View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Nottingham Trent Institutional Repository (IRep)

International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: https://www.tandfonline.com/loi/tprs20

The effect of economies-of-scale on the performance of lot-sizing heuristics in rolling horizon basis

Ramez Kian, Emre Berk, Ülkü Gürler, Hassan Rezazadeh & Baback Yazdani

To cite this article: Ramez Kian, Emre Berk, Ülkü Gürler, Hassan Rezazadeh & Baback Yazdani (2020): The effect of economies-of-scale on the performance of lot-sizing heuristics in rolling horizon basis, International Journal of Production Research, DOI: 10.1080/00207543.2020.1730464

To link to this article: https://doi.org/10.1080/00207543.2020.1730464

Published online: 26 Feb 2020.

Submit your article to this journal

View related articles

View Crossmark data

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=tprs20

International Journal of Production Research, 2020
https://doi.org/10.1080/00207543.2020.1730464

Taylor & Francis
Taylor & Francis Group
Check for updates

The effect of economies-of-scale on the performance of lot-sizing heuristics in rolling horizon basis

Ramez Kian [a,b*], Emre Berk [c], Ülkü Gürler [d], Hassan Rezazadeh [b] and Baback Yazdani [a]

[a]*Nottingham Business School, Nottingham Trent University, Nottingham, UK;* [b]*Department of Industrial Engineering, University of Tabriz, Tabriz, Iran;* [c]*Department of Management, Bilkent University, Ankara, Turkey;* [d]*Department of Industrial Engineering, Bilkent University, Ankara, Turkey*

(*Received 21 March 2019; accepted 9 February 2020*)

In this article, we consider the production planning problem in the presence of (dis)economies-of-scale in production costs on a rolling horizon basis with a fixed forecast horizon. We propose variants of three well-known and commonly used heuristics (Wagner–Whitin, Silver–Meal and Least Unit Cost) adapted for this particular setting. In an extensive numerical study with demands exhibiting stationary, increasing and decreasing trends and seasonality, we demonstrate that having longer forecast horizon is less effective in obtaining more cost effective production plans when the production cost function is convex and also when fixed setup cost is lower, which both are proxy to lack of economies-of-scale.

Keywords: lot-sizing; production planning; heuristics; rolling horizon; economies-of-scale

1. Introduction

Dynamic lot-sizing models address the problem of finding an optimal production or procurement plan in such a way that the total cost that includes fixed setup cost, storing or holding cost over the periods, and the production cost is minimised. This problem started to receive interest with the early work of Wagner and Whitin (1958) which provides an $O(T^2)$ algorithm for a planning horizon of T periods resulting from the important zero inventory optimality property which indicates that in an optimal production plan, both the production quantity and the on-hand inventory at the beginning of any period cannot be positive. Federgruen and Tzur (1991), Aggarwal and Park (1993), Wagelmans, Van Hoesel, and Kolen (1992) and Van Hoesel et al. (1994) each independently improved the complexity time of the Wagner–Whitin (WW) algorithm to $O(T \log T)$. In the presence of the complete data of the entire horizon, implementing a WW algorithm solves the problem optimally in less than a second not only in the currently available fast personal computers but also in the relatively slow computers of the last decade. However, its optimality is not guaranteed in a rolling horizon setting and heuristic algorithms are extensively used for this class of problems. Chand, Hsu, and Sethi (2002) in their extensive review paper quote from Hopp and Spearman (2011) that 'no commercial MRP package actually uses WW algorithm'. and they justify it with another quotation from the same authors stating that 'people would rather live with a problem they cannot solve than accept a solution that they do not understand'. The importance of the heuristic algorithms emerges from knowing that in the rolling horizon framework, heuristics may outperform the exact WW algorithm. (e.g. see Russell and Urban 1993). There is a solid body of work providing further evidence in that regard. In an early work of Blackburn and Millen (1980), the impact of rolling schedule on the performance of three pioneer heuristics has been examined. Their main findings is that Silver–Meal (SM) heuristic can provide cost performance superior to WW algorithm. Wemmerlöv and Whybark (1984) conduct a simulation experiment to evaluate 14 different lot-sizing rules and establish the impact of demand uncertainties on heuristic performances, Stadtler (2000) demonstrate that planning horizon lengths affect the performance, Simpson (2001) provide an extensive numerical simulation and argue their performance dominance and weaknesses. Other studies on rolling horizon problems can be found in Chand, Hsu, and Sethi (2002) with a classified review of the literature in the forecast, solution and rolling horizon problems and Sahin, Narayanan, and Robinson (2013) with a review on the rolling horizon planning literature. Among recent works, Toy and Berk (2013) examine the performance of the modified counterpart of the classic lot-sizing heuristics on special kind of lot-sizing problems called warm/cold process both in static and rolling horizon configuration. They identify operating environment characteristics where each particular heuristic is the best or

*Corresponding author. Email: ramez.kian@ntu.ac.uk

© 2020 Informa UK Limited, trading as Taylor & Francis Group

among the best. In another recent work of Baciarello et al. (2013) performances of eight well-known heuristics for the classical uncapacitated lot-sizing problem are compared with an extensive numerical simulations. Ziarnetzky, Mönch, and Uzsoy (2018) study a particular case of production planning for semiconductor wafer fabrication facilities both safety stocks at the finished goods inventory level and workload-dependent lead times. They conclude that considering forecast evolution in production planning models may improve the solution performance by exploiting the advance demand information. Saif et al. (2019) consider the mixed model production industries and propose a *drum buffer rope*-based heuristic algorithm for multi-level planning considering shifting bottleneck resource to make efficient schedules on a rolling horizon basis. They have assessed their algorithm with a real industry case.

The difference between the problem investigated in this paper from those in the vast literature of dynamic lot-sizing and rolling horizon is that here the production cost function is not linear as in the classical version of the problem. A number of production or procurement settings present such opportunities to exploit scale economies or instances to avoid scale diseconomies. In both cases, the total cost of producing or procuring a given batch of items is non-decreasing in the batch size, and the average unit cost is non-increasing for economies-of-scale and it is non-decreasing for diseconomies-of-scale (Chu, Hsu, and Shen 2005). Economies-of-scale resulting in concave costs arise in procurement settings with quantity discount schedules. Diseconomies-of-scale resulting in convex costs arise in production settings due to (i) use of overtime or variable production speeds/rates (Koca, Yaman, and Aktürk 2015), (ii) maintenance and repair activities or undesirable waste disposal activities (Kian, Gürler, and Berk 2014), (iii) penalties arising from the sustainability and ecological regulations (Heck and Schmidt 2010) or (iii) the price-dependent supply capacity and re-manufacturing (Teksan and Geunes 2016). Thus, the setting considered herein is a current and important production/procurement environment which has not been studied before in the rolling horizon basis wherein the demand forecast plays an important role. Therefore, one might be interested in comparison of forecast horizon effect in different parameter settings characterising economies-of-scale.

The convex-cost production planning problem is NP-hard (Florian, Lenstra, and Rinnooy Kan 1980; Teksan and Geunes 2016) and therefore it is believed that no exact polynomial time algorithm exists to solve it. Although a few number of exact algorithms (see Veinott 1964; Kian, Gürler, and Berk 2014; Teksan and Geunes 2015) for a special case of the problem with no setup cost, and a few heuristics (see Heck and Schmidt 2010; Kian, Gürler, and Berk 2014) have been proposed in the literature, they have not been examined in the practical rolling horizon basis. In this paper, we propose an augmentation method for modifying the classical lot-sizing algorithms to solve the convex-cost instances. Three well-known heuristics are equipped with that and numerically tested.

The remainder of this paper is organised as follows: in Section 2 we present the mathematical model of our problem and provide some analytical results. Then we discuss heuristic algorithms in Section 3 and finally in Sections 4 and 5 we provide numerical results and the conclusion.

2. The mathematical model

We consider the basic production planning setting with a single item and no capacity constraints over a finite planning horizon of T periods. Demands quantities, denoted by d_t, are forecast and known with certainty for a given *forecast horizon length* (FHL) consisting of a number of periods ahead. In any period t, a fixed setup cost is incurred if any production takes place in the period and ending inventory I_t in the period incurs a unit cost of h_t. Denoting the production quantity in the period with X_t, it is assumed that the variable production cost incurred in period t is given by the function $f(X_t) = w_t X_t^r$ where w_t is a scalar coefficient and r is a measure of the economies-of-scale for $r \leq 1$, or the diseconomies-of-scale if $r > 1$. The former refers to concave production costs, whereas the latter refers to convex production costs. The objective is to determine the minimum cost production plan for the periods u through v ($u \leq v$) over the forecast horizon. The optimisation problem is formally stated in (1)–(3).

$$(RPP) \quad \min_{X_u,\ldots,X_v} P_{u,v} = \sum_{t=u}^{v} \left[(k_t \mathbb{1}_{\{X_t>0\}} + h_t I_t + f(X_t) \right] \tag{1}$$

s.t.

$$I_t = I_{t-1} + X_t - d_t, \quad t \in \{u, \ldots, v\} \tag{2}$$

$$I_t, X_t \geq 0, \quad t \in \{u, \ldots, v\} \tag{3}$$

where $\mathbb{1}_{\{A\}}$ is the indicator function which returns 1 if A is true; or 0, otherwise. Note that given all demand quantities, by setting $u = 1$ and $v = T$, the static version of the problem with entire planning horizon is obtained. However, the demand quantities are unknown beyond the periods in the *FHL* at each decision period. That is, at the first period we have only the demand information of periods $\{1, \ldots, FHL\}$ and the decision on the production quantity of these periods is made.

