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Correction for turbulence-induced aberration by modelling the atmosphere as a multimode coupler

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ABSTRACT

A simulation method is proposed which approximates the atmospheric beam path as an extremely large aperture hollow waveguide containing a numerically sufficient subset of linearly polarized lossless propagation modes. The proposed method is shown to agree numerically with the standard angular spectrum of plane waves method in a single transverse dimension simulation and is readily expandable into two transverse dimensions but currently limited by available hardware. The method is expanded to involve coupling matrices describing the transition through each phase screen and the intermediate free-space portions. The coupling matrices are combined into a single multimode coupling matrix describing the propagation for one instance of atmosphere. The proposed matrix method has potential to evaluate multiple input beams simultaneously or condense high turbulence simulations requiring many phase screens. The input beam can be constructed by multiplying the decomposition of the desired output profile with the pseudo-inverse of the coupling matrix; however, the matrix cannot be realized in experiment. Therefore, the opportunity for beam shaping to compensate optical turbulence is evaluated by principal component analysis of the compound coupling matrix. It is shown that an average of the lowest order eigenmode across multiple simulations produces a super-Gaussian-like beam with improved power delivery and stability. The implication for optical countermeasure beam control is the potential to create any desired beam shape at the target plane.

Keywords: turbulence, multimode coupler, aberration, matrix method, pseudo inverse, compensation, countermeasure, simulation

1. INTRODUCTION

Turbulence in the atmosphere is created by refractive index variations due to inhomogeneous air pockets created by solar radiation. The solar energy is injected into the atmosphere through a variety of processes including direct absorption or convection from the ground. Convection to the air takes place as the ground cools, resulting in higher turbulence near ground level as indicated by the Hufnagel-Valley model[1]. The inhomogeneous distribution of dissipated energy creates packets of air which refract light due to refractive index changes related to their temperature and density[2].

Beam control techniques typically involve means to adapt the shape of a wavefront either based on feedback from the laser system[3] or the incoherent return after propagating through the atmospheric path[4]. The motivation of these systems is to optimize the delivery of energy to the desired location in the atmosphere which reduces power requirements and improves overall system efficacy. Established beam control techniques involve a wavefront measurement with feedback loop linked to a spatial light modulator or deformable mirror[3]. The delivery of radiation through the atmosphere has applications in laser-directed energy weapons, directional infrared countermeasure and free-space communication systems.

1.1 Available simulation tools

Theoretical predictions of laser beam propagation in the atmosphere are often based on free-space propagation coupled with transverse perturbations in the form of phase screens. These phase screens are generated statistically to represent accumulated phase retardation from refractive index variations throughout the traversed volume[4,5]. The free-space propagation step is usually performed using the angular spectrum of plane waves (ASPW) method involving a Fourier transformation, followed by multiplication with a phase term in the spatial frequency domain[6]. This approach has become a standard simulation method; however, alternative scalar finite-difference approaches include the Leontovich parabolic method[7].

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1.2 Proposed method

The atmosphere is modelled as a very large multimode waveguide using theory from hollow waveguide structures[8,9]. The method will be referred to in this proceeding as multimode coupler (MMC) method. The modes are constructed using sinusoidal functions:

$$EH_{pq}(x,y) = \sqrt{\frac{2}{L}}\frac{\cos}{\sin}\left(\frac{p\pi x}{L}\right)\frac{\cos}{\sin}\left(\frac{q\pi y}{L}\right) \tag{1}$$

where p and q are positive integer mode orders, cosine and sine are used for odd and even mode numbers respectively, L is the size of the computational array and x varies between -L/2 and L/2. The set of modes is theoretically infinite; however, for a given system there exists a numerically sufficient set of linearly-polarized lossless propagation modes which will describe how the system behaves. The propagation constants are calculated using a quadratic approximation to analytical expressions derived by Laakmann and Steier[8]. The propagation coefficient of each mode is expressed as:

$$\beta_{pq} = \frac{2\pi}{\lambda}\left[1 - \frac{1}{2}\left\{\left(\frac{p\lambda}{2L}\right)^2 + \left(\frac{q\lambda}{2L}\right)^2\right\}\right] \tag{2}$$

