-P_I^{MR}\alpha}{1-\alpha}\ if\ P_I^{MR} - \beta^M > 1\ and\ \frac{1-P_I^{MR}\alpha}{1-\alpha} \ge C_E \\ +\infty\ if\ otherwise \end{Bmatrix}.$$

If the determined contestable demand is $1-\alpha+\theta$,

$$P_E{}^* = \begin{Bmatrix} P_I^{MR} - \frac{\beta^M}{1-\alpha+\theta}\ if\ P_I^{MR} - \beta^M \le 1\ and\ P_I^{MR} - \frac{\beta^M}{1-\alpha+\theta} \ge C_E \\ \frac{1+\theta-P_I^{MR}\alpha}{1-\alpha+\theta}\ if\ P_I^{MR} - \beta^M > 1\ and\ \frac{1+\theta-P_I^{MR}\alpha}{1-\alpha+\theta} \ge C_E \\ +\infty\ if\ otherwise \end{Bmatrix}.$$

Note that $P_E = +\infty$ denotes that *E* decides not to enter.

Step 2: Optimal exclusive market-share-based rebate

According to the optimal behaviours of *B* and *E* specified in *Step 1*, the conditions that *I* can exclude *E* through the market-share-based rebate regardless of the determined size of the contestable demand are

(A3.11) $\quad P_I^{MR} - \beta^M \le 1,$

(A3.12) $\quad P_I^{MR} - \beta^M/(1-\alpha) < C_E$ and

(A3.13) $\quad P_I^{MR} - \beta^M/(1-\alpha+\theta) < C_E$.

In this situation, *I*'s expected profit is given by

$\Pi_I = 0.5(P_I^{MR} - \beta^M - 0.5) + 0.5(1+\theta)(P_I^{MR} - \beta^M - 0.5) = (1+0.5\theta)(P_I^{MR} - \beta^M - 0.5).$

According to Inequality (A3.11), the expected profit is maximised when $P_I^{MR} - \beta^M = 1$ holds. Since $\beta^M/(1-\alpha) > \beta^M/(1-\alpha+\theta)$, when Inequality (A3.13) holds, Inequality (A3.12) always holds. By substituting $P_I^{MR} - \beta^M = 1$ into Inequality (A3.13), the equations can be rewritten as

$P_I^R > \frac{1-C_E(1-\alpha+\theta)}{\alpha-\theta}.$

Step 3: Exclusive quantity-based rebate is dominated by exclusive market-share-based rebate

Step 2 shows that *I* can capture the entire surplus of the market by using the market-share-based rebate in the sense that *I* can obtain the profit that *I* could obtain, supposing that *I* is the dominant producer in the market, which is equal to $0.5(1 + 0.5\theta)$. This implies that *I* has an incentive to use a quantity-based rebate only if *I* can get a profit of $0.5(1 + 0.5\theta)$ by using a quantity-based rebate.

The discussion in the baseline model shows that in order to capture the entire surplus of the market by a quantity-based rebate when the determined contestable demand size is small, *I* must offer the rebate scheme such that the discounted price is equal to *B*'s willingness to pay ($P_I^R - \beta = 1$) and that the threshold volume to obtain the discount is equal to the demand size ($\gamma = 1$).[19] However, if *I* sets γ equal to 1, *I* cannot capture the entire surplus of the market by a quantity-based rebate when the determined contestable demand is great. This inability is due to *B* being unable to obtain a further discount when *I* purchases more than 1 unit. Hence, when the determined demand size is large, *B* will purchase only 1 unit from *I* and θ units because *E*'s price is equal to or lower than the discounted price of *I*'s product ($P_E \leq 1$). In other words, suppose *I* offers a rebate scheme such that $P_I^R - \beta = 1$ and $\gamma = 1$; the effective price for the units of *I*'s product that *B* purchases more than the threshold volume, 1, is equal to 1, so *I* cannot block the entry. In this situation, when the determined contestable demand size is large, *E* can enter the market by setting its price equal to the discounted price for *I*'s product (i.e., $P_E = P_I^R - \beta = 1$), from which it follows that *B*'s surplus is zero while *E* obtains a profit of $(1 - C_E)\theta$. In this situation, *I*'s expected profit is equal to that granted by the exclusive two-part tariff, $0.5 + 0.5\theta(C_E - 0.5)$. ■

3.8.11 Proof of Lemma 3.10

From the definition, the second period can be divided into the following two cases: $\alpha_2 = \alpha_1$ and $\alpha_2 = 0$.

If $\alpha_2 = \alpha_1$ holds in the second period, the game is equivalent to one-shot game that is analysed in Chapter 2. Hence, from Proposition 2.4, the incumbent's optimal behaviour in the second period is to use exclusive conditional rebates if $C_E \geq 0.5$ and exploitative pricing if $C_E < 0.5$.

If $\alpha_2 = 0$, the game in the second period is a simple Bertrand game because *I* no longer has an advantage such that its product is must-stock, so *B*'s options are either to purchase only from *I*, to purchase only from *E* or to purchase not at all. These options imply that in this situation, *I* can trade with *B* only if $P_{I2} < C_E$. Otherwise, *I* is excluded

[19] Moreover, in order to block the entry, $P_I^R > [1 - (1 - \alpha)C_E]/\alpha$ must hold.

from the market, because E will respond by setting P_{E2} at P_{I2}. This sequence of events implies that I's optimal pricing in this situation is setting P_{I2} at $C_E - \varepsilon$, where ε is a small positive number. Such pricing is equal to competitive linear pricing in Chapter 2. The profit from competitive linear pricing is

$$\Pi_I^{CL} = C_E - 0.5 - \varepsilon.$$

Because $C_I = 0.5$, the condition that I can trade with B and does offer its price such that $P_{I2} = C_E - 0.5 - \varepsilon$ is $C_E \geq 0.5$. If $C_E < 0.5$, I cannot trade with B, because E can respond by setting P_{E2} at P_{I2} even if I sets the lowest possible price, 0.5. Hence, in this situation, I decides to leave the market, and I's profit in the second period is equal to zero.[20] ■

3.8.12 Proof of Proposition 3.13

As analysed in Section 6.2, when the entry happens in a certain period, the next game becomes a simple Bertrand game because I no longer has an advantage such that its product is must-stock. This change implies that if I introduces a pricing scheme at period $\hat{t} \in [1, T)$ such that E can enter, the optimal behaviour thereafter is to maximise profit in the Bertrand game in each period. According to Lemma 3.10, such behaviour is to offer competitive linear pricing to obtain a profit of $C_E - 0.5 - \varepsilon$ if $C_E \geq 0.5$ and to leave the market if $C_E < 0.5$.

However, if $C_E \geq 0.5$, I has no incentive to allow E's entry, since the pricing behaviour that maximises I's profit in each period under the condition that there is no entry in the previous periods is the exclusive conditional rebate, which blocks the entry. This situation is equivalent to what happens in the case of the two-period model analysed in the previous section. Hence, if $C_E \geq 0.5$, I's most profitable behaviour is to keep introducing exclusive conditional rebates.

On the other hand, if $C_E < 0.5$, the most profitable pricing for I that allows E's entry is exploitative pricing, and the profit from such pricing exceeds the profit from exclusive conditional rebates that is the most profitable pricing, which blocks the entry ($\Pi_I^{EP} > \Pi_I^{ECCR}$). In this situation, the maximum total expected profit when I introduces exploitative pricing at period $\hat{t} \in [1, T)$ can be written as

$$\text{(A3.14)} \qquad \begin{Bmatrix} \Pi_I^{EP} \ if \ \hat{t} = 1 \\ \sum_{t=1}^{\hat{t}} \delta^{t-1} \, \Pi_I^{ECCR} + \delta^{\hat{t}} \Pi_I^{EP} \ if \ 1 < \hat{t} < T \end{Bmatrix}.$$

Moreover, if $C_E \geq 0.5$ and an entry has not happened previously until period $T - 1$, the game at period T presents the same situation as the second game in the two-period

[20] In this situation, after observing that I decided to leave the market, E will set the dominant price ($P_{E2} = 1$), so B's surplus is zero. If a certain more complex assumption with regard to the first mover of the game is introduced and, as a result, the first mover of the second period becomes E, E's optimal pricing is not 1 but 0.5, because otherwise I can trade with B. If that is the case, B can obtain a positive profit, 0.5.

model. Hence, I's optimal behaviour at period T is exploitative pricing. In this situation, I's maximum total expected profit can be written as

(A3.15) $\sum_{t=1}^{T-1} \delta^{t-1} \Pi_I^{ECCR} + \delta^T \Pi_I^{EP}$.

Therefore, if $C_E < 0.5$, I's optimal behaviour is to introduce exploitative pricing at some time between period 1 and period T. Let $\Pi_I^{\hat{t}}$ denote I's total profit when I introduces exploitative pricing at period $\hat{t} \in [1, T]$. Note that from Equations (A3.14) and (A3.15), $\Pi_I^{\hat{t}} - \Pi_I^{\hat{t}-1} = \delta^{\hat{t}-1}[\Pi_I^{ECCR} - (1-\delta)\Pi_I^{EP}]$ holds for $\hat{t} \geq 2$. This equivalence implies that the condition that I prefers to introduce exploitative pricing at period $\hat{t} \in [1, T]$ and to introduce exploitative pricing at period $\hat{t} - 1$ is equal to Inequality (3.3). It follows that when Inequality (3.3) is satisfied, I's optimal behaviours is introducing exploitative pricing only in the last period and introducing exclusive conditional rebate in the previous periods. ■

3.8.13 Proof of Proposition 3.14

As analysed in Section 8.12, if $C_E \geq 0.5$, I has no incentive to allow E's entry. This lack of incentive occurs because the pricing behaviour that maximises I's profit in each period under the condition that there is no entry in the previous periods is the exclusive conditional rebate, which blocks the entry. This result is equivalent to what happens in the case of the two-period model analysed in the previous section. Hence, if $C_E \geq 0.5$, I's most profitable behaviour is to keep introducing exclusive conditional rebates.

On the other hand, if $C_E < 0.5$, the most profitable pricing for I that allows E's entry is exploitative pricing, and the profit from such pricing exceeds the profit from excusive conditional rebates that is the most profitable and that blocks the entry ($\Pi_I^{EP} > \Pi_I^{ECCR}$).

If $C_E < 0.5$ holds and the entry has been blocked before period $\hat{t} \geq 1$ ($\alpha_{\hat{t}} = \alpha_1$), the maximum expected total profit when I keeps introducing exclusive conditional rebates, hereafter evaluated at period $\hat{t}$, can be written as

(A3.16) $\sum_{\hat{t}=1}^{\infty} \delta^{\hat{t}-1}\theta^{\hat{t}-1} \Pi_I^{ECCR} = \frac{\Pi_I^{ECCR}}{1-\delta\theta}$.

On the other hand, the maximum expected total profit when I introduces exploitative pricing at period $\hat{t}$ and will be kept out of the market thereafter evaluated at period $\hat{t}$ can be written as Π_I^{EP}.

Hence, if $C_E < 0.5$ holds and the entry has been blocked before period $\hat{t}$, the condition that I prefers to continue introducing exclusive conditional rebates rather than introducing exploitative pricing at period $\hat{t}$ is

(A3.17) $\frac{\Pi_I^{ECCR}}{1-\delta\theta} \geq \Pi_I^{EP}$.

Equation (A3.17) can be modified to

(A3.18) $\delta\theta \geq \frac{\Pi_I^{EP} - \Pi_I^{ECCR}}{\theta \Pi_I^{EP}}$.

Equation (A3.18) is equal to the equation that substitutes δ into $\delta\theta$ in Inequality (3.3). Because *I* faces the same problem in each period as long as the entry has never happened, if Equation (A3.18) holds *I* prefers to keep introducing exclusive conditional rebates rather than introducing exploitative pricing in a certain period.■

Chapter 4: Platform Competition and Effects of Platform Parity Clauses with a Monopoly Platform, a Monopoly Seller and Uninformed Consumers

4.1. Introduction

In recent years, online platforms have grown rapidly. The companies behind these platforms offer a variety of services such as e-commerce and travel bookings, and they bring significant benefits to consumers and small and small–medium-sized enterprises by making the transactions between them easier. On the other hand, it has been pointed out that such platforms tends to have dominant power due to the characteristics of this area, including network effects and some types of contracts imposed by such platforms, which can have negative impacts on competition. Among such contracts, most-favoured nation clauses (MFNs) or platform parity clauses have received much attention; these clauses prohibit companies using platforms from selling under more preferable conditions (e.g., better prices) on other platforms or their own website. National competition authorities have taken certain actions against these platforms, in particular against online booking platforms such as booking.com. In France, a law has even been established to prohibit the introduction of such clauses in the hotel industry.

Most-favoured nation clauses themselves are not new business practices. Traditionally, MFNs have been used to refer clauses that guarantee that one party of the contract will not offer better trade conditions (e.g., lower price) to third parties than the conditions applied to the other party. The MFN clauses that have received attention recently differ from the traditional MFNs in the sense that such clauses are used in vertical situations (i.e., there are some intermediaries between sellers and buyers) and are not about the trade conditions between the parties but about the trade conditions applied to third parties, in particular the trade conditions for buyers who purchase on a platform. For this reason, some commentators prefer to call such clauses “platform parity clauses”. If the restrictions imposed by such clauses are prices, those clauses are often called “price parity clauses”.

Platform parity clauses or MFNs are divided into two types, depending on scope. If the restriction imposed by the clauses regards selling under better trade conditions on other platforms, such clauses are called “wide” MFNs or parity clauses. On the other hand, if the restriction includes selling under better trade conditions on sales channels other than platforms, including direct sales by the party, such clauses are called “narrow” MFNs or parity clauses.

The debate regarding platform parity clauses or MFNs is on-going. In particular, there is a division of the views among the European competition authorities with respect to the approach towards narrow platform parity clauses. While French, Italian and Swedish authorities have concluded that narrow platform parity clauses can be justified in the hotel booking platform case,[21] the German Federal Cartel Office published a decision that not only wide-platform parity clauses but also narrow-platform parity clauses should be prohibited.[22] In its decision, the German authority denied the hotel booking platform's claim to justify the platform parity clauses: that they improve efficiency by reducing the risk of free rides, which imposes a negative effect on the incentive to make investments in the industry, because those problems did not exist at all in the case.

Moreover, the Competition and Market Authority prohibited the wide MFNs in the private motor insurance industry in 2014. In the industry, many consumers purchase their insurance through price comparison websites, which are two-sided platforms that match consumers searching for insurance with private car insurance companies. The Competition and Market Authority found that the wide MFNs imposed by the platforms against the insurance companies softened price competition between price comparison websites and was likely to increase the consumer prices. On the other hand, the Competition and Market Authority did not prohibit narrow MFNs, because their effect on competition is small.

In the United States, the Department of Justice has alleged that the contracts between Apple and the major publishers violate the antitrust law, including MFN clauses, because the contracts harm the competition by increasing the e-book prices.[23]

This chapter aims to analyse the effects of MFNs or parity clauses on competition and consumer welfare.

4.2. Literature Review

With regard to the issues related platform parity clauses or MFNs, some economic studies have been done already. According to Hviid and Fletcher (2017), the anticompetitive effects caused by parity clauses and MFNs, in particular wide-type, identified in the economic literature regarding these conduct can be summarised in the following three: (1) softening competition between retailers or platforms on the fees imposed on suppliers, (2) restricting the entrance of other retailers or platforms and (3)

[21] See the French Competition Commission's decision concerning practices implemented in the online hotel booking sector in 15-D-06 (2015). In the decision, the commission accepted the commitment proposed by Booking.com, including the removal of the wide MFNs. The commitment also includes the removal of narrow MFNs for off-line sales and loyalty customers.

[22] See the German Federal Cartel Office's decision against HRS on Best price clauses in B9-66/10 (2013).

[23] See U.S. v. Apple, Inc. et al., (S.D.N.Y. Apr. 6, 2012) (proposed final judgment as to defendants Hachette, HarperCollins, and Simon & Schuster). European Commission dealt with the similar case. See CASE AT.39847 – E-BOOKS.

eliminating price competition on platforms in the asymmetric market where suppliers set their prices on some platforms but other platforms can set the prices on their platforms, based on their previous works such as Hviid (2010) and Hviid (2015). Note that this and the next chapter cover the first effect.

First, Boik and Corts (2015) examine the case where there is one supplier who sells its product through two competing differentiated platforms and analyse the effects of platform price parity clauses. They show that the clauses lead to higher commissions imposed by the platform, followed by the higher prices set by the supplier, since under the clauses, when one platform increases the fee to use the platform, the supplier no longer diverts the sales to the other platform by setting lower price on the platform. They also analyse the effects on entry and conclude that if the demand for the entrant platform is low compared to the incumbent platform and the fixed cost to enter the market is high to some extent, the entrance that could happen without the platform parity clauses would not happen after the platform introduces the clauses.

Johanson (2017) considers a case of multiple suppliers and multiple retailers in which the retailers impose not a fixed fee per unit but a commission based on the suppliers' revenues. He found that the agency model is a weapon against the suppliers in the sense that it gives a lower price for the buyer, raises buyers' welfare higher and raises retailers' welfare higher than does the wholesale model; however, when the agency model is used with parity clauses, the prices become higher, and consumers are harmed. Rey and Verge (2016) also examine the case where there are multiple differentiated suppliers and multiple differentiated retailers and the commissions paid by the suppliers to the retailers are determined based on secret negotiations between a supplier and a seller in which they share their (maximised) joint surplus depending on the parameter of the relative bargaining power between the supplier and the retailer. They also examine the effects of price parity clauses and conclude that the clauses do not have any effect on the equilibrium prices and profits in both the agency model and the wholesale model, because the commissions depend on the retailers' costs.

Although those studies do not regard direct sales by sellers using platforms, Johanson and Verge (2016) consider the possibility of sellers' direct sales. In order to analyse the effects of platform price parity clauses, they adopt the linear inverse demand functions adapted from Ziss (1995), which include the parameters about the degrees of inter-brand competition and intra-brand competition.[24] They show that wide platform price parity clauses do not necessarily lead to higher commissions and higher final prices, because the constraints due to the possibility that the sellers prefer not to use a platform restrict the platforms' ability to set high commissions. The degree of harm depends on the sellers' ability to sale directly. When the sellers' ability to direct sell is strong, the rise of consumer prices is unlikely, because in such cases the constraints by the possibility that the seller will not use the platform that raises the commission becomes

[24] Specifically, the inverse demand for supplier j's product on sales channel i is given by, $p_{ij} = 1 - q_{ij} - \alpha \sum_{k \neq j \in J} q_{ik} - \beta(\sum_{h \neq i \in I} q_{hj} + \alpha \sum_{k \neq j \in J} q_{hk})$ where $i \in I \equiv \{A, B, D\}$ (i.e. two platforms or direct sales) and $j \in J \equiv \{1, \dots, N\}$.

stronger. They also point out that the effects of wide platform price parity clauses and that of narrow platform price parity clauses gives equivalent outcomes, and hence, the remedies that replace wide parity clauses with the narrow type, as adopted in the hotel booking platform case in France, may be ineffective.

While the above models do not regard investment, some studies have considered investment. Edelman and Wright (2018) have analysed the model that if buyers purchase a product through an intermediary rather than purchasing from the seller directly, the buyer obtains certain benefits, depending on the investment an intermediary makes. They point out that the examples of the benefits that buyers obtain by using an intermediary include the provision of complementary products, reduction of transaction costs and financial rebates referring to the credit and debit card industry. They show that an intermediary has an incentive to introduce price parity clauses (in their words, "price coherence"), when possible and that the introduction of such clauses increases investment by the intermediary to an excessive level at which the efficient outcome is not achieved, harming buyers. On the other hand, in the case where the price parity clause is not introduced, the amount of the investment that the platform makes is efficient, where the net benefit that the platform can make is maximised.

Wang and Wright (2016) have studied this topic by introducing a model based on a search model in which consumers choose whether they search horizontally differentiated sellers directly or search on a platform. In their model, it is assumed that if a buyer chooses to search on a platform, in addition to obtaining a convenience benefit, as in Edelman and Wright (2018)'s model, the buyer can search at lower cost. They also assume that when a buyer finds a seller through searching on a platform, the consumer can switch to buy the product from the seller directly when the direct sales price is cheap enough without incurring an additional search cost ("showrooming"). They show that the introduction of wide price parity clause leads to higher commission fees imposed by the platform, a higher price for the consumer and higher profits for the platform, because such clauses limit constraints through showrooming on the platform's ability to set a high commission. With regard to narrow price parity clauses, they conclude that while the clauses remove constraints by showrooming, the introduction of the narrow type can be beneficial for consumers when platforms compete and the competition is strong enough to limit the platforms' ability to set high commissions. They also analyse an extension model where the reduction of search costs when using a platform depends on the investment that the platform makes, as presented by Edelman and Wright (2018). Wang and Wright (2016) find that the introduction of price parity clauses by a platform in the presence of showrooming by buyers can increase the incentive for the platform to invest; however, the effects on the welfare is ambiguous because the investment achieved under the price parity clauses is excessive.

One of the important motivations for enterprises to use such platforms, however, is to be known by consumers, and this motivation seems not to have been be examined closely in previous research. This point is important to consider whether the claim is appropriate to justify narrow-MFNs such that it can improve the efficiency by avoiding free rides and reducing the incentive for investment. With that background, this chapter analyses economic models considering this aspect. Some economic studies have considered advertising through investment to inform consumers who may not know if the existence of products; this model is called an “informative advertising model”. First, Butters (1977) analysed the economic model that buyers know of products through advertisements, assuming monopolistic competition. Grossman and Shapiro (1984) later considered the horizontal product differentiation on the Hotelling line, assuming that only a part of consumers know about the existence of products by using an information technology similar to that of Butters (1977).

Other studies have also examined uninformed consumers, although those models do not analyse the effects of price parity clauses or MFNs. In those models, it is assumed that while some consumers knows only one firm’s price, other consumers can find all the prices in the market in some way, such as using a “price clearing house” or price comparison websites. Initially, Varian (1980) and Rosenthal (1980) analysed the models in which firms selling a homogenous good compete for informed consumers under Bertrand competition, where firms sell homogenous products although each seller is the monopoly seller for uninformed consumers who only know about that seller’s good. In the equilibrium of their models, the sellers adopt mixed-strategy pricing, which implies that price dispersion occurs. Baye and Morgan (2001) have introduced a two-sided platform called the “information gatekeeper” in their model. In the assumptions of their model, sellers can inform consumers of their existence if those consumers subscribe to the information gatekeeper for a fee, if the sellers pay the advertisement fee to the gatekeeper.

Some studies have applied Varian’s model to the case in which informed consumers get the information concerning the prices through platforms that make profit from fees to suppliers using the platform. Ronayne and Taylor (2018) extend Varian’s model by introducing a competitive sales channel, which is a price clearing house where two sellers can list their prices by paying commission, assuming that the sellers can set the prices of a homogenous good on the competitive sales channel or the direct sales channel. They show that if the competitive sales channel does not have many loyal consumers relative to the sellers (i.e., weak brand power), the competitive sales channel sets a low commission, and it follows that the sellers set low prices on the competitive sales channel to attract informed consumers and set high prices to exploit form uninformed loyal consumers. On the other hand, if the competitive sales channel has relatively strong brand power, the sales channel sets a high commission, and it follows that sellers compete with each other in their direct sales prices based on mixed strategies. It is also shown that the increase of the number of informed consumers does not always improve consumer welfare because while competition within channels is strengthened, competition between channels is weakened.

Ronayne (2015) applies Varian’s model to the case in which informed consumers can get information on sellers’ prices for homogeneous goods through homogeneous price comparison websites, which list the prices of the sellers paying fees. He shows that in the case in which price parity clauses are not allowed, while there exists a pure strategy Nash equilibrium with respect to the fees of the platforms, the sellers set prices based on mixed-strategy in the equilibrium. He also concludes that the introduction of price comparison websites makes both informed and uninformed consumers worse off, due to the rise of expected prices by the sellers caused by the fees to the sellers, imposed by the websites. He also shows that if the sellers can do price discrimination (i.e., set different prices on the direct sales and each website), the introduction of MFNs by the price comparison website harms consumers, because the clauses soften the competition between the websites regarding the fees.

Little empirical work has been done in this field. Mantovani et. al. (2017) examines the effect of the prohibition of price parity clauses in the European Union on the hotel prices. They show that the prices decreased shortly after the ban but bounced back thereafter. They conclude that the initial price decrease was compatible with the prohibition of the clauses, but the subsequent price increase may be caused by multiple factors. They suggest that one possible explanation is that after the removal of the clauses, the platforms need to compete, providing more customer experience thorough more investment and, hence, they need to pass through an increase of the investment costs to their fees. De los Santos et. al. (2019) analyse the e-book market to examine the effect of the transition from the wholesale model to the agency model, as well as MFNs in the case of e-books in the United States. They find that the switch to the agency model increased the prices on Amazon but had little effect on the prices on other e-book platform. They also carry out the counterfactual simulation and show that the introduction of MFNs would increase the prices of non-fiction books.

This chapter and the next chapter analyse the behaviours of consumers, sellers and platforms in the platform industry (especially, in online hotel booking) and the effects on competition and consumer welfare of price parity clauses or MFNs (both narrow and wide) by applying the informative advertising model to this industry. This chapter considers the case in which both a platform and a seller are monopolist and examines the effect of a narrow price parity clause. Chapter 5 considers the case in which there are two platforms and two sellers and examines the effect of wide price parity clauses.

None of the literature applying the informative advertisement model to the platform industry to analyse the effect of price parity clauses or MFNs. The closest literature to the model covered in this chapter is Wang and Wright (2016) in the sense that consumers who are not aware of product and do a searching behaviour to obtain the information about products are shed light on.

This study will shed light upon the important but not widely discussed platforms' function, building consumer awareness. This model especially suits industries in which building awareness of products or services among consumers is important and platforms have an significant tool by which to be known by consumers. Moreover, this study will clarify the effect of price parity clauses on the level of investment by platforms, which is also not covered in most of previous research. This point is also important because investment by platforms is a crucial factor in improving platform's function of getting attention from consumers and can enhance consumer welfare.

This chapter is organised as follows. Section 3 explains the assumptions of the model and the process of the game. Section 4 then analyses the special case such that there is no product differentiation. It first analyses the optimal behaviours of the platform, the seller and the consumers when the transportation cost is zero. Section 5 covers the optimal behaviours of those players when the transportation cost is positive. Both section 4 and 5 first analyse the case where the price parity clause is not allowed and, then, analyse the case where the clause is allowed. Finally, the effect of allowing a price parity clause on competition and on consumers is analysed. Section 6 offers concluding remarks and presents their implications for competition policy.

4.3. Basic Assumptions

Based on the motivations introduced in the previous section, this chapter and the next chapter aim to analyse competition such that the firms compete with each other through an intermediary platform.

The basic structure of the model analysed in this chapter and the next chapter is similar to that of some of the previous studies related to platform price parity clauses, such as Boik and Corts (2015), Johanson and Verge (2016) and Wang and Wright (2016). That is, the model uses a the vertical structure consisting of three types of players: sellers, platforms and consumers. This chapter considers the case of only one seller (Seller 1) and one platform (Platform A). In addition, like Johanson and Verge (2016) and Wang and Wright (2016), this analysis assumes that a seller can sell its product directly to consumers.

Moreover, this chapter and the next chapter aim to consider the situation in which gaining awareness from consumers who may not be informed about the existence of a product is important for sellers, because one of the important incentives to use platforms is to enhance the probability of awareness among consumers, especially in markets related to online platforms. In order to consider this point, this chapter and the next chapter analyse the theoretical model based on the informative advertising model for platforms with product differentiation between sellers, introduced by Grossman and Shapiro (1980). The main difference between the models is that the model analysed in this chapter includes a vertical relationship. While the original model considers only the interaction between the sellers and the consumers, this model considers a situation in which the sellers can use platforms to create awareness among consumers. This approach is similar to that of Wang and Wright (2016), which assumes consumers need to search to buy a product on a platform or from the seller directly and that when they searching via the platform, consumers obtain the benefits of convenience and a lower search cost. However, this model is different from their approach in that this Chapter and Chapter 5 focus on the function of platforms that sellers use to become known by consumers rather than on lower search costs or supplementary benefits.[25] This focus is well-suited to the case of online platforms, where search costs are very low and where consumers do not care considerably about the source of their goods.

In order to consider consumers' limited awareness of products and the possibility of switching of sales channel by consumers, two types of parameters are introduced in this chapter. The first parameters are about consumers' awareness of products.

In specific, it is assumed that only a share of $\lambda_1 \in [0,1)$ know about the existence of products sold directly by Seller 1 and $\lambda_A \in (0,1)$ and that consumers know about the existence of any product sold on Platform A. Variable λ_1 is assumed to be exogenous.

On the other hand, the share of the consumers who are aware of the existence of the platform (λ_A) is assumed to depend on the amount of investment Platform A has made in the present period, I_A, and the investment it has made so far, $\underline{I_A}$,[26] where $I_A \geq 0$ and $\underline{I_A} > 0$. Note that while I_A is an endogenous variable, $\underline{I_A}$ is an exogenous variable. Specifically, it is assumed that $\lambda_A = g(I_A + \underline{I_A})$, where $g'\left(I_A + \underline{I_A}\right) > 0$ and $g''\left(I_A + \underline{I_A}\right) < 0$. It is also assumed the probability to be noticed cannot be 1 even if the platform makes the investment as much as possible. In other words, $\lim_{I_A \to +\infty} g(I_A + \underline{I_A}) < 1$ holds. It can be assumed that $\underline{I_A} = 0$ without loss of generality because there is no rival platform. Hereafter, the investment function is simply written as $g(I_A)$ for simplicity.

The assumption of exogeneity about the consumers' awareness of the direct sales and the assumption that the consumers' awareness of the product sold on the platform depends on the platform's decision about investment imply that the seller cannot directly expand the number of consumers being aware of the product. This can be thought of as delegated information provision, and it is the difference from the original informative advertisement model, in which the sellers can expand the number by making their own investment.[27]

The relationship between the sellers and the platforms is a client–agent relationship. When a seller contracts with a platform, it can sell its product on the platform. The seller decides the price of its product sold on the platform, p_{1A}. In exchange, the seller needs to pay a commission, f_A, for each unit it sells. There are no further cost (e.g., upfront fee) to use the platform. In addition, the seller can also sell its product to

[25] The other (technical) difference is that in the informative advertising model, the following assumption is not required: that consumers know the distribution of the reservation values of products they do not know, which is necessary to find equilibrium in search models.

[26] Rate of depreciation with respect to the invested capital in the previous period is not included in this model, because $\underline{I_k}$ can be interpreted as taking the depreciation into account.

[27] By assuming that λ_I is an endogenous variable that depends on the amount of the seller's own investment, the seller can expand the number, although the model becomes much more complicated.

consumers directly. The price for the direct sales is given by p_{1D}. Although in this model there is only one seller, taking into account the continuity with the next chapter which assumes multiple platforms, we use p_{1A} and p_{1D}.

The other parameter is about the type of consumers' search behaviour. Specifically, it is assumed that there are two types of consumers, namely active consumers and inactive consumers, and that only active consumers undertake searching behaviours to find sales channels. The proportion of active customer is given by $\alpha \in [0,1]$. That proportion is assumed to be independent from λ_1 and λ_A. The proportion of the inactive consumers is $1-\alpha$.

