

Assessment of Compliance of Dimensional Tolerances in Concrete Slabs using TLS data and the 2D Continuous Wavelet Transform

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Abstract

While several concrete waviness assessment methods are being developed to overcome the disadvantages of one assessment method over the other, the sparseness of measurements associated with each method prevents from achieving a better understanding of how elevations and undulations change across the surface. Assessing waviness over multiple one-dimensional (1D)-survey lines may not accurately reflect the actual condition or waviness of the entire floor. The methodology presented in this paper presents a compliance-checking algorithm for detecting elements where dimensions exceed specified tolerance. It also enables assessment of a concrete surface in two-dimensional (2D) domain using the synergy of Terrestrial Laser Scanning (TLS) and Continuous Wavelet Transform (CWT). 2D CWT analysis provides information not only about the periods of the surface undulation, but also the location of such undulations. The validity of the methodology is established by running a test on point clouds obtained from a warehouse project near Gresham, Oregon. A rigorous comparison between one of the existing floor waviness measurement methods, the waviness index method, and the proposed method is made. The results showed that the proposed methodology delivers accurate results that enable the localization of surface undulations of various characteristic periods. Furthermore, the proposed method is more efficient in terms of time taken for acquiring the measurements, and is, thus, more cost efficient.

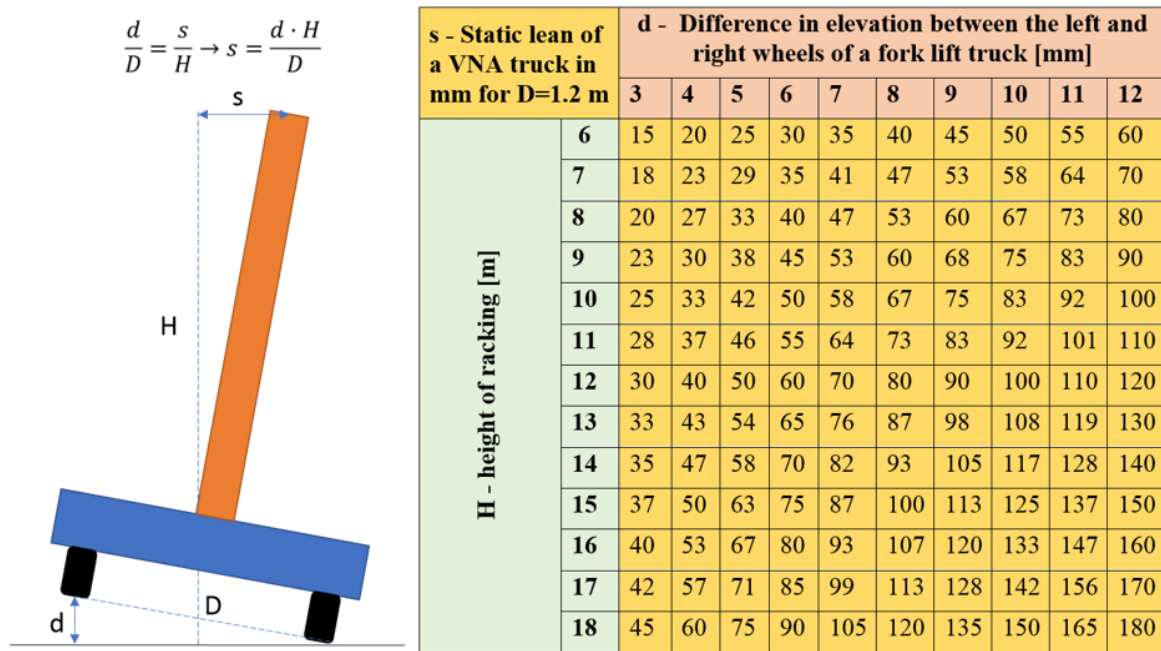
Keywords: Dimensional Quality Control, Tolerance Compliance, Terrestrial Laser Scanning, Point Cloud, Continuous Wavelet Transform, Depth Map

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1 **1 Introduction**

2 As-built dimensions of cast-in-place concrete elements often differ from the dimensions
3 originally specified in as-designed plans [1]. Dimensional Quality Control (QC) verifies that
4 elements are constructed in compliance with the specified dimensional tolerances. For instance,
5 when combining precast and cast-in-place elements, checking dimensional tolerance of all
6 elements is necessary for ensuring acceptable performance of joints and interfacing materials [2].
7 In addition, failure to detect the imperfections in newly constructed surfaces during the earlier
8 stages of construction causes delays in carrying out necessary repair works [3]. The repair,
9 demolition, removal and replacement of defective concrete elements entail additional costs that
10 could amount to as much as 12% of the project contract value [3-6]. Thus, upon the completion of
11 concrete elements, it is crucial to carry out inspections in a timely manner. Furthermore, traditional
12 inspection procedures related to dimensional QC are labor intensive and time consuming.

13 Focusing on concrete slabs, several factors influence the dimensional quality of cast-in-
14 place concrete slabs, such as sweltering temperatures, placement and finishing techniques that are
15 applied during construction. Proper regulation and control of these factors are essential for
16 achieving specified levels of waviness and levelness. The defects resulting from the waviness of
17 concrete slabs not only create aesthetic issues but also affect the efficiency of lift trucks and very
18 narrow aisle (VNA) vehicles. Even if waviness present in slabs is not immediately noticeable, the
19 waviness of the concrete slabs in industrial facilities, such as large warehouses, must be strictly
20 examined since failing to detect waviness and deviations from the specified tolerances can greatly
21 affect the operational activities that the floor is designed to handle [8]. Figure 1 illustrates how
22 irregularities on the floor affect the stability of VNA trucks. Variations in elevation between the
23 left and right wheels (d) of a VNA truck results in static lean (s) of VNA trucks. The static lean
24 can potentially increase up to ten times due to the waviness present in concrete slabs [9].