where λ is the wavelength. The phase screens are constructed using the standard von Kármán power spectrum. The inner and outer scales used in the analysis are 1 cm and 3 m respectively[1]. The simulation method (ASPW and MMC alike) produce phase screens by filtering complex Gaussian noise with the refractive index power spectrum. The inverse Fourier transformation of this spectrum produces two independent phase screens (real and imaginary parts) which constitute absolute phase values, φ which are applied in the following form as a phase perturbation to the propagating field[10]:

$$\exp[i\varphi(x,y)] \tag{3}$$

Propagation of a laser beam in the atmosphere is simulated through a sequence of propagations in vacuum alternated with the application of a phase term representing the spatial inhomogeneities contained in the traversed atmospheric subsection. This phase term can be applied after, mid-way through or before the free-space segment. The additional phase term represents the atmosphere in the free-space segment. The phase terms lead to distortion of the wavefront which develop into intensity fluctuations when the array enters the next propagation segment. The choice of positioning of the phase screen will either under- or over-estimate the resulting intensity fluctuations – with placing the screen before the segment resulting in the largest observed fluctuations[11].

The propagation of the wavefront by estimation of a multimode coupler begins with a decomposition of the input field $E_{in}(x,y)$ into the basis consisting of the waveguide modes and the field is propagated using coefficients from (2):

$$c_{pq}(z) = \left(\iint EH_{pq}(x,y)E_{in}(x,y)dxdy\right)e^{i\beta_{pq}z} \tag{4}$$

The phase screen term is applied to the reconstructed complex amplitude and then decomposed back into the basis to propagate to the next phase screen. The proposed method has been verified with a Fast Fourier Transform Beam Propagation Method simulation (FFT-BPM) in both vacuum and atmospheric paths (see Figure 1).

The proposed method is verified to adequately model the atmosphere using metrics such as beam size (see Figure 2) and relative error (see Figure 3) with comparison to FFT-BPM in a 1-dimensional like-for-like simulation. Parameters for this simulation are $C_n^2 = 10^{-13}$ over a 2 km path using 20 phase screens, wavelength is 2.1 µm and Gaussian beam waist diameter (center of path) of 40 mm. The proposed MMC method above is analogous to the FFT-BPM but does not permit analysis of the transmission modes. Analysis of such transmission modes can be achieved by considering the multimode coupling matrix.

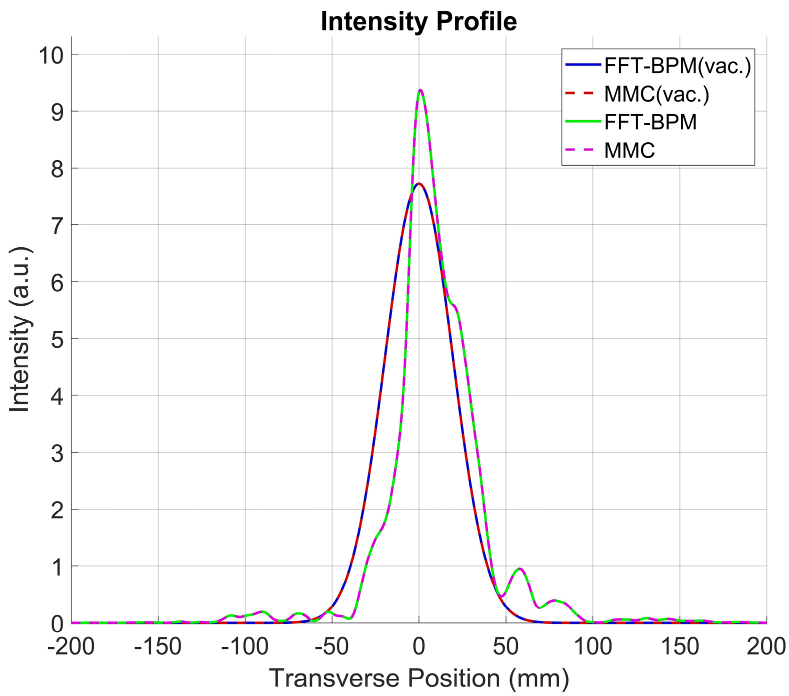

Figure 1. Intensity profile after propagation through vacuum (vac.) and highly turbulent atmosphere with refractive index structure, $C_n^2 = 10^{-13}$ through a 2 km path using 20 phase screens. Simulation executed using standard FFT-BPM (a.k.a. ASPW) and proposed MMC method for comparison. The relative error between the two results is shown in Figure 3.

