Fuzzy Interval-Valued Multi Criteria Based Decision Making for Ranking Features in Multi-Modal 3D Face Recognition

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Abstract- This paper describes an application of multi-criteria decision making (MCDM) for multi-modal fusion of features in a 3D face recognition system. A decision making process is outlined that is based on the performance of multi-modal features in a face recognition task involving a set of 3D face databases. In particular, the fuzzy interval valued MCDM technique called TOPSIS is applied for ranking and deciding on the best choice of multi-modal features at the decision stage. It provides a formal mechanism of benchmarking their performances against a set of criteria. The technique demonstrates its ability in scaling up the multi-modal features.

Keywords- 3D Face Recognition, multi-modal features, interval values, evidential reasoning under uncertainty, fuzzy fusion, TOPSIS, MCDM.

1 Introduction

Face recognition systems benefit from multi-modal feature (MMF) sets and their performance can outway that of individual modalities [1]. Multi-modal systems utilise multiple information sources enabling increased performance, reliability and filling in missing information. MMFs play a key role in fusing information towards decision making in a face recogniton system. In situations where several modalities may be identified such as multiple sensor configurations or combinations of feature sets, the problem becomes that of selecting the right modality for the application. The Cumulative Match Curve (CMC) which is a set of performance plots typically used in biometric systems may exhibit similar responses of the modalaties under the same environmental conditions or the number of parameters to deal with are large making the feature selection process a difficult task. In such cases, subjective judgements that do not have a 100% certainty or due to lack of data or incomplete information lead to decision making under uncertainty [2].

1.1 Multi Criteria-based Decision Making (MCDM)

Evidential reasoning (ER) denotes a body of techniques specifically for reasoning from evidential information [3]. ER requires two parameters namely a structure to encompass the collected evidence and a framework for evidence accumulation using fusion techniques [4]. Multi-Attribute Decision Making (MADM) or otherwise known as Multiple Criteria-based Decision Making (MCDM) refers to making decisions in the presence of multiple, usually conflicting set of criteria. Operations Research (OR) models have the capability of making decisions in the presence of multiple, usually conflciting criteria. They use mathematical programming techniques in a continuous decision space. MCDM techniques are a branch of the OR techniques except that they deal with discrete spaces where the set of decision alternatives is pre-determined. A key characteristic of MCDM techniques is that they use both qualitative and quantitative attributes for evidential reasoning which is ideal for modelling uncertanties dealing with incomplete and vague information [5-7]. MCDM techniques share certain common terminology as follows:

- **Alternatives:** Alternatives relate to the available options from which ranked selections are made.
- **Criteria or Attribute**: The MCDM is associated with a set of criteria or attributes that will impact the selection of the alternatives. An attribute is a property; quality or feature of alternatives being considered. Multiple criteria are typically organised into a set of sub-criteria or sub-attribute.
- **Weights –** Weights provide relative importance of criteria provided by decision makers.
- **Decision Makers (DMs) –** a set of experts providing weights to each criterion.

- **Decision Matrix –** a matrix that is used to make objective decisions from several options. DMs rate each criterion of each alternative.

An MCDM problem may thus be described by a decision matrix **D**. Suppose that there are m alternatives that are assessed by n attributes or criteria, then **D** is an $m \times n$ matrix. An MCDM problem is typically described as a decision matrix as follows [8]:

$$\mathbf{D} = r_{ij} = \begin{vmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{vmatrix} \tag{1}$$

The set of alternatives is denoted by $A_1, A_2, \dots, A_m$ and the criteria denoted by $C_1, C_2, \dots, C_n$ and x_{ij} represents the rating of alternative A_i with respect to criteria C_j. When the ratings are described in linguistic terms, x_{ij} is replaced by r_{ij}. From a performance evaluation perspective, r_{ij} indicates the performance of alternative A_i when evaluated against the criteria C_j. The decision maker DM determines the weights W_j of relative performance of C_j. W_j may also be described linguistically. This information is as shown in TABLE I. The MCDM problem then becomes that of determining the optimal alternative A_i given the set of criteria C_j that are to be met.

TABLE I DECISION MATRIX REPRESENTED AS A TABLE

	Criteria			
	C_1	C_2	…	C_n
Alternatives↓	W_1	W_2	…	W_n
A_1	r_{11}	r_{12}	…	r_{1n}
A_2	r_{21}	r_{22}	…	r_{2n}
…	…	…	…	…
A_m	r_{m1}	r_{m2}	…	r_{mn}

Popular MCDM techniques include ELECTRE (Elimination et Choice Translating Reality) [9], SAW (Simple Adaptive Weighting) [10], TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [8], AHP (Analytical Hierarchy Process) [8], ANP (Analytic Network Process) and SMART (Simple Multi Attribute Rating Technique) [11] to name a few. Some of these techniques have been benchmarked for a rigorous classification problem in [12]. Amongst the MCDM techniques, TOPSIS has the unique advantage of incorporating preferential group decision making given a set of alternatives. It defines a set of ideal solutions called *ideal positive* and *ideal negative* solutions which are used as reference points and with respect to which distance measures are computed as a logical process of ranking the set of alternatives [13] [14]. TOPSIS works on the basis of a-priori based weights of attributes, subjective weights of experts and individual judgement matrix [15-16]. In [15], a correlation between individual and group judgements are first established which are used to adaptively modify the weights of experts that contribute to the group's decision, thereby increasing the confidence in the individual's judgement.

1.2 Interval Valued Fuzzy MCDM

MCDM has recently been applied in the development of multi-modal face recognition systems involving multiple criteria with varying weights of importance. The decision matrix **D** can be defined by fuzzy logic by assigning each element of the matrix a degree of membership in the interval [0,1]. This proves a strong tool for representing human knowledge. A well known extension is the theory of interval-valued fuzzy set (IVFS). The membership degree of each element on an IVFS is defined on a closed subinterval of [0, 1]. MCDM deals with the concept of group decision making whereby the views of the decision makers can be captured by interval values [17]. Such interval valued judgements permit further discussion, negotiation and analysis between the members of the group before arriving at a concensus [15]. An extension of the IVFS theory led to the concept of type-2 fuzzy sets[18] in which the membership function (normally type-1) is itself represented by a fuzzy set as the interval [0,1]. See Fig. 1 and Fig. 2 for this distinction. Thus we can see the relevance of type-2 systems for information fusion due to its improved results and accomodation to uncertainty in comparison with type-1 systems. Type-2 fuzzy systems minimise any loss of information when transferring the interval-based data into into fuzzy set models.

In [19], interval type-2 fuzzy logic and fuzzy integrals are used in the feature extraction stage of training data followed by a relevance measure used in decision making for a face recognition application. A face recognition system[20] extends fuzzy TOPSIS based on Sugeno Integral type2 fuzzy system [8] for aggregating results from modular neural networks (MNN). The sytem associates a degree of uncertainty defined by the footprint of uncertainty, FOU [21] (Section 7) to the fuzzy densities of the Sugeno interval and the ouputs of the MNNs. Applying the MCDM approach for aggregating the performance measures when tested against different databases provides a ranking mechanism based on a comprehensive analysis of the feature modalities used for recognition. The system is used to benchmark an operator's performance, in this case, face databases as ORL [22], Yale [23] and FERET [24]. With the FERET database, the algorithm shows very high statistical significance for type2 when compared with type1 performance and hence its suitability for such problems.

Other applications that deal with uncertainty through effective use of type-2 fuzzy logic include 2D face recognition [20], [25], [26], emotion recognition [27] fuzzy fusion [28], IR and visible imagery fusion [29], and video face recognition [30]. In this paper, we extend the the interval valued fuzzy TOPSIS (IVFT) technique for fusing information from multi-modal features of a 3D face recognition system. The IVFT approach in [19], [20] is followed that aggregates information from MMF sets during decision making in a 3D face recognition (3DFR) system involving multi-criteria. The 3DFR system description in [31] demonstrates the complexity of performance analysis even with a partial criteria set being considered as seen from the CMC plots that are typically used for making a decision. In order to arrive at an objective measure of ranking the features in a recognition task, IVFT technique is applied to the 3DFR system that is benchmarked against a set of 3D face databases. By this process, it is shown that the IVFT has the potential for performance analysis on a full scale. The mechanism proves to be useful in decision making when the choice of alternatives of the feature sets is combinatorial and complex. It also lends itself well to data scaling.

The 3D face recognition system is presented in Section 2 with an analysis of the performance characeristics for the application of TOPSIS. The basis of TOPSIS and its algorithmic approach is presented in Section 3. It sets up the interval valued fuzzy set formalisation for TOPSIS. It also details out the algorithmic steps necessary for fuzzy TOPSIS implementation. In Section 4, the interval valued fuzzy TOPSIS is applied to 3DFR and the results are analysed. Section 4 also illustrates the technique with a numerical example. Section 5 provides a conclusion from the results and outlines further work.

2 Multi-Modal 3D Face Recognition

In this section, the 3D face recognition system proposed in [31] is considered that treats mutliple instances of features as the multi-modality. The system is revisited here and revised based on [32] in which the MCDM approach to a scaled down 3DFR is outlined. The image feature considered in the system involves the extraction of depth intensity profiles called signature profiles sampled along the depth dimension for specific angular planar intersections with the image. Fig. 3. shows sample directions in which such a slicing can take place. Consider a 3D face image in Fig. 4(a) that is sliced at regular intervals. Intuitively, the slicing [33] along a particular direction appears as shown in Fig. 4(b). The dotted lines show the intersection of parallel planes with the image. An example of 3D signatures (depth variations) along each slice as one-dimensional plots are as shown in Fig. 5. The set of slices for an image form a compressed shape signature set and act as a level-1 feature vector. This effect is shown in Fig. 6. A level-2 feature set is formed by determining central moments [34] on the signatures. These features now form the templates of a model in the feature space. Two variations to the model representation in the feature space is generated depending on whether the individual samples are combined to form a single template or retained as such. The Fisher's Linear Discrimant Analysis (FLDA) is used for classification. The system architecture is as shown in Fig. 7. An extensive performance analysis of the

MMF sets is carried out by benchmarking against two different 3D databases namely FRVT[33] and a local student database. The terms FRVT and FRGC are used intermittently to mean the same.

In this work, a distinction is made between single vs multi-modality depending on the combinations of directional feature vectors chosen. A single modality is associated with each individual direction called a unary feature. When feature vectors from multiple directions are combined, they constitute a multi-modal feature vector.

The remainder of this Section is organised into two main parts. Sections 2.1-2.5 present the multi-modal 3D Face Recognition (MM-3DFR) system. Sections 2.6-2.7 restructers the performance analysis in [31] to suit the proposed MCDM feature ranking and selection process as outlined in [32].

