The Solar and Lunar Variations in Barometric Pressure at Glasgow and Ben Nevis

by

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PREFACE.

The investigations which form the subject of the present thesis were begun in 1933, in collaboration with Dr. R. A. Robb.

In Chapter I is given a short general description of the phenomena under investigation.

Chapter II discusses the two methods employed for removing the "convexity effect" from the solar and lunar variations which are to be determined. One of these methods - namely, the "upper and lower" transit method - was developed by Dr. R. A. Robb and the writer.

Chapters III, IV, and V describe the work done, in collaboration with Dr. R. A. Robb, on the lunar atmospheric tide at Glasgow. A summary of this work has been published in the Proceedings of the Royal Society of Edinburgh, volume lix., pages 81-90.

Chapters VI and VII describe my own investigations on the solar variation on barometrically quiet days at Glasgow. In this work the "transposition" method of removing convexity is applied to the solar variation for the first time.

Chapters VIII and IX describe my own investigations on the clear-day barometric curve at Ben Nevis. In these investigations, the "upper and lower transit" method is used to separate the convexity from the periodic components.

Chapter VIII, which is a critical examination of the work of Buchan and Omond on this subject, is being published in the Proceedings of the Royal Society of Edinburgh. Chapters VI, VII, and IX are unpublished.

An investigation of the lunar atmospheric tide at Ben Nevis has led to entirely negative results, and no account of this work has been included.

I am indebted to Dr. R. A. Robb for his unfailing interest in the progress of this work.

T. R. Tannahill.

University Observatory,

Glasgow. June, 1941.

ADDITIONAL PAPERS.

The following additional papers, on work done in collaboration with Professor W. M. Smart, are submitted:-

(1). "The Constants of the Star-Streams from the Photographic Proper Motions of 1775 Stars"

Monthly Notices of the Royal Astronomical Society, Volume 98, pp. 563-570.

(2). "The Constants of the Star-Streams from the Cape Photographic Proper Motions of 18,323 Stars"

Monthly Notices of the Royal Astronomical Society, Volume 100, pp. 30-44.

(3). "Star-Streaming in Relation to Spectral Type from the Cape Photographic Proper Motions"

Monthly Notices of the Royal Astronomical Society, Volume 100, pp. 688-692.

I.

(1). An examination of the trace of a self-recording barograph at a tropical station, made during periods of sufficiently quiet weather, shows clearly a double oscillation of the atmospheric pressure during the course of a day. The maxima of this oscillation are seen to occur about 10 a.m. and 10 p.m. local time, the minima about 4 a.m. and 4 p.m; the amplitude of the oscillation being about one millimetre. At extra-tropical stations, owing to the irregular variations of pressure due to rapid weather changes, this oscillation is not apparent on the individual traces; it can, nevertheless, be easily detected by taking the average pressure at each hour over a comparatively short period of time. The amplitude is found to decrease rapidly with increasing latitude, north or south; but the times of maximum and minimum remain practically constant, in local time, from place to place, until high latitudes are reached.

The daily oscillation of the barometer was first noticed soon after the invention of the mercury barometer in 1643 by Torricelli. Since that time it has been the subject of very comprehensive studies, both from the theoretical and observational standpoints. The daily curve, taken over an entire year, is mainly semi-diurnal in character, but a diurnal component and higher harmonics also occur. The semi-diurnal component is greatest at the equator and least at the poles, and also has a seasonal variation, being in general greatest at the equinoxes, smaller at the winter solstice and least of all at the summer solstice. Table I gives the data for three typical stations, namely, Batavia, Calcutta and Glasgow, situated as follows:-

	Batavia	Calcutta	Glasgow
Latitude	6° 11'S	22° 32'N	55° 53'N
Longitude	106° 50'E	88° 20'E	4° 18'W
Height above sea-level (metres)	8	6.5	55

The data given are the constants of harmonic analysis, c_n and α_n, the hourly mean values of the barometric readings being represented by

$$y = \sum c_n \sin (nx + \alpha_n) .$$

x is the local mean time, measured from midnight. The great regularity of the semi-diurnal component, particularly in phase, is clearly seen, as is also the rapid diminution in the amplitude from its values in the tropics to those in extra-tropical regions. The diurnal component is fairly regular in amplitude and in phase at the two tropical stations

T a b l e I .

Harmonic Coefficients of the Solar Variation.

Unit 1 millibar.   Local Mean Time.

Latitude	Batavia 6°S		Calcutta 22°N		Glasgow 56°N	
	c_n	α_n	c_n	α_n	c_n	α_n
Diurnal Component						
Decr. Solstice	0.75	22°	0.94	341°	0.08	159°
Mch. Equinox	0.79	26°	1.13	338°	0.06	96°
June Solstice	0.89	28°	0.78	354°	0.15	39°
Sept. Equinox	0.99	25°	0.76	350°	0.05	172°
Semi-diurnal Component						
Decr. Solstice	1.36	160°	1.38	153°	0.26	151°
Mch. Equinox	1.36	157°	1.51	144°	0.33	152°
June Solstice	1.25	158°	1.26	142°	0.25	144°
Sept. Equinox	1.39	165°	1.37	154°	0.31	155°
Ter-diurnal Component						
Decr. Solstice	0.02	328°	0.26	358°	0.12	349°
Mch. Equinox	0.06	8°	0.01	315°	0.01	250°
June Solstice	0.09	26°	0.09	192°	0.06	157°
Sept. Equinox	0.05	29°	0.08	339°	0.04	42°

included in this table, but at Glasgow this component suffers a seasonal reversal in phase. The distribution of the diurnal component over the earth has not been greatly studied, largely owing to the fact that this component is greatly influenced by local conditions. The ter-diurnal component, likewise, has also been neglected. Figures 1 - 3 show the seasonal components of the solar variation at Glasgow. A reversal of phase, between summer and winter, in the first and third harmonics, and the virtual disappearance of these harmonics at the equinoxes, are noticeable.

(2). The distribution of the semi-diurnal component with respect to latitude is best described by a formula due to G.C. Simpson[1]. It was first suggested by A. Schmidt[2] in 1890 that the daily semi-diurnal variation at any place might be represented by the combination of two waves:-

(a) An oscillation parallel to the circles of latitude, consisting of a double wave travelling from east to west; the amplitude being maximum at the equator and zero at the poles, the phase being constant in local time.

(b) An oscillation along meridians, between poles and equator, due to a stationary wave; the amplitude being maximum at the poles, decreasing to zero at latitudes $\pm 35^\circ 16'$, and increasing again towards the equator to half its value at the poles, the phase being reversed at latitudes $\pm 35^\circ 16'$. For this oscillation, the phase is constant in universal time

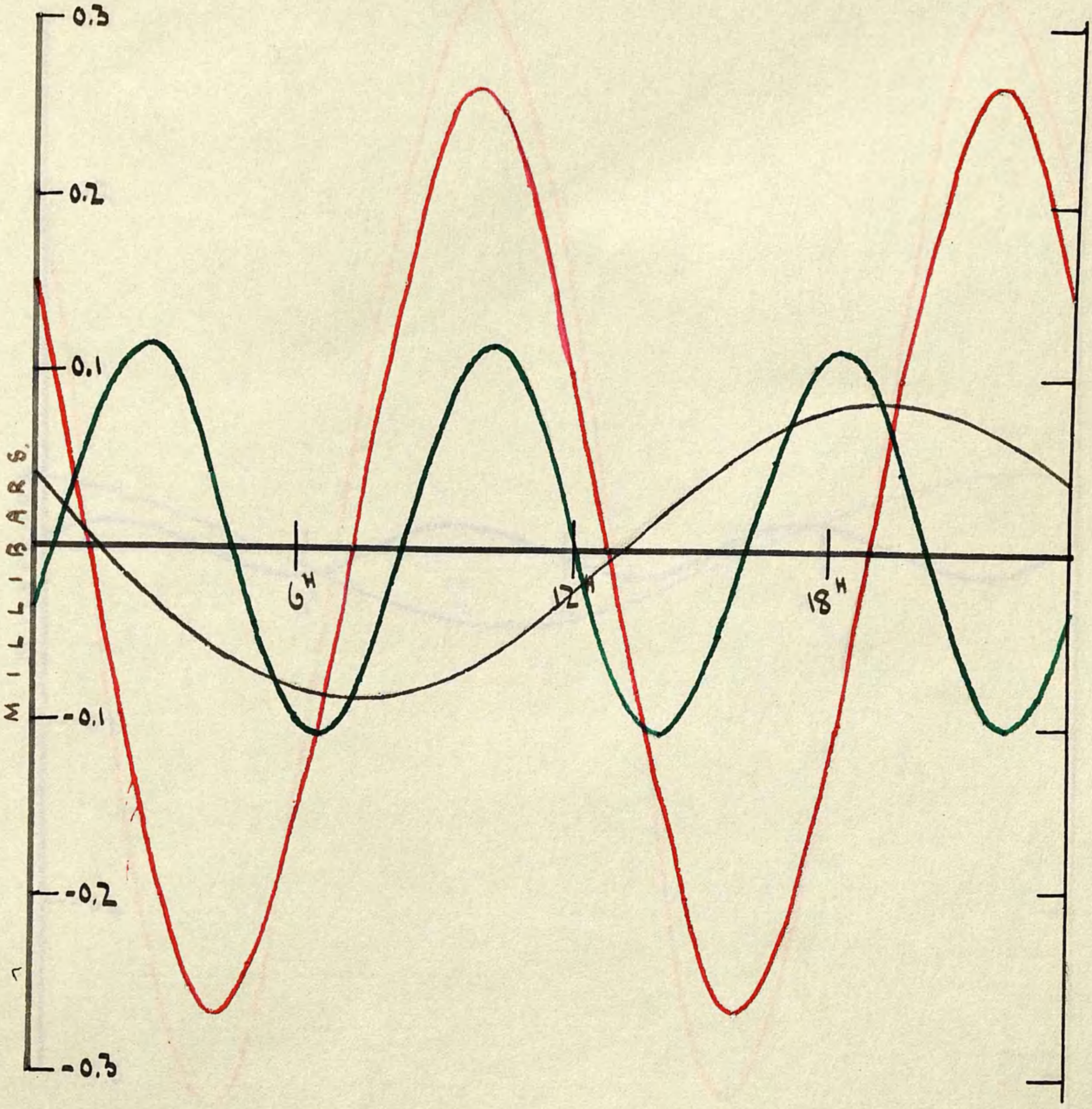

Figure 1.- The Solar variation at Glasgow (Winter).

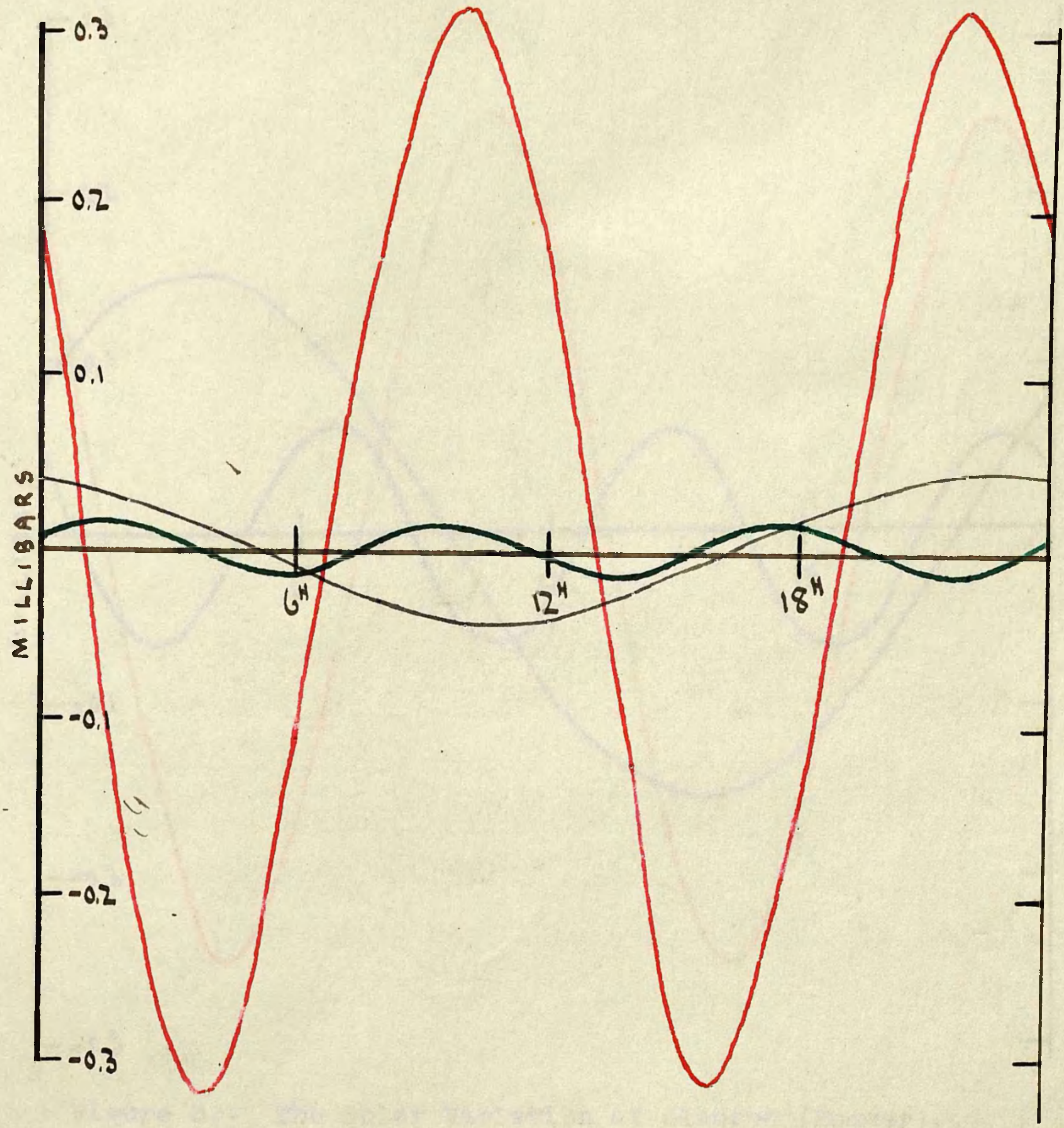

Figure 2.- The Solar Variation at Glasgow (Equinoxes).

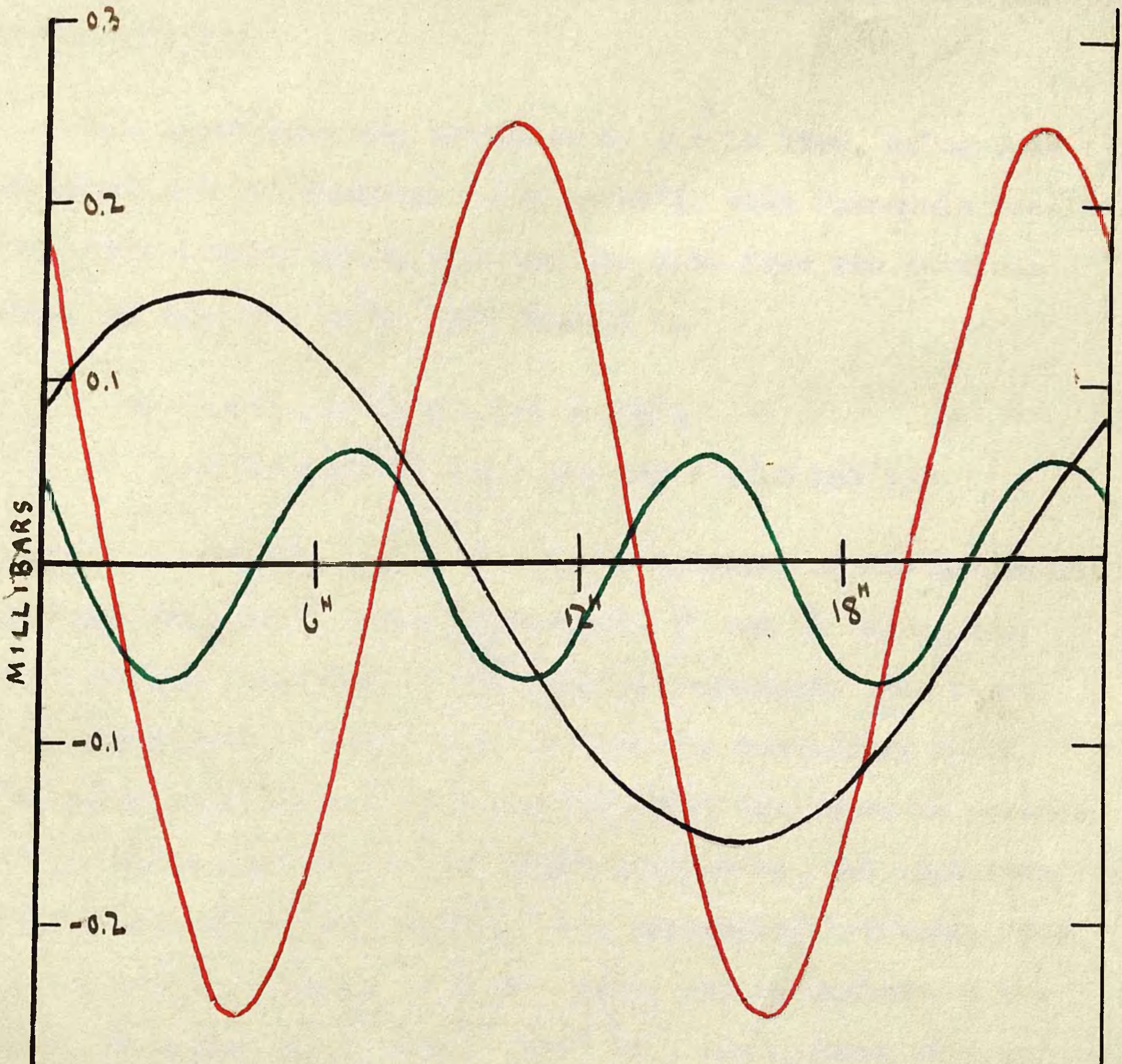

Figure 3.- The Solar Variation at Glasgow (Summer).

at all places north (south) of latitude 35° 16'N (S), and is also constant, with the phase reversed, in the zone between these latitudes.

This hypothesis was tested by E. Alt[3] in 1909, using data obtained from 49 stations north of 45°N, with favourable results. Simpson's investigation utilised the data from 190 stations north of latitude 10°S. His formula is

$$y = 1.248 \cos^3\phi \sin(2x + 154^\circ) + 0.182 (\sin^2\phi - \tfrac{1}{3}) \sin(2x - 2\lambda + 105^\circ),$$

where y, in millibars, is the mean inequality of the barometric pressure at hour x (local mean time), ϕ and λ being the latitude and longitude of the station concerned. The first term represents a double wave of pressure travelling round the earth with its maxima occurring about two hours in advance of the sun's transits at any particular place, the amplitude of the wave decreasing rapidly with increasing latitude. The second term is likewise a double wave, but is maximum at the poles, vanishes at latitudes ±35° 16', and appears with an abrupt phase change nearer the equator. The phase of this component north of latitude 35° 16'N is constant with respect to universal time, that is, the maxima occur at the same time all over the earth. The two terms may be designated the equatoreal and polar vibrations respectively. The observed amplitudes and phases, for various latitude zones, of these vibrations are shown in Tables II and III, together with the corresponding quantities calculated from the final results. As regards the equatoreal vibration, this is fairly large and can be well determined from the data for the numerous stations in tropical and temperate regions. In the case of the polar vibration, however, which is small and reaches its maximum in the polar regions, where stations are few and the existing records cover only very short periods of time, the accuracy in determining phase and amplitude is not very high, particularly towards the equator. The calculated amplitude is obtained in this case only from the four zones nearest the pole.

