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Fast cancellation of sidelobes in the pattern of a uniformly excited array using external elements

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Abstract

A method for wide null steering in the pattern of a uniformly excited linear array that utilizes the edge elements of the array is investigated. Simpler and faster algorithms for sidelobe reduction are introduced which use one or two external edge elements. Comparisons between these methods are held. Sample results are given.

Keywords

null steering, external edge elements, algorithms, sidelobe reduction

Disciplines

Computer Engineering | Computer Sciences

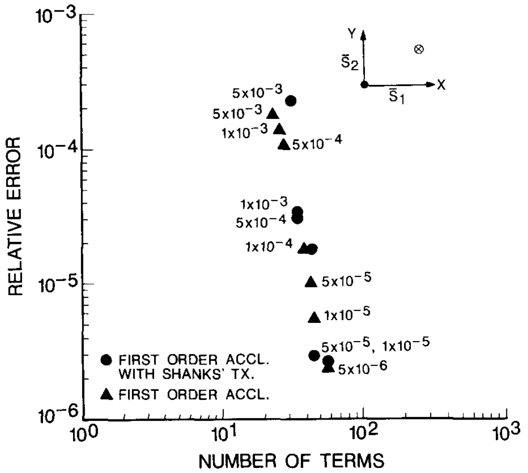

Fig. 8. Relative error versus number of terms of spatial sum, $|\bar{s}_1| = |\bar{s}_2| = 1.1\ \lambda$, $\bar{r} = 0.91\bar{s}_1 + 0.91\bar{s}_2$, $u = 2.856$, $m_0 = n_0 = 0$.

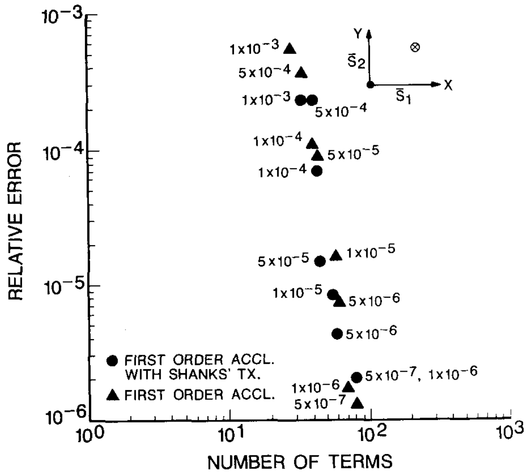

Fig. 9. Relative error versus number of terms of spatial sum, $|\bar{s}_1| = |\bar{s}_2| = 1.1\ \lambda$, $\bar{r} = 0.72\bar{s}_1 + 0.72\bar{s}_2$, $u = 2.856$, $m_0 = n_0 = 0$.

function of the number of terms required to achieve a specified degree of convergence. In each case the "error" is computed by comparing the result to a series evaluation which has been computed to machine precision. The convergence criterion ϵ_c is indicated alongside each point of the figures. It is evident from the results that the first order acceleration of the series with the Shanks' transformation converges faster or at least as fast as the first order acceleration alone in each case. It is interesting to note, as can be seen from Figs. 2 and 4, that as the observation point approaches the source point, the spectral sum converges faster.

Next we look at the convergence of the spatial sum, that is the last series in (23). The relative error versus the number of terms for the accelerated series with and without applying Shanks' transform is shown in Figs. 8 and 9. We find that for the spatial sum, the two methods yield convergence within approximately the same number of terms. Also the relative position of the source and observation points seems to have little effect on the convergence.

It is found that the choice of the smoothing parameter u has a more dramatic impact on the rate of convergence of the spatial sum than it does on the spectral sum. This is obvious from the expression for the spatial sum which has exponential decay proportional to u. A reasonable choice of u which seems to ensure good convergence for both the spatial and the spectral sum in (23) is about half the size of the maximum reciprocal lattice base vector.

Numerical experiments reveal that the rate of convergence of the series does not depend significantly upon the interelement phase shift constant m_0 and n_0. It should be pointed out that a summation of the unaccelerated series in (15) typically takes more than 10^4 terms for a convergence criterion of $\epsilon_c = 0.001$. By contrast, the accelerated series typically converges in less than 200 terms with the same criterion. Hence the acceleration methods provide considerable savings in computation time.

Finally, it has been observed previously [8, p. 372] that Shanks' transform is sensitive to round-off error. However, for the computations in this work the problem of round-off error was not encountered for the range of convergence factors used.

References

[1] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1965.

[2] W. F. Richards, "The Green's function for an infinite periodic array of transverse dyadic dipoles in free space," personal notes, July 1982.