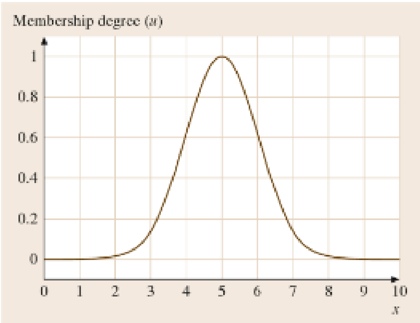


Fig. 1. Type-1 Fuzzy System [8]

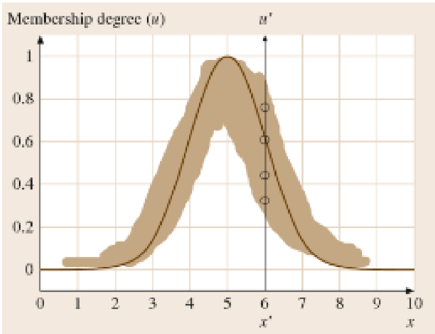


Fig. 2. Type-2 Fuzzy System [8]

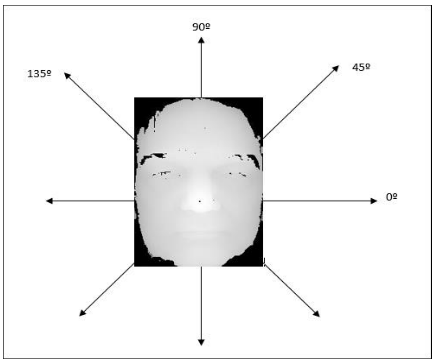
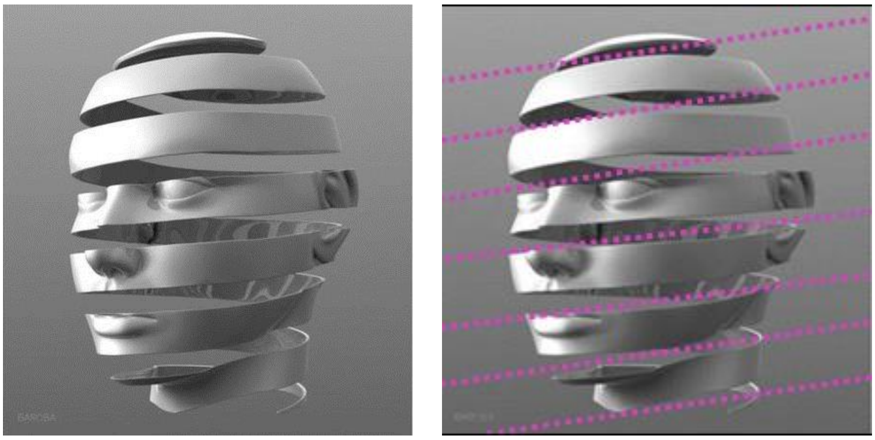


Fig. 3. Directions of Planar Intersections with a 3D Face Image



(a)

(b)

Fig. 4. Intuitive Representation of 3D Face Image Sampling in a Specific Direction[1]

[1] http://www.shapeways.com/product/J7X36WQLC/escheresque-face-peeling

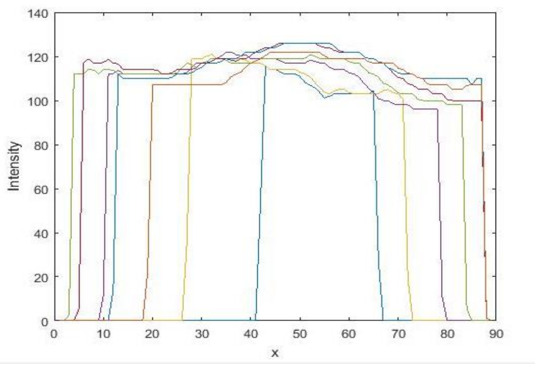
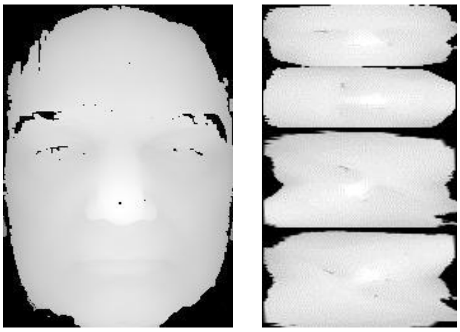


Fig. 5. Sampled Signatures: Depth Variations along the Slices



(a) 3D Face Sample (b) Directional Signatures

Fig. 6. Original FRVT Image and Resulting Directional Signatures Showing the Effect of Sampling

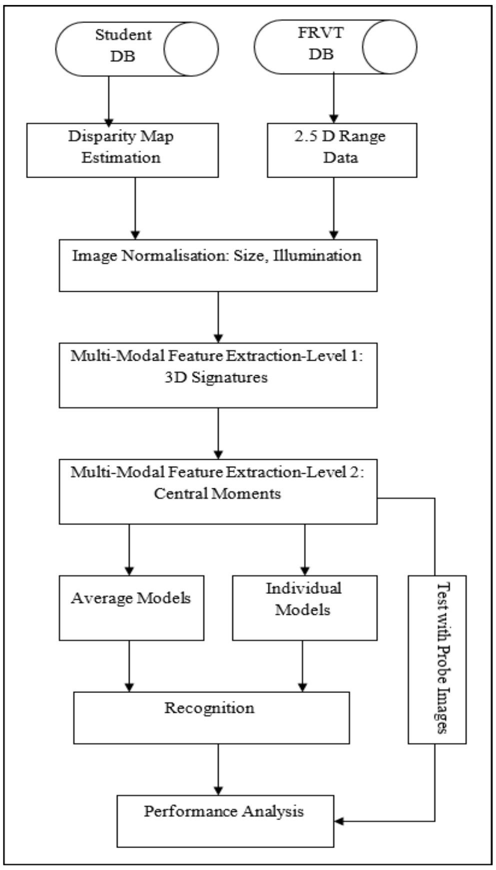


Fig. 7. 3DFR System Architecture

2.1 3D Face Databases

The proposed 3DFR system deals with two different databases namely a student database and the FRVT v1 database. They are both acquired using different 3D imaging systems. The student DB is a disparity map derived from a stereo-vision pair of left and right images from the commercial stereo-vision system [35], samples of which are shown in Fig. 7. The FRVT database consists of 3D images from a Minolta Vivid sensor and a Qlonerator sensor. A sample from the FRVT database is shown in Fig. 6(a). In this paper, the shape channels of the FRVT dataset are used [33].

a) Student Database:

A student database is captured with a Stereo Vision System (SVS) [35] under a controlled illumination lab environment. Small variations in pose were allowed. Two subsets of the student database are considered based on the focal length of the camera, namely 7.5mm and 12.5mm. Each subset consists of 100 subjects with 10 canonical views per subject (fixed sample sizes) totalling 1000 images. The canonical views span 180º and therefore an approximate 18º separation between two consecutive samples. The canonical views must have sufficient overlap to provide within class correlation. The typical disparity maps for the samples of a subject by SVS are shown in Fig. 8. The images were pre-processed for any depth discontinuities by a left-right check of the stereo pair provided by the SVS system. Any texture less areas were filled using standard morphological opening and closing operations. These steps are illustrated in Fig. 9.

b) FRVT Database:

The FRVT data base consists of 275 subjects with varying sample sizes as shown in TABLE II with a total of 943 images. The database consists only of frontal images. The images vary in illumination and scaling.

2.2 Image Normalisation

Both the student and FRVT databases were manually cropped and resized to an image size of 128x128 pixels. The student DB was acquired in an illumination controlled environment; hence did not require further illumination normalization. The FRVT database required illumination normalisation using the standard histogram equalisation technique available in MATLAB. Thus, the DBs were normalised with respect to scaling and illumination. In the face recognition system, the rest of the stages are maintained the same for both databases as shown in Fig. 9.

2.3 3D Profile Signatures Generation and Feature Extraction

To start with, signatures are derived at the intersections of the depth surface with evenly spaced vertical planes intersecting the image. The signatures act as profile curves at sample points, say along the Y-axis (90º) of the image. For convenience, a fixed set of 40 signatures is derived for each image. Sampling takes place at points of intersection of a stack of planar surfaces oriented in a particular angle with the images. An alternative means to determining the directional signatures is to extract the 3D profile signatures on a rotated image, keeping the axis of slicing planes fixed. The 3D signatures appear as a compressed image due the effect of sampling in 3D as seen from Fig. 6(b). Directional signatures from a desired direction are derived to form multiple instances of the feature sets.

Basic variations in intersecting planar angles with an image include 0º, 45º, 90ºand 135º and the corresponding signatures are used as uni-modal (unary) features. Additional feature sets are organised into three MM categories θ_i depending on the level of the linear combination of unary features. Each category has further sub-categories with permutation ${}^{n}C_{r}$.as a result of the linear combination of the fundamental uni-modal feature vector. Thus, an *N*-ary set of MM features are derived as follows:

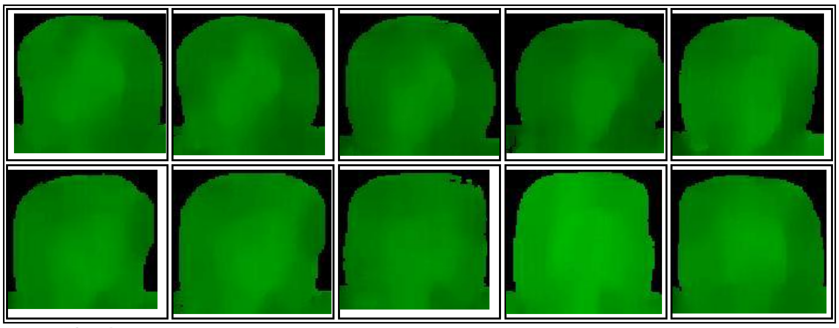


Fig. 8. Student Database: Disparity Maps from Canonical Views of a Subject

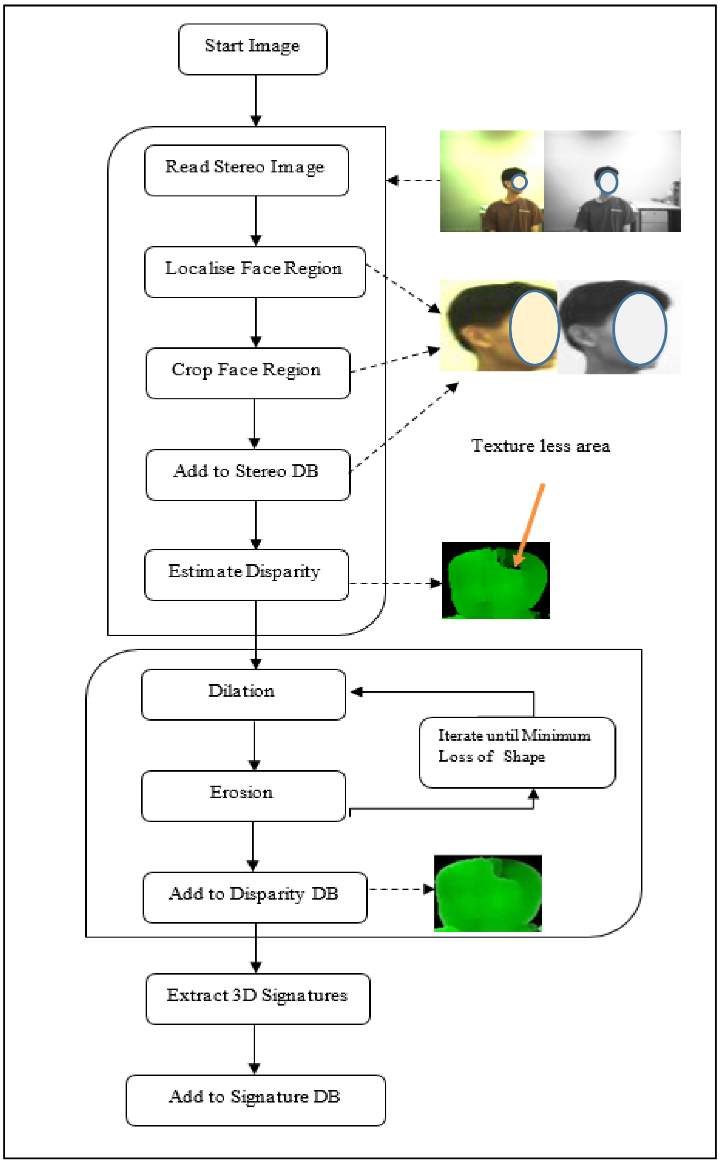


Fig. 9. Pre-processing and Feature Space Formalisation. Faces are masked to retian anonymity.