Considering the paucity of the data for polar regions, it will be seen that this formula accords well with the hypothesis of Schmidt. The main discrepancy is in the magnitude of the phase change at latitude $35^{\circ}\ 16'$, which is nearer 90° than 180°, but, as explained above, large uncertainties may be expected in the determination of this component. The hypothesis may therefore be regarded as verified.

The semi-diurnal oscillation of the atmosphere has received, from the theoretical point of view, much attention, being the subject of memoirs mainly by Laplace[4], Kelvin[5,6], Margules[7], Lamb[8] and Chapman[9], and more recently by Taylor[11] and Pekeris[10]. The theoretical view-point of the subject will not be reviewed here; summaries of the main developments are given periodically by Chapman.[9,12]

Table II.

Equatoreal Oscillation in various latitude Zones.

Unit 1 millibar.

Mean Lat. N.	Observed Phase	Observed Amplitude	Calculated Amplitude	Observed minus Calculated
0°	156° 50'	1.225	1.248	− 0.023
18°	155° 17'	1.112	1.073	+ 0.039
30°	149° 07'	0.836	0.810	+ 0.026
40°	153° 56'	0.515	0.561	− 0.046
50°	153° 01'	0.320	0.331	− 0.011
60°	158° 03'	0.128	0.156	− 0.028
74°	152° 53'	0.029	0.026	+ 0.003
Mean	154°			

Table III.

Polar Oscillation in various latitude Zones.

Unit 1 millibar.

Mean Lat. N.	Number of stations	Observed Phase	Observed Amplitude	Calculated Amplitude	Observed minus Calculated
0°	17	176° 02'	0.091	0.061	+ 0.030
18°	15	156° 47'	0.109	0.043	+ 0.066
30°	12	190° 26'	0.079	0.015	+ 0.064
40°	46	91° 04'	0.057	0.015	+ 0.042
50°	60	104° 27'	0.055	0.046	+ 0.009
60°	18	108° 23'	0.083	0.076	+ 0.007
70°	14	98° 34'	0.096	0.100	- 0.004
80°	8	116° 27'	0.107	0.116	- 0.009
Mean		105°			

(3). As has already been mentioned, the diurnal component of the daily oscillation is subject to great local variations and, so far as is known, does not vary in any regular manner over the earth's surface. A definite variation with altitude is, however, recognised. Figure 4 shows the diurnal component of the oscillation for four stations, during the summer months. The stations are as follows:-

1. Bureau Central Meteorologique, Paris. Latitude 48° 52'N, Longitude 2° 18'E, altitude 33 metres.
2. Eiffel Tower, Paris. Latitude 48° 52'N, Longitude 2° 18'E, altitude 313 metres.
3. Puy de Dome. Latitude 45° 46'N, Longitude 2° 58'E, altitude 1467 metres.
4. Sonnblick. Latitude 47° 03'N, Longitude 12° 57'E, altitude 3106 metres.

It will be seen from the diagrams that the component occurring near sea-level with a phase angle in the neighbourhood of 0° decreases with increasing altitude and disappears, reappearing with phase reversed at higher altitudes. The explanation of this component is well-known. Thus, according to Humphreys[13]:-

> "There are two classes of well-defined 24-hour pressure changes. One obtains at places of considerable elevation and is marked by a barometric maximum during the warmest hours and minimum during the coldest. The other applies to low, especially sea level, stations and is the reverse of the above, the maximum occurring during the coldest hours and the minimum during the warmest.

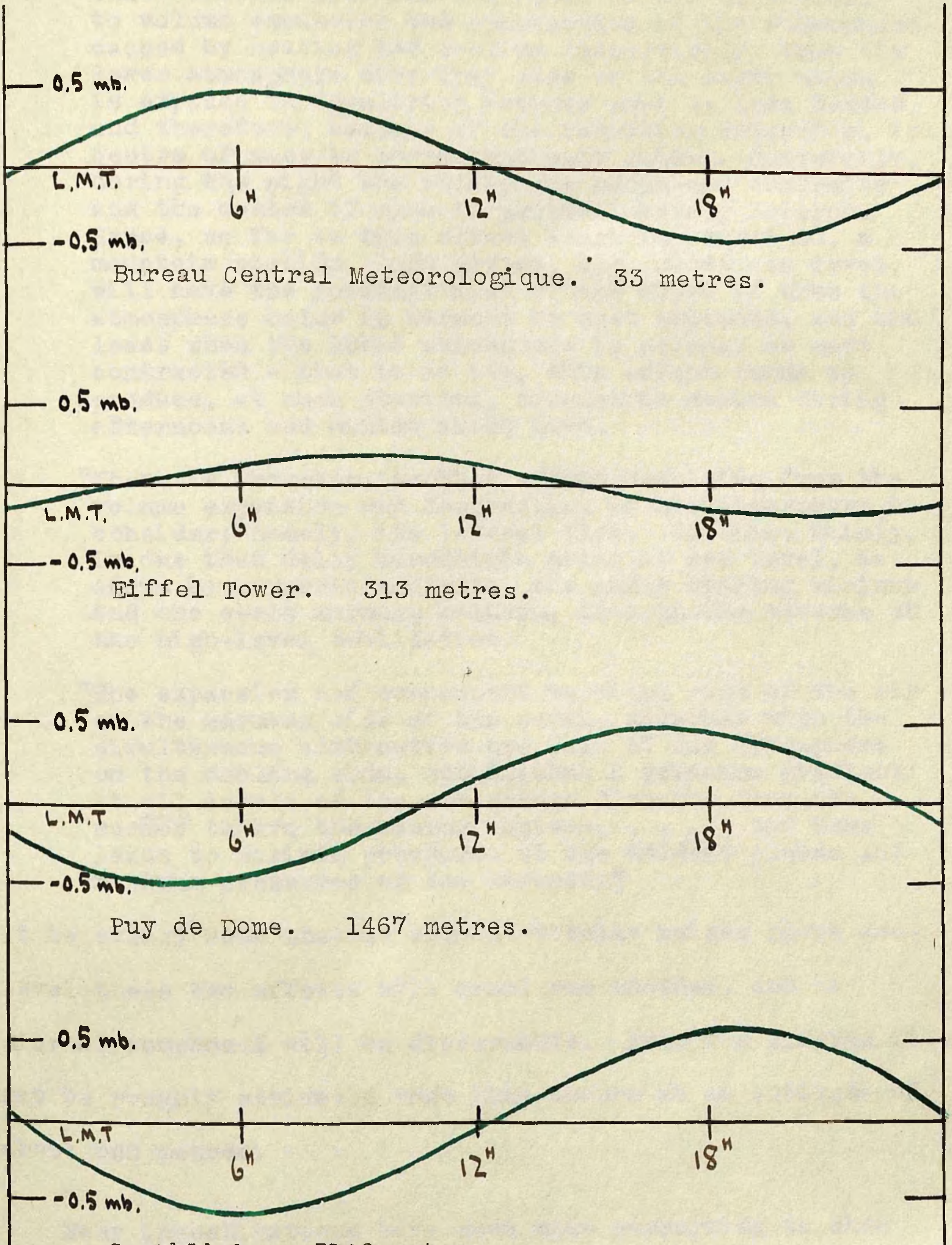

Figure 4. Variation of the diurnal component with altitude.

"The first class of changes just mentioned, the one that concerns elevated stations, is due essentially to volume expansion and contraction of the atmosphere caused by heating and cooling respectively. Thus the lower atmosphere over that side of the earth which is exposed to insolation becomes more or less heated, and therefore, because of the resulting expansion, its centre of mass is correspondingly raised. Conversely, during the night the atmosphere cools and contracts and the centre of mass is proportionately lowered. Hence, so far as this effect alone is concerned, a mountain station, 1000 metres, say, above sea level, will have the greatest mass of air above it when the atmosphere below is warmest or most expanded, and the least when the lower atmosphere is coldest or most contracted - that is to say, this effect tends to produce, at such stations, barometric maxima during afternoons and minima about dawn.

"There is, however, another effect resulting from the volume expansion and contraction of the atmosphere to consider; namely, its lateral flow. To this, mainly, is due that daily barometric swing at sea level, as shown by harmonic analysis, the early evening minimum and the early morning maximum, that is the reverse of the high-level oscillation.

"The expansion and consequent vertical rise of the air on the warming side of the earth, together with the simultaneous contraction and fall of the atmosphere on the cooling side, establishes a pressure gradient at all levels of the atmosphere directed from the warmer toward the cooler regions, ... and thus leads to maximum pressures at the coldest places and minimum pressures at the warmest."

It is easily seen that at some particular height above sea-level these two effects will annul one another, and no diurnal component will be discernable. From the diagram it may be roughly estimated that this occurs at an altitude of about 500 metres.

Many investigations have been made purporting to show the changes in this component with varying types of weather, for example, on clear and on cloudy days. These will be discussed later in connection with the investigation of the Ben Nevis data.

(4). Besides the comparatively large and easily determined solar daily variation in the barometric pressure, there is also discernable - in the mean of at least a year's observations at a tropical station - a minute variation in lunar time. The primary cause of this variation is fairly simple, being the tidal action of the moon, but there are some notable problems awaiting solution before it is thoroughly understood.

At tropical stations the lunar atmospheric tide shows itself as a purely semi-diurnal wave of pressure travelling round the earth with its maxima occurring about one hour after lunar transits. The amplitude is very small, its value at the equator being of the order of 0.1 millibar, and decreases with latitude very rapidly. In extra-tropical regions the amplitude is exceedingly minute (its value at Greenwich is only 0.012 millibar) and it is very difficult to separate the tidal effect from the large and irregular variations of pressure which occur in such regions. The lunar tide has, nevertheless, been detected at a considerable number of stations in the temperate and tropical zones, mainly through the work of Chapman[14,15,16,17] and his co-workers. It is found that the amplitude of the tide undergoes a large annual variation (whose magnitude is of the same order as that of the tide itself), being greater at the June solstice than at other times. There is also a marked lag in the time of high tide at the December solstice. This annual variation (which is not a seasonal variation, since it occurs at the same time in both the northern and southern hemispheres) has not been explained. A further variation, with varying distances of the moon, has also been noticed.

Full accounts of the progress made in the determination of the lunar tide have been given by Chapman.[12,15,24]

II.

(5). The present investigations are concerned mainly with the solar and lunar daily variations of the barometer on selected types of days. In the case of the lunar variation, it is found necessary, on account of the smallness of the variation expected (the lunar variation at Greenwich, for example, is of semi-diurnal type and is represented by $0.0120 \sin (2x + 114^\circ)$ millibar , to discard entirely those days in which the barometric record is greatly disturbed, using only those days within which the pressure does not vary more than a stated amount, say 0.1 inch, from beginning to end of the day. By this means the accidental errors of the data are reduced sufficiently to permit the disentanglement of the small periodic variation; this method is effective in spite of the great reduction in the number of days used (in the Glasgow records, only about 1/4 of the data is retained; for Greenwich the fraction is about 1/3) owing to this limitation of the pressure range. The results obtained are usually accepted as being representative of the typical variation belonging to any type of day; that is to say, the fact that the days are selected is ignored. Nevertheless, it is well to remember that this selection has taken place; and the Glasgow records, as will be shown later, give evidence that the lunar variation may, on occasions, be dependent on the particular type of days chosen.

(6) The selection of barometrically "quiet" days is a necessity in the determination of the lunar variation (for latitudes such as those of Greenwich and Glasgow). In the case of the solar variation, the selection of days, according to various criteria, is of interest in itself, and several investigations have been made to determine this variation on different types of days, for example, on days characterised by clear and cloudy skies. Thus Buchan and Omond,[18] in a paper entitled "The Diurnal Range of the Barometer in Clear and Cloudy Weather" deduced a great increase in the first harmonic of the daily variation on clear days, and gave a theoretical explanation of it.

This investigation, and many other investigations of similar type (for example, Hann,[19] "Der Täglicher Gang des Barometers") suffer from a failure on the part of the authors to appreciate the exact significance of the effect of selection. In selecting barometrically quiet days, or clear days, we are in effect restricting our choice of observations to those taken in anti-cyclonic weather. In such circumstances the barometric curve is near a maximum turning-point, the quantity $\frac{d^2p}{dt^2}$ being negative. Consequently, in selecting portions of such a curve, we are superposing on the ordinary daily periodic variations of the barometer a "convex variation" which is non-periodic, and which is entirely spurious, so far as periodic effects are concerned. If the inequalities concerned are analysed harmonically, a first harmonic is present due entirely to the spurious convexity, which simulates an additional periodic variation.

(7). Consider the barometric curve during anticyclonic weather, when the pressure is high and near a maximum turning-point. Assume for the present that there is no periodic daily variation. The curve is approximately parabolic, as shown in Fig. 5 . The selection of days near this part of the barometric curve is equivalent to the selection of 24-hour portions of the curve, such as AB, BC, CD. It is usual, in such work, to remove the "non-cyclic change", that is, to apply to each hourly reading of the barometer a correction which increases progressively (and linearly) from one end of the day to the other, thus equalising the barometer readings at the beginning and end of the day. This process does not remove the convexity; the "corrected" curve consists of a series of parabolic arcs, symmetrical about the centre of the selected portion of the record, as in Fig. 6 . In the final inequalities, obtained by addition of a large number of such portions of the record, this effect is in no way eliminated. It is to be noticed in the first place that the effect can only arise when the days are selected near a maximum turning-point. In inequalities obtained from unselected

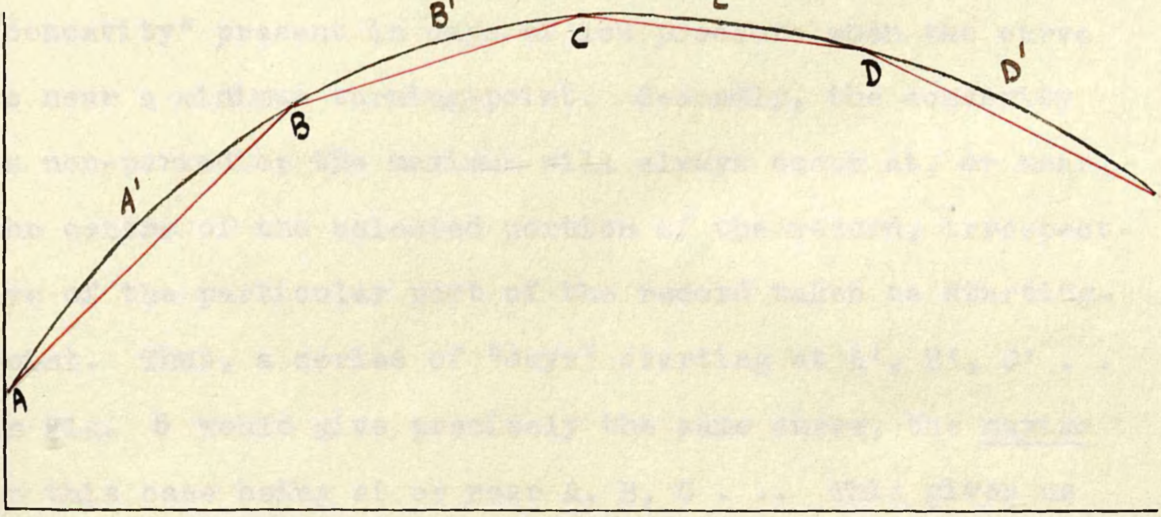

Figure 5.

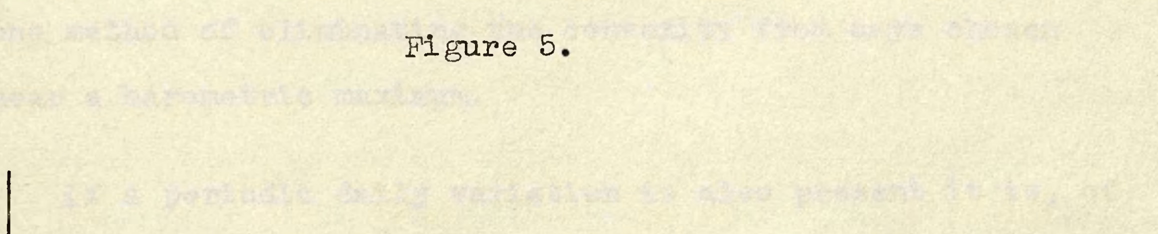

Figure 6.

days, the convexity of the anticyclonic days would be cancelled out in the final inequalities by a corresponding "concavity" present in days of low pressure when the curve is near a minimum turning-point. Secondly, the convexity is non-periodic; the maximum will always occur at, or near, the centre of the selected portion of the record, irrespective of the particular part of the record taken as starting-point. Thus, a series of "days" starting at A', B', C' . . in Fig. 5 would give precisely the same curve, the maxima in this case being at or near A, B, C . .. This gives us one method of eliminating the convexity from days chosen near a barometric maximum.

If a periodic daily variation is also present it is, of course, superposed on the parabolic arc. Suppose a series of 24-hour periods are selected from the barometric curve starting, say, at midnight; then the average periodic variation may be represented by

$$c_1 \sin(x+\alpha_1) + c_2 \sin(2x+\alpha_2) + c_3 \sin(3x+\alpha_3) \; . \; . \; .$$

where x is measured from the beginning of the period, midnight in this case. Let p_x represent the ordinate at hour x due to the convexity. The maximum of p_x will be near noon. The sum of the periodic and non-periodic effects is then

$$y_{x,M} = p_x + c_1 \sin(x+\alpha_1) + c_2 \sin(2x+\alpha_2) + \; . \; . \; . \; c_n \sin(nx+\alpha_n)$$

Let a similar series of 24-hour periods be selected according to the same criterion as before (say range of pressure over

24-hour period $\leqq$ 0.1 inch), but starting at noon. If x be again measured from the beginning of the selected "day", the convexity will remain unchanged, having in this case its maximum at midnight; but the periodic components will have a phase difference of 180^{o}, that is, odd harmonics will be reversed, even harmonics will remain unchanged. Thus

$$y_{x,N} = p_x - c_1 \sin(x+\alpha_1) + c_2 \sin(2x+\alpha_2) - \ldots c_n \sin(nx+\alpha_n)$$

where $c_n \sin(nx+\alpha_n)$ is positive for even harmonics and negative for odd harmonics. By addition and subtraction of these equations, we have

$$y_{x,M} + y_{x,N} = 2p_x + 2c_2 \sin(2x+\alpha_2) + 2c_4 \sin(4x+\alpha_4) + \ldots$$
$$+ \text{ higher even harmonics } \ldots\ldots\ldots \quad (1)$$

$$y_{x,M} - y_{x,N} = 2c_1 \sin(x+\alpha_1) + 2c_3 \sin(3x+\alpha_3) \ldots\ldots$$
$$+ \text{ higher odd harmonics } \ldots\ldots\ldots \quad (2)$$

Using equation (2), we see that it is possible to obtain the odd harmonics of the periodic variation entirely free from the spurious convexity effect. The determination of the even harmonics is not possible without making some assumption regarding the shape of the convexity. Assuming the convexity to be parabolic in shape, and symmetrical with respect to the middle ordinate of the 24-hour period selected, it may be represented by

$$y = c - a(x - 180^{o})^2.$$

(It is assumed that linear "non-cyclic change" has been removed from the inequalities concerned, so that the initial and final ordinates of the convexity are zero). This expression, developed as a harmonic series, becomes (omitting a constant term)

$$y = A \sin (x + 270^\circ) + \frac{A}{4} \sin (2x + 270^\circ) + \dots \dots + \frac{A}{n} \sin (nx + 270^\circ) \dots \dots \quad (3)$$

If equation (1) is analysed harmonically, we obtain a first "harmonic" which is entirely due to the convexity. The remaining "harmonics" of the convexity can be calculated by (3), since the amplitudes of the higher harmonics are simply related to that of the first. Thus the entire convex variation can be determined and removed from (1), giving the even harmonics of the true periodic variation, and effecting the complete separation of the true and spurious effects.