Then, implementing and fixing the decision for the first period, in the second period the production quantities for periods $2, \ldots, FHL + 1$ is updated as the demand of period $FHL + 1$ is revealed.

In a similar manner, the planned productions are amended by solving solve $P_{t,t+FHL-1}$ for all t as it is rolled until the period $T - FHL + 1$. At each step, the production quantity of the first period within each sub-problem (over FHL periods) is retained. This implementation approach is depicted in Algorithm 1.

Algorithm 1 Rolling implementation

Input: Solution algorithm, Forecast Horizon Length
Output: Provide a production plan

```
function ROLLING_IMPLEMENTATION(solution_algorithm, FHL)
    for t = 1 to T − FHL + 1 do
        Solve P_{t,t+FHL−1} for the net demands with the given algorithm
        Keep the obtained X_t.
        Update the net demands using X_t.
    end for
end function
```

2.1. Some analytical results

There are a number of key structural properties for production planning problems with non-convex production costs, which aid in constructing heuristic solutions. The first is the existence of inventory decomposition property as given below.

PROPOSITION 1 (Florian and Klein 1971) (*Inventory Decomposition Property*) *If the constraint*

$$I_k = 0 \quad \textit{for some } k \in \{1, \ldots, t-1\}, \tag{4}$$

is added to problem $P_{1,t}$, then an optimal solution to the original problem can be found by independently finding solutions to the problems for the first k periods and for the last $t - k$ periods.

This property implies that a problem horizon can be decomposed into independent sub-problems with zero starting and ending inventories. Any such sub-plan within two consecutive zero inventory periods is called a *generation* in the literature. Another fundamental result, on which the WW algorithm based, is the so-called integrality property of the production quantities as stated below.

PROPOSITION 2 (Wagner 1960) (*Zero Inventory Property*) *Given a concave production cost function f, there exists an optimal program such that $I_t\, X_t = 0$ for all t.*

Thus, the optimal production quantities can be positive only in the first period of each generation and, therefore, there is only one production period for each generation for concave production cost functions. The determination of the generation length (i.e. the number of periods whose demands are covered with the production at the beginning period) basically hinges on the trade-off between setup and holding costs if the unit product is constant (and is, therefore, ignored).

If the production cost is convex then the *Zero Inventory Property* does not hold and multiple production periods within the generations are possible in an optimal solution (Kian, Gürler, and Berk 2014; Teksan and Geunes 2016). Such a structure is illustrated in Figure 1 wherein the production quantities at each generation are depicted with different colors while the

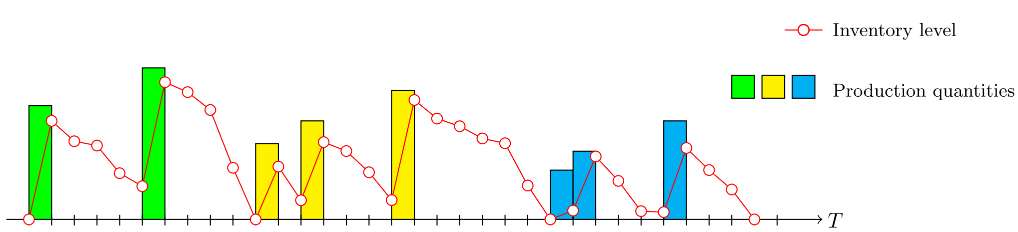

Figure 1. Illustration of the generation structure for convex production cost functions.

inventory level indicates the starting and ending periods of the generations. In this situation, the production quantities within the generations satisfy a fundamental condition given in Proposition 3.

PROPOSITION 3 (Kian, Gürler, and Berk 2014; Teksan and Geunes 2016) *In any generation of an optimal production plan for* (*RPP*), *the following holds between each pair of positive production quantities:*

$$f'(X_i^*) + \sum_{t=i}^{j-1} h_t = f'(X_j^*), \quad i < j \tag{5}$$

where f' denotes the first derivative of the f.

Proof Proof is by contradiction. Suppose that for a generation with an optimal cost this equation does not hold. Then, either (a): $f'(X_i^*) + \sum_{t=i}^{j-1} h_t > f'(X_j^*)$; or (b): $f'(X_i^*) + \sum_{t=i}^{j-1} h_t < f'(X_j^*)$. Let ϵ where $0 < \epsilon < \min_{i \le t \le j}(I_t^* + X_t^* - d_t)$ be a positive production quantity. Using asterisk and tilde respectively for denoting the values of all variables corresponding to the optimal solution and an alternative production plan, a possible improve amount Δ is calculated as follows.

In case of (a), let $\tilde{X}_i = X_i^* - \epsilon$, $\tilde{X}_j = X_j^* + \epsilon$ and $\tilde{X}_t = X_t^*$ for $t \neq i, j$. Thus, $\tilde{I}_t = I_t^* - \epsilon,\ i \le t \le j$ and for small ϵ values we have,

$$\begin{aligned}
\Delta &= f(X_i^*) + \sum_{t=i}^{j} h_t I_t + f(X_j^*) - \left[f(\tilde{X}_i) + \sum_{t=i}^{j} h_t \tilde{I}_t + f(\tilde{X}_j) \right] \\
&= \left[f(X_i^*) - f(\tilde{X}_i) \right] + \epsilon \sum_{t=i}^{j-1} h_t + \left[f(X_j^*) - f(\tilde{X}_j) \right] \\
&= \epsilon \left[\frac{f(X_i^*) - f(X_i^* - \epsilon)}{\epsilon} + \sum_{t=i}^{j-1} h_t + \frac{f(X_j^*) - f(X_j^* + \epsilon)}{\epsilon} \right] = \epsilon \left[f'(X_i^*) + \sum_{t=i}^{j-1} h_t - f'(X_j^*) \right] > 0.
\end{aligned}$$

The positiveness of Δ above which implied by assumption (a), indicates that the solution is improved which contradicts with its optimality assumption. In case of (b), by letting $\tilde{X}_i = X_i^* + \epsilon$, $\tilde{X}_j = X_j^* - \epsilon$ and $\tilde{X}_t = X_t^*$ for $t \neq i, j$, and following the similar process, the same contradiction will be carried out. ■

COROLLARY 1 *In an optimal plan for non-increasing coefficients parameters w_t and positive unit holding costs h_t the followings hold.*

(a) *The positive production quantities will be increasing in t within a generation.*
(b) *For non-increasing demand patterns, the production quantity of the last period in the generation equals zero.*

Proof Under the positiveness assumption of h_t the following arguments hold.

(a) Equation (5) implies that $f'(X_i^*) < f'(X_j^*)$. Since f is a convex function, f' is increasing which implies that $X_i^* < X_j^*$.
(b) Suppose to the contrary that $X_{t_f} > 0$. According to the balance Equation (2), we have $X_{t_f} + I_{t_f-1} - d_{t_f} = I_{t_f} = 0$ which implies that $X_{t_f} < d_{t_f}$ due to the positivity of I_{t_f-1}. Further, part (a) implies that $X_i < X_{t_f} < d_{t_f}$ for $i < t_f$ which contradicts with feasibility of solution as $\sum_{s=t_f}^{t} X_s \ge \sum_{s=t_f}^{t} d_s > \sum_{s=t_f}^{t} d_{t_f}$ should hold.

■

Remark Under the assumptions of the corollary above, for non-increasing demand patterns, a myopic production planning approach will be optimal over the entire horizon indicating that further precise demand forecast for the periods ahead, does not have any value-added to the solution quality.

3. Heuristic algorithms

We consider variants of three heuristics originally developed for the classical production planning problem – the WW, SM and Least Unit Cost (LUC). The rationale for selecting these heuristics is that they are simple to construct (thereby, to train practitioners to use them on the shopfloor) and that they are known to perform moderately well in the classical setting. Let $g_{i,t}$ denote the total cost of setup, holding and production activities of a sub-production plan corresponding to a generation

Table 1. Lot-sizing stopping rules at the first step.

