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Closed-loop Analysis with Incremental Backstepping Controller considering Measurement Bias

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Abstract: In this paper, closed loop system characteristics with an incremental backstepping controller are investigated through theoretical analysis when both measurement biases and model uncertainties exist. Incremental backstepping algorithm is proposed in previous studies to reduce model dependency of classical backstepping algorithm with additional measurements about state derivatives and control surface deflection angles. This research enables to have following critical understandings especially about the effects of biases on these additional measurements to system characteristics with incremental backstepping method. First, these biases do not affect a characteristic equation, so they do not have any influence about a condition for absolute stability. Second, these biases cause a steady state error, and model uncertainty in control effectiveness information starts to have an impact to it when these biases are additionally considered.

Keywords: Backstepping control, Incremental backstepping control, Closed-loop analysis, Measurement bias, Model uncertainty, Model-based approach, Sensor-based approach

1. INTRODUCTION

Backstepping(BKS) method has been widely applied as one of nonlinear flight controllers. Nevertheless, it has a crucial drawback to be sensitive to model uncertainties because it requires explicit model information for its implementation. In reality, it is difficult to get an accurate model, so incremental backstepping(IBKS) algorithm is proposed to reduce model dependency of BKS. Thanks to additional measurements about state derivatives and control surface deflection angles, only control effectiveness information is required to implement IBKS.

There have been several researches Gils (2016); Ali (2014); Acquatella (2013); Falconi (2016) where closed loop characteristics with IBKS under model uncertainties can be found from numerical simulations or experiments. Jeon (2018) suggested theoretical closed loop analysis results to have critical understandings about them. Previous studies indicate that IBKS has a strong advantage when model uncertainties exist; one of important characteristics obtained from the analysis in Jeon (2018) is that a system is robust with respect to an uncertainty even in control effectiveness information if a control command is calculated, transmitted and reflected fast enough to a real control surface deflection. However, there is a limitation in previous works that measurements are assummed to be ideal, which is hard to be achieved in practical applications. If measurement related issues like a bias, a noise and a delay are additionally considered, IBKS which lies in between model based and sensor based approach, might show worse performance than BKS.

In this paper, closed loop analysis with IBKS considering both measurement biases and model uncertainties, is performed. The main purpose of this research is to have critical understandings about the effects of biases to system characteristics with IBKS, which can make aimed performance and stability characteristics difficult to be acheived. As previously mentioned, measurements about state derivatives and control surface deflection angles are additionally required to implement IBKS comparing to

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BKS, so biases on them are mainly considered in this analysis. Besides, in this paper, it will be investigated whether closed loop system with IBKS is still robust with respect to model uncertainties although those biases are additionally considered.

In section 2, dynamics for control law derivation will be suggested as a preliminary. In section 3, a control algorithm using IBKS will be derived and proposed. In section 4, closed loop analysis with IBKS considering both measurement biases and model uncertainties, is performed to have critical understandings especially about the effects of biases to a system. To verify properties obtained from section 4, simulations will be carried out and following results will be suggested in section 5.

2. PRELIMINARY : DYNAMICS

For control law derivation and closed loop analysis, short period mode dynamics (1), one of the longitudinal oscillation modes with a high natural frequency, is applied. This simplified version of dynamics, not full 6-DoF dynamics, is utilized for simplicity of analysis. This short period mode is of paramount importance in flight control, because one of the main purposes of a stability augmentation system for an airplane is to improve characteristics about this mode. Since the main objective of this paper is to have critical understandings about closed loop characteristics with IBKS especially considering measurement biases, dynamics (1) is reasonable for this purpose.

$$\begin{aligned}\dot{\alpha} &= Z_\alpha^*\left(M,\alpha\right)\alpha + q + Z_\delta^*\left(M,\alpha\right)\delta \\ \dot{q} &= M_\alpha^*\left(M,\alpha\right)\alpha + M_q^*\left(M,\alpha\right)q + M_\delta^*\left(M,\alpha\right)\delta\end{aligned} \tag{1}$$

The state variables α and q indicate angle of attack and pitch rate. The control input δ represents elevator deflection angle. Z_α^*, Z_δ^*, M_α^*, M_q^* and M_δ^* denote aerodynamic derivatives where M corresponds to Mach number. Dynamics (1) can be regarded as a linear parameter-varying(LPV) system i.e., a nonlinear system which can be expressed into a parametrized linear system whose parameters change with the states.

