A level set immersed boundary model for extreme wave impacts on wave energy converters

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Abstract

Floating wave energy converters (WECs) are installed at locations with high wave energy and in relatively shallow water where wave nonlinearity is amplified. As a result, wave impact loads constitute a major design consideration for wave energy converters and the violent impact of an extreme wave onto a wave energy converter can be the criterion that determines a number of design parameters. Numerical simulation of the coupled dynamic response of WEC and mooring in storm conditions and under extreme wave loading remains a complex and difficult problem. Nevertheless, quantitative understanding of the wave impact is very important to the efficient performance and long time survivability of a wave energy converter.

A new fully nonlinear CFD technique is developed to assess the wave impacts and dynamic response on wave energy converters. Wave breaking and overtopping occur under extreme wave loading on offshore WECs. Both the water and air that may be entrained when a wave breaks or overtops a structure should be modelled, and the interface between them defined with a high resolution free surface capturing technique. In this work, a Navier-Stokes equation model is used to simulate the hydrodynamics. A level set method with the global mass correction is developed to study wave breaking and overtopping, and the immersed boundary method is employed to capture the extreme wave loading on offshore WECs.

Calculations have been made for the entry and exit of a cylinder, in which the hydrodynamic force on the cylinder during the first stage of the impact is obtained. The slamming coefficients of the cylinder entry with different entry velocities are calculated and agree well with experimental results. This problem is of importance in the design of various floating structures that experience worst case loading.

Keywords: Level set method, global mass correction, immersed boundary method, wave impacts, wave energy converter, water entry and exit.

Nomenclature

i	= 1, 2, two dimensional geometrical descriptions
u_j	= velocity
P	= pressure
x_j	= Cartesian spatial coordinates
f_i	= external body force field
τ_{ij}	= viscous term
ρ,μ	= density and viscosity appropriate for the phase occupying the particular spatial location at a given instance of time.
ϕ	= the first level set variable
ϕ'	= the second redistancing variable
$\bar{t}$	= pseudo time for the variable ϕ'
ϕ''	= the third mass conservation variable
t'	= pseudo-time for the variable ϕ''
M_{cor}	= dimensionless mass correction factor
M_o	= the original mass
M_t	= the mass of the reference phase at time t
$H(\phi)$	= a smoothed Heaviside function
ε	= a factor of the grid spacing
Ψ	= immersed boundary
u_ψ	= Dirichlet boundary condition of the immersed boundary
u_f	= value of the forcing point
u_v	= value of the virtual point
Δt	= the time step
RHS_i^k	= convective, viscous and body force of the governing equations
$\mathbf{f}$	= the force per unit area on a surface element with an outward normal $\mathbf{n}$
n_j	= the direction cosine of $\mathbf{n}$ in x_j direction
r	= radius of the circle
t ,T	= time, the dimensionless time
γ,F_r	= the dimensionless quantity
C	= slamming coefficient
F	= the total vertical hydrodynamic force
V	= entry or exit velocity

1 Introduction

Floating wave energy converters (WECs) are installed at locations with high wave energy and in relatively shallow water where wave nonlinearity is amplified. As a result, the continuous impact and slamming of waves constitute a major design consideration for wave energy converters and the violent impact and slamming of an extreme wave onto a wave energy converter can be the criterion that determines a number of design parameters of efficiency and survivability. Thus, quantitative understanding of the wave impact and interaction between the waves and WECs is very important to the efficient performance and long time survivability of a wave energy converter. Furthermore many parameters such as geometry, equipment and location must be optimized at the design stage. The need for a numerical simulation tool appears not only during the pre-conception stage, by avoiding expensive model tests, but also in the production run of the power station. Here a level set immersed boundary model is developed to optimize design parameters of WECs and provide helpful performance predictions.

Many applications of interactions between a wave and structure use potential flow theory, which assumes the fluid is incompressible and inviscid. However, in some situations, such as extreme wave conditions, viscous effects including flow separation and turbulence must be considered which means solving the full Navier-Stokes equations. Also a high resolution surface capturing scheme needs to be included in order to simulate complex free surface changes such as wave breaking and overturning and a fluid-structure interaction method needs to be incorporated to investigate the impacts of extreme waves on the structure.