25

26 Figure 1 The effect of difference in elevation between the wheels on the static lean for a VNA

27 The methods for measuring concrete slab waviness, which are currently prevalent in the
 28 construction industry, require intensive human intervention, are tedious and time-consuming, and
 29 yet are based on sparse measurements. These methods entail the surveying of 1D lines for
 30 differences in elevations and characterizing undulations of specific periods.

31 The Straightedge method involves laying a 10-ft (3.05 m) Straightedge across a survey line on
 32 the floor and measuring the distance between the Straightedge and the floor using a stainless steel
 33 slip gauge [1]. While the results obtained with this method are easily comprehensible, the process
 34 of laying out the Straightedge over large surface areas is labor-intensive and engenders random
 35 errors in the measurements [10]. In addition, this time-intensive method provides information
 36 about the deviations between as-built and as-designed points only at relatively few measured
 37 points.

38 The introduction of the F-numbers method was aimed towards eradicating random errors in
 39 measurements, via the use of instruments that enable measurement of elevation differences at fix
 40 intervals to produce more accurate results. It provides the results in the form of two numbers: Floor
 41 Flatness (FF) and Floor Levelness (FL). FF describes the flatness associated with the measured
 42 floor surface point, whereas FL describes the levelness of the measured floor surface point. As

43 described in the ASTM E1155-14 standard [11], the measurements are carried out at intervals of
44 1-ft along each survey line, and the measurements collected from multiple survey lines are
45 statistically processed to generate FF and FL numbers that describe the conditions of the entire
46 floor surface. In addition, results produced from the F-number method are in the form of flatness
47 numbers (F_F and F_L values) that are hard to comprehend.

48 The Waviness Index (WI) method, as described in ASTM E1486-14 [12], was developed later
49 because the F-number method provides information only about the floor undulations with periods
50 of 1.5 to 4-ft (0.46 – 1.22 m) and 15 to 80-ft (4.6 – 24.4 m). In contrast, the WI method identifies
51 various periods of floor undulations between 2-ft and 10-ft, which correspond to the periods of
52 surface undulation that affect the operation of forklifts [13][8][14]. The results obtained using the
53 WI method are expressed in inches and are relatively easier to comprehend.

54 Despite having a significant advantage over the Straightedge and the F-number methods, the
55 WI method shares similar drawbacks with those methods. Sparse measurements yielded by the
56 three methods fail to guarantee that the collected data is an accurate representation of the geometric
57 features of the surface. Although results may be repeatable with a certain error, they fail to capture
58 the geometric details of the entire 2D floor surface and essentially do not impart information about
59 the waviness of the 2D surface. Data collection from large surface areas using these methods
60 demands significant amount of time and manual labor. Since these methods require measurement
61 tools to be manually moved across the surface of the floor, the results obtained are prone to human
62 error. Random errors, which potentially arise due to possible carelessness exerted while handling
63 the measurement instruments, contribute toward inaccuracies in measurements. Moreover, the
64 inability to retrieve similar results between different measurement sessions is one of the prominent
65 drawbacks of these methods. And finally, applying these methods to measure the floor waviness
66 of large floor areas, such as warehouse projects, is quite difficult. It is important to note that
67 warehouse projects typically have floor surface areas that are larger than 4,000 m². Consequently,
68 using these methods for such projects generate results that are not repeatable. In addition, the
69 obtained results are limited in the orientation of the defects and the range of wavelength.
70 Furthermore, the inability to explicitly reveal the location of those undulations remains a
71 disadvantage for these methods [8].

72 The F-number and WI methods reflect the state-of-the-art practices in measuring the waviness
 73 of concrete slabs. To overcome the challenges arising with these methods, newer technologies,
 74 such as terrestrial laser scanning (TLS), can be leveraged to measure surface waviness in a more
 75 efficient manner. The ability of a TLS device to accurately capture the geometric information of
 76 concrete surfaces provides an opportunity to reconsider the assessment of surface waviness using
 77 traditional measurement instruments.

78 The objective of this study is to develop a methodology that uses TLS to obtain accurate
 79 waviness information about newly constructed and existing concrete surfaces rapidly. This paper
 80 presents a methodology that applies the two-dimensional Continuous Wavelet Transform (2D
 81 CWT) to TLS point clouds to measure concrete slab surface waviness. The proposed methodology
 82 is designed to help carry out tolerance compliance control tasks for slabs based on project
 83 specifications describing their waviness tolerances. Uniquely, the methodology is able to perform
 84 an accurate and comprehensive assessment of the surface geometry in both spatial and frequency
 85 domains. The features pertaining to the proposed and existing methods are summarized in Table
 86 1.

87 Table 1 Features and performance of existing standard methods and the proposed method for
 88 measuring floor flatness.

	Method	Straightedge	F-Number	Waviness Index	2D CWT
Features	Periods of undulation detected	10'-20' (3.05 – 6.10 m)	1.5' – 4' (0.46 – 1.22 m) (F_F) 15' – 80' (4.57 – 24.38 m). (F_L)	2' – 10' (0.61 – 3.05 m)	Any
	Output	Values in Inches	F_F and F_L values	Values in Inches	Map showing detected undulations for various periods
	Types of errors	Random and systematic error	Random error	Random error	Systematic error (± 3 to ± 6 mm)

	Data acquisition efficiency (approx.) (sec/m²)	7	4	4	2
	Point Sampling	Along 1D non-parallel survey lines in 10-foot segments	Along 1D parallel survey lines in two orthogonal directions	Along 1D parallel survey lines in two orthogonal directions	Across 2D surface
	Sparsity of measurements	No	No	No	Yes
Performance Criteria Achievement	Localization of defects	No	No	No	Yes
	Visualization of region of defects	No	No	No	Yes
	Repeatability	No	No	No	Yes

89

90 The rest of the paper is structured as follows. Section 2 provides background information
91 on the utilization of TLS point clouds for project and QC, and the application of discrete and
92 continuous wavelet transforms for surface characterization. Section 3 then introduces the proposed
93 method for characterizing surface waviness of concrete slabs and discusses the various stages of
94 pre-processing and processing the TLS point cloud data. The proposed methodology is validated
95 using data obtained from a warehouse construction project. The experimental results are provided
96 and discussed in Section 4. The final section draws conclusions and discusses directions for future
97 research.

