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A study on the variable sampling interval EWMA \bar{X} chart when the process parameters are unknown

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Abstract. The exponentially weighted moving average (EWMA) \bar{X} chart is well known in statistical process control for detecting small and moderate process mean shift. A variable-sampling-interval (VSI) control scheme of the EWMA \bar{X} chart is able to enhance the speed of detection when the process is out-of-control as the intervals switch according to the value of the previous sample statistics. Generally, the performance of the VSI EWMA \bar{X} chart is investigated under the assumption of known process parameters. However, in reality, these parameters are usually unknown and are estimated from a historical Phase-I dataset. Hence, the performance of the VSI EWMA \bar{X} chart is significantly affected when the process parameters are unknown. This is due to estimation errors. In this paper, we investigate the effects of parameter estimation on the performance of the VSI EWMA \bar{X} chart in terms of the average time to signal and standard deviation of the time to signal. Suggestions on the required number of Phase-I samples and sample sizes in designing the VSI EWMA \bar{X} chart with unknown process parameters, are provided for practitioners' fast implementation.

1. Introduction

Statistical Process Control (SPC) refers to a collection of statistical techniques, which is extensively used to maintain a product's quality at an acceptable level. Control chart are undeniably the most useful tool in SPC to detect any occurrence of process variability. The exponentially weighted moving average (EWMA) \bar{X} chart has been extensively adopted in the context of SPC as it is effective in detecting small and moderate process mean shifts. To enhance the operation of the EWMA \bar{X} chart, using an adaptive strategy, such as varying the sampling intervals, will result in a more effective and sensitive chart towards the detection of process changes. The general idea behind the VSI strategy is that the sampling interval is varied adaptively by switching between short and long intervals, depending on what is observed from the process.

The VSI EWMA \bar{X} chart was first proposed by Saccucci et al. [1]. They studied the run-length properties of the VSI EWMA \bar{X} chart when the process parameters are known. Since then, extensive



research has been focussed on the VSI EWMA-type chart. The statistical performance of the VSI R-EWMA control chart was studied by Castagliola et al. [2] to monitor the range. To monitor the process variance, Castagliola et al. [3] proposed the VSI S^2 -EWMA chart. Kazemzadeh et al. [4] developed an EWMA t chart with VSI feature, based on a control statistic having t distribution for monitoring the process mean. Moreover, the run-length properties of the multivariate VSI EWMA chart was evaluated in Lee et al. [5]. Most of the work on developing the VSI EWMA-type control charts, compare their performance with that of the standard fixed-sampling-interval (FSI) EWMA-type control charts. It is demonstrated that those EWMA-type charts with VSI feature perform better than the FSI EWMA-type charts in detecting process out-of-control conditions, see, for example, [2,3,6].

Most of the research on Phase-II control charts assume that the in-control process parameters are known. However, process parameters are seldom known in practice and thus, they are estimated from a historical in-control Phase-I dataset. A comprehensive literature review on the performance of various types of control charts when process parameters are unknown, is provided in [7] and [8]. They claimed that estimation of parameters severely affects a control chart's performance. This situation has received great interest from several authors to study the impact of parameter estimation and propose methods to reduce this impact. For instance, Khoo et al. [9] and Lim et al. [10] developed optimal designs of the double sampling \bar{X} chart and variable sample size and sampling interval \bar{X} chart with unknown process parameters, respectively. Other related studies on the impact of parameter estimation can be found in [11, 12]. The general conclusion is that the control charts' performance is undesirable when the process parameters are unknown. This is due to the additional variability of the estimates computed from Phase-I process.

The rest of this paper is organized as follow: The VSI EWMA \bar{X} chart is first briefly introduced in Section 2. Next, the run-length properties of the VSI EWMA \bar{X} chart with known process parameters are reviewed in Section 3. Section 4 examines the performance of the VSI EWMA \bar{X} chart with known versus unknown process parameters. Finally, some conclusions are drawn in Section 5.

2. The VSI EWMA \bar{X} chart

Assume that the observations $Y_{i,1}, Y_{i,2}, \dots, Y_{i,n}$ taken from a Phase-II process, are independent and identically distributed and follow a normal distribution with in-control mean (μ_0) and in-control variance (σ_0^2), i.e. $Y_{i,j} \sim N(\mu_0, \sigma_0^2)$, for $i = 1, 2, \dots$ and $j = 1, 2, \dots, n$. The graphical view of the VSI EWMA \bar{X} chart is displayed in Figure 1.