TABLE II FRVT SAMPLE SIZE FREQUENCIES

Sample Size	1	2	3	4	5	6	7	8
Frequency	7	32	47	33	28	30	15	13

$$\left.\begin{array}{c} Unary\ features, \theta_1 \in {}^1_4C = \{0,45,90,135\} \\ Binary\ features, \theta_2 \in {}^2_4C = \{0+45, 0+90, 45+90, 0+135, 45+135, 90+135\} \\ Ternary\ features, \theta_3 \in {}^3_4C = \{0+45+90, 0+45+135, 0+90+135, 45+90+135\} \\ Quadruple\ features, \theta_4 \in {}^4_4C = \{0+45+90+135\} \end{array}\right\}$$

(1)

Indices to the above feature sets are $\{[1,4],[5,10],[11,14],[15]\}$ and respective cardinalities are $\{4,6,4,1\}$.

The profile signatures in Eqn. (1) form the first level of features from the 3D image data set. A set of 7 central moments are further estimated on the shape surface of the signatures to form a second level of features which further reduce the dimensionality of the feature vectors. In the discrete domain, the two dimensional standard moment of order $(p+q)$ of a function $f(x,y)$ is given by[34],[36]:

$$m_{pq} = \sum_x \sum_y f(x,y)\, x^p y^q\,, \qquad p,q = 0,1,2,\dots,r$$

(2)

for some positive r and $x,y \in \Omega$ where Ω is the subimage within which the signature shape lies. The position invariant central moments are given by:

$$M_{pq} = \sum_x \sum_y f(x,y)\,(x-\overline{x})^p (y-\overline{y})^p\,, \qquad p,q = 0,1,2,\dots,r$$

(3)

where $\overline{x} = m_{10}/m_{00}$ and $\overline{y} = m_{01}/m_{00}$ where $\overline{x}$ and $\overline{y}$ are the co-ordinates of the centroid of the shape. From these central moments, a set of normalised central moments are defined by:

$$\eta_{pq} = \frac{M_{pq}}{{M_{pq}}^n}, \qquad n = (p+q)$$

(4)

The moment invariants are now given by:

$$\left.\begin{array}{c} \emptyset_1 = \eta_{20} + \eta_{02}, \\ \emptyset_2 = (\eta_{20} - \eta_{02})^2 + 4{\eta_{11}}^2, \\ \emptyset_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2, \\ \emptyset_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} - \eta_{03})^2, \\ \emptyset_5 = (\eta_{30} + 3\eta_{12})(\eta_{30} + \eta_{12})((\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2) + \\ (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})(3(\eta_{30} + \eta_{12})^2 - (\eta_{21} - \eta_{03})^2), \\ \emptyset_6 = (\eta_{20} - \eta_{02})((\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2) + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}), \\ \emptyset_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})((\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2) \\ -(\eta_{30} - 3\eta_{12})(\eta_{21} - \eta_{03})(3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2). \end{array}\right\}$$

(5)

2.4 Model Representation

Eqn.(2) represents a set of features that are used as templates in the feature space. These templates constitute a model representation for each subject in the database. Two approaches to this model formation are followed:

a) *Average Model:* An average image is constructed from the samples available for each subject. The average image is a blurred version of the individual images. Intuitively, such a model captures imprecision from its samples. The result is a single image model/subject. With the student DB, an average image is generated from fixed sample sizes (i.e. number of samples/subject=k, constant). However, the FRVT database, as seen from TABLE II, has varying number of samples for each subject and the average image is constructed from such samples.

b) *Individual Model:* In this representation, each sample acts a model and hence we have several models/subject. The individual models are useful when there are insufficient samples for the subjects as in the case of the FRVT dataset where the number of samples/subject is one for some part of the database (TABLE II).

Both models are used as benchmarking criteria for the feature sets in the 3DFR system. It is expected that the within-class distance is larger in the case of the average model compared to the individual model as it is a fuzzy representation encompassing the average information from all of the samples of a subject. Therefore, with the average model representation, it is not expected to produce a 100% match score between the query and the target images even for validation tests. However, this does not imply that it is a poor representation as it allows an implicit modelling of imprecision within the dataset. In contrast, the individual model representation can lead to very sensitive performance measures in a generalisation test since the query images may deviate significantly from the models.

From TABLE III, it is noted that six different database partitions are possible. These depend on the following set compositions:

(i) Number of databases, $\delta_i, i = 1{,}2$, namely Student and FRVT respectively.
(ii) Model representation, $\mathrm{M}_j, j = 1{,}2$, namely *Average* and *Individual.*
(iii) In the case of the Student DB, two focal lengths are considered, $\psi_k, k = 1{,}2$, corresponding to 7.5mm and 12.5 mm

In other words, we arrive at the following database configurations:

(i) *Student DB:* $\{\mathrm{M}\ x\ \psi\ \}$
(ii) *FRVT DB*: $\{\ \mathrm{M}\ \}$

TABLE III 3DFR DATABASES USED FOR BENCHMARKING

Database Notation	**DB δ**	**Focal Length ψ**	**Model M**	**Number of Subjects**	**Samples/ Subject**	**Size of DB**
DB_1, 7.5-Ind	Student	7.5mm	Individual	100	10	1000
DB_2, 7.5-Avg	Student	7.5mm	Average	100	10	100
DB_3, 12.5-Ind	Student	12.5mm	Individual	100	10	1000
DB_4, 12.5-Avg	Student	12.5mm	Average	100	10	100
DB_5, FRVT-Ind	FRVT	-	Individual	973	Varies [1,8]	973
DB_6, FRVT-Avg	FRVT	**-**	Average	973	Varies [1,8]	275

2.5 Fischer's Linear Discriminant Analysis

Fischer's Linear Discriminant Analysis (FLDA) as a classifier has been very successful in face recognition [37]. Such classifiers perform LDA training via scatter matrix analysis. For an *M* class classification, the within- and between-class scatter matrices S_w and S_b respectively, are computed as follows:

$$S_w = \sum_{i=1}^{M} \mathrm{PR}(\omega_i) C_i \tag{6}$$

$$S_b = \sum_{i=1}^{M} \Pr(\omega_i)(m_i - m_0)(m_i - m_0)^T \tag{7}$$

where $\Pr(\omega_i)$ is the prior class probability and usually replaced by $1/M$ in practice with the assumption of equal probability. S_w depends on the average scatter C_i of the sample vectors x of the individual classes ω_i around their means m_i:

$$C_i = \mathrm{E}[(\mathrm{x} - m_i)(x - m_i)^T],\ \omega = \omega_i \tag{8}$$

Similarly, S_b represents the scatter of the conditional mean vectors m_i around the overall mean vector m_0. Various measures are available for quantifying the discriminative power, a commonly used one being the ratio of the determinant of the between- and within-class scatter matrices of the projected samples:

$$J(\mathrm{V}) = \left|\frac{\mathrm{V}^T S_b \mathrm{V}}{\mathrm{V}^T S_w \mathrm{V}}\right| \tag{9}$$

The optimal projection matrix V which maximises J(V) can be obtained by solving the generalised eigenvalue problem:

$$S_b \mathrm{V} = \lambda S_w \mathrm{V} \tag{10}$$

The Fisher-face method uses a subspace projection prior to LDA to avoid the possible singularity in S_w. For the scatter matrices defined in Equations (6) and (7), the matrix V_{opt} cannot be found directly from Equation (9) because in general the matrix S_w is singular. This stems from the fact that the rank of S_w is less than *N-M*, and in general the number of pixels in each image is much larger than the number of images in the learning set *N*. In [8], the Fisherfaces method avoids S_w being singular by projecting the image set onto a lower dimensional space so that the resulting within class scatter is non-singular. This is achieved by using Principal Component Analysis (PCA) to reduce the dimension of feature space to *N-M* and then applying the standard linear discriminant on the resulting separation matrix defined in Equation (10) to reduce the dimension to *M-1*. Thus, we have,

$$V = V_{Fisher}\, V_{PCA} \tag{11}$$

where

$$V_{PCA} = \arg max_T |\mathrm{V}^T C \mathrm{V}| \qquad \text{and} \tag{12}$$

$$V_{Fisher} = \arg max \left|\frac{\mathrm{V}^T V_{PCA}^T\, S_b\, V_{PCA}\mathrm{V}}{\mathrm{V}^T V_{PCA}^T\, S_w\, V_{PCA}\mathrm{V}}\right| \tag{13}$$

This is the approach followed in this paper. Let a training set of *N* face images represent *M* different subjects. The face images in the training set are two dimensional arrays of disparity/range image values, represented as vectors of dimension *nx1*. Different instances of a person's face are defined to be in the same class and faces of different subjects to be from different classes.

Equation (6) forms the feature vector space for cluster analysis. Hence every sample in the set of *N* face images is projected onto this feature vector corresponding to the columns of V_{Fisher} and a set of features is extracted for each sample image in the training set. Alternatively, average of feature vectors may be determined for each class. This provides a generalised feature vector for each class and minimises the number of searches during matching. The Fisher's LDA algorithm above is applied on the central moments extracted as features $\emptyset_i$ in Eqn.(5).

2.6 Classification and Query Processing

Using Euclidean distance in the feature space performs the recognition task. This is given by:

$$D(\Gamma, E) = \sum_{v=0}^{n} \frac{(\Gamma_v - E_v)^2}{\sum_{E \epsilon S} (\Gamma_v - E_v)^2} \tag{14}$$

where Γ_v and E_v are the projections of the test sample and a template respectively on vector *v*. *S* is the set of all templates. Therefore, for a given face image Γ, the best match E^0 is given by

$$E^0 = \arg\max\{D(\Gamma, E)\} \tag{15}$$

A query image from the *Probe Feature Sets* is matched against the *Gallery* (for definiiton, see Section 8, Appendix A). The result is a ranked set of images based on the above distance measures between the query and the models. Usually, the system is designed to perform well so long as the expected result is within a rank threshold. The lower this rank value, better is the performance. For example, if a query (probe) image identifies the right subject to lie within the top few ranks, then the system is rated to be good. With practical systems, the identification rate is an estimate of the probability that a subject is identified correctly at least at rank-k. With CMC plots (**Error! Reference source not found.**), a better system is one that works within a small threshold on rank and is closest to the top left corner of the graph.