1) Inactive consumers

Inactive consumers are defined as those who choose only from the options they initially know and never do any searching. The options that a consumer initially knows are determined by whether a consumer is a part of λ_1 or λ_A consumers or not. An inactive consumer can purchase Seller 1's product directly from the seller only if the consumer falls under the share of λ_1 consumers, and such a consumer can purchase the product sold on Platform A only if the consumer falls under the share of λ_A consumers and Seller 1 contracts with the platform. Otherwise, they do not have the information on the product sold directly or the product sold on the platform.

2) Active consumers

Active consumers are defined as those who engage in searching behaviour and, as a result, may obtain information that they initially do not have. In other words, it is assumed that when such consumers have an awareness of a product sold on any sales channel at the beginning of the game (i.e., falling under a share of part of λ_1 or λ_A consumers), it will find the price of the product sold on the other sales channel to compare the prices and purchase the product at the best price (e.g., if such consumer is informed about only the Seller 1's product sold on Platform A, the consumer chooses the cheapest sales channel to purchase seller 1's product. On the other hand, if consumers of this type are unaware of a product, they will not find the product. It follows that based on this definition, a consumer who has information neither about the seller's products sold directly nor about those sold on the platforms (i.e., a share of $(1-\lambda_1)(1-\lambda_A)$ consumers) cannot purchase the product, because such a consumer does not have any information about it when deciding to purchase, regardless of consumer type.

The assumptions about the consumers' awareness of the platform and the direct sales and the existence of two types of consumers imply that eight types of consumers exist, depending on whether they initially know about the platform, initially know about the seller and are active consumers. The kinds of information that each type of consumers has after the active consumers finish searching can be summarised in Table 4.1.

Table 4.1 Information that each type of consumer has after the active consumers finish searching

Consumer Type	Initially knows Platform A	Initially knows Seller 1	Proportion	Information after Searching
Active	Yes	Yes	$\alpha\lambda_A\lambda_1$	P_{1A}, P_{1D}
Active	Yes	No	$\alpha\lambda_A(1-\lambda_1)$	P_{1A}, P_{1D}
Active	No	Yes	$\alpha(1-\lambda_A)\lambda_1$	P_{1A}, P_{1D}
Active	No	No	$\alpha(1-\lambda_A)(1-\lambda_1)$	n. a.
Inactive	Yes	Yes	$(1-\alpha)\lambda_A\lambda_1$	P_{1A}, P_{1D}
Inactive	Yes	No	$(1-\alpha)\lambda_A(1-\lambda_1)$	P_{1A}
Inactive	No	Yes	$(1-\alpha)(1-\lambda_A)\lambda_1$	P_{1D}
Inactive	No	No	$(1-\alpha)(1-\lambda_A)(1-\lambda_1)$	n. a.

As shown in the rightmost column of Table 4.1, after the active consumers finish searching, the information consumers have is one of the following four: (1) The product sold on the platform and the direct sales channel, (2) only the product sold on the platform, (3) only the product sold on the seller's own sales channel or (4) no information about the product. The proportions of those four types of consumers are $\alpha(\lambda_A+\lambda_1)+(1-2\alpha)\lambda_A\lambda_1$[28], $(1-\alpha)\lambda_A(1-\lambda_1)$, $(1-\alpha)(1-\lambda_A)\lambda_1$[29] and $(1-\lambda_A)(1-\lambda_1)$ respectively.

With regard to consumers, to maintain consistency with the next chapter (which analyses the case of are two horizontally differentiated sellers), this chapter and next chapter assume the Hotelling model. Specifically, it is assumed that a unit mass of consumers is uniformly distributed over the unit interval and the unique seller is located at zero on the Hotelling line. It is also assumed that the consumers do not have preferences on where to buy the product. Hence, the utility of consumer $x \in [0,1]$ when purchasing Seller 1's product sold on Sales Channel m is $u = r - \tau x - p_{1m}$, where $m \in A, D$. The variable $\tau \geq 0$ can be understood as the transportation cost. For example, in the online hotel booking platform case where the seller is a hotel, τ can be interpreted as the transportation cost per unit distance for the consumers from the hotel to their destinations, which differs between consumers. Note that when $\tau = 0$, all the consumers have the same destination. It is also possible to interpret that τ denotes the difference in taste between consumers. For example, when considering a situation such that a retailer sells a product on the e-commerce platform, the transportation cost can be understood as the disutility due to the difference from their taste. In this situation,

[28] If $\alpha = 0$, the share of the consumers who are aware of the product sold on both sales channels is $\lambda_A\lambda_1$. If $\lambda_1 = 0$, the share of such consumers is λ_A.

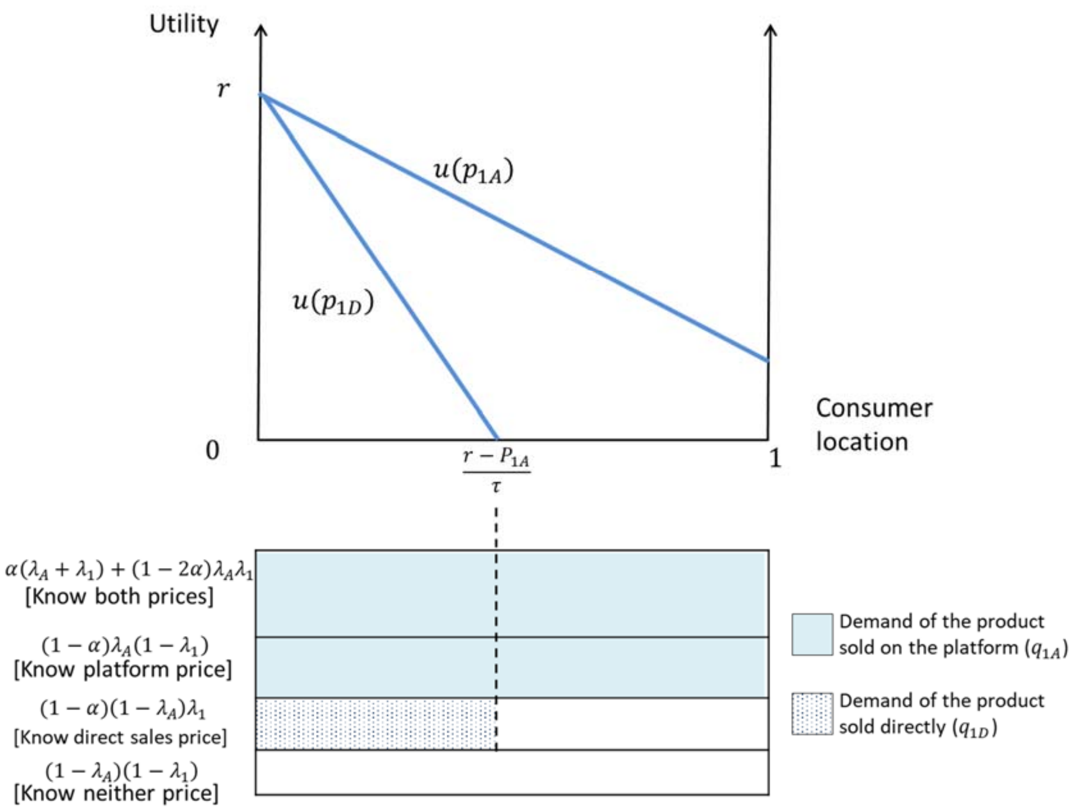
[29] If $\alpha = 0$, the share of the consumers who are aware of only the product sold on the platform or the direct sale channels is $\lambda_A(1-\lambda_1)$ or $(1-\lambda_A)\lambda_1$, respectively. If $\lambda_1 = 0$, the shares are $(1-\alpha)\lambda_A$ or 0, respectively.

$\tau = 0$ means that all the consumers have the same taste, which is equivalent to no product differentiation. The special case where $\tau = 0$ holds is covered in the next section and the general cases where such restriction is not assumed are covered in Section 5.

This implies that the utility of the most distant consumer to purchase the product at Sales Channel $m \in A, D$ is $r - \tau - p_{1m}$. It follows that if $p_{1m} \leq r - \tau$ all the consumers who are aware of the product will purchase the product at Sales Channel m or the other one when the price is cheaper there. Hereafter, such price is referred to as "full-coverage price". Note that there exist consumers such that even when the seller offers full-coverage price, because such consumers are unaware of product. On the other hand, if $p_{1m} > r - \tau$, there exist consumers who will not purchase from it even they are aware of the product (hereafter, referred to as "non-full-coverage price").

From the proportions of the four types of consumers based on their information and the assumption about the utility, the demands of the product sold on both sales channels given p_{1A} and p_{1D} can be summarised in Figure 4.1 below. [30]

Figure 4.1 Utility of consumers when the platform price is full-coverage price and the direct sales price is non-full-coverage price



[30] This illustration is based on Figure 6.1 of Belleflamme and Peitz (2015), which write a compact summary of the informative advertisement model.

The upper part of Figure 4.1 shows the utilities of consumers given p_{1A} and p_{1D} when $p_{1A} \leq r - \tau < p_{1D}$ holds. The vertical axis denotes the utilities when they purchase the product on the platform and the direct sales channel. The horizontal axis denotes the location of consumers. The lower part of the figure shows the demand size of each of the four types of consumers. The first row of the diagram in the lower part shows the demand of the consumers who know both the platform price and the direct sales price. They purchase on the platform because the price is cheaper. On the other hand, the consumers shown in the second row only know the platform price. It follows that they only have two choices, purchasing on the platform or not purchasing. In this sense, they are "captive consumers". The consumers in the third row are also captive consumers since they only know the direct sales price. However, some consumers decide not to purchase because the direct sales price is non-full-coverage price in this example. The consumers shown in the fourth row will not purchase the product because they do not have any information about the product.

Several tie-breaking rules are also assumed. First, if $p_{1A} = p_{1D}$, the consumers with the information about the product sold on the platform and that sold directly by Seller 1 will purchase the product on the platform with a probability of 0.5 and on the seller's direct sales channel with a probability of 0.5. Second, if more than one options give the same expected profit, the platform prefers the option that more consumers will purchase on the platform, and the seller prefers the option that more consumers will purchase its product.

Moreover, our interest is on the case where the seller sets full-coverage prices rather than the case where the seller sets non-full-coverage prices, because it seems that non-full-coverage price is unlikely to be taken in the real cases. Taking this into account and making the model more simple, r is assumed to be large enough to guarantee that the seller does not have an incentive to set non-full-coverage price for its direct sales in the equilibrium ($r \geq 2\tau$).

The sequence of the game is defined as follows:

a) Platform A decides the amount of investment and then decides the commission (f_A) it offers to the sellers.

b) Seller 1 decides whether they contract with the platform and, if so, decides the price sold on Platform (p_{1A}). At the same time, it decides the direct sales price (p_{1D}).

c) Consumer decide whether to purchase the product of the seller or not and on which sales channel to purchase from.

4.4. Zero Transportation Cost Case

This section analyses the case in which the transportation cost is assumed to be zero ($\tau = 0$), which means that the utility to purchase the product is same for all consumers. This assumption also implies that when the consumers purchase the product in the equilibrium, the equilibrium prices should be full-coverage prices.

Section 4.1 of this chapter analyses the case where a price parity clause are not allowed. Then, Section 4.2 analyses what happens when the clause is allowed and the effect on welfare of allowing the clause. In each subsection, the behaviours of the consumers, the sellers and the platform are analysed in turn.

4.4.1 No Possibility of Price Parity Clause

This section analyses the case in which the platform is not allowed to introduce a price parity clause.

Analysis of this model provides the following proposition.

Proposition 4.1. *Suppose the platform is not allowed to introduce a price parity clause and $\tau = 0$; there always exists a unique equilibrium. In this equilibrium, the platform chooses the investment level ${I_A}^*$ such that*

$g'({I_A}^*) = \frac{1}{(1-\alpha)(1-\lambda_1)(r-\varepsilon)}$ *and sets the commission ${f_A}^*$ and the sellers' prices $({p_{1A}}^*, {p_{1D}}^*)$ such that $({f_A}^*, {p_{1A}}^*, {p_{1D}}^*) = (r-\varepsilon, r, r-\varepsilon)$, where ε is a small positive amount.*

Proposition 4.1 shows that the investment level in the equilibrium depends on the values of r, α and λ_1. According to the proposition, the optimal investment level rises when the reservation value for the consumer r becomes larger. Moreover, the optimal investment level lowers as the proportion of active consumers (α) or the proportion of the consumers who are initially aware of the seller (λ_1) grows. This relationship occurs because when α or λ_1 becomes larger, more and more consumers purchase the product from the direct sales channel rather than through the platform, decreasing the marginal profit of the platform from the investment. This chain of events implies that the existence of direct sales decreases the platform's profitability. In this sense, it is possible to say that the free riding by the seller happens due to the existence of the active consumers who initially only aware of the platform and then notices the direct sales after searching.

With respect to the commission of the platform, the platform sets the price just below the reservation price of the consumers. It follows that the seller must set its price on the platform equal to the commission, because the consumers will not purchase on the platform if it sets the price higher than that. This implies that the margin that the seller obtains from sales through the platform ($p_{1A} - f_A$) is almost zero. On the other hand, the seller sets its direct sales price just below r in order to avoid paying the commission to the platform as much as possible.

Note that while α and $\lambda_1$affects the optimal investment level, the optimal platform's commission prices do not depend on α and λ_1.

4.4.1.1 Consumers' Behaviour

As shown in Table 4.1, eight types of consumers exist, depending on whether they initially know about the platform, initially know about the seller and are active consumers. The seller's optimal pricing is to maximise the sum of the profits from each type of consumer. The consumers' behaviour given the seller's prices are summarised in Lemma 4.1.

Lemma 4.1. *The demand for the seller's product sold on sales channel* $m \in A, D$, $q_{1m}{}^*$, *where* $n \in A, D$ *and* $m \neq n$, *given* p_{1A}, p_{1D}, f_A, *and* I_A, *can be written as follows:*

$i)$ *When* $p_{1m} < p_{1n}$,

$$q_{1m}{}^* = \begin{Bmatrix} \lambda_m + \alpha\lambda_n - \alpha\lambda_m\lambda_n \; if \; p_{1m} \leq r \\ 0 \; if \; p_{1m} > r \end{Bmatrix}.$$

$ii)$ *When* $p_{1m} = p_{1n}$,

$$q_{1m}{}^* = \begin{Bmatrix} (1 - 0.5\alpha)\lambda_m + 0.5\alpha\lambda_n - 0.5\lambda_m\lambda_n \; if \; p_{1m} \leq r \\ 0 \; if \; p_{1m} > r \end{Bmatrix}.$$

$iii)$ *When* $p_{1m} > p_{1n}$,

$$q_{1m}{}^* = \begin{Bmatrix} (1 - \alpha)\lambda_m(1 - \lambda_n) \; if \; p_{1m} \leq r \\ 0 \; if \; p_{1m} > r \end{Bmatrix}.$$

Proof: See Appendix C

Lemma 4.1 shows that no consumers purchase the product through Sales channel m if the price sold on the platform is higher than the reservation value for the consumers ($p_{1m} > r$).

Lemma 4.1 also shows that the demand depends on whether the price on one sales channel is larger than that of the other sales channel. This dependence arises because while the share of $\lambda_m(1 - \lambda_n)$ consumers is aware of the product sold only on the sales channel m (i.e., they are "captive consumers"), the share of $\lambda_m\lambda_n$ consumers is aware of the product sold on both sales channels (i.e., they are "non-captive consumers"). If the price on one sales channel is cheaper than the price on the other sales channel, all the non-captive consumers purchase the product. If the price on the two sales channel is the same, half of the non-captive consumers purchase on one platform.

4.4.1.2 Seller's Behaviour

Lemma 4.1 shows that the quantity that the consumers purchase from the seller depends on the price. If the lowest price that consumers are aware of is equal to or lower than r, the consumers purchase the product through a sales channel offering the lowest price they know.

For future reference, define

(4.1) $\nu \equiv \lambda_A + \lambda_1 - \lambda_A\lambda_1$ and

(4.2) $\xi = (1 - 0.5\alpha)\lambda_A + 0.5\alpha\lambda_1 - 0.5\lambda_A\lambda_1$,

where ν is the number of consumers who are aware of the product sold on the platform or the product sold on the direct sales channel; ξ is the number of consumers who purchase the product on the platform when $p_{1A} = p_{1D}$ holds and the prices are low enough to cover all the consumers ($p_{1A} = p_{1D} \leq r - \tau = r$).

Solving this profit-maximisation problem for the seller provides the optimal seller's behaviours given the consumers' response, summarised in Lemma 4.2.

Lemma 4.2. *Anticipating the consumers' response specified in Lemma 4.1, given f_A, and I_A, the optimal pricing policy for the seller $({p_{1A}}^*, {p_{1D}}^*)$ is*

$({p_{1A}}^*, {p_{1D}}^*) = \begin{Bmatrix} (r, r-\varepsilon)\ if\ f_A < r \\ (r+b, r)\ if\ f_A \geq r \end{Bmatrix}$, *where b is any positive amount and ε is a small positive amount.*

Proof: See Appendix C

This lemma shows that regardless of the values of f_A and r, the seller sets the prices such that $p_{1A} > p_{1D}$. When the commission is higher than the reservation value for the consumers ($f_A > r$), the seller sets the direct sales price equal to r and sets the platform price high enough to guarantee that no consumer purchase on the platform, because it incurs the loss from the sales on it. When the commission is equal to or lower than the reservation value for the consumers ($f_A \leq r$), the seller sets the platform price equal to r and sets the direct sales price a little less than the platform price in order to avoid paying the commission to the platform as much as possible. Note that in any case, the price that the consumers purchase the product is almost equal to their reservation value, which follows that the consumer surplus is almost equal to zero. This is because, while there is intra-brand competition between sales channels, there is a product monopoly and the demand is inelastic up to r.

4.4.1.3 Platform's Behaviour Regarding Commission and Investment

Lemmata 4.1 and 4.2 imply that the expected revenue that the platform obtains from the transaction on it can be written as

$$R_A = \begin{Bmatrix} (1-\alpha)\lambda_A(1-\lambda_1)f_A \; if \; f_A < r \\ 0 \; if \; f_A \geq r \end{Bmatrix}.$$

This implies that the optimal commission for the platform is setting f_A just below r. The total expected profit taking the investment into account is given by $\Pi_A = R_A\big(g(I_A)\big) - I_A$.

Solving this profit-maximisation problem directly yields the optimal commission and investment set by the platform, given the seller and consumers' response, as summarised in Lemma 4.3.

Lemma 4.3. *Anticipating the seller and consumers' response specified in Lemmata 4.1 and 4.2, the platform chooses the optimal investment level $I_A{}^*$ such that*

$g'(I_A{}^*) = \frac{1}{(1-\alpha)(1-\lambda_1)\,(r-\varepsilon)}$ *and sets the optimal commission $f_A{}^*$ such that $f_A{}^* = r - \varepsilon$.*

Proof: See Appendix C

Lemma 4.3 shows that the optimal invest level depends on the values of r, α or λ_1. The optimal investment level becomes higher when r gets higher or α or λ_1 gets lower. The optimal commission is equal to the reservation value of the consumers.

From Lemma 4.1 to 4.3, Proposition 4.1 can be obtained directly.

4.4.2 Allowing Price Parity Clause

This section considers the case in which the platform is allowed to impose the price parity clause such that the seller's prices must satisfy $p_{1A} \leq p_{1D}$ and the seller cannot use the platform unless it accepts the offer including this restriction.

Analysis of this model provides the following proposition, to be proved in the subsequent sections.

Proposition 4.2. *Suppose the platform is allowed to introduce a price parity clause and* $\tau = 0$*; there is a unique equilibrium in which the platform introduces a price parity clause. In this equilibrium, the platform chooses the investment level* ${I_A}^*$ *such that*

$g'({I_A}^*) = \frac{1}{(1-\lambda_1)\, r}$ *and sets the commission* ${f_A}^*$ *and the sellers' prices* $({p_{1A}}^*, {p_{1D}}^*)$ *such that* $({f_A}^*, {p_{1A}}^*, {p_{1D}}^*) = \left(\frac{{\lambda_A}^*(1-\lambda_1)r}{(1-0.5\alpha){\lambda_A}^*+0.5\alpha\lambda_1-0.5{\lambda_A}^*\lambda_1}, r, r\right)$*, where* ${\lambda_A}^* \equiv g({I_A}^*)$ *.*

Proposition 4.1 shows that when the platform is allowed to introduce a price parity clause, the platform does introduce it, which follows that the commission imposed by the platform the investment level in the equilibrium depends on the values of r, α and λ_1. Proposition 4.1 also shows that the optimal investment level always rises by allowing a price parity clause.

Both the platform price and the direct sales price in the equilibrium are equal to r, which is the reservation value for the consumers. This implies that allowing a price parity clause does not change the direct sales price but decreases the platform price, although the amount of the decrease is negligibly small.

On the other hand, the equilibrium commission can be higher than the equilibrium prices.[31] This implies that the margin that the seller obtains from sales through the platform ($p_{1A} - f_A$) can be negative. In this situation, the seller still chooses to use the platform because the seller can attract the active consumers who are initially only aware of the platform to purchase through the direct sales channel. The seller cannot sell to those customers unless it uses the platform. Under this equilibrium, the expected profit of the seller is same as the profit when the seller refuses the price parity clause, $\lambda_1 r$.

4.4.2.1 Consumers' Behaviour

Since the introduction of the price parity clause does not constrain the consumers' behaviours, the consumers' response is given by Lemma 4.1. Hence, the consumers' behaviours given the seller's prices are summarised in Lemma 4.4.

Lemma 4.4. *The demand for the seller's product sold on sales channel* $m \in A, D$*,* ${q_{1m}}^*$*, where* $n \in A, D$ *and* $m \neq n$*, given* p_{1A}*,* p_{1D}*,* f_A *and* I_A*, is same as the demand in the case where a price parity clause is not allowed, as specified in Lemma 4.1.*

[31] This situation happens when both $\alpha > \lambda_1$ and ${\lambda_A}^* > \alpha\lambda_1/(\alpha - \lambda_1)$ hold.

4.4.2.2 Seller’s Behaviour

In this case, the seller has two options. The first option is to accept the price parity clause imposed by the platform and set the prices such that $p_{1A} \leq p_{1D}$. The second option for the seller is to refuse the offer by the platform and to concentrate instead on direct sales.

Solving this profit-maximisation problem for the seller gives the optimal seller's behaviours given the consumers’ response, as summarised in Lemma 4.5.

Lemma 4.5. *Anticipating the consumers’ response, as specified in Lemma 4.4, given f_A and I_A, the optimal pricing policy for the seller is as follows:*

$(p_{1A}, p_{1D}) = \begin{Bmatrix} (r, r) \; if \; f_A \leq \frac{\lambda_A(1-\lambda_1)r}{\xi} \\ (+\infty, r) \; if \; f_A > \frac{\lambda_A(1-\lambda_1)r}{\xi} \end{Bmatrix}$, *where $p_{1A} = +\infty$ denotes that the seller refuses to accept the clause;*

Proof: See Appendix C

Lemma 4.5 shows that the seller’s optimal direct sales price is setting the prices equal to r, which is the reservation value for the consumers. Whether the seller refuses to use the platform in the equilibrium depends on whether the commission is higher than a certain threshold.

If the commission is lower than the threshold, the seller uses the platform and set the platform price equal to the direct sales price. It is also shown that the seller uses the platform even if the commission is higher than the optimal prices for the seller, which follows that the margin that the seller obtains from sales through the platform is negative ($p_{1A} - f_A < 0$). This is because the seller can attract the active consumers who are initially only aware of the platform to purchase through the direct sales channel. The seller cannot sell to those customers unless it uses the platform. Note that the condition that the threshold value is larger than r is $\lambda_A > \alpha\lambda_1/(\alpha - \lambda_1)$.

4.4.2.3 Platform’s Behaviour Regarding Commission and Investment

Lemmata 4.4 and 4.5 imply that the expected revenue that the platform obtains from the transaction on it can be written as follows

$$R_A = \begin{Bmatrix} \xi f_A \; if \; f_A \leq \frac{\lambda_A(1-\lambda_1)r}{\xi} \\ 0 \; if \; f_A > \frac{\lambda_A(1-\lambda_1)r}{\xi} \end{Bmatrix}.$$

This implies that the optimal commission for the platform is setting f_A equal to $\lambda_A(1 - \lambda_1)r/\,\xi$. The total expected profit taking the investment into account is given by

$$\Pi_A = (1-\lambda_1)rg(I_A) - I_A.$$

Solving this profit-maximisation problem directly yields the optimal commission and investment set by the platform, given the seller and consumers' response, as summarised in Lemma 4.6.

Lemma 4.6. *Anticipating the seller and consumers' response specified in Lemmata 4.4 and 4.5, the platform chooses the optimal investment level I_A^* such that*

$g'\left(I_A^* + \underline{I_A}\right) = \frac{1}{(1-\lambda_1)\, r}$ *and sets the optimal commission f_A^* such that* $f_A^* = \frac{(\lambda_A^* + \lambda_1 - \lambda_A^* \lambda_1)r}{(1-0.5\alpha)\lambda_A^* + 0.5\alpha\lambda_1 - 0.5\lambda_A^*\lambda_1}$, *where $\lambda_A^* \equiv g(\,I_A^*)$, where $\lambda_A^* \equiv g(\,I_A^*)$.*

From Lemmata 4.4, 4.5 and 4.6, Proposition 4.2 can be obtained directly.

4.4.2.4 Effect on Consumer Welfare and Sellers' Profits of Allowing Price Parity Clause

Taking the results of the analysis above, this section analyses the effect of introducing the platform price parity clause on the welfare for each player. Comparison between the equilibrium sellers' prices and the number of the consumers who are aware of the platforms summarised in Propositions 4.1 and 4.2 yields the Proposition 4.3.

Proposition 4.3. *If the platform is allowed to offer a price parity clause, in the unique equilibrium, the consumer welfare always improves compared to the unique equilibrium where the clause is not allowed and the profit of seller always decreases.*

Proposition 4.3 shows that allowing the price parity clause increases consumer welfare. The price parity clause increases the investment, which affords more opportunities to be aware of the product for consumers. Although it increases the equilibrium direct sales price, the effect of the increase on consumer welfare is negligible because it is just a slight increase.

On the other hand, the price parity clause decreases the profit of the seller. While the clause improves the consumer awareness of the product due to the increase in the investment, this positive effect on the profit of the seller is surpassed by the negative effect caused by the increase of the commission imposed by the platform, which is higher than the equilibrium price.

This result is in common with Wang and Wright (2016) in the sense that the price parity clause enables the platform to set a higher commission. However, while in this model the price parity clause increases the consumer welfare, in their model the clause decreases consumer surplus without competition between platforms. This is because their model takes into account the possibility that the more and more consumers are aware of the product through the investment by the platform.

In this model, the clause increases the commission because the effect of the possibility that the seller decides not to use the platform is not strong enough to limit the ability of the platform to set a high commission. On the other hand, the consumer welfare is always improved because the positive effect of increasing awareness of the product is much larger than the negative effect thorough the higher price due to the higher commission, which is negligibly small.

4.5. Positive Transportation Cost Case

This section analyses the case in which the transportation cost is assumed to be positive ($\tau \in (0, r)$), which means that the utility to purchase the product is different between consumers. This assumption also implies that when the consumers purchase the product in the equilibrium, non-full-coverage price may be adopted as an equilibrium platform price.

Like in the previous section, Section 5.1 of this chapter analyses the case where a price parity clause are not allowed. Then, Section 5.2 analyses what happens when the clause is allowed and the effect on welfare of allowing the clause.

4.5.1 No Possibility of Price Parity Clause

This section analyses the case in which the platform is not allowed to introduce a price parity clause.

Analysis of this model provides the following proposition.

Proposition 4.4. *Suppose the platform is not allowed to introduce a price parity clause; there always exists a unique equilibrium. In this equilibrium, the platform chooses the investment level ${I_A}^*$ such that*

$g'\left(I_A{}^* + \underline{I_A}\right) = \begin{cases} \frac{1}{(1-\alpha)(1-\lambda_1)\,(r-2\tau)} \; if\ r \geq 4\tau \\ \frac{8\tau}{(1-\alpha)(1-\lambda_1)\,r^2} \; if\ r < 4\tau \end{cases}$ *and sets the commission* $f_A{}^*$ *and the sellers' prices* $(p_{1A}{}^*, p_{1D}{}^*)$ *such that*

$$(f_A{}^*, p_{1A}{}^*, p_{1D}{}^*) = \begin{cases} (r - 2\tau, r - \tau, r - \tau - \varepsilon)\ if\ r \geq 4\tau \\ \left(\frac{r}{2}, \frac{3r}{4}, r - \tau\right)\ if\ 2\tau \leq r < 4\tau \end{cases}.$$

Proposition 4.4 shows that like the zero transportation cost case, the seller may set the maximum full-coverage price for the platform and set the direct sales price slightly lower than the direct sales price in the equilibrium. However, such situation does not always happen in the positive transportation cost case. There are two patterns of equilibrium depending on whether $r \geq 4\tau$ holds. The shape of the optimal investment differs depending on whether $r \geq 4\tau$ holds or not, because when r is high compared to τ, the marginal profit of the platform from the investment is high, which leads the higher investment level in the equilibrium. In any case, the investment level in the equilibrium depends on the values of r, τ, α and λ_1. According to the proposition, the optimal investment level rises when the reservation value for the consumer whose taste is closest to the seller's product (r) becomes larger or the degree of the reduction of the reservation value caused by the difference in taste (or the transportation cost (τ)) becomes smaller.

Moreover, the optimal investment level lowers as the proportion of active consumers (α) or the proportion of the consumers who are initially aware of the seller (λ_1) grows. This relationship occurs because when α or λ_1 becomes larger, more and more consumers purchase the product from the direct sales channel rather than through the platform, decreasing the marginal profit of the platform from the investment. This chain of events implies that the existence of direct sales decreases the platform's profitability. In this sense, it is possible to say that the free riding by the seller happens due to the existence of the direct sales.

With respect to the commission of the platform, if $r \geq 4\tau$ holds, like the zero transportation cost case, the platform sets the commission low enough to guarantee that the equilibrium prices are low enough that all the consumers who are aware of the product will purchase the product (i.e., "full-coverage price"). In this situation, the seller sets the price on the platform at the maximum full-coverage price, $r - \tau$ and sets the direct sales price just below the platform price.

If $r < 4\tau$ holds, the platform sets a higher commission in the equilibrium, which follows that the seller sets the equilibrium platform price so high that some consumers whose taste is at a distance from the seller's product will not purchase the product, even if they have the information on it (i.e., "non-full-coverage price"), which does not happen in the zero transportation cost case. On the other hand, the seller sets the maximum full-coverage price for its direct sales channel.

This implies that regardless of whether $r \geq 4\tau$ holds, the seller sets the direct sales price lower than the price on the platform in order to avoid paying the commission to the platform as much as possible. Moreover, while α and λ_1 affects the optimal investment level, the optimal platform's commission prices do not depend on α and λ_1.

The proposition also implies that in the equilibrium, the margin that the seller obtains from each consumer purchasing through the platform $(p_{1A} - f_A)$ is τ if $r \geq 4\tau$ and $0.25r$ if $r < 4\tau$. Hence, the margin that the platform obtains from each transaction is larger than the margin for the seller. Moreover, the result that the seller can obtain a substantial margin from the sales on the platform is the other notable difference from the zero transportation cost case where the profit of the seller from the sales on the platform is almost zero.

4.5.1.1 Consumers' Behaviour

As shown in Table 4.1, eight types of consumers exist, depending on whether they initially know about the platform, initially know about the seller and are active consumers. The seller's optimal pricing is to maximise the sum of the profits from each type of consumer. The consumers' behaviour given the seller's prices are summarised in Lemma 4.7.

