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Evolutionary Game Theory based Multi-Objective Optimization for Control Allocation of Over-Actuated System

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Abstract: This research presents multi-objective optimization for control allocation problem based on the Evolutionary Game Theory to solve distribution of redundant control input on the over actuated system in real-time. Optimizing the conflicting objectives, an evolutionary game theory based approach with replicator dynamics is used to find the optimal weighting using the weighted sum method. The main idea of this method is that the best strategy or dominant solution can be selected as a solution that survive among other non-dominant solutions. The Evolutionary Game Theory considers strategies as a player and investigates how these strategies can survive using replicator dynamics with payoff matrix. The numerical simulation results show the optimal weightings selected by Evolutionary Game and how the payoff has been changed in replicator dynamics.

Keywords: Multi-Objective Optimization, Weighted Sum Method, Evolutionary Game Theory, Replicator Dynamics, Evolutionary Stable Strategy, Control Allocation

1. INTRODUCTION

In aerospace engineering, Unmanned Aerial Vehicle (UAV) is getting more and more attention and the demand for the UAV has beed increased in various areas throughout the industry and research institute. Especially rotor type UAV such as quadrotor or this kind of multi rotor type UAV is widely used, since it is easy to operate, inexpensive, and powerful compared to the conventional aircraft. One of features of multi rotor type UAV is that it can be seen as an over-actuated system which has redundant rotors and a decision for the control combination must be determined. For this reason, as the number of rotors has increased, the control allocation (or control distribution) problem is followed.

There have been several attempts to allocate redundant control input on the over actuated system. Direct (or constrained) control allocation has beed proposed by Durham (1994) as a method that provides an optimal solution using the attainable moment set for the given control effectiveness and constraint conditions. Härkegård (2003) suggests the Dynamic Control Allocation which addresses the mixed optimization combined the error and control minimization problem into a single problem, and Oppenheimer (2006) and Johansen (2013) describe other control allocation schemes.

This research aims to solve the control allocation by formulating Multi-Objective Optimization (MOO), where the conflicting objective functions and control input are decision variables. In order for the over-actuated system to deal with the control allocation problem, the optimal solution for the formulated multi-objective optimization problem is suggested based on the weighted-sum method where the weights are determined by Game Theory.

MOO is the process of optimizing a set of more than one objective simultaneously and is also known as multi-objective programming, vector optimization. MOO has been applied in many areas of science, engineering, economics and logistics where the optimal decision making concerned with optimization problems is required in the presence of tradeoff between more than one conflicting objective functions. Cho (2016) provides a comprehensive survey on modeling and optimizing MOO problem. In aerospace engineering, MOO can be applicable to distribute control input for over actuated system which has more than one control input about one axis compared to the conventional aircraft. Jamil (2012) and Salama (2014) solve multi-objective control allocation problem with two objective functions, which utilize the minmax control allocation scheme for two objective functions. The conflicting objective functions are solved by finding the Pareto optimal solution defined as a solution to the multi-objective optimization problem.

Evolutionary Game Theory (EGT) is one of the game theory that is the study of decision theory, which tries to find the best response that a player should perform maximizing the chances of success mathematically, and widely used form to determine the optimal strategy among interactive situation and players. The main idea of EGT is to determine its observable characteristics and its payoff in a given game where organism providing greater fitness

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will tend to produce more off spring to increase their population. Leboucher (2014) first attempts to use EGT to determine the weights to solve MOO problem for weapon target assignment problem, and Lee (2016) solves MOO problem for network resilience using EGT to find the weighting of the weighted sum method. Shin (2016) proposes *a priori* MOO approach based on the EGT to determine the weights of each objectives where conflicting objectives need to be optimized. The MOO problem is formulated as an evolutionary game of optimal solutions by solving this game with replicator dynamics.

Based on this background, this paper suggests a method to deal with control allocation problem based on the combination of MOO and EGT with weighted sum approach.

This paper is organized as follows: Section 2 describes over-actuated system and control allocation algorithm, which is the optimized based method. Section 3 presents the multi-objective optimization based control allocation algorithm where objectives are used in the Section 2. A brief summary of evolutionary game theory is introduced and evolutionary game theory based control allocation algorithm is presented in Section 4. Section 5 discusses the obtained result of numerical simulation. Finally, conclusion is addressed.