[3] J. R. Zinecker, "Efficient computation of Green's functions for periodic structures," Master's thesis, Dept. Elec. Eng., Univ. Houston, Houston, TX, 1985.

[4] R. Bracewell, *The Fourier Transform and Its Applications*, 2nd ed. New York: McGraw-Hill, 1978.

[5] D. Shanks, "Non-linear transformations of divergent and slowly convergent sequences," *J. Math. Phys.*, vol. 34, pp. 1-42, 1955.

[6] P. Wynn, "On a device for computing the $e_m(S_n)$ transformation," *Math. Tables Aids to Comp.*, vol. 10, pp. 91-96, 1956.

[7] B. A. Munk and G. A. Burrell, "Plane-wave expansion of arrays of arbitrarily oriented piecewise linear elements and its application to determining the impedance of a single linear antenna in a lossy half-space," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 331-343, May 1979.

[8] C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*. New York: McGraw-Hill, 1978.

Fast Cancellation of Sidelobes in the Pattern of a Uniformly Excited Array Using External Elements

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Abstract—**A method for wide null steering in the pattern of a uniformly excited linear array that utilizes the edge elements of the array is investigated. Simpler and faster algorithms for sidelobe reduction are introduced which use one or two external edge elements. Comparisons between these methods are held. Sample results are given.**

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I. Introduction

Phased arrays are expected to have significant applications in future small earth stations [1]. In many applications, interfering signals could be incident from some directions. These could be accidently generated, or could emanate from jammers: for brevity, the sources of such signals will be referred to here as jammers, whether the interference is deliberate or not. Instead of creating a null in the exact direction of the unwanted signal only, it is sometimes more convenient to cancel, or at least reduce, the sidelobe into which the unwanted signal is coming. Such a class of arrays is the class of sidelobe cancellers. Sometimes only a few elements of the array are arranged to be controllable. Full control could be prohibitively expensive in many applications and may raise reliability problems. In a previous paper [2], a simple algorithm for canceling specific sidelobes, using the edge elements alone, was presented. In this communication, this algorithm is improved by appending two external elements to the array. The new algorithm is found to be faster and simpler than that of [2]. Another improvement suggests the use of only one external element. Comparisons between these methods are held. Sample results are given.

II. Cancellation by Two External Elements

Consider a linear array with N isotropic elements separated by equal intervals of size $h = \lambda/2$. Assume a uniform excitation function such that $w(n) = 1$ for all elements. Take the center of the array as a reference point.

The edge elements of the array produce a sine pattern with almost the same periodicity as the sidelobes of a uniformly excited array as discussed in [2]. In situations when the periodicity is not exactly the same as the periodicity of the array sidelobes, null steering is actually obtained instead of true sidelobe cancellation.

Now consider the closed form of the pattern of the uniformly excited array ($\lambda/2$ spacing):

$$f_a(\theta) = \frac{\sin \dfrac{\pi N \sin\theta}{2}}{\sin \dfrac{\pi \sin\theta}{2}}. \tag{1}$$

The main beam occurs when both the numerator and the denominator are zero. The nulls of the pattern, however, occur at the other zeros of the numerator, i.e., at

$$\frac{\pi N \sin\theta}{2} = \pm n\pi, \qquad n = 1, 2, \cdots$$

or, in other words:

$$\sin\theta_n = \pm\frac{2n}{N}, \qquad n = 1, 2, \cdots.$$

The maxima of the pattern (the sidelobes) occur, approximately, at the peaks of the numerator:

$$\sin\theta_m = \pm\frac{2m+1}{N}, \qquad m = 0, 1, 2, \cdots.$$

A. The Algorithm

The above analysis suggests that: an interferometer pattern, properly scaled, may be used for sidelobe cancellation purposes. This pattern is, simply, the cosine function formed by two elements placed at distances equal to half the spacing between the other elements outside the array, i.e., $h/2$, and phase-shifted by $-\pi/2$.

A simple search algorithm can be used to determine the sidelobe containing the angle θ_{int}.

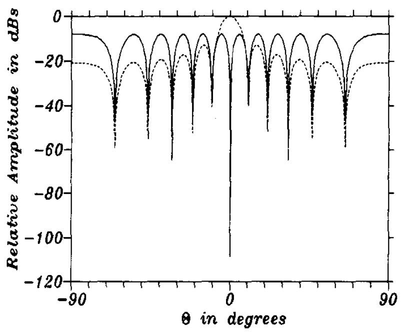

Fig. 1. The pattern of the external elements (solid curve), compared with the pattern of a uniformly excited 11-element array (broken curve).