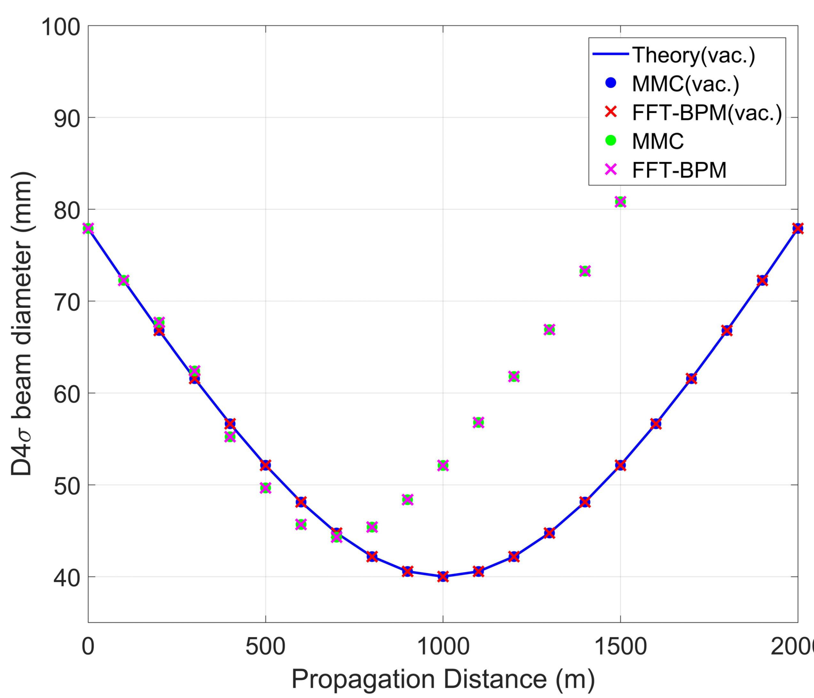

Figure 2. Beam diameter evaluated using standard D4σ monitored through a propagation path of 2 km with refractive index structure, $C_n^2 = 10^{-13}$ and 20 phase screens. Shown alongside the theoretical vacuum propagation, the simulation using both FFT-BPM and MMC methods in vacuum (vac.) and turbulent atmosphere. The beam is near-collimated with a waist of 40mm diameter located 1km from the source. The beam diverges faster when in turbulence due to a reduction in spatial coherence.

2. SIMULATION BY MULTIMODE COUPLING MATRICES

2.1 Motivation

The ASPW method relies on repeated fast Fourier transform (FFT) and element-by-element multiplicative operations on a single-array. Typically, operations with the standardized method will be done in sequence to build statistical information. Also, if the simulation is to be applied several times to a variety of input fields (for example, multiple combined emitters) the generated phase screens need to be stored and applied to each field component at each step – which scales linearly with the number of screens and input fields. In the proposed method however, the atmospheric path can be saved in one compound coupling matrix which scales with the square of the number of grid elements. The proposed method allows propagation of an arbitrary number of emitters in the atmosphere in a single matrix operation. It also permits simple inclusion of full-vectorial propagation for polarization beam combined emitters or communication systems utilizing orbital angular momentum. The simulation parameters required are now only those pertaining to the atmospheric conditions and wavelength. Other parameters such as input beam size, divergence and modal distribution are not necessary in creating the multimode coupling matrix because it is formulated on a basis of arbitrary modes.

Although the 2D method uses much more random-access memory (RAM) than typically available on a standard office computer at the time of writing (8GB); multi-core processors and RAM of 128GB or more which are commonly available on servers are well suited to this computation. For this reason, the analysis presented here is restricted to a single transverse dimension. The multimode coupling matrix can also be decomposed to investigate the transmission modes of the atmospheric path for various system apertures and target sizes.