2. CONTROL ALLOCATION

Control Allocation (CA) can be applicable on the over-actuated system in which the number of input variable is greater than that of state variables to handle redundant control input variables. It essentially distributes the virtual command to redundant actuators. This means there is an infinite number of control input u, therefore, there is no unique solution for control input, u, and control distribution algorithm is required to handle mapping problem.
Consider an over-actuated system dynamics on state space form as follows:

$$\dot{x} = Ax + B_u u \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R^m$ is the control input variable, $A \in R^{n\times n}$ is the known state matrix and $B_u \in R^{n\times m}$ is the known control effectiveness matrix, respectively. There is a nullspace of dimension $n-m$, since B_u has rank $n < m$. Thus, a number of control input can be computed, and B_u matrix can be factorized as:

$$B_u = B_\nu B \tag{2}$$

where $B_\nu \in R^{n\times k}$ and $B \in R^{k\times m}$ in which both matrices have rank k ($k < n$, $k < m$). Given control input u and B, CA problem can be formulated to determine u such that:

$$Bu = \nu \tag{3}$$

where $\nu \in R^k$ is the virtual control input produced by the outer loop controller. Therefore, system dynamics can be rewrited as:

$$\begin{aligned}\dot{x} &= Ax + B_u u \\ &= Ax + B_\nu \nu\end{aligned} \tag{4}$$

The CA problem is to achieve the solution of the (3), which can apply to the (4).

2.1 Control Energy Minimization

The objective function defined by the following equation is to minimize the total displacement of the control input relative to its trim position, neutral position, or ideal position, which is one of the simplest linearized objective function :

$$J_1(u_t) = \min_{u_t} \frac{1}{2} W_1 \|u_t - u_d\|_2 \quad \text{subject to } \nu = Bu \tag{5}$$

where u_d is designed parameter, and W_1 is a positive diagonal weighting matrix a particular actuator can be prioritized to the preference of application, respectively. The numerical solution can be obtained using the following hamiltonian function, where λ denotes a Lagrange multiplier.

$$H_1(u_t, \lambda) = \frac{1}{2}(u_t - u_d)^T W_1 (u_t - u_d) + \lambda(\nu - Bu_t) \tag{6}$$

The optimization based CA problem has the following explicit solution considering the control demand constraint condition, $\nu - Bu$.

$$u_t^* = u_d + W_1^{-1} B^T (B W_1^{-1} B^T)^{-1} (\nu - Bu_d) \tag{7}$$

2.2 Control Input Deflection Minimization

The objective function defined by the following equation is to minimize the total deflection rate of the control input at each time step. The CA minimizes the total deflection rate as opposed to the minimum deflection.

$$J_2(u_t) = \min_{u_t} \frac{1}{2} W_2 \|u_t - u_{t-T}\|_2 \quad \text{subject to } \nu = Bu \tag{8}$$

where $u(t - T)$ is the position of the actuators at the previous time step, T is the sampling time, and W_2 is the positive diagonal weighting matrix where a particular actuator can be prioritized to the preference of application, which is similar to W_1. The numerical solution can also be obtained in the same manner as to that of (6) by defining the hamiltonian function as follows:

$$H_2(u_t, \lambda) = \frac{1}{2}(u_t - u_{t-T})^T W_2 (u_t - u_{t-T}) + \lambda(\nu - Bu_t) \tag{9}$$

The optimal solution considering the demand constraint condition, $\nu - Bu$, can also be derived.

$$u_t^* = u_{t-T} + W_2^{-1} B^T (B W_2^{-1} B^T)^{-1} (\nu - Bu_{t-T}) \tag{10}$$

3. MULTI-OBJECTIVE CONTROL ALLOCATION

3.1 Multi-Objective Optimization

MOO is the multiple criteria decision making area that concerns with more than one objective to be optimized simultaneously. Optimization involves the values of decision variables that generate the maximum or minimum objective. The objectives of the MOO problem are conflicting each other and impossible to find a solution which can satisfy all objectives of MOO problem. The solution computed to optimize each objective can not optimize all objective functions and find the trivial solution of the multiple optimization problem. Therefore, MOO aims to find a point which attains to the optimal solution or a set of an appropriate tradeoff among the objective functions. As the design of objective function depends on the decision variable, the MOO problem can be formulated as:

$$\min_{u} J_1(u), J_2(u), .., J_p(u) \tag{11}$$

where the the number of objective function, p is larger than 2 , and u is the decision variable, respectively. Equality and inequality constraints can either exist or not. There are many techniques to solve the MOO problem, for example, scalarization techniques, metaheuristic method, and hybrid method. In this paper, the weighted sum method which is one of the scalarization techniques and the most widely used algorithm to solve MOO problem is used since its simplicity and intuitiveness, which integrates multiple weighted objectives into a single objective function as:

$$\begin{aligned} \min_{w} J_t &= w_1 J_1 + w_2 J_2 + \cdots + w_p J_p \\ &= \sum_{i=1}^{p} w_i J_i \end{aligned} \tag{12}$$

where each weight, w_i represents the importance of objective function. It is critical to determine appropriate weight and normalization of weight unit.