Lemma 4.7. *The demand for the seller's product sold on sales channel $m \in A, D$, ${q_{1m}}^*$, where $n \in A, D$ and $m \neq n$, given p_{1A}, p_{1D}, f_A, and I_A, can be written as follows:*

$i)$ *When* $p_{1m} < p_{1n}$,

$${q_{1m}}^* = \begin{Bmatrix} \lambda_m + \alpha\lambda_n - \alpha\lambda_m\lambda_n \ if \ p_{1m} \leq r - \tau \\ \frac{(\lambda_m + \alpha\lambda_n - \alpha\lambda_m\lambda_n)(r - p_{1m})}{\tau} \ if \ r - \tau < p_{1m} \leq r \\ 0 \ if \ p_{1m} > r \end{Bmatrix}.$$

$ii)$ *When* $p_{1m} = p_{1n}$,

$${q_{1m}}^* = \begin{Bmatrix} (1 - 0.5\alpha)\lambda_m + 0.5\alpha\lambda_n - 0.5\lambda_m\lambda_n \ if \ p_{1m} \leq r - \tau \\ \frac{[(1-0.5\alpha)\lambda_m + 0.5\alpha\lambda_n - 0.5\lambda_m\lambda_n](r - p_{1m})}{\tau} \ if \ r - \tau < p_{1m} \leq r \\ 0 \ if \ p_{1m} > r \end{Bmatrix}.$$

$iii)$ *When* $p_{1m} > p_{1n}$,

$${q_{1m}}^* = \begin{Bmatrix} (1 - \alpha)\lambda_m(1 - \lambda_n) \ if \ p_{1m} \leq r - \tau \\ \frac{(1-\alpha)\lambda_m(1-\lambda_n)(r - p_{1m})}{\tau} \ if \ r - \tau < p_{1m} \leq r \\ 0 \ if \ p_{1m} > r \end{Bmatrix}.$$

Proof: See Appendix C

Lemma 4.7 shows that no consumers purchase the product through Sales channel m if the price sold on the platform is too high ($p_{1m} > r$) and that the demand is elastic if the price is below the threshold but not too low ($r - \tau < p_{1m} \leq r$). Moreover, if the price is weakly lower than the reservation value for the consumer whose taste is the most distant from the product ($p_{1m} \leq r - \tau$), all consumers who are aware of the product sold on Sales channel m purchase the product through one of the two sales channel.

Lemma 4.7 also shows that the demand depends on whether the price on one sales channel is larger than that of the other sales channel. This dependence arises because while the share of $\lambda_m(1 - \lambda_n)$ consumers is aware of the product sold only on the sales channel m (i.e., they are "captive consumers"), the share of $\lambda_m\lambda_n$ consumers is aware of the product sold on both sales channels (i.e., they are "non-captive consumers"). If the price on one sales channel is cheaper than the price on the other sales channel, all the non-captive consumers purchase the product. If the price on the two sales channel is the same, half of the non-captive consumers purchase on one platform.

4.5.1.2 Seller's Behaviour

Lemma 4.7 shows that the quantity that the consumers purchase from the seller depends on the price. If the lowest price that consumers are aware of is equal to or lower than $r - \tau$ (i.e., full-coverage price), the consumers purchase the product through a sales channel offering the lowest price they know. However, if the lowest price that consumers are aware of is higher than $r - \tau$ (i.e., non-full-coverage price), some consumers whose location is far from the seller on the Hotelling line[32] do not purchase any product.

Recall $\nu \equiv \lambda_A + \lambda_1 - \lambda_A\lambda_1$ and

$\xi = (1 - 0.5\alpha)\lambda_A + 0.5\alpha\lambda_1 - 0.5\lambda_A\lambda_1$,

where ν is the number of consumers who are aware of the product sold on the platform or the product sold on the direct sales channel; ξ is the number of consumers who purchase the product on the platform when $p_{1A} = p_{1D}$ holds and the prices are low enough to cover all the consumers ($p_{1A} = p_{1D} \leq r - \tau$).

Solving this profit-maximisation problem for the seller provides the optimal seller's behaviours given the consumers' response, summarised in Lemma 4.8.

Lemma 4.8. *Anticipating the consumers' response specified in Lemma 4.7, given f_A and I_A, the optimal pricing policy for the seller $({p_{1A}}^*, {p_{1D}}^*)$ is*

[32] In particular, consumers $x \in ((r - p_{1m})/\tau, 1]$ will not purchase the product, where $m \in A, D$ and p_{1m} is the lowest price as far as consumer x is aware.

$$(p_{1A}{}^{*}, p_{1D}{}^{*}) = \begin{Bmatrix} (r+b, r-\tau) if\ f_A \geq r \\ \left(\frac{r+f_A}{2}, r-\tau\right) if\ r-2\tau < f_A < r, \\ (r-\tau, r-\tau-\varepsilon) if\ f_A \leq r-2\tau < r \end{Bmatrix},$$ *where b is any positive amount and ε is a small positive amount.*

Proof: See Appendix C

This lemma shows that regardless of the values of f_A, r and τ, the seller sets the prices such that $p_{1A} > p_{1D}$. If the commission is equal to or higher than the willingness to pay of the consumer whose location is the closest to the seller, r, the seller sets the direct sales price equal to the maximum full-coverage price $(r - \tau)$ and sets the platform price high enough to guarantee that no consumer purchase on the platform, because it incurs the loss from the sales on it.

When the commission is equal to or lower than the reservation value for the consumers $(f_A \leq r)$, the seller sets the platform price equal to r and sets the direct sales price a little less than the platform price in order to avoid paying the commission to the platform as much as possible. Note that in any case, the price that the consumers purchase the product is almost equal to their reservation value, which follows that the consumer surplus is almost equal to zero.

If the commission is lower than the threshold but higher than a certain threshold $(r - 2\tau < f_A < r)$, the seller sets the non-full coverage price on the platform and set the maximum full-coverage price for its direct sales. If the commission offered by the platform is weakly lower than the certain threshold $(f_A \leq r - 2\tau)$, the seller will set its price for the product sold on the platform at the maximum full-coverage price and set the direct sales price a little less than the platform price.

4.5.1.3 Platform's Behaviour Regarding Commission and Investment

Lemmata 4.7 and 4.8 imply that the expected revenue that the platform obtains from the transaction on it can be written as

$$R_A = \begin{Bmatrix} (1-\alpha)\lambda_A(1-\lambda_1) f_A\ if\ f_A \leq r-2\tau \\ (1-\alpha)\lambda_A(1-\lambda_1)\left(\frac{r-f_A}{2\tau}\right) f_A\ if\ r-2\tau < f_A < r \\ 0\ if\ f_A \geq r \end{Bmatrix},$$ and the total expected profit taking the investment into account is given by $\Pi_A = R_A\left(g\left(I_A + \underline{I_A}\right)\right) - I_A$.

Solving this profit-maximisation problem yields the optimal commission and investment set by the platform, given the seller and consumers' response, as summarised in Lemma 4.9.

Lemma 4.9. *Anticipating the seller and consumers' response specified in Lemmata 4.1 and 4.2, the platform chooses the optimal investment level* ${I_A}^*$ *such that*

$$g'\left({I_A}^* + \underline{I_A}\right) = \begin{Bmatrix} \frac{1}{(1-\alpha)(1-\lambda_1)\,(r-2\tau)} \; if\; r \geq 4\tau \\ \frac{8\tau}{(1-\alpha)(1-\lambda_1)\,r^2} \; if\; r < 4\tau \end{Bmatrix}$$ *and sets the optimal commission* ${f_A}^*$

such that

$${f_A}^* = \begin{Bmatrix} r - 2\tau \; if \; r \geq 4\tau \\ \frac{r}{2} \; if \; r < 4\tau \end{Bmatrix}.$$

Proof: See Appendix C

Lemma 4.9 shows two types of equilibrium depending on whether r is weakly larger than 4τ or not. If r surpasses the threshold, the platform sets its commission low enough to ensure that the seller sets the full-coverage price. If r is lower than the threshold, the platform sets its commission high enough to guarantee that the seller sets the non-full coverage price. In both cases, the optimal investment level becomes higher when r gets higher or τ, α or λ_1 gets lower.

From Lemma 4.7 to 4.9, Proposition 4.4 can be obtained directly.

4.5.2 Allowing Price Parity Clause

This section considers the case in which the platform is allowed to impose the price parity clause such that the seller's prices must satisfy $p_{1A} \leq p_{1D}$ and the seller cannot use the platform unless it accepts the offer including this restriction.

When the platform offers a price parity clause, the platform has two options regarding its commission. The first option is to set a commission low enough to guarantee that the seller will react to set a price such that all the consumers know the product will be covered (hereafter, "full-coverage commission"). The second option is setting a commission high enough to let the seller set a price such that certain consumers are not covered (hereafter, "non-full-coverage commission").

Since the complexity of the equilibrium in the case where a price parity clause is allowed, as discussed below, it will be useful to define the following:

$$g'\left(\widehat{I_A} + \underline{I_A}\right) = \frac{1}{(r-2\tau)(1-\lambda_1)},\ \widehat{\lambda_A} \equiv g\left(\widehat{I_A} + \underline{I_A}\right),\ \widehat{f_A} \equiv \frac{[(1-\lambda_1)\widehat{\lambda_A}+\lambda_1](r-2\tau)}{(1-0.5\alpha-0.5\lambda_1)\widehat{\lambda_A}+0.5\alpha\lambda_1},$$

$$g'\left(\widetilde{I_A} + \underline{I_A}\right) = \frac{1}{(r-\tau)(1-\lambda_1)},\ \widetilde{\lambda_A} \equiv g\left(\widetilde{I_A} + \underline{I_A}\right),\ \widetilde{f_A} \equiv \frac{(1-\lambda_1)\widetilde{\lambda_A}(r-\tau)}{(1-0.5\alpha-0.5\lambda_1)\widetilde{\lambda_A}+0.5\alpha\lambda_1},$$

$\overline{\lambda_A} \equiv \frac{\lambda_1(r-2\tau)}{(1-\lambda_1)\tau} = g\left(\overline{I_A} + \underline{I_A}\right), \overline{f_A} = \frac{\lambda_1(r-\tau)(r-2\tau)}{[(1-0.5\alpha-0.5\lambda_1)r-(2-1.5\alpha-\lambda_1+0.5\alpha\lambda_1)\tau]\tau},$

$g'\left(\acute{I_A} + \underline{I_A}\right) = \frac{8\tau}{(1-\lambda_1)r^2}, \acute{\lambda_A} \equiv g\left(\acute{I_A} + \underline{I_A}\right), \acute{f_A} \equiv \frac{[(1-\lambda_1)\acute{\lambda_A}+\lambda_1]r}{2(1-0.5\alpha-0.5\lambda_1)\acute{\lambda_A}+\alpha\lambda_1},$

$\widehat{\Pi_A} \equiv (r-2\tau)\left[(1-\lambda_1)g\left(\widehat{I_A} + \underline{I_A}\right) + \lambda_1\right], \overline{\Pi_A} \equiv \frac{\lambda_1(r-\tau)(r-2\tau)}{\tau} - \overline{I_A}$, and

$\acute{\Pi_A} \equiv \frac{r^2}{8\tau}\left[(1-\lambda_1)g\left(\acute{I_A} + \underline{I_A}\right) + \lambda_1\right] - \acute{I_A}.$

The terms with the "hat" (e.g., $\widehat{f_A}$) refer to the optimal values, if the platform sets the maximum commission, that satisfy the condition for the commission to be full-coverage commission,[33] and the seller does not have an option to refuse the price parity clause.

The terms with the "tilde" (e.g., $\widetilde{f_A}$) refer to the optimal values, if the platform sets the maximum full-coverage commission, that satisfy the condition for the commission to ensure that the seller accepts using the platform under the price parity clause.[34]

The terms with the "bar" (e.g., $\overline{f_A}$) refer to the optimal values when the condition for the commission to be a full-coverage commission is equivalent to the condition for the commission to ensure that the seller accepts use of the platform offering the full-coverage commission under the price parity clause.[35]

The terms with the "acute" (e.g., $\acute{f_A}$) refer to the optimal values, if the platform sets the commission, that maximise the platform's profit when the platform decides to offer non-full coverage commission if the seller does not have an option to refuse the price parity clause.[36]

Note that when $2\tau \leq r < 4\tau$ holds, $\widehat{\lambda_A} < \acute{\lambda_A} < \widetilde{\lambda_A}$ and $\widehat{I_A} < \acute{I_A} < \widetilde{I_A}$ hold.

Analysis of this model provides the following proposition, to be proved in the subsequent sections.

Proposition 4.5. *Suppose the platform is allowed to introduce a price parity clause; there is a unique equilibrium in which the platform introduces a price parity clause.*

a) *When $r \geq 4\tau$, the set of the platform's investment level ${I_A}^*$ and commission ${f_A}^*$ and the sellers' prices $({p_{1A}}^*, {p_{1D}}^*)$ in this equilibrium is*

[33] As discussed later, the condition for the commission to be a full-coverage commission is $f_A \leq v(r-2\tau)/\xi$.

[34] As discussed later, the condition for the commission to ensure that the seller accepts use of the platform under the price parity clause is $f_A \leq \lambda_A(1-\lambda_1)(r-\tau)/\xi$.

[35] This situation happens if $\frac{v(r-2\tau)}{\xi} = \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi} \leftrightarrow \lambda_A = \frac{\lambda_1(r-2\tau)}{(1-\lambda_1)\tau} = \overline{\lambda_A}$.

[36] As discussed later, the profit is the a concave quadratic function on f_A, which is maximised when $f_A = \frac{vr}{2\xi}$.

$$(I_A{}^*, f_A{}^*, p_{1A}{}^*, p_{1D}{}^*) = \left\{\begin{array}{c} \left(\widetilde{I_A}, \widetilde{f_A}, r-\tau, r-\tau\right) \, if \, \overline{\lambda_A} \geq \widetilde{\lambda_A} \\ \left(\overline{I_A}, \overline{f_A}, r-\tau, r-\tau\right) \, if \, \widehat{\lambda_A} < \overline{\lambda_A} < \widetilde{\lambda_A} \\ \left(\widehat{I_A}, \widehat{f_A}, r-\tau, r-\tau\right) \, if \, \overline{\lambda_A} \leq \widehat{\lambda_A} \end{array}\right\}.$$

When $2\tau \leq r < 4\tau$, *the set of the platform's investment level* $I_A{}^*$ *and commission* $f_A{}^*$ *and the sellers' prices* $(p_{1A}{}^*, p_{1D}{}^*)$ *in this equilibrium is*

$$(I_A{}^*, f_A{}^*, p_{1A}{}^*, p_{1D}{}^*) =$$
$$\left\{\begin{array}{c} \left(\widetilde{I_A}, \widetilde{f_A}, r-\tau, r-\tau\right) \, if \, \acute{\lambda}_A < \widetilde{\lambda_A} \leq \overline{\lambda_A} \\ \left(\overline{I_A}, \overline{f_A}, r-\tau, r-\tau\right) \, if \, a) \, \acute{\lambda}_A \leq \overline{\lambda_A} < \widetilde{\lambda_A} \, or \, if \, b) \, \widehat{\lambda_A} < \overline{\lambda_A} < \acute{\lambda}_A \, and \, \acute{\Pi}_A \leq \overline{\Pi_A} \\ \left(\widehat{I_A}, \widehat{f_A}, r-\tau, r-\tau\right) \, if \, \overline{\lambda_A} \leq \widehat{\lambda_A} < \acute{\lambda}_A \, and \, \acute{\Pi}_A \leq \widehat{\Pi_A} \\ \left(\acute{I}_A, \acute{f}_A, \frac{3}{4}r, \frac{3}{4}r\right) \, if \, a) \, \widehat{\lambda_A} < \overline{\lambda_A} < \acute{\lambda}_A \, and \, \acute{\Pi}_A > \overline{\Pi_A} \, or \\ if \, b) \, \overline{\lambda_A} \leq \widehat{\lambda_A} < \acute{\lambda}_A \, and \, \acute{\Pi}_A > \widehat{\Pi_A} \end{array}\right\}.$$

$b)$ *The amount of the investment may decrease compared to when the price parity clause if* $r < 4\tau, \widehat{\lambda_A} \leq \overline{\lambda_A} < \acute{\lambda}_A$ *and* $\acute{\Pi}_A \leq \overline{\Pi_A}$ *hold or if* $r < 4\tau, \overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda}_A$ *and* $\acute{\Pi}_A \leq \widehat{\Pi_A}$ *hold. Otherwise, the amount of investment always increases.*

Briefly explained, Proposition 4.5 shows that when the ratio of r to τ is high ($r \geq 4\tau$), the full-coverage equilibrium always occurs. However, when the ratio of r to τ is low ($2\tau \leq r < 4\tau$), not only full-coverage equilibrium but also the non-full-coverage equilibrium can occur, and the result depends on which type equilibrium is more profitable for the platform. Moreover, when the ratio of r to τ is low and the full-coverage equilibrium happens, the amount of the investment may be smaller than the non-price parity clause case. This is because in those situations, the platform choose to set a higher commission rather than making larger investment than the non-price parity clause case.

Specifically, Proposition 4.5 shows that when the price parity clause is introduced and the ratio of r to τ is weakly higher than a certain threshold ($r \geq 4\tau$), there are three types of the equilibrium. In any case, the seller accepts use of the platform and set both the price on the platform and the direct sales price at the maximum full-coverage price, $r - \tau$. It follows that the equilibrium seller's prices are almost equal to the equilibrium prices when the price parity clause is not allowed. The difference is that the direct sales price is slightly higher than that of the non-price parity clause case, because the price parity clause prohibits the seller from undercutting the direct sales price to avoid paying the commission.

Among the three types, the platform can set the high commission without being constrained by the possibility that the seller does not use the platform only when the optimal λ_A, if the seller does not have an option to refuse the price parity clause, is lower than a certain threshold (i.e., $\widehat{\lambda_A} \leq \overline{\lambda_A}$). Note that this case is likely to happen when τ is close to zero. The result also shows that allowing the clause just causes a negligible increase in the direct sales price but increases the amount of investment, which follows the more consumers are aware of the product. In other two types of equilibrium, $({I_A}^*, {f_A}^*) = \left(\overline{I_A}, \overline{f_A}\right)$ and $(\widetilde{I_A}, \widetilde{f_A})$, the possibility that the seller does not use the platform constrains the platform's ability to set high commissions. In those situations, the platform makes a larger investment than the most efficient investment level if the seller does not have an option to refuse the price parity clause instead of setting a high commission. If $({I_A}^*, {f_A}^*) = (\widetilde{I_A}, \widetilde{f_A})$, the equilibrium commission can be smaller than when the price parity clause is not allowed. Otherwise, the equilibrium commission is higher than when the clause is not allowed.

In any of the three types, the investment level is weakly higher than when the price parity clause is not allowed. The equilibrium investment remains the same only if both $\alpha = 0$ and $({I_A}^*, {f_A}^*) = \left(\widehat{I_A}, \widehat{f_A}\right)$ hold, since $g'\left(\widehat{I_A} + \underline{I_A}\right) = 1/(r - 2\tau)(1 - \lambda_1) \leq 1/(1 - \alpha)(r - 2\tau)(1 - \lambda_1) = g'\left({I_A}^* + \underline{I_A}\right)$ holds. Recall that in this situation, $\widehat{\lambda_A} < \widetilde{\lambda_A}$ and $\widehat{I_A} < \widetilde{I_A}$ hold and $\left(\overline{I_A}, \overline{f_A}\right)$ becomes an equilibrium only if $\widehat{I_A} < \overline{I_A} < \widetilde{I_A}$ holds.

If the ratio of r to τ is lower than a certain threshold but not too low ($2\tau \leq r < 4\tau$), the full-coverage equilibrium and the non-full-coverage equilibrium can both occur. Whether the full-coverage equilibrium happens or not is determined by which type of equilibrium is more profitable for the platform. When the platform prefers the full-coverage equilibrium, there are three types of equilibrium, all the same as in the case in which $r \geq 4\tau$ holds. It follows that in this situation, the platform can set the highest possible full-coverage commission (i.e., $({I_A}^*, {f_A}^*) = \left(\widehat{I_A}, \widehat{f_A}\right)$) without being constrained by the possibility that the seller does not use the platform only if $\widehat{\lambda_A} \leq \overline{\lambda_A}$ holds and the profit is higher than the profit when the platform prefers the non-full coverage equilibrium.

When the platform prefers the non-full-coverage equilibrium, there is only one type of equilibrium in which the seller sets both the price on the platform (i.e., $({I_A}^*, {f_A}^*) = \left(\acute{I_A}, \acute{f_A}\right)$) and the direct sales price at $3r/4$.

Note that the equilibrium seller's price when the platform prefers the full-coverage equilibrium is equal to the equilibrium seller's direct sales price when the price parity clause is not allowed. The equilibrium seller's price when the platform prefers the non-full-coverage equilibrium is equal to the equilibrium seller's platform price when the price parity clause is not allowed. It follows that allowing the price parity clause decreases the equilibrium seller's platform price when the platform prefers the full-coverage equilibrium. When the platform prefers the non-full-coverage equilibrium, allowing the price parity clause increases the equilibrium seller's direct sales price.

With respect to the effect on investment level of allowing the price parity clause, when the platform prefers the non-full-coverage equilibrium, the investment level is always weakly higher than when the price parity clause is not allowed, because $g'\left(\acute{I_A} + \underline{I_A}\right) = 8\tau/(1-\lambda_1)r^2 \leq 8\tau/(1-\alpha)(1-\lambda_1)r^2 = g'\left({I_A}^* + \underline{I_A}\right)$ holds. When the platform prefers the full-coverage equilibrium such that $({I_A}^*, {f_A}^*) = (\widetilde{I_A}, \widetilde{f_A})$, the investment level is also always weakly higher than when the price parity clause is not allowed, since $\widehat{I_A} < \acute{I_A} < \widetilde{I_A}$ holds. However, when the platform prefers the full-coverage equilibrium such that $({I_A}^*, {f_A}^*) = \left(\overline{I_A}, \overline{f_A}\right)$ or $\left(\widehat{I_A}, \widehat{f_A}\right)$, the investment level can be smaller than when the price parity clause is not allowed, because in this situation the platform chooses to get the profit by setting the higher commission.

Moreover, with respect to the effect on the equilibrium commission of allowing the price parity clause, if the non-full-coverage equilibrium arises, the equilibrium commission is always higher than when the clause is not allowed. On the other hand, if the full-coverage equilibrium happens, the commission can be larger or smaller than the equilibrium commission when the clause is allowed, because in this type of equilibrium, the platform prefers to gain profit from the larger volume of transactions rather than a high commission.

Proposition 4.5 also shows that allowing the price parity clause weakly increases the investment level except for the certain special situations which can arise when $2\tau \leq r < 4\tau$ holds and the platform prefers the full-coverage equilibrium to the non-full-coverage equilibrium. The optimal amount of the investment does not change with the introduction of the clause when $\alpha = 0$, and the gap of the investment in the two cases becomes larger as α increases. Moreover, the introduction of the clause weakly decreases the price on the platform and weakly increases the direct sales price.

The effect of allowing a price parity clause on the equilibrium investment level, commission and prices can be summarised in Table 4.2 below.

Table 4.2 Effect of allowing a price parity clause on the equilibrium investment level, commission and prices

$i)\ r \geq 4\tau$

Case	Equilibrium when price parity clause is allowed	I_A	f_A	p_{1A}	p_{1D}
$\widetilde{\lambda_A} \geq \overline{\lambda_A}$	$\left(\widetilde{I_A}, \widetilde{f_A}, r-\tau, r-\tau\right)$	+	+/-	0	+*
$\widehat{\lambda_A} < \overline{\lambda_A} < \widetilde{\lambda_A}$	$\left(\overline{I_A}, \overline{f_A}, r-\tau, r-\tau\right)$	+	+	0	+*
$\widehat{\lambda_A} \leq \overline{\lambda_A}$	$\left(\widehat{I_A}, \widehat{f_A}, r-\tau, r-\tau\right)$	+	+	0	+*

$\overline{\lambda_A}$: λ_A that the condition for the full-coverage commission ($f_A \leq \nu(r-2\tau)/\xi$) is equivalent to the condition for the seller's acceptance of the price parity clause ($f_A \leq \lambda_A(1-\lambda_1)(r-\tau)/\xi$).
$\widetilde{\lambda_A}$: The optimal λ_A when the condition for the commission to ensure that the seller accepts the price parity clause and set the full coverage price is binding.
$\widehat{\lambda_A}$: The optimal λ_A when the seller does not have an option to refuse the price parity clause.

+: Allowing price parity clause increases the equilibrium value
-: Allowing price parity clause decreases the equilibrium value
+/-: Allowing price parity clause can increase or decrease the equilibrium value
0: Allowing price parity clause does not change the equilibrium value
+*: Allowing price parity clause only slightly increases the direct sales price

$ii)\ 2\tau \leq r < 4\tau$

Case	Equilibrium when price parity clause is allowed	I_A	f_A	p_{1A}	p_{1D}
$\acute{\lambda}_A < \widetilde{\lambda_A} \leq \overline{\lambda_A}$	$(\widetilde{I_A}, \widetilde{f_A}, r-\tau, r-\tau)$	+	+/-	-	0
$\acute{\lambda}_A \leq \overline{\lambda_A} < \widetilde{\lambda_A}$	$(\overline{I_A}, \overline{f_A}, r-\tau, r-\tau)$	+	+/-	-	0
$\widehat{\lambda_A} \leq \overline{\lambda_A} < \acute{\lambda}_A$ and $\acute{\Pi}_A \leq \overline{\Pi_A}$	$(\overline{I_A}, \overline{f_A}, r-\tau, r-\tau)$	+/-	+/-	-	0
$\overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda}_A$ and $\acute{\Pi}_A \leq \widehat{\Pi_A}$	$(\widehat{I_A}, \widehat{f_A}, r-\tau, r-\tau)$	+/-	+/-	-	0
$\widehat{\lambda_A} \leq \overline{\lambda_A} < \acute{\lambda}_A$ and $\acute{\Pi}_A > \overline{\Pi_A}$	$\left(\acute{I}_A, \acute{f}_A, \frac{3}{4}r, \frac{3}{4}r\right)$	+	+	0	+
$\overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda}_A$ and $\acute{\Pi}_A > \widehat{\Pi_A}$	$\left(\acute{I}_A, \acute{f}_A, \frac{3}{4}r, \frac{3}{4}r\right)$	+	+	0	+

$\acute{\lambda}_A$: The optimal λ_A if the platform sets the non-full-coverage commission.

4.5.2.1 Consumers' Behaviour

Since the introduction of the price parity clause does not constrain the consumers' behaviours, the consumers' response is given by Lemma 4.7. Hence, the consumers' behaviours given the seller's prices are summarised in Lemma 4.10.

Lemma 4.10. *The demand for the seller's product sold on sales channel* $m \in A, D$, $q_{1m}{}^{*}$, where $n \in A, D$ *and* $m \neq n$, *given* p_{1A}, p_{1D}, f_A *and* I_A, *is same as the demand in the case where a price parity clause is not allowed, as specified in Lemma 4.7.*

4.5.2.2 Seller's Behaviour

In this case, the seller has two options. The first option is to accept the price parity clause imposed by the platform and set the prices such that $p_{1A} \leq p_{1D}$. The second option for the seller is to refuse the offer by the platform and to concentrate instead on direct sales.

Solving this profit-maximisation problem for the seller gives the optimal seller's behaviours given the consumers' response, as summarised in Lemma 4.11.

Lemma 4.11. *Anticipating the consumers' response, as specified in Lemma 4.4, given* f_A *and* I_A*, the optimal pricing policy for the seller is as follows:*

$$(p_{1A}, p_{1D}) = \left\{ \begin{array}{c} (r-\tau, r-\tau)\ if\ f_A \le min\left[\frac{v}{\xi}(r-2\tau), \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi}\right] \\ \left(\frac{r}{2}+\frac{\xi f_A}{2v}, \frac{r}{2}+\frac{\xi f_A}{2v}\right)\ if\ \frac{v}{\xi}(r-2\tau) < f_A \le \frac{v}{\xi}\left\{r-2\left[\frac{\tau\lambda_1(r-\tau)}{v}\right]^{0.5}\right\} \\ (+\infty, r-\tau)\ if\ \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi} < f_A \le \frac{v}{\xi}(r-2\tau)\ or \\ if\ f_A > max\left[\frac{v}{\xi}(r-2\tau), \frac{v}{\xi}\left\{r-2\left[\frac{\tau\lambda_1(r-\tau)}{v}\right]^{0.5}\right\}\right] \end{array} \right\},$$

where $p_{1A} = +\infty$ *denotes that the seller refuses to accept the clause.*

Proof: See Appendix C

Lemma 4.11 shows that when the seller accept the price parity clause, the seller prefers to set the maximum full-coverage price $(r-\tau)$ for both the platform price and the direct sales price if the commission is equal to or lower than a certain threshold (i.e. $f_A \le v(r-2\tau)/\xi$). In this situation, the seller does accept the price parity clause and set the full-coverage price if the commission is low enough to guarantee that the expected profit is weakly larger than the profit when the seller refuses the clause and sets the maximum full-coverage price (i.e. $f_A \le \lambda_A(1-\lambda_1)(r-\tau)/\xi$).

When $f_A > v(r-2\tau)/\xi$ holds, the expected profit for accepting the price parity clause with setting the non-full-coverage price is higher than the expected profit for accepting the clause with setting the full-coverage price. In this situation, the seller does accept the price parity clause and set the non-full-coverage price if the commission is low enough to guarantee that the expected profit is weakly larger than the profit when the seller refuses the clause and sets the maximum full-coverage price. Note that, the non-full-coverage price in equilibrium is lower than the non-full-coverage price on the platform in the previous case ($p_{1A} = \frac{r+f_A}{2}$).

4.5.2.3 Platform's Behaviour Regarding Commission and Investment

Lemmata 4.10 and 4.11 imply that the expected revenue that the platform obtains from the transaction on it can be written as follows:

$$R_A = \left\{ \begin{array}{c} \xi f_A \; if \; f_A \le \min\left[\frac{\nu}{\xi}(r-2\tau), \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi}\right] \\ \xi\left(\frac{r-\frac{\xi f_A}{\nu}}{2\tau}\right) f_A \; if \; \frac{\nu}{\xi}(r-2\tau) < f_A \le \frac{\nu}{\xi}\left\{r - 2\left[\frac{\tau\lambda_1(r-\tau)}{\nu}\right]^{0.5}\right\} \\ 0 \; if \; \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi} < f_A \le \frac{\nu}{\xi}(r-2\tau) \text{ or} \\ if \; f_A > \max\left[\frac{\nu}{\xi}(r-2\tau), \frac{\nu}{\xi}\left\{r - 2\left[\frac{\tau\lambda_1(r-\tau)}{\nu}\right]^{0.5}\right\}\right] \end{array} \right\}.$$

The total expected profit taking the investment into account is given by $\Pi_A = R_A(g(I_A)) - I_A$.

Solving this profit-maximisation problem gives the optimal commission and investment set by the platform given the seller and consumers' response, which are summarised in Lemma 4.12 below.

