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A comparative analysis between two statistical deviation–based consensus measures in Group Decision Making problems

M. J. del Moral[a], F. Chiclana[b], J. M. Tapia[c]*, C. Tapia–Garcia[d], E. Herrera–Viedma[e]

[a]Department of Statistics and Operational Research, University of Granada, 18071 Granada Spain.
[b]DIGITS, Department of Informatics, Faculty of Technology, De Montfort University, Leicester LE1 9BH, UK.
[c]Department of Quantitative Methods in Economic and Business, University of Granada, 18071 Granada Spain.
[d]IEEF, University School of Industrial Technical Engineering, 28012 Madrid Spain.
[d]Department of Computer Science and A.I, University of Granada, 18071 Granada Spain.

Abstract

The mean absolute deviation and the standard deviation, two statistical measures commonly used in quantifying variability, may become an interesting tool when defining consensus measures. Two consensus indexes which obtain the level of consensus in some problems of Group Decision Making are introduced in this paper by expanding the aforementioned statistical concepts. A comparative analysis reveals that the levels of consensus derived from these indexes are close to those obtained employing distance functions when a fuzzy preference relations frame is considered, so they turn out to be a useful tool in this context. In addition, these indexes are different from each other and with the distance functions considered. Thus, they are applicable tools in the calculation of consensus in our context and are different from those commonly used.

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Keywords: Mean Absolute Deviation; Standard Deviation; Group Decision Making problems; Consensus measures; Fuzzy preference relations.

1. Introduction

Those problems where a collection of individuals, called experts, try to find an agreement among different alternatives are known as Group Decision Making (GDM) problems and consensus is utilized to name the status of the agreement among the individuals [1], either absolute or not. Different levels of understanding among experts can be expressed by means of measures known as soft consensus measures [1,2] which use the notion of similarity among preferences.

* Corresponding author. Tel.: + 34 - 958 24 06 19.
E-mail address: jmtaga@ugr.es.

The calculation and aggregation of distance measures representing the closeness of the preferences of every couple of experts on every couple of alternatives is needed to calculate the levels of consensus [1]. In fact, the values of the level of consensus are influenced not only by the distance function but also by the aggregation operator used in the computation [3,4,5].

In order to quantify consensus, different measures based on the statistical variability have been proposed [5,6]. A high disagreement among experts is indicated by a high value of the variability.

In this paper, in a frame of fuzzy preference relations in GDM problems, two indexes of consensus are introduced by expanding the statistical concepts of mean absolute deviation and standard deviation. A comparative analysis among them and some of the most used distance functions is performed.

2. GDM problems, distance functions and consensus indexes

There are several formats with which preferences can be expressed by each expert in a GDM problem [7]. Some of them focus on utility values [8], others on preference relations [9] such as fuzzy, linguistic and multiplicative preference relations [1]. The most utilised representation format is preference relations.

In a fuzzy preference relation framework a GDM problem that considers a soft consensus degree [1,10,11] several experts, $E = \{e_1,..., e_n\}$ $(n \geq 2)$, must find the best alternative from the considered alternatives, $X = \{x_1,..., x_m\}$ $(m \geq 2)$, following their preferences. To show an expert' preference of x_i over x_j, $\mu_P : X \times X \to [0,1]$ characterizes a fuzzy preference relationship, P, on a finite collection of alternatives X [12]:

$$\mu_P\left(x_i,\, x_j\right) \;=\; P\left(x_i,\, x_j\right) = p_{ij}$$

where 0 indicates minimum preference and 1 expresses maximum preference. We usually denoted the fuzzy preference relation through a matrix $P = (p_{ij})$ and consider a popular assumption: it is reciprocal – $p_{ji} + p_{ij} = 1$, i,j in $\{1,2,..., m\}$-. It is desirable that a fixed minimum level of consensus be obtained among experts to support the decision.

The computation of the level of consensus among experts can be facilited by quantifying the distance among their preferences. Some distance functions are commonly used in this measurement [13]:

$$\textit{Euclidean} \quad d_1(A,B) = \sqrt{\sum_{j=1}^{n} \left|a_j - b_j\right|^2}$$

$$\textit{Cosine} \quad d_2(A,B) = \frac{\sum_{j=1}^{n} a_j \cdot b_j}{\sqrt{\sum_{j=1}^{n} a_j^{\,2}}\sqrt{\sum_{j=1}^{n} b_j^{\,2}}}$$

$$\textit{Jaccard} \quad d_3(A,B) = \frac{\sum_{j=1}^{n} a_j \cdot b_j}{\sum_{j=1}^{n} a_j^{\,2} + \sum_{j=1}^{n} b_j^{\,2} - \sum_{j=1}^{n} a_j \cdot b_j}$$

with $A = \{a_1,...,a_n\}$, $B = \{b_1,...,b_n\}$ collections of numbers.