3.2 Multi-Objective Optimization for Control Allocation

The generalized multi-objective optimization problem is presented in order to understand the multi-objective control allocation. In order to understand the multi-objective optimization for CA, a generalized multi-objective optimization problem must be formulated as such:

$$\min_{u\ \in \Omega} J_1(u), J_2(u), ..., J_p(u) \quad \text{subject to } \nu = Bu \tag{13}$$

where p is the number of the conflicting objectives, and Ω denotes a set of design constraints, respectively. u is the control input variable, which is the decision variable of MOO problem. All the attainable solutions are affected by the constraints condition, Ω that limits the feasible set of attainable solutions, u, among the objective functions. By combining the two objective functions (5) and (8), which is described in a previous section, the following augmented objective function can be defined. The augmented objective function, J_t, comprises two objectives, J_1 and J_2.

$$J_t = \min_{u_t} \frac{1}{2} W_1 \|u_t - u_d\|_2 + \frac{1}{2} W_2 \|u_t - u_{t-T}\|_2 \tag{14}$$

The numerical solution of the augmented objective function can be derived in the same manner as to that of (6), (9) by using Lagrange multiplier λ:

$$\begin{aligned} H_t(u_t, \lambda) &= \frac{1}{2}(u_t - u_d)^T W_1 (u_t - u_d) \\ &\quad + \frac{1}{2}(u_t - u_{t-T})^T W_2 (u_t - u_{t-T}) + \lambda(\nu - Bu_t) \end{aligned} \tag{15}$$

The optimal solution of the augmented objective function can be calculated by a similar methodology used in (7) and (10) as that of the single objective weighted pseudo-inverse method, and taking the partial derivatives of the objective function with respect to the input variable, u, and the Lagrange multiplier, λ as follows:

$$\begin{aligned} u_t^* &= W_0^{-1}(W_1 u_d + W_2 u_{t-T}) \\ &\quad + W_0^{-1} B^T (B W_0^{-1} B^T)^{-1} [\nu - B W_0^{-1} (W_1 u_d + W_2 u_{t-T})] \end{aligned} \tag{16}$$

where $W_0 = W_1 + W_2$

4. EVOLUTIONARY GAME THEORY BASED MULTI-OBJECTIVE OPTIMIZATION

4.1 Evolutionary Game Theory

Game Theory is mathematical model of strategic interaction between self-interested players (or decision makers) who are engaged in a given game. The classical game theory deals with making a decision (or strategy) with other players in order to maximize payoff (or fitness) which is a mathematical measurement that describes how much the player like or does not like a given situation. In contrast, Evolutionary Game Theory (EGT) deals with a dynamics describing how the population of players will change over time. The main insight of EGT is equilibria of games played by a population of players, where the payoff is derived from the interaction of multiple populations, and the success of any one of populations depends on how their behavior interacts with that of others.

The main evolutionary game theoretic concept is Evolutionary Stable Strategy (ESS), which is a strategy with high payoff that will spread within the population and tend to maintain once if the strategy is prevalent in the population. It means that when the whole population uses the evolutionary stable strategy, a, any populations, called mutant, using a different strategy, b, can not invade in this population. In order for a strategy to be evolutionary stable, the following conditions should be satisfied.

$$u(a, (1-\mu)a + \mu b) > u(b, (1-\mu)a + \mu b) \tag{17}$$

where u is the expected payoff, and μ is a frequency of mutants playing with strategy b in which $u(a, b)$ means the payoff playing strategy a, when mutant plays strategy b. Since a is evolutionary stable strategy, the payoff of a population following a must be greater than that of mutant following b. If frequency of mutants playing with μ is small enough, the condition (17) can be seen as:

$$u(a,a) > u(b,a) \tag{18}$$

If $u(a,a)$=$u(b,a)$, the second term of the condition (17) must meet the following condition:

$$u(a,b) > u(b,b) \quad \text{such that } u(a,a)=u(a,b) \tag{19}$$

Choi (2009), and Easely (2010) describe more details about the EGT and ESS condition.

