

Conference Paper

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# Assessment of Engineering Mathematics in the Context of Distance Learning

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**Abstract**—Mathematics is a crucial component of programmes on engineering, science and technology. In the context of computerised distance learning education, the subject of engineering mathematics is often delivered and assessed through a digital learning environment. This paper gives an overview of the history of distance learning education and its mathematics component. It also discusses the most popular on-line automated technologies applied to the assessment and evaluation of engineering mathematics including problems of the on-line representation of mathematical graphs and equations in the assessment material.

**Keywords**—engineering mathematics; distance learning; online assessment

## I. INTRODUCTION

Distance learning is referred to as “formalized instructional learning where the time/geographic situation constrains learning by not affording in-person contact between student and instructor” [1]. This method of education, sometimes called as open learning, has more than a 170-year history and was invented by Isaac Pitman in the middle of the 19th century. In 1844, he was the first educator in the UK who suggested his course on shorthand via post correspondence [2]. Although it was a simple training aimed to develop practical skills, the course delivery included fundamental educational components such as assignments and feedback. As for higher education, the University of London became the pioneer in HE distance learning shortly after Isaac Pitman's innovation. The university has adopted this method in 1858 to offer an external degree where “individuals prepared themselves for the University's examinations” [3].

The technology of distance learning in the early years was fully dependent on postal service. The industrial revolution and improvement in transportation in the second half of the 19th century dramatically reduced the postage cost and mail delivery times. Due to fast and cheap commercial correspondence many universities, colleges, and schools nationally and across the world implemented distance learning methods offering various courses and degrees. This approach made education accessible for people who could not attend the institutions on a daily basis because of geographical, work or other reasons. Distance learning in the form of correspondence study existed for a long period of time up to the invention of new technologies of communication like radio, telephone and TV [4]. However, the biggest impact on the distance learning

technology has been made by digital communication and the internet in particular.

Today, the biggest provider of distance learning in the UK is the Open University. The Open University was established in 1969 to reflect the growing demand for higher education from the people across the country who could not study for a HE qualification as full-time or part-time students [5]. The first students to be enrolled on university distance learning programmes was in 1971. Since then the university has educated more than two million students and became the largest teaching institution in the UK. About 190,000 students have participated in studies at the Open University in the academic year 2013-2014 [6]. “The OU's founding mission is to open up higher education to people who previously might have been denied access for lack of prior formal qualifications” [7].

## II. MATHEMATICS IN THE CONTEXT OF ENGINEERING EDUCATION

Engineering and applied science is mainly based on two science subjects: Physics and Mathematics. In fact, both subjects significantly interact and underpin each other to provide practical knowledge in the appropriate areas of applied science and technology. The progress in science and technology during two past centuries has made modern engineering a very complicated discipline requiring advanced level mathematics. Paul [8] stated that “Engineering can be characterized as the mathematical modelling of physical systems for the purpose of design. Hence engineers must be, in a sense, applied mathematicians.” Therefore, the mathematics for engineering purposes or, in other words, engineering mathematics becomes a crucial component of applied science and technology providing a foundation and methodology to the subject.

Another reason of the growth of mathematics in engineering subjects is the advances in computer technologies that have occurred in a few past decades. Modern computers can provide a very fast and effective computational analysis of a mathematical problem which were impossible to solve in the past. Due to significant improvements in computational resources, extremely complicated engineering problems are attempted to be solved today that requires engineers to have an advanced level of knowledge in engineering mathematics.

On the other hand, since the middle of the 19th century, when engineering became a profession, the number of professional engineers has dramatically increased expanding engineering education at different levels across the world. To reflect the growing demand in professional engineers, engineering education has employed all existing methods of education including distance learning. Therefore, engineering mathematics is a core component of engineering and applied science and an essential subject of engineering education including distance learning.

