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## **Income poverty, affluence and polarisation viewed from the median**

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# **Income Poverty, Affluence and Polarisation Viewed From The Median**

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## **Abstract**

The present paper brings together different features of the distribution of income – poverty, affluence and dispersion – in a single framework that allows ready comparisons across countries. We believe that such a unified framework contributes both to the policy debate and to the theoretical understanding of inequality. There are at present largely separate literatures on the measurement of poverty, and (to a limited degree) affluence, and on bi-polarization. In relation to the EU social indicators, the paper may be seen as providing complementary information.

Key words: poverty, inequality, polarisation, affluence

JEL classification: I31

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## 1. Introduction

EU-SILC provides a rich source of evidence about the distribution of income in different countries. At the same time, the very richness of the data is a challenge, and it is not surprising that much of the analysis has tended to focus on particular features of the distribution, such as the extent of income poverty or the tendency for the middle of the distribution to be hollowed out. But, as the recent debate about inequality has brought out, it is not enough to look at one single indicator. Our statistics have to be encompassing. This becomes even more important as policymakers become increasingly concerned with linking macro-economic outcomes with their impact on the well-being of individual citizens. It is not enough to replace GDP per head by just another number. As has been well recognised in the design of the EU social indicators, there is need for contextual information.

The aim of the present paper is to bring together different features of the distribution – income poverty, affluence and dispersion – in a single framework that allows ready comparisons across countries and across time. We believe that such a unified framework contributes both to the policy debate and to the theoretical understanding of inequality. The former is well illustrated by the recent media and political interest in inequality generated by the publication of the English translation of Thomas Piketty’s *Capital in the Twenty First Century* (Piketty, 2014). Attention in the debate has focused on the top 1 per cent, and how their share of income is racing away, particularly in Anglo-Saxon countries. But others have asked how this relates to what is happening at the bottom of the income ladder. Do rising top shares have implications for the ambitions of the European Union to reduce the number at risk of poverty or social exclusion under the Europe 2020 agenda? Are the countries with rising top incomes also those that are failing to meet the objective of reducing income poverty and social exclusion? When one turns to the academic arena, one finds too a need to bring together separate debates. There are at present largely separate literatures on the measurement of income poverty, (to a limited degree) affluence, and on bi-polarisation.<sup>1</sup>

In relation to the EU social indicators, the present paper may be seen as providing complementary information. Methods developed at Statistics Norway (set out in Aaberge and Atkinson, 2013) are applied to the EU-SILC data for 2012 to show how these tools extend the concept of contextual indicators. One major purpose of this complementary information is to test the robustness of the conclusions drawn to the choice of indicator. As has been recognised from the outset, there is a degree of arbitrariness to the choice of a particular percentage (60 per cent) of the median as the at-risk-of-poverty threshold. When comparing the progress made by different Member States towards the Europe 2020 targets, we need to know how sensitive the

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<sup>1</sup> We refer to this as ‘bi-polarisation’, to distinguish it from other concepts of polarisation, notably those pioneered by Esteban and Ray (1994, 1999 and 2012) and Duclos, Esteban and Ray (2004).

conclusions are to the percentage cut-off. Has a country achieved a substantial reduction in income poverty by ‘gaming the system’, concentrating financial help on those nearest to the cut-off? How should the evolution of the income poverty rate be seen in terms of the changes in the income distribution as a whole? For these purposes, the existing contextual EU indicators, the quintile share ratio and the Gini coefficient, while together informative, may not be sufficient. In particular, they do not address two of the issues that have surfaced in recent debate: the ‘squeezing of the middle’ and the ‘racing away’ of the top 1 per cent.

## 2. Poverty, affluence and dispersion in theory

The key elements in the approach adopted here are familiar ones. They are characterised by the keywords: graphics, dominance, and cumulation. All three have a long history, having been embodied in the Lorenz curve introduced in 1905. The Lorenz curve is a graphic device. It is based on cumulating people and incomes from the bottom; and it allows us to see whether one distribution is Lorenz-superior to (dominates) another. Where we depart is in taking the median as a point of reference. In a sense, we are following a trend. As has been widely recognised, with the rise in inequality at the top in a number of countries, notably the United States, the mean has become a less satisfactory indicator of overall progress, and attention is turning to the median. As it was put by the Stiglitz Commission, ‘median consumption (income, wealth) provides a better measure of what is happening to the ‘typical’ individual or household than average consumption (income or wealth)’ (Stiglitz et al, 2009, pages 13-14 of Executive Summary). In the literature on the ‘middle class’, this group is typically defined in terms of a range around the median. Following the recommendation of the Eurostat Task Force (1998) on social exclusion and poverty statistics, the EU social indicators for income poverty (referred to at EU level as *poverty risk*<sup>2</sup>) are based on an income threshold defined as a percentage of the median, rather than the mean as had previously been employed (see Atkinson et al, 2002, page 94).

### 2.1 *The median and poverty measurement*

How is the median taken as a point of reference? We start from the fact that, in the countries covered by EU-SILC, poverty is a minority phenomenon. No one would consider poor a person with income (by which we mean equivalised disposable income per person in the household) at the median. If we define  $z$  as income relative to the median, then the poverty line is set at  $z^*$ , where  $z^*$  is below 1, and the poverty headcount is  $F(z^*)$ , where  $F()$  is the cumulative distribution. Whereas there may be a range of views about the choice of  $z^*$ , there is general agreement that  $z^*$  should be below some  $z_+$ , where  $z_+ < 1$ .

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<sup>2</sup>

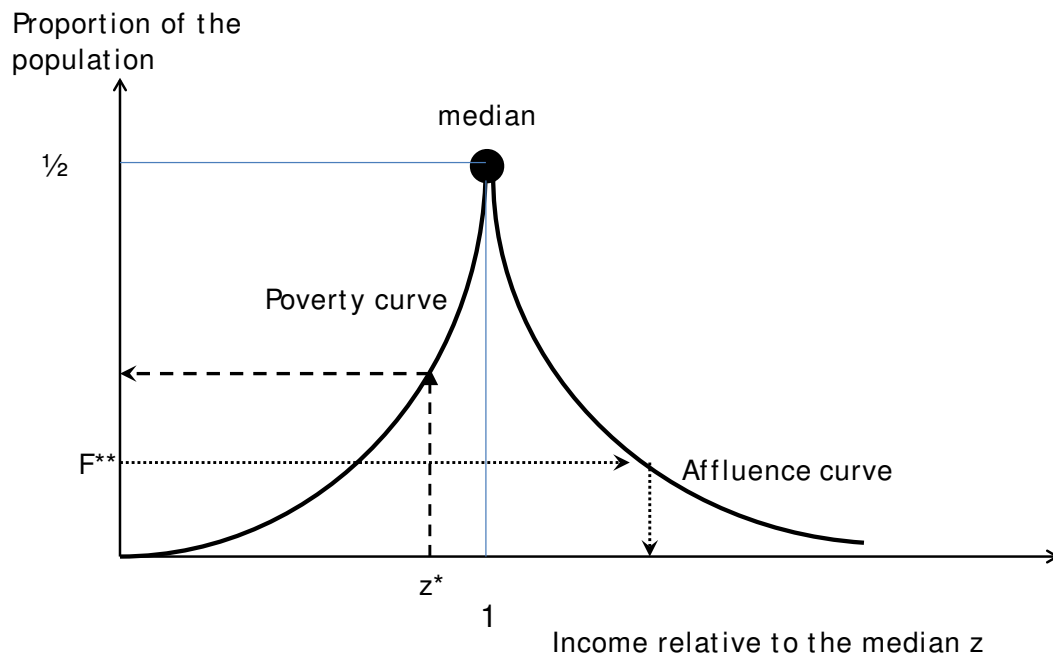
In this paper, ‘poverty’ always refers to income poverty.