4.2 Replicator Dynamics

The main difference between EGT and classical game theory is the investigation of dynamics of strategy change. EGT is interested in the dynamics of the competing strategies in the population, how the population evolves over time. In order to express the growth rate of the population using a certain strategy, the Replicator Dynamics is introduced. The Replicator Dynamics models how the proportion of players playing each strategy changes assuming that each player asexually reproduces offsprings who play the same strategy, where the number of offspring depends on their payoff. The general equation of replicator dynamics is defined as follow:

$$\dot{w_i} = w_i[u(e^i x) - u(x,x)] \tag{20}$$

where w_i is the proportion of the population following the ith strategy, $u(e^i, x)$ is the individual payoff of a player following the ith strategy, and $u(x,x)$ is the current average payoff of the population, respectively. This equation states the growth rate $\frac{\dot{w}}{w}$ of the population.

4.3 Evolutionary Game based Multi Objective Optimization

The main idea of the evolutionary game based MOO problem is a type of non-cooperative game where the decision variables and cost functions act as players. Among the objective functions, the objective that has a priority dominates at least one of the others, and the weight is applied to the objective function that is more sensitive to the one of which objective functions to be optimized, which can be compared to the concept of equilibrium in the game theory.

Finding the equilibrium solution in a non-cooperative game requires analyzing the player's fitness to formulate a payoff matrix. The decision variables and cost functions act as players, and objectives to be optimized are the possible strategies. The weightings of a MOO problem can also be considered as the Nash equilibrium of the mixed strategies, which is the solution of non-cooperative game involving players in which each player is assumed to know the equilibrium strategy and there is no reason to change their own strategy. Therefore, the payoff table can be defined such that the row is the objective function and the column is the optimal decision variables corresponding to each objective function. The payoff matrix for (11) can be defined as such:

In this paper, the payoff table can be reduced to 2 × 2 matrix since the number of objective function is 2 defined in a previous section. Thus, the payoff matrix for (13) can be composed as

where u_i^* is the optimal solution for the ith objective function.

The normalization method can affect the characteristic of the payoff matrix. Different scales of each cost function affect to relative importance, thus the cost functions are normalized. The linear normalized payoff matrix is composed as:

$$\bar{A_{ij}} = \frac{J_i(u_j{}^*)}{J_i{}^+} \tag{21}$$

Using the normalized payoff matrix, the evolving dynamics of the fitness (20) can be represented by the replicator dynamics as follows:

$$\dot{w} = w_i(e_i^T Aw - w^T Aw) \tag{22}$$

where A is the payoff matrix, e_i is a column vector with one at the ith element and zero at the others, w is the proportion of the ith strategy in the population, respectively.

The analytic solution of the evolutionary stable solution $\bar{w}$ can be derived from the derivative of the replicator dynamics, which means that the payoff difference between individual and average of population becomes zero at the stable point.

$$e_i^T A\bar{w}^T - \bar{w}^T A\bar{w} = 0 \tag{23}$$

where individual payoff converges to the average payoff of the population, k. Thus, (23) can be rearranged as follow:

$$IA\bar{w} = \bar{w}^T A\bar{w} 1_{p\times 1} = k1_{p\times 1} \tag{24}$$

The second condition for the analytic solution is that summation of the weights is always 1.

$$\bar{w}_1 + \bar{w}_2 + ... + \bar{w}_p = 1 \tag{25}$$

These two condition can be expressed as an augmented matrix, and it can be solved when the augmented matrix is invertible.

$$\begin{bmatrix} \bar{w}^T \\ k \end{bmatrix} = \begin{bmatrix} A & -1_{p\times 1} \\ 1_{1\times p} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0_{p\times 1} \\ 1 \end{bmatrix} \tag{26}$$

Table 1. The payoff matrix

	u_1	u_2	$\cdots$	u_p
J_1	$J_1(u_1)$	$J_2(u_2)$	$\cdots$	$J_1(u_p)$
J_2	$J_2(u_2)$	$J_2(u_2)$	$\cdots$	$J_2(u_p)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
J_p	$J_p(u_1)$	$J_p(u_2)$	$\cdots$	$J_p(u_p)$