Similar to the algorithm of [2], the angle θ_m of the sidelobe's center is first determined. Then, by applying a cancellation signal with conjugated phase shifts of $\pi/2$ and $-\pi/2$ to the first and the last elements, respectively, a superimposed interferometer pattern is created whose peaks coincide, approximately, with all of the peaks of the sidelobes and whose zeros coincide, exactly, with the nulls of the array pattern as shown in Fig. 1. By scaling the amplitude of the sine pattern by a factor C, say, so that it is equal in magnitude to and in antiphase with the array pattern at θ_m, the sidelobe in question would be canceled.

The arithmetic involved in the above procedure is very much simpler than that of [2] and can be summarized as follows:

1) The maxima of the sidelobes occur, approximately, at angles θ_m, where

$$\sin\theta_m = \pm\frac{2m+1}{N}, \tag{2}$$

N is the total number of elements in the array, and m, an integer such that $1 \le m \le (N-1)/2$, is the index of the sidelobe to be canceled.

2) The corresponding maxima f_m can be computed from (1)

$$f_m = \frac{\sin\left[\pi\dfrac{2m+1}{2}\right]}{\sin\left[\dfrac{\pi}{2}\dfrac{2m+1}{N}\right]}. \tag{3}$$

3) Apply conjugated phase shifts of $-\pi/2$ and $\pi/2$ to the cancellation signal of amplitude C fed to the external edge elements at $Nh/2$ and $-Nh/2$, respectively. The cancellation pattern is then given by

$$\begin{aligned} f_c(\theta_m) &= Ce^{(j(\pi/2)(2m+1)-\pi/2)} + Ce^{-(j(\pi/2)(2m+1)-\pi/2)} \\ &= 2C\sin\left((\pi/2)(2m+1)\right). \end{aligned} \tag{4}$$

4) At θ_m,

$$f_c(\theta_m) = -f_m. \tag{5}$$

The locations of the peaks of the sine pattern almost coincide with those of the sidelobes, therefore,

$$\begin{aligned} C &= f_c(\theta_m)/2 \\ &= -f_m/2 \end{aligned} \tag{6}$$

i.e., the excitation of each of the external edge elements is minus half the magnitude of the sidelobe to be cancelled.

Swapping the signs of the phase shifts and $Nh/2$ and $-Nh/2$ results in canceling the lobe at $-\theta_m$.

The result of applying this simple algorithm is shown in Fig. 2 where the third sidelobe is canceled.

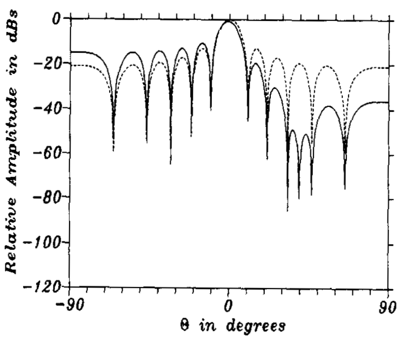

Fig. 2. The third sidelobe of a uniformly excited array with 11 elements canceled by two external edge elements (solid curve), compared with the original pattern (broken curve).

II. Sidelobe Cancellation by One External Element

It was suggested in Section II that two elements, placed at distance $h/2$ beyond the two edges of the array, can be used to form an interferometer which generates the cancellation pattern. It is clear that the pattern of such an interferometer depends on the separation, which is Nh in the present case, between the elements forming it.

Now, instead of using two external elements, one of the actual edge elements of the array together with only one external element, placed at a distance h beyond the other end can be used to produce a similar effect. The arithmetic involved in this case is slightly more complicated than the previous case.

A. The Algorithm

Similarly to the previous case, assume an array with N elements whose center is the reference point and the separation between elements is $h = \lambda/2$. A simple search algorithm can be used to determine which sidelobe contains the angle θ_{int}. Then, θ_m, the angle of the sidelobe center, is determined by (2). Let an external element be located at $(N+1)h/2$. The cancellation pattern f_c formed by the external element and the edge element at $-(N-1)h/2$, is given by

$$f_c(\theta) = Ce^{-j(\pi/2)(N-1)\sin\theta + j(\pi/2)} + Ce^{j(\pi/2)(N+1)\sin\theta - j(\pi/2)}$$
$$= Ce^{j(\pi/2)\sin\theta}\left(e^{-j(\pi/2)N\sin\theta + j(\pi/2)} + e^{j(\pi/2)N\sin\theta - j(\pi/2)}\right). \tag{7}$$

Therefore,

$$f_c(\theta) = 2Ce^{j(\pi/2)\sin\theta}\sin\left(\frac{\pi N}{2}\sin\theta\right). \tag{8}$$

Now, at θ_m:

$$f_c(\theta) = -f_m$$
$$= e^{j\pi}f_m \tag{9}$$

where m, the index of the sidelobe to be canceled, is an integer such that $1 \le m \le (N-1)/2$.