Heuristic	Stopping rule
WW	recursion (6)
SM	$\frac{g_{u,u+l}}{l} < \frac{g_{u,u+l+1}}{l+1}$
LUC	$\frac{g_{u,u+l}}{\sum_{i=u}^{u+l} d_i} < \frac{g_{u,u+l+1}}{\sum_{i=u}^{u+l+1} d_i}$

starting from period i and ending with period t. Moreover, let F_t denote the total production cost from the beginning period until period t of the entire planning horizon. Then the WW dynamic programming recursion can be written as

$$F_0 := 0, \quad F_t = \min_{1 \leq i < t} \{F_{i-1} + g_{i,t}\}, \quad 1 \leq t \leq T. \tag{6}$$

Under each heuristic, the schedule (timing) of production instances is determined by applying the corresponding classical heuristics whose constructions rest on setting consecutive sub-plans like $[u, u+l]$ which forms a generation with cost $g_{u,u+l}$. The starting and ending periods of these generations are determined based on the biggest l allowed by the stopping rules given in Table 1.

The production periods and quantities obtained hitherto the current period are prone to change as (6) iterates over t during the implementation of the WW algorithm. In contrast, SM and LUC algorithms have a pure forward structure in which the determined production quantities are fixed without further changes over the time periods. The demand integrality property is subsumed in both of them, and they are constructed based on stopping rules to decide on the number of periods whose demands being covered with the single production at the beginning of the generation. In the SM heuristic, the stopping rule for identifying generations' ending period is the *per-period averaged cost* whereas in the LUC heuristic this criterion is based on *per-unit averaged cost*. That is, $g_{it}/(t-i+1)$ is non-increasing in t within any generation constructed by SM, and similarly $g_{it}/\sum_{s=i}^{t} d_s$ is non-increasing in t for any production plan within the generations constructed by LUC.

The calculation of g_{it} differs depending of whether economies or diseconomies-of-scale is present in production. According to the demand integrality property which holds for concave cost functions, imposed by Proposition 2, the calculation of g_{it} is straightforward as,

$$g_{it} = K_i + \sum_{s=i}^{t} h_s \left[\sum_{j=s+1}^{t} d_j \right] + w_i \left[\sum_{s=i}^{t} d_s \right]^r \tag{7}$$

wherein inventory levels and the single production quantity are written in terms of demand amounts. However, in the presence of convex production costs, optimal production plans may result in multiple production periods within a generation as illustrated in Figure 1. In general, determination of the optimal schedule of production periods for a finite planning horizon problem with convex production costs has been shown to be an NP-hard combinatorial problem (Florian, Lenstra, and Rinnooy Kan 1980; Teksan and Geunes 2016). Therefore, for identifying the production periods, we propose a simple rule based on which productions are allowed to be done in a block of consecutive periods at the beginning of a generation as illustrated in Figure 2.

Thus, this class of heuristics (which we call augmented heuristics) spread production over a block of consecutive periods within a generation while production quantities meet the property given in (5). The heuristics are constructed as follows. The production periods are not predetermined but, rather, they are determined dynamically as the algorithm proceeds. To form a sub-plan within $[i, j]$, period i is designated as the sole production period and the cost of the sub-plan is calculated as $g_{i,j}^{(1)} = K + \sum_{t=i}^{j} h\hat{I}_t + f(\sum_{t=i}^{j} d_t)$. Then, production is assumed to be spread over periods i and $i+1$ and the corresponding cost is

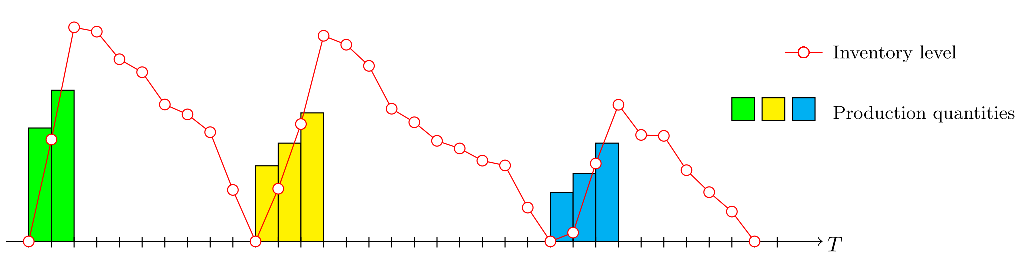

Figure 2. Lumped structure.

Table 2. Illustrative example for heuristics output ($K_t = 800, h_t = 1, f(X_t) = 0.01X^2$).

Period:	1	2	3	4	5	6	7	8	9	10	
Demand:	10	50	150	200	50	150	250	20	60	320	**Cost**
SM	60	0	150	250	0	150	330	0	0	320	8264.0
LUC	210	0	0	250	0	400	0	400	0	0	8816.0
WW	210	0	0	250	0	150	330	0	0	320	7944.0
psSM	60	0	150	250	0	150	330	0	0	320	8264.0
psLUC	210	0	0	250	0	185	235	0	165	215	8055.0
psWW	210	0	0	250	0	215	265	0	0	320	7859.5
CPLEX	210	0	0	250	0	215	265	0	0	320	7859.5

calculated as $g_{i,j}^{(2)} = 2K + \sum_{t=i}^{j} h\tilde{I}_t + f(\tilde{X}_i) + f(\tilde{X}_{i+1})$, where $f'(\tilde{X}_i) + h = f'(\tilde{X}_{i+1})$. If $g_{i,j}^{(1)} > g_{i,j}^{(2)}$ production is assumed to be spread over periods i, $i+1$ and $i+2$, and costs $g_{i,j}^{(2)}$ and $g_{i,j}^{(3)}$ are compared. Thus, starting with a single production period for the entire net demand of the generation, additional productions are successively added and production quantities are spread over the consecutive production periods according to Equation (5) as long as generation cost is decreasing by doing so. The procedure continues until no further cost improvement is observed ($g_{i,j}^{(n)} < g_{i,j}^{(n+1)}$) and the sub-plan corresponding to last solution, $g^{(n)}$, is retained as the best. This procedure results in a production plan consisting of production sub-plans – separated with regeneration points – in which the first or more consecutive beginning periods are production periods.

One may view the construction of the production plans as production splitting; hence, we denote these augmented production-splitting heuristics with the prefix 'ps'. Algorithm 2 summarises this construction procedure.

Algorithm 2 Production splitting within generations

Input: Demands and problem parameters of a generation including periods $u, u+1, \ldots, v$
Output: A consecutive production periods and quantities

$X_u \leftarrow \sum_{t=u}^{v} d_t, \quad \tau \leftarrow u$
g_{uv} ←calculate the generation cost
repeat
    $\tau \leftarrow \tau + 1$
    split production quantities from period u to τ such that (5) holds.
    $\tilde{g}_{uv}$ ←calculate cost of the augmented plan
    if (the split production plan is feasible) and ($\tilde{g}_{uv} \leq g_{uv}$) **then**
        retain the modified plan
        $g_{uv} \leftarrow \tilde{g}_{uv}$
    else
        $STOP \leftarrow True$
    end if
until $STOP = True$

With the proposed construction, we developed three heuristics: psSM where the stopping rule is based on the SM criterion, psLUC where the stopping rule is based on the Least Unit Cost criterion and psWW where the stopping rule is based on the WW criterion (Kian 2016) all summarised in Table 1.

An illustrative example of these algorithms are provided in Table 2 for a 10-period fixed horizon (i.e. $FHL = T = 10$) with the optimal solution provided by CPLEX. Clearly, the production-splitting heuristics outperform the generalised versions with a single production period. Interestingly, psWW is able to produce the optimal production plan while WW is incapable due to a single demand integrality structure imposed on it. Sub-plans are highlighted with different colours in this table.

4. Numerical study

To evaluate the performance of our proposed heuristics, we use the deviation percentage of their cost from the objective value of the static solution obtained via ILOG CPLEX 12.8 with full demand information of the entire horizon,

Table 3. Random demand realisation for $T = 100$ periods.