### III. ASSESSMENT AND FEEDBACK IN DIGITAL LEARNING ENVIRONMENT

Delivering engineering mathematics through electronic means such as the internet in the framework of distance learning is very different in comparison to conventional ways of teaching and learning. The main problem of distance learning in engineering is the limitation of forms of assessment. In fact, distance learning acting in the digital environment employs almost the same technologies used for both formative and summative assessments. Under the assessment procedure a student usually passes through the same testing methods but can have different outcomes. A test related to formative assessment usually referred to as diagnostic test provides an indication of a student's learning activity only [9]. A summative assessment test gives results on achievement of learning outcomes. Feedback is "commonly used in academia to refer to any type of information given to students on assessment performance" [10]. Feedback obtained from both formative and summative assessment tests in the digital environment is usually given in terms of scores or marks etc. This type of feedback is appropriate for summative assessment but insufficient for formative assessment where a student needs to have qualitative feedback.

Therefore, the disadvantage of distance learning based on a digital learning environment is that the methods of assessment are depleted and formalised while feedback could be quite poor. On the other hand, if an automated learning system is implemented the feedback in the digital environment are generated immediately [11],[12].

### IV. VIRTUAL LEARNING ENVIRONMENT MOODLE

Many UK universities have adopted Moodle as a Virtual Learning Environment (VLE) to provide learning support to full time and part time students. Moodle is abbreviation of Modular Object-Oriented Dynamic Learning Environment developed in 2002 "in Australia as a free alternative to commercial learning management systems" [13]. The VLE has replaced "Blackboard" previously implemented in many UK universities for resource purposes mainly. Walker et al. [14] stated that Moodle is currently leading VLE across universities and colleges in the UK and "can be used in many types of environments such as education, training and development" [14]. Pina [13] reported "the number of registered Moodle sites at over 35,000, with over 2 million courses and over 25 million users - more than any other" VLE.

Today Moodle is available for academic staff involved in teaching and learning support. Designed originally for

resource and learning management purposes Moodle now includes many technologies like Turnitin and Quizzes etc. which can be used as powerful tools in the context of distance learning. The latest versions of Moodle provide functioning of the most common pedagogical tools: (1) content creation, (2) communication, (3) assessment, and (4) administration [13].

The Moodle technologies used for student support and support for part-time students in particular are exactly the same as that used for distance learning. This is why the assignment approaches, assessment technologies and feedback methods developed to provide on-line support, for part-time students for example, can be also applicable for distance learners.

Another important feature of Moodle is the provision of communication between students and between students and academic staff. This is a very important VLE function for distance learners because "mathematical learning can be frustrating and slow when working in isolation, especially for those who struggle with mathematics in a traditional classroom" [15].

### V. ONLINE ASSESSMENT TECHNOLOGIES

As mentioned above, a very important feature of online education in the digital environment is an automated assessment. For example, The Open University declared that the "areas for further improvement include the automatic assessment of student assignments" [7]. The advantage of automated assessment is that it can provide immediate feedback to students. However, as discussed, the assessment methods of on-line assignments usually are very poor due to the nature of electronic interaction between the automated learning system and student [10]. This is why the assessment of on-line tasks on mathematic subjects is limited and represented as following [16]:

- true/false or yes/no;
- calculated numerical answer;
- multiple choice including multiple select (several possible correct answers);
- matching;
- fill-in-the-blank.

"While many of these problems can be structured in a way to use the above types of questions, some of the more interesting and important problems cannot easily be rendered in this way" [16]. The mentioned above assessment methods are widely used in a number of on-line educational and resource systems such as Moodle, WebAssign, WeBWork, LON-CAPA, etc.

Any automated assessment system requires a tutor to provide a bank of questions in advance. The system generates an assignment comprising a particular number of questions from the bank. The questions for the assignment can be selected randomly or using a predefined algorithm.

**Moodle.** Academics basically use Moodle as a repository of learning materials, submission points of assignments and tool for communication with students. However, Moodle has many features that can be used for distance learning. A very

useful component of Moodle in the context of distance learning assessment is Quizzes. The Moodle Quiz module is designed to provide online learning activity “consisting of a large variety of Question types” [17]. In terms of engineering mathematics, Quizzes are a very useful component of the VLE having properties of math, equation typing and numerical input. Quizzes based assignments can be assessed using all the mentioned above assessment methods.