2.2 Coupling matrices

A multimode coupling matrix, M is an n x n matrix which describes the coupling between n modes. The M_{ij} element of the matrix represents the transfer of field strength and phase from j^{th} to the i^{th} mode. The matrix element is calculated using the formula below:

$$M_{ij} = \iint EH_i\left(Ae^{i\theta}EH_j\right)dxdy \tag{5}$$

where $Ae^{i\theta}$ is the amplitude-phase mask (both amplitude and phase are functions of x and y) and i and j span all (p,q) modes. For the free-space propagation segments, the mode amplitudes remain constant and no mode coupling occurs. Therefore, the matrix for propagation in free-space is diagonal of the form:

$$M_{aa} = e^{i\beta_{aa}z} \tag{6}$$

where a spans all (p,q) mode indices from (1). In addition, any optical element that can be represented by an amplitude and/or phase mask can be represented by an MMC matrix constructed using the following matrix operation:

$$M = E^T\left[diag\left(Ae^{i\theta}\right)\right]Edxdy \tag{7}$$

where E is a matrix whose columns contain mode amplitude at every (x,y) value for a single mode and whose rows contain the mode amplitude at a single (x,y) point for every mode. For example, the (i,j) element of E is given by:

$$E_{ij} = EH_j(i) \tag{8}$$

where i spans all (x,y) points, and j spans all modes. Representation of the optical element in this manner allows propagation through amplitude and/or phase masks without the need to re-construct the complex amplitude. An amplitude-phase mask is now applied simply by multiplication of the MMC matrix to the decomposition vector, c:

$$c_a(z) = Mc_a(0) \tag{9}$$

where a spans all modes, and M is constructed using (7) and represents the amplitude and phase changes of modes over distance z containing any optical element represented by an amplitude-phase mask.

2.3 Compound coupling matrices

A compound coupling matrix can be formed by multiplying a sequence of MMC matrices produced from phase screens and free-space sections in the propagation path. The compound matrix of the form:

$$\boldsymbol{M} = \boldsymbol{M}_k \boldsymbol{M}_{k-1} \ldots \boldsymbol{M}_1 \tag{10}$$

represents the propagation through a complete atmospheric path consisting of k phase screens. Note: the reverse order of matrices – the first matrix in the sequence representing the final atmospheric subsection. The compound matrix therefore transforms between the input and output modal decompositions of the optical system. The accuracy of the compound coupling matrix method is verified by analysis of relative error between the simulated output intensity profiles of the FFT-BPM and MMC methods. The statistical relative error in this case is defined by the standard deviation of the absolute error divided by the mean intensity from 250 simulations of FFT-BPM and the proposed method propagation described earlier in this proceeding. The comparison can be found in Figure 3 and shows there is < 0.1% variance between the recognized FFT-BPM and the proposed MMC method across the central portion of the beam profile containing significant intensity values.

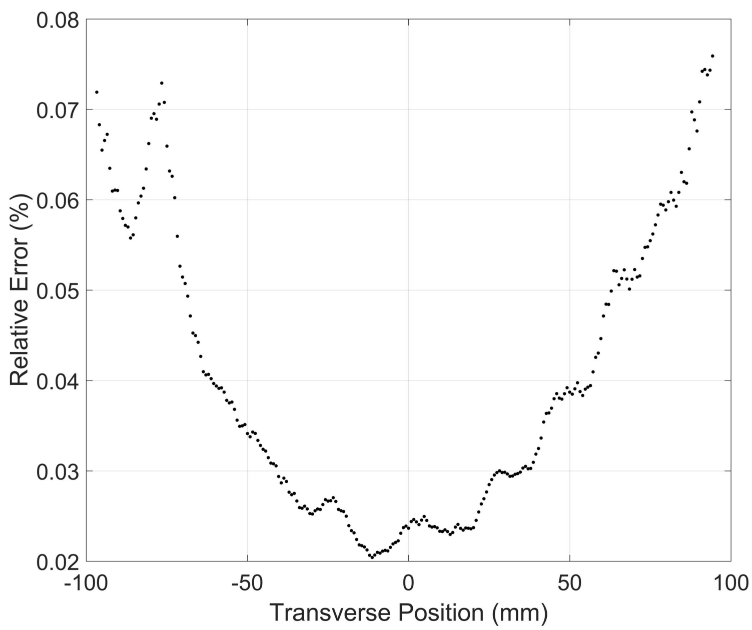

Figure 3. Statistical relative error is calculated for the output intensity profiles of the FFT-BPM and MMC methods. The relative error compares the standard deviation of the absolute error with the mean profile intensity across the sample size of 250 instances.