Demand	Construction formula	Parameters
Stationary	truncated normal $N(\mu, \sigma^2)$	$\mu = 100,\ \sigma \in \{0, 10, 22, 43\}$
Increasing	$d_t = \mu + \sigma + \tau(t-1)$	$\mu = 100,\ \sigma = 10,\ \tau \in \{1, 10, 20, 40\}$
Decreasing	$d_{T-t+1} = \mu + \sigma + \tau(t-1)$	$\mu = 100,\ \sigma = 10,\ \tau \in \{1, 10, 20, 40\}$
Seasonal	$d_t = \mu + \sigma\epsilon_t + a\sin[2\pi/c(t + c/4)]$	$\mu = 100,\ \sigma = 10,\ c = 12,\ a \in \{20, 40, 60, 80\}$

Table 4. The algorithm with minimum average optimality deviation.

d	FHL	(0,0.5)	(0,1)	(0,2)	(400,0.5)	(400,1)	(400,2)	(800,0.5)	(800,1)	(800,2)
STAT.	2	psWW	psWW,psLUC	psWW	–	–	psWW	–	–	psWW
	4	psWW	psWW,psLUC	psWW	psSM	psSM	psWW	psSM	psSM	psLUC
	6	psWW	psWW,psLUC	psWW	psWW	psWW	psWW	psSM	psSM	psWW
DEC.	2	psWW,psLUC	–	–	psWW	psLUC	psWW	psWW	psWW	psWW
	4	psWW,psLUC	–	–	psSM	psSM	psWW	psLUC	psSM	psWW
	6	psWW,psLUC	–	–	psWW	psWW	psWW	psSM	psSM	psWW
IIN.	2	–	–	psWW	psWW	psWW	psWW	psWW	psWW	psWW
	4	–	–	psWW	psSM	psSM	psWW	psSM	psSM	psWW
	6	–	–	psWW	psWW	psSM	psWW	psSM	psSM	psWW
SEAS.	2	psWW	psWW,psLUC	psWW	–	–	psLUC	–	–	psLUC
	4	psWW	psWW,psLUC	psWW	psSM	psLUC	psWW	psWW	psWW	psLUC
	6	psWW	psWW,psLUC	psWW	psWW	psWW	psWW	psSM	psSM	psLUC

calculated as

$$\Delta_i^{FHL} = \frac{TC_i^{FHL} - TC^*}{TC^*} \times 100 \tag{8}$$

where TC_i^{FHL} denotes the total cost obtained from the rolling implementation of algorithm i with a forecast horizon length of FHL periods, while TC^* denotes the optimal total cost. We have obtained the optimal costs by employing the WW algorithm for $r \le 1$, and by solving the conic quadratic reformulation of the model via CPLEX solver for $r > 1$ (see Kian, Berk, and Gürler 2019).

Following Van Den Heuvel and Wagelmans (2005), we have considered four different demand patterns: (i) stationary (as a base case), (ii) with a decreasing trend, (iii) with an increasing trend and (iv) with a seasonality component. The parameters corresponding to each of these demand groups are given in Table 3.

To compare the effect of fixed setup cost, three levels are used, i.e. $K_t \in \{0, 400, 800\}$. The economies-of-scale is tested in three levels, $r \in \{0.5, 1, 2\}$ and the coefficient of the production cost function has been set as $w_t = \mu^{1-r}$ to roughly make the same magnitude of production costs in each group (with or without economies-of-scale). For convenience, the unit holding cost is set as $h_t = 1$ and the algorithms are examined in rolling basis with $FHL \in \{2, 4, 6, 8, 10, 100\}$. All computations were implemented on a personal computer equipped with Intel(R) Core(TM) i5-7200U, 2.50 GHz and 8GB of RAM.

The computation times of the heuristics for all problem instances are very short in the order of milliseconds and, hence, not reported here. Also the execution time of CPLEX solver was set to 1 h which was sufficient enough to optimally solve the convex-cost instances ($r = 2$).

The numerical results of the heuristics are presented in Tables A1–A4 in Appendix for each demand pattern separately. The outputs are classified based on the parameters K_t, r and FHL in row blocks; and based on the demand parameters in column blocks. In the following, we briefly discuss the main observations.

4.1. Demand pattern and parameter

Although the proposed heuristics behave consistently against the setup and production costs according the examined demand patterns, the superiority of them may change in different circumstances of demand. For instance, psSM outperforms psLUC for the increasing and decreasing demand patterns (see Tables A2 and A3 in Appendix) while it is defeated against psLUC for the stationary and seasonal demand patterns particularly when there is no economies-of-scale for the production cost

($r > 1$). On the other hand a better performance of pdSM compared to psLUC is observed generally when economies-of-scale holds (see Tables A1 and A4 in Appendix). Despite the general superiority of psWW to the others, its defeat is also observed as well (for the concave production function see Table A2, $\tau = 1$, $r = 0.5$, $FHL = 4$, $K = 400{,}800$; and for the convex cost see Table A4, $a = 20$, $r = 2$, $FHL = 4$, $K = 800$).

4.2. Set-up cost (K)

Comparing optimality deviation of heuristics for different demand patterns and for their different parameters we observe that it generally deteriorates for higher setup cost. However, this sensitivity is more significant for shorter forecast horizons (up to 6 periods), and when economies-of-scale holds (concave).

4.2.1. Economies-of-scale (r)

With the higher values of r, the performance of the heuristics are slightly improved based on the augmentation procedure they are equipped with. However, for the case of zero setup cost they all show an insensitive behaviour against the existence or lack of economies-of-scale. Both psSM and psLUC heuristics perform more steadily and less sensitively to the length of the forecast horizon as they might produce more myopic solution with less use of the future data ahead. However, the quality of the solutions they generate are slightly better when the production cost function is convex. It may be due to the

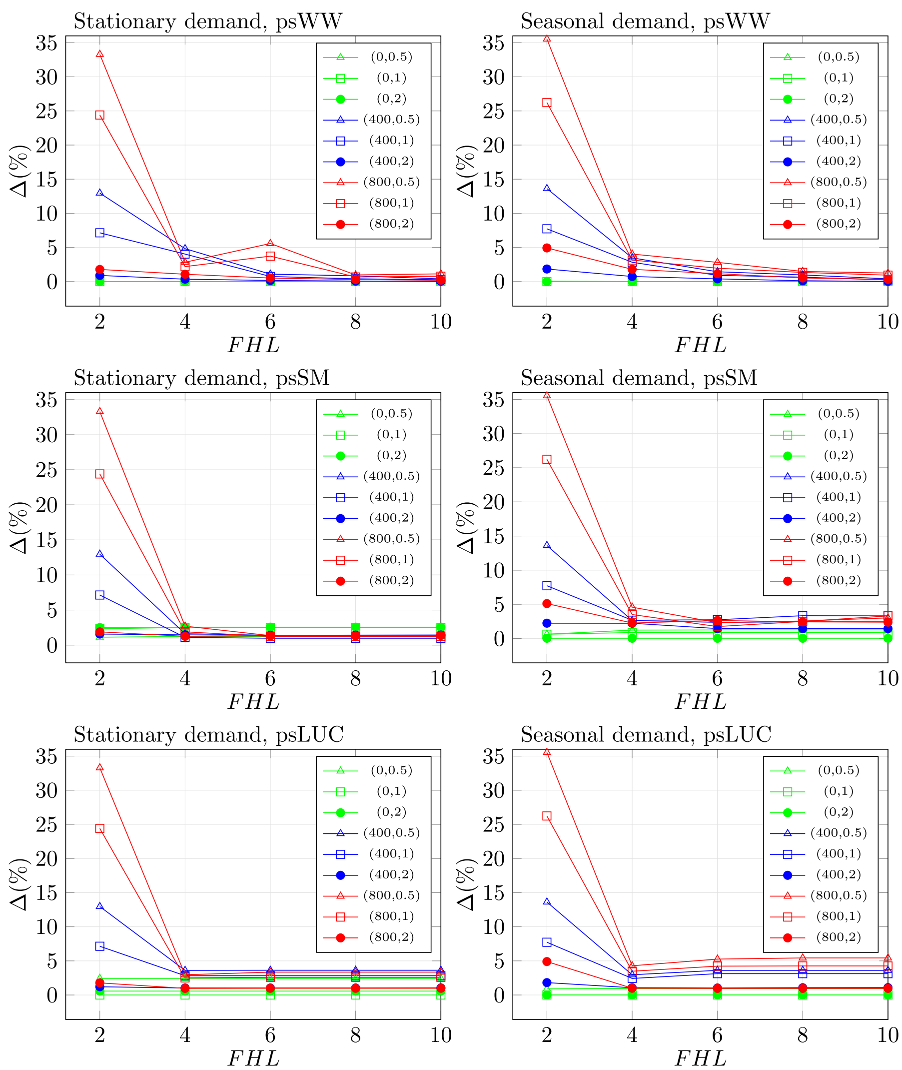

Figure 3. The trend of optimality deviation over different forecast horizon lengths for the stationary and seasonal demand patterns.

myopic manner they have, which is more appropriate when economies-of-scale does not exist because the more periods ahead to consider and the more quantities to produce, the higher cost it will incur in this case.

4.2.2. Forecast horizon (FHL)

For all the demand patterns and algorithms, an insensitive performance against the forecast horizon is observed for the case in which the setup cost is zero. Among the other configurations, psWW algorithm performs more sensitively to the horizon length while psSM and psLUC are stabilised within short forecast horizons. Despite having the same trend, their performances are deteriorating for higher demand fluctuations or variance. Also, the intensity of their performance change is more significant for the cases in which setup costs are high and/or the variable production cost benefits from economies-of-scale. Another interesting observation is that, for the increasing and decreasing demand patterns, the performance of the algorithms are more insensitive to the forecast data, even less than the case of stationary demand.