3. CONTROL LAW DERIVATION

Before derivation of a control algorithm, dynamics (1) is modified as follows. First, aerodynamic derivatives estimates $\hat{(\cdot)}$ are utilized instead of real aerodynamic derivatives $(\cdot)$, because only estimated values are available in a controller design phase. Second, $\hat{Z}_\delta^*\delta$ related to non-minimum phase is neglected. IBKS control law is also based on backstepping method, so a system is required to be in strictly feedback form. This is valid for most of aircrafts, often made in flight control systems' design process, because $\hat{Z}_\delta^*\delta$ is usually small enough comparing to the other terms in $\dot{\alpha}$ equation.

$$\begin{aligned}\dot{\alpha} &= \hat{Z}_\alpha^*\alpha + q \\ \dot{q} &= \hat{M}_\alpha^*\alpha + \hat{M}_q^*q + \hat{M}_\delta^*\delta\end{aligned} \tag{2}$$

The state errors are defined as follows.

$$\begin{aligned}z_1 &= \alpha - \alpha_c \\ z_2 &= q - q_c\end{aligned} \tag{3}$$

where subscript c represents a command.

If Lyapunov candidate function becomes positive definite and its derivative becomes negative definite, asymptotic stability for a nonlinear system can be guaranteed. To derive a control command which satisfies asymptotic stability assuming that $\hat{(\cdot)}$ have their true values, following 2 cascaded steps are performed.

First, Lyapunov function candidate V_1 considering only z_1 for an outer-loop controller design is selected as

$$V_1 = \frac{1}{2}z_1^2 \tag{4}$$

which is positive definite. The derivative of V_1 becomes

$$\begin{aligned}\dot{V}_1 &= z_1\dot{z}_1 \\ &= z_1\left(\hat{Z}_\alpha^*\alpha + q - \dot{\alpha}_c\right)\end{aligned} \tag{5}$$

In order to satisfy Lyapunov stability condition, a psedo-command q_c is derived as

$$q_c \triangleq -C_1z_1 - \hat{Z}_\alpha^*\alpha + \dot{\alpha}_c \tag{6}$$

which makes negative definite $\dot{V}_1 = -C_1z_1^2$ where C_1 is a positive design parameter.

For the outer-loop controller design, classical BKS, not IBKS, is applied in this paper. If IBKS is applied here for the outer loop control, $\dot{\alpha}_0$ measurement is additionally required instead of model information $\hat{Z}_\alpha^*$. There exist more practical ways to replace $\hat{Z}_\alpha^*$ information, so an incremental algorithm is not normally used for an outer loop control. This can be seen also in other papers Gils (2016) Acquatella (2013) Sieberling (2010) Smeur (2016) which just applied BKS or PID for it.

For the second step to design an inner-loop controller, q dynamics in (2) is modified assuming that the states α, q and the control input δ can be expressed as combination of reference points $(\cdot)_0$ and perturbations $\Delta(\cdot)$ around them. This is a valid assumption especially with a sufficiently high sampling rate.

$$\begin{aligned}\dot{q} &= \hat{M}_\alpha^*\left(\alpha_0 + \Delta\alpha\right) + \hat{M}_q^*\left(q_0 + \Delta q\right) + \hat{M}_\delta^*\left(\delta_0 + \Delta\delta\right) \\ &= \dot{q}_0 + \hat{M}_\alpha^*\Delta\alpha + \hat{M}_q^*\Delta q + \hat{M}_\delta^*\Delta\delta\end{aligned} \tag{7}$$

The increments in states, $\Delta\alpha$ and Δq, are negligible comparing to the increment in input, Δu, since a control surface deflection directly affects pitch moment, while integations are required first for states. Then, final incremental q dynamics for the inner loop controller design with IBKS is given as below.