Lemma 4.12. *Anticipating the seller and consumers' response specified in Lemmata 4.10 and 4.11, the platform chooses the optimal investment level* ${I_A}^*$ *and the optimal commission* ${f_A}^*$ *as follows.*

1) Where $r \ge 4\tau$*, the set of the platform's investment and commission and the sellers' prices is*

$$({I_A}^*, {f_A}^*) = \left\{ \begin{array}{c} (\widetilde{I_A}, \widetilde{f_A}) \; if \; \overline{\lambda_A} \ge \widetilde{\lambda_A} \\ (\overline{I_A}, \overline{f_A}) \; if \; \widehat{\lambda_A} < \overline{\lambda_A} < \widetilde{\lambda_A} \\ (\widehat{I_A}, \widehat{f_A}) \; if \; \overline{\lambda_A} \le \widehat{\lambda_A} \end{array} \right\}.$$

Where $2\tau \le r < 4\tau$*, the set of the platform's investment and commission and the sellers' prices is*

$$({I_A}^*, {f_A}^*) = \left\{ \begin{array}{c} (\widetilde{I_A}, \widetilde{f_A}) \; if \; \acute{\lambda}_A < \widetilde{\lambda_A} \le \overline{\lambda_A} \\ (\overline{I_A}, \overline{f_A}) \; if \; a) \; \acute{\lambda}_A \le \overline{\lambda_A} < \widetilde{\lambda_A} \; or \; if \; b) \; \widehat{\lambda_A} < \overline{\lambda_A} < \acute{\lambda}_A \; and \; \acute{\Pi}_A \le \overline{\Pi_A} \\ (\widehat{I_A}, \widehat{f_A}) \; if \; \overline{\lambda_A} \le \widehat{\lambda_A} < \acute{\lambda}_A \; and \; \acute{\Pi}_A \le \widehat{\Pi_A} \\ (\acute{I}_A, \acute{f}_A) \; if \; a) \; \widehat{\lambda_A} < \overline{\lambda_A} < \acute{\lambda}_A \; and \; \acute{\Pi}_A > \overline{\Pi_A} \; or \\ if \; b) \; \overline{\lambda_A} \le \widehat{\lambda_A} < \acute{\lambda}_A \; and \; \acute{\Pi}_A > \widehat{\Pi_A} \end{array} \right\}.$$

2) The amount of the investment may decrease compared to when the price parity clause is not allowed if $2\tau \le r < 4\tau, \acute{\lambda}_A > \overline{\lambda_A}$ *and* $\acute{\Pi}_A \le \overline{\Pi_A}$ *hold. Otherwise, the amount of investment always increases.*

Proof: See Appendix C

From Lemmata 4.10 to 4.12, Proposition 4.5 can be obtained directly.

4.5.2.4 Effect on Consumer Welfare of Allowing Price Parity Clause

Taking the results of the analysis above, this section analyses the effect of introducing the platform price parity clause on the welfare for each player.

Analysis on this model provides the following proposition, as proven in the following section.

Proposition 4.6. *Allowing a price parity clause improves the consumer welfare except for the following cases;*

$a)\ r \geq 4\tau,\ \alpha = 0$ *and* $\overline{\lambda_A} \leq \widehat{\lambda_A}$,

b) $2\tau \leq r < 4\tau,\ \widehat{\lambda_A} < \overline{\lambda_A} < \acute{\lambda}_A,\ \acute{\Pi}_A \leq \overline{\Pi_A}$ *and* $\overline{\lambda_A} \leq \left[\alpha + \frac{(1-\alpha)r^2}{16\tau^2}\right]{\lambda_A}^*$,

c) $2\tau \leq r < 4\tau,\ \overline{\lambda_A} \leq \widehat{\lambda_A} < \acute{\lambda}_A,\ \acute{\Pi}_A \leq \widehat{\Pi_A}$ *and* $\widehat{\lambda_A} \leq \left[\alpha + \frac{(1-\alpha)r^2}{16\tau^2}\right]{\lambda_A}^*$,

d) $2\tau \leq r < 4\tau,\ \acute{\lambda}_A \leq \overline{\lambda_A} < \widetilde{\lambda_A},\ \acute{\Pi}_A > \overline{\Pi_A}$ *and* $\acute{\lambda}_A \leq \frac{[\lambda_1+\alpha{\lambda_A}^*(1-\lambda_1)](16\tau^2-r^2)}{(1-\lambda_1)r^2} + {\lambda_A}^*$ *and*

e) $2\tau \leq r < 4\tau,\ \overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda}_A,\ \acute{\Pi}_A > \widehat{\Pi_A}$ *and* $\acute{\lambda}_A \leq \frac{[\lambda_1+\alpha{\lambda_A}^*(1-\lambda_1)](16\tau^2-r^2)}{(1-\lambda_1)r^2} + {\lambda_A}^*$.

Proof: See Appendix C

Proposition 4.6 shows that allowing the price parity clause can improve or harm consumer welfare in contrast to the case where the utility to purchase the product is same for all consumers (zero transportation cost), in which the clause simply improves the consumer welfare and decreases the profit of the seller. The clause can do either because it can have both two positive effects and a negative effect on consumer welfare. The increase of investment by the introduction of the clause has a positive effect on the consumer surplus by creating more opportunities to be aware of the product (consumer awareness effect). Moreover, the decrease of the seller's platform price also has a positive effect (platform price effect). On the other hand, the consumer surplus from the rise of the direct sales price through the introduction of the clause increases after the introduction of the clause (direct sales price effect), having a negative effect on the consumer surplus. This negative direct sales price effect may exceed the positive effect, consumer awareness effect and platform price effect.

In the zero transportation cost case, there is no platform price effect and negligibly small direct sales effect. On the other hand, there is a substantial consumer awareness effect. Hence, in this case, the price parity clause always improve the consumer welfare. However, in the positive transportation cost case, the total effect is ambiguous.

According to Proposition 4.6, if $r \geq 4\tau$ (which is equivalent to case i) in Table 4.2), the introduction of the price parity clause harms consumer welfare only on the extremely limited occasion that the investment does not change with the introduction of the clause. In this situation, the clause only slightly increases the seller's direct sales price. It follows that when the decrease of the consumer welfare happens, the effect is negligibly small.

If $2\tau \leq r < 4\tau$, when the full-coverage equilibrium happens in the case where the price parity clause is allowed (equivalent to the first four rows of case ii) in Table 4.2), the introduction of the price parity clause harms consumer welfare if the equilibrium investment level is smaller than when the clause is not allowed and the amount of the decrease of the investment level is large enough to exceed the positive effect through the decrease in the seller's platform price (Proposition 4.6 *b) and c)*).

If $2\tau \leq r < 4\tau$ and the non-full-coverage equilibrium happens when the price parity clause is allowed (as in the last two rows of case ii) in Table 4.2), the introduction of the price parity clause harms consumer welfare if the negative effect through the increase of the seller's direct sales price is large enough to exceed the positive effect through the increase in the consumers' awareness of the product (Proposition 4.6 *d) and e)*).

Note that unlike the effect on consumer welfare, the effect on the seller's profit in allowing the price parity clause is ambiguous. This effect remains unclear because the price parity clause have the following four effects that can change the seller's profit: (1) the change in consumers' awareness due to the change in the investment level, (2) the change in the behaviours of the consumers who are aware of both the platform and the direct sales channel (i.e. only half of such consumers purchases directly), (3) the change in the platform's commission and (4) the change in the seller's prices. As shown in Table 4.2, the net effect of the four effects can be positive or negative to the seller's profit, the total effect on the seller's profit is ambiguous.

4.6. Policy Implications and Conclusions

The analysis on the monopoly platform case in this chapter shows that in the case where the utility to purchase the product is same for all consumers (zero transportation cost), allowing the price parity clause simply improves the consumer welfare, because it improves the consumer awareness and only slightly increases the equilibrium direct sales price. However, the clause decreases the profit of the seller, because its positive effect on the profit of the seller thorough the increased consumer awareness is surpassed by the negative effect caused by the increase of the commission imposed by the platform, which is higher than the equilibrium price.

This chapter also shows that in the case where the utility to purchase the product is different between consumers (positive transportation cost), the effect of allowing the price parity clause is not straightforward as the zero transportation cost case. The introduction of the price parity clause generally increases the platform's investment, which has a positive effect on the consumer surplus by affording more opportunities to be aware of the product. On the other hand, it also has the negative effect on the consumer surplus by the rise of the direct sales price, and in some situations the net effect on the consumer welfare can be negative.

These result imply that the claim that narrow price parity clauses by platforms is justified because it could be necessary to prevent free rides by sellers using a platform. However, at the same time, the result of the analysis also shows that the effect on consumer welfare from the introduction of price parity clauses is not always positive, because the rise of the direct sales price may offset the positive effect, which implies that prohibition of narrow price parity clause may have positive effects on consumer welfare.

Moreover, the comparison between the zero transportation cost case and the positive transportation cost case shows that when the transportation cost is small, allowing price parity clause is more likely to increase the consumer welfare and decrease the profit of the seller. The margin of the seller can obtain from the sales on the platform depends on the amount of transportation cost, which follows that the more diversity of the consumers means the more room for making profit for the seller.

Those results are in common with the other previous literature analysing the effect of narrow price parity clauses in a dominant platform situation, Wang and Wright (2016), in the sense that the price parity clause enables the platform to set a higher commission and can harm the consumer welfare. However, this study shows that the consumer welfare can be improved by the clause. This is because the positive effect through improving the consumer awareness may exceed its negative effect. The point that the more consumer awareness enabled by the price parity clause are not taken into account in the previous studies including Wang and Wright (2016).

The results in the case of more than one platform and one seller and the effects of wide price parity clauses are discussed in the next chapter.

4.7. Appendix C

4.7.1 Proof of Lemma 4.1

If a consumer has the information on the seller's product sold on sales channel m**,** where $m \in A\,, D$ and p_{1m} is the lowest price as far as the consumer is aware of it, the utility of the consumer to purchase the seller's product on the platform is $r - p_{1m}$. Hence, the condition that the consumer purchases on sales channel m is written as $r - p_{1m} \geq 0$.

This implies that, supposing all the consumers with the information that the product sold are on sales channel m when they make a decision about purchasing, and supposing p_{1m} is the unique lowest price as far as they know, the demand for the seller's product sold on sales channel m is 1 if $p_{1m} \leq r$ and 0 if $p_{1m} > r$.

According to Table 4.1, the share of the consumers who are aware of the product sold only on sales channel m when they make a decision about purchasing is $(1 - \alpha)\lambda_m(1 - \lambda_n)$. The share of the consumers who are aware of the product sold on both sales channels is given by $\alpha(\lambda_m + \lambda_n) + (1 - 2\alpha)\lambda_m\lambda_n$.

If $p_{1m} < p_{1n} \leq r$ or $p_{1m} < r < p_{1n}$ holds, all of the consumers who are aware of the product sold on both sales channels and all of the consumers who only are aware of the product sold on sales channel m will purchase the product on sales channel m, which yields that the demand in this case is equal to $\lambda_m + \alpha\lambda_n - \alpha\lambda_m\lambda_n$.

If $p_{1n} < p_{1m} \leq r$ holds, only the consumers who only are aware of the product sold on sales channel m will purchase the product on the sales channel m as long as the utility for the consumer is positive or equal to zero. Hence, the demand in this case is $(1 - \alpha)\lambda_m(1 - \lambda_n)$.

If $p_{1m} = p_{1n} \leq r$ holds, half of the consumers who are aware of the product sold on both sales channels and all of the consumers who only are aware of the product sold on sales channel m will purchase the product on the sales channel m as long as the utility for the consumer is positive or equal to zero, meaning that the demand for each sales channel is equal to $(1 - 0.5\alpha)\lambda_m + 0.5\alpha\lambda_n - 0.5\lambda_m\lambda_n$.

If $p_{1m} > r$ and $p_{1n} > r$ hold, the demand for each sales channel is zero, because the prices the consumer face is larger than their reservation value. ■

4.7.2 Proof of Lemma 4.2

Lemma 4.1 implies that the expected profit for the seller can be written as follows:

$i)\ p_{1A} \leq r, p_{1D} \leq r$

$$\Pi_1 = \begin{Bmatrix} \nu p_1 - \ \xi f_A \ if \ p_{1A} = p_{1D} \\ (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1 p_{1D} \ if \ p_{1A} < p_{1D} \\ (1-\alpha)\lambda_A(1-\lambda_1)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D} \ if \ p_{1A} > p_{1D} \end{Bmatrix}.$$

$ii) \ p_{1A} \leq r < p_{1D}$

$\Pi_1 = (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A).$

$iii) \ p_{1D} \leq r < p_{1A}$

$\Pi_1 = (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}.$

$iv) \ p_{1A} > r, p_{1D} > r$

$\Pi_1 = 0.$

Lemma 4.2 can be obtained by solving this profit-maximisation problem. The proof is divided into two steps. In *Step 1* we find the prices for the seller in each of the four cases: (i) both p_{1A} and p_{1D} do not exceed r, (ii) only p_{1A} does not exceed r, (iii) only p_{1D} does not exceed r, and (iv) both p_{1A} and p_{1D} exceed r. In *Step 2*, we compare the profits in each case to find the equilibrium price.

Step 1: Optimal prices in each of four cases

In this step, we find the optimal prices for the seller in each of the four cases. *Step 1* is divided into the four substeps, each covering one of the four cases.

Substep 1.1: Optimal prices if $p_{1A} = p_{1D} \leq r$

This case can be divided into three sub-cases depending on the relationship between p_{1A} and p_{1D}.

$a) \ p_{1A} = p_{1D} = p_1 \leq r$

In this case, the expected profit is $\Pi_1^{p_{1A}=p_{1D}\leq r} = \nu p_1 - \ \xi f_A$.

Since this profit is monotonically increasing on p_1, which is subject to $p_1 \leq r$, this profit is maximised when setting $p_1 = \ r$. The maximum expected profit for the seller can be written as

$\Pi_1^{*\,p_{1A}=p_{1D}\leq r-\tau} = \nu r - \xi f_A.$

$b)\ p_{1A} < p_{1D} \le r$

In this case, the expected profit is

$$\Pi_1^{p_{1A}<p_{1D}\le r-\tau} = (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1 p_{1D}.$$

Since this profit is monotonically increasing on p_{1A} and p_{1D}, this profit is maximised when setting $(p_{1A}, p_{1D}) = (\ r - \varepsilon, r)$. The maximum expected profit for the seller can be written as

$$\Pi_1^{*\,p_{1A}<p_{1D}\le r} = (\lambda_A + \lambda_1 - \lambda_A\lambda_1)r - (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(f_A + \varepsilon) = \nu r - (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)f_A - \varepsilon.$$

$c)\ p_{1D} < p_{1A} \le r$

In this case, the expected profit is

$$\Pi_1^{p_{1D}<p_{1A}\le r} = (1-\alpha)\lambda_A(1-\lambda_1)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}.$$

Since this profit is monotonically increasing on p_{1A} and p_{1D}, this profit is maximised when setting $(p_{1A}, p_{1D}) = (r, r-\varepsilon)$. The maximum expected profit for the seller can be written as

$$\Pi_1^{*\,p_{1D}<p_{1A}\le r} = (\lambda_A + \lambda_1 - \lambda_A\lambda_1)(r-\tau) - (1-\alpha)\lambda_A(1-\lambda_1)f_A - (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)\varepsilon = \nu r - (1-\alpha)\lambda_A(1-\lambda_1)f_A - \varepsilon.$$

The results of the three cases imply that the revenues are almost same because the seller sets the price equal to or just below r. It is also shown that $\Pi_1^{*\,p_{1D}<p_{1A}\le r} > \Pi_1^{*\,p_{1A}=p_{1D}\le r} > \Pi_1^{*\,p_{1A}<p_{1D}\le r}$, since the commission that the seller need to pay is the lowest when $p_{1D} < p_{1A}$.

Hence, if the seller decides to set prices such that both prices are full-coverage price, this profit is maximised when setting $(p_{1A}, p_{1D}) = (r, r-\varepsilon)$.

Substep 1.2: Optimal prices if $p_{1A} \le r < p_{1D}$

In this case, the expected profit is

$$\Pi_1 = (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A).$$

Since this is monotonically increasing on p_{1A}, this profit is maximised when setting $p_{1A} = r$. The maximum expected profit for the seller can be written as

$(\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(r - f_A)$. However, setting such prices will not be chosen by the seller because setting $(p_{1A}, p_{1D}) = (r, r - \varepsilon)$ is more profitable.

Substep 1.3: Optimal prices if $p_{1D} \leq r < p_{1A}$

In this case, the expected profit is

$\Pi_1 = (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}$.

Since this is monotonically increasing on p_{1D}, this profit is maximised when setting $p_{1D} = r$. The maximum expected profit for the seller can be written as

$(\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)r$.

Substep 1.4: Optimal prices if $p_{1A} > r, p_{1D} > r$

This case can be chosen in the equilibrium because the profit is always zero.

Step 2: Comparing the four cases

Step 1 shows that there are two types of pricing that can be optimal for the seller. The first one is setting $(p_{1A}, p_{1D}) = (r, r - \varepsilon)$. The second one is setting p_{1D} at r and p_{1A} high enough to guarantee that no consumers purchase through the platform ($p_{1A} > r$).

The condition that the seller prefers to set $(p_{1A}, p_{1D}) = (r, r - \varepsilon)$ can be written as

$vr - (1 - \alpha)\lambda_A(1 - \lambda_1)f_A > (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)r$.

The condition can be modified as $f_A < r$.■

4.7.3 Proof of Lemma 4.5

In this case, the seller has two options. The first option is to accept the price parity clause imposed by the platform and set the prices such that $p_{1A} \leq p_{1D}$. As shown in Section 4.7.2, if the seller sets the prices such that $p_{1A} \leq p_{1D}$, the seller's expected profit is maximised when it sets the prices such that $p_{1A} = p_{1D} = p_1$. The expected profit is $vr - \xi f_A$.

The second option for the seller is to refuse the offer by the platform and to concentrate on its direct sales. If the seller takes this option, the seller's expected profit is given by

$$\Pi_1 = \begin{Bmatrix} \lambda_1 p_{1D} \; if \; p_{1D} \leq r \\ 0 \; if \; p_{1D} > r \end{Bmatrix}.$$

Since this profit is monotonically increasing on p_{1D}, this profit is maximised when setting p_{1D} at r. The maximum expected profit for the seller is $\lambda_1 r$.

Hence, the condition that the seller prefers to accept the price parity clause can be written as

$vr - \xi f_A \geq \lambda_1 r.$

This condition can be modified as $f_A \leq \lambda_A(1 - \lambda_1)r/\ \xi$. ■

4.7.4 Proof of Lemma 4.7

If consumer x has the information on the seller's product sold on sales channel m, where $m \in A\ , D$ and p_{1m} is the lowest price as far as consumer x is aware of it, the utility of consumer x to purchase the seller's product on the platform is $r - \tau x\ - p_{1m}$. Hence, the condition that consumer x purchases on sales channel m is written as $r - \tau x\ - p_{1m} \geq 0$. This inequality can be modified to $x \leq (r -\ p_{1m})/\tau$.

This implies that, supposing all the consumers with the information that the product sold are on sales channel m when they make a decision about purchasing, and supposing p_{1m} is the unique lowest price as far as they know, the demand for the seller's product sold on sales channel m is 1 if $p_{1m} \leq r - \tau$, $(r -\ p_{1m})/\tau$ if $r - \tau < p_{1m} \leq r$ and 0 if $p_{1m} > r$.

According to Table 4.1, the share of the consumers who are aware of the product sold only on sales channel m when they make a decision about purchasing is $(1 - \alpha)\lambda_m(1 - \lambda_n)$. The share of the consumers who are aware of the product sold on both sales channels is given by $\alpha(\lambda_m + \lambda_n) + (1 - 2\alpha)\lambda_m\lambda_n$.

If $p_{1m} < p_{1n}$ holds, all of the consumers who are aware of the product sold on both sales channels and all of the consumers who only are aware of the product sold on sales channel m will purchase the product on sales channel m as long as the utility for the consumer is positive or equal to zero, which yields that the total share is equal to $\lambda_m + \alpha\lambda_n - \alpha\lambda_m\lambda_n$. Hence, the demand in this case can be obtained through multiplying $\lambda_m + \alpha\lambda_n - \alpha\lambda_m\lambda_n$ by the demand for the seller's product sold on sales channel m suppose all the consumers only have the information about the product sold on sales channel m.

If $p_{1m} > p_{1n}$ holds, only the consumers who only are aware of the product sold on sales channel m will purchase the product on the sales channel m as long as the utility for the consumer is positive or equal to zero. Hence, the demand in this case can be obtained through multiplying $(1 - \alpha)\lambda_m(1 - \lambda_n)$ by the demand for the seller's product sold on sales channel m suppose all the consumers only have the information about the product sold on sales channel m.

If $p_{1m} = p_{1n}$ holds, half of the consumers who are aware of the product sold on both sales channels and all of the consumers who only are aware of the product sold on sales channel m will purchase the product on the sales channel m as long as the utility for the consumer is positive or equal to zero, meaning that the total share is equal to $(1-0.5\alpha)\lambda_m + 0.5\alpha\lambda_n - 0.5\lambda_m\lambda_n$. Hence, the demand in this case can be obtained by multiplying $(1-0.5\alpha)\lambda_m + 0.5\alpha\lambda_n - 0.5\lambda_m\lambda_n$ by the demand for the seller's product sold on sales channel m suppose all the consumers only have the information about the product sold on sales channel m.■

4.7.5 Proof of Lemma 4.8

Lemma 4.7 implies that as long as $p_{1A} \le r$ and $p_{1D} \le r$ hold the expected profit for the seller can be written as follows:

$i)\ p_{1A} \le r-\tau, p_{1D} \le r-\tau$

$$\Pi_1 = \begin{Bmatrix} vp_1 - \xi f_A \ if\ p_{1A} = p_{1D} \\ (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1 p_{1D} \ if\ p_{1A} < p_{1D} \\ (1-\alpha)\lambda_A(1-\lambda_1)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D} \ if\ p_{1A} > p_{1D} \end{Bmatrix}.$$

$ii)\ p_{1A} \le r-\tau < p_{1D} \le r$

$$\Pi_1 = (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1 \left(\frac{r-p_{1D}}{\tau}\right) p_{1D}.$$

$iii)\ p_{1D} \le r-\tau < p_{1A} \le r$

$$\Pi_1 = (1-\alpha)\lambda_A(1-\lambda_1)\left(\frac{r-p_{1A}}{\tau}\right)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}$$

$iv)\ r-\tau < p_{1A} \le r, r-\tau < p_{1D} \le r$

$$\Pi_1 = \begin{Bmatrix} \left(\frac{r-p_1}{\tau}\right)(vp_1 - \xi f_A) \ if\ p_{1A} = p_{1D} \\ (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)\left(\frac{r-p_{1A}}{\tau}\right)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1\left(\frac{r-p_{1D}}{\tau}\right)p_{1D} \ if\ p_{1A} < p_{1D} \\ (1-\alpha)\lambda_A(1-\lambda_1)\left(\frac{r-p_{1A}}{\tau}\right)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)\left(\frac{r-p_{1D}}{\tau}\right)p_{1D} \ if\ p_{1A} > p_{1D} \end{Bmatrix}.$$

Lemma 4.8 can be obtained by solving this profit-maximisation problem. The proof is divided into two steps. In *Step 1* we find the optimal prices for the seller in each of the four cases when $p_{1A} \le r$ and $p_{1D} \le r$ hold: (i) both p_{1A} and p_{1D} are full-coverage price, (ii) only p_{1A} is full-coverage price, (iii) only p_{1D} is full-coverage price, and (iv) both p_{1A} and p_{1D} are non-full-coverage price. In *Step 2*, we find the optimal price for the seller when $p_{1A} > r$ or $p_{1D} > r$ holds and compare with the optimal profits when $p_{1A} \leq r$ and $p_{1D} \leq r$ hold.

Step 1: Optimal prices when $p_{1A} \leq r$ and $p_{1D} \leq r$ hold

In this step, we find the optimal prices for the seller in each of the four cases, which is divided based on whether the prices are full-coverage price (i.e., weakly lower than $r - \tau$) or not. *Step 1* is divided into the four substeps, each covering one of the four cases.

Substep 1.1: Optimal prices if $p_{1A} = p_{1D} \leq r - \tau$

This case can be divided into three sub-cases depending on the relationship between p_{1A} and p_{1D}.

$a)\ p_{1A} = p_{1D} = p_1 \leq r - \tau$

In this case, the expected profit is $\Pi_1^{p_{1A}=p_{1D}\leq r-\tau} = \nu p_1 - \xi f_A$.

Since this profit is monotonically increasing on p_1, which is subject to $p_1 \leq r - \tau$, this profit is maximised when setting $p_1 = r - \tau$. The maximum expected profit for the seller can be written as

$$\Pi_1^{*\,p_{1A}=p_{1D}\leq r-\tau} = \nu(r - \tau) - \xi f_A.$$

$b)\ p_{1A} < p_{1D} \leq r - \tau$

In this case, the expected profit is

$$\Pi_1^{p_{1A}<p_{1D}\leq r-\tau} = (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + (1 - \alpha)(1 - \lambda_A)\lambda_1 p_{1D}.$$

Since this profit is monotonically increasing on p_{1A} and p_{1D}, this profit is maximised when setting $(p_{1A}, p_{1D}) = (r - \tau - \varepsilon, r - \tau)$. The maximum expected profit for the seller can be written as

$$\Pi_1^{*\,p_{1A}<p_{1D}\leq r-\tau} = (\lambda_A + \lambda_1 - \lambda_A\lambda_1)(r - \tau) - (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(f_A + \varepsilon).$$

$c)\ p_{1D} < p_{1A} \leq r - \tau$

In this case, the expected profit is

$$\Pi_1^{p_{1D}<p_{1A}\leq r-\tau} = (1 - \alpha)\lambda_A(1 - \lambda_1)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}.$$

Since this profit is monotonically increasing on p_{1A} and p_{1D}, this profit is maximised when setting $(p_{1A}, p_{1D}) = (r - \tau, r - \tau - \varepsilon)$. The maximum expected profit for the seller can be written as

$\Pi_1^{*\,p_{1D}<p_{1A}\leq r-\tau} = (\lambda_A + \lambda_1 - \lambda_A\lambda_1)(r-\tau) - (1-\alpha)\lambda_A(1-\lambda_1)f_A - (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)\varepsilon.$

The results of the three cases imply that $\Pi_1^{*\,p_{1D}<p_{1A}\leq r-\tau} > \Pi_1^{*\,p_{1A}=p_{1D}\leq r-\tau} > \Pi_1^{*\,p_{1A}<p_{1D}\leq r-\tau}$, since

$\Pi_1^{*\,p_{1D}<p_{1A}\leq r-\tau} - \Pi_1^{*\,p_{1D}=p_{1A}\leq r-\tau} = 0.5[\alpha(\lambda_A+\lambda_1) + (1-2\alpha)\lambda_A\lambda_1]f_A - (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)\varepsilon > 0$ and

$\Pi_1^{*\,p_{1D}=p_{1A}\leq r-\tau} - \Pi_1^{*\,p_{1A}<p_{1D}\leq r-\tau} = 0.5[\alpha(\lambda_A+\lambda_1) + (1-2\alpha)\lambda_A\lambda_1]f_A + (1-\alpha)\lambda_A(1-\lambda_1)\varepsilon > 0$ hold.

Hence, if the seller decides to set prices such that both prices are full-coverage price, this profit is maximised when setting $(p_{1A}, p_{1D}) = (r-\tau, r-\tau-\varepsilon)$. Note that if $f_A \geq r-\tau$, the profit from the sales through the platform cannot be positive. In this situation, setting $(p_{1A}, p_{1D}) = (r-\tau, r-\tau-\varepsilon)$ will not be chosen in the equilibrium, because setting the platform price high enough to guarantee that no consumers purchase on the platform is more profitable.

Substep 1.2: Optimal prices if $p_{1A} \leq r-\tau < p_{1D} \leq r$

In this case, the expected profit is

$\Pi_1^{p_{1A}\leq r-\tau<p_{1D}} = (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1\left(\frac{r-p_{1D}}{\tau}\right)p_{1D}.$

$(\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A)$ is monotonically increasing on p_{1A}, and $(1-\alpha)(1-\lambda_A)\lambda_1\left(\frac{r-p_{1D}}{\tau}\right)p_{1D}$ is a concave quadratic function on p_{1D}. However, since it is assumed that $r \geq 2\tau$, $\left(\frac{r-p_{1D}}{\tau}\right)p_{1D}$ is maximised when setting p_{1D} at $r-\tau+\varepsilon$ if $p_{1A} \leq r-\tau$ and $p_{1D} > r-\tau$ must hold. Hence, the set of (p_{1A}, p_{1D}) that maximises $\Pi_1^{p_{1A}\leq r-\tau<p_{1D}}$ is $(r-\tau, r-\tau+\varepsilon)$.

However, $(p_{1A}, p_{1D}) = (r-\tau, r-\tau+\varepsilon)$ will not be chosen by the seller in the equilibrium because setting $(p_{1A}, p_{1D}) = (r-\tau, r-\tau-\varepsilon)$ is more profitable.

Substep 1.3: Optimal prices if $p_{1D} \leq r-\tau < p_{1A} \leq r$

In this case, the expected profit is

$\Pi_1^{p_{1A}\leq r-\tau<p_{1D}} = (1-\alpha)\lambda_A(1-\lambda_1)\left(\frac{r-p_{1A}}{\tau}\right)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}.$

Note that $(\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}$ is monotonically increasing on p_{1D}, and $\left(\frac{r-p_{1A}}{\tau}\right)(\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A)$ is a concave quadratic function on p_{1A}. Since

$p_{1D} \leq r - \tau$ and $r - \tau < p_{1A} \leq r$ must hold, the set of (p_{1A}, p_{1D}) that maximises $\Pi_1^{p_{1A} \leq r-\tau < p_{1D}}$ is $(\max[(r + f_A)/2, r - \tau + \varepsilon], r - \tau)$.

Note that $\max[(r + f_A)/2, r - \tau + \varepsilon]$ is equal to $(r + f_A)/2$ if $f_A > r - 2\tau$ and $r - \tau + \varepsilon$ if $f_A \leq r - 2\tau$. However, if $f_A \leq r - 2\tau$ holds, $(p_{1A}, p_{1D}) = (r - \tau + \varepsilon, r - \tau)$ will not be chosen by the seller because setting $(p_{1A}, p_{1D}) = (r - \tau, r - \tau - \varepsilon)$ is more profitable.

Moreover, even if $f_A > r - 2\tau$ holds, the profit from the sales through the platform cannot be positive when $f_A > r$ holds. In this situation, setting $(p_{1A}, p_{1D}) = ((r + f_A)/2, r - \tau)$ will not be chosen in the equilibrium, because setting the platform price high enough to guarantee that no consumers purchase on the platform is more profitable.

Substep 1.4: Optimal prices if $r - \tau < p_{1A} \leq r, r - \tau < p_{1D} \leq r$

This case can be divided into three sub-cases depending on the relationship between p_{1A} and p_{1D}: $a)\ r - \tau < p_{1A} = p_{1D} = p_1 \leq r$, $b) r - \tau < p_{1A} < p_{1D} \leq r$ and $c)\ r - \tau < p_{1D} < p_{1A} \leq r$.

The profits of the first three cases is

$$\Pi_1^{p_{1A} = p_{1D} > r-\tau} = \left(\frac{r - p_1}{\tau}\right)(\nu p_1 - \xi f_A).$$

$$\Pi_1^{p_{1D} > p_{1A} > r-\tau} = (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)\left(\frac{r - p_{1A}}{\tau}\right)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1\left(\frac{r - p_{1D}}{\tau}\right)p_{1D}.$$

$$\Pi_1^{p_{1A} > p_{1D} > r-\tau} = (1-\alpha)\lambda_A(1-\lambda_1)\left(\frac{r - p_{1A}}{\tau}\right)(p_{1A} - f_A) + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)\left(\frac{r - p_{1D}}{\tau}\right)p_{1D}.$$

Note that those profits depend on $\left(\frac{r-p_{1A}}{\tau}\right)(p_{1A} - f_A)$ and $\left(\frac{r-p_{1D}}{\tau}\right)p_{1D}$, and they are concave quadratic functions on p_{1A} and p_{1D}.