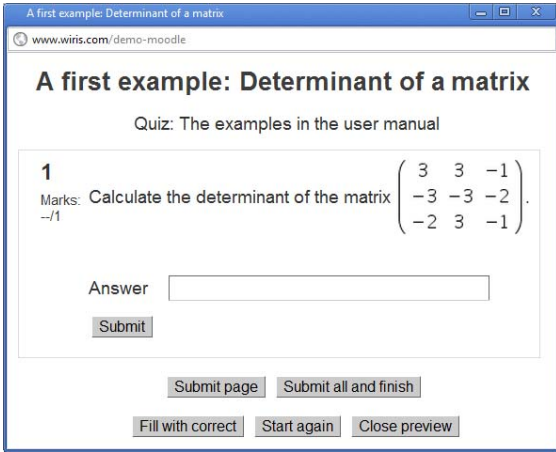


Fig. 1. Example of Quiz question in Moodle [18]

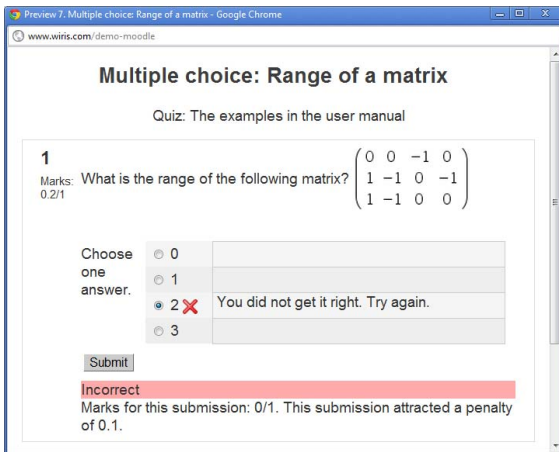


Fig. 2. Example of Moodle Quiz feedback [18]

Fig. 1 shows an example of Moodle Quiz question on matrix. The task is designed to use a “fill-in-the-blank” method of assessment. Fig 2 demonstrates an example of Moodle Quiz feedback obtained from a multiple-choice question.

In order to prevent student plagiarism, all Quiz based questions must not be repeated. However, the design of large number of Quiz questions is quite time consuming and this is why it is impossible to create the full set of tasks using unique questions. Instead of the set of unique tasks the Moodle Quiz module can create the set of repeated tasks having unique attributes. It means that the tasks can be the same in terms of question formulation but the constant parameters in the task can be different leading to different solutions. The variation of constant parameters can be generated randomly within installed boundaries. Under this approach the tutor needs to design only a certain number of questions related to

appropriate sections of learning material. This approach also significantly reduces the time spent for the Quiz questions preparation and improves the efficiency of teaching.

The discussed approach is a very common solution in the area of education in digital the environment including distance learning. For example, Trussell [16] stated “that the parameters of the mathematical problems can be randomized in such a way that students are given different problems. This prevents simply copying, but also allows the students to ask for additional variations on the same problem, which can be used for practice.”

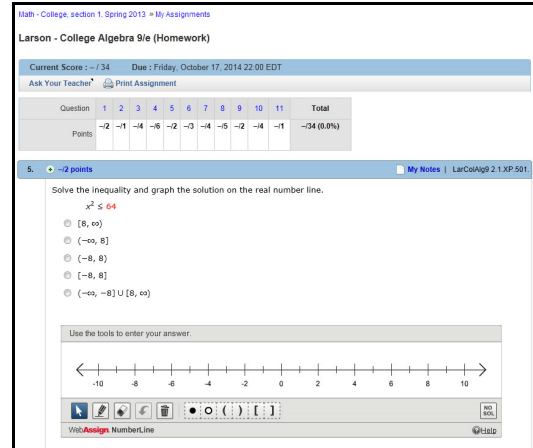


Fig. 3. Example of WebAssign online task [22]