$$\dot{q} \simeq \dot{q}_0 + \hat{M}_\delta^*\Delta\delta \tag{8}$$

In the second step, Lyapunov function candidate V_2 considering both z_1 and z_2 is selected as

$$V_2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{9}$$

which is positive definite. The derivative of V_2 becomes

$$\begin{aligned}\dot{V}_2 &= z_1\dot{z}_1 + z_2\dot{z}_2 \\ &= z_1\left(\hat{Z}_\alpha^*\alpha + q - \dot{\alpha}_c\right) + z_2\left(\dot{q}_0 + \hat{M}_\delta^*\Delta\delta - \dot{q}_c\right)\end{aligned} \tag{10}$$

Using the pseudo-command (6), $\dot{V}_2$ becomes

$$\dot{V}_2 = z_1\left(-C_1z_1 + z_2\right) + z_2\left(\dot{q}_0 + \hat{M}_\delta^*\Delta\delta - \dot{q}_c\right) \tag{11}$$

To satisfy Lyapunov stability condition, $\Delta\delta$ is derived as

$$\Delta\delta \triangleq \frac{1}{\hat{M}^*_\delta}\left(-C_2 z_2 - z_1 - \dot{q}_0 + \dot{q}_c\right) \tag{12}$$

which makes negative definite $\dot{V}_2 = -C_1 z_1^2 - C_2 z_2^2$ where C_1 and C_2 are positive design parameters.

Final form of the control law can be suggested as follows.

$$\begin{aligned} q_c &= -C_1 z_1 - \hat{Z}^*_\alpha \alpha + \dot{\alpha}_c \\ \delta &= \delta_0 + \Delta\delta \\ &= \frac{1}{\hat{M}^*_\delta}\left(-C_2 z_2 - z_1 - \dot{q}_0 + \dot{q}_c\right) + \delta_0 \end{aligned} \tag{13}$$

δ makes q to achieve q_c, and α goes to its desired value α_c by q_c. For implementation of a control algorithm, only $\hat{Z}^*_\alpha$ and $\hat{M}^*_\delta$ are required, and $\hat{M}^*_\alpha$ and $\hat{M}^*_q$ are not necessary because the incremental dynamics about q is utilized for a derivation process of IBKS as an inner loop controller. Hence, comparing to BKS only controller, model information is less required and a system becomes robust with respect to the uncertainties in $\hat{M}^*_\alpha$ and $\hat{M}^*_q$. Instead, the additional measurements δ_0 and $\dot{q}_0$ are required to compensate them.

4. CLOSED-LOOP ANALYSIS

Closed-loop analysis considering both biases on additional measurements and model uncertainties, is performed. As in Lee (2016), analysis is carried out in piece-wise way to easily apply existing analysis framework for a linear time-invariant(LTI) system. The effects of measurement biases, which can make aimed performance and stability characteristics in a controller design phase difficult to be acheived, are investigated.

Dynamics (1) with $Z^*_\delta = 0$ can be expressed as a state space equation (14) below. In general, $\hat{Z}^*_\delta$ is small enough to be neglected, especially for large airplanes.

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad \mathbf{y} = \mathbf{C}\mathbf{x} \\ &\text{where} \\ &\mathbf{x} = [\,\alpha\ \ q\,]^T \qquad \mathbf{u} = \delta \\ &\mathbf{A} = \begin{bmatrix} Z^*_\alpha & 1 \\ M^*_\alpha & M^*_q \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ M^*_\delta \end{bmatrix} \quad \mathbf{C} = [\,1\ \ 0\,] \end{aligned} \tag{14}$$

Comparing to the dynamics (2) for the control law derivation, real aerodynamic derivatives, not estimates, are considered in this dynamics (14) for the analysis.