The distribution of income below the median is illustrated in left hand part of Figure 1, which shows  $F(z)$  from 0 to  $\frac{1}{2}$  at  $z = 1$  (the median). For any  $z^*$  we can read off the headcount from the vertical axis, as shown by the dashed lines. The maximum poverty line  $z_+$  demarcates the range of permissible poverty lines. If for two countries the curves do not intersect in the range from 0 to  $z_+$ , then the lower curve dominates and we can conclude that there is a lower rate of income poverty for all permissible poverty lines. The first important point to be noted is that the poverty line is defined in primal space: i.e. income. We define poverty in terms of income below a specified level and the unknown is the percentage of people. An alternative would be to define poverty as people in the bottom  $x$  per cent, when the unknown would then be the income at the  $x$ -th percentile. This ‘dual’ approach is not one that has been adopted in the EU at-risk-of-poverty indicators, although it is widely used when investigating the distribution of earnings, when the OECD and others report the earnings at the bottom decile (as a proportion of the median). In what follows, though, the distinction between primal and dual approaches runs through the paper.

## 2.2 *Affluence*

The left hand part of Figure 1 is familiar. Eurostat publishes the Dispersion around the at-risk-of-poverty threshold, taking cut-offs of 40, 50, 60 and 70 per cent. The right hand part of Figure 1 is less familiar. This construction, which is due originally to Foster and Wolfson (1992/2010) (we have simply turned their Figure 9 upside down) shows the half of the distribution above the median in the form of  $(1-F(z))$  for  $z \geq 1$ . In effect, this inverts the upper half of the cumulative distribution, showing the proportion of people *above* any given threshold. Concern with ‘affluence’ is commonly presented in terms of the top 10 per cent or the top 1 per cent. In terms of the distinction drawn in the previous paragraph, this approaches the measurement of affluence from the perspective of the dual. In Figure 1, as shown by the dashed lines, it means starting from a given percentage on the vertical axis, such as  $F^{**}$ , and reading across to the income required to enter this group. For example, from the World Top Incomes Database (<http://topincomes.g-mond.parisschoolofeconomics.eu>) one can see that, in France in 2009, to appear in the top 1 per cent of gross incomes it was necessary to have an income 4.8 times the mean.

**Figure 1: Poverty and affluence curves**



Reading Note: The left hand ('poverty') curve shows the proportion of the population with income less than or equal to a poverty defined relative to the median; the right hand ('affluence') curve shows the proportion of the population with income equal to or above an affluence threshold defined relative to the median.

There are however good reasons for considering a primal approach to measuring affluence. Not only does this parallel the approach adopted in the measurement of poverty, but defining a cut-off above which people can be classified as 'rich' allows the proportion of rich people to vary. There will always be a top 1 per cent, but a society concerned about the distance between the top and the bottom may seek to reduce the number of people with incomes above the 'affluence' cut-off.<sup>3</sup> Such an approach to defining affluence has been adopted by Peichl, Schaefer and Scheicher (2010, page 608), who take the richness line to be twice the median, describing it as 'arbitrary but common practice', whereas Brzezinski (2010) also considers lines equal to three and four times the median.

Again, we may apply a dominance test to the affluence curve,  $1-F$ . Suppose that we are agreed that the affluence threshold  $z^{**}$  is no lower than  $z_-$ . Where the curve for one country lies everywhere below that for another country for all  $z \geq z_-$ , then the affluence score is lower for all cut-offs. This is important, since, as the examples above suggest, there is less agreement about the appropriate threshold. It may for example be agreed that a person cannot be rich unless they have at least twice the median ( $z_- = 2$ ), but people disagree whether  $z^{**}$  should be 2, 3, 4 or higher.

<sup>3</sup> Reasons why societies may be concerned with the top of the distribution are discussed in Atkinson (2007).



### 2.3 *Intersection and cumulation*

Application of the principle of dominance only allows us to make definite comparisons in cases where the relevant curves do not intersect. The ranking can only be extended by attaching a weighting. In the inequality measurement literature, this has proceeded by cumulation, based on the assumption that a higher weight is attached to those who appear earlier in the sum (or integral). This allows us to move from first-degree dominance (of the cumulative distribution) to second-degree dominance (of the Lorenz curve). The crucial question then concerns the starting point for the cumulation. When measuring poverty, it is natural to cumulate from the lowest income, attaching most weight to the poorest. This is the procedure embodied in the Lorenz curve, and the basis for Lorenz dominance is that it ranks distributions in the same way as all social welfare functions where the marginal valuation of income falls (or does not increase) with income. It follows that, if the poverty curve for country A starts out above that for country B, then it can never dominate.

In contrast, when measuring affluence we may wish to attach most weight to transfers affecting those at the top of the income scale.<sup>4</sup> This means cumulating downwards, as proposed in Aaberge (2009). In terms of Figure 1, it means integrating from the right. If the affluence curve for country A ends above that for country B, then it can never dominate.

### 2.4 *Specific measures*

In order to make a complete ranking, and attach numerical values, further assumptions have to be made so as to yield a specific indicator. Table 1 shows the different indicators employed here, where, as already signalled, we consider both primal and dual approaches. In arriving at specific indicators, the first key assumption is an independence axiom, which ensures linearity of the indicator in the relevant variable ( $F$  in the case of the first line in Table 1). The axiom takes a different form in the primal and dual cases. The second assumption is that the remaining part of the indicator should be a power function, leaving the choice of the parameter  $k$  that determines how rapidly the weights fall away. The effect of weighting may be seen in the case of the first indicator, which is the integral of poverty headcounts measured at each value of  $z$  ( $z$  is equal to  $x/M$ ) from 0 to 1, weighted by the gap from the median  $(1-z)$  to the power of  $(k-1)$ . This primal indicator of poverty may therefore be viewed as corresponding to the Foster-Greer-Thorbecke (FGT) poverty measure; the dual indicator shown in the second line of Table 1 corresponds to the Sen (1976) poverty measure.<sup>5</sup> Where  $k=1$ , the two indicators are equal, but for  $k$  greater than 1 the two indicators diverge. In both cases, the weights vary between  $k$  (at  $z=0$ ,  $F=0$ ) and 0 (at  $z=1$ ,  $F=1/2$ ), but the pattern of weighting is different. With the primal indicator, with  $k=2$ , a person with zero income has a weight of 2, but a person with an income equal to

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<sup>4</sup> It should be noted that, in contrast to Peichl, Schaefer and Schleicher (2010), we are assuming that the principle of transfers applies.

<sup>5</sup> See Aaberge and Atkinson (2013). By replacing the median  $M$  with a poverty threshold  $T$  less than  $M$ ,  $\Psi_k$  coincides with the FGT poverty measure of power  $k$  and  $\Pi_k$  coincides with a modified version of the Sen poverty measure.

half the median has half the weight. With the dual indicator, half weight would be reached when we are at the lower quartile, which is typically further up the distribution. If this is the case, then it explains why primal measures may be more sensitive to outliers than dual measures.

**Table 1: Summary indicators**

**Poverty**

$\Psi_k$	$k \int_0^1 (1-z)^{k-1} F(Mz) dz$	Primal: weight = income gap from median
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$\Pi_k$	$k \int_0^1 (1-2F(Mz))^{k-1} (1-z) dF(Mz)$	Dual: weight = rank in bottom half of distribution (median = 0)
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**Affluence**

$A_k$	$k \int_1^\infty (z-1)^{k-1} (1-F(Mz)) dz$	Primal: weight = income minus median
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$\Gamma_k$	$k \int_1^\infty (2F(Mz)-1)^{k-1} (z-1) dF(Mz)$	Dual: weight = rank from top down in top half of distribution (median = 0)
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Notes:

1.  $x$  denotes income;  $M$  denotes the median;  $k$  is a parameter;  $F$  denotes the cumulative distribution;  $t$  denotes rank.
2. The formulae for the affluence indices apply only to values of  $k$  for which the integral converges.