As discussed in *Substep* 1.3, $\left(\frac{r-p_{1A}}{\tau}\right)(p_{1A} - f_A)$ is maximised by setting p_{1A} at $\frac{r+f_A}{2}$ if $f_A > r - 2\tau$ and setting p_{1A} at $r - \tau + \varepsilon$ if $f_A \leq r - 2\tau$. Moreover, as discussed in *Substep* 1.2, $\left(\frac{r-p_{1D}}{\tau}\right)p_{1D}$ is maximised by setting p_{1D} at $r - \tau + \varepsilon$.

Those imply that if $f_A > r - 2\tau$ hold, while $\left(\frac{r-p_{1A}}{\tau}\right)(p_{1A} - f_A)$ is maximised by setting p_{1A} at $\frac{r+f_A}{2}$ and $\left(\frac{r-p_{1D}}{\tau}\right)$ p_{1D} is maximised by setting p_{1D} at $r - \tau + \varepsilon$.

However, such pricing will not be chosen by the seller in the equilibrium because setting $(p_{1A}, p_{1D}) = \left(\frac{r+f_A}{2}, r-\tau\right)$ is more profitable in this situation.

Similarly, $f_A \leq r - 2\tau$ holds, the profit is maximised when setting the platform price and the direct sales price slightly higher than $r - \tau$. However, such pricing will not be chosen by the seller in the equilibrium because setting $(p_{1A}, p_{1D}) = (r-\tau, r-\tau-\varepsilon)$ is more profitable.

In summary, in this case, the set of optimal prices for the seller satisfying $p_{1A} \leq r$ and $p_{1D} \leq r$ is $(p_{1A}, p_{1D}) = (r-\tau, r-\tau-\varepsilon)$ if $f_A \leq r - 2\tau$ holds.

If $r - 2\tau < f_A \leq r$ holds, the optimal set of the prices is $(p_{1A}, p_{1D}) = \left(\frac{r+f_A}{2}, r-\tau\right)$ or $(r-\tau, r-\tau-\varepsilon)$ In this situation, the seller prefers to set $(p_{1A}, p_{1D}) = \left(\frac{r+f_A}{2}, r-\tau\right)$, since $(1-\alpha)\lambda_A(1-\lambda_1)\left[\frac{(r-f_A)^2}{4\tau} - (r-\tau-f_A)\right] + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)\varepsilon = (1-\alpha)\lambda_A(1-\lambda_1)\frac{(r-f_A-2\tau)^2}{4\tau} + (\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)\varepsilon > 0$.

When $f_A > r$, any prices satisfying $p_{1A} \leq r$ and $p_{1D} \leq r$ will not be chosen in the equilibrium.

Step 2: Optimal prices when $p_{1A} > r$ or $p_{1D} > r$

If the seller sets $p_{1D} > r$, the revenue from the direct sales price become zero, which follows that the seller does not have an incentive to set such prices because the cost from direct sales is zero.

If the seller sets $p_{1A} > r$, the sales on the platform is zero. The expected profit is equal to $(\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)p_{1D}$, which is maximised at $(\alpha\lambda_A + \lambda_1 - \alpha\lambda_A\lambda_1)(r-\tau)$ when setting p_{1D} at $r-\tau$ since $r \geq 2\tau$ holds. Hence, the set of optimal prices satisfying $p_{1A} > r$ or $p_{1D} > r$ is $(p_{1A}, p_{1D}) = (r+b, r-\tau)$, where b is any positive amount.

Then, compare this pricing with the optimal prices satisfying $p_{1A} \leq r$ *and* $p_{1D} \leq r$. If $f_A \leq r$ holds, $(p_{1A}, p_{1D}) = \left(\frac{r+f_A}{2}, r-\tau\right)$ or $(r-\tau, r-\tau-\varepsilon)$ is chosen in the equilibrium, because by setting $(p_{1A}, p_{1D}) = (r+b, r-\tau)$ the seller will lose the sales thorough the platform.

If $f_A > r$, $(p_{1A}, p_{1D}) = (r+b, r-\tau)$ is chosen in the equilibrium, because any prices satisfying $p_{1A} \leq r$ and $p_{1D} \leq r$.■

4.7.6 Proof of Lemma 4.9

If the platform decides to set its commission such that $f_A \leq r - 2\tau$, Seller 1's expected profit is maximised when $f_A = r - 2\tau$. In this situation, the expected profit is given by $(1-\alpha)\lambda_A(1-\lambda_1)(r-2\tau)$.

If the platform to set its commission such that $r - 2\tau < f_A \leq r$, the expected profit is given by $(1-\alpha)\lambda_A(1-\lambda_1)\left(\frac{r-f_A}{2\tau}\right)f_A$, which is a concave quadratic function that is maximised when $f_A = \frac{r}{2}$, since the FOC is $(1-\alpha)\lambda_A(1-\lambda_1)\frac{r-2f_A}{2\tau} = 0$. The profit is $\frac{(1-\alpha)\lambda_A(1-\lambda_1)r^2}{8\tau}$.

Note that

$$\frac{(1-\alpha)\lambda_A(1-\lambda_1)r^2}{8\tau} - (1-\alpha)\lambda_A(1-\lambda_1)(r-2\tau) = \frac{(1-\alpha)\lambda_A(1-\lambda_1)}{8\tau}[r^2 - 8\tau r + 16\tau^2] = \frac{\lambda_A}{8\tau}(r-4\tau)^2 \geq 0.$$

It follows that the expected profit when setting $\frac{r}{2}$ is always equal to or larger than the profit when setting $r - 2\tau$. However, that commission is not chosen when $\frac{r}{2} \leq r - 2\tau$ holds, because the value is below the assumed range ($r - 2\tau < f_A \leq r$). The condition can be modified to $r \geq 4\tau$. If this condition is satisfied, Seller 1's profit increases as the price decreases, which implies that setting $f_A = r - 2\tau$ is more profitable.

With regard to the optimal amount of investment, the total expected profit for the seller can be rewritten as

$$\Pi_A = \begin{cases} (1-\alpha)(1-\lambda_1)g(I_A + \underline{I_A})(r-2\tau) - I_A \ if \ r \geq 4\tau \\ (1-\alpha)(1-\lambda_1)g(I_A + \underline{I_A})\frac{r^2}{8\tau} - I_A \ if \ r < 4\tau \end{cases}.$$

Solving this profit-maximisation problem gives Lemma 4.9. ■

4.7.7 Proof of Lemma 4.11

In this case, the seller has two options. The first option is to accept the price parity clause imposed by the platform and set the prices such that $p_{1A} \leq p_{1D}$. If the seller takes this option and sets the prices such that $p_{1A} = p_{1D} = p_1$, the seller's expected profit can be written as

$$\Pi_1^{p_{1A}=p_{1D}} = \begin{cases} vp_1 - \xi f_A \ if \ p_1 \leq r - \tau \\ \left(\frac{r-p_1}{\tau}\right)(vp_1 - \xi f_A) \ if \ r - \tau < p_1 \leq r \\ 0 \ if \ p_1 > r \end{cases}.$$

If the seller and sets the prices such that $p_{1A} < p_{1D}$, the seller's expected profit can be written as

$$\Pi_1{}^{p_{1A}<p_{1D}} = \begin{Bmatrix} (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + (1-\alpha)(1-\lambda_A)\lambda_1 p_{1D} \; if \; p_{1A} < p_{1D} \le r-\tau \\ (\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + \left(\frac{r-p_{1D}}{\tau}\right)(1-\alpha)(1-\lambda_A)\lambda_1 p_{1D} \; if \; p_{1A} \le r-\tau < p_{1D} \le r \\ \left(\frac{r-p_{1A}}{\tau}\right)(\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + \left(\frac{r-p_{1D}}{\tau}\right)(1-\alpha)(1-\lambda_A)\lambda_1 p_{1D} \; if \; r-\tau < p_{1A} < p_{1D} \le r \\ \left(\frac{r-p_{1A}}{\tau}\right)(\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) \; if \; r-\tau < p_{1A} \le r < p_{1D} \\ 0 \; if \; r < p_{1A} < p_{1D} \end{Bmatrix}.$$

The second option for the seller is to refuse the offer by the platform and to concentrate on its direct sales. If the seller takes this option, the seller's expected profit is given by

$$\Pi_1{}^{REFUSE} = \begin{Bmatrix} \lambda_1 p_{1D} \; if \; p_{1D} \le r-\tau \\ \lambda_1 \left(\frac{r-p_{1D}}{\tau}\right) p_{1D} \; if \; r-\tau < p_{1D} \le r \\ 0 \; if \; p_{1D} > r \end{Bmatrix}.$$

Lemma 4.11 can be obtained by solving this profit-maximisation problem. The proof is divided into two steps. In *Step 1,* we find the optimal prices when the seller accepts the price parity clause. *Step 1* is divided into two parts. In the first part, we find the optimal prices when the seller sets the prices such that $p_{1A} = p_{1D}$. Then, we show that the prices such that $p_{1A} < p_{1D}$ are not optimal for the seller. In *Step 2,* we find the optimal prices when the seller refuses the price parity clause. In *Step 3,* we compare the profits in each case to find the equilibrium price.

Step 1: Optimal seller's prices when the seller accepts the price parity clause

With regard to the profit when the seller accepts the offer by the platform and the seller sets the prices such that $p_{1A} = p_{1D} = p_1$, if the seller decides to set the price such that $p_1 \le r - \tau$, the profit is given by $\nu p_1 - \xi f_A$. Hence, the price that maximises the profit is $p_1 = r - \tau$, and the maximised expected profit is $\nu(r-\tau) - \xi f_A$.

If the seller decides to set the price such that $p_1 > r - \tau$, the profit is given by $[(r - p_1)/\tau](\nu p_1 - \xi f_A)$. Hence, the price that maximises the profit is $p_1 = r/2 + \xi f/2\nu$, since in this situation the expected profit is a quadric concave function and the FOC is $(r\nu - 2\nu p_1 + \xi f_A)/\tau = 0$. The maximised expected profit is $\nu(r - \xi f_A/\nu)^2/4\tau$. However, that price is not chosen when $r/2 + \xi f/2\nu \le r - \tau$ holds, because the price is below the assumed range. The condition can be modified to $r \ge \xi f_A/\nu + 2\tau$. If $r < \xi f_A/\nu + 2\tau$ holds, setting $p_1 = r/2 + \xi f/2\nu$ is more profitable than setting $p_1 = r - \tau$, since $\nu(r - \xi f_A/\nu)^2/4\tau - \nu(r-\tau) - \xi f_A = \nu(r - \xi f_A/\nu - 2\tau)^2/4\tau > 0$.

Then, consider the case where when the seller accepts the offer and the seller decides to set the prices such that $p_{1A} < p_{1D}$. If $p_{1A} < p_{1D} \le r - \tau$, the price that maximises the profit is $(p_{1A,}p_{1D}) = (r - \tau - \varepsilon, r - \tau)$, where ε is a small non-negative amount.

However, this pricing is dominated by $p_{1A} = p_{1D} = r - \tau$, since in this situation the commission that the seller needs to pay is smaller.

If the seller decides to set the prices such that $p_{1A} \leq r - \tau < p_{1D}$ holds, the seller's profit is given by

$$(\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + \left(\frac{r - p_{1D}}{\tau}\right)(1 - \alpha)(1 - \lambda_A)\lambda_1 p_{1D}.$$

Since it is assumed $r \geq 2\tau$, that the second term of this profit increases as p_{1D} approaches to $r - \tau$. Hence, the prices that maximises the profit is $(p_{1A,}p_{1D}) = (r - \tau, r - \tau + \varepsilon)$ when setting such prices such that $p_{1A} \leq r - \tau < p_{1D}$ holds. However, $(p_{1A,}p_{1D}) = (r - \tau, r - \tau + \varepsilon)$ cannot be an equilibrium because this is dominated by $(r - \tau, r - \tau)$.

If the seller decides to set the prices such that $r - \tau < p_{1A} < p_{1D}$ holds, the seller's profit is given by

$$\left(\frac{r - p_{1A}}{\tau}\right)(\lambda_A + \alpha\lambda_1 - \alpha\lambda_A\lambda_1)(p_{1A} - f_A) + \left(\frac{r - p_{1D}}{\tau}\right)(1 - \alpha)(1 - \lambda_A)\lambda_1 p_{1D}.$$

The first term and the second term of this profit are quadric concave functions on p_{1A} and p_{1D} respectively, which are maximised when the direct sales price is $(r + f_A)/2$ and $r/2$, respectively. It follows that if the seller sets the direct sales price at $\hat{p}_{1D} > r - \tau$, p_{1A} that maximises the profit in this situation is $p_{1A} = \hat{p}_{1D} - \varepsilon$ as long as $\hat{p}_{1D} \leq r/2 + \xi f/2\nu$ holds. However, these pricings are dominated by $(p_{1A}, p_{1D}) = (\hat{p}_{1D}, \hat{p}_{1D})$, since in this situation the commission that the seller needs to pay is smaller. In addition, the prices such that $\hat{p}_{1D} > r/2 + \xi f/2\nu$ will not be chosen in the equilibrium since in this situation, the profit increases as $\hat{p}_{1D}$ becomes smaller.

Hence, the set of prices that satisfies $p_{1A} < p_{1D}$ will not be chosen by the seller in the equilibrium.

Step 2: Optimal seller's prices when it refuses the price parity clause

Since it is assumed that r is high enough to guarantee that the seller does not have an incentive to set non-full-coverage price for its direct sales in the equilibrium ($r \geq 2\tau$). ($r \geq 2\tau$), the profit when the seller refuses the offer and sells only through direct sales is maximised by setting p_{1D} at $r - \tau$.

Step 3: Comparing the two cases

According to the results in *Step 1* and *Step 2,* the prices that maximise the seller's expected profit in the two cases (i.e., accepting the price parity clause and refusing the clause) are summarised Table A4.1.

Table A4.1 (p_{1A}, p_{1D}) that maximises the seller's expected profit

Case	$2\tau \leq r < \frac{\xi f_A}{v} + 2\tau$	$r \geq \frac{\xi f_A}{v} + 2\tau$
Accept	$(\frac{r}{2} + \frac{\xi f_A}{2v}, \frac{r}{2} + \frac{\xi f_A}{2v})$	$(r-\tau, r-\tau)$
Refuse	$(+\infty, r-\tau)$	$(+\infty, r-\tau)$

Table A4.2 shows that if $r \geq \xi f_A/v + 2\tau$, the condition that the seller accept the condition if $r \geq \xi f_A/v + 2\tau$ is

$$v(r-\tau) - \xi f_A - \lambda_1(r-\tau) \geq 0.$$

This inequality can be modified to

(A4.1) $f_A \leq \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi}$.

In addition, if $2\tau \leq r < \xi f_A/v + 2\tau$, the condition that the seller accepts the price parity clause is

$$\frac{v\left(r - \frac{\xi f_A}{v}\right)^2}{4\tau} - \lambda_1(r-\tau) \geq 0 \leftrightarrow \left(r - \frac{\xi f_A}{v}\right)^2 \geq 4\tau\lambda_1(r-\tau).$$

This condition can be modified to

(A4.2) $f_A \leq \frac{v}{\xi}\left\{r - 2\left[\frac{\tau\lambda_1(r-\tau)}{v}\right]^{0.5}\right\}$. ■

4.7.8 Proof of Lemma 4.12

The proof of Lemma 4.12 is divided into three steps. In *Step 1*, we find the equilibrium investment level and commission if the platform introduces a price parity clause. In *Step 2*, we show that the platform's profit increases if the platform introduces a price parity clause and, hence, the platform has an incentive to introduce it when it is allowed.

In the case covered in *Step 1*, the platform can set both full-coverage commission and non-full-coverage commission. Hence, we first find the optimal investment level and commission when the platform decides to set full-coverage commission and non-full-coverage commission, respectively (*Substep 1.1* and *1.2*). Compare the optimal to find the equilibrium and analyse whether the equilibrium investment level is the level in the case where the price parity clause is not allowed (*Substep 1.3*).

Step 1: Equilibrium investment level and commission if the platform offering a price parity clause

If the platform offers a price parity clause, the platform has two options regarding its commission. The first option is setting a full-coverage commission such that $r \geq \xi f_A/\nu + 2\tau$. The second option is setting a non-full-coverage commission.

Substep 1.1: Optimal full-coverage commission

If the platform decides to set a full-coverage commission such that $r \geq \xi f_A/\nu + 2\tau \leftrightarrow f_A \leq \nu(r-2\tau)/\xi$, the platform's profit-maximisation problem given λ_A, determined by its investment, is

$\max_{f_A} \xi f_A$ subject to $f_A \leq \min\left[\frac{\nu}{\xi}(r-2\tau), \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi}\right]$.

Since this revenue is monotonically increasing on f_A, the commission that maximises the platform's revenue is $f_A = \min\left[\frac{\nu}{\xi}(r-2\tau), \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi}\right]$. In this situation, the maximised expected revenue is $\min[\nu(r-2\tau), \lambda_A(1-\lambda_1)(r-\tau)]$.

The condition that $f_A \leq \nu(r-2\tau)/\xi$ is binding is

$\frac{\nu}{\xi}(r-2\tau) \leq \frac{\lambda_A(1-\lambda_1)(r-\tau)}{\xi}$.

This condition can be modified to

(A4.4) $\lambda_A \geq \frac{\lambda_1(r-2\tau)}{(1-\lambda_1)\tau} \equiv \overline{\lambda_A}$.

If Inequality (A4.4) holds, the commission that maximises the platform's revenue given λ_A is $f_A = \nu(r-2\tau)/\xi$. In this situation, the total expected profit for the platform taking the investment into account is given by

$\Pi_A = \nu(r-2\tau) - I_A = (r-2\tau)\left[(1-\lambda_1)g\left(I_A + \underline{I_A}\right) + \lambda_1\right] - I_A$.

Let

$\widehat{I_A} \equiv \arg\max_{I_A} (r-2\tau)\left[(1-\lambda_1)g\left(I_A + \underline{I_A}\right) + \lambda_1\right] - I_A$,

$\widehat{\lambda_A} \equiv g\left(\widehat{I_A} + \underline{I_A}\right)$ and

$\widehat{f_A} \equiv \frac{\nu(\widehat{\lambda_A})}{\xi(\widehat{\lambda_A})}(r-2\tau) = \frac{[(1-\lambda_1)\widehat{\lambda_A}+\lambda_1](r-2\tau)}{(1-0.5\alpha-0.5\lambda_1)\widehat{\lambda_A}+0.5\alpha\lambda_1}$.

$(\widehat{I_A}, \widehat{f_A})$ is the optimal commission and the investment when the platform decides to set a commission low enough to guarantee the full-coverage price by the seller if $\widehat{\lambda_A} \geq \overline{\lambda_A}$. If $\widehat{\lambda_A} < \overline{\lambda_A}$, $(\widehat{I_A}, \widehat{f_A})$ is no longer optimal because in this situation the optimal commission is bounded by Inequality (A4.1) rather than $f_A \leq \nu(r-2\tau)/\xi$.

If $\lambda_A < \overline{\lambda_A}$ holds, the commission that maximises the platform's revenue given λ_A is $f_A = \lambda_A(1-\lambda_1)(r-\tau)/\xi$. In this situation, the total expected profit for the platform taking the investment into account is given by

$$\Pi_A = \lambda_A(1-\lambda_1)(r-\tau) - I_A = (r-\tau)(1-\lambda_1)g\left(I_A + \underline{I_A}\right) - I_A.$$

Let

$$\widetilde{I_A} \equiv \arg\max_{I_A} \ (r-\tau)\left[(1-\lambda_1)g\left(I_A + \underline{I_A}\right) + \lambda_1\right] - I_A \,,$$

$$\widetilde{\lambda_A} \equiv g\left(\widetilde{I_A} + \underline{I_A}\right) \text{ and}$$

$$\widetilde{f_A} \equiv \frac{\widetilde{\lambda_A}(1-\lambda_1)(r-\tau)}{\xi(\widetilde{\lambda_A})} = \frac{(1-\lambda_1)\widetilde{\lambda_A}(r-\tau)}{(1-0.5\alpha-0.5\lambda_1)\widetilde{\lambda_A}+0.5\alpha\lambda_1}.$$

$(\widetilde{I_A}, \widetilde{f_A})$ is the optimal commission and the investment when the platform decides to set a commission low enough to guarantee the full-coverage price by the seller if $\widetilde{\lambda_A} \le \overline{\lambda_A}$. If $\widetilde{\lambda_A} > \overline{\lambda_A}$, $(\widetilde{I_A}, \widetilde{f_A})$ is no longer optimal because in this situation because in this situation the optimal commission is bounded by $f_A \le \nu(r-2\tau)/\xi$ rather than Inequality (A4.1).

Note that $\widehat{I_A} < \widetilde{I_A}$ and $\widehat{\lambda_A} < \widetilde{\lambda_A}$ hold because

$$g'\left(\widehat{I_A} + \underline{I_A}\right) = \frac{1}{(r-2\tau)(1-\lambda_1)} > \frac{1}{(r-\tau)(1-\lambda_1)} = g'\left(\widetilde{I_A} + \underline{I_A}\right).$$

It follows that if $\widehat{\lambda_A} < \overline{\lambda_A} < \widetilde{\lambda_A}$ holds, neither $(\widehat{I_A}, \widehat{f_A})$ nor $(\widetilde{I_A}, \widetilde{f_A})$ is the optimal pair of commission and investment. In this situation, the platform has an incentive to increase the investment from $\widehat{I_A}$ and an incentive to decrease the investment from $\widetilde{I_A}$. The optimal investment is the investment such that the two constraints on the commission, $f_A \le \nu(r-2\tau)/\xi$ and Inequality (A4.1), are equivalent. In this situation, the optimal investment $\overline{I_A}$ satisfies $g\left(\overline{I_A} + \underline{I_A}\right) = \overline{\lambda_A}$, and the optimal commission is

$$\overline{f_A} = \frac{\nu(\overline{\lambda_A})}{\xi(\overline{\lambda_A})}(r-2\tau) = \frac{\overline{\lambda_A}(1-\lambda_1)(r-\tau)}{\xi(\overline{\lambda_A})} = \frac{\lambda_1(r-\tau)(r-2\tau)}{[(1-0.5\alpha-0.5\lambda_1)r-(2-1.5\alpha-\lambda_1+0.5\alpha\lambda_1)\tau]\tau}.$$

In summary, when the platform decides to set a full-coverage commission, the optimal investment is either $\widehat{I_A}$, $\widetilde{I_A}$ or $\overline{I_A}$.

Substep 1.2: Optimal non-full-coverage commission

If the platform decides to set a non-full coverage commission such that $f_A > \nu(r-2\tau)/\xi$, the platform's profit-maximisation problem given λ_A determined by its investment is

$$\max_{f_A} \xi\left(\frac{r-\frac{\xi f_A}{\nu}}{2\tau}\right) f_A \text{ subject to } \frac{\nu}{\xi}(r-2\tau) < f_A \leq \frac{\nu}{\xi}\left\{r-2\left[\frac{\tau\lambda_1(r-\tau)}{\nu}\right]^{0.5}\right\}.$$

It follows that if $(\nu/\xi)\{r-2\{[\tau\lambda_1(r-\tau)]/\nu\}^{0.5}\} \leq \nu(r-2\tau)/\xi$ holds, there does not exist f_A that satisfies both Inequality (A4.2) and $f_A > \nu(r-2\tau)/\xi$. The condition can be modified to $\lambda_A \leq \overline{\lambda_A}$. Hence, if $\lambda_A \leq \overline{\lambda_A}$ holds, the platform will not set a commission such that $r < \xi f_A/\nu + 2\tau$.

If $\lambda_A > \overline{\lambda_A}$ holds, because the platforms profit is a concave quadratic function on f_A, the commission that maximises the platform's revenue given λ_A is $f_A = \nu r/2\xi$. Note that $f_A = \nu r/2\xi$ satisfies Inequality (A4.2) because

$$\frac{\nu r}{2\xi} \leq \frac{\nu}{\xi}\left\{r-2\left[\frac{\tau\lambda_1(r-\tau)}{\nu}\right]^{0.5}\right\} \leftrightarrow \frac{r}{2} \geq 2\left[\frac{\tau\lambda_1(r-\tau)}{\nu}\right]^{0.5} \leftrightarrow r^2-\frac{4\tau\lambda_1}{\nu}r+\frac{4\tau^2\lambda_1}{\nu}$$

$$\leftrightarrow \left(r-\frac{2\tau\lambda_1}{\nu}\right)^2+(\nu-\lambda_1)\frac{4\tau^2\lambda_1}{\nu^2} > 0.$$

Moreover, $\nu r/2\xi > \nu(r-2\tau)/\xi$ must hold because the optimal commission must be high enough to let the seller set a non-full-coverage price. The condition can be modified to $r < 4\tau$. If $r < 4\tau$ holds, the commission that maximises the platform's revenue given λ_A is $f_A = \nu r/2\xi$. In this situation, the total expected profit for the platform taking the investment into account is given by

$$\Pi_A = \frac{\nu r^2}{8\tau} - I_A = \frac{r^2}{8\tau}\left[(1-\lambda_1)g\left(I_A+\underline{I_A}\right)+\lambda_1\right]-I_A.$$

Let

$$\acute{I_A} \equiv \arg\max_{I_A} \frac{r^2}{8\tau}\left[(1-\lambda_1)g\left(I_A+\underline{I_A}\right)+\lambda_1\right]-I_A,$$

$\acute{\lambda_A} \equiv g\left(\acute{I_A}+\underline{I_A}\right)$ and

$$\acute{f_A} \equiv \frac{\nu(\acute{\lambda_A})r}{2\xi(\acute{\lambda_A})} = \frac{[(1-\lambda_1)\acute{\lambda_A}+\lambda_1]r}{2(1-0.5\alpha-0.5\lambda_1)\acute{\lambda_A}+\alpha\lambda_1}.$$

$(\acute{I_A}, \acute{f_A})$ is the optimal commission and the investment when the platform decides to set a non-full-coverage commission if $\acute{\lambda_A} > \overline{\lambda_A}$. If $\acute{\lambda_A} \leq \overline{\lambda_A}$, $(\acute{I_A}, \acute{f_A})$ is no longer optimal because when the platform makes the investment of $\acute{\lambda_A}$, there exists no f_A that satisfies both Inequality (A4.2) and $f_A > \nu(r-2\tau)/\xi$. In this situation, the platform can set a non-full-coverage commission by making the investment more than the most efficient level, $\acute{I_A}$. If the platform takes this option, the total expected profit is maximised when it makes the investment just above $\overline{I_A}$. However, the platform never chooses this option since setting $(\overline{I_A}, \overline{f_A})$ is more profitable than setting $(\overline{I_A} + \varepsilon, \acute{f}_A)$.

Substep 1.3: Comparing the two types of the equilibria

This part compares the platform's profit when the platform decides to set the optimal full-coverage commission and the optimal non-full-coverage commission.

If $r \geq 4\tau$ holds, because the platform never sets the commission high enough to let the seller set a non-full-coverage price, the pair of the equilibrium commission and investment in the equilibrium is

$$({f_A}^*, {I_A}^*) = \begin{Bmatrix} (\widetilde{I_A}, \widetilde{f_A})\ if\ \widetilde{\lambda_A} \leq \overline{\lambda_A} \\ (\overline{I_A}, \overline{f_A})\ if\ \widehat{\lambda_A} < \overline{\lambda_A} < \widetilde{\lambda_A} \\ (\widehat{I_A}, \widehat{f_A})\ if\ \widehat{\lambda_A} \geq \overline{\lambda_A} \end{Bmatrix}.$$

Note that $g'\left(\widehat{I_A} + \underline{I_A}\right) = 1/(r - 2\tau)(1 - \lambda_1) \leq 1/(1 - \alpha)(r - 2\tau)(1 - \lambda_1)$ holds. Moreover, $\widehat{I_A} < \widetilde{I_A}$ and $\overline{I_A}$ become equilibrium if $\widehat{I_A} < \overline{I_A} < \widetilde{I_A}$. Hence, if $r \geq 4\tau$ holds, the equilibrium investment is always equal to or higher than the case where the price parity clause is not allowed.[37]

If $2\tau < r < 4\tau$ holds, the platform may set the commission high enough to let the seller set a non-full-coverage price if $\acute{\lambda}_A > \overline{\lambda_A}$ holds. In this situation, the platform prefers to set $(\acute{I}_A, \acute{f}_A)$ if the expected profit is larger than the expected profit when it sets the commission low enough to guarantee the full-coverage price. Suppose if $\acute{\lambda}_A > \overline{\lambda_A}$, the pair of the optimal commission and investment when the platform decides to set a commission low enough to guarantee the full-coverage price by the seller is $(\overline{I_A}, \overline{f_A})$ if $\widehat{\lambda_A} < \overline{\lambda_A} < \acute{\lambda}_A$ and $\left(\widehat{I_A}, \widehat{f_A}\right)$ if $\overline{\lambda_A} \leq \widehat{\lambda_A} < \acute{\lambda}_A$, because $\acute{I}_A < \widetilde{I_A}$ and $\acute{\lambda}_A < \widetilde{\lambda_A}$ hold if $2\tau < r < 4\tau$. These two inequlaities can be proved as follows:

$g'\left(\widetilde{I_A} + \underline{I_A}\right) - g'\left(\acute{I}_A + \underline{I_A}\right) = \frac{1}{(r-\tau)(1-\lambda_1)} - \frac{8\tau}{(1-\lambda_1)r^2} = \frac{r^2 - 8\tau r + 8\tau^2}{(r-\tau)(1-\lambda_1)r^2}$ and $r^2 - 8\tau r + 8\tau^2 < 0$ if $4\tau - 2\sqrt{2}\tau < r < 4\tau + 2\sqrt{2}\tau$ holds.

In this situation, the condition that the platform prefers to set $(\acute{I}_A, \acute{f}_A)$ when $\widehat{\lambda_A} < \overline{\lambda_A} < \acute{\lambda}_A$ holds can be written as

$$\acute{\Pi}_A = \frac{r^2}{8\tau}\left[(1 - \lambda_1) g\left(\acute{I}_A + \underline{I_A}\right) + \lambda_1\right] - \acute{I}_A > \frac{\lambda_1 (r-\tau)(r-2\tau)}{\tau} - \overline{I_A} = \overline{\Pi_A}.$$

The condition can be modified to

[37] If $\alpha > 0$, the equilibrium investment is strictly higher than in the case where the price parity clause is not allowed.

(A4.5) $\acute{I_A} - \overline{I_A} < \frac{r^2}{8\tau}\left[(1-\lambda_1)g\left(\acute{I_A} + \underline{I_A}\right) + \lambda_1\right] - \frac{\lambda_1(r-\tau)(r-2\tau)}{\tau}$.