In (13), δ can be rewritten as follows by substituting q_c, under the assumption of constant α_c (i.e. $\dot{\alpha}_c = \ddot{\alpha}_c = 0$).

$$\begin{aligned} \delta =& \frac{1}{\hat{M}^*_\delta}\left\{-\left(C_1 + \hat{Z}^*_\alpha\right)(C_2 + Z^*_\alpha)\alpha - \left(C_1 + C_2 + \hat{Z}^*_\alpha\right) q \right. \\ &\left. + C_1 C_2 \alpha_c - z_1\right\} - \frac{1}{\hat{M}^*_\delta}\dot{q}_0 + \delta_0 \end{aligned} \tag{15}$$

As mentioned in previous section, measurements δ_0 and $\dot{q}_0$ are additionally required to implement IBKS. Hence, if there exist biases on these additional measurement, as an innerloop controller, IBKS might show worse performance than BKS. In this paper, biases on δ_0 and $\dot{q}_0$ measurements, $b_{\dot{q}_0}$ and b_{δ_0}, are considered in closed loop analysis as follows, to have critical understandings about the effects of them to system characteristics with IBKS.

$$\begin{aligned} \dot{q}_0 &= \dot{q}_{0,true} + b_{\dot{q}_0} \\ \delta_0 &= \delta_{0,true} + b_{\delta_0} \\ &\text{where} \\ \dot{q}_{0,true} &= M^*_\alpha \alpha + M^*_q q + M^*_\delta \delta_{0,true} \\ \delta_{0,true} &= \delta(t - \tau) \end{aligned} \tag{16}$$

From a piece-wise version of (1), the model for $\dot{q}_{0,true}$ in (16) is suggested. Under the assumption of an ideal actuator, a control surface deflection becomes the same as a generated control command. Then, $\delta_{0,true}$ can be regarded as a control command generated in previous step, where τ indicates a step size.

By substituting (16) to (15), δ can be rearranged as below.

$$\begin{aligned} \delta =& -\frac{1}{\hat{M}^*_\delta}\nu_{2,\alpha}\alpha - \frac{1}{\hat{M}^*_\delta}\nu_{2,q} q + \frac{1}{\hat{M}^*_\delta}\left(C_1 C_2 + 1\right)\alpha_c \\ &+ \left(1 - \frac{M^*_\delta}{\hat{M}^*_\delta}\right)\delta\left(t - \tau\right) - \frac{1}{\hat{M}^*_\delta} b_{\dot{q}_0} + b_{\delta_0} \\ &\text{where} \\ &\nu_\alpha = \left\{\left(C_1 + \hat{Z}^*_\alpha\right)(C_2 + Z^*_\alpha) + M^*_\alpha + 1\right\} \\ &\nu_q = \left(C_1 + C_2 + M^*_q + \hat{Z}^*_\alpha\right) \end{aligned} \tag{17}$$

Applying Laplace transform to (17) and arranging this equation with respect to δ,

$$\begin{aligned} \delta(s) =& \left[-\tfrac{1}{\phi(s)}\nu_\alpha(s)\ -\tfrac{1}{\phi(s)}\nu_q(s)\right]\mathbf{X}(s) \\ &+ \frac{1}{\phi(s)}\left(C_1 C_2 + 1\right)\alpha_c(s) - \frac{1}{\phi(s)} b_{\dot{q}_0} + \frac{\hat{M}^*_\delta}{\phi(s)} b_{\delta_0} \\ &\text{where} \\ &\phi(s) = \hat{M}^*_\delta\left(1 - e^{-\tau s}\right) + M^*_\delta e^{-\tau s} \end{aligned} \tag{18}$$

If Laplace transform is applied to (14) and $\delta(s)$ in (18) is substituted into that equation, a closed loop system can be suggested, as follows.