2.7 Performance Evaluation and Analysis

Performance evaluation of the multi-modal 3DFR system is carried out using (i) CMC chart of Rank Vs Identification Rate and (ii) Equal Error Rate (EER) plots of False Acceptance Rate (FAR) Vs False Rejection Rate (FRR) [38]. The performance metrics are explained in Appendix A. For performance evaluation, the following configurations are adopted:

(i) Identification tests were carried out on each of the database in TABLE III independently.

(ii) A unified set of features were extracted for each of the database in TABLE III. It is to be noted that the templates thus formed are maintained separate for each model as indicated in the table.

(iii) With each database, two different tests were conducted namely validation and cross-validation (generalisation) tests. With the validation test, all samples in the database belong to both the gallery and probe sets. Hence, they are the same sets. In the case of the cross-validation test, a four-folder test was conducted in which the database is partitioned in the ratio of 0.7:0.3 amongst the gallery and probe tests making them mutually exclusive. Repeating this process by choosing the elements randomly leads to four different folders for cross-validation. It is ensured that all samples are selected in the process. However, since we consider a closed-set, any probe sample is expected to be an element of the gallery set. This is achieved by considering 70% of the total number of samples for each subject as elements of the gallery and the remaining 30% of the samples in the probe set. For the Student DB, since there are 10 samples/subject, each cross-validation folder contained 7 samples in the gallery and 3 in the probe set. However, with the FRGC dataset, the number of samples/subject varies.

A specific issue related to the FRGC dataset is the lack of sufficient samples/subject. Where it is not possible to divide the total number of samples per subject into the training to testing ratio of 0.7:0.3, the closest possible numbers are selected. For example, where there are only 2 samples/subject, one of it is chosen as the training sample and the other as the test. During the next repetition, this sequence is alternated where the test sample becomes the training sample and vice-versa. In this way, the four-folder cross validation may be attempted to its best.

For brevity, the results of validation test alone are considered in this paper. Further, we consider only the CMC plots. The results of benchmarking against the FRGC *Individual Model (FRGC-Ind)* and *Average Model* (*FRGC-Avg*) corresponding to $\emptyset_i = \{\emptyset_1, \emptyset_2, \emptyset_3, \emptyset_4\}$ are shown in **Error! Reference source not found.**. In order to facilitate the ranking mechanism of the feature sets employed in the system, the CMC plots have been further analysed by using the notion of Transient, Cut-off and Steady State responses. These correspond to the performance measures at key rank positions. Such a sampling reduces the cost of traversing the CMC in arriving at a decision. The **transient response** determines a rank threshold $T_{\mathcal{R}}$ at which an acceptable performance is attained. Experiments based on heuristics suggests $T_{\mathcal{R}}$ =5. The **cut-off** is the rank at which good performance is expected which is much higher than $T_{\mathcal{R}}$. This is denoted and assigned as T_{cutoff} =10. Lastly, the **steady state response** (SSR) is the rank at which the performance reaches saturation. This is denoted and assigned as T_{ssr} = 17. These response points $T = \{T_{\mathcal{R}}, T_{cutoff}, T_{ssr}\}$ are indicated in **Error! Reference source not found.**. A summary of the results is reproduced in TABLE VI-TABLE VI in which rows marked {1-4, 5-10, 11-13, 14} correspond to $\{\emptyset_1, \emptyset_2, \emptyset_3, \emptyset_4\}$ respectively. A local manual ranking is also indicated in these tables to show the relative performance of the features against the individual and average models. Using these performance criteria, inferences are drawn similar to [32].

The following notations are used:

$\Theta_{i\text{-}avg}$ and $\Theta_{i\text{-}ind}$ represent identical feature modality corresponding to *FRGC-Avg* and *FRGC-Ind* databases respectively and $\Theta_{i\text{-}avg}, \Theta_{i\text{-}ind} \in \emptyset_i$.

1. A mapping of feature sets as best performers across the board are as follows:

   a) *Unary feature performance:* $\Theta_1 \rightarrow \{V\}$. That is, amongst the unary features, {V} performs well on both DBs. i.e. $\Theta_{1\text{-}avg} = \Theta_{1\text{-}ind}$. Further:

   (i) The unary feature {V} works better on *FRGC-Avg* DB than *FRGC-Ind* DB. Refer to rows 1-4 in TABLE VI where the features are manually ranked to show this relative performance difference based on $\{T_{\mathcal{R}}, T_{cutoff}, T_{ssr}\}$.

   (ii) The unary feature {V} attains 100% performance against the *FRGC-Avg* DB, $\forall T(T_{\mathcal{R}}, T_{cutoff}, T_{ssr})$. It is sub-optimal for *FRGC-Ind* DB in that it does not reach a 100% performance.

TABLE IV RANK VS CUMULATIVE MATCH: RESPONSE CHARACTERISTICS OF UNARY FEATURES

$\Theta_i \downarrow$	**Alternatives, $\Theta_{ij} \downarrow$**		**FRGC *Individual Model***				**FRGC *Average Model***			
		Criteria→	$T_{\mathcal{R}}$ | **Rank=5**	T_{cutoff} | **Rank = 10**	T_{ssr} | **Rank=17**	**Ranking (manual)**	$T_{\mathcal{R}}$ | **Rank=5**	T_{cutoff} | **Rank = 10**	T_{ssr} | **Rank=17**	**Ranking (manual)**
Unary	1.	H	0.91304	0.93955	0.94274		0.99275	0.90646	1	
	2.	V	0.93531	0.96607	0.96607	2	1	1	1	1
	3.	D_{45}	0.82185	0.9035	0.92259		0.93478	0.98551	0.9052	
	4.	D_{135}	0.33298	0.4772	0.54083	Very poor	0.7971	0.87681	0.91304	Poor

The CMC plot in Fig. 10 shows the detailed performance of rank Vs score for each unary feature. D_{135} is clearly a consistently poor performer and V consistently the best performer for both models.

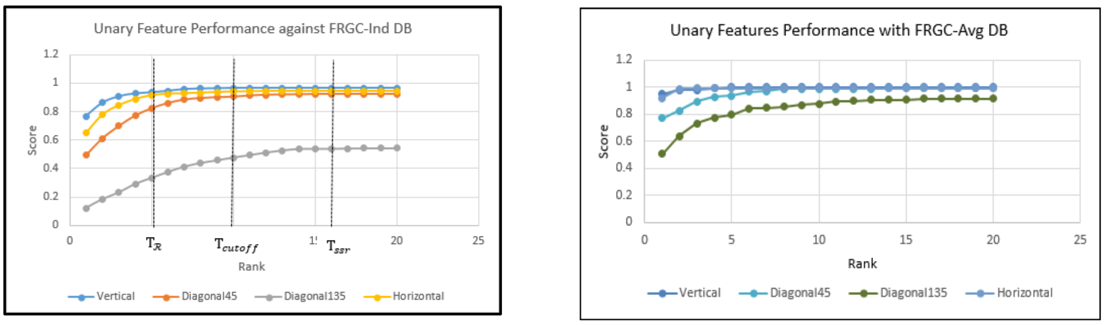


Fig. 10. Performance Evaluation of Unary Features Θ_1 against a) Individual and b) Average Models of FRGC-DB

   b) *Binary feature performance:* $\Theta_{2\text{-}ind} \rightarrow \{VH\}$; $\Theta_{2\text{-}avg} \rightarrow \{D_{135}H, H\}$.

   (i) From the ranking in TABLE V in rows 5-10, it is seen that the top 3 ranks are occupied by $\Theta_{2\text{-}avg}$ and the next 3 ranks by $\Theta_{2\text{-}ind}$. This implies that Θ_2 favours *FRGC-Avg* DB.

   (ii) $\Theta_{2\text{-}avg}$ attains 100% performance against the *FRGC-Avg* DB, $\forall T(T_{\mathcal{R}}, T_{cutoff}, T_{ssr})$. This is true for both of its feature subsets. However, even though $\Theta_{2\text{-}ind}$ is the best performer for FRGC-Ind, it exhibits sub-optimal performance.

The CMC plot in Fig. 11 shows the detailed performance of rank Vs score for each binary feature. The following inferences are drawn:

(i) The binary feature Θ_2 that includes D_{135} is clearly a consistently poor performer whilst a combination with V is consistently the best performer for both models. Their minimum performances are slightly over 0.4/$\Theta_{2\text{-}ind}$ and 0.8/$\Theta_{2\text{-}avg}$ which is a big difference. This clearly indicates that $\Theta_{2\text{-}avg}$ is far superior to $\Theta_{2\text{-}ind}$.

(ii) Θ_2 reaches saturation far sooner with the *FRGC-Avg* DB and sooner than Θ_1.

TABLE V RANK VS CUMULATIVE MATCH: RESPONSE CHARACTERISTICS OF BINARY FEATURES

$\Theta_i \downarrow$	Alternatives, $\Theta_{ij} \downarrow$		FRGC *Individual Model*				FRGC *Average Model*			
	Criteria→		$T_{\mathcal{R}}$ | Rank=5	T_{cutoff} | Rank = 10	T_{ssr} | Rank=17	Ranking (manual)	$T_{\mathcal{R}}$ | Rank=5	T_{cutoff} | Rank = 10	T_{ssr} | Rank=17	Ranking (manual)
Binary	5.	$D_{45}H$	0.92153	0.98091	0.98834	5	0.99275	1	1	
	6.	$D_{135}H$	0.79321	0.9141	0.94804		1	1	1	1
	7.	VH	0.98091	0.99046	0.99046	3	1	1	1	1
	8.	VD_{135}	0.89608	0.95122	0.97349		0.99275	1	1	
	9.	VD_{45}	0.93743	0.97773	0.98621	4	0.99275	1	1	2
	10.	$D_{45}D_{135}$	0.74337	0.87381	0.93955	Poor	0.99275	0.992	1	Poor

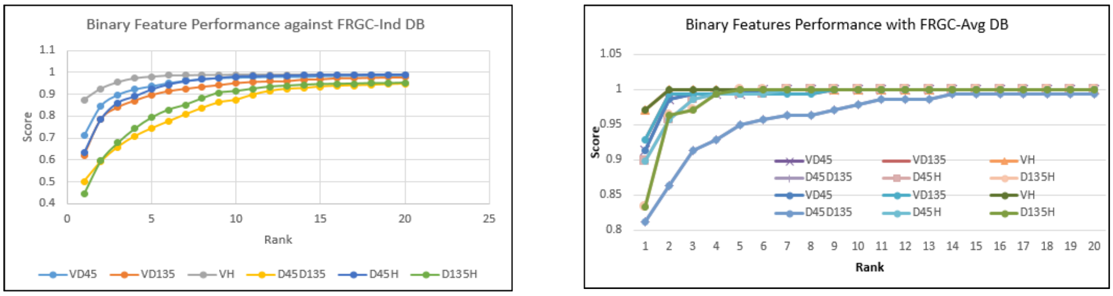


Fig. 11. Performance Evaluation of Binary Features Θ_2 against a) Individual and b) Average Models of FRGC-DB

c) Ternary feature performance: $\Theta_{3\text{-}ind} \rightarrow \{VD_{135}H\}$; $\Theta_{3\text{-}avg} \rightarrow \{VD_{135}H, D_{45}D_{135}H\}$.

(i) From the ranking in TABLE VI in rows 11-13, it is seen that $\Theta_{3\text{-}avg}$ works better than $\Theta_{3\text{-}ind}$. This implies that Θ_3 favours *FRGC-Avg* DB.