The condition that the platform prefers to set $(\acute{I_A}, \acute{f_A})$ when $\overline{\lambda_A} \le \widehat{\lambda_A} < \acute{\lambda_A}$ holds can be written as

$$\acute{\Pi_A} = \frac{r^2}{8\tau}\left[(1-\lambda_1)g\left(\acute{I_A} + \underline{I_A}\right) + \lambda_1\right] - \acute{I_A} > (r-2\tau)\left[(1-\lambda_1)g\left(\widehat{I_A} + \underline{I_A}\right) + \lambda_1\right] - \widehat{I_A} = \widehat{\Pi_A}.$$

The condition can be modified to

(A4.6) $\acute{I_A} - \widehat{I_A} < \frac{r^2}{8\tau}\left[(1-\lambda_1)g\left(\acute{I_A} + \underline{I_A}\right) + \lambda_1\right] - (r-2\tau)\left[(1-\lambda_1)g\left(\widehat{I_A} + \underline{I_A}\right) + \lambda_1\right]$.

If $\acute{\lambda_A} \le \overline{\lambda_A}$ holds, the platform never sets the commission high enough to let the seller set a non-full-coverage price. In this situation, the pair of the equilibrium commission and investment in the equilibrium is $(\widetilde{I_A}, \widetilde{f_A})$ if $\acute{\lambda_A} < \widetilde{\lambda_A} \le \overline{\lambda_A}$ and $(\overline{I_A}, \overline{f_A})$ if $\acute{\lambda_A} \le \overline{\lambda_A} < \widetilde{\lambda_A}$.

Note that $\acute{I_A} > \widehat{I_A}$ and $\acute{\lambda_A} > \widehat{\lambda_A}$ hold if $2\tau < r < 4\tau$, since

$$g'\left(\acute{I_A} + \underline{I_A}\right) - g'\left(\widehat{I_A} + \underline{I_A}\right) = \frac{8\tau}{(1-\lambda_1)r^2} - \frac{1}{(r-2\tau)(1-\lambda_1)} = \frac{-(r-4\tau)^2}{(r-2\tau)(1-\lambda_1)r^2} < 0.$$

It follows that $(\widehat{I_A}, \widehat{f_A})$ cannot be an equilibrium if both $2\tau < r < 4\tau$ and $\acute{\lambda_A} \le \overline{\lambda_A}$ hold.

We next examine whether I_A and λ_A increase by allowing a price parity clause. Note that $g'\left(\acute{I_A} + \underline{I_A}\right) = 8\tau/(1-\lambda_1)r^2 \le 8\tau/(1-\alpha)(1-\lambda_1)r^2$. It follows that $\acute{I_A}$ is equal to or larger than the equilibrium investment in the case where the clause is not allowed. In addition, $\acute{I_A} < \widetilde{I_A}$ holds. Hence, suppose $2\tau < r < 4\tau$, the equilibrium investment is always equal to or greater than the case in which the price parity clause is not allowed if $\acute{\lambda_A} \le \overline{\lambda_A}$ holds, if $\widehat{\lambda_A} \le \overline{\lambda_A} < \acute{\lambda_A}$ holds and Inequality (A4.5) does not hold, or if both $\overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda_A}$ holds and Inequality (A4.6) does not hold. On the other hand, if both $\widehat{\lambda_A} \le \overline{\lambda_A} < \acute{\lambda_A}$ and Inequality (A4.5) hold or if both $\overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda_A}$ and Inequality (A4.6) hold, the equilibrium investment can be smaller than the case where the price parity clause is not allowed.

Step 2: Improvement of the platform's profit by the introduction of a price parity clause

Recall that $\overline{\Pi_A}$ is the platform's profit when the pair of the equilibrium investment and commission is $\left(\overline{I_A}, \overline{f_A}\right)$. If $r \ge 4\tau$, $\overline{\Pi_A}$ is strictly larger than the platforms profit in the case where the clause is not allowed, because while the equilibrium price on the platform does not change with the introduction of the clause, the introduction of the clause raises the equilibrium investment and the commission. When the equilibrium pair is $(\widetilde{I_A}, \widetilde{f_A})$ or $(\widehat{I_A}, \widehat{f_A})$, the option is the most profitable that ensures the profit is greater than $\overline{\Pi_A}$.

Similarly, if $r \le 4\tau$, $\acute{\Pi}_A$ is strictly larger than the platform's profit in the case where the clause is not allowed and when the equilibrium pair is other than $(\acute{I}_A, \acute{f}_A)$, the platform's profit is larger than $\acute{\Pi}_A$. This result implies that if the platform is allowed to introduce the clause, it always prefers to introduce it. ■

4.7.9 Proof of Proposition 4.6

With regard to consumer surplus (CS), if all consumers have the information only on the product sold on sales channel m and $p_{1m} \le r - \tau$ holds, the surplus of such consumers is

$$\int_0^1 (r - \tau x - p_{1m}) dx = r - 0.5\tau - p_{1m}.$$

If all consumers have the information only on the product sold on sales channel i and if $p_{1m} > r - \tau$ holds, the surplus of such consumers is

$$\int_0^{\frac{r - p_{1m}}{\tau}} (r - \tau x - p_{1m}) dx = \frac{(r - p_{1m})^2}{2\tau}.$$

Let CS^* and CS^{**} denote the consumer surplus in the equilibrium when the price parity clause is not allowed and the consumer surplus in the equilibrium when the clauses are allowed respectively. CS^* can be written as

$$CS^* = \begin{cases} 0.5(\lambda_A{}^* + \lambda_1 - \lambda_A{}^*\lambda_1)\tau + \lambda_1\varepsilon \; if \; r \ge 4\tau \\ \frac{(1-\alpha)(\lambda_A{}^* - \lambda_A{}^*\lambda_1)r^2}{32\tau} + 0.5(\alpha\lambda_A{}^{**} + \lambda_1 - \alpha\lambda_A{}^*\lambda_1)\tau \; if \; 2\tau \le r < 4\tau \\ \frac{(1-\alpha)(\lambda_A{}^* - \lambda_A{}^*\lambda_1)r^2}{32\tau} + \frac{(\alpha\lambda_A{}^* + \lambda_1 - \alpha\lambda_A{}^*\lambda_1)r^2}{8\tau} \; if \; r < 2\tau \end{cases}, \quad \text{where}$$

$\lambda_A{}^*$ is λ_A that is realised in equilibrium in the case where the price parity clause is not allowed.

Next, we compare CS^{**} with CS^*.

Where $r \ge 4\tau$,

$$CS^{**} = \begin{cases} 0.5(\widetilde{\lambda_A} + \lambda_1 - \widetilde{\lambda_A}\lambda_1)\tau \; if \; \widetilde{\lambda_A} \ge \overline{\lambda_A} \\ 0.5\lambda_1(r - \tau) \; if \; \widehat{\lambda_A} < \overline{\lambda_A} < \widetilde{\lambda_A} \\ 0.5(\widehat{\lambda_A} + \lambda_1 - \widehat{\lambda_A}\lambda_1)\tau \; if \; \widehat{\lambda_A} \le \overline{\lambda_A} \end{cases}.$$

Note that ${\lambda_A}^* \le \widehat{\lambda_A} < \widetilde{\lambda_A}$ and $\overline{\lambda_A}$ achieve equilibrium only if $\widehat{\lambda_A} < \overline{\lambda_A} < \widetilde{\lambda_A}$, as shown in the proof of Lemma 4.6.[38] Moreover, the introduction of the price parity clause does not change the price on the platform and increases the direct sales price only slightly. Those result imply that $CS^{**} \ge CS^*$ holds if $r \ge 4\tau$ except when both $\alpha = 0$ and $\widehat{\lambda_A} \le \overline{\lambda_A}$ hold.

Where $2\tau \le r < 4\tau$,

$$CS^{**} = \begin{Bmatrix} 0.5\left(\widetilde{\lambda_A} + \lambda_1 - \widetilde{\lambda_A}\lambda_1\right)\tau \ if\ \acute{\lambda}_A < \widetilde{\lambda_A} \le \overline{\lambda_A} \\ 0.5\lambda_1(r-\tau)\ if\ a)\ \acute{\lambda}_A \le \overline{\lambda_A} < \widetilde{\lambda_A}\ or\ if\ b)\ \widehat{\lambda_A} \le \overline{\lambda_A} < \acute{\lambda}_A\ and\ \acute{\Pi}_A \le \overline{\Pi_A} \\ 0.5\left(\widehat{\lambda_A} + \lambda_1 - \widehat{\lambda_A}\lambda_1\right)\tau\ if\ \overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda}_A\ and\ \acute{\Pi}_A \le \widehat{\Pi_A} \\ \frac{(\acute{\lambda}_A + \lambda_1 - \acute{\lambda}_A\lambda_1)r^2}{32\tau}\ if\ a)\ \acute{\lambda}_A \le \overline{\lambda_A} < \widetilde{\lambda_A}\ and\ \acute{\Pi}_A > \overline{\Pi_A}\ or \\ if\ b)\ \overline{\lambda_A} < \widehat{\lambda_A} < \acute{\lambda}_A\ and\ \acute{\Pi}_A > \widehat{\Pi_A} \end{Bmatrix}.$$

If $\acute{\lambda}_A < \overline{\lambda_A}$ holds $CS^{**} < CS^*$ can happen only if the equilibrium investment level is lower than the equilibrium investment when the clause is not allowed, because the introduction of price parity clause does not change the direct sales price and decreases the price on the platform in this situation.

If $\acute{\lambda}_A \ge \overline{\lambda_A}$ holds, $CS^{**} < CS^*$ can happen if the introduction of the clause increases the direct sales price without decreasing the platform price or decreases the equilibrium investment.

Where $r < 2\tau$,

$$CS^{**} = \begin{Bmatrix} \frac{(\acute{\lambda}_A + \lambda_1 - \acute{\lambda}_A\lambda_1)r^2}{32\tau}\ if\ \widetilde{\lambda_A} \ge \frac{3\lambda_1}{1-\lambda_1} \\ \frac{\lambda_1 r^2}{8\tau}\ if\ \widetilde{\lambda_A} < \frac{3\lambda_1}{1-\lambda_1} \end{Bmatrix}.$$

In this situation, $CS^{**} < CS^*$ can happen if $\widetilde{\lambda_A} \ge 3\lambda_1/1 - \lambda_1$ holds, because the introduction of the clause increases the direct sales price without decreasing the platform price. If $\widetilde{\lambda_A} < 3\lambda_1/1 - \lambda_1$, $CS^{**} < CS^*$ always holds. This is because

$$CS^{**} - CS^* = \frac{\lambda_1 r^2}{8\tau} - \frac{(1-\alpha)\left({\lambda_A}^* - {\lambda_A}^*\lambda_1\right)r^2}{32\tau} - \frac{\left(\alpha{\lambda_A}^* + \lambda_1 - \alpha{\lambda_A}^*\lambda_1\right)r^2}{8\tau} =$$
$$-\frac{(1-\alpha)\left({\lambda_A}^* - {\lambda_A}^*\lambda_1\right)r^2}{32\tau} - \frac{\alpha{\lambda_A}^*(1-\lambda_1)r^2}{8\tau} < 0.$$

Comparing CS^* with CS^{**} gives Proposition 4.3. ■

[38] ${\lambda_A}^* = \widehat{\lambda_A}$ holds only if $\alpha = 0$.

Chapter 5: Platform Competition and Effects of Platform Parity Clauses Where There Exist Duopoly Platforms, Duopoly Sellers and Uninformed Consumers

5.1. Introduction

Chapter 4 analyses the case in which there is one monopoly platform and one monopoly seller in order to analyse the effect of the price parity clause such that the seller must not set its direct sales price lower than its price sold on the platform, called a "narrow price parity clause". By contrast, this chapter analyses a model where there exists competition between two platforms (Platform A and B) and between two sellers (Seller 1 and 2) in order to analyse the effect of wide price parity clauses, which prohibit sellers from selling at lower prices on other platforms. To simplify the discussion and focus on the effects of wide price parity clauses, in this section, the possibility of direct sales to consumers by the sellers are excluded.

This chapter is organised as follows. Section 2 explains the differences between the assumptions from Chapter 4 and the assumptions made in this chapter. Section 3 then analyses the optimal behaviours of the platforms, the sellers and the consumers when there are two platforms and two sellers and when price parity clauses are not allowed. The effect on competition and the consumers when the two platforms can impose price parity clauses on the seller is analysed in Section 4. Finally, Section 5 presents concluding remarks and the implications for competition policy that derive from this chapter's analysis.

5.2. Basic Assumptions

Most basic assumptions of this chapter are common to Chapter 4's assumptions, as follows. In order to consider the effect of wide price parity clauses, it is assumed that there are three types of players: two sellers (Seller 1 and Seller 2), two platforms (Platform A and Platform B) and consumers.

Moreover, as with Chapter 4, this chapter analyses the theoretical model based on the informative advertising model for platforms with product differentiation between sellers. This model covers the situation in which getting awareness from consumers who may not be informed about the existence of a product is important for sellers.

In order to consider consumers' limited awareness of products and the possibility of consumers switching of sales channels, parameters regarding consumers' awareness of products are introduced.

In particular, it is assumed that only a share of $\lambda_k \in (0,1)$ consumers know about the existence of any product sold on Platform k, where $k = A$ or B. It is assumed that λ_k depends on the amount of investment Platform k has made, I_k, where $I_k \geq 0$. Note that while I_k is an endogenous variable, $\underline{I_k}$ is an exogenous variable. Specifically, it is assumed that the two platforms have the same investment function such that $\lambda_k = g(I_k)$, where $g'(I_k) > 0$ and $g''(I_k) < 0$. It is also assumed the probability to be noticed cannot be 1 even if it makes the investment as high as possible. In other words, $\lim_{I_k \to +\infty} g(I_k) < 1$ holds.

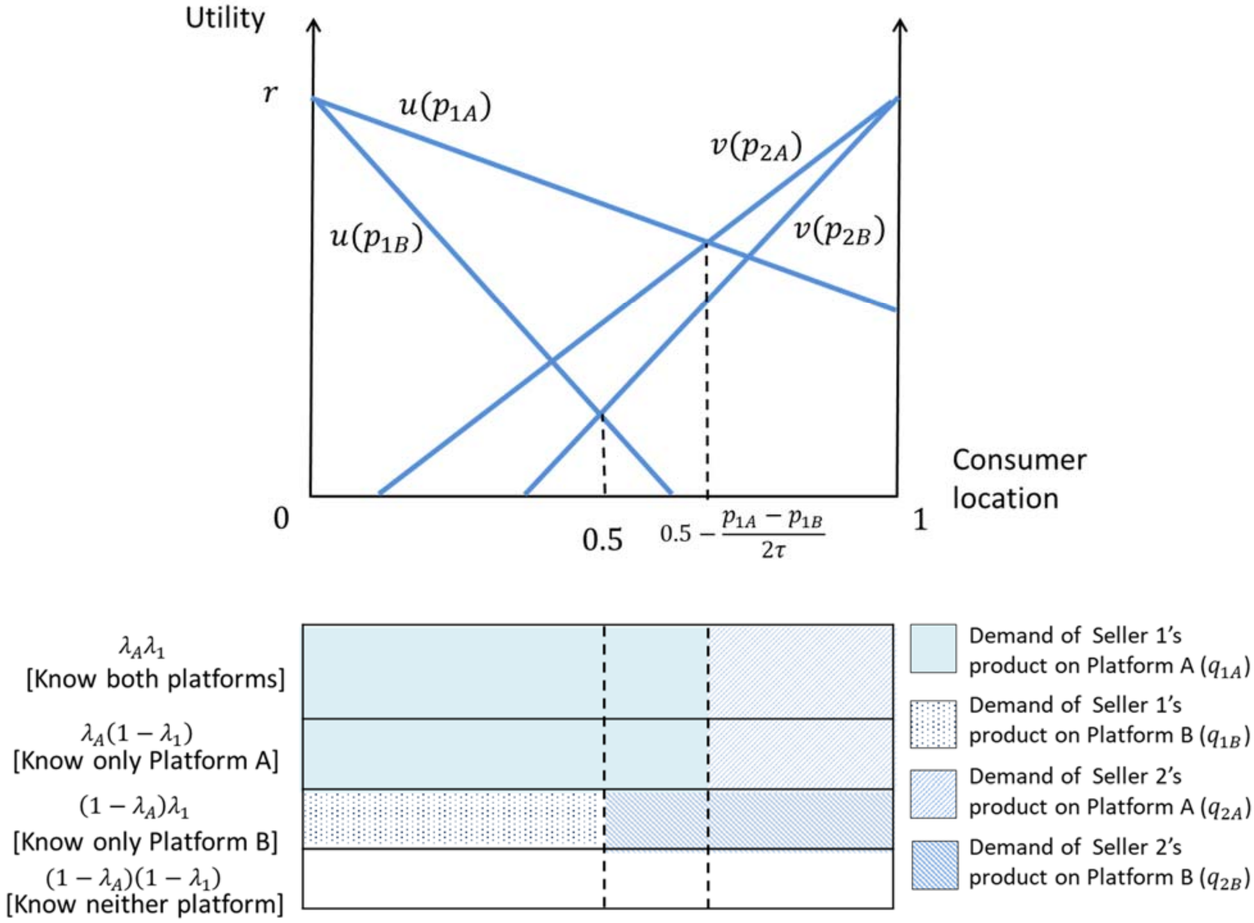
The assumptions about the consumers' awareness of the platforms imply that four types of consumers exist, depending on the information consumers have: (1) The product(s) sold on both platforms, (2) only the product(s) sold on Platform A, (3) only the product sold on Platform B or (4) no information about either product. The proportions of those four types of consumers are $\lambda_A\lambda_1$, $\lambda_A(1-\lambda_1)$, $(1-\lambda_A)\lambda_1$ and $(1-\lambda_A)(1-\lambda_1)$ respectively.

With regard to consumers, it is assumed that a unit mass of consumers are uniformly distributed over the unit interval (Hotelling model). The relationship between the sellers and the platforms is a client–agent relationship. When a seller contracts with a platform, it can sell its product on the platform. The seller decides the price of its product sold on the platform, p_{ik} where $i = 1$ or 2, $k = A$ or B. In exchange, the seller needs to pay a commission, f_k, for each unit it sells. There are no further cost (e.g., upfront fee) to use the platform. It is assumed that Seller 1 is located at 0 and Seller 2 is located at 1. The utility of consumer $x \in [0,1]$ when purchasing Seller 1's product sold on the sales channel k is $u = r - \tau x - p_{1k}$, and the utility when purchasing Seller 2's product is $v = r - \tau(1-x) - p_{2k}$. The variable $\tau > 0$[39] can be understood as the transportation cost. The definition of the utility functions implies that the utility of the most distant consumer to purchase the product of Seller $i \in 1,2$ at Sales Channel $k \in A, B$ is $r - \tau - p_{ik}$.

[39] When it is assumed that $\tau = 0$, the equilibrium behaviours of the sellers and platforms are different from the $\tau > 0$ case. This is because in this situation the competition the sellers are facing is Bertrand model rather than Hotteling model.

From the proportions of the four types of consumers based on their information and the assumption about the utility, the demands of the product sold on both sales channels given p_{1A}, p_{1B}, p_{2A} and p_{2B} can be summarised in Figure 5.1 below. [40]

Figure 5.1 Utility of consumers when $p_{1A} < p_{2A} < p_{1B} = p_{2B}$



The sequence of the game is defined as follows.
1. Two platforms decide the amount of investment, and then decide the commission (f_k) they offer to the sellers.
2. Two sellers decide whether they contract with the platform and, if so, decide the price sold on each platform (p_{ik}).
3. Consumers decide whether they purchase the product of the sellers or not and through which sales channel they purchase.

In addition, the following assumptions are specific to this chapter.

[40] This illustration is also based on Figure 6.1 of Belleflamme and Peitz (2015).

- [No direct sales] Chapter 4 assumes that there are two types of consumers, active consumers and inactive consumers, and only active consumers carry out searching behaviours to obtain information about prices that they initially do not have. In this chapter, for simplicity, it is assumed that all the consumers are inactive consumers, who choose only from the options they initially know and never search. It follows that direct sales by the sellers never happen. This assumption also implies that a seller can sell the product only if it contracts with at least one platform.

- [Large r to guarantee full-coverage price equilibrium] Variable r is assumed to be large enough to guarantee that all the consumers who have information about at least one of the seller's products purchase the product in the equilibrium ($r \geq 2\tau$). The reason is that if it does not hold, there will be no competition between sellers.

- [Tie break rule] It is assumed that if a seller sets the same price on the two platforms ($p_{iA} = p_{iB}$), the consumers with the information about the product sold on Platform A and that sold on Platform B will purchase the product on Platform A with the probability of 0.5 and on Platform B with the probability of 0.5.
If there is more than one set of equilibrium prices for the sellers, the seller prefers to set prices such that it can sell more on the platform offering the lower commission.

5.3. No Possibility of Price Parity Clause

This section analyses the case where the platform is not allowed to introduce a price parity clause.

Analysis on this model by the backward induction approach provides Proposition 5.1, which will be proved in the following sections.

Proposition 5.1. *Suppose the platform is not allowed to introduce a price parity clause, there always exists a unique equilibrium. In this equilibrium, the platforms choose the investment level* $({I_A}^*, {I_B}^*)$ *such that*

${I_A}^* = {I_B}^* = I^*$ *and*

$$g'(I^*)\big(1 - g(I^*)\big) = \frac{1}{r - \tau}.$$

Then, each platform sets a commission based on the cumulative distribution function, $H(f_k)$ *such that*

$$H(f_k) = \begin{Bmatrix} 0 \; for \; f_k \in [0, (1-\lambda^*)(r-\tau)] \\ \frac{1}{\lambda^*}\left[1 - \frac{(1-\lambda^*)(r-\tau)}{f_k}\right] for \; f_k \in ((1-\lambda^*)(r-\tau), r-\tau) \\ 1 \; for \; f_k \in [(r-\tau), +\infty) \end{Bmatrix},$$

where $k \in A, B$ *and* $\lambda^* = g(I^*)$.

The sellers set their prices such that

$$p_{1k}{}^{*} = p_{2k}{}^{*} = \left\{ \begin{array}{c} f_k + \tau \;\; if\ 0 \le f_k \le r - 1.5\tau \\ r - 0.5\tau\ if\ r - 1.5\tau < f_k \le r - \tau < \ f_l\ or \\ \qquad if\ r - 1.5\tau < f_k \le r - \tau\ and\ f_l\ \le f_k \\ r - 0.5\tau - \varepsilon\ if\ r - 1.5\tau < f_k < f_l \le r - \tau \end{array} \right\},$$

where ε is a small positive amount.

Proposition 5.1 shows that in the equilibrium the two platforms make the same amount of investment. Note that if r is high or τ is low, the optimal amount of investment, I^* rises, so the equilibrium share of the consumers who are aware of each platforms, λ^*, also rises.

The result can be summarised in Corollary 5.1.

Corollary 5.1. *If there are two sellers and two platforms, the amounts of the investment made by the two platforms and the share of the consumers being aware of the two platforms in the equilibrium increases as r increases and decreases as τ increases.*

Recall that τ is a transportation cost parameter and if τ is high, the utility for the consumer located at a distance from a seller is smaller. It follows that low τ compared to r means that the two sellers' products are similar to each other, and, hence, the product is less differentiated. It follows that there will be fierce competition between the two sellers. Corollary 5.1 shows that when there is a fiercer competition, the amount of the investment and the share of the consumers being aware of each platform are smaller.

Moreover, the effectiveness of the investment determined by the form of the investment function and the share of the consumers who are aware of each platform before they make investment $\underline{I}$ also have the influence on the values of I^* and λ^* . However, there is no straightforward relationship between these factors and the values of I^* and λ^*.

Proposition 5.1 also shows that while the sellers adopt pure strategies like the results in Chapter 4, the platforms adopt mixed strategies, whose support is $f_k \in \big((1-\lambda^*)(r - \tau), r - \tau\big)$ in the equilibrium, because each platform has an incentive to undercut its commission as long as the other platform's commission is higher than a certain threshold. It follows that dispersion of the platforms' commissions is found in the equilibrium. The range of the dispersion of the platforms' commissions becomes wider as the utility of the farthest consumer $(r - \tau)$ gets larger, because the equilibrium consumer awareness (λ^*) depends on $r - \tau$. This growth in the support reflects that if λ^* is close to 1, there are few captive consumers for each platform. On the other hand, if λ^* is close to zero, almost all consumers who are aware of the platform are captive consumers to it.

Note that by differentiating the cumulative distribution function, the following probability distribution function holds:

$$h(f_k) = \frac{(1-\lambda^*)(r-\tau)}{\lambda^* {f_k}^2}.$$

The result implies that the probability decreases in f_k.

Moreover, the average commission under this mixed-strategy equilibrium is given by

$$\bar{f} = \int_{(1-\lambda^*)(r-\tau)}^{r-\tau} h(f_k) f_k df_k = \left[\frac{(1-\lambda^*)(r-\tau)}{\lambda^*} ln\, f_k + C\right]_{(1-\lambda^*)(r-\tau)}^{r-\tau}$$
$$= \frac{(1-\lambda^*)(r-\tau)}{\lambda_l}[ln(r-\tau) - ln(1-\lambda^*)(r-\tau)]$$
$$= \frac{(1-\lambda^*)(r-\tau)}{\lambda_l} ln\left(\frac{1}{1-\lambda^*}\right).$$

This shows that the expected commission is increasing in $r-\tau$. It also shows that the expected commission rises when λ_l lessens, since

$$\partial\bar{f}/\partial\lambda_l = -(r-\tau)(1-\lambda^*)/\lambda^{*2} < 0.$$

The popularity of the other platform drives lower commission because when λ^* is high, the number of captive consumers for each platform is small. In this situation, the necessity to undercut the commission increases.

On the other hand, the two sellers' prices on one platform are equal. The prices depend on the commission that a platform actually imposed. When the commission which a platform proposes is low, the sellers set prices such that they can get a margin of τ, because setting the price is the best response for each seller, given the other seller's price. However, when the commission is high, they cannot transfer the high commission to the consumers to get that margin, because the willingness to pay of the consumer who is located in the middle now works as a constraint.

5.3.1 Consumers' Behaviour

In this case, there are four types of consumers: those who know both Platform k and Platform l, those who know only Platform k, those who know only Platform l, and those who do not know any platform.

Let Q_i, Q_j denote the total demand for each seller's product and $q_i(\underline{p_i}, \underline{p_j})$ denote the demand for Seller i's product when all consumers have the same information such that the lowest prices of Seller i's product and Seller j's product they know are $\underline{p_i}$ and $\underline{p_j}$, respectively. Note that $p_{ik} = +\infty$ if Seller i does not sell its product on Platform k. Similarly, $\underline{p_i} = +\infty$ if the consumers are not aware of Seller i's product.

The demand of the three types of consumers can be summarised in Lemma 5.1.

Lemma 5.1. *i) [Product demand when all consumers have the same information]*

The product demand when all consumers have the same information can be written as follows:

$a)$ if $\underline{p_j} < r - \tau$

$$q_i(\underline{p_i}, \underline{p_k}) = \begin{cases} 1 & if\ \underline{p_i} \le \underline{p_j} - \tau \\ 0.5 - \frac{\underline{p_i} - \underline{p_j}}{2\tau} & if\ \underline{p_j} - \tau < \underline{p_i} \le \underline{p_j} + \tau \\ 0 & if\ \underline{p_i} > \underline{p_j} + \tau \end{cases}$$

$b)$ if $r - \tau \le \underline{p_j} < r$

$$q_i(\underline{p_i}, \underline{p_k}) = \begin{cases} 1 & if\ \underline{p_i} \le \underline{p_j} - \tau \\ 0.5 - \frac{\underline{p_i} - \underline{p_j}}{2\tau} & if\ \underline{p_j} - \tau < \underline{p_i} \le 2r - \tau - \underline{p_j} \\ \frac{r - \underline{p_i}}{\tau} & if\ 2r - \tau - \underline{p_j} < \underline{p_i} \le r \\ 0 & if\ \underline{p_i} > r \end{cases}$$

$c)$ if $\underline{p_j} \ge r$

$$q_i(\underline{p_i}, \underline{p_k}) = \begin{cases} 1 & if\ \underline{p_i} \le r - \tau \\ \frac{r - \underline{p_i}}{\tau} & if\ r - \tau < \underline{p_i} \le r \\ 0 & if\ \underline{p_i} > r \end{cases}.$$

ii) [Total product demand from consumers who have different information] The demand for Seller i's product can be written as

$$Q_i = \lambda_k[1 - \lambda_l]q_i(p_{ik}, p_{jk}) + [1 - \lambda_k]\lambda_l q_i(p_{il}, p_{jl}) + \lambda_k \lambda_l q_i\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right).$$

iii) [Platform demand] Let Q_k denote the quantity of products sold on Platform k. The quantity can be written as

$$Q_k = \lambda_k[1 - \lambda_l][q_i(p_{ik}, p_{jk}) + q_j(p_{ik}, p_{jk})] + \lambda_k \lambda_l[\gamma_{ik} q_i\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right) + \gamma_{jk} q_j\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right)],$$

$$where\ \gamma_{ik} = \begin{cases} 1\ if\ p_{ik} < p_{il} \\ 0.5\ if\ p_{ik} = p_{il} \\ 0\ if\ p_{ik} > p_{il} \end{cases}.$$

Proof: See Appendix D

This lemma shows that depending on the prices, the constraint that binds the demand of a seller's product varies. If the price of the other seller's product is relatively high or if the consumers do not have the information about the other seller's product, the demand is determined by the possibility that the consumers decide not to buy the seller's product. On the other hand, if the difference between the prices of the two sellers' products is small, the demand is constrained by the price of the other seller's product. This result reflects price competition under the standard Hotelling model.

5.3.2 Seller's Behaviour

The two sellers are symmetric in the sense that they face an equivalent demand function as shown in Lemma 5.1, and their marginal costs are the same (f_A for the transaction through Platform A and f_B for the transaction through Platform B). Therefore, it is natural to consider that there exists a symmetric equilibrium with respect to the two sellers' pricing as long as the commissions are not too high.

The sellers' optimal pricing are summarised in Lemma 5.2.

Lemma 5.2. *[Optimal sellers' platform choices and prices] Anticipating the consumers' response specified in Lemma 5.1, given f_A, f_B, I_A and I_B, the optimal behaviour of the seller is as follows:*

i) If $f_A \leq r$ and $f_B \leq r$ hold, both sellers decide to use both platforms.

ii) If $f_A \leq r$ and $f_B \leq r$ hold, the sellers' optimal pricing policy is

$$p_{1k}{}^{*} = p_{2k}{}^{*} = \left\{ \begin{array}{c} f_k + \tau \qquad if\ 0 \leq f_k \leq r - 1.5\tau \\ r - 0.5\tau\ if\ r - 1.5\tau < f_k \leq r - \tau < \ f_l\ or \\ \qquad if\ r - 1.5\tau < f_k \leq r - \tau\ and\ f_l\ \leq f_k \\ r - 0.5\tau - \varepsilon\ if\ r - 1.5\tau < f_k < f_l \leq r - \tau \end{array} \right\},$$

$where\ k, l \in A, B\ and\ k \neq l.$

Proof: See Appendix D

The optimal price on Platform k given f_k can be summarised in Figure 5.2.

Figure 5.2 Seller's price on a platform under the full-coverage symmetric equilibrium

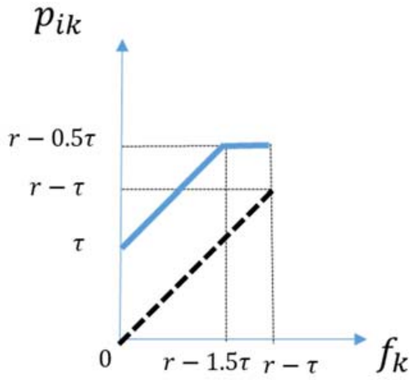


Figure 5.2 shows that when the commission is low, the sellers' optimal price increases as the commission grows, because in this case, the optimal price is such that each seller obtains a margin of τ. However, when the commission is higher than a certain point ($f_k \geq r - 1.5\tau$), such pricing is no-longer full-coverage price in the sense that the consumers whose location is distant from the sellers (i.e., consumers located near the middle of the Hotelling line) prefer not to purchase the product. In this situation, the optimal price is constrained by the willingness to pay of the consumer located the middle of the Hotelling line. If a platform sets a commission higher than $r - \tau$, the optimal reaction by the seller to set such a high price that consumers who are in the middle of the Hotelling line are not covered and the two sellers are not in the direct competition, which is the case we ruled out by assumption. In this situation, the optimal price for the sellers is $(r + f_k)/2$.