### 3. Poverty and affluence in EU-SILC

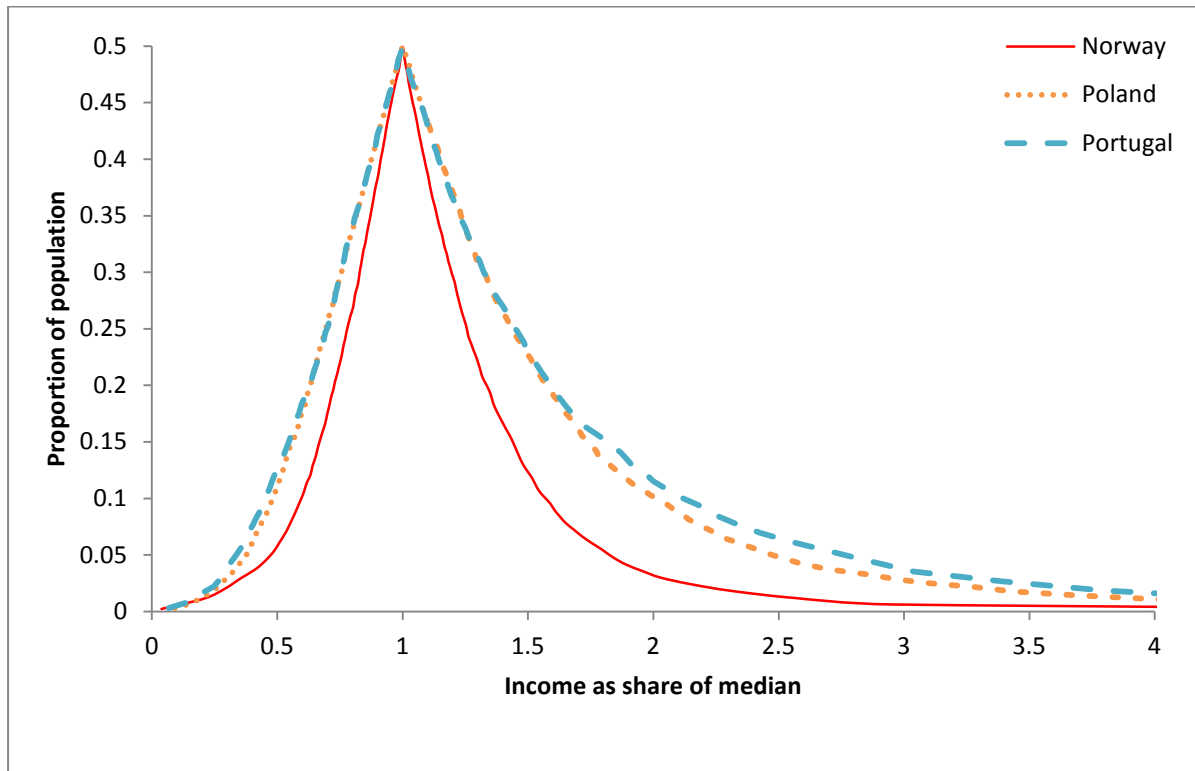
The approach described above has been implemented using the EU-SILC data for 2012. These data refer in general to the income year 2011 (exceptions are Ireland and the UK; see Paper 2). Negative incomes have been set to zero. All households with missing income data and those consisting only of students have been excluded. The fact that we use one year as the analytical period instead of a life-cycle perspective means that we are unable to capture the full economic value of being a student. Students partly live on loans justified by higher expected income in the future. Students' low cash income is temporary and thus will not be considered to be associated with poverty. This practice is consistent with the (national) official poverty statistics in several countries.

#### 3.1 Dominance

We begin with Figure 2, which illustrates well three considerations. It shows the poverty and affluence curves for Norway, Poland and Portugal. In each case, the poverty curves are for the full range  $z$  from 0 to 1, and the affluence curves from  $z = 1$  to  $z = 5$ . The curves meet at  $(1, \frac{1}{2})$ . The first two considerations are methodological. First, there is considerable 'noise' at the tails of the distribution. The same occurs (but is less obvious in Figure 2) as the median is approached. From the standpoint of

considering dominance, this suggests that the dominance condition should be applied to a restricted range. On a primal approach, we should limit the range of  $z$  over which dominance is tested.

**Figure 2: Poverty and affluence curves for Norway, Poland and Portugal, 2012**



Reading Note: The solid curves relate to Norway and show that over much of the income range poverty and affluence are both lower than in Poland or Portugal.  
 Source: EU-SILC cross-section 2012 UDB from August 2014.

The second point concerns the statistical criterion for ranking. As may be seen from Figure 2, the poverty curves for Poland and Portugal are virtually indistinguishable over much of the range and we would not expect a statistical test, taking account of the sampling error, to reject the hypothesis that the poverty curves coincide (over a restricted range). However, as argued in Atkinson, Marlier, Montaigne and Reinstadler (2010), sampling error is not the only consideration when considering the policy significance of differences in poverty rates. When examining changes over time, Atkinson et al (2010) took a two percentage point difference as salient, and the same practice is followed here. A country is said to dominate another where the poverty/affluence curve is at least 2 percentage points lower at some point and is nowhere more than 2 percentage points higher. No ranking can be made where the differences are everywhere less than 2 percentage points ('identical'), or where both countries are at some point at least 2 percentage points lower ('intersecting'). (Alternatively the dominance condition could be stated in terms of differences measured horizontally.)

The third point is substantive. It may be seen by eye from Figure 2 that the curves for Norway dominate over most of the range in both directions. Fewer people proportionately in Norway are below any poverty threshold; and fewer people are above any affluence threshold. The poverty curve for Poland is slightly below that for Portugal. Also, the affluence curve for Poland lies clearly inside that for Portugal for much of the income range. We have therefore a clear picture of the differences between the distributions in the three countries, which can be summarised as follows, where the Table 2 should be read horizontally.

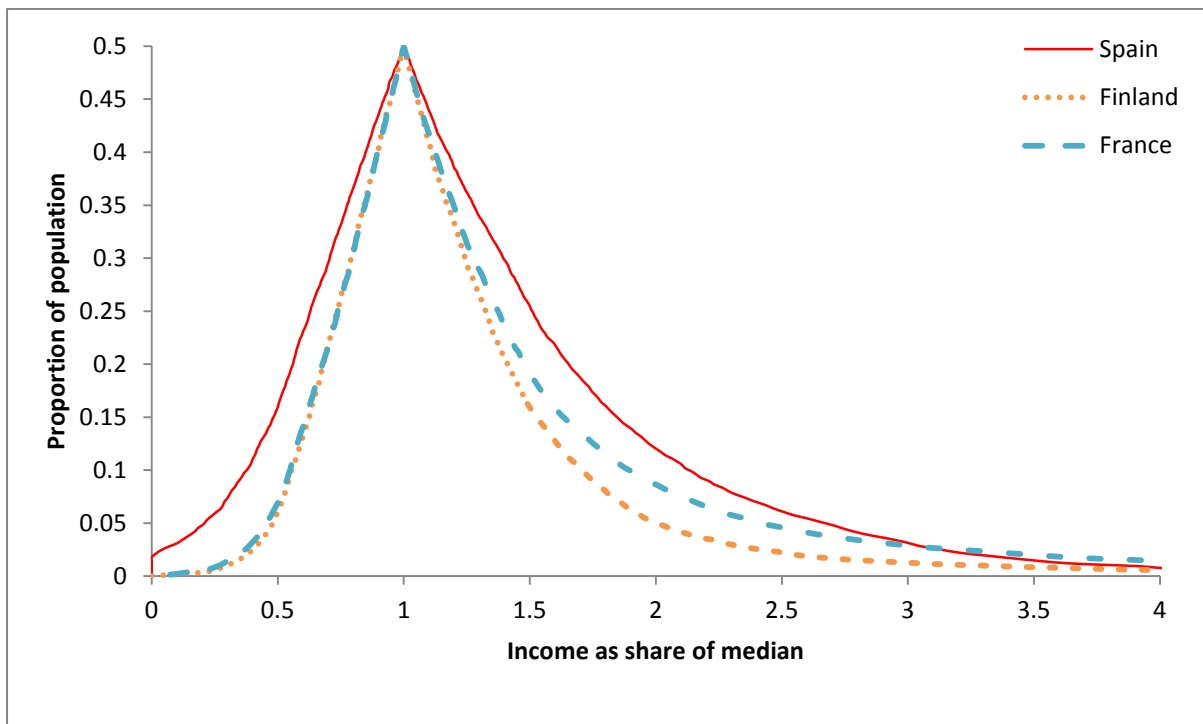
**Table 2: Ranking of Norway, Poland and Portugal, 2012**

	Poland	Portugal
Norway	Dominant on poverty Dominant on affluence	Dominant on poverty Dominant on affluence
Poland		Dominant on poverty Dominant on affluence

Reading Note: Dominant on X means less X.