$$\begin{aligned} &s\mathbf{X}(s) = \mathbf{A}(s)\mathbf{X}(s) + \mathbf{B}(s)\alpha_c(s) + \mathbf{D}(s)\mathbf{b}(s) \\ &\mathbf{Y} = \mathbf{C}(s)\mathbf{X}(s) \\ &\text{where} \\ &\mathbf{A}(s) = \begin{bmatrix} a_{2,11}(s) & a_{2,12}(s) \\ a_{2,21}(s) & a_{2,22}(s) \end{bmatrix} \\ &\quad = \begin{bmatrix} Z^*_\alpha & 1 \\ M^*_\alpha - \frac{M^*_\delta}{\phi(s)}\nu_{2,\alpha}(s) & M^*_q - \frac{M^*_\delta}{\phi(s)}\nu_{2,q}(s) \end{bmatrix} \\ &\mathbf{B}(s) = \begin{bmatrix} 0 \\ \frac{M^*_\delta}{\phi(s)}\left(C_1 C_2 + 1\right) \end{bmatrix} \\ &\mathbf{C}(s) = [\,1\ \ 0\,] \\ &\mathbf{D}(s) = \begin{bmatrix} 0 & 0 \\ -\frac{M^*_\delta}{\phi(s)} & \frac{M^*_\delta \hat{M}^*_\delta}{\phi(s)} \end{bmatrix} \\ &\mathbf{b}(s) = \begin{bmatrix} b_{\dot{q}_0}(s) \\ b_{\delta_0}(s) \end{bmatrix} \end{aligned} \tag{19}$$

Then $\alpha(s)$ can be derived as below.

$$\alpha(s) = \mathbf{C}(s)\{s\mathbf{I} - \mathbf{A}(s)\}^{-1}\{\mathbf{B}(s)\alpha_c(s) + \mathbf{D}(s)\mathbf{b}(s)\}$$
$$= \frac{1}{s^2 - (a_{2,11} + a_{2,22})s + (a_{2,11}a_{2,22} - a_{2,12}a_{2,21})}$$
$$\left\{\frac{M_\delta^*}{\phi(s)}(C_1C_2+1)\alpha_c(s) - \frac{M_\delta^*}{\phi(s)}b_{\dot{q}_0}(s) + \frac{M_\delta^*\hat{M}_\delta^*}{\phi(s)}b_{\delta_0}(s)\right\} \tag{20}$$

(20) can be simplified as (21) assuming $\tau \simeq 0$ for analysis purpose. Thanks to enhanced computation power and reduced transmission time in recent avionics systems, this assumption is reasonable.

$$\alpha(s) = \frac{T(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where

$$T(s) = (C_1C_2+1)\alpha_c(s) - b_{\dot{q}_0}(s) + \hat{M}_\delta^* b_{\delta_0}(s) \tag{21}$$
$$2\zeta\omega_n = (C_1 + C_2) + \left(\hat{Z}_\alpha^* - Z_\alpha^*\right)$$
$$\omega_n^2 = (C_1C_2+1) + C_2\left(\hat{Z}_\alpha^* - Z_\alpha^*\right)$$

ζ and ω_n represent a damping ratio and a natural frequency for the closed loop system.

Absolute stability is normally guaranteed for a damped system, so a condition $\mathscr{G}$ to maintain stability under measurement biases and model uncertainties can be proposed from $2\zeta\omega_n > 0$ (Cond.1) under $\omega_n^2 > 0$ (Cond.2) as follows.

$$\begin{aligned}\mathscr{G} &= \{C_1, C_2 \in \mathbb{R}_{>0} | \text{Cond. 1 \& Cond. 2}\}\\ \text{Cond. 1} &: C_1 + C_2 > -Z_\alpha^*\Delta_{Z_\alpha^*}\\ \text{Cond. 2} &: C_1C_2 + C_2Z_\alpha^*\Delta_{Z_\alpha^*} > -1\end{aligned} \tag{22}$$

$\Delta_{(\cdot)}$ indicates an uncertainty in aerodynamic derivative estimates $\hat{(\cdot)} = (\cdot)\{1 + \Delta_{(\cdot)}\}$.