2.4 Pseudo-inverse

The aberrations introduced by the atmosphere that distort the wavefront as it propagates through the atmosphere can be corrected for by methods such as the Gerchberg-Saxton algorithm[12]. Multiple propagations and projections onto the source amplitude are required to build up a phase profile which corrects for the atmospheric turbulence. However, this result typically only corrects for phase aberrations and does not fully re-construct the desired amplitude profile at the target plane. The proposed method can be used to reverse the propagation through the Moore-Penrose pseudo-inverse. This is analogous to propagating backwards through the atmospheric instance as shown in Figure 4. The conjugate field determined via MMC pseudo-inverse is re-propagated in FFT-BPM to show agreement between methods in Figure 5.

The resulting beam profile from a pseudo-inverse or reverse propagation contains the source complex amplitude required to fully compensate for the given atmospheric instance. In practice the phase screens are unknown, and it is unlikely the MMC matrix will be easily retrievable from information at the target plane; however, a principal component analysis of the multimode coupling matrix can be used to extract the optimum transmission mode of a given optical path.

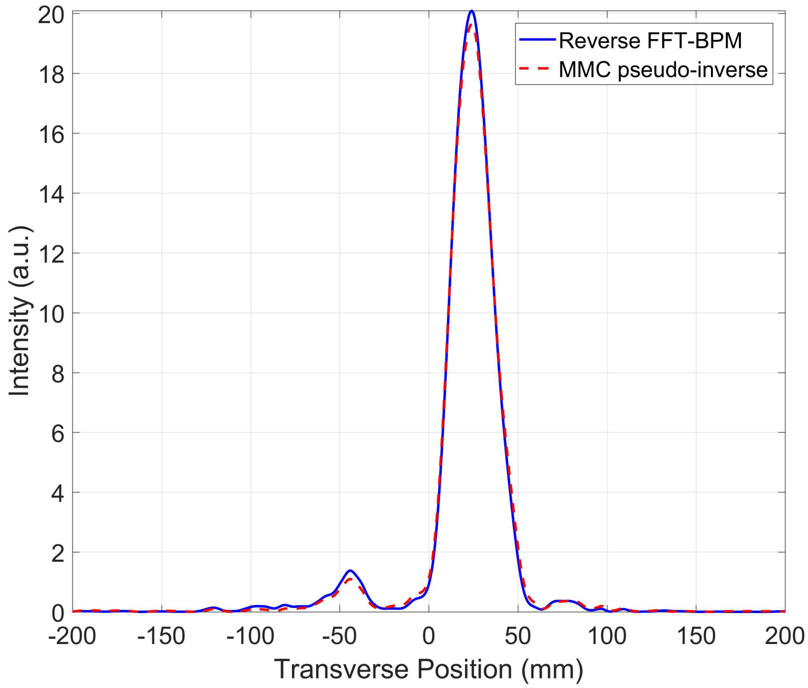

Figure 4. The reverse propagation and multi-mode coupler (MMC) pseudo-inverse input fields which are conjugate fields for phase screen aberrations through the atmospheric path. The output field of the above input field is shown below in Figure 5.