A mathematical function is employed to quantify the similarity among different preferences. Usually the similarity function is $s = 1 - d$ [3].

A matrix of similarity, $SM^r = (sm^r_{ij})$ is then obtain through $sm^r_{ij} = s(p^r_{ij}, p_{ij})$. This expression gives us a mesurement of the similarity among the preferences through a comparison among the opinion of every expert with the others couples of alternatives.

A matrix of consensus, $CM = (cm_{ij})$, is derived through the use of the OWA operator by aggregating the set of similarity matrices. This is performed by [14]:

$$p_{ij}{}^{c} = \phi_Q(p_{ij}{}^{1}, \dots, p_{ij}{}^{m}) = \sum_{k=1}^{m} w_k \cdot p_{ij}{}^{\sigma(k)}$$

with σ a function that allows permutation:

$$p_{ij}^{\sigma(k)} \geq p_{ij}^{\sigma(k+1)}, \quad \forall k \in \{1, 2, ..., n-1\},$$

Q a fuzzy linguistic quantifier of fuzzy majority that allows the calculation of the weights vector, $W = [w_1, \dots, w_n]$.

The notion of fuzzy majority and its alternative representations has been discussed in the literature [1].

A matrix with the degree of consensus on every pair (x_i, x_j), $CM = (cm_{ij})$, $i, j \in \{1,2,\dots, m\}$, is obtained:

$$cm_{ij} = \phi\left(sm_{ij}^{1}, \dots, sm_{ij}^{n}\right)$$

To computate the degree of consensus on the relation, cr, the group agreement among experts, it is performed an operation of aggregation of the consensus degrees at the level of pairs of alternatives:

$$cr = \phi\left(cm_{ij} : \ \ i \neq j \ \ \& \ \ i, j = 1, 2, \dots, m\right)$$

It is common to use an OWA operator. The Average operator ($W = [1/n, \dots, 1/n]$) is one of these operators.

Figure 1 shows the consensus model.

Widely used in descriptive statistics, Standard Deviation and Mean Absolute Deviation shows the magnitude of the data variability. They are presented as possible consensus measures[4,5].

Definition 1 (Standard deviation consensus index on (x_i, x_j) with fuzzy preferences).

Let (x_i, x_j), $i,j \in \{1,2,\dots,m\}$ be a couple of alternatives and $\{p_{ij}{}^{1},...,p_{ij}{}^{n}\}$ be the experts' preferences on them. We define the standard deviation for (x_i, x_j) as

$$SD_{ij} = +\sqrt{Var_{ij}} = +\sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(p_{ij}^{k} - \overline{p}_{ij}\right)^2} \quad i, j \in \{1, 2, ..., m\}$$

with $\overline{p}_{ij} = \dfrac{1}{n}\sum_{k=1}^{n} p_{ij}{}^{k}$ is the average value.

These values can be summarized in the matrix:

$$SDC = \left(SDC_{ij}, \quad i, j = 1, 2, \dots, m\right)$$

with $SDC_{ij} = 1 - \dfrac{1}{\overline{p}_{ij}\sqrt{n-1}} \cdot SD_{ij} \quad \forall i, j \in \{1, 2, ..., m\}$

Definition 2 (Standard deviation consensus index on the relation).

$$C_{SDC} = \frac{\sum_{i=1}^{m}\sum_{j>i}^{m} SDC_{ij}}{\sum_{k=1}^{m-1}(m-k)}$$

Definition 3 (Mean absolute deviation consensus index on (x_i, x_j) with fuzzy preferences).

Let (x_i, x_j), $i,j \in \{1,2,\ldots,m\}$, a couple of alternatives and $\{p_{ij}^1,\ldots,p_{ij}^n\}$ be the experts' preferences on them. We define the mean absolute deviation (*MAD*) consensus index for (x_i, x_j) as

$$MAD_{ij} = 1 - \frac{2}{n}\sum_{k=1}^{n}\left| p_{ij}^{\;k} - \frac{1}{n}\sum_{r=1}^{n} p_{ij}^{\;r} \right|$$

These values can be summarized in the matrix:

$$MAD = \left(MAD_{ij}, \quad i, j = 1, 2, \ldots, m \right)$$

Definition 4 (Mean absolute deviation consensus index on the relation).

$$C_{MAD} = \frac{\sum_{i=1}^{m}\sum_{j>i}^{m} MAD_{ij}}{\sum_{k=1}^{m-1}(m-k)}$$

The consensus model with statistical deviation–based functions is represented in Figure 2.