$\alpha_c(s) = \frac{\alpha_c}{s}$, $b_{\dot{q}_0}(s) = \frac{b_{\dot{q}_0}}{s}$, and $b_{\delta_0}(s) = \frac{b_{\delta_0}}{s}$ for a step input and constant biases. Then, $\alpha(s)$ becomes

$$\alpha(s) = \frac{(C_1C_2+1)\alpha_c - b_{\dot{q}_0} + \hat{M}_\delta^* b_{\delta_0}}{s^2 + 2\zeta\omega_n s + \omega_n^2}\frac{1}{s} \tag{23}$$

Steady state error e_{ss} can be derived from

$$e_{ss} = \alpha_c - \lim_{t\to\infty}\alpha(t) = \alpha_c - \lim_{s\to 0} s\alpha(s) \tag{24}$$

Then, e_{ss} can be suggested as below.

$$e_{ss} = \frac{\eta_2}{\eta_1+\eta_2}\alpha_c + \frac{1}{\eta_1+\eta_2}b_{\dot{q}_0} + \frac{\eta_3}{\eta_1+\eta_2}b_{\delta_0}$$

where

$$\eta_1 = C_1C_2 + 1 \tag{25}$$
$$\eta_2 = C_2\left(\hat{Z}_\alpha - Z_\alpha\right) \qquad \eta_3 = -\hat{M}_\delta^*$$

From the closed loop analysis above, following two main observations can be found, and they can be further understood by comparing to Jeon (2018) where closed-loop analysis with the same control structure was carried out only considering model uncertainties under the assumption of perfect measurements.

First, $b_{\dot{q}_0}$ and b_{δ_0} do not affect the characteristic equation which is the same as the one in Jeon (2018) without considering biases. Hence, the condition for absolute stability (22) becomes the same with the one in Jeon (2018) to maintain stability just under the model uncertainties. Unlike $\Delta_{Z_\alpha^*}$ from BKS for the outerloop, $\Delta_{M_\delta^*}$ from IBKS for the innerloop doesn't have any impact.

Second, $b_{\dot{q}_0}$ and b_{δ_0} additionally cause the second and the third terms in steady state error (24) where the first term is identical to the one in Jeon (2018). One of important characteristics obtained from the analysis in Jeon (2018) is that the system is robust with respect to $\Delta_{M_\delta^*}$ if a control command is calculated, transmitted and reflected fast enough to a real control surface deflection, even though M_δ^* information is required for implementation of IBKS. However, if $b_{\dot{q}_0}$ and b_{δ_0} are considered, $\Delta_{M_\delta^*}$ starts to have an impact to the steady state error, as it can be seen especially in b_{δ_0} related term of (25).

5. SIMULATION

Simulations are carried out to verify theoretical analysis results suggested in previous section. With a piece-wise approach as in Lee (2016), several points for each grid were simulated, and as an example, results when altitude is 7.6200km and $U_0 = 185.9280$m/s are suggested in this paper. The corresponding aerodynamic derivatives are $Z_\alpha^* = -1.963$, $Z_\delta^* = 0$, $M_\alpha^* = -4.749$, $M_q^* = -3.933$ and $M_\delta^* = -26.68$.

Simulation parameters such as an angle of attack command, design parameters, level of a model uncertainty in control effectiveness information, and biases on additional measurements are suggested in table 1. The initial values for α and q are 0° and 0°/s. Small enough $\tau = 0.001sec$ is applied for the simulation. The effects of $\Delta_{Z_\alpha^*}$ to this closed loop system comes from BKS for the outerloop. Since the main objective of this research is to have critical understandings with IBKS, it is assumed in this simulation that there is no model uncertainty in $\hat{Z}_\alpha^*$. As suggested in previous section, the condition to guarantee absolute stability for the system is only affected by $\Delta_{Z_\alpha^*}$. Without $\Delta_{Z_\alpha^*}$, absolute stability can be accomplished just with positive design parameters, as intended in previous controller design process. Theoretical analysis results indicate that a primary effect of biases on additional measurements is in a steady state error, so stable cases with $\Delta_{Z_\alpha^*} = 0$ are examined in this simulation.