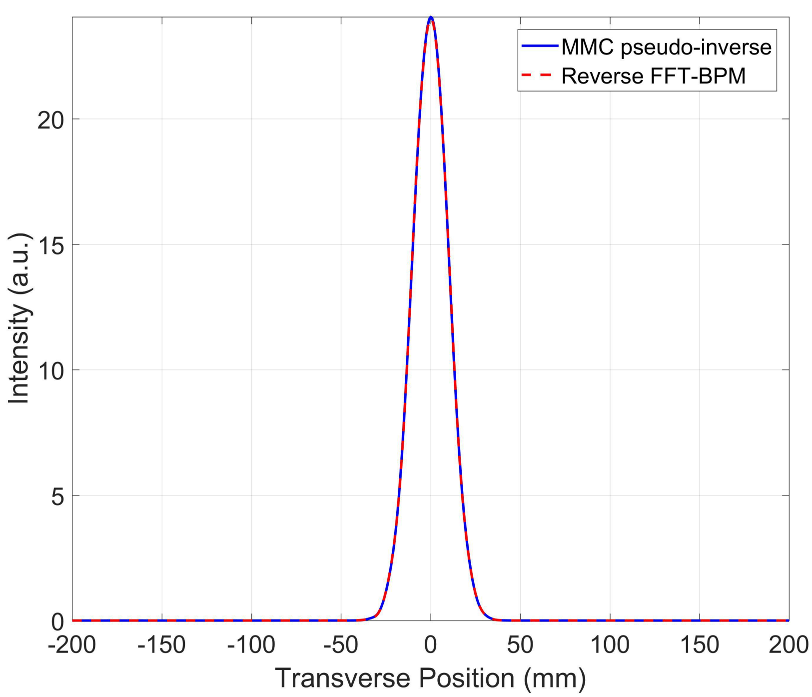

Figure 5. Intensity profiles after propagating the reverse FFT-BPM and MMC pseudo-inverse input conjugate field solutions back through the same instance of atmosphere. Both conjugate input fields are verified using FFT-BPM showing the MMC pseudo-inverse method produces an analogous solution which is compatible with FFT-BPM.

3. PRINCIPAL COMPONENT ANALYSIS

3.1 Motivation

The constraints of the atmosphere can only be modelled statistically and therefore, the pseudo-inverse method described previously would not work in practice since the phase screens are unknown in experiment – only the intensity or wavefront phase at the target can be measured.

A closed loop system would correct outgoing fields using the phase correction algorithms and tilt compensation for beam wander based on the returning field[3]. But in the case of an open loop system, the output of the system must be optimized for the given atmospheric instance and no further correction can be made. The expectation is a Gaussian distribution filling the system aperture would obtain the best performance – measured on power delivery to target. The principal component analysis of the multimode coupling matrix will present the highest power delivery modes for the given atmospheric instance.

3.2 PCA of multimode coupling matrix

Principal component analysis (PCA) can be used to decompose a matrix into a set of p principal components where p is the rank of the matrix. With the addition of hard apertures representing the system aperture and target size, the multimode coupling matrix has less than full rank i.e. $p<n$ where n is the original basis size.

The resulting distortions produce a degradation of the eigenvalues corresponding to each of these modes. For high turbulence simulations, few modes have transmission 0.5 or higher – the transmission of an eigenmode being the square of its corresponding eigenvalue with unity being the largest obtainable value. An example is shown in Figure 6 where singular value decomposition is applied to the multimode coupling matrix. Singular value decomposition represents the MMC matrix M as a sequence of 3 matrices: M = USV* where U and V are unitary matrices constructed of eigenvectors and S is the diagonal matrix of corresponding eigenvalues. The eigenvectors are superpositions of the original basis from which the corresponding field can be constructed. An example of the first five field modes of a decomposed matrix for high turbulence and free-space is shown in Figure 6. Parameters used for system aperture is 200 mm, target aperture is 100 mm, distance is 1 km using 6 phase screens and $C_n^2 = 10^{-13}$.