(ii) $\Theta_{2\text{-}avg}$ attains 100% performance against the *FRGC-Avg* DB, $\forall T(T_{\mathcal{R}}, T_{cutoff}, T_{ssr})$ for both of its feature subsets. Even though $\Theta_{2\text{-}ind}$ is the best performer for FRGC-Ind, it exhibits sub-optimal performance.

(iii) $\Theta_{3\text{-}avg} \rightarrow \{VD_{135}H, D_{45}D_{135}H\}$ performs optimally $\forall T(T_{\mathcal{R}}, T_{cutoff}, T_{ssr})$.

TABLE VI RANK VS CUMULATIVE MATCH: RESPONSE CHARACTERISTICS OF TERNARY AND QUADRUPLE FEATURES

$\Theta_i \downarrow$	**Alternatives, $\Theta_{ij} \downarrow$**		**FRGC *Individual Model***				**FRGC *Average Model***			
		Criteria→	$T_{\mathcal{R}}$ | **Rank=5**	T_{cutoff} | **Rank = 10**	T_{ssr} | **Rank=17**	**Ranking (manual)**	$T_{\mathcal{R}}$ | **Rank=5**	T_{cutoff} | **Rank = 10**	T_{ssr} | **Rank=17**	**Ranking (manual)**
Ternary	11.	$VD_{135}H$	0.95758	0.98515	0.9894	2	0.94928	0.97826	0.99275	3
	12.	$D_{45}D_{135}H$	0.88123	0.95228	0.98834	Poor	1	1	1	1
	13.	$VD_{45}D_{135}$	0.92683	0.96819	0.98409	4	1	1	1	1
All	14.	$VD_{45}D_{135}H$	0.96394	0.9894	0.99152	2	1	1	1	1

The CMC plot in Fig. 12 shows the detailed performance of rank Vs score for each ternary feature. The following inferences are drawn:

(i) The ternary feature Θ_3 that includes D_{135} is clearly a consistently poor performer whilst a combination with V is consistently the best performer for both models. Their minimum performances are slightly over 0.6/$\Theta_{3\text{-}ind}$ and 0.9/$\Theta_{3\text{-}avg}$ which again is a big difference.

(ii) Θ_3 reaches saturation far sooner with the *FRGC-Avg* DB and sooner than Θ_2. This is an indication that with increasing level of multi-modality performances can be improved.

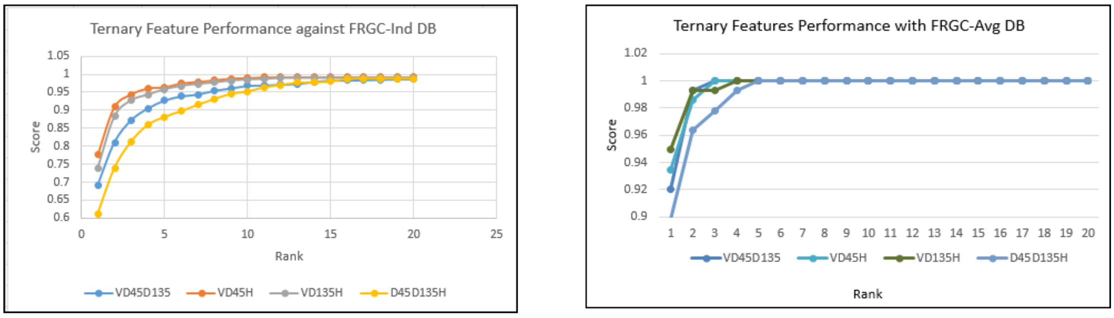


Fig. 12. Performance Evaluation of Ternary Features Θ_3 against a) Individual and b) Average Models of FRGC-DB

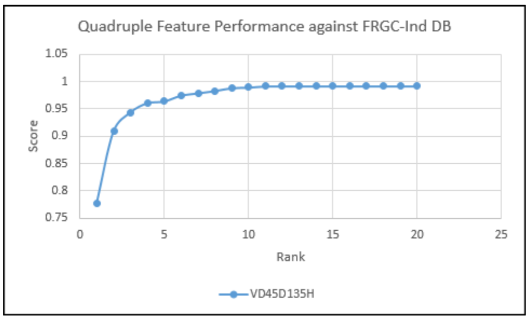
d) Quadruple feature performance: Θ_4 → $\{VD_{45}\ D_{135}H\}$ performs well for both DBs.

(i) From the ranking in TABLE VI in rows 14, it is seen that $\Theta_{4\text{-}avg}$ works better than $\Theta_{4\text{-}ind}$. Once again, Θ_4 favours *FRGC-Avg* DB.

(ii) $\Theta_{4\text{-}avg}$ attains 100% performance against the *FRGC-Avg* DB, $\forall T(T_{\mathcal{R}}, T_{cutoff}, T_{ssr})$. $\Theta_{4\text{-}ind}$ exhibits sub-optimal performance.

(iii) $\Theta_{4\text{-}avg}$ → $\{VD_{45}\ D_{135}H\}$ performs optimally $\forall T(T_{\mathcal{R}}, T_{cutoff}, T_{ssr})$.

It may be concluded that $\Theta_{i\text{-}avg}$ performs consistently optimally on *FRGC-Avg* DB for $\forall T$ and $\Theta_{i\text{-}ind}$ less optimally on *FRGC-Ind* DB. The CMC plot in Fig. 13 shows the detailed performance of rank Vs score for the quadruple feature Θ_4. It significantly improves performances in both models of FRGC. Their minimum performances are slightly over 0.75/$\Theta_{4\text{-}ind}$ and 0.93/$\Theta_{4\text{-}avg}$ which again is a big difference.



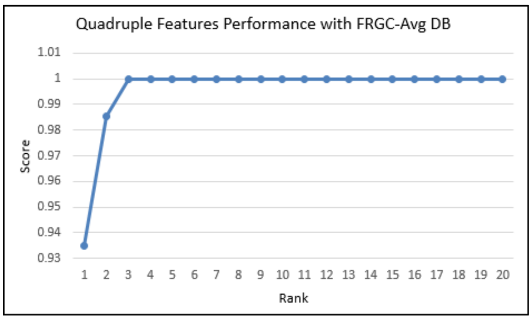


Fig. 13. Performance Evaluation of Quadruple Features Θ_4 against a) Individual and b) Average Models of FRGC-DB

2. The worst performers across the board of multi-modalities include: $\{D_{135}/\Theta_1, D_{45}D_{135}/\Theta_2\}$. This is the same for both FRGC DB models.

3. Overall, the *Average Model* has more winners than the *Individual Model* enabling more options to choose from. Higher scores of match are also produced by the *Average Model.*

4. Further analysis can be carried out for each individual response characteristic namely, $T_{\mathcal{R}}$, T_{cutoff} and T_{ssr}. For instance, the best $T_{\mathcal{R}}$ is obtained for $\{VH, VD_{45}D_{135}H\}/\Theta_{ind}$ and $\{V, D_{135}H, VH, VD_{135}H\}/\Theta_{avg}$.

5. Further analysis can be carried out separately for the Student-DB [32]. For brevity, these details are not listed out here. The final performance values alone are considered in the rest of the sections.

2.8 Impact of Average and Individual Models on Identification Performance

From the above experimental analysis, the multi-modal feature sets perform consistently better using an average model rather than maintaining an individual template. The key reason for such improved performance against the average models is that an average model captures imprecision from each sample during the modelling process. Hence, a query image is matched against an approximate model. The individual models are looking for exact matches as against approximations and hence the not so perfect performance. Further detailed analysis on this aspect may be established through a fuzzy modelling process whereby the average models are described by fuzzy sets and the individual models as crisp sets. This will provide a formal basis for establishing a theory for the reasons for such a variation in their performance characteristics. Such an analysis will be treated as a future work.

The performance analysis so far has been carried by sampling the CMC at $C = \{T_{\mathcal{R}}, T_{cutoff}, T_{ssr}\}$ as otherwise this would lead to a far more laborious and inaccurate subjective inference. The complexity of performance analysis varies significantly based on the following factors: $\{\text{number of databases} \times \text{feature modality} \times \text{model representation}\}$. The aim of this work is to formalise the above evaluation analysis through a fuzzy interval based TOPSIS technique to arrive at an objective measure of relative performance analysis when we have several choices of features and criteria to satisfy. The rest of the paper deals with this aspect.

3 Interval Valued Fuzzy Topsis (IVFT)

In this Section, the mathematical approach presented in [39] for modelling uncertainty through the use of interval valued fuzzy sets is considered. According to [21], type-1 fuzzy sets in general address uncertainties due to ambiguities in words used in rules, when there are multiple consequents in a rule due to experts' varied voting, uncertainty due to measurement noise, and parametric noise. In such cases, type-2 fuzzy sets that handle linguistic uncertainties better by modelling vagueness and unreliability of information are one of the options. By blurring type-1 systems [40] [41] through a shift in points for example, on a triangular membership function on either side but not necessarily by the same amount leads to a type-2 fuzzy system definition.

3.1 TOPSIS Formalisation

A type-1 fuzzy system is represented by a fuzzy membership function as shown in Fig.1. A type-2 system is derived by blurring the type-1 membership function to the left and right as shown in Fig.2 and is defined as follows [41-42]:

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \,|\, \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$$

(16)

where $0 \le \mu_{\tilde{A}}(x,u) \le 1, J_x \subseteq [0,1]\}$ represents the primary membership of x and $\mu_{\tilde{A}}(x,u)$ is a type-1 fuzzy set called the secondary set that defines the possibilities for the primary membership. Uncertainty is defined by a region called the footprint of uncertainty (FOU) as depicted by the blurred regions in Fig.2. The FOU can be described in terms of upper and lower membership functions as follows :

$$\left.\begin{array}{c} A = \{((x, [\mu_A^L(x), \mu_A^U(x)]\} \\ \mu_A^L, \mu_A^U : \longrightarrow [0,1]\ \forall x \in X, \quad \mu_A^L < \mu_A^U \\ \bar{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)] \\ \therefore A = \{((x, \bar{\mu}_A(x)\,]\}, \quad x \in [-\propto, \propto] \end{array}\right\}$$

(17)

where $\mu_A^L(x), \mu_A^U(x)$ form the upper and lower bounds respectively for $\bar{\mu}_A(x)$.

A fuzzy logic system that has at least one of its sets to be of type-2 is defined as type-2 fuzzy system. Its IF-THEN rules will contain type-2 antecedent or consequent sets. In this paper, we are concerned about the inference mechanism at the point of decision making (output stage).

3.2 Distance Measures in TOPSIS

In the Pattern Recognition domain, distance measures serve as as a measure of similarity between a probe and the gallery. The best match is based on the minimum distance criteria. Similarly, in the case of TOPSIS, a similarity measure based on minimum distance from an ideal (positive) solution and furthest distance from an ideal negative solution determines the choice of the alternative. A fuzzy approach to TOPSIS in applications of fuzzy group decision making using triangular membership functions and determining closeness of such numbers using fuzzy interval arithmetic is proposed in [8] in which the application is for selecting one among several candidates during an interview process. This approach uses an interval valued fuzzy TOPSIS for multi-criteria decision making wherein criteria values and their weights are treated as linguistic terms and described using interval valued fuzzy numbers. This approach is adopted here.