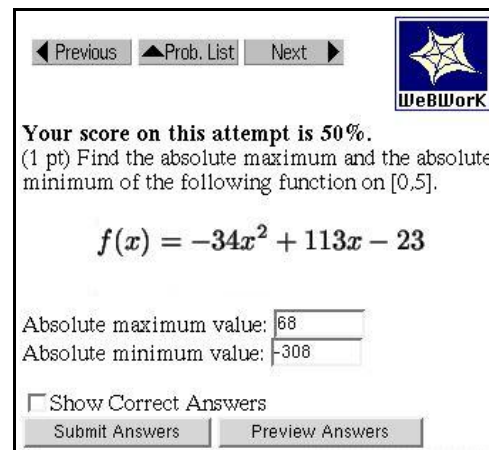


Fig. 4. Example of WeBWork task [24]

**WebAssign.** Another online assessment tool of math related subjects is WebAssign defined as “a powerful online instructional system designed by educators to enrich the teaching and learning experience. WebAssign provides extensive content, instant assessment, and superior support” [19]. WebAssign has been created in 1997 in North Carolina State University and currently very popular across the USA. Today many publishers of text books have established partnerships with WebAssign. “WebAssign loads textbook end-of-chapter questions into its system so teachers can assign homework and students can go online and do the work. Teachers also can load their own questions into the computer” [20]. For example, the math text book “Applied Calculus” published by Cengage Learning provides 1193 questions

available online through the WebAssign platform [21]. Fig. 3 shows an example of WebAssign online task based on the method of multiple choice question.

**WeBWorK.** This assessment tool is defined as “an open-source online homework system for math and sciences courses” [23]. The WeBWorK software was developed in 2007 in the University of Rochester. The advantage of this tool is that the current maintenance and operation are supported by Mathematical Association of America and National Science Foundation. “Supported courses include college algebra, discrete mathematics, probability and statistics, single and multivariable calculus, differential equations, linear algebra and complex analysis” [23]. Another advantage is that this tool is distributed under GNU General Public License and can be available free of charge. Fig. 4 shows an example of a problem to be assessed using the WeBWorK tool. The example demonstrates the “fill-in-the-blank” assessment technique.

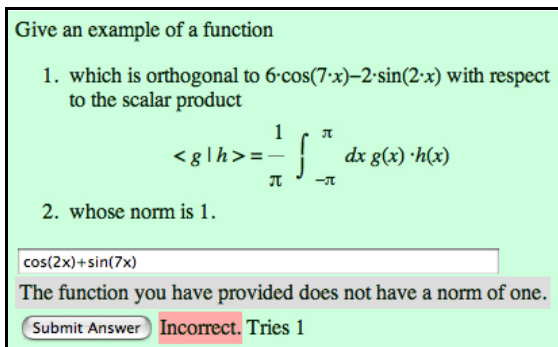


Fig. 5. Example of LON-CAPA task [27]

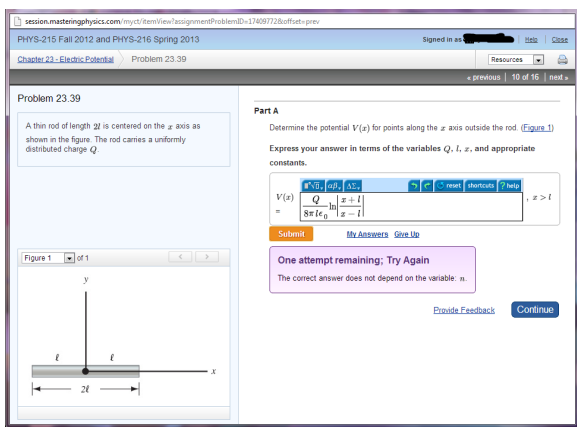


Fig. 6. Example of MasterEngineering task [28]

**LON-CAPA** is abbreviation of Learning Online Network with Computer-Assisted Personalized Approach referred to as “a full-featured course management, learning content management, and assessment system” [25]. This e-learning platform is also known as a Course Management System (CMS) or Learning Management System (LMS) [26]. LON-CAPA can be considered as a network having more than 160 domains across the world where every domain is operated by a university or college. The platform is also an open source, free of charge software. An example of task in LON-CAPA platform used a “fill-in-the-blank” assessment method is shown in Fig. 5.