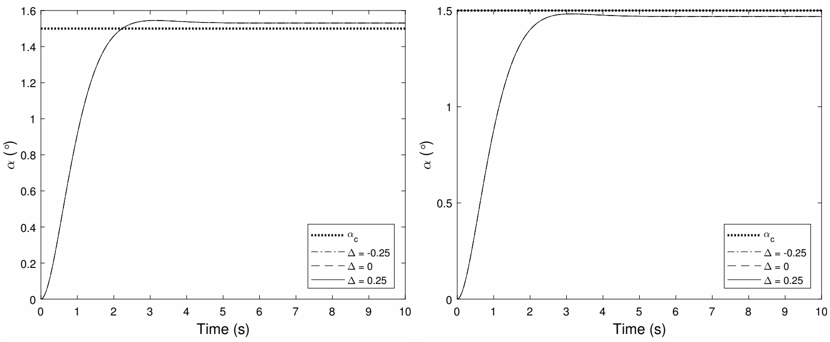
Table 1. Simulation Parameters

Parameter	Value
α_c	1.5°
C_1, C_2	1.5
$\Delta_{\hat{M}_\delta^*}$	$[-0.25,\ 0,\ 0.25]$
$b_{\dot{q}_0}, b_{\delta_0}$	$[-0.1,\ 0.1]$

Closed loop system responses obtained from simulations considering only $b_{\dot{q}_0}$ are suggested in Fig.1, and e_{ss} values predicted by (25) for corresponding cases are summarized in Table 2. When b_{δ_0} is only considered, simulation results are proposed in Fig.2 and predicted steady state errors by (25) are suggested in Table 3. Since $\hat{Z}_\alpha^*$ information is assumed not to have any uncertainty in this simulation, ζ and ω_n of the system doesn't change depending on cases as expected, resulting in the same rising and settling time for Fig.1 and Fig.2. Steady state errors identified from simulations and predicted from the analysis result (25)

Table 2. Predicted Steady State Error e_{ss} by (25) with $b_{\dot{q}_0}$

$\Delta_{M_\delta^*}$ \ $b_{\dot{q}_0}$	$-0.1°/s$	$0.1°/s$
-0.25	$-0.0308°$	$0.0308°$
0	$-0.0308°$	$0.0308°$
0.25	$-0.0308°$	$0.0308°$

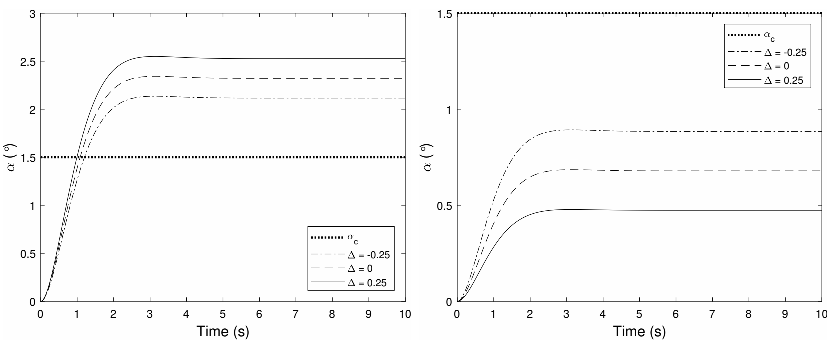


(a) Time response when $b_{\dot{q}_0} = -0.1$ (b) Time response when $b_{\dot{q}_0} = 0.1$

Fig. 1. Closed-loop System Response with $b_{\dot{q}_0}$

Table 3. Predicted Steady State Error e_{ss} by (25) with b_{δ_0}

$\Delta_{M_\delta^*}$ \ b_{δ_0}	$-0.1°$	$0.1°$
-0.25	$-0.6158°$	$0.6158°$
0	$-0.8211°$	$0.8211°$
0.25	$-1.0263°$	$1.0263°$



(a) Time response when $b_{\delta_0} = -0.1$ (b) Time response when $b_{\delta_0} = 0.1$

Fig. 2. Closed-loop System Response with b_{δ_0}

are shown to be the same for every cases. Additionally, following phenomena observed in Fig.1 and Fig.2 can be also understood from (25). While $\Delta_{M_\delta^*}$ has an impact on steady state errors induced by b_{δ_0}, steady state errors induced by $b_{\dot{q}_0}$ are not affected by $\Delta_{M_\delta^*}$. Besides, the effect of b_{δ_0} is appeared to be bigger than of $b_{\dot{q}_0}$ in steady state error point of view, because $|\hat{M}_\delta^*| > 1$ in this simulation.