98 **2 Research Background**

99 **2.1 Dimensional Tolerances for Floor Flatness and Waviness**

100 The dimensions of newly constructed and existing building elements can vary, slightly or
101 significantly, from the dimensions specified in the design documents [1]. Tolerances, or allowable
102 deviations in those dimensions, are typically specified during the design phase for different
103 measures such as length, width, thickness, perpendicularity, or verticality. Standard ACI 117-90,
104 for example, provides a comprehensive list of tolerance criteria for cast-in-place concrete
105 elements, such as vertical, lateral, and level alignments, and cross-sectional dimensions. The
106 specified dimensional tolerances are an output of economical and practical considerations [15].
107 The role of QC inspectors is to ensure that the appropriate/specified tolerance values are achieved
108 as construction progresses. Inaccuracies in the geometry of concrete elements during construction
109 arise from improper establishment of a reference system for controlling the alignment, manual
110 measurements, poor workmanship and in some cases, the method used for measurement [16].

111 **2.2 TLS Point Clouds for Project Control**

112 Acquisition of accurate project as-built data is crucial for dimensional quality control
113 measurements, so that informed decisions can be made in a timely manner. TLS is a modern
114 surveying technology that has been gaining increasing popularity in the Architectural,
115 Engineering, Construction and Facilities Management (AEC&FM) industry. The versatility of the
116 TLS technology has been tested in various fields of AEC&FM. For example, TLS has been used
117 in remotely assessing the conditions of environments where human access is difficult or dangerous
118 [17], for generating BIMs that represent the as-built conditions of building facilities [18], as well
119 as construction progress control [15–19].

120 **2.2.1 TLS Point Clouds for Quality Control**

121 The use of TLS for dimensional QC is gaining interest due to its ability to rapidly provide
122 inspectors with project as-built data in the form of both dense and accurate 3D point clouds (sub
123 mm to mm-level accuracy) [24]. Using TLS not only solves the problems associated with accuracy
124 and repeatability, but also enables the acquisition of data that represents the geometry of entire
125 surfaces, thereby addressing the data sparsity limitations of existing surveying methods. Focusing

126 on flatness measurement, compared to existing measurement tools employed in current standard
127 flatness measurement methods, TLS thus offers an efficient way of collecting dense as-built data
128 covering entire slab surface.

129 Regarding the processing of TLS data for QC, Fuchs et al. [25] and Shafer and Weber [26]
130 developed deformation monitoring algorithms to find the differences in positions of TLS data
131 points with respect to a reference surface. In [27], a color map generated from the TLS data was
132 used to assess the flatness of facades in a multi-story building and the additional costs arising from
133 placing excess mortar on these facades was evaluated based on the volumetric quantities derived
134 from TLS data. In [28], a methodology that identifies the geometric irregularities in precast
135 concrete elements by comparing as-built data obtained from TLS and as-designed data obtained
136 from BIM was developed. Tang et al. [7] developed three algorithms which helped in finding the
137 difference in elevation of the points in the point cloud with respect to a plane taken from a BIM
138 model or a plane specified by the user. The method described in [29] used an elevation map where
139 each interval in height were represented with different colors. This approach represented the height
140 of each point with respect to a reference plane obtained from a BIM model. All the proposed
141 methodologies and existing approaches detected areas where different degrees of deviations
142 occurred, however they have failed to characterize the waviness or the periods of surface
143 undulations.

144 Using 3D point clouds obtained from TLS and as-designed geometry information from BIM,
145 Bosché and Guenet [30] developed an approach based on BIM and TLS data to assess whether the
146 geometry of as-is elements adhere to the specified surface flatness tolerances. An experiment was
147 conducted to compare the results obtained using the proposed method with the ones obtained from
148 Straightedge and F-number methods. [The digital application of Straightedge and F-Number, as
149 presented in that study, significantly reduces the time required for data collection and analysis,
150 compared to traditional methods. However, flatness analysis remains conducted along sparsely
151 surveyed 1D survey lines, which delivers results limited in spatial and wavelength resolution.](#)
152 Bosché and Biotteau [8] developed a method that applies 1D CWT to TLS data to characterize
153 surface undulation periods. [That approach addresses the limitation of previous works in
154 wavelength resolution, i.e. the approach examines surface waviness at a wider range of
155 wavelengths or characteristic periods. However, the study remains based on measurements along](#)

156 1D lines that do not enable an analysis of flatness in all possible directions, and still provide results
157 with limited spatial resolution. Valero and Bosche [31] presented preliminary results on the
158 application of the 2D CWT to TLS data and compared the results with the WI method using
159 laboratory experiments. The work presented here expands on this and delivers a 2D analysis of
160 surface data in a more comprehensive manner using a larger and more representative concrete slab
161 as case study.

162 **2.3 Wavelet Transform for Surface Characterization**

163 Wavelet Transform (WT) has a wide range of applications in engineering, some of which
164 include seismic signal analysis [32], sound pattern analysis [33] and quantum mechanics [34]. WT
165 is also widely used in the area of surface texture characterization, where it is used to break down
166 the 2D profiles of surfaces into the roughness and waviness components [35]. WT has been applied
167 to characterize surface roughness and waviness in several studies, and it has many applications in
168 the field of point cloud processing such as point cloud de-noising and rock surface roughness
169 quantification [31–33]. A technique using 1D WT for characterizing different types of surfaces
170 was introduced by Chen et al. [39]. Josso et al. [40] performed 2D multi-scaled decomposition
171 using images instead of profiles. Stepień and Makiela [41] applied 2D WT to analyze the
172 deviations of cylindrical surfaces. Jiang et al. [42], Coiffman and Maggioni [43] and Hussein et al.
173 [44] described the concepts of lifting wavelets and diffusion wavelets and used them for surface
174 filtering. WT can be extended from 1D analysis to multi-dimensional signals as well [8][45]. Some
175 applications of 2D CWT include characterizing the wavelengths of landslide areas and identifying
176 the regions having high risk of landslides using topographic images [46]. Additional information
177 about the different types of wavelets and the application of wavelet transform, can be found in [47]
178 and [48]. And, as reviewed above, the CWT has been previously suggested in [8] and [31] for use
179 in construction surface flatness assessment.