It will be noticed that the harmonic analysis of (1) gives a determination of the first "harmonic" of convexity quite independent of the particular form (3) chosen to represent the convexity as a whole. It is generally found (as will be seen later) that the phase angle of the first "harmonic" is not 270°, but rather greater. This means that the assumed simple parabolic form is not an exact representation of the convexity, which is not symmetrical about $x = 180^\circ$. For this reason it is preferable in some cases to use the expression

$$y = A \sin (x + \beta) + \frac{A}{4} \sin (2x + 2\beta + 90^\circ) + \frac{A}{9} \sin (3x + 3\beta + 180^\circ) + \dots \dots \quad (4)$$

to represent the convexity, leaving β to be determined by the actual analysis of (1). By this means the first "harmonic" of convexity is obtained free from all assumptions as to the actual form of the convexity. The form (4) is not, of course, an exact representation of convexity, but it takes account of the asymmetry mentioned, and represents exactly the first "harmonic".

The convexity effect was first pointed out by J. Bartels,[20] whose determination of it from the data of Potsdam is shown in Figure 7. Bartels defines the convexity as the difference of the barometric curve for "quiet days" and that for "all days", in the sense "quiet minus all". There is implicit in this definition the assumption that, if it were not for the presence of the convexity, the solar variation obtained from un-selected days ("all days") would be the same as that from the anti-cyclonic days chosen; in other words, the possibility of a <u>real</u> solar effect on barometrically quiet days is ignored. In the earlier papers by Buchan and Omond and by Hann already mentioned, the presence of the convexity effect is not recognised, and the entire difference "anti-cyclonic days minus all days" is considered as a real solar effect, attributed to actual physical processes in the atmosphere. It is evident that neither of these assumptions is justifiable. The preceding method gives a separation of the convexity effect from any true solar effect which may be present, without any previous assumptions.

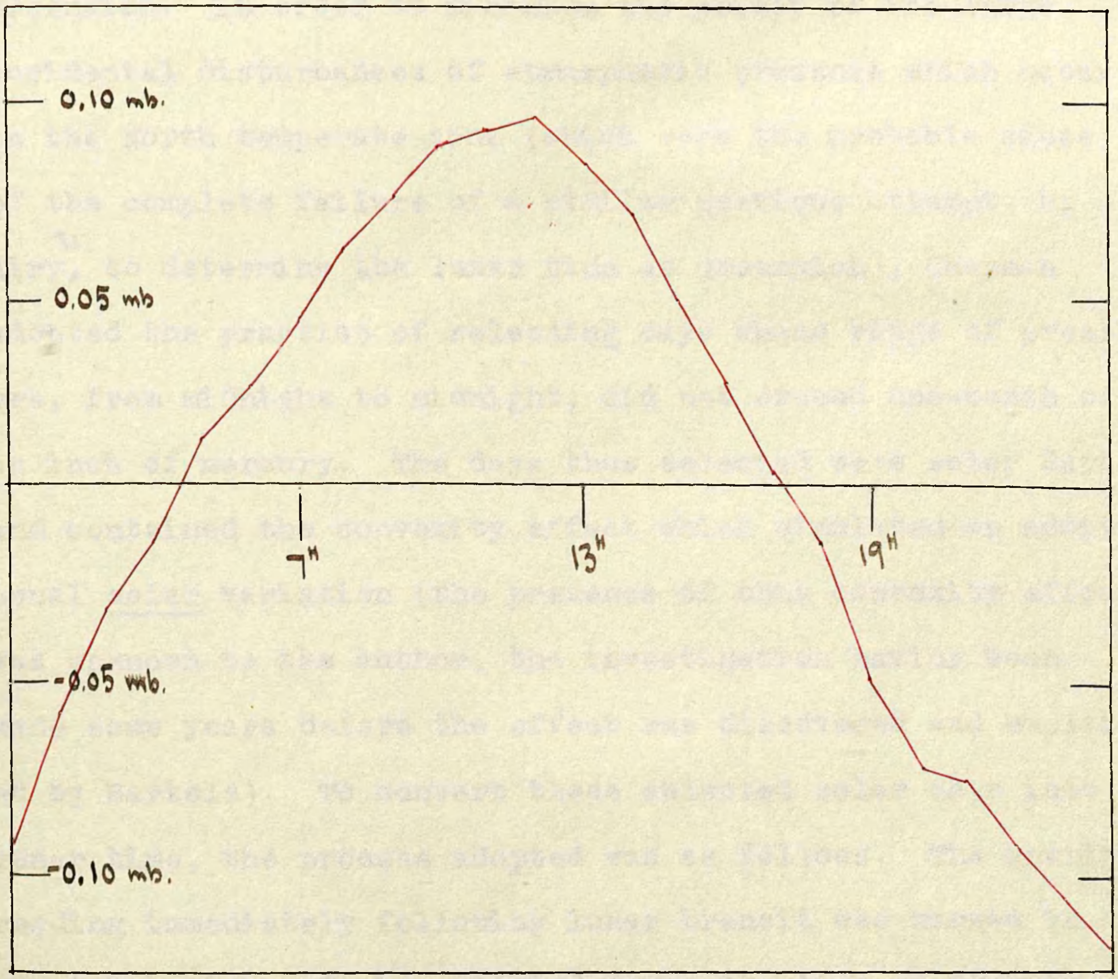

Figure 7.- Bartels' determination of "convexity" from the Potsdam data.

(8). Another method of removing the convexity effect depends on a process evolved - for quite another purpose - by Chapman[14] in his evaluation of the lunar atmospheric tide at Greenwich. In order to minimise the effect of the large accidental disturbances of atmospheric pressure which occur in the North temperate zone (which were the probable cause of the complete failure of a similar previous attempt, by Airy,[21] to determine the lunar tide at Greenwich), Chapman adopted the practice of selecting days whose range of pressure, from midnight to midnight, did not exceed one-tenth of an inch of mercury. The days thus selected were solar days, and contained the convexity effect which simulated an additional solar variation (the presence of this convexity effect was unknown to the author, the investigation having been made some years before the effect was discovered and explained by Bartels). To convert these selected solar days into lunar time, the process adopted was as follows. The hourly reading immediately following lunar transit was marked in the tabulations of the selected days, the time of transit being obtained from the Nautical Almanac. This reading was then taken as the first hourly reading of a lunar day, the second, third, . . . hourly readings being those immediately following on the original record, until the reading corresponding to 23^h (solar time) was reached. This was followed by the readings for 23^h, 24^h of the preceding day, and then by the readings for 1^h, 2^h, . . . of the selected day. By this means a sequence of readings, corresponding to a "lunar day" of 25 solar hours, and starting on the average half an hour after lunar transit, was built up. The following typical extract shows the method clearly.

Solar Hour	23^h	24^h
Barometer Reading	30.151	.149

1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h
.146	.144	.131	.123	.122	.123	.134	.122	.108	.108	.106	.112
13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	24^h
.102	.104	.096	.100	.119	.128	.142	.161	.173	.170	.174	.176

The lunar transit occurs, say, at 13^h 45^m. The first reading of the lunar sequence is that immediately following this time, namely, 30.104. The lunar sequence is therefore

Lunar Hour	1^h	2^h	3^h	4^h	5^h
Barometer Reading	30.104	.096	.100	.119	.128

6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h
.142	.161	.173	.170	.174	.151	.149	.146	.144	.131	.123	.122
18^h	19^h	20^h	21^h	22^h	23^h	24^h	25^h				
.123	.134	.122	.108	.108	.106	.112	.102				

There is generally a pronounced break in the lunar sequence between the double reading at 23^h solar time, due to the linear non-cyclic change during the day. This break, since it always occurs at the same solar time, may occur at any lunar hour, and is assumed, in the Greenwich calculations, to cancel out in the final inequalities. Further details of this "transposition" method will be discussed later in connection with the Glasgow data. Meanwhile, it is sufficient to note that the periodic components of any lunar variation present will not be affected by the transposition. The convexity effect, however, is zero at the beginning and end of the solar sequence used, that is, at 23^h on the preceding and on the selected day, and maximum about 11^h on the selected day. When the transposition takes place into lunar time, the convexity is, in the average of many days with transit-times at different solar hours, eliminated just as if it were a solar periodic effect. Thus the process of transposition used by Chapman, intended merely to simplify the numerical work of selecting and tabulating the data, is effective in eliminating an unwanted effect only recognised some years later.

Both the above methods for eliminating the convexity can be applied with equal success to the determination of either solar or lunar daily variations. Thus the method of transposition has been applied (in the present work) to the determination of the lunar tide at Glasgow, and to the investigation of the "quiet-day" solar variation and its changes with increasing quietness of day. The method of "upper and lower transits" has also been applied to the determination of the lunar tide at Glasgow, and in a discussion of the "clear-day" solar variation on Ben Nevis.

III.

(9). In this chapter the lunar variation at Glasgow is investigated by the transposition method. The records consist of hourly readings of barometric pressure taken at the former Dowanhill observatory of the University of Glasgow. The sequence is unbroken (except for minor interruptions) from 1868 till 1912, when the observations were discontinued. There are thus 45 years data. The following description is given by Becker[22] of the method adopted in registering the pressure:-

> "The atmospheric pressure was photographically registered by the following method. Immediately behind the top of the mercury barometer a narrow slit was fixed parallel to the tube of the barometer, and a beam of light was sent through this slit with the result that the length of the illuminated portion of the slit depended on the height of the mercury. This luminous line was photographed on paper stretched on a drum rotating once round in 48 hours. A zero mark, compensated for temperature, and placed immediately behind the slit, constantly cut off the light from a short piece of the slit. Every second hour the light from the barometer was automatically shut off during four minutes, 58 m. to 2 m. Greenwich Mean Time. The developed photograph shows a black band, whose breadth depends on the height of the barometer. The band is crossed lengthwise by the white zero line and crosswise by the equi-distant two-hour lines. The scale of the photograph is about double that of the barometer. The cistern of the barometer is 184 feet above sea-level. Readings were taken five times a day - at 10 a.m., noon, 2, 6, and 10 p.m. Greenwich Mean time, and also at 9 a.m. and 9 p.m. local time. . . .

"The photographic trace was measured by means of a measuring apparatus. This apparatus had a pointer which was moved by means of a rack and pinion in the direction of the hour-lines, and whose position was read by a scale and vernier to 0.001 inch. The scale gave very nearly the barometer reading in inches. The eye-observations of the standard barometer determine the corrections of the measurements. The corrected measurements for each hour were tabulated . . . 329 traces, 2.0 per cent of the total number, are incomplete, and of these two-thirds belong to the six years 1868-73. There are no missing traces in the last six years. In some cases, not included in this number, the trace is defective at hours for which standard readings are available, and it can, therefore, be utilised as if it were complete; and in a few cases where one to three readings are missing, the curve can be interpolated with all desirable accuracy."

(10). The procedure used in selecting and tabulating days is very similar to that used by Chapman for the Greenwich data, which has already been described; but there are several minor differences:-

(a). In the Greenwich reduction the days selected were solar days, of 24 hours duration, from midnight to midnight, the criterion of selection being that the difference between the highest and lowest reading for the hours 0^h, 9^h, 12^h, 15^h, 21^h and 0^h of the following day should not exceed 0.09 inch. 6457 days, in the period 1854-1917, were obtained by this method. It was inferred that on the majority of these days the total range would not exceed 0.1 inch.

In the present investigation the "day" selected is of 25 hours duration, starting at 23^h and including the 23^h reading of the following day. The criterion of selection is that the difference of the highest and lowest readings of this 25-hour sequence, rounded to the nearest 0.01 inch, (in the manner described below), should not exceed 0.1 inch. It is evident that the maximum possible range is thus 0.110 inch.

(b). In the Greenwich reduction the hourly readings were tabulated to the nearest 0.01 inch, as follows:-

> " . . . the second decimal figure is raised by one unit when the third figure exceeds 5. When the third figure was 5, the next even figure was adopted for the second decimal; thus 29.875 would be read as 29.88, and 29.865 as 29.86."

In the Glasgow reduction the third decimal place has been retained in the tabulations. The effect of "rounding" is discussed later, in section 22.

(c). In the Greenwich reduction, the transposition into lunar time introduces a "break" in the readings between the two solar hours 23^h. This break is due to the fact that, in general, the barometer does not return to its original reading after completing its periodic variations in the course of a day. Superposed on the periodic variations, there is a trend, usually described as the "non-cyclic change". In the present work, in tabulating the hourly readings, a 26th. hourly reading was added at the end of the sequence, in order to permit of the removal of this non-cyclic change.

(11). The tabulation of the data virtually consisted of making a "card-catalogue" of the selected days. The readings were not tabulated exactly as they appeared in the original records, but a quantity, constant for each sequence, was subtracted from each before tabulation, so that no entry exceeded 205, the unit being one-thousandth of an inch. Thus, if the lowest reading of a sequence was 29.582 inches and the highest 29.665, the corresponding tabulated quantities were 82 and 165, the constant quantity 29.500 having been subtracted.

The tabulation was done on a Burroughs adding and tabulating machine. Each "card" consisted of a paper slip containing a series of 26 hourly readings, the first reading being that immediately following the lunar (upper) transit, which had been marked, as previously described (section 8) in the original records. The sum of the 26 entries for each sequence was obtained, the addition being done by the machine during the process of tabulation. On each slip was entered the solar hour of the first reading of the lunar sequence, or "transit-hour", as we shall call it, and the date (year, month and day). A specimen slip is shown in Figure 8.

(12). The sequences as tabulated contain, in addition to the lunar variation which is to be derived from them, (a) the non-cyclic change mentioned above (b) the periodic solar variation (c) the convex variation as described in the previous chapter.

Year	Month and Date	Transit Hour	
79	I 23	14H	15 ← Semi-diameter

	Solar Hour
66	14
56	15
52	16
43	17
35	18
35	19
41	20
38	21
45	22
44	23
137	23
135	24
120	1
117	2
114	3
105	4
97	5
99	6
97	7
107	8
105	9
107	10
109	11
96	12
83	13
38	24
2121 *	Daily Total

Figure 8 .

Each of these variations greatly exceeds the lunar variation expected. The non-cyclic change can, at the most, amount to a difference of 0.11 inch between the beginning and end of a selected sequence. The solar variation, at Glasgow, has a range, between maximum and minimum values, of about 0.020 inch. The convex variation has a range of about 0.005 inch. Small as these last two quantities are, they are still much larger than the expected lunar variation, whose extreme range, at Greenwich, was found by Chapman to be less than 0.001 inch. It is therefore important that the utmost care be taken in eliminating these unwanted variations from the lunar inequalities.

The method finally adopted, in the present investigation, for the treatment of the data was as follows. The lunar sequences were collected into groups characterised by the same transit-hour. The Burroughs slips belonging to such a group, each containing 26 hourly readings, were then attached to long pieces of card, each capable of holding about 40 slips; the slips being arranged so that corresponding lunar hours were in lines. Cross-addition of these lines gave the total inequality for each hour of the lunar day, a check on the addition being obtained by means of the "daily" totals at the bottom of each slip. By this means, 25 series of hourly totals were obtained, each representing a different transit-hour. (It is to be noticed that there are <u>two</u> sets of inequalities with the transit-hour 23^h, since the 25-hour sequences selected begin and end at this hour. Thus one of these sets of inequalities has its transit-hour at the 23^h at the beginning of the selected sequence; the order of solar hours in the tabulation is therefore 23^h, 24^h, 1^h, 2^h, . . . 23^h. The second set has its transit-hour at the 23^h at the end of the selected sequence; the order of solar hours is therefore 23^h, 23^h, 24^h, 1^h, 2^h, . . . 22^h. These transit-hours are designated 23^h(b) and 23^h(a) respectively. It will be seen that sequences with transit-hour 23^h(b) need no transposition - they start at the lunar transit-hour without further change. Sequences with transit-hour 23^h(a) have, in common with all other transit-hours, a break between the two 23^h readings.

The first step in the treatment of these 25 sets of inequalities was the removal of the non-cyclic change. This is effected by applying a progressively (and linearly) increasing correction to all readings except the first (that is, the initial 23^h) reading of the solar sequence, of such magnitude that this first reading becomes equal to the twenty-sixth (additional) reading. Such corrections were in general applied to each transit-hour total separately. The corrected transit-hour totals are now added together, giving a result which would, if the number of sequences belonging to each transit-hour were the same, be the required lunar inequalities. There is in general, however, a somewhat uneven distribution of lunar transit-hours, and it is necessary to calculate the residue of the solar variation and convexity (which may be treated together, since owing to the method of selection the convexity is equivalent to an additional solar variation).

The solar variation used for this purpose is that calculated from the actual data, and includes the convexity. It is easily obtained from the separate transit-hour totals, which can readily be re-converted into solar time to form solar sequences of 26 hours starting at 23^h and ending at 24^h on the following day. Linear non-cyclic change is removed by equalising the two 23^h readings, the last reading being ignored as superfluous. The part of each transit-hour total due to solar variation and convexity is obtained by multiplying the solar variation by the number of sequences in the particular transit-hour set concerned, and arranging the resulting corrections in lunar time. This is done for each of the 25 transit-hours, and the final total gives the correction to be subtracted from the lunar totals. The corrections thus obtained are surprisingly small, and are very insensitive to uncertainties in the solar variation curve used to calculate them. A full discussion of the magnitude of such corrections will be given later (section 17).

(13). The above separation of the lunar sequences into transit-hour groups was made merely to facilitate the handling of the data. In addition, the total data, comprising 4358 days in all, was treated in sub-divisions according to seasons, the distance of the moon, and also in periods; each sub-division being separately treated in the manner described above. The seasonal groupings were:- Winter months (November, December, January, February), Equinoctial months (March, April, September, October), and Summer months (May, June, July, August). The sub-division according to the lunar distance was effected by classifying the lunar sequences according to the moon's semi-diameter as given in the Nautical Almanac for the date concerned, viz., semi-diameters not exceeding 14!99, between 15!00 and 15!99, and greater than 16!00. The total period of 45 years, 1868-1912, was divided into three periods, 1868-1882, 1883-1897, 1898-1912.

Days selected according to the present criterion have, for convenience of reference, been designated α-days. The final lunar inequalities are shown in Table IV, and the results of the harmonic analysis in Table V. In analysing the inequalities, a 24-ordinate scheme has been used; this involved interpolation of the inequalities to obtain the necessary 24 ordinates. It is assumed that the inequalities can be represented by the series

$$\sum_{r=1} a_r \cos rx + b_r \sin rx ,$$

or by

Table IV.

Lunar Inequalities from Transposed Solar Days (α-days).

Unit .0001 millibar.