Given two interval valued triangular fuzzy numbers (IVTFNs), $N_x = [N_x^-, N_x^+]$ and $M_y = [M_y^-, M_y^+]$, the following definitions from [14] are considered:

- If $\circ \in (+,-,\times,\div)$, then $N_x \circ M_y = [N_x^- \circ M_y^-, N_x^+ \circ M_y^+]$

(18)

- The normalised Euclidean distance between N_x and M_y is given by

$$d(\tilde{N}, \tilde{M}) = \sqrt{\frac{1}{6}\sum_{i=1}^{3}\left[\left(N_x^- - M_y^-\right)^2 + \left(N_x^+ - M_y^+\right)^2\right]}$$

(19)

3.3 IVFT Formalisation

The IVFT proposed in [8] is adopted here as follows:

Step 1: The fuzzy decision matrix r_{ij} in (1) and weights ω_j are assumed to be interval valued triangular fuzzy numbers (IVTFN) defined generally by $\tilde{x} = \{(x_1, x_2, x_3), (x_1', x_2', x_3')\} = [(x_1, x_1');\ x_2;\ (x_3, x_3')]$ whose average values are given by:

$$\left.\begin{aligned} \tilde{x}_{ij} &= \frac{1}{K}\left[\tilde{x}_{ij}{}^{1} + \tilde{x}_{ij}{}^{2} + \cdots + \tilde{x}_{ij}{}^{K}\right] \\ \widetilde{w}_{ij} &= \frac{1}{K}\left[\widetilde{w}_{ij}{}^{1} + \widetilde{w}_{ij}{}^{2} + \cdots + \widetilde{w}_{ij}{}^{K}\right] \end{aligned}\right\} \tag{20}$$

where $\tilde{x}_{ij}$ and $\widetilde{w}_{ij}$ are the rating connected by the operation in (9) and importance weight of the Kth decision maker.

Step 2: Given $\tilde{x}_{ij} = \left[(a_{ij}, a_{ij}'); b_{ij}; (c_{ij}', c_{ij})\right]$, the normalised decision matrix **D** according to [14] is given by:

$$\left.\begin{aligned} \tilde{r}_{ij} &= \left[\left(\frac{a_{ij}}{c_j^+}; \frac{a_{ij}'}{c_j^+}\right)\left(\frac{b_{ij}}{c_j^+}\right); \left(\frac{c_{ij}'}{c_j^+}; \frac{c_{ij}}{c_j^+}\right)\right], \mathrm{i} = 1, \ldots, \mathrm{n},\ \ \mathrm{j} \in \Omega_b \ \ or \\ \tilde{r}_{ij} &= \left[\left(\frac{a_j^-}{a_{ij}'}; \frac{a_j^-}{a_{ij}}\right)\left(\frac{a_j^-}{b_{ij}}\right); \left(\frac{a_j^-}{c_{ij}}; \frac{a_j^-}{c_{ij}'}\right)\right], \mathrm{i} = 1, \ldots, \mathrm{n},\ \ \mathrm{j} \in \Omega_c \\ c_j^+ &= Max_i c_{ij}, \mathrm{j} \in \Omega_b \\ a_j^- &= Min_i a_{ij}', \mathrm{j} \in \Omega_c \end{aligned}\right\} \tag{21}$$

where either c_j^+, the global maxima or a_j^-, the global minima is used depending on whether the benefit or the cost crietria is applicable. Using Operations Research terminology, this implies an objective function with maximisation or minisation goal.

Step 3: A weighted normalised fuzzy decision matrix ***D*** matrix is constructed using the importance measure of each criterion and as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}{}^2}}, \mathrm{i} = 1,2, \cdots n; j = 1,2, \cdots m \tag{22}$$

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}, \text{ where } \tilde{v}_{ij} = \tilde{r}_{ij} \times \widetilde{w}_j\,; \tag{23}$$

Using the × operator in (18),

$$\tilde{v}_{ij} = \left[(\tilde{r}_{1ij} \times \widetilde{w}_{1j}, \tilde{r}_{1ij} \times w_{1j}'); \tilde{r}_{2ij} \times \widetilde{w}_{2j}; (\tilde{r}_{3ij}' \times \widetilde{w}_{3j}', \tilde{r}_{3ij} \times \widetilde{w}_{3j})\right] = \left[(g_{ij} \times g_{ij}');\ h_{ij}; (l_{ij}, l_{ij}')\right] \tag{24}$$

Ideal and negative ideal solutions are given by:

$$\left.\begin{aligned} A^+ &= [(1,1); 1. (1,1)], \mathrm{j} \in \Omega_b \\ A^- &= [(0,0); 0. (0,0)], \mathrm{j} \in \Omega_c \end{aligned}\right\} \tag{25}$$

Step 4: A set of primary and secondary distance measures as defined in (19) with respect to (25) is determined by the interval values as follows:

$$\left.\begin{aligned} D^-\big(\widetilde{N}, \widetilde{M}\big) &= \sqrt{\tfrac{1}{3}\textstyle\sum_{i=1}^{3}\left[\left(N_x^- - M_y^-\right)^2\right]} \\ D^+\big(\widetilde{N}, \widetilde{M}\big) &= \sqrt{\tfrac{1}{3}\textstyle\sum_{i=1}^{3}\left[\left(N_x^+ - M_y^+\right)^2\right]} \end{aligned}\right\} \tag{26}$$

Step 5: From (19), the primary and secondary distances from the ideal and are defined as:

$$\left.\begin{aligned} D_{i1}^{\mp} &= \sqrt{\frac{1}{3}\sum_{i=1}^{3}\left[\left(g_{ij}-1\right)^2+\left(h_{ij}-1\right)^2+\left(l_{ij}-1\right)^2\right]} \\ D_{i2}^{+} &= \sqrt{\frac{1}{3}\sum_{i=1}^{3}\left[\left(g'_{ij}-1\right)^2+\left(h_{ij}-1\right)^2+\left(l'_{ij}-1\right)^2\right]} \end{aligned}\right\} \tag{27}$$

Step 6: The interval valued primary and secondary distances from the negative ideal solution are defined as follows:

$$\left.\begin{aligned} D_{i1}^{-} &= \sqrt{\frac{1}{3}\sum_{i=1}^{3}\left[\left(g_{ij}-0\right)^2+\left(h_{ij}-0\right)^2+\left(l_{ij}-0\right)^2\right]} \\ D_{i2}^{-} &= \sqrt{\frac{1}{3}\sum_{i=1}^{3}\left[\left(g'_{ij}-0\right)^2+\left(h_{ij}-0\right)^2+\left(l'_{ij}-0\right)^2\right]} \end{aligned}\right\} \tag{28}$$

Step 7: An aggregated distance measure is given by:

$$RC_1 = \frac{D_{i1}^{-}}{D_{i1}^{+} + D_{i1}^{-}}, RC_2 = \frac{D_{i2}^{-}}{D_{i2}^{+} + D_{i2}^{-}}, RC = \frac{RC_1 + RC_2}{2} \tag{29}$$

This results is a coefficient that can be used to rank all the alternatives and choose the best one from the available options.

4 Application of IVFT in Ranking the Performance of Multi-Modal 3DFR System

In this Section, the fuzzy TOPSIS formalisation in Section 3 is applied to the 3DFR system described in Section 2. For convenience of empirical evaluation, we transpose ***D*** so that the DMs are the feature sets whilst the alternatives are the database subsets. Thus, the following parameters are identified:

1. **MCDM Alternatives (Databases):** $A = DB_i, i = 1 \ldots 6 \,\big|\, C_j$.
2. **MCDM Criteria (Response Characteristics):** $\mathrm{C} = \{T_{\mathcal{R}}, T_{\mathrm{cutoff}}, T_{\mathrm{ssr}}\}$.
3. **Weights of Importance of Criteria:** Linguistically defined based on the performance against DB_i in previous experiments [31].
4. **Decision Makers (DMs) (Multi-Modal Feature Sets):** Denoted by DM_i and refer to Θ.

The testing conditions vary based on the relation $\{\Theta x \ \mathrm{C}\}$. We consider only those that are representative of data scaling and response characteristics namely:

Condition 1. $\{\Theta_1, T_{\mathcal{R}}\}$ refers to the unary set of features tested for *Transient* response charcteristcs.

Condition 2. $\{\Theta_2, T_{cutoff}\}$ refers to the binary set of features tested for *Cutoff* response charcteristcs.

Condition 3. $\{\Theta, C\}$ refers to the full set of MM features defined in Eqn.(5) and all response characteristics, $C = \{\mathrm{T}_{\mathcal{R}}, \mathrm{T}_{cutoff}, \mathrm{T}_{ssr}\}$.

4.1 IVFT for Ranking Binary Feature Sets

In [32], we consider in detail the computation of $\boldsymbol{D}$ for Condition 1. Call this $\boldsymbol{\mathcal{D}}_1$. The results of extending the algorithm for Condition 3 were also reported without any illustration. Without loss of generality, by implementing Algorithm A in [32] on Condition 2 fuses binary feature sets. Call this $\boldsymbol{\mathcal{D}}_2$. Except for the dimensionality of the feature vector, there is no difference between $\boldsymbol{\mathcal{D}}_1$ and $\boldsymbol{\mathcal{D}}_2$. This approach is shown to extend easily for a full system performance evaluation by implemeting Algorithm B (or A) for Condition 3. Call this $\boldsymbol{\mathcal{D}}$.

Algorithm A. IVFT based Decision Making with Binary Features

Parametric Assumptions: Condition 2 is tested with $\{\Theta_2, T_{cutoff}\}$.

Step 1. Identify the alternatives A_i and the criteria C. For the 3DFR system, these include the relation $\Theta_i \times \{T_{\mathcal{R}}, T_{cutoff}, T_{ssr}\}$. In this work, we conisder a subset namely $\{\Theta_2 \times T_{cutoff}\}$.

Step 2. Assign the lingusitic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to the criteria. This results in the heuristics based lingusitic decision matrix $\boldsymbol{D}$ of ratings for Θ_2.

Step 3. Map the importance weight of each criterion and the ratings of qualitative criteria to a fuzzy decision matrix $\widetilde{D}$ based on IVTFNs. The IVTFNs are derived from previous experiments of the same datasets where information fusion was carried out manualy.

Step 4. Aggregate i) DMs fuzzy rating $\tilde{x}_{ij}$ of Alternative A_i and ii) the weight $\widetilde{w}_j$ of criterion C_j both using IVTFNs.

Step 5. Normalise the above fuzzy decision matrix, $\boldsymbol{D}$.

Step 6. Determine the ideal and negative ideal solutions. Calculate the distance of each alternative A_i to the the ideal and negative ideal solutions.

Step 7. Calculate the distance measures. Aggregate the distance measures suitably for ranking the alternatives.

4.2 Numerical Illustration of Fuzzy TOPSIS on Binary Feature Sets

Here we consider Condition 2. Hence the MCDM alternatives relate to Θ_2.

1) Consider the first two columns of TABLE VII that lists the linguistic variables for rating the DMs namely Θ_2. The lingusitc variables are defined from the performance data from TABLE IV - TABLE VI. The rest of the columns contain IVTFNs determined apriori.

2) Consider the decision matrix $\boldsymbol{\mathcal{D}}_2$ in TABLE VIII determined by the lingustic ratings from **Error! Reference source not found.**.