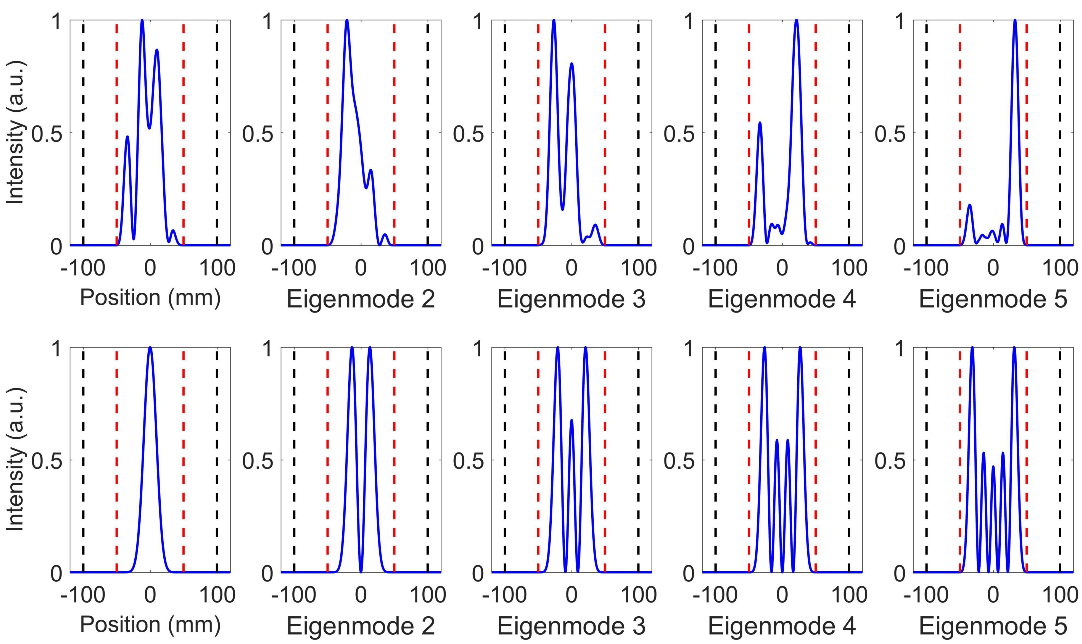

Figure 6. Example of the first five eigenmodes with $C_n^2 = 10^{-13}$ (top row) and free-space (bottom row) using principal component analysis. The black dashed line indicates the system aperture and the red dashed line indicates the target aperture. In this case, these were 200 mm and 100 mm respectively. Path is 1 km with 6 phase screens.

The corresponding eigenvalues show power loss of each mode through the atmospheric path from system aperture to target area/aperture. As the system or target aperture increases, the number of transmission modes with non-negligible transmission increases. The optimal mode size is limited by the smaller of the two aperture sizes assuming the waist of the beam is mid-path.

3.3 Statistically optimal transmission mode

Each instance of the atmosphere has a set of optimum transmission modes extracted from the multimode coupling matrix through principal component analysis. The highest power transmission mode of repeated simulations can help produce a statistical representation of the best mode to use in an open loop beam delivery system for a given atmospheric situation (see Figure 7 and 8). The highest power transmission mode was found to reduce beam wander on the target. In example, from 100 simulations using the first eigenmode, transmission fell from 94.1% to 93.2% (delivery to target aperture) however, rms stability improved for the super-Gaussian beam with 3.95% down from 5.22%. Losses were due to beam wander out of the target aperture indicating a reduction in walk-off (RMSD found to be 11 mm down from 12.2 mm).

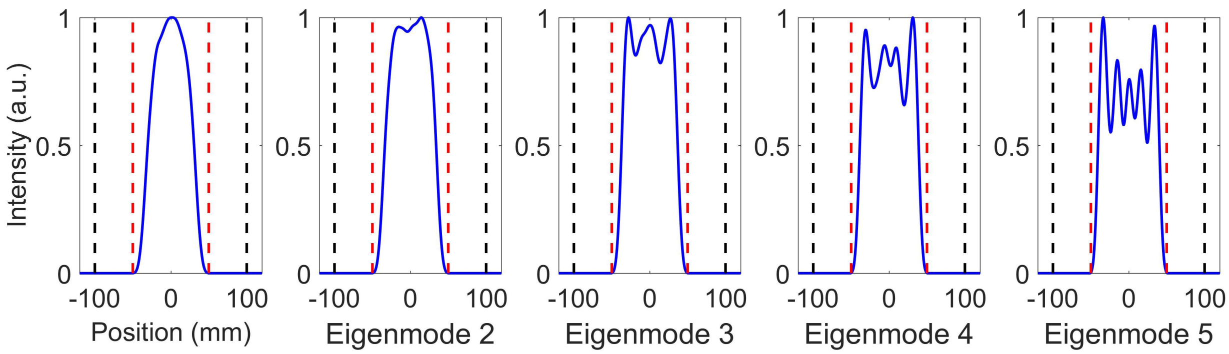

Figure 7. Average of the first five eigenmodes of the system modelled through 500 instances of atmosphere with structure coefficient, $C_n^2 = 10^{-13}$. The intensities of the modes are added together – the phase is ignored. It is worth noting the optimal modes resemble the free-space modes but are wider and super-Gaussian in shape and fill the target aperture.