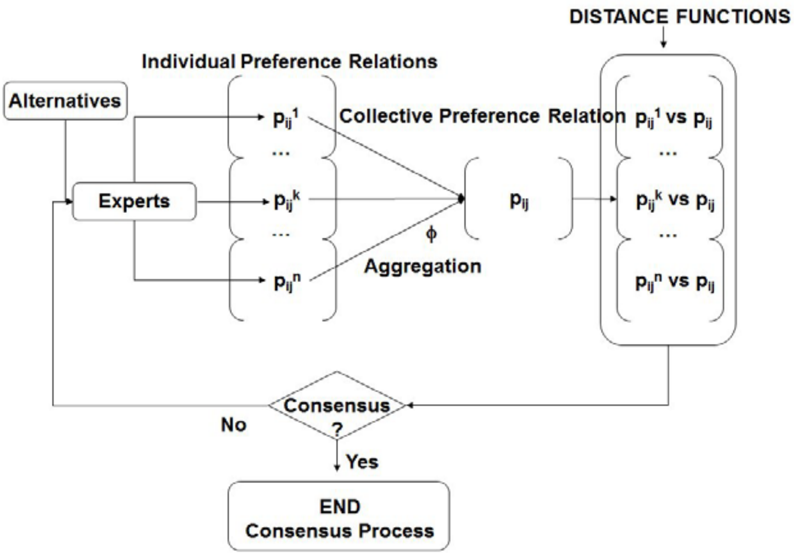

Fig. 1. Consensus model with distance functions and aggregation operators.

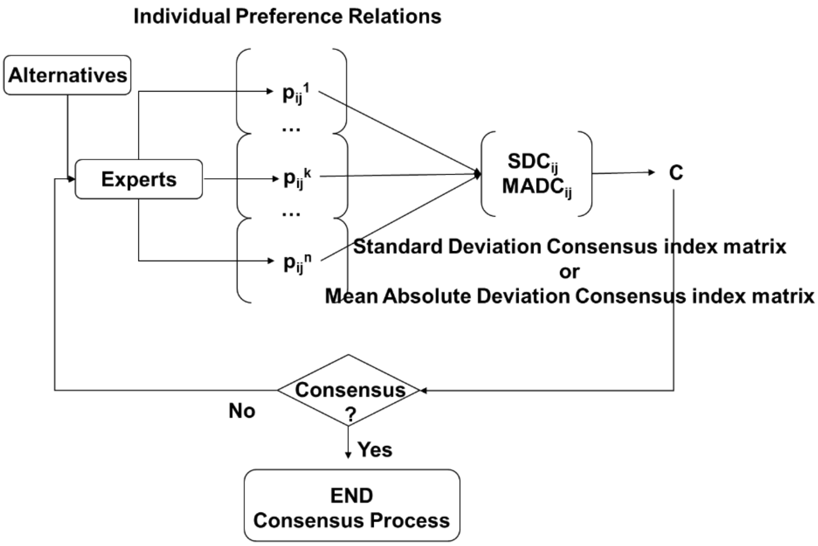

Fig. 2. Consensus model with statistical deviation–based functions.

3. Comparative Study

Using the strategy that we already employed in previous documents [3,4,5] we carried out a comparative study consisting in contrasting a hypothesis (H_0) by means of a statistical test. Assuming a fuzzy preference relations frame in GDM problems, the hypothesis can be stated as:

H_0: Consensus measures based on SDC and MAD indexes do not produce significantly differences facing the use of a distance d_i.

For the comparative analysis we generated 50 random GDM problems with a specific number of experts (3) and a fixed number of alternatives (4).

The OWA operator that was chosen for the distances d_i introduced in section 2, i.e., Euclidean distance –d_1– Cosine distance –d_2– and Jaccard distance –d_3–, was the Average operator. Since in this operator the weights are equal, the corresponding weights vector was $W = [0.3333,0.3333,0.3333]$.

The two proposed measures, SDC and MAC, were compared with each of the distances d_i and these new measures were also compared between them.

The statistical test used to compare the different measures has been the Wilcoxon test [3]. The results of this testing process are shown in Table 1.