The deviation of our heuristics from optimality averaged over the instances, is plotted with respect to the forecast horizon length in Figures 3 and 4 for (K, r) pairs. We observe that the peak deviations correspond to those instances where unit setup cost is high and $r \leq 1$. This combination leads to high economies-of-scale: The higher unit setup cost is, the bigger batch size is desirable to amortise it; and the more concave the production cost function is, the smaller the cost margin is for bigger batch sizes. Thus, it is more likely in these combinations to cover the demand of several periods via each individual production and consequently have longer generations in the optimal solution. Therefore, myopic forecast

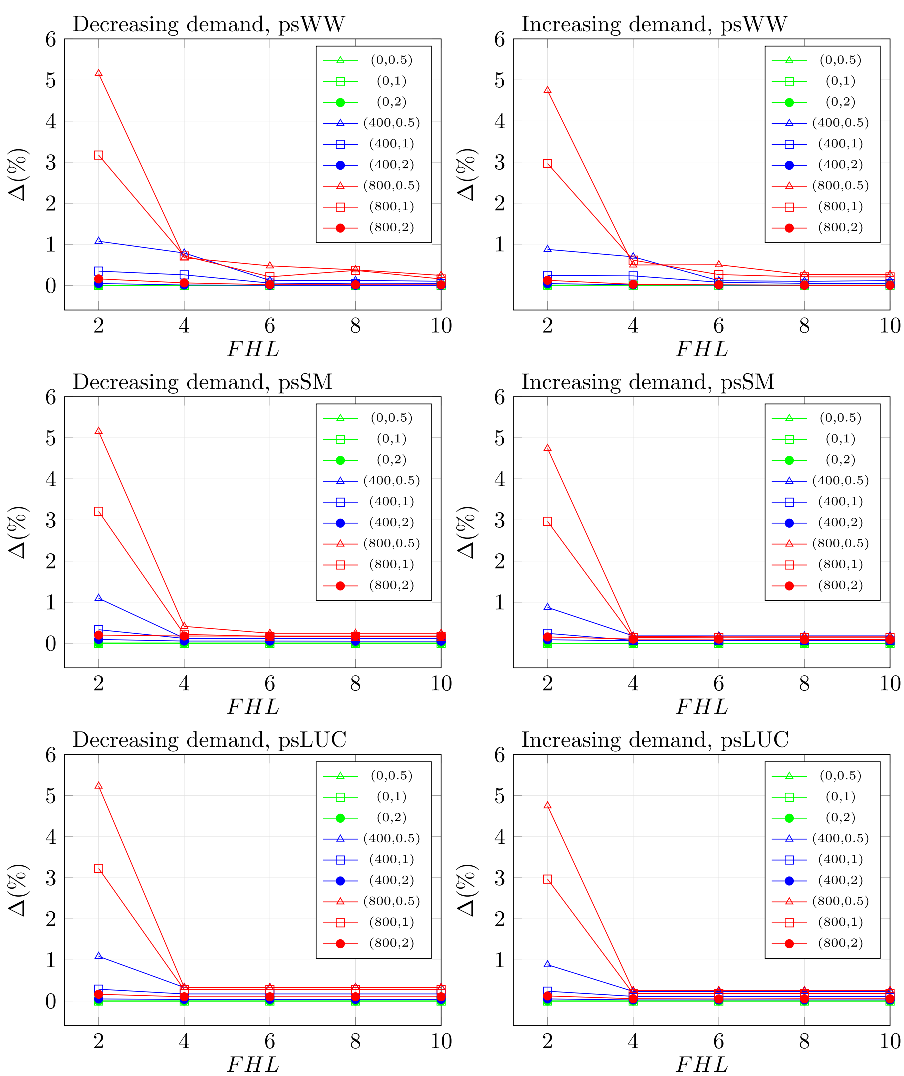

Figure 4. The trend of optimality deviation over different forecast horizon lengths for the decreasing and increasing demand patterns.

horizons in the rolling horizon settings are prone to significantly deviate from the optimal production plan. On the other hand, a lower setup cost and a more convex production cost function induce the production lots to cover the demand of fewer periods (and possibly producing lot-for-lot) in an optimal production plan. Therefore, a short forecast horizon might provide sufficient information to achieve a satisfactory production plan on a rolling basis without further investment on forecasting future demand. The winner algorithms with minimum average deviation for each (K, r) combination and FHL are given in Table 4. The equally performed cases are specified by dashes; and as the winners do not change after $FHL = 6$, the longer forecast horizons dropped from the table.

5. Conclusion and future directions

In this paper, using the solution optimality structure of the convex-cost production planning problem, we have proposed an augmentation method for the lot-sizing heuristics in the presence of diseconomies-of-scale. Then, modifying the three well-known lot-sizing algorithms with our augmentation method for solving convex-cost production planning problems which lack the economies-of-scale property, we have examined their performance with different demand patterns and parameters in the rolling horizon schema to see the importance of demand forecasts as well.

Our numerical study suggests that even though the length of forecast horizon does not change the dominance order of the algorithms, the performance of algorithms appear with different magnitudes and they converge after a quite short forecast horizon (4–6 periods), which entails that excess of information about the future demands does not necessarily result in a better production plan. This threshold after which the deviation percentages are stabilised, are shorter when the problem lacks economies-of-scale, suggesting more myopic actions and less investments on the precisely forecasting later future demands.

In addition, our numerical tests demonstrated that the for the convex-cost production planning problems per-unit-based algorithms (LUC) perform better than per-period-based ones (SM). Because the per-unit stopping rule can capture the diseconomies-of-scale when the total cost is dividend by the total units. Nevertheless, for the concave-cost production costs this is the opposite way around and SM has the superior performance.

All of the algorithms on average perform better for the monotone demand patterns either increasing or decreasing. However, the seasonal and stationary demand patterns may lead to significantly poorer performance of the heuristics with short forecast horizon lengths.

Another interesting observation within our study is that irrespective of the algorithm we use, the forecast horizon is more important when the objective function benefits from economies-of-scale. That is, when the production cost function is more concave or the setup cost is high. As an example, the concave instances for which all the algorithms have resulted in almost 40% of optimality deviation, are solved with 11% deviation for their convex counterparts.

Although the investigation of lot-sizing algorithms is a classical topic in the production planning, in this study we revisited it from a different aspect which has not been addressed before. Our managerial insight in investigating the effect of economies-of-scale with considering convex or concave cost function in the rolling horizon framework might be of interest in practice as the manufacturing sector faces new paradigms under sustainability, workers' rights or any other legislation which might change the production margin.

We have not been able to obtain worst case performance bounds similar to the linear cost cases (Axsäter 1982, 1985) and conjecture that the worst case performance is unbounded for convex production costs. Development of performance bounds for the proposed heuristics would be an area for future study. In this work, we focused on a single item setting. However, many of our motivating particulars apply to production environments with multiple items. Another very interesting area of future study is to consider multi-item production planning.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by Iran's National Elites Foundation, I.R. IRAN and The Scientific and Technological Research Council of Turkey (TUBITAK Grant ID:110M307).

ORCID

Ramez Kian http://orcid.org/0000-0001-8786-6349
Emre Berk http://orcid.org/0000-0001-9674-5194
Ülkü Gürler http://orcid.org/0000-0002-7273-3743
Baback Yazdani http://orcid.org/0000-0001-9484-8552

References

Aggarwal, A., and J. K. Park. 1993. "Improved Algorithms for Economic Lot Size Problems." *Operations Research* 41 (3): 549–571.

Axsäter, S. 1982. "Worst Case Performance for Lot Sizing Heuristics." *European Journal of Operational Research* 9 (4): 339–343.

Axsäter, S. 1985. "Note – performance Bounds for Lot Sizing Heuristics." *Management Science* 31 (5): 634–640.

Baciarello, L., M. D'Avino, R. Onori, and M. M. Schiraldi. 2013. "Lot Sizing Heuristics Performance." *International Journal of Engineering Business Management* 5 (6): 1–10.

Blackburn, J. D., and R. A. Millen. 1980. "Heuristic Lot-sizing Performance in a Rolling-schedule Environment*." *Decision Sciences* 11 (4): 691–701.

Chand, S., V. N. Hsu, and S. Sethi. 2002. "Forecast, Solution, and Rolling Horizons in Operations Management Problems: a Classified Bibliography." *Manufacturing & Service Operations Management* 4 (1): 25–43.

Chu, L. Y., V. N. Hsu, and Z. -J. M. Shen. 2005. "An Economic Lot-sizing Problem with Perishable Inventory and Economies of Scale Costs: Approximation Solutions and Worst Case Analysis." *Naval Research Logistics (NRL)* 52 (6): 536–548.

Federgruen, A., and M. Tzur. 1991. "A Simple Forward Algorithm to Solve General Dynamic Lot Sizing Models with N Periods in 0 (n Log N) Or 0 (n) Time." *Management Science* 37 (8): 909–925.

Florian, M., and M. Klein. 1971. "Deterministic Production Planning with Concave Costs and Capacity Constraints." *Management Science* 18 (1): 12–20.