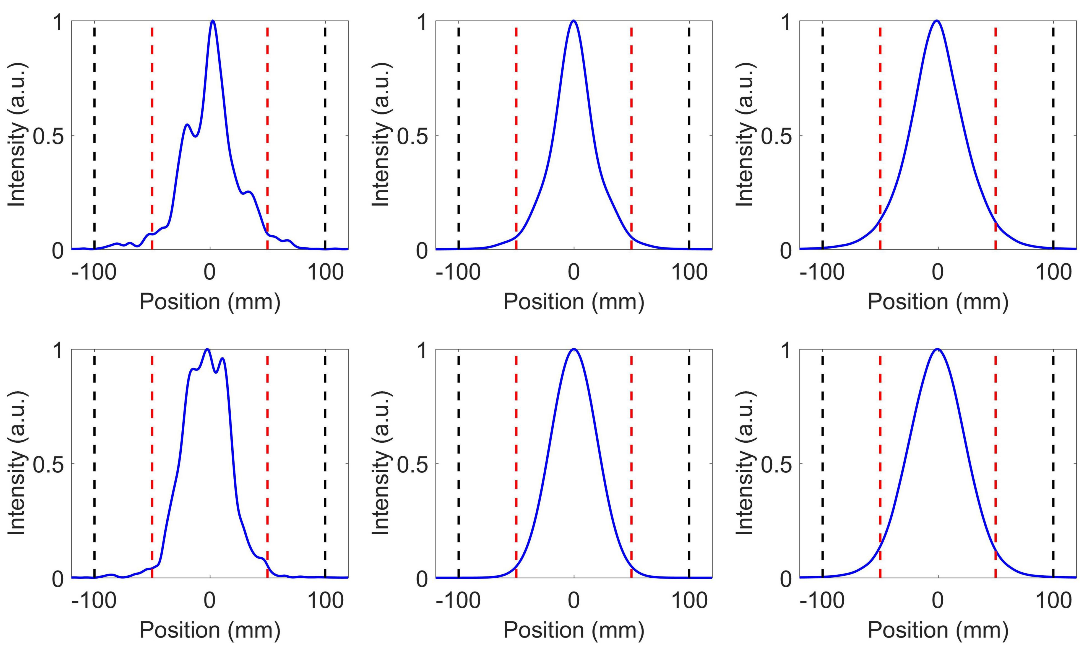

Figure 8. Performance of highest transmission eigenmode from left to right for (i) a single instance ($C_n^2 = 10^{-13}$) (ii) free-space and (iii) ensemble average of 500 instances at $C_n^2 = 10^{-13}$. Top row is performance of the super-Gaussian averaged eigenmode and bottom row is the eigenmode found from free-space and apertures alone. Path is 1 km with 6 phase screens. The spatial sampling was 0.8 mm where the required spatial resolution was found to be <1.2 mm.

4. DISCUSSION

Unfortunately, the inaccessibility of the phase screen structure means that the conjugate field cannot be realized in practice through the pseudo-inverse of the multimode coupler (MMC) matrix. Nor is it simple or perhaps even possible to construct the MMC matrix from intensity or phase information at the target or from the return signature in a closed-loop system. However, the MMC method does provide a representation of the atmospheric path to enable PCA to extract the highest transmission modes of the system.

The choice of laser profile is important in optimizing beam delivery through the atmosphere in an open-loop system. It is shown here that a modified Gaussian may be the optimal transmission mode as opposed to a fundamental mode from a typical stable resonator. The super-Gaussian output field exhibits a reduction in beam walk-off at the target. Whether an improvement in scintillation is observed is still to be investigated. From PCA, the averaged mode was found to resemble a super-Gaussian – which have previously been reported to produce circular Gaussian beams when subject to atmospheric turbulence[13].

The analysis should readily apply to a variety of other optical systems such as laser resonators and amplifiers. Resonator design can be accomplished using this technique through calculating round trip mode coupling coefficients. Since the analysis starts with an arbitrary set of orthogonal modes, PCA permits construction of the eigenmodes of any system.

5. CONCLUSION

The multimode coupler (MMC) method is shown to approximate the widely recognized FFT-BPM. The simulation routine enables the expression of an entire atmospheric instance as a single matrix. The conjugate field required to compensate for the atmospheric turbulence can be found using the pseudo-inverse of this matrix. However, in practice this matrix cannot be determined. Therefore, principal component analysis can be utilized to find the statistically optimal transmission mode for an open-loop system. It is shown here that construction of the optimal transmission mode using an averaged PCA provides a beam profile which delivers more consistent power on target. The proposed method is likely to enable further beam shaping at the target plane by analyzing transmission modes created using amplitude filters instead of apertures.

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