3) Expand $\boldsymbol{\mathcal{D}}_2$ in TABLE VIII using IVTFNs from **Error! Reference source not found.**, columns 3-7. Every linguistic variable is mapped to an IVTFN. Result is as shown in TABLE IX.

TABLE VII FUZZY INTERVAL VALUED LINGUISTIC VARIABLES FOR RATING

		g	g'	h	l	l'
Very Poor	VP	0	0	0	1	1.5
Poor	P	0	0.5	1	2.5	3.5
Moderately Poor	MP	0	1.5	3	4.5	5.5
Fair	F	2.5	3.5	5	6.5	7.5
Moderately Good	MG	4.5	5.5	7	8	9.5
Good	G	5.5	7.5	9	9.5	10
Very good	VG	8.5	9.5	9.5	10	10
Excellent	EX	8.5	9.5	10	10	10

TABLE VIII DECISION MATRIX: LINGUISTIC RATINGS OF BINARY FEATURE SET Θ_2 UNDER CRITERIA C_2= {T_{CUTOFF}}

DM→	DM5	DM6	DM7	DM8	DM9	DM10
Alternatives ↓	VD45	VD_{135}	VH	$D45D_{135}$	D45H	$D_{135}H$
7.5-Ind	G	G	G	MG	MG	MG
7.5-Avg	VG	VG	VG	VG	VG	VG
12.5-Ind	EX	EX	EX	VG	VG	VG
12.5-Avg	EX	EX	EX	VG	VG	VG
FRGC-Ind	VG	VG	EX	MG	VG	G
FRGC-Avg	EX	EX	EX	VG	EX	EX

TABLE IX IVFTN MAPPING FOR DECISION MATRIX OF BINARY FEATURES

Criteria	Alternative	DM5					DM6					DM7				
#M	#N	g	g'	h	l	l'	g	g'	h	l	l'	g	g'	h	l	l'
C2-Cutoff at Rank=10	7.5-Ind	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10
	7.5-Avg	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	12.5-Ind	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
	12.5-Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
	FRGC-Ind	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10
	FRGC-Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10

Criteria	Alternative	DM8					DM9					DM10				
#M	#N	g	g'	h	l	l'	g	g'	h	l	l'	g	g'	h	l	l'
C2-Cutoff at Rank=10	7.5-Ind	4.5	5.5	7	8	9.5	4.5	5.5	7	8	9.5	4.5	5.5	7	8	9.5
	7.5-Avg	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	12.5-Ind	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	12.5-Avg	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	FRGC-Ind	4.5	5.5	7	8	9.5	8.5	9.5	9.5	10	10	5.5	7.5	9	9.5	10
	FRGC-Avg	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10

4) Since our aim is to maximise the recognition performance, consider the profit factor Ω_b and hence determine the global maxima c_j^+ of the TFNs using (21). It is the global maxima of TABLE IX. $c_j^+ = 10$.

5) Using (20), aggregate DM_j and normalise by c_j^+. Result is as shown in TABLE X.

6) The ideal and negative solutions are defined in (25). Using (27), calculate the primary distances of each alternative A_i to the the ideal negative solution. See **Error! Reference source not found.** for the output.

TABLE X BINARY FEATURES DECISION SPACE BASED ON AVERAGE OPERATOR

Criteria	Alternatives	DM Average				
M	N	g	g'	h	l	l'
C2-Cutoff	7.5-Ind	5.0	6.5	8.0	8.8	9.8
	7.5-Avg	8.5	9.5	9.5	10.0	10.0
	12.5-Ind	8.5	9.5	9.8	10.0	10.0
	12.5-Avg	8.5	9.5	9.8	10.0	10.0
	FRGC-Ind	7.3	8.5	9.1	9.6	9.9
	FRGC-Avg	8.5	9.5	9.9	10.0	10.0

TABLE XI BINARY FEATURES IFVT BASED DISTANCE FROM IDEAL NEGATIVE SOLUTION

Criteria	Alternatives	Distance from A-(0,0,0,0,0)				
M	N	g	g'	h	l	l'
C2-Cutoff, Rank=10	7.5-Ind	0.08	0.24	0.52	0.69	0.95
	7.5-Avg	0.22	0.51	0.73	0.90	1.00
	12.5-Ind	0.22	0.51	0.77	0.90	1.00
	12.5-Avg	0.22	0.51	0.77	0.90	1.00
	FRGC-Ind	0.16	0.41	0.67	0.83	0.98
	FRGC-Avg	0.22	0.51	0.80	0.90	1.00

7) Similarly, using (28), calculate the secondary distances of each alternative A_i to the the ideal solution. See TABLE XII for the output.

8) Finally, using (29), aggregate the above distance measures in 7) and 8) to arrive at a ranking of the alternatives. See **Error! Reference source not found.** for the output. The last column shows the ranking of the binary features.

TABLE XII BINARY FEATURES IFVT BASED DISTANCE FROM IDEAL SOLUTION

Criteria	Alternatives	Distance from A*(1,1,1,1,1)				
M	N	g	g'	h	l	l'
C2-Cutoff, Rank=10	7.5-Ind	0.53	0.26	0.08	0.03	0.00
	7.5-Avg	0.28	0.08	0.02	0.00	0.00
	12.5-Ind	0.28	0.08	0.02	0.00	0.00
	12.5-Avg	0.28	0.08	0.02	0.00	0.00
	FRGC-Ind	0.36	0.13	0.03	0.01	0.00
	FRGC-Avg	0.28	0.08	0.01	0.00	0.00

TABLE XIII INTERVAL VALUED FUZZY NORMALISED, WEIGHTED, AVERAGED DECISION MATRIX FOR BINARY FEATURE SETS, $\{\Theta_2 \times T_{CUTOFF}\}$

Alternatives	Di1+	Di1-	Di2+	Di2-	RC1	RC2	RC	Rank
7.5-Ind	1.40	1.11	1.85	2.10	0.53	0.44	0.49	6
7.5-Avg	1.06	0.72	2.22	2.46	0.53	0.40	0.46	4
12.5-Ind	1.02	0.65	2.30	2.53	0.52	0.39	0.46	2
12.5-Avg	1.02	0.65	2.30	2.53	0.52	0.39	0.46	2
FRGC-Ind	1.15	0.80	2.14	2.41	0.53	0.41	0.47	5
FRGC-Avg	0.93	0.56	2.37	2.60	0.52	0.38	0.45	1

In summary, numerical performance measures are mapped to fuzzy linguistic variables which are in turn mapped to fuzzy intervals. Aggrgative measures are then defined suitably and their distance from positive and negative ideal solutions help finalise the rank of each performance measure leading to ranking of the feature sests as alternatives.

In this Section, only T_{cutoff} criteria is considered for which the feature sets are ranked. It is seen that the features work best with the FRGC-Avg based on rank=1 and performs the least with the Student7.5-Ind DB based on rank=6. The ranking is in agreement wit the ground truth from the CMC.

4.3 IVFT for Ranking Full Set of Multi-Modal Feature

Algorithm A for decision making with binary feature sets Θ_2 that contains the DMs linguistic descriptions can now be easily extended to any of the other feature sets in $\Theta_i = \{\Theta_1, \Theta_2, \Theta_3, \Theta_4\}$ and criteria $C = \{T_{\mathcal{R}}, T_{\text{cutoff}}, T_{\text{ssr}}\}$. The subset $\boldsymbol{D}_2 \in \boldsymbol{D}$ is highlighted in TABLE XIV. The main change comes from this expansion. The rest of the procedure is simply carried with each of the other feature sets. The procedure is listed out in Algorithm B.

Algorithm B. IVFT based Decision Making with Full Set of Multi-Modal Features

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Step 1. Expand $\boldsymbol{D}_2$ to include $\{\Theta_1, \Theta_2, \Theta_3, \Theta_4\}$ by including all columns from TABLE XIV.
Step 2. Include all criteria $C = \{T_{\mathcal{R}}, T_{\text{cutoff}}, T_{\text{ssr}}\}$ by including all rows from TABLE XIV.
Step 3. Apply Algorithm A to $\boldsymbol{D}$. Results of IVTFNs are shown in TABLE XV.
Step 4. The extended result closeness and ranking results are shown in TABLE XVI.

Using the full set of features produces a different set of rankings of the alternatives for each database. This implies that certain feature modalities favour specific DBs. Further, Algorithm B is applied to all of the criteria whereas Algorithm A is applied to only one of the criteria. Hence, comparing the two ranking mechanisms is not appropriate here. As a follow up, the relative ranking for all criteria and all possible feature modalities have been computed and provided in TABLE XVI and TABLE XV. It is seen that a more consistent and uniform ranking is achieved in the process.

4.4 Inference on the Ranking of Multi-Modal Features

With reference to TABLE XVI, the following inferences are made on the relative performance of the feature modalities:

- All modalities perform the same against Student 7.5-Ind DB with a rank of 6. Similarly, their performance against FRGC-Ind DB is the same. That is, the rank is 5. This is equivalent to a unanimous voting system.

- Based on the top ranks, it is inferred that the unary features perform the best against Student 12.5-Ind DB, the binary and ternary features perform the best against FRGC-Avg DB. Further, they both rank the same against both databases namely Student 12.5-Ind and Student 7.5-Avg databases.
- The quadruple performs the best against Student12.5-Avg DB.
- From the overall ranking, it is to be noted that all of the feature sets show their suitability towards the FRGC-Avg DB with rank 1 and Student-12.5-Avg DB and Student 12.5-Ind with rank 2. Likewise, they don't favour Student7.5-Avg DB and ranks lie between 5 & 6. This can be verified from the ground truth plots in Fig. 14.
- A reason for the multi-modal sets performing well against the *average model* better than the *individual model* is that the extent of overlap is not sufficient to maximise the within-class correlation between the canonical views of the *individual model.*

5 Conclusion and Further Work

In this paper, the fuzzy interval-valued TOPSIS (IVFT) approach to multi-criteria decision making for fusing multi-modal features in a 3D face recognition system is proposed. The novelty lies in the elegance with which IVFT can be applied to either a single or multi-modal feature set. The approach leads to a mechanism for ranking features based on multi-criteria used for benchmarking biometric systems. The ranked matrix provides a clear picture of the relative performance of the multi-modal features and their behavioural characteristics can be well interpreted at a higher level of decision making. Such decision making is based on low level detailed information processing coming from various sources of performance metrics.

The sensitivity of the system to linguistic descriptions are a key factor that will decide on the success of the IVFT. The initial settings come from the experts' knowledge by analysis of the CMCs. In this work, linguistic descriptions such as in TABLE XIV are defined manually. An adaptive mechanism may well be employed.

In summary, the system has sufficient alternatives of feature sets to suit each of the database and their varied models. This knowledge can be used to perform query matching differently (varying the feature set) depending on which partition of the database is being searched.

As part of future work, it is proposed to identify the suitability of the IVFT technique for Big Data applications in Face Recognition. It is envisaged to integrate existing 3D face databases in the public domain and test the approach against these databases. The heterogeneous nature of the databases in terms of the sensors used, features extracted and recognition algorithms deployed as well as gender, age, geography and the like will act as a true measure of test for IVFT. Further, recognising faces in social media is a big challenge. The variety in these databases and their sheer volumes of face samples will require powerful features that can discriminate well. By grouping the samples based on the feature ranking process and having a set of alterative multi-modal features will help in pruning the databases. Hence, the usefulness of features used for such a challenging problem will also be tested using the IFVT.