Florian, M., J. K. Lenstra, and A. Rinnooy Kan. 1980. "Deterministic Production Planning: Algorithms and Complexity." *Management Science* 26 (7): 669–679.

Heck, M., and G. Schmidt. 2010. Lot-Size Planning with Non-Linear Cost Functions Supporting Environmental Sustainability. In *Networked Digital Technologies, Part II: Second International Conference, NDT 2010, Prague, Czech Republic, July 7-9, 2010 Proceedings*, edited by Filip Zavoral, Jakub Yaghob, Pit Pichappan and Eyas El-Qawasmeh, vol 88, 1–6. Springer.

Hopp, W. J., and M. L. Spearman. 2011. *Factory Physics*. Long Grove, IL: Waveland Press. ISBN:1-57766-739-1.

Kian, R. 2016. "Production Decisions with Convex Costs and Carbon Emission Constraints." PhD thesis, Bilkent University.

Kian, R., E. Berk, and Ü. Gürler. 2019. "Minimal Conic Quadratic Reformulations and An Optimization Model." *Operations Research Letters* 47 (6): 489–493.

Kian, R., Ü. Gürler, and E. Berk. 2014. "The Dynamic Lot-sizing Problem with Convex Economic Production Costs and Setups." *International Journal of Production Economics* 155: 361–379.

Koca, E., H. Yaman, and M. S. Aktürk. 2015. "Stochastic Lot Sizing Problem with Controllable Processing Times." *Omega* 53: 1–10.

Russell, R., and T. Urban. 1993. "Horizon Extension for Rolling Production Schedules: Length and Accuracy Requirements." *International Journal of Production Economics* 29 (1): 111–122.

Sahin, F., A. Narayanan, and E. P. Robinson. 2013. "Rolling Horizon Planning in Supply Chains: Review, Implications and Directions for Future Research." *International Journal of Production Research* 51 (18): 5413–5436.

Saif, U., Z. Guan, C. Wang, C. He, L. Yue, and J. Mirza. 2019. "Drum Buffer Rope-based Heuristic for Multi-level Rolling Horizon Planning in Mixed Model Production." *International Journal of Production Research* 57: 1–28.

Simpson, N. 2001. "Questioning the Relative Virtues of Dynamic Lot Sizing Rules." *Computers & Operations Research* 28 (9): 899–914.

Stadtler, H. 2000. "Improved Rolling Schedules for the Dynamic Single-level Lot-sizing Problem." *Management Science* 46 (2): 318–326.

Teksan, Z. M., and J. Geunes. 2015. "A Polynomial Time Algorithm for Convex Cost Lot-sizing Problems." *Operations Research Letters* 43 (4): 359–364.

Teksan, Z. M., and J. Geunes. 2016. "Production Planning with Price-dependent Supply Capacity." *IIE Transactions* 48 (10): 938–954.

Toy, A. Ö., and E. Berk. 2013. "Dynamic Lot Sizing for a Warm/cold Process: Heuristics and Insights." *International Journal of Production Economics* 145 (1): 53–66.

Van Den Heuvel, W., and A. P. Wagelmans. 2005. "A Comparison of Methods for Lot-sizing in a Rolling Horizon Environment." *Operations Research Letters* 33 (5): 486–496.

Van Hoesel, S., R. Kuik, M. Salomon, and L. N. Van Wassenhove. 1994. "The Single-item Discrete Lotsizing and Scheduling Problem: Optimization by Linear and Dynamic Programming." *Discrete Applied Mathematics* 48 (3): 289–303.

Veinott, A. F. 1964. "Production Planning with Convex Costs: A Parametric Study." *Management Science* 10 (3): 441–460.

Wagelmans, A., S. Van Hoesel, and A. Kolen. 1992. "Economic Lot Sizing: An O (n Log N) Algorithm that Runs in Linear Time in the Wagner-Whitin Case." *Operations Research* 40 (1-supplement-1): S145–S156.

Wagner, H. M. 1960. "A Postscript to 'dynamic Problems in the Theory of the Firm'." *Naval Research Logistics Quarterly* 7 (1): 7–12.

Wagner, H. M., and T. M. Whitin. 1958. "Dynamic Version of the Economic Lot Size Model." *Management Science* 5 (1): 89–96.

Wemmerlöv, U., and D. C. Whybark. 1984. "Lot-sizing Under Uncertainty in a Rolling Schedule Environment." *The International Journal of Production Research* 22 (3): 467–484.

Ziarnetzky, T., L. Mönch, and R. Uzsoy. 2018. "Rolling Horizon, Multi-product Production Planning with Chance Constraints and Forecast Evolution for Wafer Fabs." *International Journal of Production Research* 56 (18): 6112–6134.

Appendix

Table A1. Stationary demand.

			σ = 0			σ = 10			σ = 22			σ = 43		
K	**r**	**FHL**	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC
0	0.5	2	0.000	0.018	0.000	0.000	0.018	0.000	0.000	1.307	0.405	0.013	2.057	6.915
		4	0.000	0.018	0.000	0.000	0.018	0.000	0.000	1.307	0.405	0.000	2.396	6.915
		6	0.000	0.018	0.000	0.000	0.018	0.000	0.000	1.307	0.405	0.000	2.396	6.915
		8	0.000	0.018	0.000	0.000	0.018	0.000	0.000	1.307	0.405	0.000	2.396	6.915
		10	0.000	0.018	0.000	0.000	0.018	0.000	0.000	1.307	0.405	0.000	2.396	6.915
		100	0.000	0.018	0.000	0.000	0.018	0.000	0.000	1.307	0.405	0.000	2.396	6.915
	1	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.302	0.000	0.000	5.673	0.000
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.302	0.000	0.000	6.251	0.000
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.302	0.000	0.000	6.251	0.000
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.302	0.000	0.000	6.251	0.000
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.302	0.000	0.000	6.251	0.000
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.302	0.000	0.000	6.251	0.000
	2	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.090	0.090	0.019	7.433	1.639
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.090	0.090	0.000	7.588	1.639
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.090	0.090	0.000	7.588	1.639
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.090	0.090	0.000	7.588	1.639
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.090	0.090	0.000	7.588	1.639
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.090	0.090	0.000	7.588	1.639
400	0.5	2	10.747	10.747	10.747	10.747	10.747	10.747	12.686	12.686	12.686	15.453	15.453	15.453
		4	3.739	0.647	1.195	3.739	0.647	1.195	4.496	1.721	2.820	6.225	2.501	6.809
		6	0.599	0.647	1.195	0.599	0.647	1.195	1.080	1.714	2.820	1.578	1.837	6.861
		8	1.042	0.647	1.195	1.042	0.647	1.195	0.903	1.714	2.820	0.571	1.837	6.861
		10	0.403	0.647	1.195	0.403	0.647	1.195	0.572	1.714	2.820	0.192	1.837	6.861
		100	0.000	0.647	1.195	0.000	0.647	1.195	0.000	1.714	2.820	0.000	1.837	6.861
	1	2	5.421	5.421	5.421	5.421	5.421	5.421	6.756	6.756	6.756	9.246	9.246	9.246
		4	4.382	0.687	0.870	4.382	0.687	0.870	4.125	1.048	2.188	3.570	1.586	5.414
		6	0.558	0.687	0.870	0.558	0.687	0.870	0.741	1.005	2.188	0.930	1.237	5.414
		8	0.533	0.687	0.870	0.533	0.687	0.870	0.421	1.005	2.188	0.258	1.237	5.414
		10	0.614	0.687	0.870	0.614	0.687	0.870	0.177	1.005	2.188	0.008	1.237	5.414
		100	0.000	0.687	0.870	0.000	0.687	0.870	0.000	1.005	2.188	0.000	1.237	5.414
	2	2	0.176	0.262	0.229	0.176	0.262	0.229	0.854	1.032	1.166	1.656	3.427	2.219
		4	0.162	0.713	0.167	0.162	0.713	0.167	0.464	1.398	0.614	0.416	2.278	2.301
		6	0.120	0.723	0.167	0.120	0.723	0.167	0.231	1.355	0.614	0.133	2.089	2.296
		8	0.102	0.733	0.167	0.102	0.733	0.167	0.103	1.428	0.614	0.005	2.108	2.296
		10	0.094	0.733	0.167	0.094	0.733	0.167	0.051	1.489	0.614	0.001	2.108	2.329
		100	0.000	0.775	0.167	0.000	0.775	0.167	0.000	1.595	0.551	0.001	2.409	2.388
800	0.5	2	30.872	30.872	30.872	30.872	30.872	30.872	33.445	33.445	33.445	35.552	35.552	35.552
		4	0.653	0.653	0.653	0.653	0.653	0.653	2.574	2.999	2.574	5.260	4.457	5.675
		6	5.800	0.687	0.737	5.800	0.687	0.737	6.128	1.333	3.004	4.801	2.028	6.306
		8	0.676	0.687	0.737	0.676	0.687	0.737	1.224	1.399	3.004	1.015	1.825	6.306
		10	1.453	0.687	0.737	1.453	0.687	0.737	1.161	1.399	3.004	0.768	1.825	6.306
		100	0.000	0.687	0.737	0.000	0.687	0.737	0.000	1.399	3.004	0.000	1.825	6.306
	1	2	22.543	22.543	22.543	22.543	22.543	22.543	24.520	24.520	24.520	26.147	26.147	26.147
		4	0.376	0.826	0.376	0.376	0.826	0.376	1.906	1.721	2.111	4.291	3.137	5.019
		6	3.930	0.645	0.376	3.930	0.645	0.376	3.999	1.305	1.855	3.244	1.719	5.542
		8	0.399	0.645	0.376	0.399	0.645	0.376	0.749	1.347	1.855	0.852	1.719	5.542
		10	1.269	0.645	0.376	1.269	0.645	0.376	0.650	1.347	1.855	0.417	1.719	5.542
		100	0.000	0.645	0.376	0.000	0.645	0.376	0.000	1.347	1.855	0.000	1.719	5.542
	2	2	0.292	0.292	0.292	0.292	0.292	0.292	1.491	1.491	1.491	3.541	3.882	3.541
		4	0.292	0.815	0.449	0.292	0.815	0.449	1.225	1.266	0.842	1.692	1.683	1.617
		6	0.255	0.822	0.449	0.255	0.822	0.449	0.658	1.288	0.842	0.624	1.824	1.665
		8	0.211	0.822	0.449	0.211	0.822	0.449	0.419	1.275	0.842	0.256	1.867	1.665
		10	0.160	0.822	0.449	0.160	0.822	0.449	0.314	1.429	0.842	0.125	1.867	1.665
		100	0.000	0.826	0.449	0.000	0.826	0.449	0.001	1.487	0.842	0.011	1.810	1.716