Basically there are two strategies to handle a moving or deforming boundary problem with topological change. They are body conforming moving grids [1, 2] and embedded fixed grids [3-5]. For the body conforming moving grids, the grid can be efficiently deformed in an arbitrary Lagrangean-Eulerian (ALE) frame of reference to minimize distortion if a geometric variation is quite modest. Boundary conditions can be applied at the exact location of the rigid boundary. However, if the change of topology is very complex, it will be very difficult and time consuming to regenerate the mesh. Also difficulties arise in the form of grid skewness and additional numerical dissipation may be a consequence of the redistribution of the variables in the vicinity of the boundary.

An alternative to body conforming moving grids is embedded fixed grids where the boundaries do not conform to the grid and the governing equations are usually discretized on fixed Cartesian grids. There are two major classes based on the specific treatment of the boundary cells; (1) Cartesian cut cell methods [3] and (2) Immersed boundary methods [4-5]. Cartesian cut cell methods cuts solid bodies out of a background Cartesian mesh and their boundaries are represented by different types of cut cells. It has the potential to significantly simplify and automate the difficulty of mesh generation. However, there are still a number of disadvantages inherent in the use of this method. Arbitrarily small cells arising near solid boundaries due to the Cartesian mesh intersecting a solid body can restrict the stability and reduce the efficiency of the Cartesian solvers [3].

An immersed boundary with arbitrary shape can be modelled on a fixed grid by an external force field such that a desired velocity distribution can be assigned over a boundary. The main advantage of this method is that the external force filed can be prescribed on a fixed mesh so that the accuracy and efficiency of the solution procedure on simple grids are maintained.

In this work, the finite volume method is used to discretize Navier-Stokes equations with the two step projection method on a staggered grid. The free surface is solved on a fixed grid in which the free surface is captured by the zero level set. A global mass correction scheme in a novel combination with third order essentially non-oscillatory schemes and a five stage Runge-Kutta method is used to accomplish the advection of the level set function and re-distancing [6]. The immersed boundary method is used to simulate water exit and entry of a cylinder. This problem is of importance in the design of floating structures that experience extreme wave loading.

2 Governing equations

Governing equations for an incompressible fluid flow are the mass conservation equation and the Navier-Stokes momentum conservation equations written as

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1}$$

and

$$\frac{\partial u_i}{\partial t} + \frac{\partial \left(u_i u_j\right)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} + f_i \tag{2}$$

τ_{ij} is the viscous term given by

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

The evolution of the level-set function is governed by

$$\frac{\partial \phi}{\partial t} + u_j \frac{\partial \phi}{\partial x_j} = 0 \tag{4}$$

A redistancing function is performed by solving for ϕ' given by Eq. (5):

$$\frac{\partial \phi'}{\partial \bar{t}} + s(\phi)\left(\left|\nabla \phi'\right| - 1\right) = 0 \tag{5}$$

The initial condition is $\phi'(\vec{x},0) = \phi(\vec{x})$ and $s(\phi)$ is the smoothed sign function defined as

$$s(\phi) = \frac{\phi}{\sqrt{\phi^2 + \left(\left|\nabla \phi\right| \varepsilon\right)^2}} \tag{6}$$

The steady state solution to ϕ'' is obtained using Eq. (7):

$$\frac{\partial \phi''}{\partial t'} = M_{cor} \tag{7}$$

A dimensionless mass correction term is introduced to ensure the mass conservation, written as

$$M_{cor} = sign(\phi_{ref}) \frac{M_o - M_t}{M_o} \tag{8}$$

A smoothed Heaviside function is defined.

$$H(\phi) = \begin{cases} 0 & \phi < -\varepsilon \\ \dfrac{\phi+\varepsilon}{2\varepsilon} + \dfrac{1}{2\pi}\sin\left(\dfrac{\pi\phi}{\varepsilon}\right) & -\varepsilon \le \phi \le \varepsilon \\ 1 & \phi > \varepsilon \end{cases} \tag{9}$$