A selection of results for other countries is shown in Figures 3 to 5, where in each case we compare three countries. Figure 3 compares Finland, France and Spain. As would be expected from the published Eurostat figures, the poverty curve for Spain is well outside those for the other two countries at 60 per cent of the median, and this is true throughout the range of  $z$ . The poverty curves for Finland and France, on the other hand, seem indistinguishable. In contrast, the affluence curve for Finland lies inside those for the other two countries for most of the range. On the other hand, the affluence curves for France and Spain intersect, suggesting that there are more rich households in France for cut-offs above 3 times the median. Figure 4 compares Austria, Bulgaria and Germany. In this case, Bulgaria clearly lies outside on both sides of the median. Above the median, Austria and Germany appear to be indistinguishable, but below the median the poverty curves intersect. At low levels of the poverty cut-off, Germany has a lower poverty rate, so that it cannot be dominated by Austria. However, the poverty curves intersect well before we reach 50 per cent of the median, leading to Austria performing better on the AROP indicator. Figure 5 compares Slovakia and the UK. Slovakia clearly performs better in terms of the affluence curve.

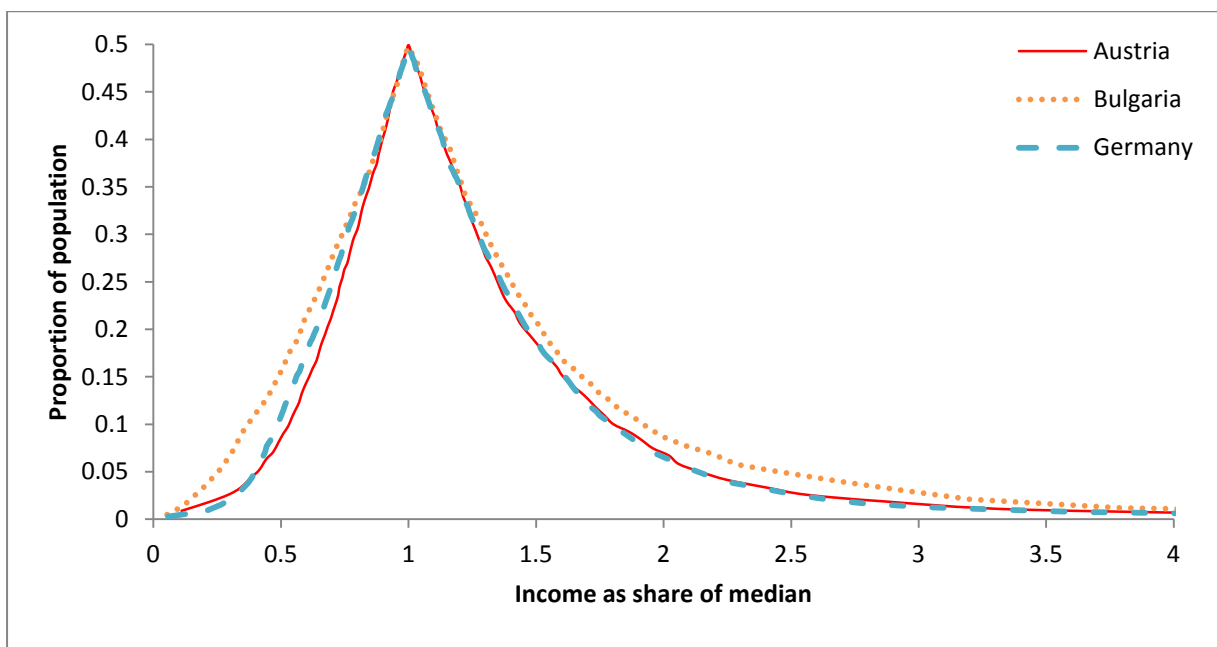
**Figure 3: Poverty and affluence curves for Spain, Finland and France, 2012**



Reading Note: The solid curves relate to Spain and show that over all of the income range poverty is higher than in Finland or France, and that affluence is higher over much of the income range.

Source: EU-SILC cross-section 2012 UDB from August 2014.

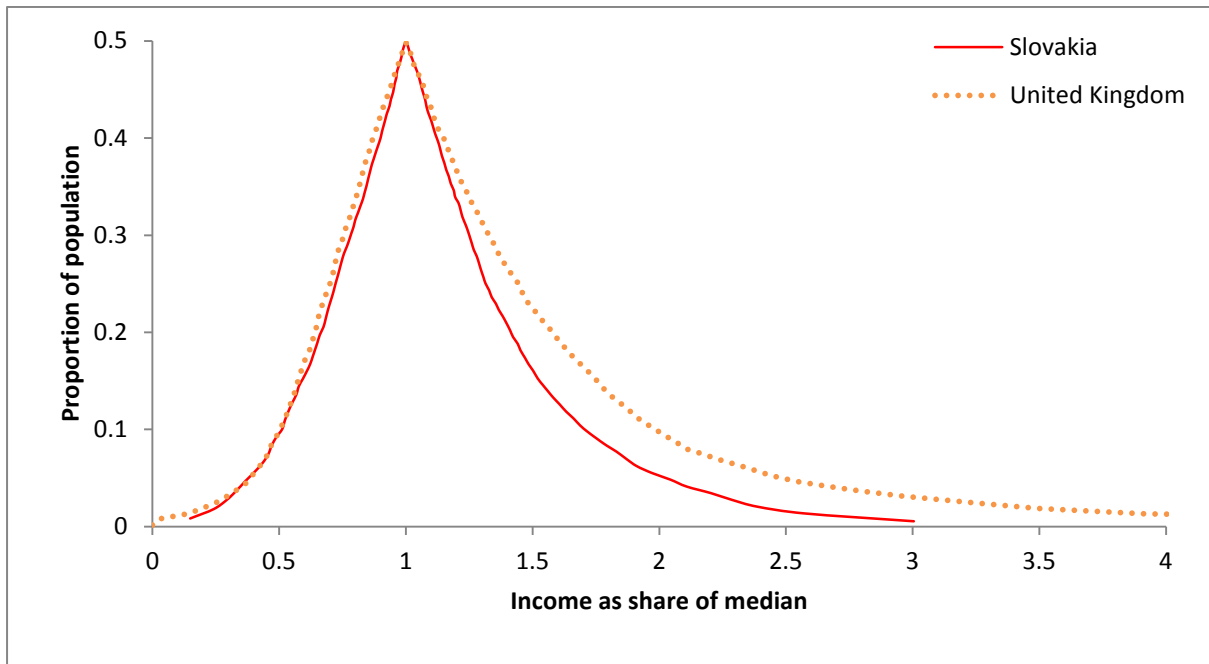
**Figure 4: Poverty and affluence curves for Austria, Bulgaria and Germany, 2012**



Reading Note: The dotted curves relate to Bulgaria and show that over much of the income range poverty and affluence are higher than in Austria and Germany.

Source: EU-SILC cross-section 2012 UDB from August 2014.

**Figure 5: Poverty and affluence curves for Slovakia and the UK, 2012**



Reading Note: The solid curves relate to Slovakia and show that over much of the income range poverty and affluence are lower than in the United Kingdom.

Source: EU-SILC cross-section 2012 UDB from August 2014.

As the above examples show, in some cases we have clear rankings, but not in others. Table 3 summarises the rankings obtained applying the criteria described above for 29 countries, where dominance is defined over the ranges  $0 \leq z \leq 0.75$  (poverty) and  $1.5 \leq z \leq 5$  (affluence).<sup>6</sup> The 29 countries are 26 of the 28 EU Member States (no data were available for Belgium and Ireland) plus Iceland, Norway and Switzerland. The first conclusion is that the application of these dominance criteria yields a clear ranking in the great majority of cases: for the 406 possible comparisons for each of poverty and affluence, there are simply 50 question marks for the poverty curves, and only 26 in the case of the affluence curves. In the case of Norway, for example, there is a clear dominance with regard to the poverty curve over all countries apart from Finland, Iceland, Luxembourg, Malta, and the Netherlands. The second conclusion is that there are a surprising number of cases (57 in all) where there is a clear ranking but it is in opposite directions for poverty and affluence. The UK, for example, has less poverty, but more affluence, than Italy and Estonia.