Hour	Periods			Seasons			Semi-diameters			Total
	1868 to 1882	1883 to 1897	1898 to 1912	Winter	Equinox	Summer	14'	15'	16'	
$0\frac{1}{2}$	- 81	37	1	- 23	-102	- 5	244	- 64	-117	- 12
$1\frac{1}{2}$	11	32	- 7	9	113	- 75	364	- 64	-190	13
$2\frac{1}{2}$	47	- 64	0	148	- 38	- 75	277	26	-300	- 7
$3\frac{1}{2}$	60	- 45	102	92	98	- 15	327	42	-215	37
$4\frac{1}{2}$	30	-116	50	- 29	2	5	126	70	-256	- 14
$5\frac{1}{2}$	- 67	- 85	- 15	-127	48	- 77	110	- 22	-247	- 56
$6\frac{1}{2}$	- 54	-126	66	- 90	97	- 81	26	41	-206	- 38
$7\frac{1}{2}$	1	12	88	- 53	182	- 9	47	150	-129	33
$8\frac{1}{2}$	- 95	21	129	4	162	- 45	- 80	32	81	21
$9\frac{1}{2}$	69	36	39	- 29	205	23	-136	66	154	46
$10\frac{1}{2}$	162	26	81	-201	114	68	- 48	- 31	354	88
$11\frac{1}{2}$	214	71	105	109	207	116	- 85	82	353	127
$12\frac{1}{2}$	120	198	87	181	106	126	- 56	77	367	136
$13\frac{1}{2}$	172	171	8	218	107	82	- 70	36	371	117
$14\frac{1}{2}$	80	209	50	217	87	83	- 60	109	261	115
$15\frac{1}{2}$	18	214	78	187	23	129	- 57	174	146	107
$16\frac{1}{2}$	55	125	- 20	170	- 42	82	- 41	122	43	55
$17\frac{1}{2}$	99	84	-209	135	-150	- 5	-124	42	6	- 12
$18\frac{1}{2}$	40	30	-249	110	-233	- 27	-194	- 36	8	- 63
$19\frac{1}{2}$	18	- 94	-218	-138	-186	- 56	-263	- 93	23	-101
$20\frac{1}{2}$	- 83	-163	- 75	-159	-138	- 40	-213	-138	32	-108
$21\frac{1}{2}$	-232	-220	- 43	-177	-115	-101	-228	-169	- 87	-164
$22\frac{1}{2}$	-154	-247	- 75	-203	-197	-117	-109	-194	-131	-159
$23\frac{1}{2}$	-191	- 98	- 6	-151	-173	- 57	70	- 98	-198	- 96
$24\frac{1}{2}$	-218	- 28	28	-217	-157	38	208	-160	-134	- 68
No. of days	1358	1506	1494	962	1383	2013	1086	1883	1389	4358

Table V.

Harmonic Analysis of Lunar Inequalities from Transposed Solar Days (α-days).

Unit .0001 millibar.

Hour	Periods 1868 to 1882	Periods 1883 to 1897	Periods 1898 to 1912	Seasons Winter	Seasons Equinox	Seasons Summer	Semi-diameters 14'	Semi-diameters 15'	Semi-diameters 16'	Total
a_1	-117	-128	- 25	-107	- 95	- 77	138	- 90	-257	- 88
b_1	34	0	86	- 12	158	- 1	125	56	- 53	46
c_1	122	128	90	108	184	77	186	106	262	99
α_1	279°	263°	336°	256°	321°	261°	40°	294°	251°	290°
a_2	31	82	55	57	34	48	106	- 1	93	57
b_2	55	65	- 14	129	- 10	18	116	72	- 69	34
c_2	63	105	57	141	35	51	157	72	116	66
α_2	14°	37°	89°	9°	91°	55°	27°	344°	112°	44°

$$\sum_{r=1} c_r . \sin (rx + \alpha_r) .$$

The first two harmonics are given. Owing to the method of marking transit-hours on the original records, the average "lunar" time of the first entry on the Burroughs slips is not 0^h, but $0\frac{1}{2}^h$. A correction of $-7^\circ.5r$ has therefore been made to each phase angle α_r in Table V.

(14). In section 19 it is shown that the standard deviation of a single barometer reading, due to accidental error, on a day selected according to the criterion here used, namely, that the range of pressure over 25 solar hours should not exceed 0.110 inch, is 0.032260 inch, or 1.0923 millibar. Assuming this result we may calculate the probable errors of the harmonic coefficients in Table V. For a mean hourly inequality determined from n readings each with standard deviation ϵ, the probable error is $0.6745\, \epsilon/\sqrt{n}$. The probable errors of the harmonic coefficients a_r, b_r, c_r are therefore $0.6745\, \epsilon/\sqrt{12n}$, and those of the coefficients α_r are

$$0.6745 \frac{1}{\sqrt{12n}} . \frac{\epsilon}{c_r}$$

The probable errors thus calculated are shown in Table VI.

(15). An examination of Table V reveals the surprising fact that, while the second harmonics in the period and seasonal groupings are probably significant they are smaller and on the whole less regular than the first harmonics, which ought, theoretically, to be zero. The results for the total data

Table VI.

Probable Errors of the α-day Harmonic Coefficients.

Unit .0001 millibar.

	Periods			Seasons			Semi-diameters			Total
	1868 to 1882	1883 to 1897	1898 to 1912	Winter	Equinox	Summer	14'	15'	16'	
a_γ, b_γ, c_γ	58	55	55	69	57	47	65	49	57	32
α_1	27°	25°	35°	37°	18°	35°	20°	26°	12°	19°
α_2	53°	30°	55°	28°	95°	54°	24°	39°	28°	28°

confirm this. According to the result here found, we have present, in addition to the expected semi-diurnal tide, with its maxima about one hour and a half after the lunar transits, a diurnal "tide", with its maximum near lower lunar transit. There is no theoretical basis for expecting such a tide. In the grouping according to lunar distance, the diurnal components for apogee and perigee are both large, but opposite in phase. A comparison with the corresponding Greenwich[14] results for the diurnal components is given in Table VII. The agreement in phase is striking, the only outstanding discrepancy being in the Summer groups, and even this is not excessive when the magnitude of the probable error is taken into consideration.

In the Greenwich investigation, this diurnal component was dismissed as accidental by Chapman. The accordance here showm, however, between the results at Greenwich and at Glasgow, make it desirable that this component should be further investigated.

Table VII.

Comparison of the first harmonics of the lunar variation at Greenwich and at Glasgow.

Unit .0001 millibar.

	Greenwich		Glasgow	
	c_1	α_1	c_1	α_1
Winter	145	290°	108	256°
Equinoxes	55	346°	184	321°
Summer	161	167°	77	261°
Apogee 14'	291	66°	186	40°
Mean 15'	36	299°	106	294°
Perigee 16'	351	228°	262	251°
Total	43	214°	99	290°

IV.

(16). In the previous chapter the Glasgow records were used in an attempt to find the lunar atmospheric tide by the transposition method. The present chapter describes an attempt to find the tide by the method of "upper and lower transits" (section 7).

The initial step in the investigation is the selection of days. The procedure is as follows. The transit-hours for all days are first marked in the original records, the times of transit as before being obtained from the Nautical Almanac. The records are then examined and those sequences which satisfy the criterion for quietness, that is, whose range of pressure over 25 hours, starting from lunar transit, does not exceed 0.110 inch, are suitably marked, and later copied (with the modifications described in section 12) on to Burroughs slips. A 26th. reading is added at the end of the sequence to facilitate correction for non-cyclic change. Each transit is treated separately; that is, all sequences starting at upper transit are first selected, marked, and tabulated; a fresh start is then made, and lower transit sequences are selected, marked and tabulated without reference to the upper transit sequences already selected. Two sets of sequences are thus obtained, one starting at upper lunar transit, and the other at lower lunar transit. Such sequences are designated, for convenience of reference, as β-days and γ-days respectively. The two sets are not entirely independent; it is apparent that, as β-days and γ-days are selected independently of one another, certain portions of the records will be common to both sets of tabulations.

Each sequence contains, in addition to the required lunar variation, the following:-

(a). the solar variation, which, in the mean of many sequences with transit-hours occurring at different solar hours, will cancel out very nearly in the final inequalitiies. A residue of the solar variation will remain, however, owing to the imperfect distribution of the transit-hours in solar time. To correct for this residue, it is necessary to find the solar variation appropriate to the actual data used. This is obtained by adding all sequences with the same transit-hour, thus obtaining 24 sets of inequalities, each of which may be readily transposed into solar time. Addition of these 24 transposed sets gives the hourly solar inequalities, from which the required solar variation is obtained. This solar variation, having been obtained from the transposition of sequences selected to begin and end at a given <u>lunar</u> time, is free from convexity. Consequently this solar variation is the same whether β-days or γ-days are used to determine it. The solar variation from the β-days has not therefore been calculated; but that from the γ-days has been used in calculating the corrections to both the β-day and the γ-day inequalities. These are discussed in section 17.

(b). Each sequence contains a linear non-cyclic element. This is very easily removed from the final inequalities by equalising the first and twenty-sixth hourly readings.

(c). Owing to the method of selection used, the convexity appears in each sequence; and since each sequence begins and ends at a lunar transit, it appears as a lunar effect, and is not cancelled out in the final inequalities.

(17). The final inequalities, corrected as above described, are shown in Tables VIII and IX. The same grouping has been adopted as for the transposed α-day inequalities. Figure 9 gives a comparison of the total inequalities for the β-day and γ-day inequalities. For each curve the zero hour is the appropriate transit. It will be seen that the difference in the two curves is of the same order of magnitude as the γ-day inequalities themselves. If the lunar variation were purely semi-diurnal, as required by theory, the β-day and γ-day curves, each starting from the appropriate transit, would be identical; any diurnal component present would in such a case be due to the convexity. The fact that this difference exists between the two curves shows that a real lunar diurnal component is present.

Table VIII.

Lunar Inequalities from Days commencing at Upper Transit of the Moon (β-days).

Unit .0001 millibar.

Hour	Periods			Seasons			Semi-diameters			Total
	1868 to 1882	1883 to 1897	1898 to 1912	Winter	Equinox	Summer	14'	15'	16'	
$0\frac{1}{2}$	-1479	-776	-1068	-1477	-1256	-809	-1072	-986	-1266	-1096
$1\frac{1}{2}$	-1124	-619	- 836	-1174	- 964	-624	- 719	-787	-1062	- 851
$2\frac{1}{2}$	- 885	-414	- 601	- 820	- 693	-489	- 508	-568	- 821	- 625
$3\frac{1}{2}$	- 589	-305	- 377	- 601	- 458	-307	- 249	-374	- 648	- 418
$4\frac{1}{2}$	- 326	-195	- 169	- 328	- 223	-185	- 100	-148	- 446	- 228
$5\frac{1}{2}$	- 168	-151	- 41	- 184	- 96	-105	19	- 71	- 319	- 119
$6\frac{1}{2}$	- 4	- 48	77	- 89	105	- 14	113	61	- 165	8
$7\frac{1}{2}$	169	66	200	- 49	320	113	174	214	7	144
$8\frac{1}{2}$	346	178	297	110	487	198	254	276	265	272
$9\frac{1}{2}$	541	209	363	123	553	347	336	305	461	365
$10\frac{1}{2}$	727	337	456	423	631	444	392	444	657	499
$11\frac{1}{2}$	828	436	484	516	722	502	443	509	787	575
$12\frac{1}{2}$	858	508	574	679	740	553	450	570	905	640
$13\frac{1}{2}$	774	434	521	774	670	408	368	544	784	570
$14\frac{1}{2}$	698	473	485	758	644	389	339	545	736	548
$15\frac{1}{2}$	650	416	476	851	508	364	313	541	645	510
$16\frac{1}{2}$	640	363	428	785	426	360	354	449	622	473
$17\frac{1}{2}$	527	317	266	680	309	258	326	318	456	364
$18\frac{1}{2}$	390	241	153	622	164	154	219	184	407	260
$19\frac{1}{2}$	234	58	82	376	100	17	68	106	219	121
$20\frac{1}{2}$	5	- 35	11	266	- 83	- 78	- 24	- 21	48	- 6
$21\frac{1}{2}$	- 323	-173	- 161	- 98	- 327	-195	- 90	-273	- 228	- 217
$22\frac{1}{2}$	- 521	-309	- 334	- 354	- 548	-285	- 174	-454	- 461	- 384
$23\frac{1}{2}$	- 858	-429	- 525	- 716	- 768	-422	- 497	-625	- 651	- 596
$24\frac{1}{2}$	-1115	-584	- 753	-1066	- 960	-584	- 739	-759	- 932	- 807
No. of days	1347	1492	1451	921	1370	1999	1106	1850	1334	4290

Table IX.

Lunar Inequalities from Days commencing at Lower Transit of the Moon (♈-days).

Unit .0001 millibar.

Hour	Periods			Seasons			Semi-diameters			Total
	1868 to 1882	1883 to 1897	1898 to 1912	Winter	Equinox	Summer	14'	15'	16'	
0½	-326	-570	-467	-727	-448	-346	-512	-526	-335	-459
1½	-275	-378	-339	-388	-335	-307	-436	-315	-264	-331
2½	-242	-329	-202	-178	-263	-295	-367	-243	-195	-257
3½	-220	-171	-189	- 19	-272	-216	-245	-181	-160	-191
4½	-223	- 47	-195	27	-152	-237	-264	-216	13	-151
5½	-234	17	-122	111	-145	-194	-174	-157	54	-107
6½	-232	- 4	-109	72	-111	-196	-299	- 60	- 10	-110
7½	-213	78	32	102	14	-125	-241	33	75	- 26
8½	-184	139	2	153	- 27	- 70	-144	37	58	- 7
9½	-113	183	54	139	12	38	- 49	136	34	48
10½	-100	234	90	124	- 12	107	62	123	35	82
11½	- 28	282	19	- 7	39	184	157	171	- 87	97
12½	40	284	49	44	113	180	174	205	1	129
13½	95	300	118	137	163	204	296	245	- 10	176
14½	129	326	136	232	153	202	318	226	46	200
15½	158	253	139	240	97	226	343	250	- 31	185
16½	182	218	86	162	60	208	311	156	2	162
17½	229	144	142	249	92	198	361	202	- 23	171
18½	265	136	126	165	124	212	359	160	40	174
19½	261	40	160	125	182	145	287	179	- 12	151
20½	245	- 53	263	81	250	88	193	95	60	150
21½	240	-160	199	- 49	215	75	168	- 55	219	88
22½	249	-199	165	-175	173	97	41	- 80	257	66
23½	205	-314	13	-238	105	- 28	- 67	-152	182	- 39
24½	97	-412	-173	-380	- 16	-147	-273	-239	48	-172
No. of days	1326	1488	1473	927	1355	2005	1078	1856	1353	4287

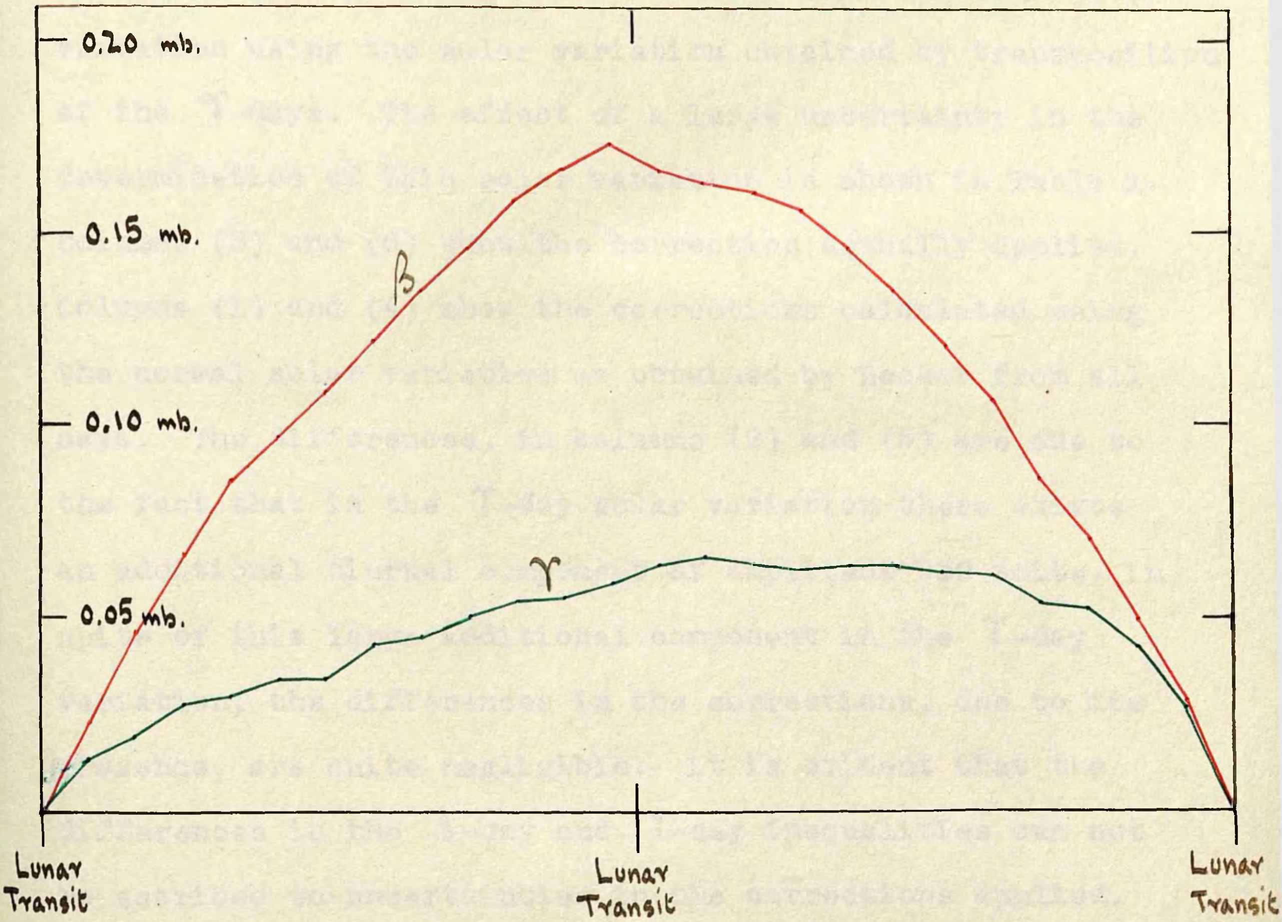

Figure 9.- Comparison of the β-day and γ-day inequalities. β-days start at upper transit, γ-days at lower transit.

As explained above (section 16), both the β-day and the γ-day inequalities were corrected for the residue of solar variation using the solar variation obtained by transposition of the γ-days. The effect of a large uncertainty in the determination of this solar variation is shown in Table X. Columns (3) and (6) show the correction actually applied. Columns (1) and (4) show the corrections calculated using the normal solar variation as obtained by Becker from all days. The differences, in columns (2) and (5) are due to the fact that in the γ-day solar variation there exists an additional diurnal component of amplitude 950 units. In spite of this large additional component in the γ-day variation, the differences in the corrections, due to its presence, are quite negligible. It is evident that the differences in the β-day and γ-day inequalities can not be ascribed to uncertainties in the corrections applied.

A comparison of Tables VIII and IX shows that these differences exist also in the sub-groups. It should be remarked here that as no lower transit times are tabulated in the Nautical Almanac for the period 1868-1882, interpolated values of the inequalities are given for this period.

The presence of two distinct effects in these inequalities is thus established: firstly, the convexity effect, which reaches its maximum, in the β-day inequalities, at lower transit, and in the γ-day inequalities at upper transit: secondly, the lunar variation, with a diurnal component with its maximum at lower transit. This diurnal component is in

Table X.

Corrections to γ-day and to β-day inequalities.

Total data.

Unit .0001 millibar.