The VSI EWMA \bar{X} chart is divided into three regions, which are the safe region, warning region and out-of-control region. The operation of the VSI EWMA \bar{X} chart is illustrated as follows:

Step 1: A random sample of n observations is collected.

Step 2: The upper (UCL) and lower (LCL) control limits, $UCL / LCL = \pm K_2 \sqrt{\lambda/2 - \lambda}$, as well as the upper (UWL) and lower (LWL) warning limits, $UWL / LWL = \pm K_1 \sqrt{\lambda/2 - \lambda}$ of the VSI EWMA \bar{X} chart are computed respectively, where λ is a smoothing constant; while $K_1 > 0$ and $K_2 > K_1$ are the warning limit and control limit coefficients, respectively.

Step 3: The standardized sample mean, $W_i = (\bar{Y}_i - \mu_0) / (\sigma_0 / \sqrt{n})$ of subgroup i and the VSI EWMA statistic, $Z_i = \lambda W_i + (1 - \lambda) Z_{i-1}$, for $i = 1, 2, \dots$, are computed.

Step 4: If $Z_i \in [LWL, UWL]$, the process is declared as in-control, then the next sample is taken after a long sampling interval, h_1 .

Step 5: If $Z_i \in (UWL, UCL]$ or $Z_i \in [LCL, LWL)$, the process is also declared as in-control, but the next sample is taken after a short sampling interval, h_2 .

Step 6: If $Z_i \notin [UCL, LCL]$, the process is declared as out-of-control and assignable cause(s) must be searched and omitted.

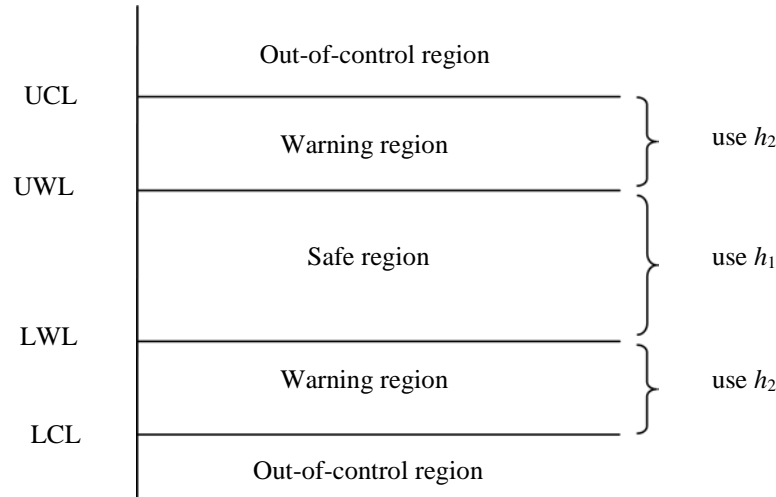


Figure 1. A graphical view of the VSI EWMA \bar{X} chart's operation.

3. The run-length properties of the VSI EWMA \bar{X} chart with known process parameters

As shown in [1], the approach to evaluate the run-length properties of the VSI EWMA \bar{X} chart can be modelled using the Markov Chain approach. Let \mathbf{R} be the transition probability matrix for the transient states, i.e.

$$\mathbf{R} = [R_{k,\ell}]_{(2g+1) \times (2g+1)}, \tag{1}$$

for $k, \ell = -g, \dots, -1, 0, 1, \dots, g$. The Markov Chain approach involves the procedure of dividing the interval between UCL and LCL into $2g + 1$ subintervals, each of width $2d$, where $2d = (UCL - LCL) / (2g + 1)$. Let H_ℓ represents the midpoint of the ℓ^{th} subinterval. Then the entries $R_{k,\ell}$ of matrix \mathbf{R} are

$$R_{k,\ell} = \Phi\left(\frac{H_\ell + d - (1 - \lambda)H_k}{\lambda} - \delta\sqrt{n}\right) - \Phi\left(\frac{H_\ell - d - (1 - \lambda)H_k}{\lambda} - \delta\sqrt{n}\right), \tag{2}$$

where δ is the standardized mean shift which is defined as $\delta = |\mu_1 - \mu_0| / \sigma_0$, with μ_1 is the out-of-control mean and $\Phi(\cdot)$ represents the standard normal cumulative distribution function (cdf).