Using the smoothed Heaviside function, these properties are calculated using

$$\beta = (1-H)\beta_1 + H\beta_2 \tag{10}$$

A 2D immersed boundary $\boldsymbol{\psi}$ coincides with a Cartesian grid node (i, j) on which a Dirichlet boundary condition u_ψ needs to be enforced at this point. If u_{ij} is an approximation to the solution of the governing equations, the discrete form can be written as

$$\frac{u_{ij}^{k+1} - u_{ij}^k}{\Delta t} = \mathrm{RHS}_i^k + f_i^k \tag{11}$$

The external force function that will enforce the above boundary condition can be obtained from Eq.(12) by setting $u_{ij}^{k+1} = u_\psi$ and solving for f_i^k

$$f_i^k = \frac{u_\psi - u_{ij}^k}{\Delta t} - \mathrm{RHS}_i^k \tag{12}$$

The force $\mathbf{f}$ per unit area on a surface element with an outward normal $\mathbf{n}$ can be written as

$$f_i = \tau_{ji} n_j = \left[-p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] n_j \tag{13}$$

$\dfrac{\partial u_i}{\partial x_j}$ can be computed using the stencil and interpolation coefficients that were used to construct the velocity field near the interface.

3 Numerical Method

The finite volume method is used to discretize the Navier-Stokes equations on a non-uniform staggered Cartesian grid. A two step projection method is employed for velocity-pressure coupling, in which a pressure Poisson equation is solved to enforce the continuity equation. The QUICK scheme with deferred correction is used for the convective terms and central differencing is used for the viscous terms. Generalized minimum residual (GMRES) method with incomplete LU factorization for preconditioning [7] is applied to solve linear systems of the form: A**x**=b.

Mass conservation is improved significantly by applying a global mass correction scheme, in a novel combination with third order essentially non-oscillatory schemes and a five stage Runge-Kutta method, to accomplish the advection and re-distancing of the level set function.

For the immersed boundary treatment, the grid-interface relation with an immersed boundary is established. Thus all Cartesian grids can be classified into three categories as shown in Fig. 1: (1) forcing points, which are grid points in the solid phase that have one or more neighbouring points in the fluid phase; (2) fluid points, which are all the points in the fluid phase; (3) solid points, which are all the remaining points in the solid phase. It is proposed to compute u_f by extrapolating along the well-defined line normal to the boundary as shown in Fig. 1. The value of the virtual point u_v can be interpolated from the surrounding grid points.

4 Model and Results

4.1Water exit of a cylinder

The problem of water exit of a cylinder is very significant in variant practical applications. Understanding such complicated physical processes, including breaking up of the free surface, body-fluid interaction and free surface-vortex interaction is useful to understand the impacts of the wave energy converters under the extreme waves. The studies of water exit of a horizontal circular cylinder can be traced back to Greenhow and Lin [8] who conducted experiments to show the free surface deformation in the entry and exit processes.

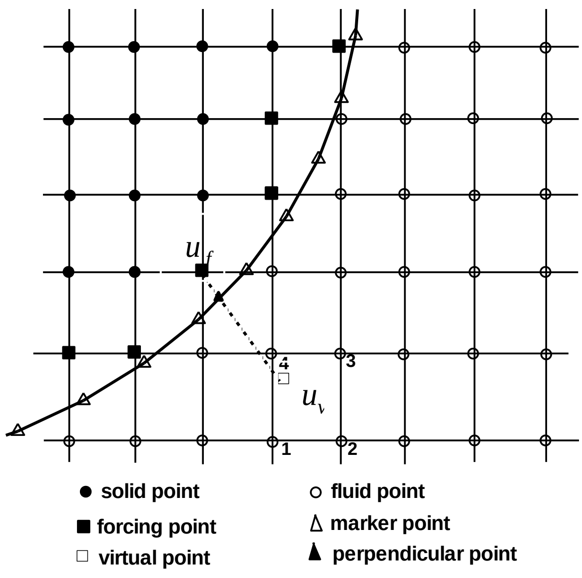
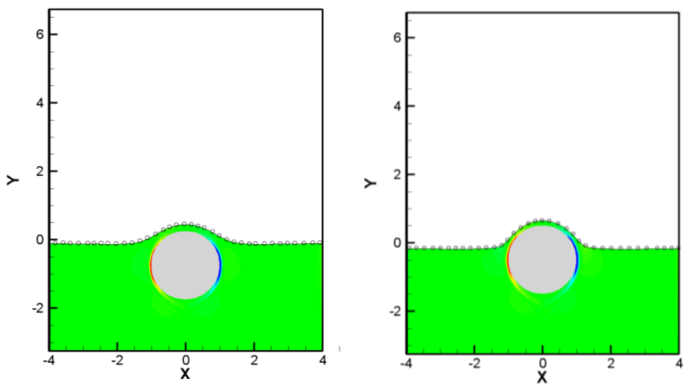