	γ - days			β - days		
	(1)	(2)	(3)	(4)	(5)	(6)
$0\frac{1}{2}$	14	3	17	42	8	50
$1\frac{1}{2}$	0	8	8	15	5	20
$2\frac{1}{2}$	-19	5	-14	-15	0	-15
$3\frac{1}{2}$	-34	3	-31	-32	1	-31
$4\frac{1}{2}$	-37	4	-33	-38	-6	-44
$5\frac{1}{2}$	-38	2	-36	-31	-4	-35
$6\frac{1}{2}$	-28	5	-23	-29	-7	-36
$7\frac{1}{2}$	-15	2	-13	-23	-8	-31
$8\frac{1}{2}$	14	1	15	-16	-10	-26
$9\frac{1}{2}$	44	-4	40	4	-13	-9
$10\frac{1}{2}$	68	0	68	21	-11	10
$11\frac{1}{2}$	64	-5	64	37	-7	30
$12\frac{1}{2}$	74	-2	72	29	-3	26
$13\frac{1}{2}$	42	-1	41	19	-2	17
$14\frac{1}{2}$	21	-3	18	-10	-1	-11
$15\frac{1}{2}$	-4	-1	-5	-20	-1	-21
$16\frac{1}{2}$	-21	-6	-27	-37	5	-32
$17\frac{1}{2}$	-35	-2	-37	-39	6	-33
$18\frac{1}{2}$	-44	-4	-48	-34	7	-27
$19\frac{1}{2}$	-42	-1	-43	-14	9	-5
$20\frac{1}{2}$	-31	-4	-35	4	9	13
$21\frac{1}{2}$	-15	-1	-16	40	8	48
$22\frac{1}{2}$	3	-4	- 1	63	7	70
$23\frac{1}{2}$	16	2	18	64	3	67
$24\frac{1}{2}$	14	3	17	42	8	50

phase with the convexity in the β-day inequalities, and tends to neutralise the convexity in the γ-day inequalities.

(18). The separation of these two effects follows the method described in Chapter II. In the present case, the β-day and γ-day inequalities are represented by

$$y_{x,U} = p_x + c_1 \sin(x + \alpha_1) + c_2 \sin(2x + \alpha_2) \ . \ . \ .$$

$$y_{x,L} = p_x - c_1 \sin(x + \alpha_1) + c_2 \sin(2x + \alpha_2) \ . \ . \ . \ ,$$

where $y_{x,U}$, $y_{x,L}$ are the ordinates at hour x, and p_x is the convexity at hour x, x being measured in both cases from the beginning of the selected sequence, that is, from upper transit in the case of the β-days, and from lower transit in the case of the γ-days. Only the first and second harmonics are retained. By addition and subtraction of these equations,

$$y_{x,U} + y_{x,L} = 2p_x + 2c_2 \sin(2x + \alpha_2) \ . \ . \ . \ . \ (1)$$

$$y_{x,U} - y_{x,L} = 2c_1 \sin(x + \alpha_1) \ . \ . \ . \ . \ . \ (2)$$

Analysis of (1) gives a first harmonic which is entirely due to convexity. Assuming that p_x may be represented by a parabola

$$p_x = a_1' \sin(x + \beta_1) + \frac{a_1'}{4} \sin(2x + 2\beta_1 + 90^\circ) \ . \ . \ . \ (3)$$

we may compute the amplitude and phase of the second harmonic of convexity. The second harmonic of the lunar effect is now obtained from the analysis of (1). The first harmonic of the lunar effect is obtained directly from the analysis of (2).

These lunar harmonics are shown in Table XI.

(19). The probable errors of the coefficients have been determined by the method of variance. The total variance to which a single barometer reading is subject may be regarded as composed of three components, viz.,

(a). that due to the regular hour-to-hour fluctuation of the barometer.

(b). that due to the change of daily mean value of the barometer on the selected days.

(c). that due to accidental causes.

In the present determination of probable error, we neglect variances due to non-cyclic change and to solar variation. Of the three mentioned above, the first two can be separately computed. By removing these from the total variance the component due to accidental causes can be determined. This has been done for the β-day period grouping 1883-1897, the details of the calculation being shown in Table XII. The resulting standard deviation for a single barometer reading is 0.032260 inch, or 1.0923 millibar. The probable error of a single mean hourly inequality obtained from n readings is therefore

$$0.6745 \cdot \frac{1.0923}{\sqrt{n}} \text{ millibar.}$$

It may be assumed that this holds for all days selected according to the same criterion as β-days - for example, γ-days and α-days, whose range over 25 hours does not exceed

Table XI.

Harmonic Coefficients of Lunar Inequalities from Combined β-days and γ-days.

Unit .0001 millibar.

	Periods			Seasons			Semi-diameters			Total
	1868 to 1882	1883 to 1897	1898 to 1912	Winter	Equinox	Summer	14'	15'	16'	
c_1	449	86	255	322	363	183	197	201	422	255
α_1	276°	246°	276°	240°	280°	288°	306°	276°	256°	272°
c_2	51	48	56	131	24	41	73	18	12	31
α_2	325°	22°	254°	307°	180°	45°	308°	49°	132°	338°

Table XII.

Standard Deviation for β-days.

	Degrees of Freedom	Sum of Squares	Mean Square	Standard Deviation Unit 1 inch
Hours	24	.0400		
Days	1491	3449.039		
Remainder	35784	37.2409	.0010407	.032260
Total	37299	3486.3201		

0.110 inch - and this result has already been used (section 14) to find the probable errors of the α-day harmonic coefficients. In the present case, the mean hourly inequalities involved are those obtained by combining β-days and γ-days. If n_1 and n_2 are the numbers of β-days and γ-days involved in a certain mean hourly inequality, and if we suppose that the β-day and γ-day tabulations are entirely independent, the probable error of the mean hourly inequality is

$$0.6745 \cdot \frac{1.0923}{\sqrt{n_1 + n_2}} \text{ millibar.}$$

This latter supposition is not quite justified, however, as certain portions of the barometric record are common to both sets of tabulations. Assuming, as a rough estimate, that half of the tabulations are the common property of both β-days and γ-days, we may take, as an estimate of the probable error of a single mean hourly inequality derived from the combination of β-days and γ-days,

$$0.6745 \cdot \frac{1.0923}{\sqrt{\frac{3}{4}(n_1 + n_2)}} \quad .$$

The probable errors of the harmonic coefficients may now be easily derived. Their values are shown in Table XIII.

(20). An examination of Tables XI and XIII shows that the only component of the lunar variation which can be considered significant is the diurnal, whose amplitude for the total data is 255 units. The semi-diurnal components appear to be entirely accidental throughout all the groups. These results

Table XIII.

Probable Errors of the Harmonic Coefficients from Combined β-days and γ-days.

Unit .0001 millibar.

	Periods			Seasons			Semi-diameters			Total
	1868 to 1882	1883 to 1897	1898 to 1912	Winter	Equinox	Summer	14'	15'	16'	
c_γ	47	45	46	57	48	39	53	40	47	27
α_1	6°	30°	10°	18°	8°	12°	15°	11°	6°	6°
α_2	53°	54°	47°	45°	115°	55°	42°	127°	224°	50°

are different from those obtained by the transposition method (Table V) where it appeared that the semi-diurnal component, though small, was probably significant. The striking feature of the present results, however, is the large increase in the diurnal component from its value as obtained from the α-day inequalities. This increase is seen in all groups, except for the period 1883-1897, where the value of the amplitude is inexplicably low. The phase of the diurnal component in all groups is near 270°; this component therefore represents a wave with its single daily maximum near lower lunar transit.

The difference between the α-day determination of the diurnal component and that obtained in the present chapter will be discussed in Chapter V.

(21). The insignificance of the semi-diurnal component of the lunar variation in the combined β-day and γ-day inequalities has been noted above. Equation (1) of section 18 therefore represents the convexity effect alone. A parabola has been fitted by the method of least squares to the combined inequalities represented by this equation, and the results, for the three seasonal groups and for the total data, are shown in Table XIV, while Figure 10 gives a comparison of this convexity for the total data with the corresponding β-day and γ-day inequalities. This convexity will be used later in connection with the investigation of solar inequalities.

(22). In section 10 it was pointed out that, in the Greenwich

Table XIV.

25-hour Convexity derived from Combined β-days and γ-days.

Unit .0001 millibar.

Hour	Winter	Equinoxes	Summer	Total
1	-1044	- 849	- 632	- 842
2	- 817	- 663	- 491	- 657
3	- 607	- 492	- 362	- 487
4	- 417	- 337	- 244	- 333
5	- 244	- 196	- 136	- 192
6	- 90	- 70	- 40	- 67
7	46	41	45	44
8	164	137	119	139
9	263	218	181	220
10	344	283	233	286
11	407	334	273	337
12	451	370	303	374
13	477	390	328	398
14	484	396	323	401
15	474	387	308	389
16	445	362	282	362
17	397	323	244	321
18	332	268	195	265
19	248	199	135	193
20	145	115	65	108
21	25	15	- 20	7
22	- 115	- 100	- 111	- 109
23	- 272	- 229	- 216	- 239
24	- 448	- 374	- 331	- 384
25	- 642	- 533	- 458	- 544

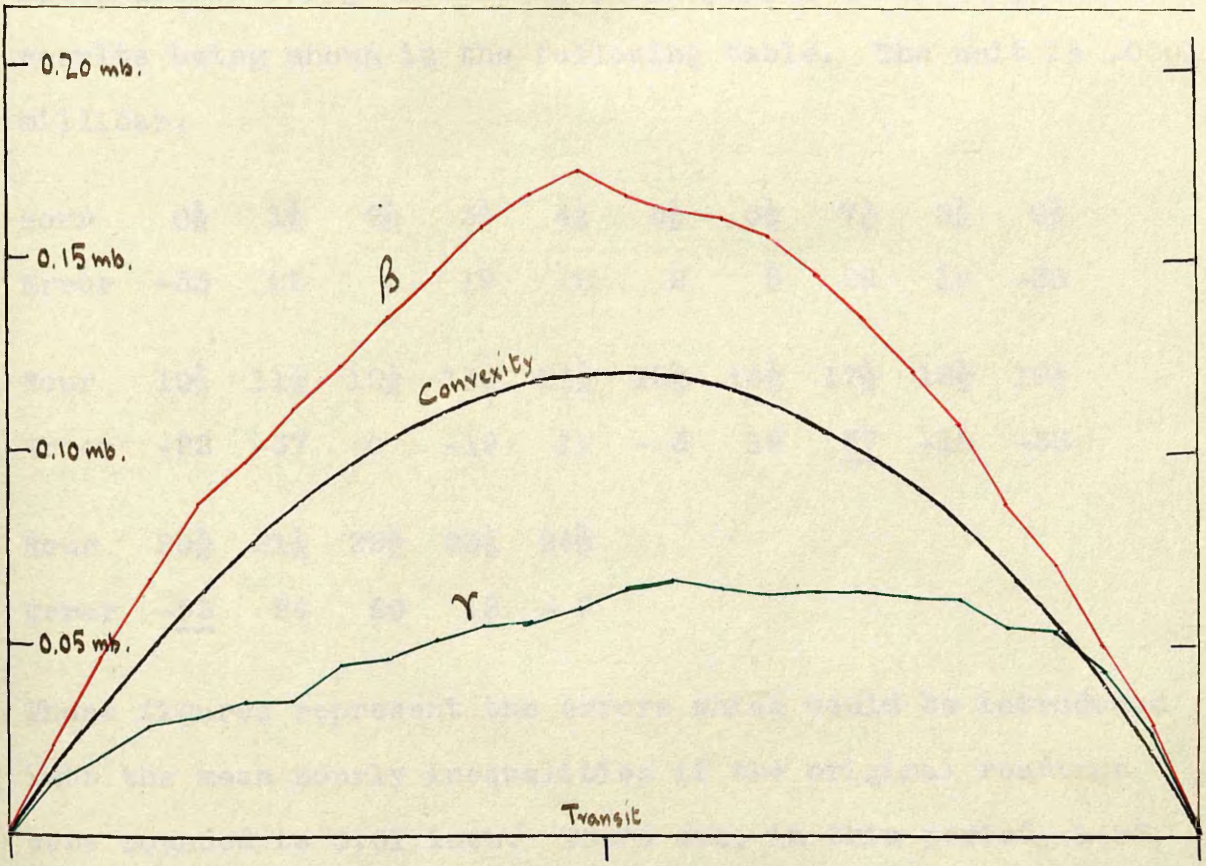

Figure 10.- Comparison of the convexity with the β-day and γ-day inequalities.

reductions by the transposition method, the barometer readings, tabulated to an accuracy of 0.001 inch in the original records, had been "rounded" in the re-tabulation to 0.01 inch. The effect of this rounding has been investigated for the β-day inequalities using the data for the period 1883-1897, the results being shown in the following table. The unit is .0001 millibar.

Hour	0½	1½	2½	3½	4½	5½	6½	7½	8½	9½
Error	-33	22	5	19	18	2	5	29	10	-36

Hour	10½	11½	12½	13½	14½	15½	16½	17½	18½	19½
Error	-22	-37	0	-19	11	- 3	19	67	-18	-33

Hour	20½	21½	22½	23½	24½
Error	-55	24	20	12	- 9

These figures represent the errors which would be introduced into the mean hourly inequalities if the original readings were rounded to 0.01 inch. There are, in this period, 1492 days. The theoretical standard deviation liable to occur in this way in the mean of 1492 numbers is 0.002531 millibar. It will be noted that all the above errors lie within three times the standard deviation, the greatest being 67. The actual standard deviation of a mean hourly inequality in this group is $1.0923/\sqrt{1492}$ millibar, or 0.02827 millibar. The extra variance introduced by "rounding" has therefore a negligible effect on the calculated standard deviation. Nevertheless, the extreme range of the errors is 122 units - from -55 to +67 - and this is of the same order of magnitude as the amplitude of the expected semi-diurnal component of the tide. It seems to be desirable, therefore, that the full accuracy of the original tabulations should be retained in the re-tabulation of the selected days.

V.

(23). The explanation of the large increase in the diurnal component of the lunar variation obtained by the "upper and lower transit" method of the previous chapter has proved very difficult to find. No assumptions are made in the treatment of the data which would lead one to expect such a divergence (it has already been pointed out in section 7 that the determination of the first "harmonic" of convexity is quite independent of the particular form used to represent the convexity as a whole). The only explanation which can be found (and which is later verified by a discussion of the solar variation) is that the mode of selection of the days gives rise to the discrepancy. The β-days and γ-days are selected according to the criterion that the range of pressure over 25 solar hours from transit to transit should not exceed 0.110 inch. They are, therefore, "quiet lunar days" in the same sense that we might call days selected from midnight to midnight or from noon to noon "quiet solar days" if their range of pressure did not exceed a certain amount. Let us accept this strict definition of a quiet lunar day - namely, that its range of pressure <u>from transit to transit</u> should not exceed 0.110 inch. Consider now an α-day transposed into lunar time.

It is, in the first place, selected according to the criterion that its range of pressure, from 23^h solar time to the following 23^h solar time should not exceed 0.110 inch. Transposition does not affect this range of pressure. From this point of view, therefore, the transposed α-day is exactly equivalent to the β-day or γ-day. Consider Figure 11, which represents schematically a portion of the original records. Let us suppose that an α-day, represented by ABCD in the figure, is selected, the lunar transit being at L. After transposition, the sequence consists of two parts, LD followed by ABL, so that the new sequence begins and ends at lunar transit. Each of these parts is part of the quiet <u>solar</u> sequence represented by ABCD; but they are not, necessarily, parts of a quiet <u>lunar</u> sequence as defined above. Let L_1 and L_2 be the lunar transits preceding and following L. Then ABCL is part of the lunar day L_1ABCL, and LD is part of the lunar day LDEFL_2. Neither of these lunar days is necessarily "quiet". Thus the lunar sequence obtained by transposition of a (quiet) α-day into lunar time does not necessarily produce the equivalent of a β-day or a γ-day, even although the range of pressure is less than 0.110 inch. The transposed α-day, in general, corresponds to a "less quiet" type of sequence than the β-day or γ-day; in other words, transposition raises the effective maximum range of the pressure.

If, in Figure 11, the lunar days L_1ABCL and LDEFL_2 are both quiet lunar days, for example, β-days, each part of the

```
1h       6h          12h         18h         24h

                L1                             A B
x x x x x x x | x x x x x x x x x x x x x x x x x

C                 L                            D E
x x x x x x x x | x x x x x x x x x x x x x x x x

F                   L2
x x x x x x x x x | x x x x x x x x x x x x x x x
```

Figure 11.

transposed α-day is then part of a quiet lunar day. In this case the transposed α-day should be directly comparable with a β-day or a γ-day. α-days situated in this way - so that the two parts are each parts of β-days - have been designated ε-days for brevity.

In practice, it is found that the ε-days and the β-days or γ-days are not, in fact, strictly comparable. It will be seen that an ε-day requires to be situated between two β-days. That is, two successive 25-hour quiet sequences are required before an ε-day can be selected from the records. This is obviously a more stringent requirement than that applicable to the selection of a β-day, for which only one 25-hour quiet sequence is required. Thus ε-days are actually quieter than β-days, that is, their effective maximum range of pressure is less.

It is found that this latter difference is not important in dealing with the lunar variation, owing to the large probable errors involved. But it is noticeable in dealing with the corresponding solar variation, which is treated later.

The main conclusion of this section, namely, that transposition raises the effective range of pressure, may be used to explain the difference in the α-day and the combined β- and γ-day variation if we suppose that the lunar diurnal component increases as the effective range of pressure is diminished. This supposition is tested in the following section.

(24). The inequalities obtained from ϵ-days, selected according to the method described in the previous section, are shown in Table XV. These days are treated, as regards corrections for non-cyclic change and residue of solar variation, exactly in the same manner as the α-days - they are, indeed, α-days with the special restriction that they lie within β-days. The solar variations used in calculating the corrections for the imperfect distribution of transit-hours are those derived from the particular groups of ϵ-days concerned.

The harmonic analysis of the inequalities is shown in Table XVI. The probable errors of the coefficients have been calculated by the method of variance. The procedure in this case is similar to that employed in the calculation of the probable errors of the β-day inequalities, but the variances due to non-cyclic change and to the solar variation have also been removed from the total variance, while the lunar hour-to-hour variation has been ignored in this respect. The details of the calculation are shown in Table XVII, and the deduced probable errors of the coefficients in Table XVIII.

Having regard to the probable errors involved, it will be seen, by comparing Tables XVI and XI, that the ϵ-day coefficients agree well with those obtained from the combined β-days and γ-days. In particular, we may note that the large diurnal component, with its maximum at lower lunar transit, is present. The second harmonics are again rather

Table XV.

Lunar Inequalities from Transposed ϵ-days.

Unit .0001 millibar.

Hour	Winter	Equinoxes	Summer	Total
$0\frac{1}{2}$	- 259	- 441	- 127	- 247
$1\frac{1}{2}$	- 142	- 219	- 187	- 189
$2\frac{1}{2}$	- 135	- 156	- 125	- 136
$3\frac{1}{2}$	- 83	- 117	- 3	- 52
$4\frac{1}{2}$	- 170	29	40	0
$5\frac{1}{2}$	- 31	- 16	6	- 8
$6\frac{1}{2}$	- 67	77	44	35
$7\frac{1}{2}$	107	171	10	77
$8\frac{1}{2}$	131	284	23	123
$9\frac{1}{2}$	65	382	20	140
$10\frac{1}{2}$	202	300	125	193
$11\frac{1}{2}$	216	354	106	202
$12\frac{1}{2}$	387	275	131	220
$13\frac{1}{2}$	478	173	57	165
$14\frac{1}{2}$	538	256	115	231
$15\frac{1}{2}$	540	113	126	194
$16\frac{1}{2}$	357	19	126	132
$17\frac{1}{2}$	232	82	19	76
$18\frac{1}{2}$	- 20	16	9	6
$19\frac{1}{2}$	- 314	28	- 35	- 64
$20\frac{1}{2}$	- 373	- 43	- 24	- 90
$21\frac{1}{2}$	-531	- 229	- 81	- 205
$22\frac{1}{2}$	- 413	- 385	- 111	- 248
$23\frac{1}{2}$	- 420	- 514	- 117	- 292
$24\frac{1}{2}$	- 299	- 447	- 150	- 268
No. of days	302	542	907	1751

T a b l e X V I .