Table 1: P–values obtained for Wilcoxon tests

Measures	*SDC* & d_1	*SDC* & d_2	*SDC* & d_3	*SDC* & *MAD*
P–value	0.000	0.073	0.023	0.000

Measures	*MAD* & d_1	*MAD* & d_2	*MAD* & d_3
P–value	0.000	0.000	0.813

These results corroborate those obtained in previous studies [4,5] and show that MAD and SDC are significantly different between them (P-value < 0.000 < 0.05).

It is also observed that MAD and SDC are significantly different from the Euclidean distance – d_1 – at a significance level 0.05 (P-value < 0.000 < 0.05) and MAD is significantly different from the Cosine distance – d_2 – (P-value < 0.000 < 0.05). In addition, SDC is significantly different from the Jaccard distance – d_3 – (P-value = 0.023 <0.05).

The analysis reveals that the levels of consensus derived from SDC and MAD indexes are close to those obtained using distance functions d_i when it is considered a fuzzy preference relations framework, turning them out to be a useful and simple tool in this context. In addition, both SDC and MAD measurements are significantly different between them and with some of the proposed distance functions, which provides different but acceptable alternative calculations, being the values very close in some cases.

This way, SDC and MAD indexes provide consensus measurement methods applicable in the context of fuzzy preference relations for group decision making problems, different from those commonly used with the distance functions considered.

Using the degrees of consensus obtained in the 50 random GDM problems for each of the different measures, we can establish a classification of these measures. This classification gets by comparing, for the same problem, the different degrees of consensus and ordering them from higher value to lower value. These values are summarized, normalized and, finally, expressed as percentages for a simpler interpretation. Table 2 shows the final results. It is observed that the higher the percentage value, the greater the degree of consensus. These results are depicted in Figure 3.

Table 2. Consensus degrees (percentage)

Measures	d_1	d_2	d_3	*MAD*	*SDC*
Consensus degree	63	100	79	79	97

The ranking obtained according to the degree of consensus from highest percentage to lowest percentage is: Cosine distance – d_2 – > SDC > MAD & Jaccard distance – d_3 – > Euclidean distance – d_1 –.

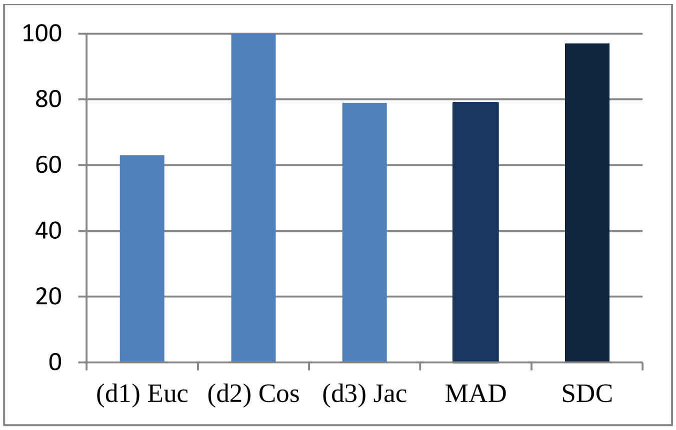

Fig. 3. Consensus degree in percentages.

In a consensus calculation that is in the situation of the 50 random GDM problems that we have used in this, the highest consensus value –maximum– would be obtained using the Cosine distance –d_2– and the lowest value –minimum– with the Euclidean distance –d_1–. In this situation, the proposed measures, SDC and MAD, would get values between them. SDC would get a value close to the maximum and MAD would obtain a lower value but above the minimum.

4. Conclusion

We have introduced two consensus indexes –SDC and MAD– based on the measurement of statistical variability. A comparative study among them and with three well–known distance functions is carried out. To manage the distance functions the average operator has been used as an aggregator operator.

The results of the comparative study reveals two interesting issues. Firstly, the levels of consensus derived from SDC and MAD indexes measures behave in a similar way to those obtained by using the distance functions considered in this study and, therefore, they could be used in this context, providing a new and simpler way of consensus calculation. Secondly, SDC and MAD measurements are significantly different from each other and with some commonly used distance functions, which provides different alternative calculations.

In addition, SDC index has a similar behavior to Cosine distance while MAD index is similar to the Jaccard distance.

Thus, SDC and MAD indexes are tools which could be applicable in the calculation of consensus in a fuzzy preference relations context for group decision making problems, different from those used with common distance functions.

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