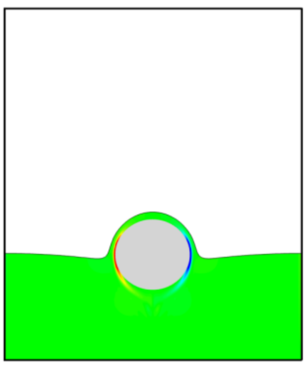
Figure 1: Grid classification and interpolation for u_f

A circle of radius r = 1 m is placed in calm water in a rectangular tank, width = 8 m, height = 10 m and the distance of its centre to the free surface is d = 1.25m. The water has dynamic viscosity 1 × 10^{-3} kg/m/s and the air 1.8×10^{-5} kg/m/s, the density of water is 1000 kg/m3 and air 1 kg/m3. The cylinder is given a constant upward velocity, V = 0.39 m/s. Thus the dimensionless parameters are $\gamma = r/d = 0.8$, $F_r = V/\sqrt{gr}$ =0.39 and $T = Vt/d$. The comparison of free surface profiles between the present numerical method and those presented by Greenhow and Moyo [9] are shown in Fig. 2 at two non-dimensional time instants, T = 0.4 and 0.6. Good agreement can be seen.



(a) (b)

Figure 2: Comparison of free surface profiles with boundary element simulation [7] and the shading represents vorticity strength (-10~10 with intervals 0.4) at two non-dimensional time instants (a) T = 0.4 ; (b) T = 0.6.



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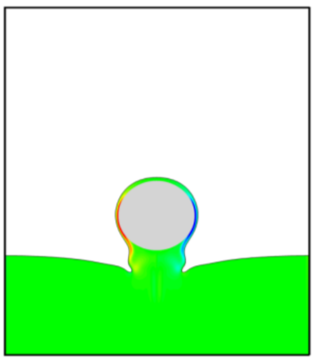

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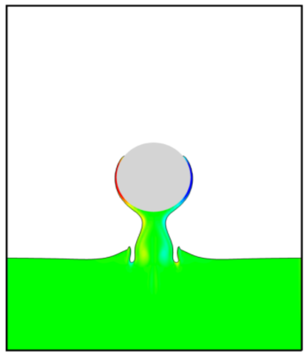

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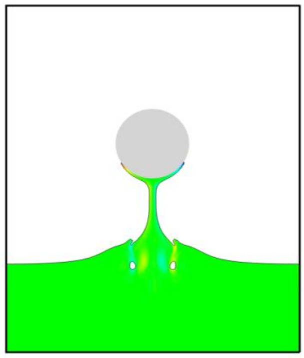


(d)

Figure 3: Water exit of a cylinder. The free surface position (solid black line) and shading represents vorticity strength (-20~20 with intervals 0.2) at non-dimensional time (a) T = 0.8; (b) T = 1.6; (c) T = 2.4; (d) T = 3.2.

Snapshots of the interaction between the cylinder and interface are shown in Fig. 3 at exit velocity V = 1m/s. As the cylinder moves upward, two vortices are formed along the left and right sides of the cylinder. As the cylinder rises further, the two vortices interact with the free surface. Waves are generated in the exit process and propagate towards both sides of the cylinder. Breaking can occur during exit due to strongly negative pressures arising on the cylinder surface.

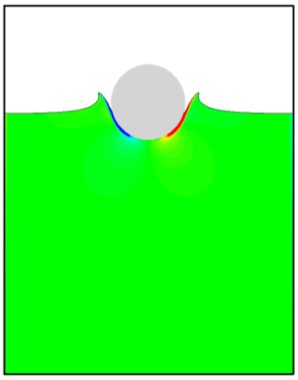
4.2 Water entry of a cylinder

Water entry/impact problems have been studied by many researchers [8-13]. The same parameters to the water exit of a cylinder are used. The cylinder starts its downward motion from a height of d=1.25m to the calm water surface with a constant velocity V = -1m/s. A series of snapshots are shown in Fig. 4. As the cylinder impinges on the free surface, there are jets generated on both sides of the cylinder. When the cylinder moves downward, a large amount of water is pulled downward and surface depression persists. As the cylinder is fully submerged in the water, there is a water jet in the centre of the water surface. The results are very close to those reported by Lin [10]. The air entrainment and water jet are captured very well due to the two phase model used here. The slamming coefficient is given by