Harmonic Analysis of Lunar Inequalities from ϵ-days.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_1	- 375	- 318	- 118	- 223
b_1	59	113	16	50
c_1	380	337	119	228
α_1	271°	282°	270°	275°
a_2	76	- 68	- 16	- 16
b_2	166	9	28	42
c_2	183	69	32	45
α_2	9°	262°	315°	324°

Table XVII.

Standard Deviation for ϵ-days.

	Degrees of Freedom	Sum of Squares	Mean Square	Standard Deviation Unit 1 inch
Hours:				
Solar Var.	25	0.2656		
Non-cyclic change		1.8437		
Days	301	1089.1218		
Remainder	7525	1.8862	.0002507	0.01583
Total	7851	1093.1173		

Table XVIII.

Probable Errors of the ϵ-day Coefficients.

Unit .0001 millibar.

	Winter	Equinoxes	Summer	Total
a_r, b_r, c_r	60	45	35	25
α_1	9°	8°	17°	6°
α_2	19°	37°	63°	32°

small and irregular, although the component for Winter may possibly be of significance (the corresponding component in the combined β-day and γ-day inequalities also borders on significance).

We have already explained that the difference between the combined β-day and γ-day inequalities and the transposed α-day inequalities is probably due to the mode of selection, the transposed α-days being more disturbed, in effect, than the β-days or γ-days; the implication being that increased quietness of day produces a larger diurnal component. Now we have shown that, by directly choosing ϵ-days from the quietest parts of the α-day records, we obtain, similarly, an increased diurnal component. The explanation we have given to explain the discrepancy between transposed α-days and β- and γ-days is therefore confirmed; and we have proved, directly, that there exists on barometrically quiet days a diurnal pressure wave, with its maximum at lower lunar transit, whose amplitude increases with increasing quietness of the days concerned.

(25). The fact must therefore be accepted that the lunar variation on a quiet day is composed of two separate periodic effects, namely:-

(1). the normal lunar tide, which must be purely semi-diurnal in character, and which is presumably of constant amplitude on all types of day.

(2). the abnormal quiet-day variation, which is mainly diurnal in character, but probably also contains a semi-diurnal component of phase opposite to that of the normal tide. We may deduce the presence of such a semi-diurnal component from a consideration of the behaviour of the second harmonics in α-days and in ϵ-days. For, in α-days the second harmonic is probably significant and in phase with the expected tide. In ϵ-days, where the abnormal second harmonic may be presumed greater, the total second harmonic is reduced, and is not significant. That is, the presumed abnormal second harmonic tends to neutralise the normal tide in quiet days.

An attempt has been made to separate the two effects by the following method. We have found that the first harmonic of the 1751 ϵ-days is 228 $\sin(x+275^{\circ})$. The total first harmonic for these days is therefore 399228 $\sin(x+275^{\circ})$. In the same way, the 4358 α-days have a total first harmonic 431442 $\sin(x+272^{\circ})$. Thus, for the 2607 α-days which remain after the ϵ-days have been removed, the total first harmonic is 112101 $\sin(x+356^{\circ})$, giving a diurnal component of 43 $\sin(x+356^{\circ})$. This is quite negligible. The abnormal first harmonic is thus confined to the ϵ-days and it is the inclusion of these days in the α-days which accounts for the diurnal component in the α-days. If we assume that the abnormal second harmonic is similarly confined to the ϵ-days, we see that the remaining 2607 α-days should yield the normal lunar tide. Applying the above process to the first and second harmonics we obtain the results XIX, in which the second harmonics may be taken as representing the normal lunar tide.

The probable errors of these coefficients may be approximately estimated from the data already available in Tables XII and XVII. Assuming that the standard deviation 1.0923 millibar calculated for the 1883-1897 group of β-days is applicable also to the Winter group of α-days, we can recalculate, for the winter group of α-days which remain after the ϵ-days have been removed the necessary "sums of squares" to obtain the standard deviation appropriate to such days. The hour-to-hour variation may be neglected. The resulting standard deviation for a single hourly reading applicable to such days is found to be 1.2166 millibar, which may be assumed applicable to the other groups. This standard deviation is probably slightly over-estimated, as no allowance has been made for the variance due to non-cyclic change and solar variation. The probable errors of the harmonic coefficients calculated from this value are given in Table XX. As might be expected from the fact that only the most disturbed of the α-days are used in this determination of the tide, the probable errors are very high. Nevertheless, the second harmonics of Table XIX show - having regard to the probable errors - a reasonable degree of accordance in amplitude and in phase, and there is every reason to believe that the normal lunar

Table XIX.

Harmonic Coefficients of the Lunar Tide.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_1	16	49	- 43	3
b_1	- 44	187	- 15	43
c_1	47	193	46	43
α_1	153°	7°	102°	356°
a_2	48	100	100	106
b_2	112	- 22	10	29
c_2	123	102	100	110
α_2	8°	87°	69°	60°
No. of days	660	841	1106	2607

Table XX.

Probable Errors of the Coefficients of the Lunar Tide.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_ν, b_ν, c_ν	92	82	71	47
α_1	112°	24°	88°	63°
α_2	43°	46°	41°	24°

tide has now been separated from the abnormal quiet-day variation. The value for the total data is

$$0.0110 \sin (2x + 60^\circ) \text{ millibar.}$$

The corresponding result for Greenwich[14] is

$$0.0120 \sin (2x + 114^\circ) \text{ millibar,}$$

and for Hongkong[15]

$$0.060 \sin (2x + 60^\circ) \text{ millibar.}$$

The agreement between the present results for Glasgow and those found elsewhere is satisfactory.

This concludes the investigations on the lunar atmospheric variation at Glasgow.

VI.

(26). The present investigation deals with the solar variation in the barometric pressure at Glasgow on barometrically quiet days. On such days, as has already been explained, the convexity effect is present, and must be eliminated before any change in the periodic variation with quietness of day can be detected.

The solar variation for all days has been obtained by Becker.[22] The inequalities are shown in Table XXI. Three seasonal groups are shown, the grouping of the months being the same as that adopted in the discussion of the lunar variation. The harmonic analysis for these groups and for the total data is given in Table XXII. It is assumed that the inequalities can be represented by

$$\sum_{r=1} a_r \cos rx + b_r \sin rx ,$$

x being measured from 1^h G.M.T., or by

$$\sum_{r=1} c_r \sin (rx + \alpha_r).$$

Acorrection of $-15^{\circ}r$ has been applied to the phase angles calculated from the a and b coefficients. This brings the zero hour to Greenwich mean midnight.

<u>Table XXI.</u>

<u>Solar Inequalities from all days.</u>

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
Hour				
1	466	804	1661	846
2	- 127	- 635	561	- 170
3	- 1227	- 2159	- 878	- 1524
4	- 2497	- 3175	- 1386	- 2201
5	- 3259	- 3344	- 1132	- 2540
6	- 3259	- 2074	53	- 1863
7	- 2582	- 381	981	- 508
8	- 296	1312	2000	846
9	1397	2158	1915	1862
10	3090	2666	1661	2539
11	3429	2243	1153	2201
Noon	1820	1481	476	1185
13	- 550	127	- 455	- 170
14	- 2158	- 1312	- 1386	- 1524
15	- 2920	- 3005	- 2571	- 2878
16	- 2243	- 3598	- 3502	- 3217
17	- 1396	- 3175	- 4095	- 2878
18	466	- 1482	- 3502	- 1524
19	1228	212	- 2317	- 170
20	2074	2243	- 32	1523
21	2244	2751	1746	2201
22	2498	3174	3016	2878
23	1990	2751	3100	2539
24	1820	2412	2931	2539

Table XXII.

Harmonic Analysis of Solar Inequalities from all days.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_1	147	367	1150	538
b_1	- 812	- 233	974	- 27
c_1	825	435	1507	539
α_1	155°	107°	35°	78°
a_2	371	296	607	403
b_2	-2589	-3154	-2399	-2712
c_2	2615	3168	2475	2742
α_2	142°	145°	136°	142°
a_3	419	35	- 92	91
b_3	1077	148	- 614	213
c_3	1156	152	621	232
α_3	336°	328°	144°	338°
a_4	- 187	303	85	85
b_4	- 189	184	169	51
c_4	266	354	189	99
α_4	165°	359°	327°	359°

(27). In a paper "Uber die Atmosphärischen Gezeiten", J. Bartels[20] has defined the "convexity" effect as the differences in the hourly inequalities for "quiet days" and "all days". It has already been pointed out (section 7) that this definition assumes the absence of any additional periodic effect on barometrically quiet days. Figure 12 shows the "convexity" effect as determined by Bartels from the Potsdam data, 1893-1922, compared with a 24-hour convexity obtained from Table XIV (total data) by interpolation. It is plain from this diagram that the Potsdam "convexity" is not a very good approximation to a parabola; the diagram, indeed, suggests that in the Potsdam "convexity" some additional effect is present, such as a possible additional periodic component on quiet days. This matter will now be fully investigated using the Glasgow data.

In section 12 we selected, for the purpose of finding the lunar variation by the transposition method, all sequences of 25 hours starting at 23^h solar time over which the extremes of pressure did not differ by more than 0.110 inch. These sequences we designated α-days. The solar variation determined from these α-days contains the convexity, and is given in Table XXIII. The 25th. hour of the solar sequence, which, like the first, is at 23^h solar time, has been used to eliminate the non-cyclic change in the usual way. It is to be noted that in Table XXIII, since the α-days are chosen to start at 23^h, the convexity is zero for this hour. Since the results have to be compared with those from "all days", in

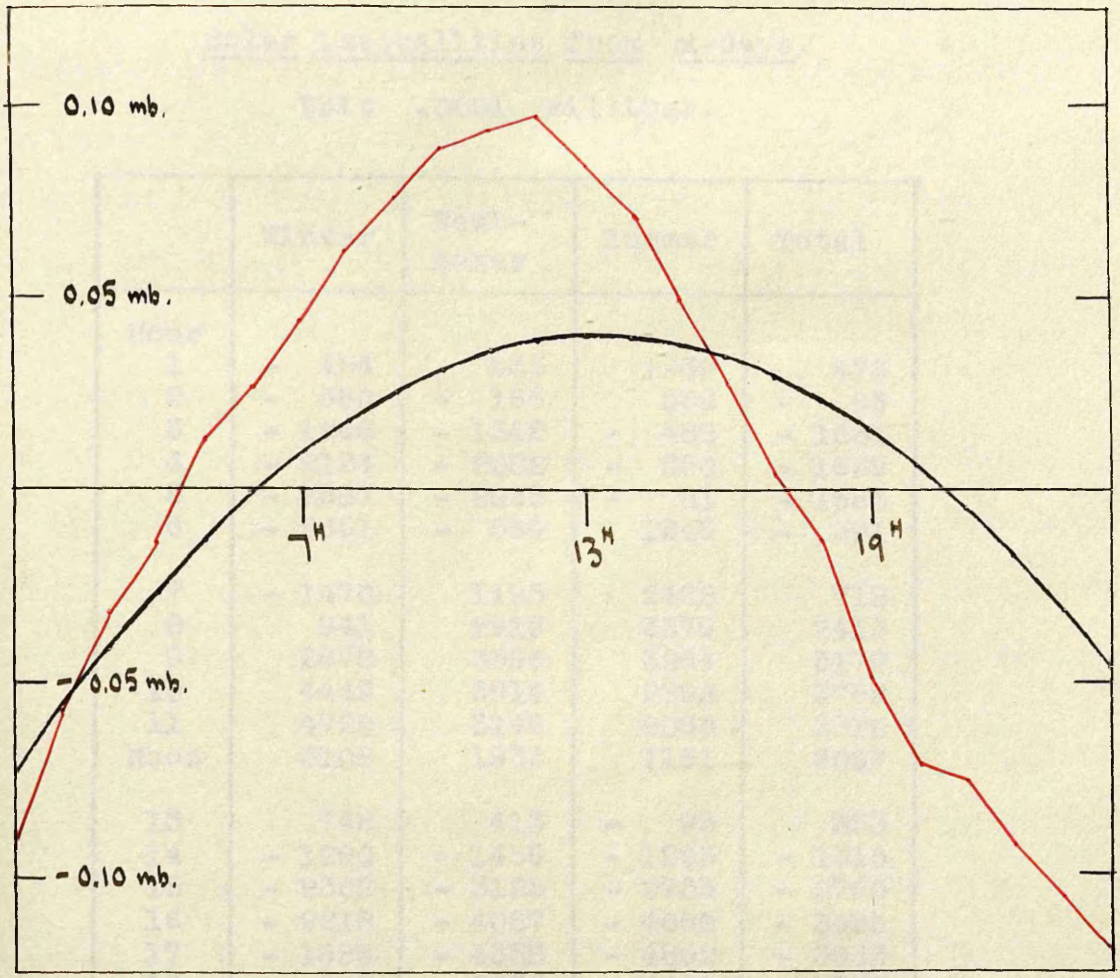

Figure 12.- Comparison of the "Bartels" convexity (Potsdam) with the convexity obtained from combined β-days and γ-days.

Table XXIII.

Solar Inequalities from α-days.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
Hour				
1	- 454	633	1239	473
2	- 660	- 166	569	- 86
3	- 1266	- 1348	- 488	- 1034
4	- 2184	- 2082	- 650	- 1639
5	- 2587	- 2028	- 81	- 1565
6	- 2451	- 569	1246	- 591
7	- 1470	1195	2428	718
8	941	2919	3379	2413
9	2678	3596	3264	3179
10	4449	3914	2892	3752
11	4720	3146	2065	3310
Noon	3108	1933	1161	2067
13	742	413	- 95	353
14	- 1290	- 1436	- 1205	- 1310
15	- 2302	- 3125	- 2733	- 2720
16	- 2218	- 4087	- 4060	- 3455
17	- 1686	- 4358	- 4862	- 3635
18	- 369	- 2875	- 4486	- 2577
19	220	- 1337	- 3538	- 1552
20	613	593	- 1561	- 118
21	579	1273	318	723
22	545	1500	1676	1240
23	- 30	1114	1608	897
24	366	1199	1910	1158
No. of days	960	1381	2014	(4355)

which all the seasons have approximately equal weight, the inequalities in the column headed "Total" has been obtained by equally weighting the seasonal groups.

The convexity effect, as defined by Bartels, namely, the differences "quiet" minus "all" days, is shown in Table XXIV. It is compared (for the total data) with the 24-hour convexity derived from Table XIV in Figure 13. It is at once apparent that the "convexity" as defined by Bartels does not agree with the convexity as derived from the β-day and γ-day inequalities.

Removing the 24-hour convexity derived from Table XIV from the α-day inequalities of Table XXIII, we obtain the inequalities of Table XXV, which represent the true solar variation on α-days freed from convexity. The harmonic analysis of these inequalities follows in Table XXVI.

(28). In calculating the probable errors of these α-day coefficients, it is to be noted that the periodic α-day inequalities are derived by the subtraction of the convexity effect (which is itself obtained by combining the β-day and γ-day inequalities) from the α-day solar inequalities of Table XXIII. The standard deviations necessary have already been obtained. Thus the standard deviation for a mean hourly inequality of the convexity is the same as that for a combined β-day and γ-day lunar inequality, namely,

$$\frac{1.0932}{\sqrt{\frac{3}{4}(n_1 + n_2)}} \text{ millibar},$$

Table XXIV.

The Bartels Convexity Effect: α-days minus all days.

Unit .0001 millibar.

Hour	Winter	Equinoxes	Summer	Total
1	- 920	- 171	- 422	- 373
2	- 533	469	8	84
3	- 39	811	390	490
4	313	1093	736	562
5	672	1316	1051	975
6	808	1505	1193	1272
7	1112	1576	1444	1226
8	1237	1607	1379	1567
9	1281	1438	1349	1317
10	1359	1248	1231	1213
11	1291	903	912	1109
Noon	1288	452	685	882
13	1292	286	360	523
14	868	- 124	181	214
15	618	- 120	- 162	158
16	25	- 489	- 558	- 238
17	- 290	- 1183	- 767	- 757
18	- 835	- 1393	- 984	- 1053
19	- 1008	- 1549	- 1221	- 1382
20	- 1461	- 1650	- 1529	- 1641
21	- 1665	- 1478	- 1428	- 1478
22	- 1953	- 1674	- 1340	- 1638
23	- 2020	- 1637	- 1492	- 1842
24	- 1454	- 1213	- 1021	- 1381

NOTE: Convexity starts at 23^h.

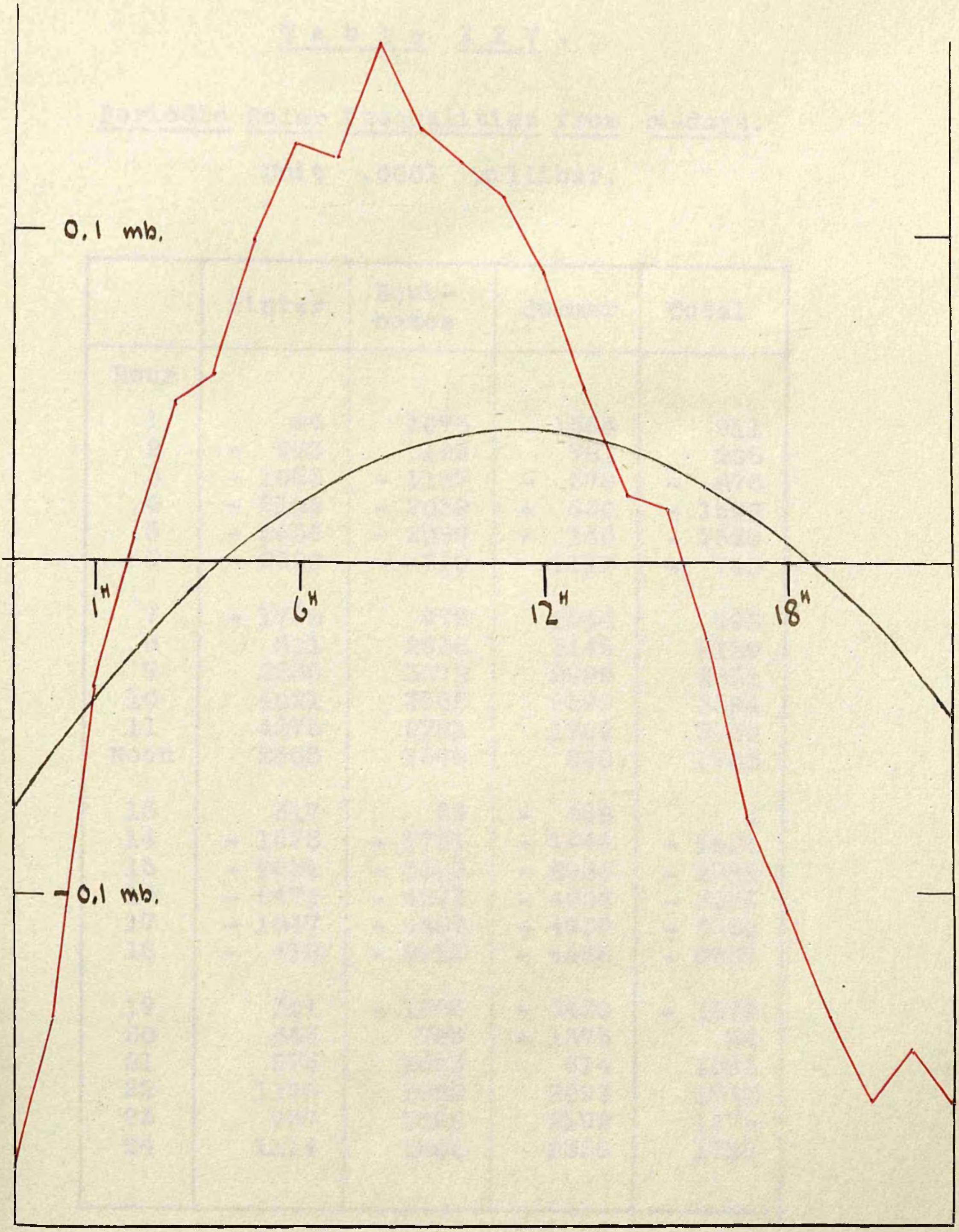

Figure 13.- Comparison of the "Bartels" convexity with the convexity obtained from combined β-days and γ-days.