Table A2. Decreasing demand.

			$\tau = 1$			$\tau = 10$			$\tau = 20$			$\tau = 40$		
K	**r**	**FHL**	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC
0	0.5	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.012	0.000
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.012	0.000
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.012	0.000
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.012	0.000
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.012	0.000
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.012	0.000
	1	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
400	0.5	2	3.442	3.442	3.442	0.427	0.430	0.424	0.225	0.195	0.250	0.201	0.307	0.236
		4	2.696	0.273	0.696	0.387	0.104	0.205	0.042	0.047	0.231	0.024	0.061	0.189
		6	0.368	0.273	0.696	0.043	0.104	0.205	0.051	0.047	0.231	0.018	0.061	0.189
		8	0.391	0.273	0.696	0.064	0.104	0.205	0.013	0.047	0.231	0.018	0.061	0.189
		10	0.316	0.273	0.696	0.037	0.104	0.205	0.013	0.047	0.231	0.035	0.061	0.189
		100	0.000	0.273	0.696	0.000	0.104	0.205	0.000	0.047	0.231	0.000	0.061	0.189
	1	2	0.997	0.997	0.997	0.216	0.179	0.066	0.091	0.089	0.071	0.082	0.049	0.016
		4	0.976	0.273	0.567	0.034	0.103	0.089	0.011	0.051	0.018	0.001	0.044	0.016
		6	0.157	0.273	0.567	0.054	0.103	0.089	0.008	0.051	0.018	0.001	0.044	0.016
		8	0.122	0.273	0.567	0.019	0.103	0.089	0.008	0.051	0.018	0.002	0.044	0.016
		10	0.138	0.273	0.567	0.019	0.103	0.089	0.008	0.051	0.018	0.000	0.044	0.016
		100	0.000	0.273	0.567	0.000	0.103	0.089	0.000	0.051	0.018	0.000	0.044	0.016
	2	2	0.168	0.352	0.194	0.011	0.010	0.007	0.000	0.003	0.001	0.000	0.000	0.000
		4	0.037	0.205	0.154	0.000	0.005	0.003	0.000	0.000	0.001	0.000	0.000	0.000
		6	0.012	0.205	0.154	0.000	0.005	0.003	0.000	0.000	0.001	0.000	0.000	0.000
		8	0.007	0.193	0.154	0.000	0.005	0.003	0.000	0.000	0.001	0.000	0.000	0.000
		10	0.000	0.193	0.154	0.000	0.005	0.003	0.000	0.000	0.001	0.000	0.000	0.000
		100	0.000	0.114	0.154	0.000	0.005	0.003	0.000	0.000	0.001	0.000	0.000	0.000
800	0.5	2	17.471	17.471	17.471	1.774	1.809	1.832	0.895	0.951	0.978	0.480	0.395	0.647
		4	1.522	0.527	0.516	0.452	0.332	0.370	0.462	0.270	0.241	0.252	0.508	0.192
		6	1.567	0.337	0.516	0.157	0.274	0.370	0.075	0.258	0.241	0.097	0.098	0.192
		8	0.981	0.337	0.516	0.360	0.274	0.370	0.140	0.258	0.241	0.030	0.098	0.192
		10	0.770	0.337	0.516	0.116	0.274	0.370	0.053	0.258	0.241	0.030	0.098	0.192
		100	0.000	0.337	0.516	0.000	0.274	0.370	0.000	0.258	0.241	0.000	0.098	0.192
	1	2	11.357	11.357	11.357	0.739	0.877	1.014	0.359	0.451	0.414	0.230	0.154	0.129
		4	2.330	0.399	0.604	0.285	0.188	0.234	0.073	0.168	0.094	0.174	0.105	0.173
		6	0.705	0.301	0.604	0.063	0.188	0.234	0.041	0.111	0.094	0.024	0.057	0.173
		8	1.158	0.301	0.604	0.247	0.188	0.234	0.041	0.111	0.094	0.011	0.057	0.173
		10	0.538	0.301	0.604	0.050	0.188	0.234	0.022	0.111	0.094	0.012	0.057	0.173
		100	0.000	0.301	0.604	0.000	0.188	0.234	0.000	0.111	0.094	0.000	0.057	0.173
	2	2	0.565	0.720	0.619	0.044	0.055	0.022	0.016	0.011	0.013	0.005	0.005	0.003
		4	0.214	0.666	0.371	0.015	0.023	0.038	0.009	0.005	0.003	0.000	0.004	0.001
		6	0.051	0.666	0.371	0.013	0.021	0.038	0.000	0.007	0.003	0.000	0.004	0.001
		8	0.054	0.666	0.371	0.006	0.021	0.038	0.000	0.007	0.003	0.000	0.004	0.001
		10	0.050	0.666	0.371	0.000	0.021	0.038	0.000	0.007	0.003	0.000	0.004	0.001
		100	0.000	0.529	0.371	0.000	0.021	0.038	0.000	0.007	0.003	0.000	0.004	0.001

Table A3. Increasing demand.