Table 2. The payoff matrix setting for two objectives

	u_1^*	u_2^*
J_1	$J_1(u_1^*)$	$J_2(u_2^*)$
J_2	$J_2(u_2^*)$	$J_2(u_2^*)$

The stability of the stable point can be determined when the following relationship is accomplished.

$$\begin{cases} w^T A\bar{w} < \bar{w}^T A\bar{w} \\ w^T A w < \bar{w}^T A w \quad \text{if } w^T \mathrm{A}\bar{w} = \bar{w}^T \mathrm{A}\bar{w} \end{cases} \tag{27}$$

The first condition means that $u(a,a) > u(b,a)$, which is the evolutionary equilibrium condition: no invader does better than the resident. The second condition means that if $u(a,a) = u(b,a)$, then $u(a,b) > u(b,b)$, which is the stability condition: if the invader does as well as the resident against the resident, then it does less than the resident against the invader, which corresponds to the ESS. Lee (2016), Ohtsuki (2006), and Hofbauer (2006) describe more details about this relationship, and these equilibrium and stability condition of replicator dynamics.

5. THE NUMERICAL SIMULATION

5.1 System Dynamics

The over-actuated dynamics modeled and simulated here is the multi-rotor system, which has more than four rotors. The following equation of motion describes the attitude dynamics of the multirotor UAV:

$$\begin{aligned} \dot{p} &= \frac{I_y - I_z}{I_x} qr + \frac{1}{I_x} l \\ \dot{q} &= \frac{I_z - I_x}{I_y} rp + \frac{1}{I_y} m \\ \dot{r} &= \frac{I_x - I_y}{I_z} pq + \frac{1}{I_z} n \end{aligned} \tag{28}$$

where I_x, I_y, I_z are inertia term, p, q, r are angular velocity and $l.m.n$ are angular moment in the body fixed frame, respectively. Furthermore, consider the z axis velocity in the inertial frame.

$$\dot{z} = -g + \frac{cos\phi cos\theta}{m} T \tag{29}$$

where g gravitational acceleration, ϕ, θ are angular position in the roll and pitch axis, m is mass of the UAV, and T is total thrust force, respectively. (28) and (29) can be formulated in the state space form as:

$$\begin{bmatrix} \dot{z} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -g \\ (I_y - I_z)/I_x qr \\ (I_z - I_x)/I_y rp \\ (I_x - I_y)/I_z pq \end{bmatrix} + \begin{bmatrix} cos\phi cos\theta/m \\ 1/I_x \\ 1/I_y \\ 1/I_z \end{bmatrix} \begin{bmatrix} T \\ l \\ m \\ n \end{bmatrix} \tag{30}$$

where (30) can be seen as $\dot{x} = f(x) + g_\nu(x)\nu$ which is equal to (4) in terms of virtual control input matrix, B_ν and virtual control input, ν. Furthermore, the actuator model in the following equations:

$$\begin{bmatrix} T \\ l \\ m \\ n \end{bmatrix} = \begin{bmatrix} c_{t1} & c_{t2} & \cdots & c_{tm} \\ c_{t1} d_l & c_{t2} d_l & \cdots & c_{tm} d_l \\ c_{t1} d_m & c_{t2} d_m & \cdots & c_{tm} d_m \\ c_{n1} & c_{n2} & \cdots & c_{nm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \tag{31}$$