Table XXV.

Periodic Solar Inequalities from α-days.

Unit .0001 millibar.

	Winter	Equinoxes	Summer	Total
Hour				
1	94	1076	1564	911
2	- 293	129	781	205
3	- 1063	- 1187	- 378	- 876
4	- 2126	- 2039	- 630	- 1599
5	- 2656	- 2089	- 140	- 1629
6	- 2629	- 719	1119	- 743
7	- 1738	972	2244	493
8	601	2638	3149	2129
9	2285	3272	2999	2851
10	4021	3562	2599	3394
11	4275	2781	1762	2939
Noon	2665	1569	868	1700
13	317	66	- 368	5
14	- 1675	- 1751	- 1446	- 1624
15	- 2631	- 3393	- 2930	- 2985
16	- 2472	- 4293	- 4202	- 3656
17	- 1847	- 4487	- 4939	- 3758
18	- 418	- 2912	- 4486	- 2607
19	301	- 1266	- 3450	- 1473
20	843	786	- 1375	84
21	975	1603	614	1063
22	1126	1982	2093	1732
23	937	1898	2192	1675
24	1114	1805	2359	1758

Table XXVI.

Harmonic Analysis of Periodic Inequalities from α-days.

Unit .0001 millibar.

	Winter	Equinoxes	Summer	Total
a_1	- 445	428	980	323
b_1	128	1195	2154	1159
c_1	463	1269	2366	1203
α_1	271°	5°	9°	1°
a_2	517	338	588	482
b_2	- 2443	- 2986	- 2321	- 2584
c_2	2497	3005	2394	2629
α_2	138°	144°	136°	139°
a_3	413	89	- 64	146
b_3	1088	154	- 612	210
c_3	1164	178	615	256
α_3	336°	345°	141°	350°
a_4	- 136	269	85	76
b_4	- 165	307	193	108
c_4	214	408	211	132
α_4	160°	341°	324°	335°

and the standard deviation for an α-day mean hourly inequality is $1.0923/\sqrt{n}$. The probable error of a mean hourly inequality of the periodic α-day variation is therefore

$$0.6745.\ 1.0923\sqrt{\left\{\frac{1}{n}+\frac{1}{\frac{3}{4}(n_1+n_2)}\right\}}\ \text{millibar},$$

from which the probable errors of the coefficients may be derived. These are shown in Table XXVII.

Comparison of Tables XXII and XXVI shows that the main difference between the "all day" coefficients and the α-day coefficients consists in a large change in the diurnal component. The difference, amounting to 0.1205 sin $(x + 335^{\circ})$ millibar for the total data, and well marked in each season, is significant on any reasonable assumption as to the probable erros of the "all day" coefficients.

It is therefore clear that Bartels' definition of convexity as the difference "quiet days - all days" disregards entirely the presence of an effect which, in the Glasgow data, is much greater than the convexity itself. The additional periodic component, which appears on quiet days, appears to be almost purely diurnal in character, the changes in the second and higher harmonics being negligible, having regard to the probable errors involved.

The additional "quiet-day variation" will be further investigated in the next chapter.

Table XXVII.

Probable Errors of the α-day Coefficients.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_γ, b_γ, c_γ	89	74	61	44
α_1	11°	3°	1°	2°
α_2	2°	1°	1°	1°
α_3	4°	24°	6°	10°
α_4	24°	10°	17°	19°

VII.

(29). In the previous chapter, the determination of the "quiet-day excess" in the solar variation depends on the 24-hour convexity derived from a discussion of the β-day and γ-day lunar inequalities. In the present chapter, this quiet-day variation is obtained by the transposition of lunar days into solar time. By this means convexity is eliminated without the necessity of using a previously determined value.

The data used are the γ-days of section 16. These days are selected to begin at lower lunar transit, and continue for 25 solar hours, the maximum range of pressure being 0.110 inch. Owing to the method of selection, the convexity is zero at lower lunar transit, and maximum at upper transit; that is, in the sequences as selected it appears as a lunar effect. After transposition of the sequences into solar time, and in the addition of many such transposed sequences with different transit hours, the convexity (and also the lunar variation) is eliminated, except for the slight residue which remains owing to the irregularity in the distribution of transit hours. The method is thus similar in principle to that used in determining the lunar variation from the transposed α-days. In that method, the α-days (which are days selected to begin at a fixed <u>solar</u> hour - 23^h) were transposed into <u>lunar</u> time; in this, the Υ-days (which are days selected to begin at a fixed <u>lunar</u> hour - lower lunar transit) are transposed into <u>solar</u> time. The result in both cases is to eliminate convexity. The treatment of the data is very similar to that described in connection with the determination of the lunar variation by the transposition method, and few details need be given. No correction for the residue of lunar variation and convexity was applied, as it was found by actual trial that this correction was negligible. As the Υ-days are 25-hour sequences, the 25th. hour of the sequence was used to determine the non-cyclic change. The final inequalities are shown in Table XXVIII, and the harmonic analysis in Table XXIX.

(30). The probable errors of the coefficients are easily obtained from the data already available. The standard deviation of a single reading of the barometer on a day whose range of pressure over 25 hours does not exceed 0.110 inch has already been found to be 1.0923 millibar. This figure is applicable in the present case. The probable errors calculated are given in Table XXX.

Comparing the coefficients of the all day variation (Table XXII) with those given in Table XXIX for the transposed Υ-days, we see that the diurnal component is greatly

Table XXVIII.

Solar Inequalities from Transposed ♈-days.

Unit .0001 millibar.

Hour	Winter	Equi-noxes	Summer	Total
1	156	1322	2128	1206
2	- 278	287	1224	414
3	- 1258	- 1167	- 47	- 821
4	- 2290	- 2067	- 473	- 1609
5	- 2843	- 2171	- 164	- 1721
6	- 2774	- 978	1007	- 913
7	- 1937	525	1809	133
8	369	2206	2690	1750
9	2187	2923	2517	2536
10	3876	3297	2106	3087
11	4241	2537	1235	2674
Noon	2622	1456	358	1473
13	206	- 128	- 753	- 227
14	- 1741	- 1918	- 1866	- 1845
15	- 2663	- 3684	- 3199	- 3183
16	- 2378	- 4532	- 4348	- 3753
17	- 1811	- 4403	- 5041	- 3750
18	- 386	- 2833	- 4491	- 2569
19	386	- 966	- 3351	- 1310
20	1022	1206	- 1061	392
21	1207	1969	1104	1427
22	1518	2544	2697	2253
23	1317	2393	2964	2228
24	1251	2181	2957	2129
No. of days	927	1355	2005	(4287)

Table XXIX.

Harmonic Analysis of Solar Inequalities from Transposed ♈-days.

Unit .0001 millibar.

	Winter	Equinoxes	Summer	Total
a_1	- 240	703	1498	621
b_1	- 62	942	1888	920
c_1	248	1175	2410	1110
α_1	241°	22°	23°	19°
a_2	542	384	680	533
b_2	- 2540	- 3090	- 2353	- 2664
c_2	2597	3114	2449	2717
α_2	138°	143°	134°	139°
a_3	409	34	- 85	121
b_3	1041	206	- 635	203
c_3	1118	209	641	236
α_3	336°	324°	143°	346°
a_4	- 169	260	45	48
b_4	- 198	260	183	85
c_4	260	368	188	98
α_4	160°	345°	314°	329°

Table XXX.

Probable Errors of Υ-day Coefficients.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_Υ, b_Υ, c_Υ	70	58	47	34
α_1	16°	3°	1°	2°
α_2	2°	1°	1°	1°
α_3	4°	16°	4°	8°
α_4	15°	9°	14°	20°

changed, the difference for the total data being

$$951 \sin (x + 350^{\circ}) \text{ millibar.}$$

This increase is of the same order of magnitude as that found in the previous chapter from the ~~transposed~~ α-days. The extra quiet-day variation is again purely diurnal in character, the changes in the other components being negligible when the probable errors are taken into consideration.

(31). It has already been noted (section 23) in connection with the lunar diurnal components obtained from transposed α-days and from combined β- and γ-days, that transposition raises the effective range of pressure. In the present case, the γ-days (which are according to their mode of selection lunar days) on being transposed into solar time, result in a solar variation which corresponds to a lesser degree of quietness than that appropriate to the α-days, which are initially selected as quiet solar days. If, as we have seen, quiet days produce an additional diurnal component in the solar variation, we should expect the α-days to produce a greater additional component than the transposed γ-days, since they are in effect quieter than the transposed γ-days.

That this is the case may be shown as follows. We may define as a "disturbed day" any solar sequence whose range is greater than 0.110 inch. The harmonic coefficients of the variation on such days may be obtained directly from those for all days and for the α-days. Taking summer for example, there are in the 45 years data 5535 days. Of these 2014 are α-days, and the remainder - 3521 - are disturbed days. Thus, for the a_1 coefficient of the disturbed days we have

$$3521\, a_1 = 5535 \cdot 1150 - 2014 \cdot 980$$

from which $a_1 = 1247$.

For the b_1 coefficient,

$$3521\, b_1 = 5535 \cdot 974 - 2014 \cdot 2154$$

from which $b_1 = 299$.

The disturbed days may be considered as days in which the quiet day variation is entirely absent (unlike "all days", which include the quiet days). Thus for summer, we find the additional quiet day variation on α-days to be

$$-\,267 \cos x + 1855 \sin x$$

$$\text{or} \quad 1874 \sin (x + 337^{\circ}),$$

and on γ-days,

$$251 \cos x + 1589 \sin x$$

$$\text{or} \quad 1609 \sin (x + 354^{\circ}),$$

x being measured from midnight in the case of the c_1 and α_1 coefficients.

The corresponding results from the other groups of data are given in Table XXXI. An inspection of this table shows that the α-days have, in all cases, a larger extra diurnal component than the transposed γ-days.

(32). The variation of the extra diurnal component with quietness of day may be shown by selecting "δ-days",

Table XXXI.

The additional diurnal component on ♈-days and on α-days.

Unit .0001 millibar.

		Winter	Equi-noxes	Summer	Total
♈-days	a_1	- 515	357	251	- 2
	b_1	953	1655	1589	1363
	c_1	1083	1693	1609	1363
	α_1	317°	357°	354°	345°
α-days	a_1	- 720	82	- 267	- 300
	b_1	1143	1908	1855	1602
	c_1	1351	1910	1874	1630
	α_1	313°	347°	337°	334°

according to the criterion that they must be γ-days each of whose parts, before and after the 23^h reading which occurs during the γ-day, should be part of an α-day. Thus δ-days are days lying completely within α-days, and are therefore analogous to the ϵ-days (section 23) which are α-days lying completely within β-days. The same remarks may be made about these δ-days as about the ϵ-days. In particular we may notice that before a δ-day can be selected from the records, two successive α-days are required. This is a more stringent requirement than that appropriate to the α-days, which require only one quiet 25-hour sequence. Thus δ-days are quieter than α-days. They are also, of course, quieter than the γ-days record as a whole, since they are selected from the quietest portions of this record.

The solar inequalities obtained from the transposed δ-days are shown in Table XXXII, and the harmonic analysis of the inequalities follows in Table XXXIII.

It has already been shown that the standard deviation of a single hourly reading on an ϵ-day is 0.01583 inch, or 0.536 millibar (section 24). This standard deviation is also applicable to the δ-days, which are selected according to an exactly similar criterion to the ϵ-days. The probable errors of the coefficients of the δ-day variation, calculated from this value of the standard deviation for a single reading, are given in Table XXXIV.

Comparing the coefficients of Table XXXIII with those obtained from the transposed γ-days, we see at once that

Table XXXII.

Solar Inequalities from Transposed δ-days.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
Hour				
1	312	1372	2117	1267
2	- 66	405	1227	522
3	- 982	- 1041	128	- 632
4	- 2109	- 1846	- 145	- 1367
5	- 2698	- 1886	247	- 1446
6	- 2564	- 469	1554	- 493
7	- 1622	1027	2397	601
8	666	2685	3311	2221
9	2486	3343	3155	2995
10	4112	3572	2521	3402
11	4398	2573	1593	2855
Noon	2739	1528	519	1595
13	324	- 130	- 755	- 187
14	- 1923	- 2105	- 2068	- 2032
15	- 2844	- 4054	- 3586	- 3495
16	- 2845	- 5037	- 4868	- 4250
17	- 2200	- 4933	- 5599	- 4244
18	- 673	- 3375	- 5101	- 3050
19	40	- 1443	- 3913	- 1772
20	729	947	- 1534	47
21	1003	1886	672	1187
22	1243	2424	2427	2032
23	1228	2361	2792	2130
24	1252	2200	2908	2120
No. of days	345	537	944	(1826)

Table XXXIII.

Harmonic Analysis of Solar Inequalities from Transposed δ-days.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_1	- 312	797	1546	677
b_1	271	1334	2459	1355
c_1	413	1554	2905	1515
α_1	296°	16°	17°	12°
a_2	595	360	614	523
b_2	- 2592	- 3204	- 2459	- 2752
c_2	2659	3224	2534	2801
α_2	137°	144°	136°	140°
a_3	419	- 8	- 1	137
b_3	1082	153	- 655	193
c_3	1160	153	655	266
α_3	336°	312°	135°	350°
a_4	- 121	299	53	80
b_4	- 170	295	173	99
c_4	208	420	181	127
α_4	155°	345°	317°	339°

Table XXXIV.

Probable Errors of the δ-day Coefficients.

Unit .0001 millibar.

	Winter	Equi-noxes	Summer	Total
a_γ, b_γ, c_γ	56	45	34	27
α_1	8°	2°	1°	1°
α_2	1°	1°	1°	1°
α_3	3°	17°	3°	6°
α_4	15°	6°	11°	12°

the diurnal component has greatly increased. The best means of showing the increase of the diurnal component with quietness is as follows. We may define three types of day thus:-

(a) "Disturbed days" : these are the days which remain after the γ-days have been abstracted from the original records.

(b) "Quiet days" : these are the γ-days which remain after the δ-days have been removed from them.

(c) "Very quiet days" : these are the δ-days themselves.

These three groups are mutually exclusive. We may expect in the disturbed days no trace of the additional diurnal component. The solar variation on these days, and on the "quiet days" as defined above, may be obtained from the data already available by the process explained in section 31. Table XXXV gives the results, and also contains the coefficients of the extra diurnal components in "quiet days" and "very quiet days". The increase in this component as the selected days become quieter is clearly shown; the phase of the extra component remains approximately constant.

(33). In Table XXXI the differences between the diurnal components from the α-day and the transposed γ-day inequalities are shown using as the basis of comparison the "disturbed" days remaining after the α-days have been removed from the total data. The disturbed days thus defined include parts of the record which are also parts of the γ-day tabulations;

Table XXXV.

The changes in the Diurnal Component with Quietness.

Unit .0001 millibar.

		Winter	Equinoxes	Summer	Total
Disturbed days	a_1	227	257	952	477
	b_1	- 967	- 618	455	- 377
Quiet days (γ')	a_1	- 197	641	1455	633
	b_1	- 259	685	1380	602
Very quiet days (δ)	a_1	- 312	797	1546	677
	b_1	271	1334	2459	1355
Extra diurnal in γ'-days	a_1	- 424	384	503	155
	b_1	708	1303	925	979
	c_1	825	1358	1053	991
	α_1	314°	1°	14°	354°
Extra diurnal in δ-days	a_1	- 539	540	594	200
	b_1	1238	1952	2004	1732
	c_1	1350	2025	2090	1744
	α_1	322°	0°	2°	352°

that is, these disturbed days and the γ-days are not mutually exclusive. In the present section, we have used as disturbed days those days which remain after the γ-days have been removed from the total data; in this case the disturbed days and α-days are not mutually exclusive. It is evident that neither of the two ways of defining a disturbed day gives a satisfactory basis for comparing the diurnal components on α-days and γ-days. The best basis of comparison is probably to use, as the coefficients of the diurnal component on disturbed days, the means of the corresponding components as obtained by the two definitions of "disturbed day" mentioned above. Table XXXVI shows the coefficients of the diurnal component of "disturbed day" thus calculated, and also the additional diurnal components of α-days and transposed γ-days calculated using this disturbed day variation as basis. This table may be considered as superseding Table XXXI. It is to be noticed, however, that considering the probable errors involved, it is likely that there is no significant difference between the results given in the two Tables XXXI and XXXVI. Both agree in showing that the α-day variation has a greater diurnal component than the transposed γ-day variation. It may therefore be regarded as proved that transposition of a selected quiet sequence produces a sequence which corresponds to a lesser degree of quietness. In the case of the solar inequalities, the γ-days, transposed from lunar into solar time, produce a smaller diurnal component

Table XXXVI.

The additional diurnal component on ϒ-days and on α-days.

Unit .0001 millibar.

		Winter	Equi-noxes	Summer	Total
Disturbed days	a_1	251	301	1100	550
	b_1	- 991	- 615	377	- 410
	c_1	1016	685	1163	686
	α_1	151°	139°	56°	112°
Extra diurnal in ϒ-days	a_1	- 491	402	398	71
	b_1	929	1557	1511	1330
	c_1	1051	1608	1563	1332
	α_1	317°	359°	0°	348°
Extra diurnal in α-days	a_1	- 696	127	- 120	- 227
	b_1	1119	1810	1777	1569
	c_1	1318	1814	1781	1585
	α_1	313°	349°	341°	337°

than the α-days, which are selected originally as solar sequences. We have already seen (section 20) the same effect produced in the lunar inequalities, where the α-days, transposed from solar into lunar time, produce a smaller diurnal component than the β-days and γ-days, which are selected originally as lunar sequences. The explanation of this curious effect has already been suggested (section 23).

(34). The main result of this investigation of the solar variation at Glasgow is that an additional diurnal component occurs on quiet days, and that the amplitude of this component increases as the effective range of barometric pressure during the course of a day diminishes. This change is well shown by Table XXXV, where the additional diurnal components corresponding to the "quiet" and "very quiet" days defined in section 32 are given. The most striking feature of the change in the solar variation with quietness of day is the fact that it is purely diurnal in character. Reference to Tables XXII, XXVI, XXIX and XXXIII shows that, having regard to the probable errors involved, the second, third and fourth harmonics are quite constant in amplitude and in phase, throughout the various types of days. The maximum of the additional diurnal component for the total data occurs at about 6.30 a.m. G.M.T.; there is however, a pronounced lag of about two hours during the winter months, accompanied by a decrease in the amplitude.

This constancy in all harmonics except the first is rather unusual in meteorological phenomena, in which variations are not generally found occurring in the pure sinusoidal form. The additional variation here found to exist on barometrically quiet days is probably associated with the "clear-day" variation believed to exist on days with clear skies. Unfortunately, hourly cloud records do not exist for Glasgow, and it is consequently not possible to test this suggestion with the Glasgow data.

VIII.

(35). In 1902 Buchan and Omond[18] discussed the differences in the mean daily barometric variation on days characterised by clear and by cloudy skies. The data for this investigation comprised hourly barometric readings for nine stations, well distributed in latitude, for which hourly records of cloud amount or of sunshine were also available. The main results were as follows:-

(a) The daily curve at each station is distorted in the same way, the forenoon maximum and afternoon minimum being increased on clear days, the evening maximum and early morning minimum diminished. On cloudy days the opposite effects are observed.