TABLE XIV DECISION MATRIX D FOR MULTI-MODAL FEATURE SETS Θ_I UNDER CRITERIA C= {T_R, T_{CUTOFF}, T_{SSR}}

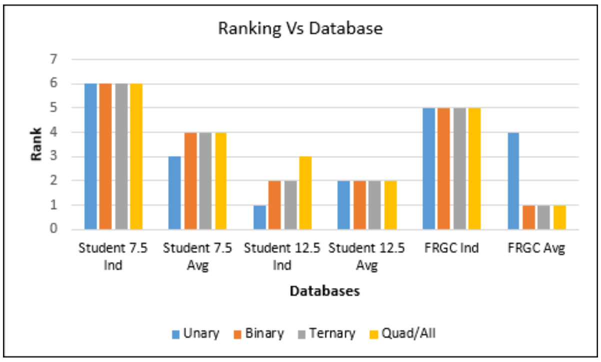
Criteria	Alternatives	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		V	**D45**	**D135**	**H**	**VD45**	**VD135**	**VH**	**D45D135**	**D45H**	**D135H**	**VD45D135**	**VD45H**	**VD135H**	**D45D135H**	**VD45D135H**
C1-Transient Response at Rank=5	**7.5-Ind**	F	P	F	F	F	F	F	F	MP	F	F	F	F	F	F
	7.5-Avg	G	MP	G	G	G	G	G	MG	MG	MG	G	G	G	G	G
	12.5-Ind	G	VG	G	G	G	G	G	G	G	G	G	G	G	G	G
	12.5-Avg	G	VG	VG	G	G	G	G	G	G	G	G	G	G	G	G
	FRGC-Ind	G	MG	MP	G	G	MG	G	MG	G	MG	G	G	G	MG	G
	FRGC-Avg	EX	MG	MP	EX	EX	EX	EX	G	EX	EX	EX	EX	EX	EX	EX
C2-Cutoff at Rank=10	**7.5-Ind**	G	MG	G	G	G	G	G	MG	MG	MG	G	G	G	MG	G
	7.5-Avg	VG	G	VG	VG	VG	VG	VG	VG	VG	VG	VG	VG	VG	VG	VG
	12.5-Ind	VG	VG	VG	VG	EX	EX	EX	VG	VG	VG	VG	VG	VG	VG	VG
	12.5-Avg	VG	VG	VG	VG	EX	EX	EX	VG	VG	VG	VG	VG	VG	VG	VG
	FRGC-Ind	VG	G	MP	G	VG	VG	EX	MG	VG	G	VG	VG	VG	G	VG
	FRGC-Avg	EX	VG	MG	G	EX	EX	EX	VG	EX	EX	EX	EX	EX	EX	EX
C3-SSR at Rank=17	**7.5-Ind**	EX	EX	EX	EX	EX	EX	EX	VG	VG	VG	VG	VG	EX	VG	VG
	7.5-Avg	EX	EX	EX	EX	EX	EX	EX	VG	VG	VG	VG	VG	EX	VG	VG
	12.5-Ind	VG	VG	VG	VG	EX	EX	EX	VG	EX	EX	EX	EX	EX	EX	EX
	12.5-Avg	VG	VG	VG	VG	EX	EX	EX	VG	EX	EX	EX	EX	EX	EX	EX
	FRGC-Ind	VG	EX	F	G	VG	VG	EX	G	VG	G	VG	EX	VG	VG	EX
	FRGC-Avg	EX	EX	G	EX	EX	EX	EX	EX	EX	EX	EX	EX	EX	EX	EX

TABLE XV FUZZY INTERVAL VALUED DECISION MATRIX D FOR MULTI-MODAL FEATURE SETS Θ_I UNDER CRITERIA C= {T_R, T_{CUTOFF}, T_{SSR}}

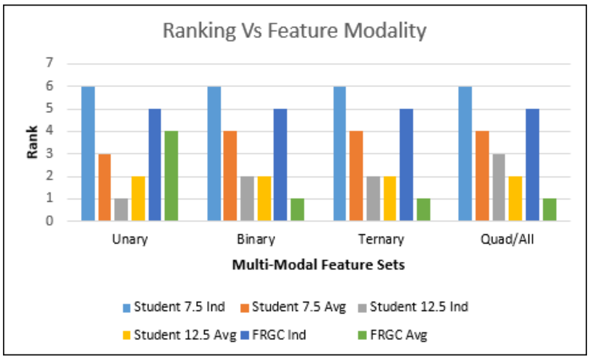
Criteria	**Alternative**	**DM5**					**DM6**					**DM7**					**DM8**					**DM9**					**DM10**				
#M	**#N**	**g**	**g'**	**h**	**l**	**l'**	**g**	**g'**	**h**	**l**	**l'**	**g**	**g'**	**h**	**l**	**l'**	**g**	**g'**	**h**	**l**	**l'**	**g**	**g'**	**h**	**l**	**l'**	**g**	**g'**	**h**	**l**	**l'**
C1-Transient Response at Rank=5	**7.5-Ind**	2.5	3.5	5	6.5	7.5	2.5	3.5	5	6.5	7.5	2.5	3.5	5	6.5	7.5	2.5	3.5	5	6.5	7.5	0	1.5	3	4.5	5.5	2.5	3.5	5	6.5	7.5
	7.5-Avg	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	4.5	5.5	7	8	9.5	4.5	5.5	7	8	9.5	4.5	5.5	7	8	9.5
	12.5-Ind	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10
	12.5-Avg	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10
	FRGC-Ind	5.5	7.5	9	9.5	10	4.5	5.5	7	8	9.5	5.5	7.5	9	9.5	10	4.5	5.5	7	8	9.5	5.5	7.5	9	9.5	10	4.5	5.5	7	8	9.5
	FRGC-Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	5.5	7.5	9	9.5	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
C2-Cutoff at Rank=10	**7.5-Ind**	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	5.5	7.5	9	9.5	10	4.5	5.5	7	8	9.5	4.5	5.5	7	8	9.5	4.5	5.5	7	8	9.5
	7.5-Avg	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	12.5-Ind	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	12.5-Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	FRGC-Ind	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10	4.5	5.5	7	8	9.5	8.5	9.5	9.5	10	10	5.5	7.5	9	9.5	10
	FRGC-Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
C3-SSR at Rank=17	**7.5-Ind**	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
	7.5-Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
	12.5-Ind	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
	12.5-Avg	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10
	FRGC-Ind	8.5	9.5	9.5	10	10	8.5	9.5	10	10	10	2.5	3.5	5	6.5	7.5	5.5	7.5	9	9.5	10	8.5	9.5	9.5	10	10	8.5	9.5	9.5	10	10
	FRGC-Avg	8.5	9.5	10	10	10	8.5	9.5	10	10	10	5.5	7.5	9	9.5	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10	8.5	9.5	10	10	10

TABLE XVI RANKING FOR EACH COMBINATION OF MULTI-MODALITY AND THEIR AGGREGATION

	Unary	Binary	Ternary	Quadruple	Overall
Student 7.5 Ind	6	6	6	6	6
Student 7.5 Avg	3	4	4	4	3
Student 12.5 Ind	1	2	2	3	2
Student 12.5 Avg	2	2	2	2	2
FRGC Ind	5	5	5	5	5
FRGC Avg	4	1	1	1	1



(a) Ranking Analysis at Database Level



(b) Ranking Analysis at Feature Level

Fig. 14. Ranking of Multi-Modalities

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7 Glossary

MCDM	Multi Criteria Based Decision Making
MADM	Multi Attribute Decision Making
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
IVFS	Interval-Valued Fuzzy Sets
IVFT	Interval Valued Fuzzy TOPSIS
MMFs	Multi-Modal Features
MM	Multi-Modal
FOU	Footprint Of Uncertainty
3DFR	3D Face Recognition
MM-3DFR	Multi-Modal 3D Face Recognition
FRVT	Face Recognition Vendor Technology
FRGC	Face Recognition Grand Challenge
DB	Database
LDA	Linear Discriminant Analysis
FLDA	Fisher's Linear Discriminant Analysis
PCA	Principal Component Analysis
CMC	Cumulative Match Charactersitics
SSR	Steady State Response

8 Appendix A: Performance Metrics Notation

In this Appendix, we consider the notations and terminologies commonly used to evaluate biometric systems [38] and used in this paper.

- *Gallery and Probe Sets*: For purpose of performance evaluation, the feature set F is divided into partitions of gallery, G that forms the database of templates of enrolled subjects and probe, P that forms the set of query samples. Depending on the specific performance metric to be determined, the elements of the gallery and probe sets, $g \in G$ and $p \in P$ respectively will vary. For example, the probe set could be a subset of the gallery during the training phase of a face recognition system, and mutually exclusive during the testing phase.

- *Identification:* Identification in a biometric system is the process of determinig the identification of an individual from the database. The identification process matches a probe as a query against the gallery and returns similarity scores, $\forall g \in G$. The scores are usually normalised in the range [0,1].

- *Verification* is the process of confirming that a claimed identity is correct by comparing the probe with one or more enrolled templates.

- *Open-set and close-set identification:* Identification is close-set if a person is assumed to be previously enrolled and open-set otherwise (as in the case of a watch-list whose identity is not known previously).

- *False Acceptance Rate (FAR):* an empirical estimate of the probablity that an impostor has been falsely verified to bear a correct identification.

- *False Rejection Rate (FRR):* an empirical estimate of the probability that a person with true identification has been falsely rejected by the system.

- *Equal Error Rate (EER):* The rate at which FAR=FRR.

- *Identity function:* A function $id(g)$ that returns the identity as an integer indexing the database templates and given by $id\colon\ \mathcal{X} \longrightarrow \mathcal{U}$ where $\mathcal{U}$ is a set of unique identities. Let U_{g} denote thes set of identities in G, and U_{p} the identities in P. As mentioned before, for some testing conditions of training and testing phases, $U_{\mathrm{g}} \cap U_{\mathrm{p}} = \emptyset$.

- *Identification Rate:* Closed-set performance evaluation requires the sorting of similarity scores during a matching process of the probe against the gallery which are now in a natural increasing order of ranking. The identification rate $I(k)$ is defined as the fraction of probes at rank k or below:

$$I(k) = \frac{\left|\{b \middle| rank(b) \leq k, \forall_{b \in B}\}\right|}{|U_{\mathrm{p}}|},$$

where $\left|U_{\mathrm{p}}\right|$ is the size of the probe set.

- *Cumulative Match Curve (CMC):* The CMC chart is a plot of k Vs $I(k)$. It is a non-decreasing function as shown in **Error! Reference source not found.**. The example in [38] is quoted here. If there are 100 probes and a system has 50 outputs with 50 rank-1 outcomes, 40 rank-2 outcomes, 5 rank-3 outcomes, 3 rank-4 outcomes, and 2 rank-5 outcomes, then, the number of elements with rank k or less is {50, 90, 95, 98, 100} for ranks k ={1, 2, 3, 4, 5}, respectively. Hence, the identification rate is 50% for rank-1 performance, 90% for rank-2 performance, and so on. As k increases, the identification rate increases and eventually attains 100%.