The average time to signal (ATS) and standard deviation of the time to signal (SDTS) are computed as [1]:

$$ATS = \mathbf{q}^T \mathbf{Q} \mathbf{b} - \mathbf{q}^T \mathbf{b} \tag{3}$$

and

$$SDTS = \sqrt{\mathbf{q}^T \mathbf{Q} \mathbf{B} (\mathbf{2Q} - \mathbf{I}) \mathbf{b} - (\mathbf{q}^T \mathbf{Q} \mathbf{b})^2}, \tag{4}$$

respectively, where \mathbf{B} is a diagonal matrix with ℓ^{th} element equal to b_ℓ , $\mathbf{Q} = (\mathbf{I} - \mathbf{R})^{-1}$ is the fundamental matrix, \mathbf{I} is an identity matrix, $\mathbf{q} = (0, \dots, 1, \dots, 0)^T$ is the initial probability vector, and \mathbf{b} represents the vector of sampling intervals corresponding to the discretized states of the Markov chain. Here, b_ℓ is the sampling interval when Z_i is in state H_ℓ .

In order to have a fair comparison of the performance between the VSI EWMA \bar{X} chart with other FSI control charts, the average sampling interval (ASI) is necessary to be taken into consideration. The ASI is given as follows:

$$\text{ASI} = h_1 p_1 + h_2 p_2, \quad (5)$$

where p_1 and p_2 represent the proportion of times the long and short sampling intervals are used, respectively. Note that $p_1 + p_2 = 1$. The in-control ASI (ASI_0) is set to be 1 time unit as it is common to assume that $h_1 = h_2 = h_F = 1$ time unit for FSI-type control charts. Here, h_F denotes the sampling interval for the FSI-type control charts.

In real practical applications, the in-control mean, μ_0 and in-control standard deviation, σ_0 are unknown and need to be estimated from an in-control Phase-I dataset that comprises of $i = 1, 2, \dots, m$ subgroups $\{X_{i,1}, X_{i,2}, \dots, X_{i,n}\}$, each of n observations. The commonly used estimator for parameter μ_0 is

$$\hat{\mu}_0 = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n X_{i,j}, \quad (6)$$

and the estimator for parameter σ_0 is

$$\hat{\sigma}_0 = \frac{S_{\text{pooled}}}{c_{4,m}}, \quad (7)$$

$$\text{where } S_{\text{pooled}} = \sqrt{\frac{\sum_{i=1}^m \sum_{j=1}^n (X_{i,j} - \bar{X}_i)^2}{m(n-1)}} \text{ and } c_{4,m} = \frac{\sqrt{2}\Gamma[(m(n-1)+1)/2]}{\sqrt{m(n-1)}\Gamma[(m(n-1))/2]}.$$

4. Performance of the VSI EWMA \bar{X} chart with known versus unknown process parameters

The performance of the VSIEWMA \bar{X} chart is evaluated in terms of ATS and SDTS. They are denoted as ATS_1 and SDTS_1 when the process is out-of-control; whereas, ATS_0 and SDTS_0 are employed for the case when the process is in-control. The optimization model for the VSI EWMA \bar{X} chart with known process parameters for minimizing the ATS_1 can be modelled as follows:

$$\text{Minimize } \text{ATS}_1(\delta_{\text{opt}}), \quad (8)$$

λ, K_1, K_2

subject to the constraints

$$\text{ATS}_0 = 500 \quad (9)$$

and

$$\text{ASI}_0 = 1, \quad (10)$$

where δ_{opt} is the desired magnitude of process mean shift, for which a quick detection is needed.

In this paper, the combination of sampling intervals, i.e. $(h_1, h_2) = (1.5, 0.5)$ is chosen. Table 1 displays an overview of the ATS_0 and SDTS_0 values for the VSI EWMA \bar{X} chart with known and unknown process parameters when the process is in-control ($\delta = 0$). Note that four $\lambda \in \{0.1, 0.2, 0.5, 1.0\}$ are considered in table 1. Their corresponding chart's parameters (K_1, K_2) are searched in order to fulfil constraints (9) and (10), of the VSI EWMA \bar{X} chart with known process parameters. These chart's parameters (λ, K_1, K_2) are listed at the top three rows of table 1. The $(\text{ATS}_0, \text{SDTS}_0)$ values obtained in the fifth to 15th rows of the table are calculated with these specific combinations of (λ, K_1, K_2) .