**MasterEngineering.** This assessment tool belongs to the publisher Pearson Higher Education. The tool is integrated into the publisher's online text books and provides “collection of online homework, tutorial, and assessment products designed with a single purpose in mind: to improve the results of higher education students” [28]. An example of a math related task in the MasterEngineering environment is shown in Fig. 6.

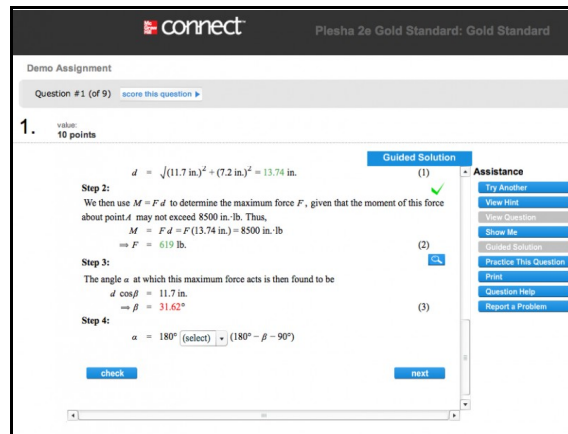


Fig. 7. Example of ConnectEngineering task [29]

**ConnectEngineering** is designed by publisher McGraw-Hill to provide learning support for their online text books. The publisher declares that ConnectEngineering is “innovative, adaptive technology which engages students in the course content” [29]. Fig. 7 demonstrates an example of ConnectEngineering assignment.

## VI. GRAPHS AND EQUATIONS IN MATH ASSIGNMENTS

Graphs and equations are very important for the presentation of any math text. During early years of IT both components were very difficult to incorporate into computerised math texts due to their graphical nature. In the past the equations have been written using a pseudo-text description adopted from various programming languages. Table I below gives some examples of math equation descriptions used in the period of time of text interfaces domination. As can be seen from the table the equations presented using pseudo-text description are very difficult in understanding by an inexperienced reader.

It was even more difficult to incorporate graphics in math text to be displayed on the monitor screen using text characters. Fig. 8 shows two graphs of the same function: the graph above has been drawn using text characters to be displayed on the screen of old fashion monitor; the graph below is designed to be shown on a modern display with graphical interface.

In fact, poor presentation of graphs and equations, essential components of any math text, made unrealistic propagation of science and technology text using digital media during a few early decades of the electronic revolution. Therefore, in the past, it was impossible to use math related texts for distance learning purposes via the first versions of the digital learning environment having no graphical capabilities; the students might not understand the tasks and give correct answers. This

is why mathematics was outside the process of distance learning development and implementation in the beginning of the digital era.

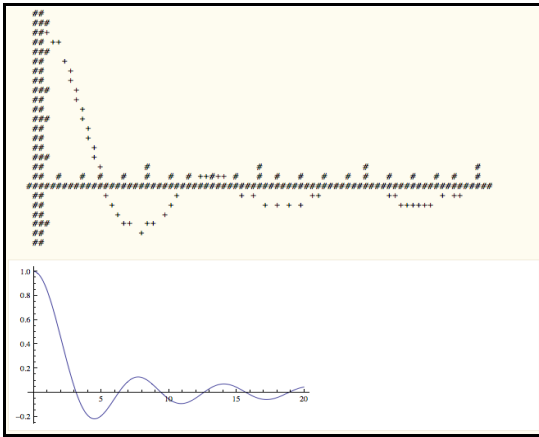


Fig. 8. The graph presented using text letters (above) and graphic interface (below) [30]

TABLE I. MATHEMATICAL EQUATION DESCRIPTIONS USING TEXT CHARACTERS

$y = 12x^{2t} (3x + 8)$	<code>y=12*x^(2*t)*(3*x+8)</code>
$z = 6e^{2t} \cos(\omega t + 180)$	<code>z=6*exp(2*t)*(omega*t+180)</code>

A significant improvement in the quality of electronic texts' reproduction brought the mathematics in the line of other educational subjects being already delivered via distance learning using digital technologies. In term of presentation, today there are no barriers and limitations for math and related subjects to be taught and learnt through the means of digital technologies. VLE assignments have exactly the same layout and presentation as it would be published in a paper text book. These assignments completely excluded misunderstandings becoming clear and familiar for students. The assignments also reflected students' expectations and improved students' experience.