(b) The effects are larger at temperate and Arctic stations than in the tropics.

At the time of Buchan and Omond's investigation, the convexity effect had not been discovered. Consequently, no precautions were taken by these authors to eliminate the effect of convexity. It is apparent that clear days, being selected in anticyclonic weather, must occur near barometric maxima in the same way as the barometrically quiet days already discussed in connection with the Glasgow data. Thus, in the clear-day variation as found by Buchan and Omond, there is also included the spurious component of convexity, which can in no way be considered as a real solar effect depending upon the atmospheric conditions on clear days. On cloudy days, which occur mainly in cyclonic weather, the barometric curve is near a minimum turning-point, and the opposite effect is obtained. It will be seen that the results obtained by Buchan and Omond, as indicated above, may be easily explained as being caused by convexity, which is maximum near the centre of the selected day (thus causing an apparent increase in the forenoon maximum) and which may be expected to be most noticeable in the temperate and Arctic regions, where alternate anticyclones and depressions are frequent.

The object of the investigation described in this chapter is to establish the presence of the convexity effect in the variation as obtained by Buchan and Omond, and to separate it from any periodic effect which may be present. For this purpose, the records of only one of the stations considered by Buchan and Omond have been utilised, namely, that on the summit of Ben Nevis (latitude 56° 48'N, longitude 5° 1'W, height above sea-level 4407 feet).

(36). The data used by Buchan and Omond comprised three years' observations of hourly barometric readings and the corresponding hourly estimates of cloud amount. The particular years used have not been stated, nor is any definite criterion given in their paper for the selection of clear days. The selected days were grouped according to months. In the present investigation only the January and July records for the years 1884-1904 have been used.

The purpose of the investigation is to examine the results obtained by Buchan and Omond, in the light of the fact that, owing to their method of selection of the data, convexity must necessarily be present in their inequalities. It is therefore desirable to use, as far as possible, the same criterion for "clearness" as used by these authors; but this is not stated explicitly in their paper. In order to find this criterion, we have the following facts: That three years' data was used, the actual years being unspecified; and that there were 18 clear January days in that period, and 36 clear July days.

Table XXXVII summarises, for July, the numbers of days of varying clearness in the entire 20 years 1884-1903. The last column gives the sum for three successive years, for average cloudiness not greater than 7 on the Ben Nevis scale. If Buchan and Omond had used, say, as criterion of a clear day, that the average cloudiness should be not greater than 7, it is apparent that at least one "36" should appear in this column. Inspection of the table shows that only by using the years 1896-98, or 1897-99, could the number of days selected have been as great as 34, and this only if a day of average cloudiness 7 be accepted as a clear day.

(37). In the present investigation a clear day is defined as a period of 24 hours, starting either at 1 a.m. or at 1 p.m., during which the average cloudiness on the Ben Nevis scale does not exceed 7; that is, during the day the calotte of the sky above 30^{o} altitude is on the average not more than $\frac{3}{4}$ covered with cloud. This criterion appears to be at least as stringent as that used by Buchan and Omond. Days thus selected starting at 1 a.m. are for brevity called M_J-days; those starting at 1 p.m. N_J-days. In tabulating these days, the same procedure as that indicated for the Glasgow data was followed; in particular, an additional hourly reading - that immediately preceding the beginning of the day - was added to facilitate correction for non-cyclic change; this correction being applied to the totals for each set of days.

Selection of the two types of day was made independently. All the M_J-days having been selected and tabulated, the cloud records were again examined, and the barometric record of all the 24-hour sequences satisfying the condition for N_J-days tabulated. Many portions of the barometric record are thus common to the M_J-day and N_J-day tabulations.

The M_J-day and N_J-day inequalities are shown in Table XXXVIII.

Table XXXVIII.

M_J-day and N_J-day inequalities for January and July.

Unit .001 millibar.

Hour	M_J - days January	M_J - days July	Hour	N_J - days January	N_J - days July
1	- 364	- 368	13	- 433	66
2	- 329	- 469	14	- 400	217
3	- 288	- 612	15	- 292	254
4	- 351	- 639	16	- 196	224
5	- 390	- 646	17	- 94	204
6	- 364	- 526	18	43	209
7	- 217	- 360	19	199	244
8	2	- 190	20	376	323
9	258	0	21	447	430
10	453	114	22	427	445
11	552	272	23	408	355
12	399	393	24	307	218
13	195	480	1	210	21
14	112	561	2	155	- 123
15	103	509	3	96	- 354
16	131	429	4	- 17	- 455
17	130	317	5	- 184	- 521
18	130	248	6	- 272	- 510
19	138	226	7	- 248	- 395
20	175	194	8	- 122	- 294
21	123	208	9	- 43	- 199
22	- 41	136	10	18	- 164
23	- 178	- 32	11	- 55	- 116
24	- 379	- 237	12	- 327	- 79
No. of days	106	131		104	130

(38). The separation of the convexity from any real periodic effect is made exactly as described in Chapter II and need not be elaborated here. The coefficients of harmonic analysis for the convexity and for the clear-day solar variation are shown in Table XXXIX, together with the normal (all days) variation for the months concerned. Coefficients in parentheses are those derived indirectly assuming a parabolic form for the convexity.

The probable errors of the coefficients have been determined using the variance method. The total variance to which a single barometer reading is subject may be regarded as composed of three components, viz.,

(a) that due to the hour-to-hour change of the barometer, which is partly periodic, and partly non-cyclic.

(b) that due to the change of daily mean value of the barometer on the selected days.

(c) that due to accidental causes.

The first two variances, (a) and (b), can be separately computed. By removing these from the total variance the component due to accidental causes can be determined. This has been done for the January N_J-days only, the details of the calculation being shown in Table XL. Assuming that the standard deviation so obtained, namely 0.02598 inch, is also applicable to January M_J-days and to July M_J-days and N_J-days, the probable errors of the harmonic coefficients in columns (2),(3),(5) and (6) of Table XXXIX can be calculated. They

Table XXXIX.

Comparison of Clear-day and Normal Solar Variations.

Unit .001 millibar.

	January			July		
	Con-vexity	Clear-day Varn.	Normal Varn.	Con-vexity	Clear-day Varn.	Normal Varn.
a_1	-290	- 26	- 8	-193	-219	-203
b_1	38	-126	-248	28	-363	-363
a_2	(- 71)	(47)	42	(- 47)	(114)	109
b_2	(19)	(-188)	-201	(14)	(-133)	-136
a_3	(- 30)	28	14	(- 20)	- 4	4
b_3	(12)	88	82	(9)	- 55	- 58
a_4	(- 15)	(- 48)	- 40	(- 10)	(- 1)	2
b_4	(9)	(15)	- 11	(7)	(26)	23
a_5	(- 10)	13	14	(- 6)	- 4	- 4
b_5	(7)	9	17	(6)	4	- 2

Table XL.

Standard Deviation for January N-days.

	Degrees of Freedom	Sum of Squares	Mean Square	Standard Deviation Unit 1 inch Hg.
Hours	24	0.1515		
		10.4075		
Days	103	360.0650		
Remainder	2472	1.6681	0.000675	0.02598
Total	2599	372.2921		

are, for January, 0.012 millibar, and for July 0.011 millibar.

(39). Comparison of the coefficients of the clear-day variation with those of the normal variation shows that, with one exception, the differences are insignificant. The first harmonic for January shows an increase, the additional component being

$$0.123 \sin (x + 352^\circ) \text{ millibar,}$$

x being measured from 1 a.m. This component, however, is much smaller than that due to convexity, the "amplitude" of which is 0.292 millibar. The first harmonic of the "clear-day excess" found for January by using M_J-days alone, which is directly comparable with the clear-day excess found by Buchan and Omond, and which also includes the convexity, has an amplitude

$$0.356 \pm 0.017 \text{ millibar;}$$

the corresponding amplitude obtained from the data of Buchan and Omond is

$$0.385 \pm 0.04 \text{ millibar.}$$

It is evident, from the last two figures stated, that the criterion here adopted for a clear day is a sufficient approximation, for January at least, to that used by Buchan and Omond, since these two amplitudes are practically equal. It is also apparent that the convexity is responsible for by far the greater part of this "clear-day excess".

The true first harmonic in the clear-day excess for January appears to be real enough, judging by the probable errors, although no such effect is present in the July coefficients. The explanation of this anomaly, which is investigated in the next chapter, appears to be that the criterion of selection is not sufficiently stringent for the July data; that is, a day of average cloudiness 7 can not be considered, in July, as a clear day, so far as the effect on the solar barometric variation is concerned.

The conclusion reached in this chapter is that the greater part of the clear-day effect as found by Buchan and Omond is due to the non-periodic convexity; although, in the January records, a real additional periodic component is undoubtedly present.

IX.

(40). The investigation of the previous chapter is inconclusive in the respect that while January shows a real additional component in the solar variation on clear days no such effect is present in the July components. The object of the investigation of the present chapter is to establish the presence or absence of such an effect, in all seasons, using a somewhat more stringent criterion of clearness.

For this purpose the Ben Nevis data have been divided into three seasonal groups, similar to those used in the discussion of the Glasgow data. The method of selection of clear days is as follows. A clear day is defined, in the first place, as a day during which the average cloudiness on the Ben Nevis scale does not exceed 3. Such a day may start either at 1 a.m. - in which case it is called an M-day - or at 1 p.m. - in which case it is called an N-day. All such days, however, were not used. First of all sequences of at least three consecutive M-days or three consecutive N-days were marked on the records; we have thus sequences of 72 hours in duration. The first 12 hours and the last 12 hours of such a sequence were ignored, leaving a sequence of 48 hours at least of clear weather, preceded and followed by 12 hours of clear weather. Such a 48-hour sequence contains either two M-days and one N-day or one M-day and two N-days, depending on whether the original 72-hour sequence started at 1 p.m. or at 1 a.m. These are the days for which the barometric readings are used in this investigation. They are distinguished from the M_7-days and N_7-days of the previous chapter by the fact that, besides having a maximum average cloudiness 3 instead of 7, they are preceded and followed by at least 12 hours of clear weather. It is evident from the method of selection that a large part of the barometric record used is common to both the M-day and the N-day tabulations. The addition of a 25th. reading in the tabulation for each day assists as before in the removal of non-cyclic change.

(41). The hourly inequalities for all days, in the three seasonal groups and for the total data 1884-1903, are given in Table XLI, and the harmonic analysis in Table XLII. It will be seen that the normal Ben Nevis variation is of the typical "high level" form, with a diurnal component having its maximum in the afternoon, as described in section 3.

The hourly inequalities for the selected M-days and N-days are shown in Tables XLIII and XLIV. In these tables the "total" inequalities are obtained by equally weighting the seasonal inequalities.

The treatment of the inequalities in order to obtain the

Table XLI.

Ben Nevis - Hourly Inequalities from all days.

Unit .001 millibar.

Hour	Winter	Equinoxes	Summer	Total
1	63	- 29	- 72	- 13
2	- 65	- 188	- 265	- 172
3	- 164	- 391	- 452	- 335
4	- 316	- 510	- 557	- 460
5	- 401	- 564	- 608	- 524
6	- 408	- 507	- 526	- 480
7	- 340	- 368	- 387	- 365
8	- 177	- 198	- 242	- 202
9	- 4	- 66	- 113	- 60
10	131	63	2	65
11	216	165	111	164
Noon	124	233	209	191
13	6	246	294	181
14	- 79	222	358	167
15	- 126	155	324	116
16	- 48	90	263	99
17	19	100	202	106
18	131	161	165	150
19	216	229	182	211
20	267	317	230	272
21	280	294	307	293
22	280	263	287	276
23	233	188	199	204
24	165	104	80	116

Table XLII.

Ben Nevis - Harmonic Coefficients of the Normal Solar Variation.

Unit .001 millibar.

	Winter	Equinoxes	Summer	Total
a_1	- 8	- 143	- 193	- 114
b_1	- 204	- 297	- 342	- 280
a_2	51	88	108	81
b_2	- 199	- 185	- 148	- 178
a_3	31	3	3	13
b_3	76	8	- 50	9
a_4	- 13	26	4	8
b_4	- 9	20	25	13
a_5	1	2	- 1	1
b_5	6	- 2	7	3

Table XLIII.

Hourly Inequalities from M-days.

Unit .001 millibar.

Hour	Winter	Equinoxes	Summer	Total
1	- 48	- 181	- 104	- 111
2	- 120	- 299	- 270	- 230
3	- 202	- 378	- 415	- 332
4	- 262	- 469	- 508	- 413
5	- 308	- 476	- 477	- 420
6	- 299	- 376	- 420	- 365
7	- 217	- 181	- 312	- 237
8	19	- 32	- 191	- 68
9	134	128	- 76	62
10	257	235	46	179
11	371	297	155	274
12	236	336	272	281
13	85	320	332	246
14	- 101	200	343	147
15	- 195	109	328	81
16	- 189	51	256	39
17	- 81	- 31	111	0
18	48	24	70	47
19	126	127	79	111
20	133	218	161	171
21	182	209	223	205
22	174	140	218	177
23	134	71	151	119
24	117	- 41	36	37
No. of days.	52	69	97	

Table XLIV.

Hourly Inequalities from N-days.

Unit .001 millibar.

Hour	Winter	Equinoxes	Summer	Total
13	35	110	230	125
14	- 125	49	258	61
15	- 198	- 24	255	11
16	- 194	- 58	203	- 16
17	- 97	- 52	84	- 22
18	22	0	64	29
19	111	153	96	120
20	110	269	174	184
21	183	297	258	246
22	192	279	272	248
23	177	237	200	205
24	150	148	105	134
1	20	15	- 10	8
2	- 102	- 148	- 179	- 143
3	- 171	- 266	- 355	- 264
4	- 287	- 388	- 450	- 375
5	- 332	- 432	- 452	- 405
6	- 313	- 358	- 424	- 365
7	- 217	- 212	- 322	- 250
8	22	- 74	- 207	- 86
9	142	48	- 98	31
10	275	122	0	132
11	366	134	95	198
12	227	149	196	191
No. of days.	50	73	94	

true clear-day variation freed from convexity is exactly the same as accorded to the M_ζ-day and N_ζ-day inequalities of the previous chapter. The results for the clear-day variation are shown in Table XLV, the coefficients derived indirectly by assuming the parabolic form for the convexity being enclosed in parentheses.

The probable errors of the coefficients have been obtained by the method of variance. To obtain the standard deviation of a single barometer reading the N-days for the equinoctial group, comprising 73 days, have been used. The variances which have been removed from the total variance to obtain that due to accidental causes are

(a) the hour-to-hour variance, partly due to the periodic daily swing of the barometer, and partly to the non-cyclic change in the course of a day,

(b) the variance due to the change of mean value from day to day.

The details of the calculation are shown in Table XLVI, which gives, as the standard deviation for a single barometer reading, 0.01486 inch. In calculating the probable errors of the coefficients, it is assumed that this value of the standard deviation is appropriate to all sub-divisions of the data; also it is assumed that one-half the barometric record used is common to the M-day and N-day tabulations, and that the probable errors are given by

Table XLV.

The Clear-Day Variation.

Unit .001 millibar.

	Winter	Equinoxes	Summer	Total
a_1	- 52	- 133	- 170	- 118
b_1	- 72	- 167	- 260	- 167
a_2	(43)	(72)	(122)	(78)
b_2	(- 235)	(- 219)	(- 134)	(- 198)
a_3	27	- 9	- 13	3
b_3	81	6	- 50	8
a_4	(- 4)	(19)	(1)	(7)
b_4	(- 24)	(22)	(19)	(6)
a_5	- 11	- 1	11	0
b_5	2	- 2	9	3

Table XLVI.

Standard Deviation for Equinoctial N-days.

Unit .001 millibar.

	Degrees of Freedom	Sum of Squares	Mean Square	Standard Deviation Unit 1 inch Hg.
Hours	24	0.0656 1.8733		
Days	72	55.5075		
Remainder	1728	0.3814	0.0002207	0.01486
Total	1824	57.8278		

Table XLVII.

The Additional Clear-Day Variation and the Convexity.

Unit .001 millibar.

	Winter	Equinoxes	Summer	Total
	Clear-Day Variation			
a_1	- 44	10	23	- 4
b_1	132	130	82	113
c_1	139	130	85	113
α_1	327°	349°	1°	343°
a_2	- 8	- 16	14	- 3
b_2	- 36	- 34	14	- 20
c_2	37	38	20	20
α_2	163°	175°	15°	158°
	Convexity			
a_1	- 10	- 84	- 39	- 44
b_1	- 1	12	- 1	4

$$0.01486 \cdot \frac{0.6745}{\sqrt{9(n_1+n_2)}} \cdot 33.86 \text{ millibar.}$$

The probable errors thus calculated are: for Winter, 11; for Equinoxes, 9; for Summer, 8; and for the total data, 5; the unit in each case being 0.001 millibar.

(42). The differences of the clear-day and normal variations are shown, for the first and second harmonics, in Table XLVII, together with the first harmonics of convexity for the various groups. The first harmonic is undoubtedly significant in all groups. The non-appearance of this harmonic in the July group of the previous chapter may be attributed to the fact that the criterion of clearness there used - namely, that the average cloud amount should not exceed 7 on the Ben Nevis scale- was not sufficiently stringent. In the present case, using a more exacting criterion of clearness, we find a significant diurnal component. The seasonal variation in phase is quite marked; the maximum in each group being (with due regard to the probable errors, which for a group are probably of the order of 12°) near the time of sunrise appropriate to the group. Thus the extra clear-day component agrees in phase with the component normally experienced at sea-level stations (section 3) and is opposite to that normally experienced at mountain stations such as Ben Nevis itself. In other words, the clear-day variation at this station is equivalent to the normal variation of a station at a considerably lower altitude.

The amplitude for summer is noticeably smaller than the amplitudes for the other seasonal groups. The second harmonic is much less distinct, but appears to be significant. It is to be remarked that the first harmonic of convexity is much smaller than that previously obtained, in Chapter VIII, using a less exacting criterion of clearness; it is very doubtful, indeed, if this component is at all significant for the Winter group.

A comparison of Table XLVII with Table XXXV reveals the fact that the first harmonic of the "clear-day excess" at Ben Nevis and that of the "quiet-day excess" at Glasgow have the same phase, while the amplitudes are of the same order of magnitude; indicating a common origin for the two effects.

We may conclude from the present investigation that the results of Buchan and Omond's analysis were erroneous for two main reasons:-

(1) the convexity effect was not eliminated. This leads to the introduction of a non-periodic "diurnal component" with its maximum near the middle of the day, tending to increase the forenoon maximum which occurs shortly before noon.

(2). the criterion for "clearness" adopted (which, as we have shown in the previous chapter, is nearly equivalent to selecting days on which the average cloudiness does not exceed 7) was not sufficiently exacting, in the case of the July records at least, to give a real additional effect. For January, the adopted criterion appears to be suitable, but the clear-day effect in this case is affected by the presence of the convexity.

It is evident that Buchan and Omond's results for the other stations discussed are liable to be affected in the same way. So, also, are those investigations made on similar lines in which days are selected in anticyclonic weather without regard to the possible presence of convexity. Thus Hann[19] has analysed the clear-day and cloudy-day variations for various pairs of mountain and valley stations, following a method similar to that used by Buchan and Omond in selecting days (equivalent, in effect, to using M-days alone in determining the variation). It is clear that, unless suitable precautions, such as those described in the present investigations, are taken, such a selection of days will inevitably introduce the non-periodic convexity effect.

It may be concluded that theories put forward to explain the additional effects found by these authors, resting as they do on faulty evidence, will probably require to be modified - particularly as regards the phase of the additional effect - in the light of the information obtained in the preceding investigations.

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