180 **2.4 Continuous Wavelet Transform**

181 The continuous wavelet transform (CWT) of a given function is the inner product of the
182 function with the scaled and shifted versions of the mother wavelet [47]. The output of the inner
183 product is the wavelet coefficient at a specific time or location and scale [48]. In order for a

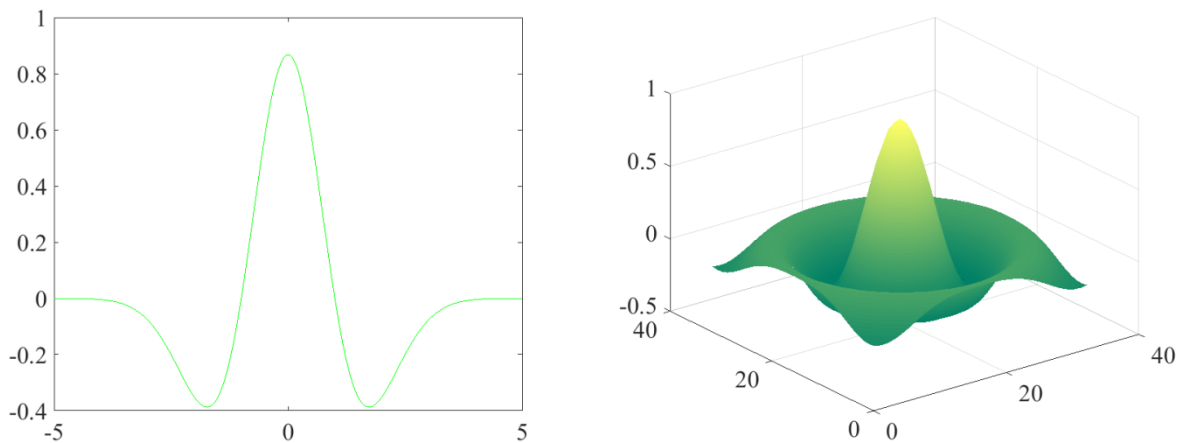
184 function, $\psi(t) \in L^2(\mathbb{R})$, to qualify as a mother wavelet, it has to satisfy a condition known as the
 185 admissibility condition as stated below [49]:

$$186 \quad 0 < C_\psi = \int_{-\infty}^{\infty} \frac{|\psi(\omega)|}{|\omega|} d\omega < \infty \quad (1)$$

187 where, C_ψ is the admissibility condition and ω is the angular (or radian) frequency. This
 188 condition can also be written as [49]:

$$189 \quad \Psi(0) = \int_{-\infty}^{\infty} \psi(t) dt = 0 \quad (2)$$

190 This implies that the function $\psi(t)$ has to move above and below the t-axis in a wave-like
 191 manner with decaying properties. Figure 2 demonstrates such properties with the example of a
 192 typical wavelet function commonly known as the Mexican Hat wavelet.



193

194 Figure 2 1D (left) and 2D (right) Mexican Hat Wavelet

195 CWT can be used to describe the time and frequency components of a temporal signal in
 196 detail. 1D CWT involves taking the original function and displaying the output function in terms
 197 of two variables, which are time and scale. The spectral information about a 2D signal for any
 198 scale s and location (x, y) is given by the 2D CWT [50], that is an extension of the 1D CWT and
 199 can be represented as follows [51]:

$$200 \quad CWT(a, b, s) = \frac{1}{s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \psi_{abs}(x, y) dx dy \quad (3)$$

201 where, $\psi_{abs}(x, y)$ is the mother wavelet, $g(x, y)$ is the continuous 2D signal, s is the scale
 202 (dilating parameter) and (a, b) represents the location (translating parameter).

203 Different values of the translating and dilating parameters of the mother wavelet help in
 204 describing the different frequencies of undulations present in the surface [52]. The convolution of
 205 ψ and g provides the wavelengths (or periods) of the undulations present in the surface. The
 206 coefficients $CWT(a, b, s)$ quantify the degree of correlation between the wavelet ψ and the
 207 function g at each point. In this way, apart from analyzing signals in time and frequency, the CWT
 208 can be extended to analyze signals, together in space and scale (space-scale analysis).

209 Different wavelets can be used as the mother wavelet to detect different types of
 210 undulations. The selection of an appropriate type of wavelet determines how efficiently the
 211 different components of a signal are extracted [53]. The geometric shapes of the wavelet is
 212 considered an important criteria when selecting the type of mother wavelet in [54]. The
 213 resemblance between the shape of the wavelet and the geometric features of a signal provides a
 214 cue for the selection of an appropriate wavelet. The defects present in as-built or as-is concrete
 215 surfaces resembles waves in the form of small bumps and dips, and the shape of 2D Mexican Hat
 216 wavelet closely resembles the shape of the surface undulations present on concrete surfaces, as
 217 shown in Figure 2. The Mexican Hat wavelet is a real and isotropic wavelet that is good for
 218 detecting contour features [55][56]. The use of different values of the translation and scale
 219 parameters of the mother wavelet enable the detection of undulations corresponding to different
 220 characteristic periods in the point cloud data. Therefore, it is chosen as the mother wavelet in this
 221 study. 2D Mexican Hat wavelet in spatial domain is depicted as follows:

$$222 \quad \psi(x,y) = \frac{1}{2\pi}(2 - x^2 - y^2) * e^{-\frac{1}{2}(x^2 + y^2)} \quad (4)$$

223

224 **2.5 Contribution**

225 Floor flatness testing is typically performed to assess how well the contractor has performed
 226 the work based on the specifications. Failure to carry out floor flatness measurements within a
 227 specific time window may yield inaccurate results and does not accurately reflect the contractor's
 228 performance. This can be attributed to the increase in curling of the joints and cracks in the concrete
 229 slabs with the age of concrete. Therefore, the American Concrete Institute (ACI 117-10) [57]
 230 requires that floor flatness testing on any concrete slab should be performed within 24 hours for