5.3.3 Platform's Behaviour regarding Commission and Investment

Lemma 5.2 implies that if the commission that one platform sets is higher than the other platform's commission, the price that the sellers set for the platform also be higher than the other. It follows that the transactions on the platform are smaller.

Taking this into account, the profits for Platform k can be written as follows:

$a)\ f_k \leq r - \tau$

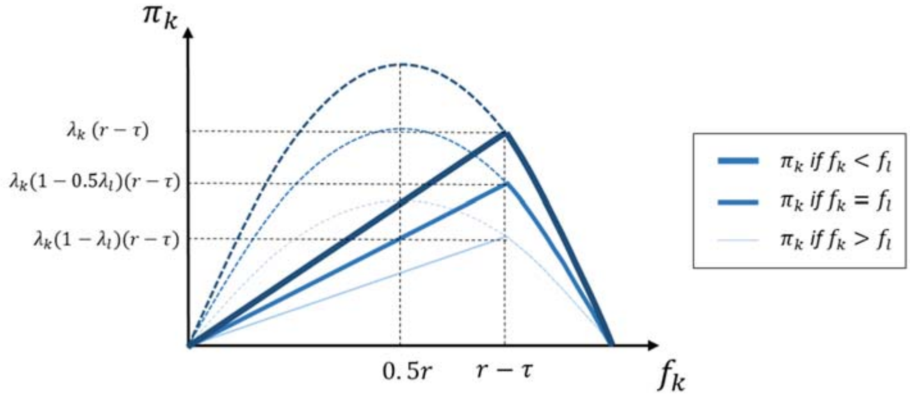
$$\Pi_k = \left\{ \begin{matrix} \lambda_k f_k \ if\ f_k < f_l \\ \lambda_k(1 - 0.5\lambda_l) f_k \ if\ f_k = f_l \\ \lambda_k(1 - \lambda_l) f_k \ if\ f_k > f_l \end{matrix} \right\}$$

$b)\ r - \tau < f_k \leq r$

$$\Pi_k = \begin{Bmatrix} \lambda_k \frac{(r-f_k)f_k}{\tau} \; if \; f_k < f_l \\ \lambda_k(1-0.5\lambda_l)\frac{(r-f_k)f_k}{\tau} \; if \; f_k = f_l \\ \lambda_k(1-\lambda_l)\frac{(r-f_k)f_k}{\tau} \; if \; f_k > f_l \end{Bmatrix}.$$

The profit of a platform is shown by Figure 5.3 below.

Figure 5.3 Platform's profits



The inverted U-shaped graphs in Figure 5.3 are Platform k's profit when it sets a commission high enough to guarantee that the sellers respond to set such high prices that some consumers prefer not to purchase the products through the platform (i.e., non-full-coverage equilibrium happens). Because the number of the consumers who prefer not to purchase increases as the commission increases, the profit decreases when the commission is low and when the commission is higher than a certain point, $0.5r$.

The upward-sloping straight lines in Figure 5.3 are Platform k's profit when it sets a commission low enough that the sellers set prices such that the consumers who are aware of at least one platform will purchase a product. The platform's profit is directly proportional to f_k, because the consumers who are aware of at least one platform will purchase a product (i.e., full-coverage equilibrium happens). In other words, the value of f_k does not affect the demand.

As shown in Figure 5.3, when the platform's commission is lower than $r-\tau$, the full-coverage equilibrium happens, and the platform's profit is given by one of the upward-sloping straight lines. When the commission is higher than $r-\tau$, the non-full-coverage equilibrium occurs, and the profit is given by one of the inverted U-shaped graphs. These results are consistent with Lemma 5.2. Since the profit when the commission is high enough to result in the non-full-coverage equilibrium decreases in f_k, the platforms do not have an incentive to set commissions higher than $r-\tau$. Hence, hereafter, we consider only the cases in which the platforms set the commissions equal to or less than $r - \tau$, which are represented as the upward-sloping straight lines in Figure 5.3.

Figure 5.3 also shows three pairs of inverted U-shaped graphs and upward-sloping straight lines. The graphs show that the profit differs depending on the relationship between the two platforms' commissions (i.e., higher, lower or equal). The top graph of the three inverted U-shaped graphs and the top graph of the three upward-sloping straight lines are the profits when Platform k offers a commission lower than Platform l's commission. In this situation, the platform gains the transactions of all the consumers who aware of both platforms, whose number is given by $\lambda_k \lambda_l$. The number of the consumers who purchase through the platform is given by λ_k.

The middle graphs are the profits when Platform k offers the same commission as Platform l's commission. In this situation, the transactions related to the consumers aware of both platforms are equally divided, and the number of consumers purchasing through the platform is $\lambda_k(1 - 0.5\lambda_l)$.

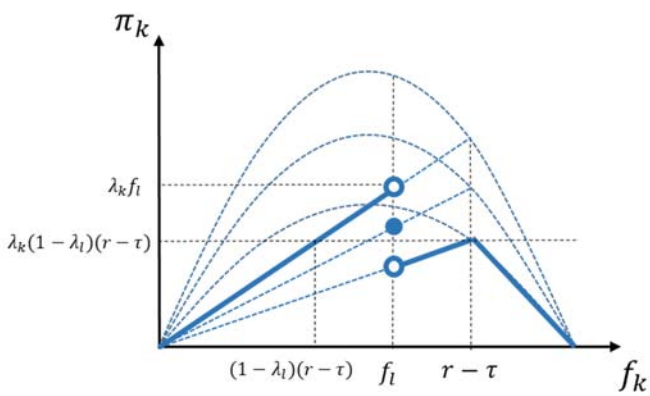
The bottom graphs represent the profit when one platform offers a commission higher than the another platform's commission. In this situation, only the consumers who are aware of only the platform purchase the product through the platform, and the number of consumers purchasing through the platform is $\lambda_k(1 - \lambda_l)$.

These figures imply that as long as the relation with the other platform's commission is not changed, the profit is maximised when the commission is equal to $r - \tau$. It follows that the platform can obtain at least a profit of $\lambda_k(1 - \lambda_l)(r - \tau)$ by setting its commission at $r - \tau$. When the other platform's commission is equal to or higher than $r - \tau$, the profit rises to $\lambda_k(1 - 0.5\lambda_l)(r - \tau)$ or $\lambda_k(r - \tau)$, respectively.

The discussion above implies that Platform k's profit given the other platform's profit can be summarised in Figure 5.4 below.

Figure 5.4 Platform k's profit given f_l

1) $(1 - \lambda_l)(r - \tau) < f_l \leq r - \tau$



2) $f_l \leq (1-\lambda_l)(r-\tau)$

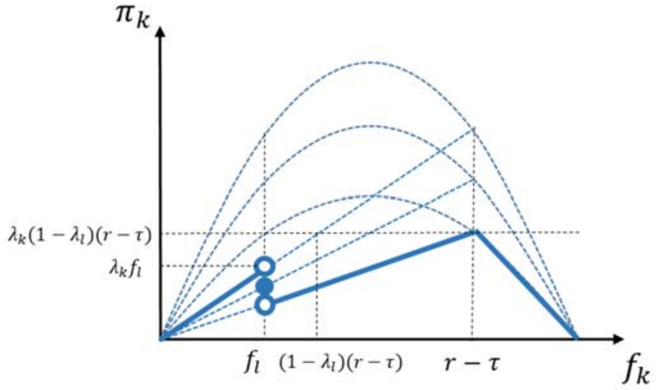


Figure 5.4.1 shows the profit of Platform k if the competing platform's commission is high (i.e., equal to or close to $r-\tau$). In this situation, if the platform decides to set a commission higher than that of the other platform, the platform's profit is maximised $\lambda_k(1-\lambda_l)(r-\tau)$ when it sets its commission at $r-\tau$. However, in this case, the platform can secure greater profit by setting its commission lower than that of the other platform (i.e., $f_k = f_l - \varepsilon$, where ε is a small non-negative amount). In this situation, the platform can obtain a profit of $\lambda_k(f_l - \varepsilon)$.

However, the profit that Platform k can obtain by setting its commission slightly lower than that of the competing platform decreases as f_l diminishes. If the competing platform's commission is lower than $(1-\lambda_l)(r-\tau)$ as shown in Figure 5.4.2, giving up the consumers who are aware of both platforms, charging $r-\tau$, and securing the profit of $\lambda_k(1-\lambda_l)(r-\tau)$ is more profitable. This situation arises if $f_l \leq (1-\lambda_l)(r-\tau)$.

The result shows that a platform does not have an incentive to set a commission such that $f_k > r-\tau$ or $f_k < (1-\lambda_l)(r-\tau)$ regardless of the value of the other platform's commission. Platform k's best response given Platform l's commission can be written as follows:

$$f_k = \begin{Bmatrix} r-\tau \ if \ f_l \leq (1-\lambda_l)(r-\tau) \ or \ if \ f_l > \ r-\tau \\ f_l - \varepsilon \ if \ (1-\lambda_l)(r-\tau) < f_l \leq r-\tau \end{Bmatrix}.$$

Moreover, the above discussion shows that each platform has an incentive to undercut the commission if the other platform's commission satisfies $(1-\lambda_l)(r-\tau) < f_l \leq r-\tau$. However, if the other platform's commission decreases to $(1-\lambda_l)(r-\tau)$, the platform's profit is then maximised by raising the commission to $r-\tau$. This result implies that there does not exist a pure strategy equilibrium with respect to platforms' commissions, meaning that the only possible equilibrium is a mixed-strategy equilibrium. Solving the profit-maximisation problem for the platforms gives the optimal platforms' commissions and investment, as summarised Lemma 5.3.

Lemma 5.3. *[Optimal platforms' investment and commissions] Anticipating the responses by the consumers and the sellers specified in Lemmata 5.1 and 5.2, the optimal behaviours of the two platforms are as follows:*

i) Both platforms choose the investment level such that

${I_A}^* = {I_B}^* = I^*$ *and*

$$g'(I^*)\big(1 - g(I^*)\big) = \frac{1}{r - \tau}.$$

ii) Both platforms set their commissions based on the cumulative distribution function, $H(f_k)$ *such that, where*

$$H(f_k) = \begin{cases} 0 \; for \; f_k \in [0, (1 - \lambda^*)(r - \tau)] \\ \frac{1}{\lambda^*}\left[1 - \frac{(1-\lambda^*)(r-\tau)}{f_k}\right] for \; f_k \in ((1 - \lambda^*)(r - \tau), r - \tau) \\ 1 \; for \; f_k \in [(r - \tau), +\infty) \end{cases},$$

where $\lambda^* = g(I^*)$.

iii) Each platform's expected profit is given by $\lambda^*(1 - \lambda^*)(r - \tau)$.

Proof: See Appendix D

Lemma 5.3 shows a unique equilibrium with respect to the platforms' commission in which each of the two platforms chooses the mixed strategy whose support is $f_k \in \big((1 - \lambda_l)(r - \tau), r - \tau\big)$. Therefore, the commissions will disperse.

This lemma also shows that the expected profit of each platform in this equilibrium is equal to the maximum profit that it can obtain when its commission is higher than the rival platform's commission. This is because, under the mixed-strategy equilibrium, all commissions included in the support of the mixed strategy must give the same expected profit, and when a platform sets its commission at $r - \tau$, the expected profit is the profit when the other platform sets the commission lower than the platform.

From Lemmata 5.1, 5.2 and 5.3, Proposition 5.1 can be obtained directly.

Note that this result also shows that the existence of the competition lowers the platform's commission although the effect on the sellers' prices in the equilibrium are ambiguous. According to Lemma 5.2, if there exists only one platform, the platform's optimal commission is $r - \tau$, because in this situation the monopoly platform does not need to care about the possibility that it loses profit to the other platform's undercutting of the commission. In this situation, both sellers' prices are equal to $r - 0.5\tau$. This equality implies that if there is only one platform, the entry of a platform decreases the commission, but the extent of the decrease is uncertain because the optimal commissions after the entry are determined based on the mixed strategy. It follows that the entry may not change the optimal prices from the sellers, because if the realised commissions of the two platforms are larger than $r - 1.5\tau$, the optimal sellers' prices remain $r - 0.5\tau$.

Moreover, the result that while there is no price dispersion in the sellers' prices, there exists a price dispersion in the platforms' commissions may look unrealistic, which is caused by the uncertainty about the rival's behaviour. In real cases, each platform may deal with this uncertainty in various ways. One possibility is that platforms introduce certain actions to reduce the uncertainty. Section 4 discusses that introduction of price parity clauses to reduce such uncertainty.[41]

5.4. Allowing Price Parity Clause

5.4.1 Assumptions

This section considers what happens when two platforms are allowed to impose the price parity clauses such that the seller's prices must satisfy $p_{ik} \leq p_{il}$ for both sellers.

To simplify the discussion, in this section it is also assumed that sellers already decide to use both platforms and that platforms have strong power enough to guarantee that when a platform imposes a price parity clause, the sellers must accept it and sell on both platforms (i.e., $q_{1k} > 0$ and $q_{2k} > 0$ hold as long as $f_k \leq r$ holds). In other words, the options such that a seller decides not to use one of the two platforms are assumed to be excluded. Recall that in the no-price-parity-clause case, the sellers always choose to use the platforms. This choice implies that the additional assumption that the sellers decide to use both platforms does not change the results in the case where the price parity clause is not allowed. What difference ensues if this additional assumption is relaxed is discussed in Section 4.5.

Note that the dominant platform model analysed in Chapter 4 shows that in some situations (e.g., when $\widetilde{\lambda_A} \geq \overline{\lambda_A}$), the possibility that the seller decides not to use a platform does not effectively change the platform's behaviour in the equilibrium.

5.4.2 Analysis

Analysis on this model provides Proposition 5.2, which will be proved in this section.

[41] Another possibility is that platforms try to acquire a competitive advantage against their rivals by reducing the costs for sellers other than commissions or providing consumers some benefits such as lowering search costs.

Proposition 5.2. *Suppose the two platforms are allowed to offer a price parity clause. In the unique equilibrium, the two platforms introduce price parity clauses, and the two platforms make the same amount of investment, which is strictly larger than the equilibrium investment in the case where the clause is not allowed. Then the platforms sets commissions as if they are colluding with each other, whose amount is strictly higher than the equilibrium commission in the case where the clause is not allowed. The equilibrium sellers' prices are equal to or higher than the highest possible prices that can be realised in the case where the clause is not allowed.*

Proposition 5.2 shows that if the price parity clause is allowed, the platforms always introduce them. It also shows that the equilibrium investments and the commissions are strictly higher than those when the clause is not allowed. The two platforms set the same commissions, whose amount is equal to the level that they would set if they colluded with each other.

This result implies that the price parity clause excludes the possibility that the transactions of the consumers who are aware of both platforms are dominated by the other platform through undercutting its commission (which is the cause of the mixed-strategy equilibrium in the case where the clause is not allowed); hence, the two platforms can set their highest possible price. In this way, the price parity clause enables the platforms to avoid the commission-cutting condition and to set their commissions to the level that they would set if they colluded with each other. The higher commission enabled by the price parity clauses guarantees that they can invest more.

The result shows that in contrast to the equilibrium prices when the price parity clause is not allowed, the four prices, the prices of the two sellers on the two platforms, in the equilibrium are all equal ($p_{1A} = p_{1B} = p_{2A} = p_{2B} = r - 0.5\tau$), because the commissions of the two platforms are equal. The equilibrium prices can be higher than those when the clause is not allowed. On the other hand, one of the striking results in this study is that the sellers' equilibrium prices may be unchanged even when the clause is not allowed. This result is same as Johanson and Verge (2016). They show that the possibility that the sellers refuse to use platforms offering price parity clauses constrains the platform's ability to set high commissions (they call it as "supplier's participation constraint"), which follows that the equilibrium prices of the sellers do not change. In our, the situation such that the prices do not change by allowing the clauses happen because the sellers' equilibrium prices are constrained by the willingness to pay of the consumer, whose location is in the middle of the Hoteling line when the commissions are high. This constraint is binding at the equilibrium commission in the case where the clause is allowed. Even in the case where the clause is not allowed, the constraint is binding when the realised commission under the mixed-strategy equilibrium is high.

As shown Section 4.3 of this chapter, the prohibition of the price parity clauses is likely to raise the sellers' prices if the equilibrium share of the consumers who are aware of each platform in the non-price-parity-clauses case, λ^*, is high or if r is high compared to τ.

In order to prove Proposition 5.2, first we consider the case where both platforms offer the price parity clause; we then shows that this case is the equilibrium.

Since the introduction of the price parity clause does not constrain the consumers' behaviours, the consumers' behaviour given the seller's prices are summarised in Lemma 5.4.

Lemma 5.4. *The demand for the seller's product sold on sales channel* $m \in A, B$, $q_{1m}{}^*$ and $q_{2m}{}^*$, where $n \in A, B$ *and* $m \neq n$, *given* p_{1A}, p_{1B}, p_{2A}, p_{2B}, f_A, f_B, I_A *and* I_B, *are same as the demand in the case where a price parity clause is not allowed specified in Lemma 5.1.*

Then, consider the optimal prices of the sellers when the platforms offer the price parity clauses. In this situation, the sellers need to set $p_{iA} = p_{iB} = p_1$. The seller's optimal pricing if the platforms offer the price parity clause is summarised in Lemma 5.5.

Lemma 5.5. *[Optimal sellers' prices] Anticipating the consumers' response specified in Lemma 5.4, given* f_A, f_B, I_A *and* I_B, *if the platforms offer the price parity clause the optimal behaviour of the seller is as follows: If* $f_A \leq r$ *and* $f_B \leq r$ *hold, the sellers' optimal pricing policy is*

$$p_{1A}{}^* = p_{1B}{}^* = p_{2A}{}^* = p_{2B}{}^* = \begin{Bmatrix} \overline{f} + \tau \ \ if\ 0 \leq \overline{f} \leq r - 1.5\tau \\ r - 0.5\tau \ \ if\ r - 1.5\tau < \overline{f} \leq r - \tau \end{Bmatrix},$$

$$where\ \overline{f} = \frac{\lambda_A(1-0.5\lambda_B)f_A + (1-0.5\lambda_A)\lambda_B f_B}{\lambda_A + \lambda_B - \lambda_A\lambda_B}.$$

Proof: See Appendix D

Lemma 5.5 shows that when the weighted average of the two platforms' commission, $\overline{f}$, is weakly lower than $r - 1.5\tau$, the sellers' optimal price increases as commission becomes larger. The weight is based on the share of transactions on each platform. The optimal price is equal to the sum of the weighted average of the commissions and τ. It follows that each seller obtains a margin of τ. When the commissions are relatively high (i.e., the weighted average of the commissions are higher than $r - 1.5\tau$), setting $\overline{f} + \tau$ is no-longer full-coverage price in the sense that the consumers whose location is distant from the sellers (i.e., consumers who locate near the middle of the Hotelling line) prefer not to purchase the product. In this situation, the assumption to guarantee a full-coverage equilibrium ensures that all consumers aware of at least one platform purchase the product for the price of $r - 0.5\tau$, which is equal to the willingness to pay of the consumer located the middle of the Hotelling line.

Consider next the optimal behaviours of the platforms. Lemma 5.5 shows that both sellers sets the same prices on both platforms when the weighted average of the two platforms commissions is equal to or less than $r - \tau$. This finding implies that as long as the weighted average of the two platforms' commissions is equal to or less than $r - \tau$, each platform's profit does not change based on the magnitude relation of the two platforms' commissions. In this situation, the transactions through Platform k are given by $\lambda_k(1 - 0.5\lambda_l)$, because the price parity clause guarantees that the prices on the two platforms are equal, and the expected profit is $\lambda_k(1 - 0.5\lambda_l)f_k$. Note that each platform does not have an incentive to set a commission above $r - \tau$, because if the platform sets such a high commission, the full-coverage price equilibrium is not maintained, so the high commission results in the loss of transactions. Hence, the optimal commission for each platforms is $r - \tau$. Moreover, no platform has an incentive to deviate from this equilibrium by ceasing to offer the price parity clause: If a platform stops offering the clause, there is a possibility that the competing platform lowers its commission to capture the transactions for the consumers who are aware of both platforms. Hence, the expected profit cannot be larger than the profit when both platforms offer the price parity clause.

Note that when both platforms do this pricing, the joint profits of the platforms are maximised. This mutual benefit implies that when the price parity clause is allowed, the equilibrium commissions are the same as the commissions when the two platforms collude with each other.

Given this optimal commission, the total expected profit for Platform k can be written as

$$\Pi_k = g(I_k)(1 - 0.5\lambda_l)(r - \tau) - I_k.$$

Solving this profit-maximisation problem yields

$$g'(I_k)(1 - 0.5\lambda_l) = \frac{1}{r-\tau}.$$

Since the two platforms are symmetric, the optimal investment for both platform is the same. Hence,

$$g'(I^{**})\big(1 - 0.5g(I^{**})\big) = \frac{1}{r-\tau}, \text{ where } I_A = I_B = I^{**}.$$

These results are summarised Lemma 5.6.

Lemma 5.6. *[Optimal platforms' investment and commissions] Anticipating the responses by the consumers and the sellers specified in Lemmata 5.4 and 5.5, the optimal behaviours of the two platforms are as follows:*

i) Both platforms introduce price parity clauses.

ii) Both platforms choose the investment level such that

$I_A{}^* = I_B{}^* = I^{**}$ *and*

$g'(I^{**})\big(1 - 0.5g(I^{**})\big) = \frac{1}{r-\tau}.$

iii) Both platforms set their commissions such that

$f_A{}^{**} = f_B{}^{**} = r - \tau.$

iv) Each platform's expected profit is given by $\lambda^{**}(1 - 0.5\lambda^{**})(r - \tau)$, *where* $\lambda^{**} = g(I^{**})$.

Lemma 5.6 shows that when the two platforms are allowed to introduce price parity clauses, both platforms introduce the clauses to avoid the loss of the transactions through the reduction of the other platform's commission.[42] It is also shown that the equilibrium commission is equal to the supremum of the support of the platforms' mixed strategy in the equilibrium, if the price parity clause is not allowed. Since there is no point mass in the density of the mixed-strategy equilibrium, the equilibrium commission is higher than the commission realised when the clause is not allowed.

In this way, the price parity clause makes it possible for the platforms to avoid the commission-cutting condition and to set their commissions to the level that they would set if they colluded with each other. Hence, in the equilibrium, the sellers choose their prices based on the pure strategy given that the platforms set the commissions based on the pure strategy. On the other hand, when the price parity clause is not allowed, the equilibrium prices of the sellers depend on the platforms' commissions, which are realised based on the mixed-strategy equilibrium. The equilibrium sellers' prices on a platform are equal to the equilibrium sellers' prices where the clause is allowed, $r - 1.5\tau$, only if the realised commission on the platform satisfies $r - 1.5\tau \leq f_k \leq r - \tau$ and $f_k \leq f_l$. Otherwise, the equilibrium sellers' prices on the platform are lower than the equilibrium prices where the clause is allowed.

Note that as shown in the case where the price parity clause is not allowed, the equilibrium investment, I^{**}, and the equilibrium share of the consumers who are aware of each platform, λ^{**} , are high when r is high compared to τ. This correlation implies that Corollary 5.1 holds even when the two platforms are allowed to use price parity clauses.

[42] This result is consistent with the online booking platform case in Europe. For example, according to the German Federal Cartel Office, three booking platforms, HRS, Booking.com and Expedia, introduced the clauses.

With regard to the equilibrium investment, the equilibrium investment where the price parity clause is allowed is strictly higher than the equilibrium investment where the clause is not allowed. This rule can be proved as follows.

According to Lemmata 5.5 and 5.6,

$g'(I^*)\big(1-g(I^*)\big) = g'(I^{**})\big(1-0.5g(I^{**})\big).$

This equation can be modified as follows:

(5.1) $\quad g'(I^*)\big(1-0.5g(I^*)\big) - g'(I^{**})\big(1-0.5g(I^{**})\big) = 0.5g'(I^*)g(I^*) > 0.$

If it is assumed that $I^* \geq I^{**}$, according to the assumptions, $g(I^*) > g(I^{**})$ and $g'(I^*) < g'(I^{**})$ hold. It follows that $g'(I^*)\big(1-g(I^*)\big) - g'(I^{**})\big(1-g(I^{**})\big) < 0$. However, this inequality contradicts Equation (5.1).

From this result and Lemmata 5.4, 5.5 and 5.6, Proposition 5.2 can be obtained directly.

5.4.3 Effect on Consumer Welfare of Allowing Price Parity Clause

Taking the results of the analysis above, this section analyses the effect of introducing the platform price parity clause on consumer welfare. As shown in Chapter 4, two elements affect consumer welfare. The first element is the sellers' prices, and consumer welfare decreases as the prices become higher. The second element is the number of the consumers who are aware of the platforms. Consumer welfare increase as this number increases (i.e., high λ_A and λ_B), because more and more consumers can purchase the product. Note that according to Proposition 5.2, allowing price parity clauses always increases the number of the consumers who are aware of the platforms.

Comparison between the equilibrium sellers' prices and the number of the consumers who are aware of the platforms summarised in Propositions 5.1 and 5.2 yields the Proposition 5.3.

Proposition 5.3. *If the platform is allowed to offer a price parity clause, in the unique equilibrium, the consumer welfare improves compared to the unique equilibrium where the clause is not allowed when both of the realised commissions under this mixed strategy, in the case where the clause is not allowed, are equal to or greater than* $r - 1.5\tau$. *Otherwise, the consumer surplus can be larger or smaller.*

Proposition 5.3 implies the rise of the commissions, which always occurs when allowing price parity clauses may not always be followed the rise of the sellers' prices, meaning that allowing the clause may improve the consumer welfare. This is because in the case where the price parity clause is not allowed, the platforms adopt the mixed strategy whose support is $f_k \in \left((1-\lambda^*)(r-\tau), r-\tau\right)$, where λ^* denotes λ realised in the equilibrium in the case where the price parity clause is not allowed. When the realised commissions under this mixed strategy are greater than the thresholds that the willingness to pay of the consumer whose location is in the middle of the Hotteling line starts binding (i.e., $f_k \geq r - 1.5\tau$), the equilibrium sellers' prices are the same as those in the case in which the clause is not allowed. The probability that the realised commission is lower than the threshold is

$$H(r-1.5\tau) = \frac{1}{\lambda_l}\left[1 - \frac{(1-\lambda_l)(r-\tau)}{r-1.5\tau}\right] = \frac{\lambda^*(r-\tau)-0.5\tau}{\lambda^*(r-1.5\tau)}.$$

This equation implies that the equilibrium sellers' prices in the non-price-parity-clauses case are always equal to those in the case where the clauses are allowed if

(5.2) $\quad \lambda^*(r-\tau) - 0.5\tau \leq 0.$

In other words, if Inequality (5.2) holds, the prohibition of the price parity clauses always raise the sellers' prices. Note that Inequality (5.2) is likely to hold when λ^* or r is low or when τ is high.

Moreover, even if Inequality (5.2) does not hold, the probability that the rise of the sellers' prices by the introduction of price parity clauses will happen increases when λ^* or r is low or when τ is high. This increase occurs because

$$\frac{\partial H(r-1.5\tau)}{\partial \lambda^*} = \frac{0.5\tau}{\lambda^{*2}(r-1.5\tau)} > 0,$$

$$\frac{\partial H(r-1.5\tau)}{\partial r} = \frac{0.5(1-\lambda^*)\tau}{\lambda^*(r-1.5\tau)} > 0, \text{ and}$$

$$\frac{\partial H(r-1.5\tau)}{\partial \tau} = -\frac{(\lambda^*+0.5)(r-1.5\tau)+1.5[\lambda_l(r-\tau)-0.5\tau]}{\lambda^*(r-1.5\tau)^2} \text{ hold.}$$

Note that if Inequality (5.2) does not hold, $\partial H(r-1.5\tau)/\partial\tau < 0$.

Therefore, the prohibition of the price parity clauses is likely to raise the sellers' prices (and, hence, improve the consumer welfare) if the equilibrium share of the consumers who are aware of each platform in the non-price-parity-clauses case, λ^*, is high or if r is high compared to τ. Note that λ^* is high when the technology related to investment is sophisticated or r is high compared to τ.

Note that when the equilibrium sellers' prices in non-price-parity-clauses case is equal to the equilibrium prices in the case when the clauses are allowed, the sellers suffer losses instead of consumers. These losses imply that if it is assumed that sellers can also make an investment that improves consumer welfare, such as improving the quality of their service, the introduction of price parity clauses may harm consumer welfare through underinvestment by the sellers even when the sellers' prices after the introduction of the clauses do not change.

With regard to the number of the consumers who are aware of the platforms, the introduction of price parity clauses increases investment, and more consumers are aware of the two platforms. However, due to the decreasing return of scale of the investment, this effect diminishes when the equilibrium shares of the consumers who are aware of the platforms are high in the case where the clauses are not allowed.

In summary, allowing the price parity clause has the two opposite effects on consumer welfare, the negative effect through higher prices and the positive effect through the improvement of consumer awareness. Moreover, the negative effect through the rise of the sellers' prices does not always ensue. It follows that the consumer welfare (and the sellers' profits) can improve with the introduction of the price parity clauses.

However, as shown in Corollary 5.1, the improvement of consumer welfare through the introduction of price parity clauses is unlikely to occur when there is a fierce competition between the two sellers (i.e., high r compared to τ), since the increase of the sellers' price is likely to occur when r is high compared to τ.

5.4.4 Effect on Sellers' Profits of Allowing Price Parity Clause

This section analyses the effect of allowing a price parity clause on the seller's surplus. The following two elements affect the sellers' profits. The first element is the sellers' margins in selling a unit. Allowing price parity clauses always decreases the sellers' margins. Proposition 5.2 implies that the equilibrium margin when the clauses are allowed is 0.5τ. On the other hand, according to Proposition 5.1 and Lemma 5.2, the equilibrium margin when the clauses are not allowed is larger than 0.5τ.

The second element is the number of the consumers who are aware of the platforms. Consumer welfare increases as the number becomes larger (i.e., high λ_A and λ_B), because more and more consumers can purchase the product. Note that according to Proposition 5.2, allowing price parity clauses always increases the number of the consumers who are aware of the platforms.

Comparison between the equilibrium platforms' commissions and the number of the consumers who are aware of the platforms, summarised in Propositions 5.1 and 5.2, yields Proposition 5.4.

Proposition 5.4. *If the platforms are allowed to offer a price parity clause, in the unique equilibrium, the sellers' profits can be larger or smaller.*

Proposition 5.4 shows that the effect on the sellers' profits of allowing price parity clauses is always ambiguous. This ambiguity is always present is because allowing price parity clauses always has two opposite effects on the sellers' profits: the negative effect through higher commission and the positive effect through the improvement of consumer awareness.

As discussed in Section 4.3, due to the decreasing return of scale of the investment, this effect becomes smaller when the equilibrium shares of the consumers who are aware of the platforms are high in the case where the clauses are not allowed.