<sup>6</sup> It should be noted that sample sizes may be small at high values of  $z$ .

**Table 3: Dominance of affluence (upper row) and poverty curves (lower row), 2012**

	IS	NO	NL	CZ	SI	FI	FR	CY	SE	AT	CH	LU	HU	MT	DK	SK	DE	UK	PL	PT	LT	HR	IT	EE	BG	LV	ES	RO	GR	
IS	-	+	+	-	+	+	+	-	+	+	+	+	+	+	?	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
NO	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
NL			-	-	?	+	+	-	+	+	+	+	+	+	-	?	+	+	+	+	+	+	+	+	+	+	+	+	+	+
CZ				+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
SI					-	+	+	+	?	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
FI						+	+	-	+	+	+	+	+	+	-	?	+	+	+	+	+	+	+	+	+	+	+	+	+	+
FR							+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	?	+	+	+	+	+	+	?	+
FR							+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
CY								-	-	-	-	-	-	-	-	-	-	+	+	+	+	?	-	+	?	+	+	+	+	
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Reading Note: The first row compares Iceland with other countries. The entry in the second column of the first row compares Iceland and Norway: the minus sign in the upper part means that affluence is higher in Iceland; the plus sign in the lower row means that poverty is lower in Iceland.

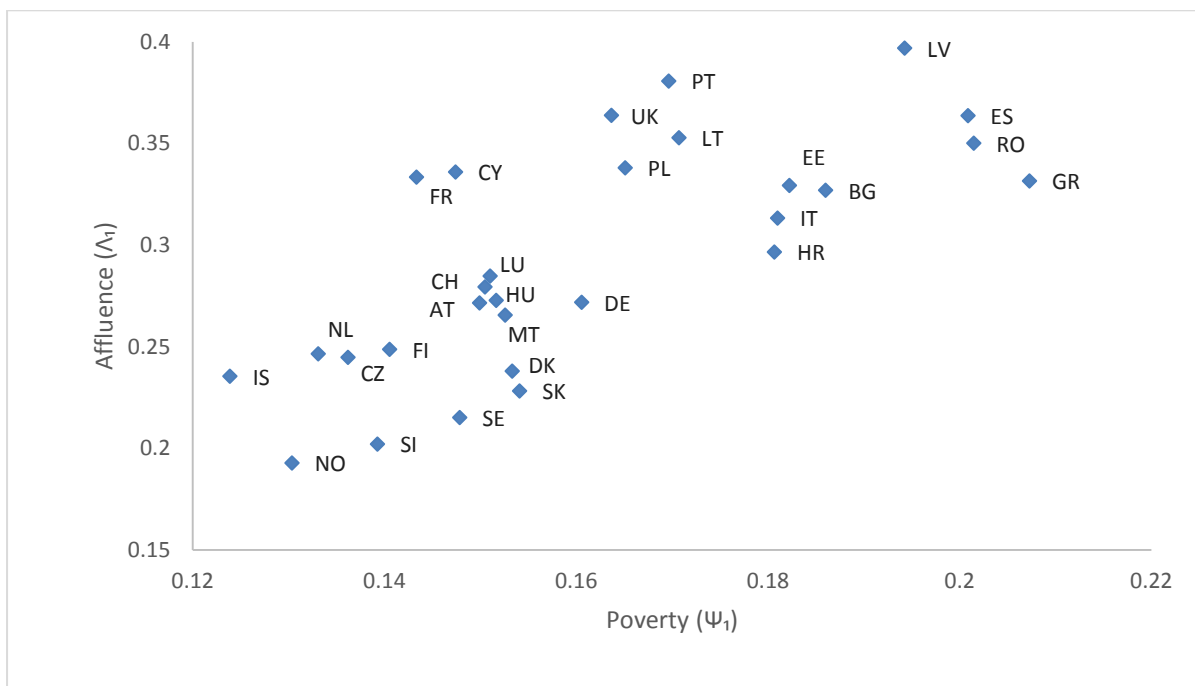
Source: EU-SILC cross-section 2012 UDB from August 2014.

### 3.2 Summary measures

The indices described in Section 2 (Table 1) may be used to summarise the performance of different countries in the poverty and affluence dimensions. Figure 6 shows the ranking of the 29 countries using values of  $k=1$ , which is the gap measure (where the primal and dual coincide). Many countries are ranked similarly for poverty and affluence. These include Norway and Slovenia, with low scores (high rankings), Austria, Germany and Switzerland in the middle, and Romania and Spain with high scores (low rankings). But there are countries that perform better on poverty than on affluence. Portugal and the UK, for example, have high affluence scores but do better in terms of their poverty ranking. France and Cyprus are much better performers in terms of poverty than of affluence.

How sensitive are these rankings to the choice of index? Figures 7a and 7b show the primal and dual indices for poverty starting from  $k=1$ , but then considering the higher values of  $k=2$ ,  $k=3$  and  $k=4$ . As may be seen, there are some changes in rankings, and there is some indication that the dual measures are less sensitive to the choice of  $k$ . Figure 8 shows that the primal measures of affluence are much more affected. (In considering these results, one has to ask how far they are influenced by the use of different data sources. It is possible that the register countries have more extensive coverage of higher incomes.)

**Figure 5.6: Comparing measures of affluence and poverty, 2012**

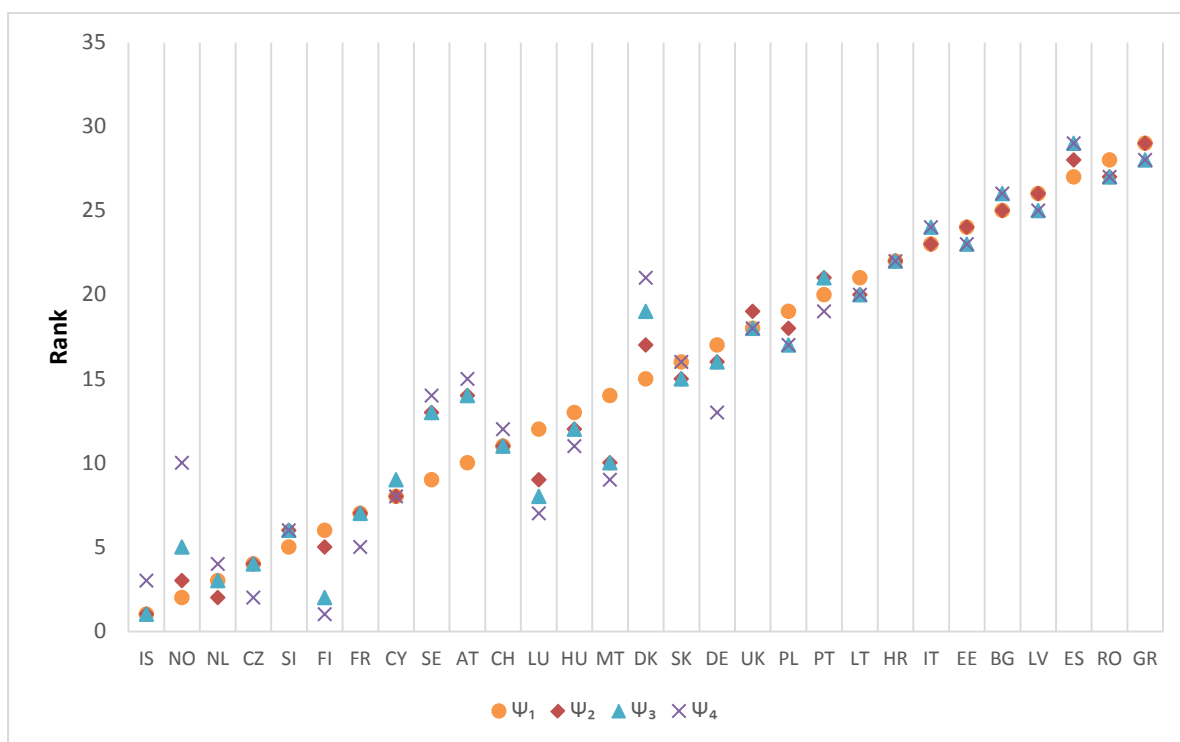


Reading Note: Iceland is ranked first with the poverty measure, whereas Norway is ranked first with the affluence measure.