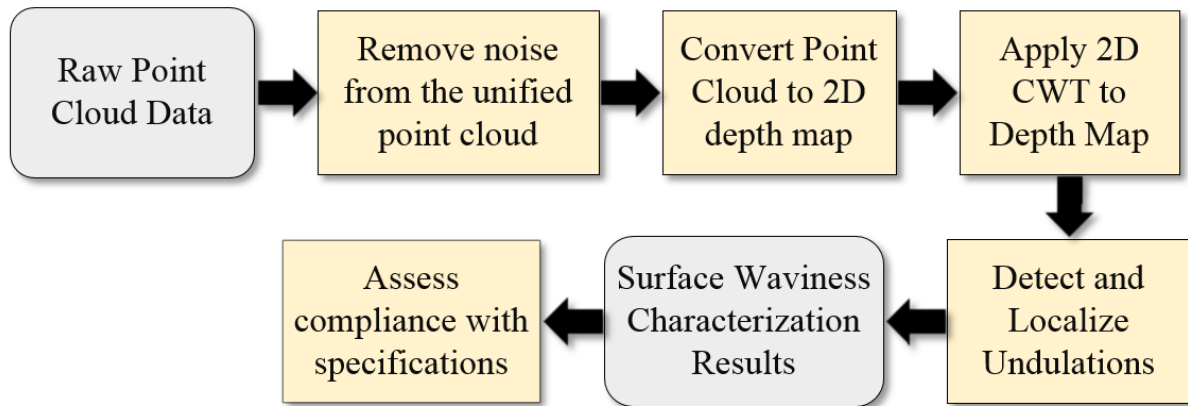
231 best results, and no later than 72 hours, after the concrete placement, unless clearly stated otherwise
232 in the specifications.

233 The major contribution of this paper is to present a new approach for floor flatness control
234 using the 2D CWT applied to TLS data. The approach builds on preliminary works published in
235 [33], but presents and analyses it in a more comprehensive manner. In particular, a warehouse
236 concrete slab having been scanned within 10 hours of having been poured is used as a
237 representative real-life case study. The overall time required for scanning the worksite is less than
238 the time associated with data collection using the traditional methods of waviness measurement.
239 The waviness results obtained using the proposed approach are validated by establishing a
240 correlation between results obtained using the proposed approach and the WI method. The
241 correlation shows that the proposed method generates results that supersede those that are obtained
242 using the WI method.

243 **3 Proposed Methodology**

244 The proposed methodology, as summarized in Figure 3, follows the approaches developed in
245 [31] in order to characterize surface waviness. The raw 3D point cloud consists of data points from
246 a concrete slab as well as its surrounding environment, including workers, equipment and
247 surrounding buildings. The point cloud is typically the result of multiple laser scans co-registered
248 using a standard (reliable) target-based approach. First, the raw point cloud is pre-processed to
249 isolate the area of interest, the concrete slab in this case, from the raw point cloud. The next step
250 is to develop a depth map, which is used as input to the 2D CWT. The areas with undulations
251 corresponding to various characteristic periods are identified after applying the 2D CWT with the
252 Mexican Hat wavelet.

253



254

255

Figure 3 Overview of the research methodology

256 3.1 Data pre-processing

257 As stated in the overview of the research methodology, the raw point cloud data should be
 258 pre-processed before 2D CWT can be applied to it. The input is a raw point cloud that may be the
 259 result of the co-registration of multiple scans collected during the scanning process. The noise
 260 present in the registered point cloud data is removed using a corresponding functionality provided
 261 by a commercial point cloud processing software, leaving a clean point cloud of the area of interest
 262 (i.e. concrete slab). The pre-processed point cloud corresponding to the slab surface is aligned
 263 (parallel) to the xy plane, which is likely to be the case already. Accordingly, z coordinates
 264 represent the elevation of each point, facilitating further analysis.

265 In order to analyze the frequencies of undulations present on the surface, the point cloud
 266 data should have equispaced rows and columns along the x and y axes [58]. Because raw point
 267 cloud data from slabs typically has a random arrangement, it is converted into a regular grid, with
 268 intervals along the x and y axes were both set to $\delta_p = 1$ cm. This sampling interval ensures robust
 269 localization of defects across the 2D surface. Triangulation-based linear interpolation is used to
 270 obtain the values of z -coordinates at each grid point. Consequently, a 2D depth map is created,
 271 which represents the height of the surface for points at $\delta_p = 1$ cm intervals along the x - and y -axes.

272 3.2 Detection of Undulations using 2D Continuous Wavelet Transform

273 The depth map resulting from the previous operation is used as the input “signal” to the 2D
274 CWT. The scale a at which the CWT is applied relates to a few parameters, as in Equation 5 [48]:

$$275 \quad a = \frac{f_c}{f \cdot \delta_p} = \frac{T \cdot f_c}{\delta_p} \quad (5)$$

276 where f represents the frequency of the undulation, T the characteristic period of the signal,
277 and f_c , the main frequency component of the Fourier Transform of the mother wavelet. For the
278 Mexican Hat wavelet, $f_c = 0.252 \text{ cm}^{-1}$.

279 The output of applying the 2D CWT is a series of scalograms that report the CWT response at each
280 grid point on the depth map. These maps are meaningful to some extent, but should be further
281 processed to accurately define the exact characteristic period at each location. Indeed, a wavy
282 region will result in peak responses at several scales, i.e. frequencies, as can be seen in Figure 77
283 for example. However, not all those peaks correspond to defects whose size matches the period
284 associated to that scale. For this, we follow the strategy initially suggested in Valero and Bosché
285 in [31]. First, peak values (i.e. local maxima) are detected in each 2D CWT response map. Next,
286 different isolines are then calculated around each peak, which connect pixels with the same CWT
287 response. These isolines may describe irregular shapes whose mathematical analysis, and further
288 comparison with other defects, can be truly complex. Therefore, areas enclosed by isolines are
289 described by means of ellipses, and the two main axes of each ellipse are determined to be used as
290 reference values. If any of the axes matches the period associated to the scale of interest, a surface
291 deviation is detected in that area for that particular period.