As noticed in table 1, the performance of the VSI EWMA \bar{X} chart when the process parameters are unknown is significantly different from that of the case of known process parameters, especially when m is small. This difference is due to the existence of variability in the estimation of process parameters. However, the effect of parameters estimation obviously reduces when m increases. We

clearly observe that both values of ATS_0 and $SDTS_0$ converge to the desired value of 500 and approximately 500, respectively, when m increases. For instance, when $(h_1, h_2) = (1.5, 0.5)$, the charting-parameter combination of $(\lambda, K_1, K_2) = (0.1, 0.621, 2.821)$ and $m = 25$, the value of ATS_0 is 294.31. This value of ATS_0 increases to 420.30 when $m = 200$, i.e. improving for about 42.81% (see table 1). When m becomes large, the difference between ATS_0 s and the desired value of 500 becomes negligible. Meanwhile, the $SDTS_0$ s decrease as m increases. Obviously, large ($m > 1000$) are needed for the chart with unknown process parameters to achieve similar performance as the chart with known process parameters.

Table 1. The ATS_0 and $SDTS_0$ values of the VSI EWMA \bar{X} chart when $n = 5$ and $m \in \{25, 50, 100, 150, 200, 500, 1000, 1500, 2000, 5000, +\infty\}$, together with the (λ, K_1, K_2) combinations corresponding to the case of known process parameter when $(h_1, h_2) = (1.5, 0.5)$.

λ	0.1	0.2	0.5	1.0
K_1	0.621	0.661	0.647	0.663
K_2	2.821	2.963	3.074	3.093
m	$(ATS_0, SDTS_0)$	$(ATS_0, SDTS_0)$	$(ATS_0, SDTS_0)$	$(ATS_0, SDTS_0)$
25	(294.31, 514.18)	(360.92, 640.18)	(482.16, 873.85)	(589.18, 1050.39)
50	(333.96, 452.30)	(387.62, 522.74)	(471.64, 636.14)	(535.41, 715.99)
100	(378.82, 443.25)	(420.75, 488.79)	(478.01, 554.81)	(515.52, 597.24)
150	(404.06, 447.37)	(439.30, 484.07)	(483.98, 534.04)	(511.02, 564.14)
200	(420.30, 452.00)	(450.86, 483.67)	(487.75, 524.78)	(508.98, 548.40)
500	(460.40, 469.46)	(477.76, 488.88)	(496.29, 510.59)	(505.65, 521.40)
1000	(479.13, 480.73)	(489.37, 493.64)	(499.77, 506.79)	(504.66, 512.78)
1500	(486.37, 485.71)	(493.69, 495.80)	(501.02, 505.65)	(504.34, 509.94)
2000	(490.23, 488.52)	(495.91, 497.02)	(501.66, 505.11)	(504.18, 508.53)
5000	(497.64, 494.26)	(500.17, 499.47)	(502.84, 504.19)	(503.90, 506.01)
$+\infty$	(500.00, 495.99)	(500.00, 498.20)	(500.00, 500.09)	(500.00, 500.74)

Table 2. The ATS_1 and $SDTS_1$ values of the VSI EWMA \bar{X} chart when $n = 5$, $m \in \{25, 50, 100, 150, 200, 500, 1000, 1500, 2000, 5000, +\infty\}$ and $\delta_{opt} \in \{0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0\}$, together with the optimal (λ, K_1, K_2) combinations corresponding to the case of known process parameters for $(h_1, h_2) = (1.5, 0.5)$.