Source: EU-SILC cross-section 2012 UDB from August 2014; no data for BE and IE.



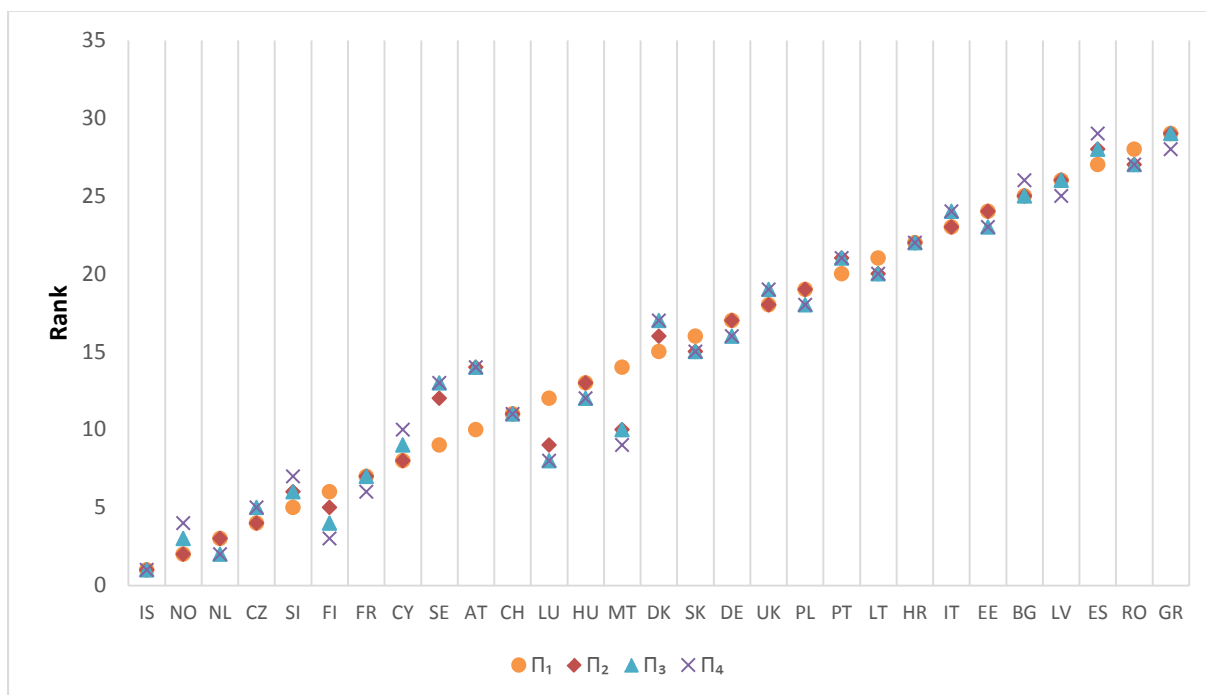
**Figure 7a: Ranking by primal measures of poverty, 2012**



Reading Note: Iceland is ranked first with the primal poverty measures  $\Psi_1, \Psi_2$  and  $\Psi_3$ , whereas Finland is ranked first with  $\Psi_4$ .

Source: EU-SILC cross-section 2012 UDB from August 2014.

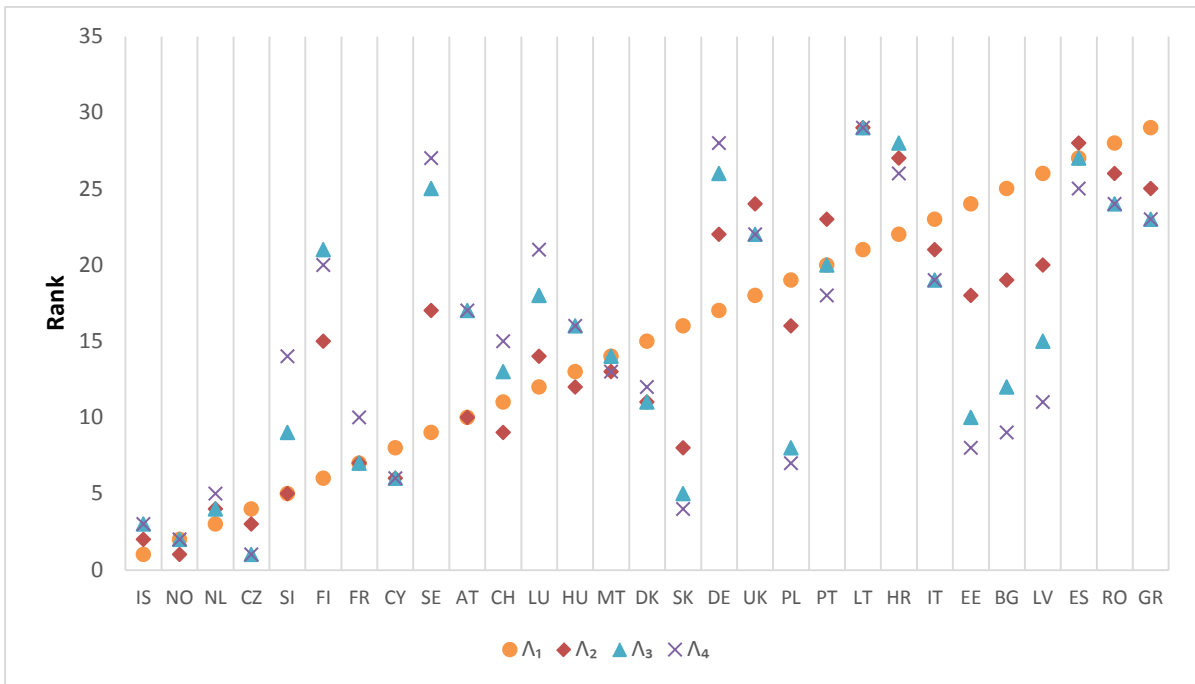
**Figure 7b: Ranking by dual measures of poverty, 2012**



Reading Note: Iceland is ranked first with all four dual poverty measures.

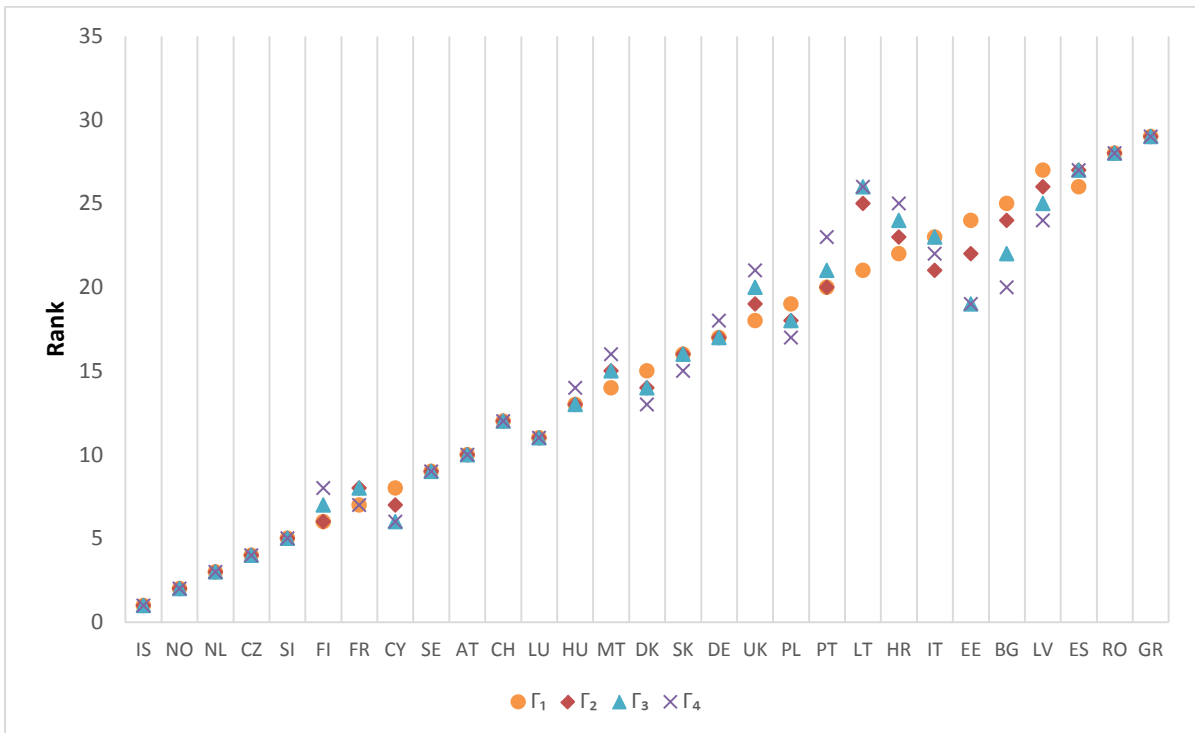
Source: EU-SILC cross-section 2012 UDB from August 2014.