292 The result of this process is a set of clear waviness defect detections at all the scales/periods
293 considered, which can be combined in a single diagram.

294 3.3 Correspondence between WI and 2D CWT methods.

295 The correspondence between the WI method and the 2D CWT method in terms of their
296 response to similar surface wavelengths (or periods) is shown in Table 2. The k values of the WI
297 method correspond to characteristic periods of different lengths. The corresponding CWT scales
298 for each of these k values are calculated using equation 5 with $\delta_p = 1 \text{ cm}$. The characteristic

299 periods (T) selected in this table represent the floor undulations, with periods of 2, 4, 6, 8 and 10
 300 ft, that are the focus of the WI method.

301 Table 2 Continuous Wavelet Transform scales and equivalent Waviness Index [8]

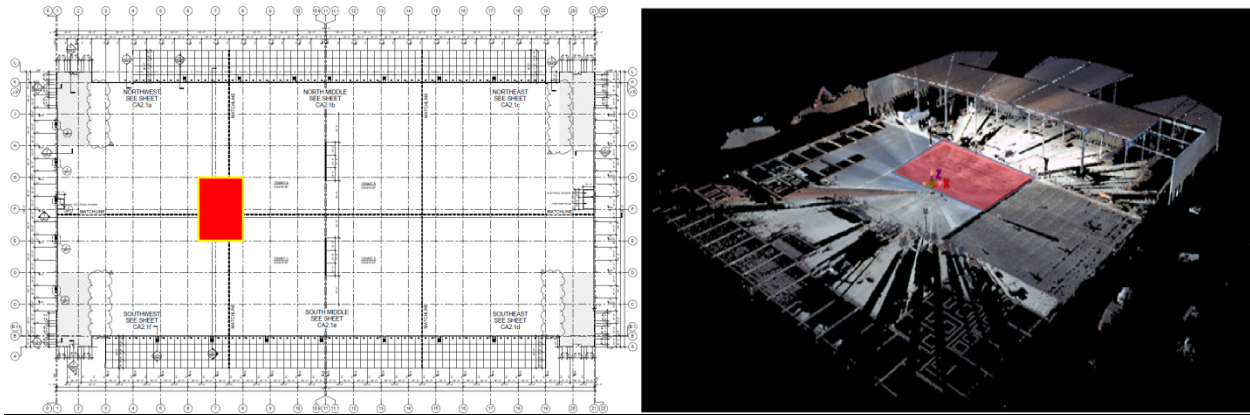
Characteristic period (T) [cm]	CWT scale (a)	Waviness Index (k values)
61	15	1
121.9	30	2
182.9	45	3
243.8	60	4
304.8	75	5

302 4 Experimental Results

303 4.1 Data Collection and Pre-Processing

304 An in-situ concrete slab from a warehouse project in Gresham, Oregon was scanned after
 305 5-6 hours of placement. The surface of the concrete slab was sturdy enough for foot traffic and for
 306 setting up the tripod of the scanner. The concrete slab of the warehouse building was scanned using
 307 a Leica ScanStation P40 3D laser scanner. The scanner has 8" horizontal and 8" vertical angular
 308 accuracy. The 3D position accuracy is ± 3 mm at 50 m and ± 6 mm at 100 m [59].

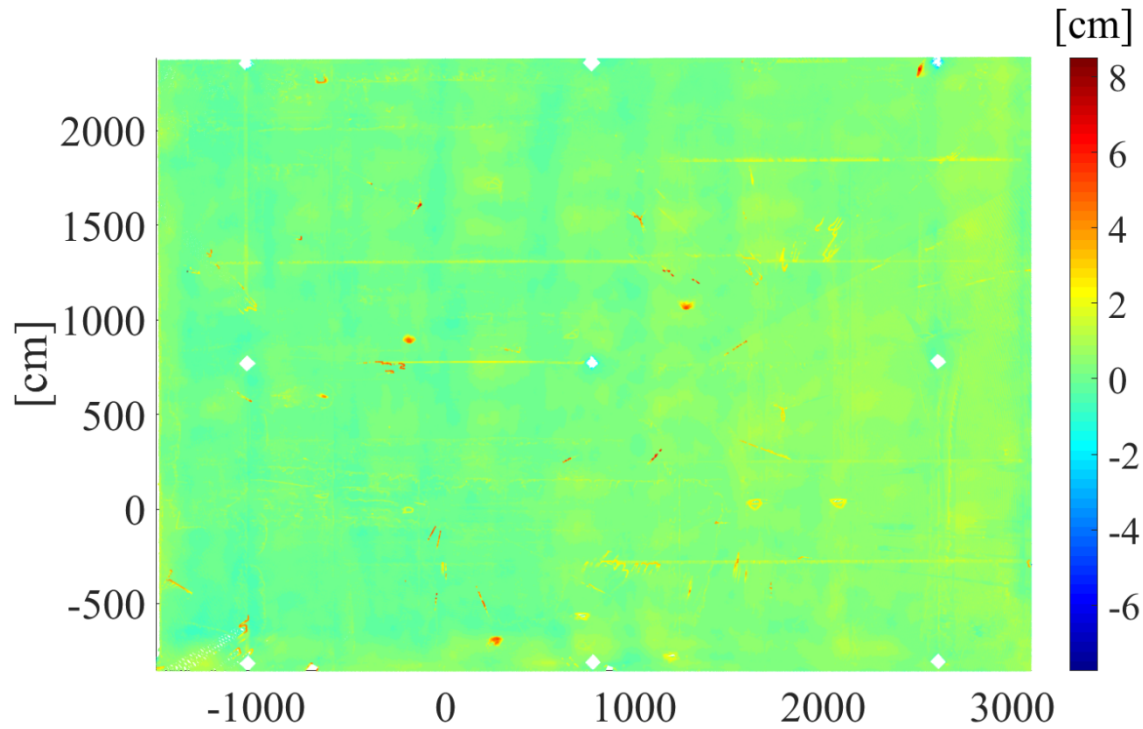
309 Figure 4 shows the plan view of the concrete slab as well as one of the 3D point clouds captured.
 310 The area of interest with a surface area of approximately 1500 m² is highlighted in both the plan
 311 view and the point cloud. It was determined that scans taken from four different locations would
 312 be sufficient to capture the surface with sufficient detail. Six targets were placed at different
 313 locations on site to facilitate the point cloud registration process.