Therefore, from (8), (9), and (3):

$$C = \frac{e^{j\pi - j(\pi/2)(2m+1)/N}}{2\sin\left[\frac{\pi}{2}\frac{2m+1}{N}\right]}. \tag{10}$$

Now, f_c is superimposed with the original excitation of the array. This means that the excitation of the external element is $Ce^{-j\pi/2}$ and the new excitation of the edge element is $1 + Ce^{j\pi/2}$.

Cancellation of sidelobes where θ is negative is computed by substituting $-(2m+1)/N$ for $(2m+1)/N$ in (10).

The plots of applying this algorithm are very similar to those described in Section II.

IV. Comparison of the Methods

The methods described in [2] and the current paper share many common features. All support the concept of partial adaptivity without greatly affecting the main lobe gain since only two elements of the whole array are used. Additionally, the individual nulls produced are deep and wide enough to accommodate frequency fluctuations, usually overcome in conventional techniques by placing two adjacent nulls in the radiation pattern [3].

However, the most important advantage of the algorithms that use two elements is that they are suitable for real-time implementation, since, knowing the excitation fluctuation and the number of elements of the array, the values of the required excitations can be computed in real time by the closed form (1). Alternatively, these values can be precomputed and stored in short look-up tables as their number is less than half the total number of elements in the array. This means that, once the sidelobe to be canceled has been determined, the weights (excitations) of the (external) edge elements may be issued immediately without delays due to matrix operations or iterations. For a nonsteerable array these algorithms also give a great reduction in the number of RF devices required. For steerable arrays all elements must have active devices in the feeds, but the algorithm is simpler than that for full control because it involves direct control for the edge elements alone whereas the interior elements may be controlled indirectly using PROM's.

The algorithm which uses external elements is better than that of [2] in many respects. For instance:

1) Using the actual edge elements effectively results in wide null steering whereas using external elements results in sidelobe cancellation (or reduction).
2) The second algorithm requires full control of both the amplitude and phase of feeds to the edge elements whereas the first algorithm requires control of the amplitude only while the phase remains fixed (just $\pi/2$ and $-\pi/2$).
3) Processing the weights for the external elements is very easy (half the maximum of the sidelobe in question) whereas the weights of the edge elements in the second algorithm require further processing and time consuming evaluation of additional mathematical functions (e.g., the square root and inverse trigonometric functions).

The only advantage of the second algorithm over the first is that the edge elements are used during both the normal operation and cancellation modes whereas the external elements remain idle during the noncancellation mode in the first algorithm.

On the other hand, one external element produces very similar results to those produced by two external elements. However, the amplitudes of the relevant elements are not equal as the case in the other algorithms. Furthermore, the phases applied to these elements are no longer conjugates.

V. Conclusion

The concept of using the edge elements adds some physical insight to the contribution of the different parts of the array to the radiation pattern. comparing the procedure of sidelobe cancellation by the actual edge elements with that by external edge elements, it is found that using external elements is easier (because it requires fewer mathematical operations) with reduced hardware requirements.

REFERENCES

[1] K. A. Al-Mashouq, I. El-Azhary, M. S. Afifi, A. S. Al-Ruwais, and P. S. Excell, "Multiple nulls formation and low interference of future arrays for small earth stations," in *Proc. ARABSAT Symp.*, King Saud Univ., Riyadh, Saudi Arabia, 1987, pp. 434-443.

[2] I. El-Azhary, M. S. Afifi, and P. S. Excell, "A simple algorithm for sidelobe cancellation in a partially adaptive linear array," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1484-1486, Oct. 1988.

[3] P. Owen and J. C. Mason, "The use of linear programming in the design of antenna patterns with prescribed nulls and other constraints," *Int. J. Comput. Math. Elec. Electron. Eng.*, vol. 3, no. 4, pp. 201-215, 1984.