Hartley [31] stated that the presentation of any material for distance learning should follow two rules: (1) "the layout should always be clear and simple, and not confusing" and (2) "... the detailing of typography should always be consistent" through whole document. Moreover, it has been found that the "paper based teaching material [for distance learners] may look old fashioned but is often preferred by the students" [32].

### VII. UK PROVIDERS OF MATHEMATICS VIA DISTANCE LEARNING

Many universities in the UK offer various programmes via distance learning including degrees and certificates in the areas of science, engineering and technology where the subject of mathematics is a key component. The biggest provider of math via distance learning in the UK is the Open University. The university portfolio includes a number of math related degrees and modules incorporated into other programmes. The list of math related Degrees offered by the Open University is given below.

- BSc (Honours) Mathematics and Statistics

- BSc (Honours) Economics and Mathematical Sciences
- BSc (Honours) Mathematics
- BSc (Honours) Mathematics and Physics
- BSc (Honours) Mathematics and its Learning

As for math modules within various programmes the Faculty of Mathematics, Computing and Technology provides through the website MathsChoices the guidance on math modules content and gives the advice regarding an appropriate choice of the modules in terms of level of study and required qualification.

Another big provider of distance learning in the UK is the University of London. As it mentioned above, the university was the first HE institution that started teaching students using distance learning. Today the university offers degrees and modules through distance learning mainly to international students. However, the university portfolio on distance learning has no Engineering or Math related degrees and just a few math related modules.

University of Leicester is the biggest provider of post graduate education via distance learning. Although the university offers an MSc in Actuarial Science, a math based degree, the majority of distance learning provision is not math related.

According to Complete University Guide [33], other significant providers of distance learning in the UK are University of Liverpool, University of East London, University of Essex, Edinburgh Napier University, and University of Derby. This list cannot be completed because almost every university in the UK is intensively working in the area of development of programmes via distance learning.

Apart from provision of conventional programmes via distance learning many universities are currently involved in another activity based on methods of distance learning. This activity is called Massive Open Online Courses (MOOCs).

### VIII. MASSIVE OPEN ONLINE COURSES

A few years ago, MOOCs were introduced to provide free of charge online programmes on a variety of subjects including mathematics. The MOOC programmes are usually delivered by top universities and include online lectures, practical exercises and assessments. Some programmes can provide interaction between tutor and students. Although the content of most of MOOCs are at HE level, the completion of a MOOC does not lead to a degree. The main player in the area of MOOCs is an education platform "Coursera" that offered, for example, 12 math related programmes in 2015.

It is evident that very respectable, high ranked universities are involved in delivery of math related subjects. It means that they consider mathematics as an important and essential component of HE education and engineering education in particular.

A UK based MOOCs platform is called FutureLearn. This education platform is owned by the Open University and operated by a partnership of 24 UK universities and a few

universities from Australia, New Zealand, South Africa and Ireland. It has been reported that 1,245,854 people have joined FutureLearn since September 2013 [34]. Although FutureLearn offers a wide range of on-line courses there are currently no math specialised programmes in the platform portfolio.

## IX. CONCLUSION

It has been proven that mathematics is a key component of courses on engineering, science and technology. In the context of distance learning the subject of engineering mathematics can be delivered through a digital learning environment using virtual lectures, tutorials and assignments. The assessment within VLE can be automated using a variety of software to provide immediate feedback to students. The most popular automated assessment technologies have been discussed to highlight the advantages and disadvantages. The distance learning and MOOCs providers in the UK have been overviewed and discussed.

It is obvious that further progress in distance learning depends on development of information technologies including digital communication and human-machine interface technologies. The most important components on online education required special attention in terms of improvement and development are assignments design, assessment technologies and feedback efficiency.

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