314

315 Figure 4 The floor plan (left) and the 3D point cloud (right) of the warehouse building. The area
316 of interest is highlighted in red.

317 The overall scanning process, including setup, scanning, dismantling and re-locating, took
318 approximately 45 minutes. The data pre-processing stage, comprising of registering the point
319 clouds in the same coordinate system and removing the noise, took 50 minutes. The raw point
320 clouds, i.e. laser scans including noise, were first imported into a commercial point cloud
321 processing software. The four laser scans were registered under the same coordinate system using
322 the targets placed at strategic locations on the construction site. After the registration was
323 complete, the point cloud of the area of interest was manually isolated from the rest. The point
324 cloud corresponding to the slab section of interest had approximately 100,000,000 points. Figure
325 5 shows an image of the scan after registration and noise removal.

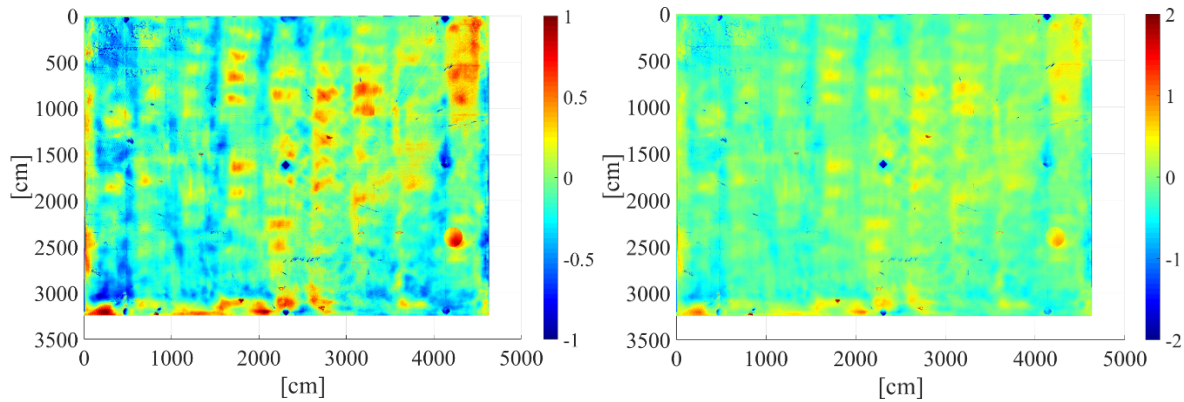


326

327 Figure 5 Top view of the point cloud of the area of interest obtained after the registration of the
 328 four scans and noise removal. The color scale represents elevation values in cm.

329 4.2 Data Processing

330 Following pre-processing, the point cloud is converted into a 3242 x 4629 depth map with 1
 331 cm intervals in both horizontal and vertical directions, with the z -coordinates at each grid point
 332 calculated as described in Section 3.1. Figure 6 shows the resulting depth map. As seen in Figure
 333 6, the height of the concrete slab with respect to the xy plane, varies most in the interval of -2 cm
 334 to 2 cm. Thus, the color map was adjusted accordingly to highlight the height differences between
 335 various areas across the floor.



336

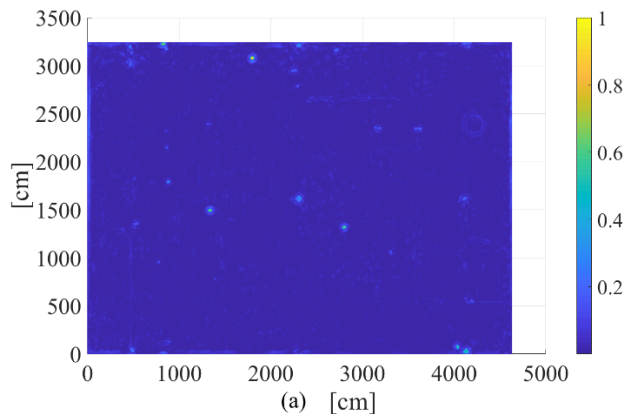
337 Figure 6 Depth map derived from the TLS data, with color map limits set to $[-1, 1]$ cm (left) and
 338 $[-2, 2]$ cm (right)

339 4.3 CWT Results

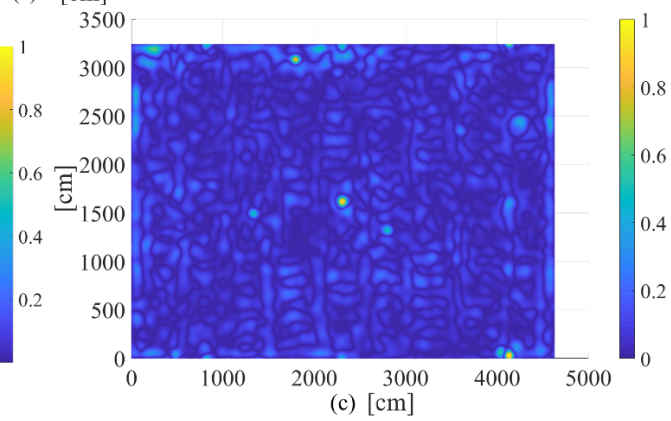
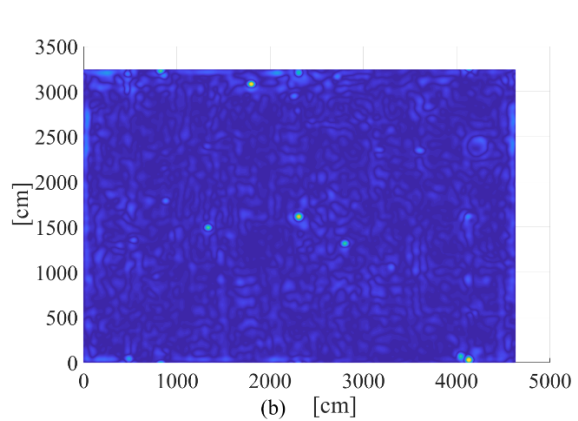
340 4.3.1 CWT Scalogram

341 Figure 7 presents the results obtained using the scales (a) 15, 30, 45, 60 and 75 of the
 342 mother wavelet. The regions where the input “signal” strongly correlates with the mother wavelet
 343 applied at the scales above are highlighted in yellow in Figure 7.