6. CONCLUSION

In this paper, closed loop analysis with IBKS considering both measurement biases and model uncertainties, is performed to have critical understandings especially about measurement bias effects. In previous study where closed loop characteristics with IBKS considering only model uncertainties are investigated, it is shown that a system is robust with respect to an uncertainty even in control effectiveness information if a control command is calculated, transmitted and reflected fast enough to a real control surface deflection. However, if measurement biases are additionally considered, the analysis results in this paper indicate that a model uncertainty in control effectiveness information starts to have an impact to a steady state error. These biases on additional measurements cause a steady state error, but they do not have any impact to the characteristic equation. These properties obtained from the analysis are verified through simulations.

REFERENCES

P. V Kokotovie. The Joy of Feedback: Nonlinear and Adaptive. *IEEE Control Syst.*, pp. 717, 1992.

T. Lee and Y. Kim. Nonlinear Adaptive Flight Control Using Backstepping and Neural Networks Controller. *Journal of Guidance*, Control, and Dynamics, Vol. 24, No. 4, pp. 675-682, 2001.

P. Van Gils, E. van Kampen, C. C. de Visser, and Q. P. Chu. Adaptive Incremental Backstepping Flight Control for a High-Performance Aircraft with Uncertainties. *AIAA SciTech Forum*, no. January, pp. 126, 2016.

A. Ait Haddou Ali, Q. P. Chu, E.-J. Van Kampen, and C. C. de Visser. Exploring Adaptive Incremental Backstepping using Immersion and Invariance for an F-16 aircraft. *AIAA Guid. Navig. Control Conf.*, no. January, 2014.

P. Acquatella, E.-J. van Kampen, and Q. P. Chu. Incremental Backstepping for Robust Nonlinear Flight Control. *CEAS Conf. Guid. Navig. Control*, no. April 2013, pp. 14441463, 2013.

G. P. Falconi, V. A. Marvakov, and F. Holzapfel. Fault tolerant control for a hexarotor system using Incremental Backstepping. *2016 IEEE Conf. Control Appl. CCA 2016*, no. 1, pp. 237242, 2016.

B. Jeon, M. Seo, H. Shin, and T. Antonios. Understandings of the Incremental Backstepping Control through Theoretical Analysis under the Model Uncertainties *IEEE Conference on Control Technology and Applications*, no. August, pp.318323, 2018.

S. Sieberling, Q. P. Chu, and J. A. Mulder. Robust Flight Control Using Incremental Nonlinear Dynamic Inversion and Angular Acceleration Prediction *J. Guid. Control. Dyn.*, vol. 33, no. 6, pp. 17321742, 2010.

E. J. J. Smeur, Q. Chu, and G. C. H. E. de Croon. Adaptive Incremental Nonlinear Dynamic Inversion for Attitude Control of Micro Air Vehicles. *J. Guid. Control. Dyn.*, vol. 39, no. 3, pp. 450461, 2016.

X. Wang, E. van Kampen, Q. P. Chu, and P. Lu. Stablity Analysis for Incremental Nonlinear Dynamic Inversion Control. *AIAA SciTech Forum*, no. January, pp. 116, 2018.

C. H. Lee, B. E. Jun, and J. I. Lee. Connections Between Linear and Nonlinear Missile Autopilots via Three-Loop Topology *J. Guid. Control. Dyn.*, vol. 39, no. 6, pp. 14241430, 2016.

J. E. Slotin and W. Li. Applied Nonlinear Control. *Prentice-Hall*, Upper Saddle River, 1991.

H. K. Khalil. Nonlinear System. *Prentice-Hall*, Upper Saddle River NJ, 1996.

M. Krstic, I. Kanellakopoulos and P. Kokotovic. Nonlinear and Adaptive Control. *John Wiley and Sons, Inc.*, 1995.

B. S. Kim, Y. D. Kim, H. C. Bang, M. J. Tahk, and S. K. Hong. Flight Dynamics and Control. *Kyung Moon Sa*, 2004.

F. Golnaraghi, and B. C. Kuo. Automatic Control Systems. *Wiley*, 2009.