**Figure 8a: Ranking by primal measures of affluence; k=1 to 4, 2012**



Reading Note: Iceland is ranked first with the affluence measure  $A_1$ , Norway is ranked first with the affluence measure  $A_2$  and the Czech Republic is ranked first with the affluence measures  $A_3$  and  $A_4$ .  
 Source: EU-SILC cross-section 2012 UDB from August 2014.

**Figure 8b: Ranking by dual measures of affluence, 2012**



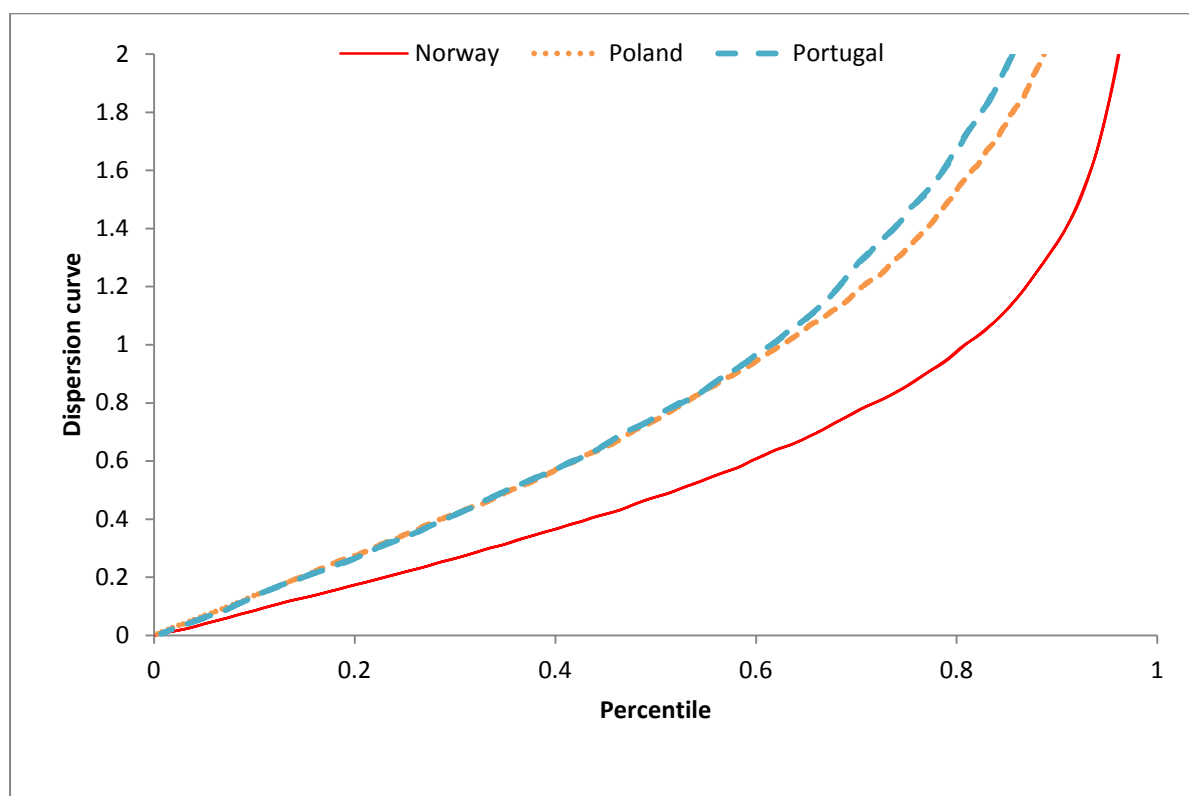
Reading Note: Iceland is ranked first on all four of the values of the dual affluence measure.  
 Source: EU-SILC cross-section 2012 UDB from August 2014.

#### 4. Dispersion, bi-polarisation and tail-heaviness

In this section, we bring together the two curves shown earlier in Figure 1. We focus on a special case of the general notion of dispersion given by Bickel and Lehmann (1979, page 34). We define dispersion in terms of the distance between the affluence and poverty curves. The distance in terms of income (defined relative to the median) between percentiles equi-distant from the median, indexed by  $t$ , where  $t$  runs from 0 (at the median) to 0.5, gives a measure of the spread of the income distribution. Since this dispersion curve is defined in terms of the percentiles, we refer to it as a dual measure. For formal definitions of dispersion, bi-polarisation and tail-heaviness curves and associated summary measures we refer to Aaberge and Atkinson (2013).

The dispersion curve combines what we have learned separately from the poverty and affluence curves, so that it is not surprising that they confirm what we have already found. In Figure 9a, the dispersion curves show that Norway is less dispersed than Poland, and Poland in turn is less dispersed than Portugal. In Figure 9b, Finland is the least dispersed, and Spain the most dispersed, with the dispersion curve for France moving from one towards the other.

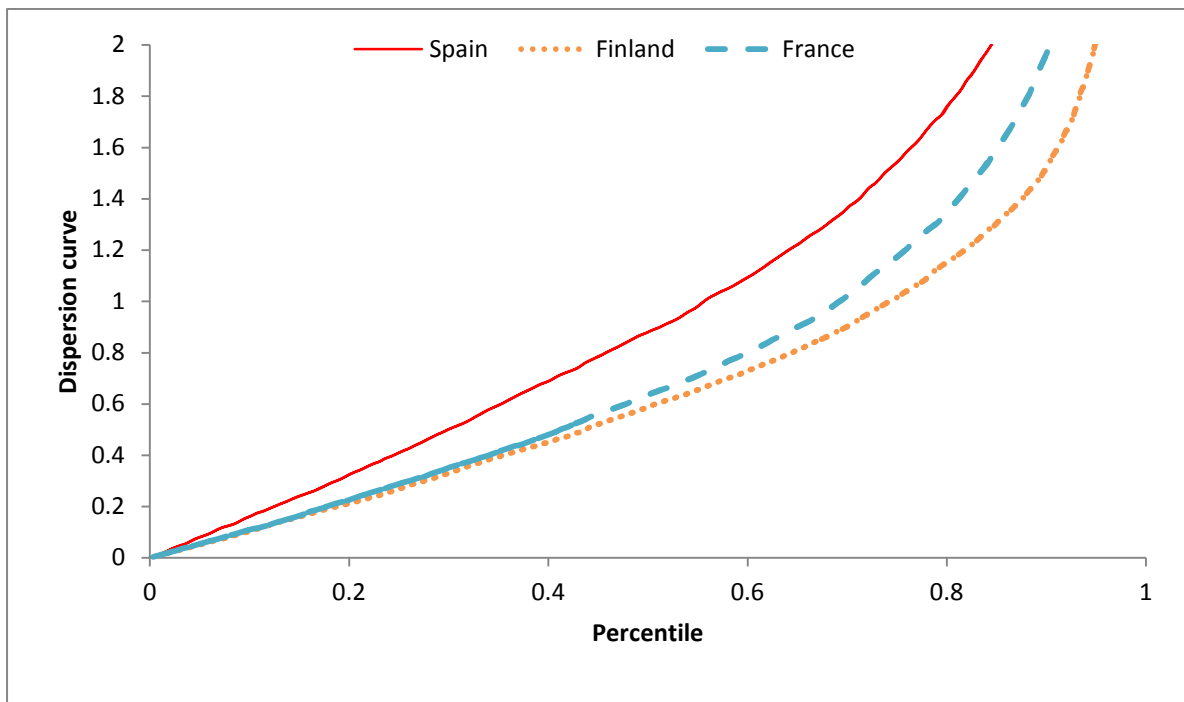
**Figure 9a: Dispersion curves for Norway, Poland and Portugal, 2012**



Reading Note: The solid curve relates to Norway and show that over all of the income range dispersion is lower than in Poland and Portugal.