5.4.5 Effect of the Possibility That a Seller Decides Not to Use a Platform under the Price Parity Clause

In the model analysed in this section, to simplify the discussion, it is assumed that platforms have strong enough power to guarantee that when a platform imposes a price parity clause, the sellers must accept it and sell on both platforms. This section considers what change can happen in the case that this assumption is relaxed.

If this assumption is relaxed, a seller may decide not to use a platform when the platforms impose the price parity clauses on the sellers. This situation can arise when the commissions of the two platforms are not equal. Each seller prefers to use only one platform if the commission of one platform is high compared to that of the other platform, such that each seller obtains a larger profit when using only the platform offering the commission lower than the profit it would get when using both platforms. In other words, selling only on the platform offering the lower commission to avoid paying the high commission can compensate for the loss of the sales to the consumers who are only aware of the platform offering the higher commission. It follows that the price parity clauses may not eliminate the possibility that the other platform undercuts its commission to deprive transactions.

In order to analyse this possibility, we define $\widetilde{f_k}\ (f_l)$ as the maximum commission of Platform k such that the sellers decides to use both Platform k and l under the price parity clauses rather than using only Platform k, given the other platform's commission, f_l**.** This definition implies that when Platform k wants to dominate the transactions of non-captive consumers, it needs to set its commission lower than $\widetilde{f_k}\ (f_l)$.

Analysis of this model provides the following proposition, to be proved in this section.

Proposition 5.5. *Suppose two platforms are allowed to offer a price parity clause without a compulsory power.*

1) If $\widetilde{f_k}\ (r-\tau) \leq [1-0.5\lambda_l][r-\tau]$ *holds, both platforms set their commissions at* $r-\tau$*, and both sellers set their prices for both platforms at* $r-0.5\tau$ *in the equilibrium.*

2) If $\widetilde{f_k}\,(r-\tau) > [1-0.5\lambda_l][r-\tau]$ holds and there exists $\widehat{f_l} \in ([1-\lambda_l][r-\tau], r-\tau)$ such that $\widetilde{f_k}(\widehat{f_l}) = [1-\lambda_l][r-\tau]$, both platforms set the commission based on the mixed strategy whose support is $f_k \in (\widehat{f_l}, r-\tau)$. Further, $\widehat{f_l}$ is higher than the lower bound of the support of the mixed strategy in the case where price parity clauses are not allowed, $[1-\lambda_l][r-\tau]$.

Proposition 5.5 shows that there are two possible scenarios such that the introduction of price parity clauses increases the equilibrium commissions and sellers' prices, which are explained in the first half and the second half, respectively. The first scenario is that the threat of the possibility that the undercutting of the commission happens is not strong enough to change the optimal behaviours of the platforms, and hence, setting the collusive commissions (i.e., $f_A = f_B = r-\tau$) is still an equilibrium. The second scenario is that the threat of such possibility is strong enough to change the optimal behaviours of the platforms, and, hence, in the equilibrium, there is no pure Nash equilibrium for the two platforms' commissions. On the other hand, the support of the mixed-strategy equilibrium is smaller than that in the case where price parity clauses are not allowed, in the sense that the lower bound of the support is higher. It follows that the two platforms are more likely to set higher commissions compared to the case in which they are not allowed to offer a price parity clause.

Proposition 5.5 suggests that the implication in the previous sections—that the price parity clauses have the same effect as that of cartels in the sense that the clauses enable firms to set commissions as if they collude—may be too strong, because if there is a possibility that the sellers choose to use only one platform, the increase of the commissions is limited or does not happen. At the same time, this proposition implies that price parity clauses can still harm consumers or sellers in the sense that the clauses may raise the commissions to the collusive level or raise the lesser support of the mixed strategy regarding the commissions.

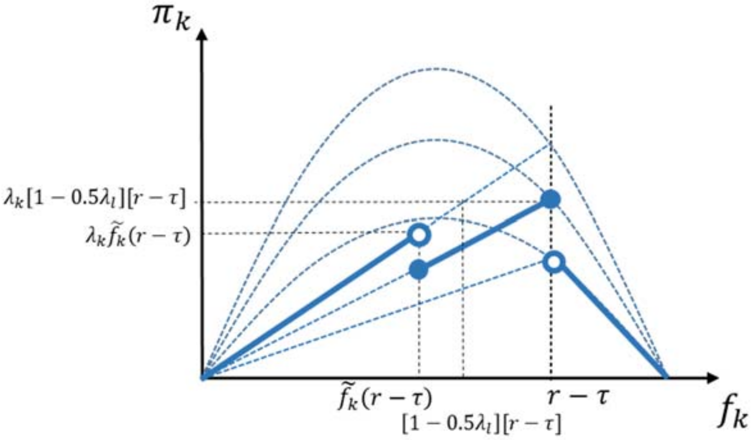
The first half of Proposition 5.5 can be proved as follows.

The first scenario refers to the situation where each platform does not have an incentive to deviate from where both platforms set their commissions as if they are colluding with each other, which is equal to the equilibrium commission in the case where the price parity clauses have compulsory power. The condition that this situation happens is that a platform's profit when both platforms set their commission at $r-\tau$ is equal to or larger than the profit when one platform undercuts the commission to encourage the sellers not to use the other platform. The condition can be written as $\lambda_k \widetilde{f_k}\,(r-\tau) \leq \lambda_k(1-0.5\lambda_l)(r-\tau)$. This condition can be modified to

(5.3) $\widetilde{f_k}(r-\tau) \leq (1-0.5\lambda_l)(r-\tau)$.

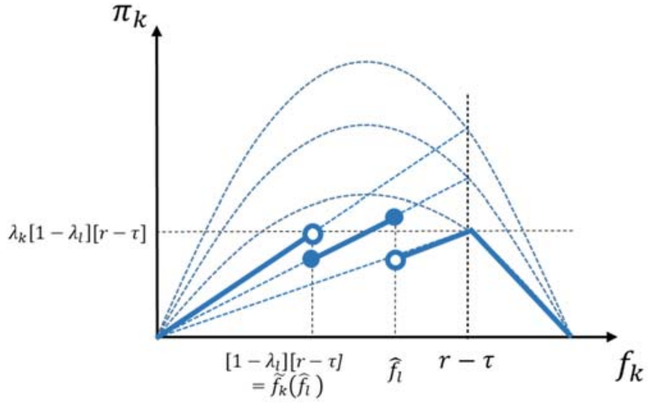
If Inequality (5.3) holds, both platforms set their commissions at $r-\tau$ in the equilibrium, as shown in Figure 5.5, meaning that the optimal behaviours of the platforms are the same as in the case where the platforms introduce compulsory price parity clauses.

Figure 5.5 Platform k's profit given that $f_l = r - \tau$



Next, we prove the second half of Proposition 5.5. The second scenario refers to the situation in which each platform no longer has an incentive to set a commission close to the lower bound of the support of the mixed-strategy equilibrium in the case where the price parity clauses are not allowed, $(1-\lambda_l)(r-\tau)$. This situation happens when there exists $f_l \in ((1-\lambda_l)(r-\tau), r-\tau)$ such that $\widetilde{f_k}(f_l) = (1-\lambda_l)(r-\tau)$. Let $\widehat{f_l}$ denote $f_l \in ((1-\lambda_l)(r-\tau), r-\tau)$ that satisfies $\widetilde{f_k}(\widehat{f_l}) = (1-\lambda_l)(r-\tau)$. In this situation, if Platform l's commission is equal to $\widehat{f_l}$, Platform k's profit decreases when it undercuts the commission to expand its consumer base as shown in Figure 5.5. Hence, a platform does not have an incentive to sets a commission such that $(1-\lambda_l)(r-\tau) < f_k < \widehat{f_l}$. This lack of incentive implies that $f_k \in ((1-\lambda_l)(r-\tau), \widehat{f_l})$ is no longer included in the support of the mixed-strategy equilibrium. In this situation, the support is $f_k \in (\widehat{f_l}, r-\tau)$. Therefore, in this scenario, the expected commissions are higher than in the case where the price parity clauses are not allowed, and this rise may furthermore increase the sellers' prices and harm consumers.

Figure 5.6 Platform k's profit given that $f_l = \widehat{f_l}$



This result can be summarised in the second half of Proposition 5.5.

5.5. Policy Implications and Conclusions

This chapter covers a duopoly platform case and analyses the effect of wide price parity clauses. The models analysed in this chapter show that wide price parity clauses enable platforms to set higher commissions compared to when the clauses are not allowed, and these higher commissions cause two different effects: the increase of sellers' prices and the increase of investment by platforms. While the former has a negative impact on consumer welfare, the latter has a positive impact on consumer welfare.

The analysis of the case where the price parity clauses are not allowed shows that while the sellers adopt pure-strategies, the platforms adopt mixed strategies in the equilibrium. This analysis differs from the results in the models of Varian (1980) and the models extending it to a two-sided platform case, such as Ronayne (2015), in which the platforms adopt the pure strategies and the sellers adopt mixed strategies in the equilibrium. This difference reflects the fact that while those models assume homogeneous sellers and platforms, in this model only the platforms are homogenous; the sellers are horizontally differentiated. On the other hand, the analysis on the case where both sellers and platforms are differentiated is a challenge for the future study.

The result of the analysis in this chapter also show that wide price parity clauses have a more straightforward effect on competition in the sense that the clauses make it possible for the platforms to avoid the competition by undercutting commissions and setting their commissions to the level at which they would set them if colluding with each other, in contrast to the effect of narrow price parity clauses analysed in Chapter 4. The result is consistent with one of the most popular explanations on the anticompetitive effect of (wide) price parity clauses, softening the competition regarding platforms' commissions in economic literature such as Boik and Corts (2015) and Ronayne and Taylor (2018). The analysis in this chapter clarifies the mechanism of such a competitive effect in the sense that it shows the following: Without wide price parity clauses, the platforms try to undercut each other's commissions until setting the highest full-coverage commission is more profitable and the clauses eliminate the possibility of undercutting by the other platform.

With respect to the effect of price parity clauses on consumer welfare, when the sellers' prices rise with the introduction of the price parity clauses, consumer welfare is harmed if the negative effect of the rise of the sellers' prices outweighs the positive effect of the increase of consumer awareness through the larger investment by the platforms. It is also shown that the improvement of net consumer welfare through the introduction of price parity clauses is unlikely to occur when the equilibrium shares of the consumers who are aware of the platforms are high, as in the case where the clauses are not allowed or when the consumers are highly differentiated. Moreover, given the possibility that sellers can make investment that improves consumer welfare, the introduction of price parity clauses may harm the consumer welfare through underinvestment by the sellers even when the sellers' prices after the introduction of the clauses does not change.

The result is similar to the dominant platform case analysed in Chapter 4. The analysis in Chapter 4 shows that the narrow price parity clause increases the investment of the platform by preventing the seller's free riding by setting a higher price on the platform, which then increases the platform's commission. With respect to the seller's prices, while the narrow price parity clause weakly increases the direct sales price, it weakly decreases the price of the seller's product sold on the platform. Considering that the possible price rise by the narrow parity clause is limited to the direct sales price and that preventing free riding is a more plausible reason for introducing price parity clauses than avoiding the competition through undercutting commissions, the results of these two chapters are favourable to the approach that narrow price parity clauses should be treated in a more generous manner than are wide price parity clauses, an approach that has been adopted in some European countries such as the United Kingdom and France.

The result also clarifies the limitation of this type of theory of harm in the sense that the rise of platforms' commissions through softening competition by price parity clauses does not always cause the rise of sellers' prices. This is because the willingness of consumers to pay can constrain high prices. This result is same as Johanson and Verge (2016), while other previous studies such as Wang and Wright (2016) and Boik and Corts (2015) find that price parity clauses always raise the prices of sellers. This is because in Johanson and Verge (2016) the platform's ability to set high commissions is bound by supplier's participation constraint", which follows that the equilibrium prices of the sellers do not change. The result in this study shows that when the price parity clauses does not increase the prices, the consumer welfare increases with the introduction of price parity clauses because there is no negative effect from the rise of sellers' prices and a positive effect through the larger investment by the platforms.

However, in this situation, the sellers' welfare can be harmed, because the sellers cannot pass on the rise of the commissions to the consumers.[43]

In this way, according to the result, wide price parity effects have effects similar to those of cartels. However, the result also shows that the negative effect can be offset by the positive effect through the larger investment[44] and the analysis in Section 4.5 shows that the negative effects on consumers and sellers may not be as strong as those of cartels. It follows that the effect-based approach is preferable to *per se* illegal approach, which is adopted in many jurisdictions to judge cartels and in which almost all conduct is regarded as illegal. The result also suggests that when competition authorities considers whether they should intervene in platforms' wide price parity clauses, the fierceness of the competition between the two sellers can be an important factor to evaluate the effects of such clauses on consumers. Although an effect of narrow price parity clauses in the context where there are multiple platforms and direct sales is possible, this investigation falls outside of the scope of this study. Furthermore, the difference between the monopoly platform case and other types of anticompetitive effects of price parity clauses, such as blocking entry, is outside of the scope of this thesis. These topics are worth considering for future research.

[43] This implies that if investment by sellers is taken into account, the negative effect on consumer welfare through the smaller investment by sellers can offset the positive effect on the consumer welfare by the larger investment from the platforms.
[44] However, in most jurisdictions, it is extremely difficult or impossible to justify cartels with this kind of claim.

5.6. Appendix D

5.6.1 Proof of Lemma 5.1

The proof of Lemma 5.1 is divided into three steps. In *Step 1* we find the demand of Seller i's product when all consumers have the same information such that such that the lowest prices of Seller i's product and Seller j's product of which they know are $\underline{p_i}$ and $\underline{p_j}$ respectively. In *Step 2*, we find the total product demand of each seller's product from the consumers who have different information. In *Step 3*, we find the quantity of the products sold on each platform.

Step 1: Product demand when all consumers have the same information

Let $\hat{x}$ denote the location of an indifferent consumer for whom the utility of purchasing Seller i's product at the lowest available price is equal to the utility of purchasing Seller j's product at the lowest available price. In such situation, $q_i = \hat{x}$ and $q_j = 1 - \hat{x}$ hold, implying that $r - \tau\hat{x} - \underline{p_i} = r - \tau(1 - \hat{x}) - \underline{p_j}$ holds. This equation can be modified to $\hat{x} = 0.5 - \frac{\underline{p_i} - \underline{p_j}}{2\tau}$. Substituting this modification into $q_i = \hat{x}$ and $q_j = 1 - \hat{x}$ gives; $q_i = 0.5 - \frac{\underline{p_i} - \underline{p_j}}{2\tau}$.

Note that $0 \leq q_i \leq 1$ must be satisfied. It follows that $q_i = 0.5 - (\underline{p_i} - \underline{p_j})/2\tau$ does not hold if $\underline{p_i} \leq \underline{p_j} - \tau$ or if $\underline{p_i} > \underline{p_j} + \tau$. If $\underline{p_i} \leq \underline{p_j} - \tau$, $q_i = 1$ and if $\underline{p_i} > \underline{p_j} + \tau$, $q_i = 0$.

Moreover, as shown in the dominant platform case, if $\underline{p_i}$ is higher than a certain threshold ($\underline{p_i} > r - \tau$), q_i decreases as $\underline{p_i}$ increases because the consumers whose locations are far from the seller decide not to buy. In that case, q_i is given by $(r - \underline{p_j})/\tau$. It follows that if $(r - \underline{p_j})/\tau < 0.5 - (\underline{p_i} - \underline{p_j})/2\tau$ holds, this constraint is binding. In other words, not purchasing is the more attractive alternative option than switching to buy the other seller's product. The inequality can be modified as $\underline{p_i} > 2r - \tau - \underline{p_j}$. Note that if $r > \underline{p_i}$, q_i becomes zero because $(r - \underline{p_i})/\tau$ becomes negative. This relationship implies that if $r > \underline{p_j} + \tau$ holds, this situation cannot happen, because in this case, the constraint by the other seller's price is always binding. Moreover, if $r - \tau \leq \underline{p_j} - \tau \leftrightarrow \underline{p_j} \geq r$ holds, the constraint by the other seller's price is never binding.

Step 2: Total product demand from consumers who have different information

There are four types of consumers; those who know both Platform k and Platform l, those who know only Platform k, those who know only Platform l, and those who do not know any platform. The proportion of the four types are given by $\lambda_k\lambda_l$, $\lambda_k(1-\lambda_l)$, $(1-\lambda_k)\lambda_l$ and $(1-\lambda_k)(1-\lambda_l)$, respectively.

The demand of i's product from the consumers who know the products sold on both Platform k and Platform l is equal to $\lambda_k\lambda_l q_i\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right)$. The demand from the consumers who know the products sold on only Platform k is equal to $\lambda_k[1-\lambda_l]q_i(p_{ik}, p_{jk})$. The demand from the consumers who know the products sold on only Platform l is equal to $[1-\lambda_k]\lambda_l q_i(p_{il}, p_{jl})$. The demand from the consumers who do not know any platform is zero.

The total product demand of each seller's product from the consumers is the sum of that demand.

Step 3: Platform demand

The consumers who know only Platform k purchase only through the platform. The demand of such consumers is $\lambda_k[1-\lambda_l][q_i(p_{ik}, p_{jk}) + q_j(p_{ik}, p_{jk})]$. The consumers who know only Platform l and the consumers who do not know any platform do not purchase through Platform k.

The consumers who know both Platform k and Platform l will buy through the sales channel that offers the lower price. In the case where the seller sets the same price on the two platforms, the demand is equally divided according to the tie-break rule. Hence, the number of consumers who know both platforms and purchase Seller i's product through Platform k is

$\lambda_k\lambda_l q_i\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right)$ if $p_{ik} < p_{il}$

and $0.5\lambda_k\lambda_l q_i\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right)$ if $p_{ik} = p_{il}$. If $p_{ik} > p_{il}$, the demand is zero.

Similarly, Seller j's product through Platform k is

$\lambda_k\lambda_l q_j\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right)$ if $p_{jk} < p_{jl}$

and $0.5\lambda_k\lambda_l q_j\left(min(p_{ik}, p_{il}), min(p_{jk}, p_{jl})\right)$ if $p_{jk} = p_{jl}$.

The total platform demand is the sum of those demand. ■

5.6.2 Proof of Lemma 5.2

The proof of Lemma 5.2 is divided into two steps. In *Step 1* we prove paragraph *i)* of the lemma, which is about the optimal sellers' platform choices. In *Step 2*, we prove paragraph *ii)*, which is about the optimal sellers' prices.

Step 1: Optimal sellers' platform choices prices

If $f_A \leq r$ and $f_B \leq r$ holds, a seller can obtain non-negative profits through the sales on each platform by setting the price below the marginal cost. In other words, there exist p_{ik} such that $p_{ik} \leq r$ and $q_{ik} > 0$. Taking the symmetry of the two sellers, this implies that both sellers decide to use both platforms.

Step 2: Optimal sellers' prices

The assumptions imply four types of consumers: those who know both Platform k and Platform l, those who know only Platform k, those who know only Platform l, and those who know neither. The proportions of the four types are given by $\lambda_k\lambda_l$, $\lambda_k(1-\lambda_l)$,$(1-\lambda_k)\lambda_l$ and $(1-\lambda_k)(1-\lambda_l)$, respectively.

Under the symmetric equilibrium, the consumers who know the product sold on only Platform k, whose proportion is given by $\lambda_k(1-\lambda_l)$, will purchase Seller i or j's product sold on Platform k, and the consumers who know the product sold on both platforms, whose proportion is given by $\lambda_k\lambda_l$, will purchase Seller i or j's product through the sales channel that offers the lowest price.

The expected profit of the seller under the symmetric equilibrium can be simplified as

$$\Pi_i = \left\{\begin{matrix} \lambda_A q_i(p_{iA}, p_{jA})[p_{iA} - f_A] + (1-\lambda_A)\lambda_B q_i(p_{iB}, p_{jB})[p_{iB} - f_B] & if\ p_{iA} < p_{iB} \\ \lambda_A(1-0.5\lambda_B) q_i(p_{iA}, p_{jA})[p_{iA} - f_A] + (1-0.5\lambda_A)\lambda_B q_i(p_{iB}, p_{jB})[p_{iB} - f_B] & if\ p_{iA} = p_{iB} = p_i \\ \lambda_A[1-\lambda_B] q_i(p_{iA}, p_{jA})[p_{iA} - f_A] + \lambda_B q_i(p_{iB}, p_{jB})[p_{iB} - f_B] & if\ p_{iA} > p_{iB} \end{matrix}\right\}.$$

This profit consists of two terms: the profit on Platform A and that on Platform B, which implies that the optimal price on Platform A must be the price that maximises $q_i(p_{iA}, p_{jA})[p_{iA} - f_A]$, and the optimal price on Platform B must be the price that maximises $q_i(p_{iB}, p_{jB})[p_{iB} - f_B]$.

The analysis of dominant platform case in Chapter 4 implies that there are two types of equilibrium: the equilibrium where sellers set the prices such that all consumers who are aware of at least one seller's product purchase the product (full-coverage price equilibrium) and the equilibrium where sellers set the prices such that all the consumers are not fully covered. Because it is assumed that the second type of equilibrium never happens, the only possible equilibrium is the full coverage price equilibrium. It follows that the platforms will not set commissions higher than $r - \tau$, since if they set such prices, the full coverage price equilibrium will not occur. Hence, hereafter, only consider the case where $f_A \leq r - \tau$ and $f_B \leq r - \tau$ hold.

According to Lemma 5.1, under the full-coverage price equilibrium, $q_i = 0.5 - \frac{p_{ik} - p_{jk}}{2\tau}$ holds. This implies that the expected profits of the two sellers depends on $(\frac{1}{2} - \frac{p_{1k} - p_{2k}}{2\tau})(p_{1k} - f_k)$ and $(\frac{1}{2} - \frac{p_{2k} - p_{1k}}{2\tau})(p_{2k} - f_k)$ respectively.

Those functions are concave quadratic functions of p_{1k} and p_{2k} respectively as long as $r - \tau\hat{x}_k - p_{ik} \geq 0$ holds. The FOCs are given by

$\frac{1}{2} - \frac{2p_{1k} - p_{2k} - f_k}{2\tau} = 0$ and $\frac{1}{2} - \frac{2p_{2k} - p_{1k} - f_k}{2\tau} = 0$.

From these equations, the following response functions can be obtained;

(A5.1) $p_{1k} = \frac{p_{2k} + f_k + \tau}{2\tau}$,

(A5.2) $p_{2k} = \frac{p_{1k} + f_k + \tau}{2\tau}$.

By solving Equation (A5.1) and (A5.2), the symmetric equilibrium price $p_{1k} = p_{2k} = f_k + \tau$ can be obtained.

Note that if $p_{1k} = p_{2k} = f_k + \tau$, $q_{1k} = q_{2k} = 0.5$ holds. Hence, the condition that this equilibrium exists is

$r - \tau\hat{x} - p_{1k} = r - \tau\hat{x} - p_{2k} = r - 0.5\tau - f_k - \tau \geq 0$.

This is because some consumers located near the middle of the Hotelling line will not purchase the product.

This condition can be modified to

(A5.3) $f_k \leq r - 1.5\tau$.

If Inequality (A5.3) does not hold, the maximum price that $q_{1k} = q_{2k} = 0.5$ holds must satisfy $r - 0.5\tau - p_{ik} = r - 0.5\tau - p_{jk} = 0$. Hence, the symmetric equilibrium price is $p_{1k} = p_{2k} = r - 0.5\tau$. In this situation, the equilibrium price does not increase as f_k increases. It follows that if $r - 1.5\tau < f_k < f_l \leq r - \tau$, by setting $p_{1k} = p_{2k} = r - 0.5\tau - \varepsilon$ and $p_{1l} = p_{2l} = r - 0.5\tau$, where ε is small non-negative amount, the two sellers can get more profit than setting $p_{1k} = p_{2k} = p_{1l} = p_{2l} = r - 0.5\tau$.

From the discussion above, the optimal pricing when the sellers sell on both platforms and set the same price on each platform can be written as

$$p_{1k} = p_{2k} = \left\{\begin{array}{cc} f_k + \tau & if\ f_k \leq r - 1.5\tau \\ r - 0.5\tau & if\ r - 1.5\tau < f_k \leq r - \tau\ and\ f_l \leq f_k\ or\ f_l > r - \tau \\ r - 0.5\tau - \varepsilon & if\ r - 1.5\tau < f_k < f_l \leq r - \tau \end{array}\right\}.$$

Since such pricing is the best response to the other seller, there is no incentive for each seller to change their prices. Moreover, if a seller stops using one platform (Platform l), the profit for the seller becomes $\lambda_k(1-\lambda_l)q_{ik}(p_{ik},p_{jk})(p_{ik}-f_k)+\lambda_k\lambda_l(p_{ik},p_{jk})(p_{ik}-f_k)$, which is strictly smaller than the profit when the seller sets the optimal pricing under the symmetric equilibrium. This is because while the profit from the consumers who are aware of only Platform k is the same as the profit under the symmetric equilibrium, in this case, the profit from the consumers who are aware of only Platform l becomes zero, and the profit from the consumers who are aware of both platforms is equal to or smaller than the profit under the symmetric equilibrium. Hence, neither seller has an incentive to stop using one of the two platforms. Therefore, the symmetric equilibrium is the pure strategy equilibrium.■

5.6.3 Proof of Lemma 5.3

The proof of Lemma 5.3 is divided into two steps. In *Step 1* we prove the optimal commissions set by the platforms. In *Step 2*, we prove the optimal investment levels chosen by the platforms.

Step 1: Optimal commissions set by the platforms

Figure 5.4 implies that Platform k's best response given Platform l's commission can be written as follows:

$$f_k=\begin{Bmatrix} r-\tau\ if\ f_l\le(1-\lambda_l)(r-\tau)\ or\ if\ f_l>\ r-\tau \\ f_l-\varepsilon\ if\ (1-\lambda_l)(r-\tau)<f_l\le r-\tau \end{Bmatrix}.$$

This shows that there is no pure Nash equilibrium for the two platforms' commissions. It also shows that $f_k\in[0,(1-\lambda_l)(r-\tau)]$ or $f_k\in(r-\tau,+\infty]$ will never be chosen by the platforms because there exist at least one $f_k\in(r-\tau,+\infty]$ that they can obtain the larger profit when they set such a commission.

Then, we examine the mixed strategy equilibrium in this model. Since $f_k\in([1-\lambda_l][r-\tau],r-\tau)$ only can be equilibrium commission, considering the cumulative distribution function, $H(f_k)$, that is continuous on $([1-\lambda_l][r-\tau],r-\tau)$. Let $h(f_k)$ denote the density function.

In this situation, the expected profit for platform k can be written as

(A5.4) $\pi_k=\int_{(1-\lambda_l)(r-\tau)}^{r-\tau}[(1-H(f_k))\,\lambda_k f_k+H(f_k)\lambda_k[1-\lambda_l]f_k]h(f_k)df_k.$

Note that $h(f_k)\ge 0$, $h(f_k)=0$ for $f_k\notin([1-\lambda_l][r-\tau],r-\tau)$

and $\int_{(1-\lambda_l)(r-\tau)}^{r-\tau}h(f)df\ =1$ must hold.

In the mixed-strategy equilibrium, the expected profit when the platform sets $f_k \in ([1-\lambda_l][r-\tau], r-\tau)$ should be constant. This implies that the following equation holds:

$[1-H(f_k)]\lambda_k f_k + H(f_k)\lambda_k[1-\lambda_l]f_k - c = 0$, where c is a constant which has a positive value, since the platform obtains a positive profit by setting $f_k \in ([1-\lambda_l][r-\tau], r-\tau)$.

This equation can be modified to

(A5.5) $H(f_k) = \frac{1}{\lambda_l}[1 - \frac{c}{\lambda_k f_k}]$.

Since the cumulative distribution function when f_k is equal to the upper bound of the mixed strategy must be 1, $H(r-\tau) = 1$ must hold.

According to Equation (A5.5),

$H(r-\tau) = \frac{1}{\lambda_l}\left[1 - \frac{c}{\lambda_k[r-\tau]}\right] = 1.$

This equation can be modified to

(A5.6) $c = \lambda_k(1-\lambda_l)(r-\tau)$.

By substituting Equation (A5.6) into Equation (A5.5), $H(f_k)$ can be obtained.

Step 2: Optimal investment levels chosen by the platforms

The analysis of the optimal commission shows that the expected profit of each platforms is $\lambda_k(1-\lambda_l)(r-\tau)$.

With regard to the optimal amount of investment, the total expected profit for the platforms can be written as

$\Pi_k = g(I_k)(1-\lambda_l)(r-\tau) - I_k$.

Solving this profit-maximisation problem produces

$g'(I_k)(1-\lambda_l) = \frac{1}{r-\tau}$.

Since the two platforms are symmetric and since the equilibrium investment for both platforms is same, Lemma 5.3 can be obtained. ■

5.6.4 Proof of Lemma 5.5

According to Lemma 5.4, under the full-coverage price equilibrium, $q_{iA} = q_{iB} = 0.5 - \frac{p_i - p_j}{2\tau}$ holds. It follows that the expected profits of the two sellers can be written as

$$\Pi_i = \lambda_A(1-\lambda_B)\left(\frac{1}{2} - \frac{p_i - p_j}{2\tau}\right)(p_i - f_A) + (1-\lambda_A)\lambda_B\left(\frac{1}{2} - \frac{p_i - p_j}{2\tau}\right)(p_i - f_B) + \lambda_A\lambda_B\left(\frac{1}{2} - \frac{p_i - p_j}{2\tau}\right)(p_i - 0.5f_A - 0.5f_B).$$

The FOC is

$$\lambda_A(1-\lambda_B)\left(\frac{1}{2} - \frac{2p_i - p_j - f_A}{2\tau}\right) + (1-\lambda_A)\lambda_B\left(\frac{1}{2} - \frac{2p_i - p_j - f_B}{2\tau}\right) + \lambda_A\lambda_B\left(\frac{1}{2} - \frac{2p_i - p_j - 0.5f_A - 0.5f_B}{2\tau}\right) = 0.$$

Since the two sellers are equivalent, the equilibrium price is supposed to be symmetric. Hence, the equation can be modified as

$$p_1 = p_2 = \tau + \frac{\lambda_A(1-0.5\lambda_B)f_A + (1-0.5\lambda_A)\lambda_B f_B}{\lambda_A + \lambda_B - \lambda_A\lambda_B} = \tau + \overline{f},$$

where $\overline{f} = \frac{\lambda_A(1-0.5\lambda_B)f_A + (1-0.5\lambda_A)\lambda_B f_B}{\lambda_A + \lambda_B - \lambda_A\lambda_B}$.

Note that if $p_1 = p_2 = \overline{f} + \tau$, $q_{1A} = q_{1B} = q_{2A} = q_{2B} = 0.5$ holds. Hence, the condition under which this equilibrium exists is

$$r - \tau\hat{x} - p_1 = r - \tau(1-\hat{x}) - p_2 = r - 0.5\tau - \overline{f} - \tau \geq 0.$$

This inequality holds, because some consumers located near the middle of the Hotelling line will not purchase the product.

This condition can be modified to

(A5.7) $\overline{f} \leq r - 1.5\tau$.

If Inequality (A5.7) does not hold, the maximum price that $q_{1A} = q_{1B} = q_{2A} = q_{2B} = 0.5$ holds must satisfy $r - 0.5\tau - p_1 = r - 0.5\tau - p_2 = 0$. Hence, the symmetric equilibrium price is $p_1 = p_2 = r - 0.5\tau$. In this situation, the equilibrium price does not increase as $\overline{f}$ increases. ■

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