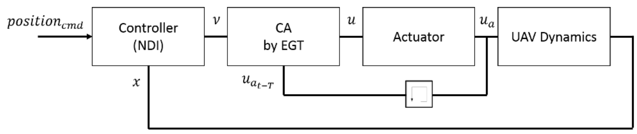

Fig. 1. Block diagram of the controller and control allocation

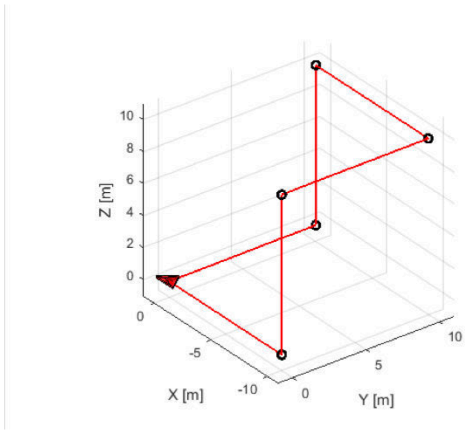

Fig. 2. UAV Trajectory

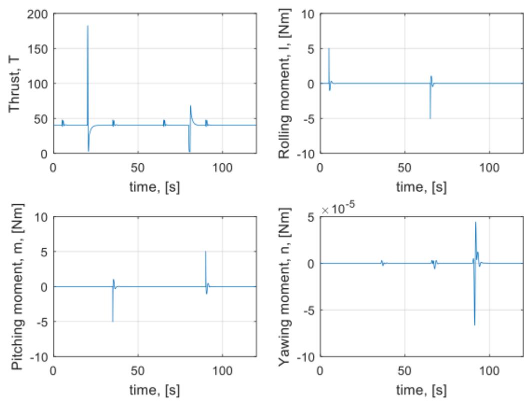

Fig. 3. Time history of virtual command

where (31) can be seen as $\nu = Bu$ which is equal to (3), and c_t is the thrust coefficient, c_n is the torque coefficient, d_l, d_m are moment arm with respect to each body axis frame.

5.2 Simulation Results

The presented method has been applied to the waypoint navigation of multi rotor UAV. This simulation is carried out with inner-loop and outer-loop controller to control waypoint and attitude of UAV using Nonlinear Dynamic Inversion (NDI) method for generating virtual control input (ν), which is shown in Fig. 1.

The Fig.2 ∼ Fig.5 show the result of the numerical simulation. As figures shown, the weights has been changed when the virtual command (ν) is generated. The physical meaning is that the weights generally try to minimize control energy, which seems to be a single objective optimization problem, because without virtual command, the minimization of control deflection rate does not need to be considered as control deflection does not exist. How-

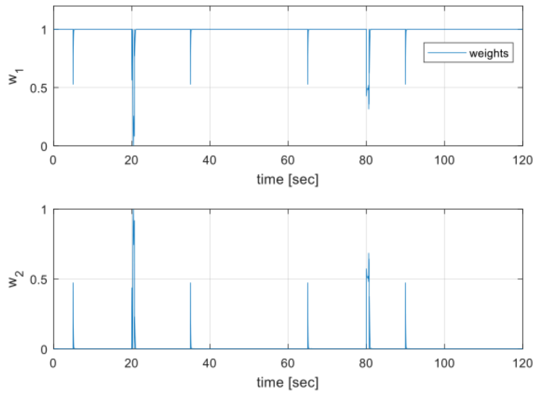

Fig. 4. Time history of weight variation

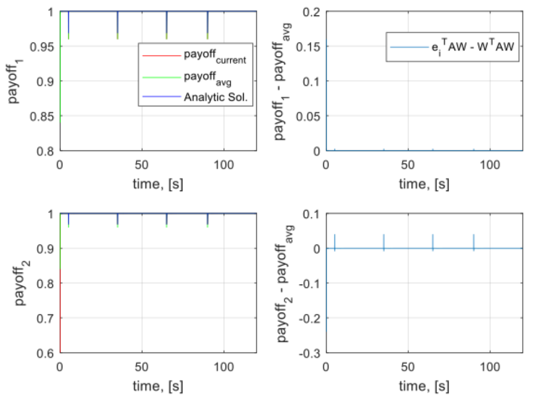

Fig. 5. Time history of payoff variation

ever, when the virtual control is generated, the weight for control energy minimization decreases to satisfy the command, and the weight for control deflection rate increases to prevent abrupt deflection of the control input.

As mentioned in the section 4.3, individual payoff converges to average payoff of the population at the stable weight point. The Fig.5 shows the results of augmented matrix (26), individual payoff and average payoff, respectively. It can be seen that these three values converge and the difference between individual and average payoff becomes zero. Thus, the inner term of right side term of (22) becomes zero as shown in the right two figures of Fig.5. Furthermore, the derivative of weight converge to zero and the replicator dynamics becomes stable.

6. CONCLUSION

This paper proposes Multi-Objective Optimization approach to solve control allocation problem for over actuated system that has redundant control input. Based on the Evolutionary Game Theory with replicator dynamics to express multi-objective optimization problems, the proposed method shows reliable result in terms of control allocation. The proposed method shows reliable result in terms of two objectives such as minimization of control energy and control deflection rate. The results of this method depend on the payoff matrix which is used to solve replicator dynamics and analytical solution, thus, the key point of this method is to find a proper payoff matrix.

This study is expected to suggest a method for designing a solution of control allocation problem in any over actuated system. Such a method, taking advantage of the concept of Evolutionary Game Theory with replicator dynamics, can be extended to deal with more objectives and resolves conflicting objectives.

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