Source: EU-SILC cross-section 2012 UDB from August 2014.

**Figure 9b: Dispersion curves for Spain, Finland and France, 2012**



Reading Note: The solid curve relates to Spain and show that over all of the income range dispersion is higher than in Finland and France.

Source: EU-SILC cross-section 2012 UDB from August 2014.

Suppose however that we wish to go further and to cumulate the distance measure. As noted in Section 2, the cumulation can be from the bottom or from the median. Cumulating from the bottom is equivalent to cumulating from the tails, and this is in the same direction as for the separate poverty and affluence measures. As discussed in Aaberge and Atkinson (2013), this is related to the concept of tail-heaviness (Doksum, 1969, page 1169): the measures of tail-heaviness are the sum of the measures of poverty and affluence. Put differently, we can see the measures of poverty and affluence as decomposing total tail-heaviness. In Norway in 2012 for example total tail heaviness, with  $k = 1$  (when the primal and dual measures coincide), was 0.32 and this was made up of 0.13 from poverty and 0.19 from affluence (figures rounded). The Czech Republic has a similar score for poverty but 0.49 for affluence.

Table 4 shows the decomposition for the 29 countries for  $k=1$ , ranked in order of tail-heaviness. The results provide valuable diagnostic information. For 10 of the 29, the tail-heaviness score exceeds 1. Of these, three countries (Spain, Latvia and Romania) have both a relatively high poverty score (in excess of 0.17) and a high affluence score (in excess of 0.33). Three (Bulgaria, Estonia and Greece) have a relatively high poverty score; the remaining four (Lithuania, Poland, Portugal and the UK) are tail-heavy on account of their relatively high affluence score. At the same time, it is clear that countries are in general ranked very similarly for poverty and affluence. It is not the case that countries can score well on poverty while be quite ‘relaxed’ about high levels of affluence.

**Table 4: Decomposition of tail-heaviness with respect to poverty and affluence, 2012**

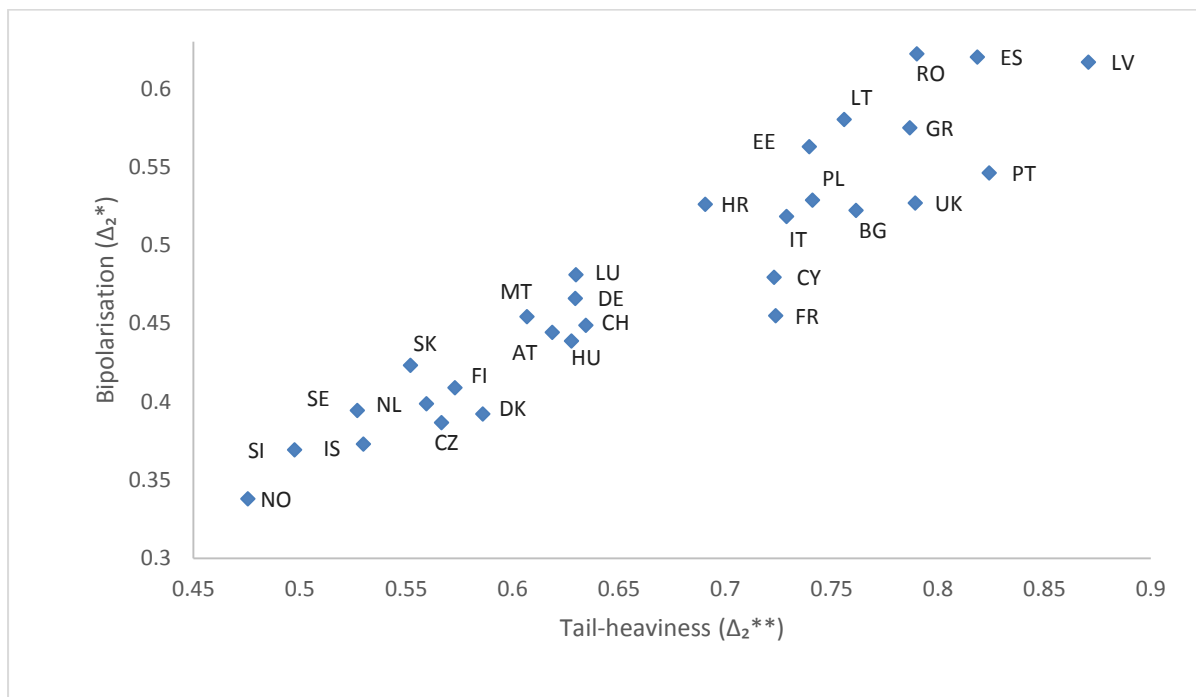
Country	Poverty ( $\psi_1$ )	Affluence ( $\lambda_1$ )	Tail-heaviness
Norway	0.13	0.19	0.32
Slovenia	0.14	0.20	0.34
Iceland	0.12	0.24	0.36
Sweden	0.15	0.22	0.36
Czech Republic	0.13	0.25	0.38
Netherlands	0.14	0.24	0.38
Slovakia	0.15	0.23	0.38
Denmark	0.14	0.25	0.39
Finland	0.15	0.24	0.39
Malta	0.15	0.27	0.42
Austria	0.15	0.27	0.42
Hungary	0.15	0.27	0.42
Switzerland	0.15	0.28	0.43
Germany	0.16	0.27	0.43
Luxembourg	0.15	0.28	0.44
France	0.14	0.33	0.48
Bosnia	0.18	0.30	0.48
Cyprus	0.15	0.34	0.48
Italy	0.18	0.31	0.49
Poland	0.17	0.34	0.50
Bulgaria	0.18	0.33	0.51
Estonia	0.19	0.33	0.51
Lithuania	0.17	0.35	0.52
United Kingdom	0.16	0.36	0.53
Greece	0.21	0.33	0.54
Portugal	0.17	0.38	0.55
Romania	0.20	0.35	0.55
Spain	0.20	0.36	0.56
Latvia	0.19	0.40	0.59

Reading Note: The poverty and affluence measures are defined in Table 5.1, with  $k=1$ . Tail-heaviness is the sum of these two measures. For Latvia, the poverty measure is 0.19 and the affluence measure is 0.40, giving a tail-heaviness measure of 0.59.

Source: EU-SILC cross-section 2012 UDB from August 2014.

The tail-heaviness measure cumulates from the tails. Cumulating from the median, on the other hand, yields the measures of bi-polarisation (Foster and Wolfson, 1992/2010), which give more weight to differences close to the median. Where the dispersion curves intersect, these tell a different story about the relative ranking of different countries. Figure 10 provides an illustration. It shows the dual measures, with  $k=2$ , of tail-heaviness and bi-polarisation. While for many countries their rankings remain the same, there are a number of countries with similar scores for tail-heaviness that score quite differently on bi-polarisation. This is the case with France and Italy, and with Bulgaria and Lithuania.

**Figure 10: Comparing measures of bi-polarisation and tail-heaviness, 2012**



Reading Note: Norway is ranked first with the tail-heaviness measure  $\Delta_2^{**}$  as well as with the bi-polarisation measure  $\Delta_2^*$  and Slovenia is ranked second with both measures.

Source: EU-SILC cross-section 2012 UDB from August 2014.

## 5. Conclusions

The paper has brought together different features of the income distribution – poverty, affluence and dispersion – in a single framework that allows one to see the relation between different concepts. The framework helps us see, for example, the difference between primal and dual measures (Foster-Greer-Thorbecke versus Sen poverty measures) and between tail-heaviness and bi-polarisation. It has shown how the at-risk-of-poverty measures embodied in the EU social indicators can be related to the wider distribution of income, allowing the full range of the EU-SILC income data to be exploited. We have focused on cross-country comparisons that allow one to identify the sources of differing performance across countries without reducing the

analysis to a single indicator. As we have seen, some countries perform better at the bottom and some at the top of the income distribution, but in general the two move closely together. The different parts of the income distribution story cannot be separated.

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