δ_{opt}	0.2	0.4	0.6	0.8	1.0	1.5	2.0
λ	0.044	0.127	0.228	0.330	0.441	0.764	0.942
K_1	0.639	0.644	0.625	0.655	0.657	0.670	0.664
K_2	2.576	2.876	2.991	3.039	3.066	3.091	3.093
m	$(ATS_1, SDTS_1)$	$(ATS_1, SDTS_1)$	$(ATS_1, SDTS_1)$	$(ATS_1, SDTS_1)$	$(ATS_1, SDTS_1)$	$(ATS_1, SDTS_1)$	$(ATS_1, SDTS_1)$
25	(53.21, 142.34)	(9.25, 13.86)	(3.73, 3.18)	(1.94, 1.56)	(1.12, 0.95)	(0.30, 0.43)	(0.05, 0.17)
50	(35.51, 65.13)	(8.13, 6.56)	(3.54, 2.56)	(1.88, 1.39)	(1.10, 0.87)	(0.29, 0.41)	(0.05, 0.16)
100	(28.61, 29.26)	(7.76, 5.34)	(3.46, 2.35)	(1.85, 1.32)	(1.08, 0.84)	(0.29, 0.40)	(0.05, 0.16)
150	(27.04, 22.63)	(7.66, 5.05)	(3.44, 2.28)	(1.85, 1.30)	(1.08, 0.83)	(0.29, 0.39)	(0.05, 0.16)
200	(26.37, 20.30)	(7.61, 4.91)	(3.43, 2.25)	(1.84, 1.29)	(1.08, 0.83)	(0.29, 0.39)	(0.05, 0.16)
500	(25.32, 17.20)	(7.52, 4.69)	(3.41, 2.20)	(1.83, 1.27)	(1.07, 0.82)	(0.28, 0.39)	(0.05, 0.16)
1000	(25.00, 16.37)	(7.50, 4.62)	(3.40, 2.18)	(1.83, 1.26)	(1.07, 0.81)	(0.28, 0.39)	(0.05, 0.15)
1500	(24.90, 16.11)	(7.48, 4.59)	(3.40, 2.18)	(1.83, 1.26)	(1.07, 0.81)	(0.28, 0.39)	(0.05, 0.15)
2000	(24.85, 15.99)	(7.48, 4.58)	(3.40, 2.17)	(1.83, 1.26)	(1.07, 0.81)	(0.28, 0.39)	(0.05, 0.15)
5000	(24.77, 15.77)	(7.47, 4.56)	(3.39, 2.17)	(1.83, 1.26)	(1.07, 0.81)	(0.28, 0.39)	(0.05, 0.15)
$+\infty$	(24.68, 15.61)	(7.45, 4.54)	(3.39, 2.16)	(1.83, 1.25)	(1.07, 0.81)	(0.28, 0.39)	(0.05, 0.15)

Table 2 presents the ATS_1 and $SDTS_1$ values of the VSI EWMA \bar{X} chart for different combinations of m , δ_{opt} and chart's parameters (λ, K_1, K_2) . The optimal charting parameters (λ, K_1, K_2) , which are displayed in the second to fourth rows of Table 2, is computed from the optimization model (8) – (10) corresponding to the VSI EWMA \bar{X} chart with known process parameters. All the $(ATS_1, SDTS_1)$ values for $m \in \{25, 50, 100, 150, 200, 500, 1000, 1500, 2000, 5000, +\infty\}$ are computed using these optimal (λ, K_1, K_2) combinations. For example, when $\delta_{opt} = 0.6$, the optimal chart's parameters for minimizing the ATS_1 are $(\lambda, K_1, K_2) = (0.228, 0.625, 2.991)$, corresponding to the VSI EWMA \bar{X} chart with known process parameters (see Table 2). These chart's parameters yield the smallest ATS_1 value, i.e. $ATS_1 = 3.39$ when $m = +\infty$. Meanwhile, these chart's parameters also give $(ATS_1, SDTS_1) = (3.44, 2.28)$ for sample size $m = 150$. Similarly, when m becomes large, for fixed δ_{opt} value, the performance for the case of unknown process parameters converges to the case of known process parameters. This difference reduces significantly when m increases. For small shifts $\delta_{opt} \leq 0.4$, at least 500 Phase-I samples are needed for the chart with unknown process parameters to be within 3% errors from the chart with known process parameters.

5. Conclusions

This paper clearly demonstrates that the performance of the VSI EWMA \bar{X} chart is significantly deteriorated when the process parameters are unknown, especially for small number of Phase-I samples, m . From the results shown in Tables 1 and 2, large number of Phase-I samples are needed for the VSI EWMA \bar{X} chart with unknown process parameters to achieve similar performance as the case with known process parameters. For in-control ($\delta = 0$) and out-of-control ($\delta_{opt} \leq 0.4$) cases, at least 500 to 1000 Phase-I samples are required for the VSI EWMA \bar{X} chart with unknown process parameters to perform satisfactory. In real life applications, taking a large number of m is impractical. The optimal charting parameters specially designed for the case of known process parameters cannot be used in the case of unknown process parameters, when there is limited number of Phase-I sample m available. Hence, a vital investigation to be conducted in our future paper is optimally designing the VSI EWMA \bar{X} chart when the process parameters are unknown.

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