Exact Analysis of Radiation Patterns Using the Expansion of the Fourier Sum

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Abstract—**The time required to compute radiation patterns of linear arrays, given in a form of a Fourier sum, depends on the number of array elements. In this communication, a new fast algorithm for computing Fourier sums is presented. The radiation pattern, given by this sum, can be replaced by an infinite series whose terms depend on the envelop of the excitation function, $w(x)$, and its derivatives at the edges of the linear array. In cases when $w(x)$ has a few nonzero derivatives, this infinite series can be replaced by a finite sum which can be evaluated faster than the original Fourier sum. This makes this new method more suitable for real-time applications. The effect of critical point is also investigated. Some sample case studies are included.**

I. INTRODUCTION

The radiation pattern of a linear array is given by a Fourier sum of the form:

$$f_a(\theta) = \sum_{i=0}^{N} w(a + ih) e^{jk(a+ih)\sin\theta}$$

where the excitation function $w(x)$ is a real analytic function, $k = 2\pi/\lambda$ is the wavenumber, and λ is the wavelength.

The time required to obtain the above sum, in different directions (θ), by computing the terms and adding them up, is proportional to the number of the array elements $N + 1$.

In this communication, a new fast algorithm for computing the above Fourier sum is presented. The radiation pattern $f_a(\theta)$, given by this sum, can be replaced by an infinite series whose terms depend on the envelope of the excitation function w and its derivatives at the edges of the linear array.

Such an expression represents a closed form for the radiation patterns for a wide class of excitation functions. The new method makes the evaluation of radiation patterns more convenient for real-time applications in the sense that it requires fewer operations. The effect of critical points in the excitation function is investigated. A few case studies of specific excitation functions are included.

II. FAST ALGORITHM FOR COMPUTATION OF PATTERNS OF LINEAR PHASED ARRAYS

Let an array with $N + 1$ elements be located along the x-axis with the zeroth element at point a. The spacing between the elements is h, and the excitation of the nth element, located at $x_n = a + nh$, is given by the real number $w(x_n) = \delta(x - x_n)w(x)$, where $w(x)$ is a nonnegative real analytic function and $\delta(x)$ is the Kronecker delta function given by

$$\delta(x) = \begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{otherwise.} \end{cases} \qquad (1)$$

Then, the array factor, $f_a(\theta)$, is given by the Fourier sum:

$$f_a(\theta) = \sum_{n=0}^{N} w(x_n) e^{jkx_n \sin\theta} = \sum_{n=0}^{N} w(a + nh) e^{jk(a+nh)\sin\theta}. \qquad (2)$$

Direct computer methods may be used to compute $f_a(\theta)$ in any direction θ. The time required to compute the above sum, term by term, increases with the number of the array elements $N + 1$. The problem is even worse if computation of the pattern in more than one direction is required.

A. The Expansion of the Fourier Sum

Using the Euler–Maclaurin sum formula (see [1], [5]) together with the expression for asymptotic expansions for the Fourier integral (see [3], [4]), it can be proved that

$$\begin{aligned} f_a(\theta) &= \sum_{n=0}^{N} w(a + nh) e^{jk(a+nh)\sin\theta} \\ &= \frac{1}{2}\left(w(a + Nh) e^{jk(a+Nh)\sin\theta} + w(a) e^{jka\sin\theta} \right) \\ &\quad + \frac{1}{2j} \sum_{m=0}^{\infty} \left(\frac{h}{2j} \right)^m \frac{1}{m!} \frac{d^m}{dt^m} \cot t \Big|_{t=\frac{kh\sin\theta}{2}} \\ &\quad \cdot \left[w^{(m)}(x) e^{jkx\sin\theta} \right]_{x=a}^{x=a+Nh} \end{aligned} \qquad (3)$$

that is, the finite sum can be replaced by an infinite series whose terms depend on the excitation function $w(x)$ and its derivatives at the edges of the linear array. Equation (3) will be called, hereafter, the expansion of the Fourier sum.

B. The Expansion as a Fast Algorithm

The expansion of the Fourier sum provides a fast algorithm for computing the radiation pattern levels of linear phased arrays due to a wide class of excitation functions. For instance, the polynomial-tapered excitation functions have zero derivatives for all $m \geq M$ for some integer M and, hence, the infinite series can be replaced by a finite series which gives the exact sum with a number of terms ($M + 1$ terms) very much less than the original sum ($N + 1$ terms) in the middle expression of (3).

III. THE EFFECT OF CRITICAL POINTS

It has been assumed until now that the excitation $w(x)$ is an analytic function. Consider now an excitation function with critical points, i.e., points at which the excitation function $w(x)$ is continuous but the derivatives do not exist. So, let $w(x)$ have a critical point at $c = N_1 h$ such that

$$w(x) = \begin{cases} w_1(x), & \text{if } a \leq x \leq c = a + N_1 h \\ w_2(x), & \text{if } c \leq x \leq c + N_2 h = a + Nh \\ w_1(x) = w_2(x), & \text{if } x = c \end{cases} \qquad (4)$$

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