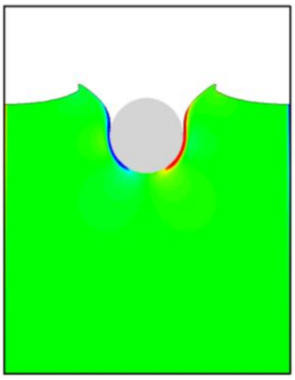
$$C = \frac{F}{\rho r V^2} \qquad (14)$$

Based on the potential flow theory, the hydrodynamic slamming force is given by

$$F = \frac{\pi}{2}\rho V\left(2Vr - 2V^{2}t\right) \tag{15}$$



(a)



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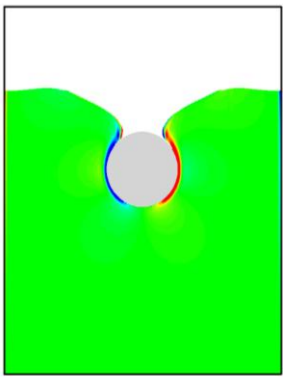

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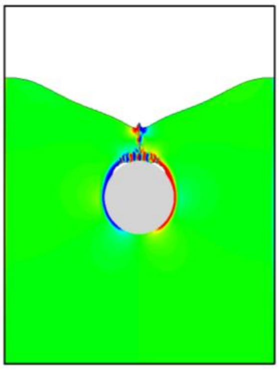

(d)

Figure 4. Water entry of a cylinder. The free surface position (solid black line) and shading represents vorticity strength (-5~5 with intervals 0.01) at non dimensional time (a) T = 0.9; (b) T = 1.8; (c) T = 2.7; (d) T = 3.6.

Fig. 5 shows the comparison of the slamming coefficient at different entry velocity versus the penetration depth between theoretical results [11], experimental results of Campbell and Weynberg [12], numerical results obtained using ComFlow [13] and present model results.

The comparison between theory, the experiments of Campbell and Weynberg, ComFlow numerical simulation and the present simulations is reasonably good. At the beginning, the initial impact reaches 4.0 and later it is around 1.6 for entry velocity V = 1 m/s and 2 m/s. The results agree with ComFlow model very well. The slamming coefficient predicted by the presented model does not reach the initial peak value measured in the experiment, but the variation in slamming coefficient with penetration depth agrees better in the later stage of the impact with the experiment. The slamming coefficients oscillate along the penetration depth at the initial impacting stages, which is due to the lack of stability of the pressure distribution near body boundary in this method.

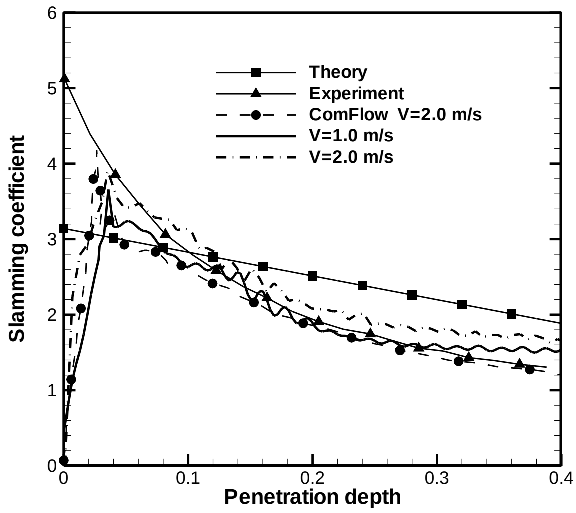

Figure 5. Comparison of slamming coefficient between theory [11], experiment [12], numerical simulation by Comflow [13] and present model.

5 Conclusions

A level set immersed boundary method is shown to be a valuable tool for investigating complex cases of fluid-structure interaction and wave impacts. Surface elevation changes with structure are predicted well. The slamming coefficient predicted by the present model agrees well with the experiment and previous numerical simulations. We are currently extending the method to simulate the coupled dynamic response of the wave energy converter and mooring system under extreme wave loading. The dynamic response of the mooring lines is strongly coupled with the hydrodynamic motion of the wave energy converter and may affect its performance.

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