			$\tau = 1$			$\tau = 10$			$\tau = 20$			$\tau = 40$		
K	**r**	**FHL**	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC
0	0.5	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
400	0.5	2	3.249	3.249	3.249	0.172	0.172	0.197	0.041	0.041	0.054	0.021	0.022	0.033
		4	2.554	0.644	0.789	0.149	0.041	0.056	0.040	0.005	0.046	0.021	0.022	0.047
		6	0.374	0.644	0.789	0.016	0.041	0.056	0.020	0.005	0.046	0.021	0.022	0.047
		8	0.316	0.644	0.789	0.016	0.041	0.056	0.020	0.005	0.046	0.021	0.022	0.047
		10	0.402	0.644	0.789	0.016	0.041	0.056	0.020	0.005	0.046	0.001	0.022	0.047
		100	0.000	0.644	0.789	0.000	0.041	0.056	0.000	0.005	0.046	0.000	0.022	0.047
	1	2	0.913	0.913	0.913	0.034	0.036	0.036	0.003	0.004	0.006	0.000	0.005	0.000
		4	0.872	0.234	0.375	0.029	0.033	0.027	0.003	0.004	0.018	0.000	0.005	0.036
		6	0.272	0.234	0.375	0.010	0.033	0.027	0.003	0.004	0.018	0.000	0.005	0.036
		8	0.162	0.234	0.375	0.010	0.033	0.027	0.003	0.004	0.018	0.000	0.005	0.036
		10	0.147	0.234	0.375	0.010	0.033	0.027	0.003	0.004	0.018	0.000	0.005	0.036
		100	0.000	0.234	0.375	0.000	0.033	0.027	0.000	0.004	0.018	0.000	0.005	0.036
	2	2	0.150	0.331	0.200	0.002	0.003	0.003	0.000	0.000	0.000	0.000	0.000	0.000
		4	0.030	0.241	0.106	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		6	0.009	0.241	0.106	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		8	0.001	0.241	0.106	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		10	0.001	0.241	0.106	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		100	0.000	0.135	0.118	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000
800	0.5	2	17.049	17.049	17.049	1.252	1.253	1.266	0.473	0.474	0.480	0.190	0.190	0.214
		4	1.319	0.560	0.703	0.385	0.060	0.135	0.171	0.034	0.054	0.091	0.017	0.135
		6	1.785	0.513	0.703	0.130	0.060	0.135	0.034	0.034	0.054	0.037	0.017	0.135
		8	0.854	0.513	0.703	0.095	0.060	0.135	0.034	0.034	0.054	0.037	0.017	0.135
		10	0.896	0.513	0.703	0.070	0.060	0.135	0.047	0.034	0.054	0.037	0.017	0.135
		100	0.000	0.513	0.703	0.000	0.060	0.135	0.000	0.034	0.054	0.000	0.017	0.135
	1	2	11.059	11.059	11.059	0.572	0.574	0.579	0.171	0.172	0.177	0.055	0.062	0.055
		4	2.121	0.498	0.574	0.210	0.039	0.057	0.057	0.006	0.037	0.047	0.005	0.055
		6	0.976	0.479	0.574	0.051	0.039	0.057	0.005	0.006	0.037	0.001	0.005	0.055
		8	0.787	0.479	0.574	0.012	0.039	0.057	0.005	0.006	0.037	0.001	0.005	0.055
		10	0.735	0.479	0.574	0.058	0.039	0.057	0.005	0.006	0.037	0.001	0.005	0.055
		100	0.000	0.479	0.574	0.000	0.039	0.057	0.000	0.006	0.037	0.000	0.005	0.055
	2	2	0.470	0.605	0.493	0.004	0.006	0.004	0.000	0.000	0.000	0.000	0.000	0.000
		4	0.085	0.397	0.224	0.000	0.003	0.003	0.000	0.000	0.002	0.000	0.000	0.000
		6	0.034	0.397	0.202	0.000	0.003	0.003	0.000	0.000	0.002	0.000	0.000	0.000
		8	0.000	0.350	0.217	0.000	0.003	0.003	0.000	0.000	0.002	0.000	0.000	0.000
		10	0.000	0.350	0.217	0.000	0.003	0.003	0.000	0.000	0.002	0.000	0.000	0.000
		100	0.000	0.244	0.217	0.000	0.003	0.003	0.000	0.000	0.000	0.000	0.000	0.000

Table A4. Seasonal demand.

			a = 20			a = 40			a = 60			a = 80		
K	**r**	**FHL**	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC	psWW	psSM	psLUC
0	0.5	2	0.000	0.210	0.000	0.000	0.531	0.008	0.017	0.797	1.265	0.212	0.998	2.328
		4	0.000	0.210	0.000	0.000	0.531	0.008	0.000	1.283	1.265	0.000	2.895	2.545
		6	0.000	0.210	0.000	0.000	0.531	0.008	0.000	1.283	1.265	0.000	2.983	2.545
		8	0.000	0.210	0.000	0.000	0.531	0.008	0.000	1.283	1.265	0.000	2.983	2.545
		10	0.000	0.210	0.000	0.000	0.531	0.008	0.000	1.283	1.265	0.000	2.983	2.545
		100	0.000	0.210	0.000	0.000	0.531	0.008	0.000	1.283	1.265	0.000	2.983	2.545
	1	2	0.000	0.000	0.000	0.000	0.109	0.000	0.000	0.516	0.000	0.000	1.866	0.000
		4	0.000	0.000	0.000	0.000	0.109	0.000	0.000	0.516	0.000	0.000	2.874	0.000
		6	0.000	0.000	0.000	0.000	0.109	0.000	0.000	0.516	0.000	0.000	2.874	0.000
		8	0.000	0.000	0.000	0.000	0.109	0.000	0.000	0.516	0.000	0.000	2.874	0.000
		10	0.000	0.000	0.000	0.000	0.109	0.000	0.000	0.516	0.000	0.000	2.874	0.000
		100	0.000	0.000	0.000	0.000	0.109	0.000	0.000	0.516	0.000	0.000	2.874	0.000
	2	2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.008	0.000	0.140	0.020
		4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.008	0.000	0.140	0.020
		6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.008	0.000	0.140	0.020
		8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.008	0.000	0.140	0.020
		10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.008	0.000	0.140	0.020
		100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.008	0.000	0.140	0.020
400	0.5	2	11.039	11.039	11.039	12.112	12.112	12.112	13.952	13.952	13.952	17.380	17.380	17.380
		4	3.676	0.679	0.897	3.330	1.321	2.021	2.736	2.517	2.926	4.143	6.005	6.004
		6	0.407	0.729	0.897	1.426	1.420	2.021	1.864	2.966	3.166	2.046	4.358	8.363
		8	1.589	0.729	0.897	1.055	1.503	2.021	0.569	3.345	3.166	0.678	4.200	8.363
		10	0.539	0.729	0.897	0.403	1.503	2.021	0.262	3.345	3.166	0.399	4.200	8.363
		100	0.000	0.729	0.897	0.000	1.503	2.021	0.000	3.345	3.166	0.000	4.200	8.363
	1	2	5.580	5.580	5.580	6.333	6.333	6.333	8.074	8.074	8.074	10.931	10.931	10.931
		4	3.437	0.719	0.894	2.545	1.537	1.881	2.105	3.173	2.282	3.230	5.266	4.632
		6	0.349	0.719	0.894	0.906	2.031	1.881	1.040	4.241	2.437	1.332	3.996	7.294
		8	0.665	0.719	0.894	0.798	2.031	1.881	0.466	5.029	2.437	0.671	5.576	7.294
		10	0.452	0.719	0.894	0.202	2.031	1.881	0.185	5.029	2.437	0.074	5.494	7.294
		100	0.000	0.719	0.894	0.000	2.031	1.881	0.000	5.029	2.437	0.000	5.494	7.294
	2	2	0.547	0.875	0.575	0.700	1.160	0.678	2.027	2.338	1.822	4.074	4.592	4.177
		4	0.311	0.929	0.434	0.464	1.264	0.697	1.307	2.059	0.666	0.883	4.675	2.444
		6	0.254	0.892	0.434	0.217	1.456	0.697	0.734	2.128	0.613	0.362	1.345	2.342
		8	0.191	0.888	0.434	0.131	1.416	0.730	0.146	2.128	0.662	0.012	1.353	2.416
		10	0.114	0.888	0.434	0.002	1.416	0.730	0.000	2.128	0.662	0.000	1.353	2.486
		100	0.000	0.895	0.450	0.000	2.024	0.958	0.000	2.734	0.828	0.000	1.505	3.009
800	0.5	2	31.736	31.736	31.736	33.095	33.095	33.095	36.308	36.308	36.308	40.999	40.999	40.999
		4	1.017	1.452	1.017	2.258	2.443	2.258	4.449	5.213	4.449	8.394	9.243	9.399
		6	4.255	0.665	1.588	2.152	1.449	2.124	2.055	3.312	6.285	2.786	3.869	11.044
		8	0.938	0.665	1.588	1.172	2.304	2.124	2.188	3.745	6.285	1.643	3.543	11.715
		10	1.532	0.665	1.588	1.131	2.304	2.124	1.057	4.801	6.285	1.266	4.287	11.715
		100	0.000	0.665	1.588	0.000	2.304	2.124	0.000	4.801	6.285	0.000	4.287	11.715
	1	2	23.198	23.198	23.198	24.308	24.308	24.308	26.887	26.887	26.887	30.531	30.531	30.531
		4	0.618	1.245	0.618	1.693	2.278	1.963	3.522	4.000	4.209	6.773	6.514	7.138
		6	3.212	0.861	1.404	1.563	2.183	2.632	1.392	2.351	4.867	1.662	1.607	8.003
		8	0.496	0.861	1.404	0.724	2.667	2.632	1.970	3.408	4.867	2.153	3.060	8.119
		10	1.322	0.861	1.404	1.087	2.667	2.632	0.681	5.299	4.867	0.667	4.392	8.119
		100	0.000	0.861	1.404	0.000	2.667	2.632	0.000	5.299	4.867	0.000	4.392	8.119
	2	2	1.287	1.287	1.287	3.259	3.259	3.259	5.658	5.871	5.658	9.499	10.058	9.385
		4	1.235	1.124	0.475	1.624	2.245	0.398	1.680	2.730	1.366	2.630	2.964	1.710
		6	0.861	1.171	0.475	1.211	2.361	0.398	1.101	3.557	1.358	1.445	3.486	1.633
		8	0.484	1.174	0.475	0.632	2.309	0.398	0.470	3.117	1.358	0.511	3.146	1.633
		10	0.395	1.174	0.475	0.414	2.294	0.398	0.285	3.097	1.358	0.202	2.998	1.633
		100	0.000	1.398	0.475	0.000	4.289	0.398	0.005	5.322	1.358	0.050	3.527	1.633