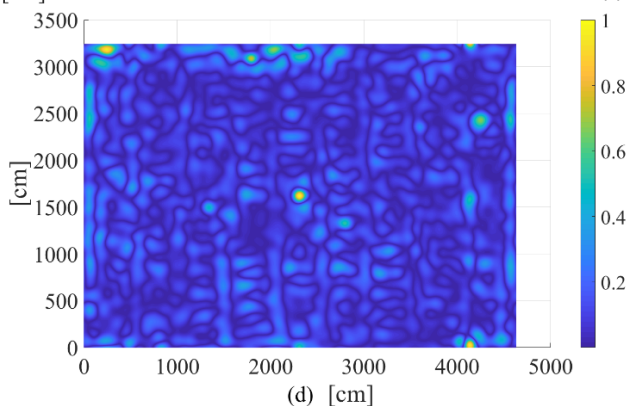
344 The point cloud in Figure 5 and the depth maps in Figure 6 show that the surface of the investigated
 345 slab is relatively flat with a few “peaks”. For the entire slab surface, the average value of the
 346 deviation in the z-axis was 0.0 cm with a standard deviation of 0.3 cm. Figure 7(a) represents the
 347 regions where the wavelength of the undulations present on the surface correlates with the mother
 348 wavelet of scale 15. The scalogram shows that the regions near (1900 cm, 3100 cm) has an
 349 undulation of this characteristic period present on the slab surface. Figure 7 (b) shows regions near
 350 (1900 cm, 3100 cm) and (2400 cm, 1700 cm) have undulations that correspond to the mother
 351 wavelet of scale 30. Similarly, the scalograms in Figure 7 (c), (d) and (e) show that the region near
 352 (2400 cm, 1700 cm), (300 cm, 3400 cm) and (4200 cm, 2400 cm) have undulations corresponding
 353 to scales 45, 60 and 75. The region near (2400 cm, 1700 cm) shows responses for all these three
 354 scales. Thus, the identification of the scale which has the top response at that location is necessary.
 355 Furthermore, such analysis is an added advantage of using the proposed 2D CWT method and
 356 cannot be done using the WI method.



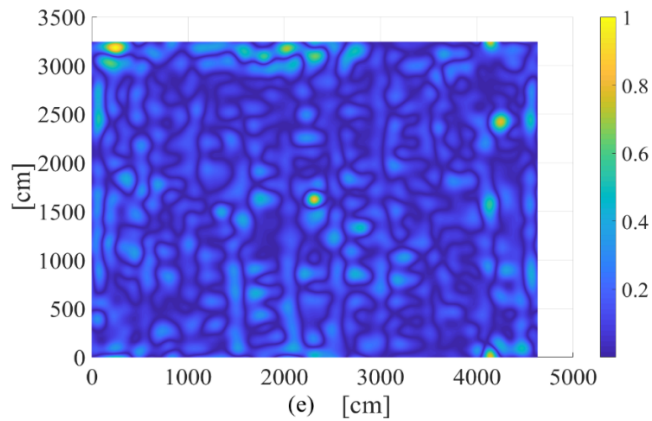
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358



359

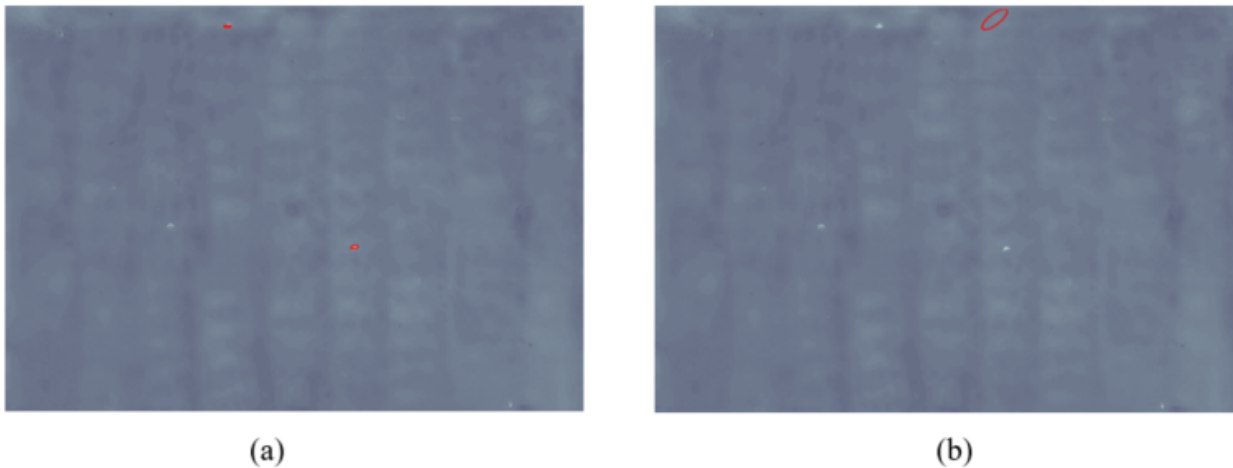


360

361 Figure 7 The coefficients obtained from the wavelet transformation corresponding to scales 15
 362 (a), 30 (b), 45 (c), 60 (d) and 75 (e) are plotted on the map. The areas in the slabs where
 363 undulations corresponding to these scales are present are shown.

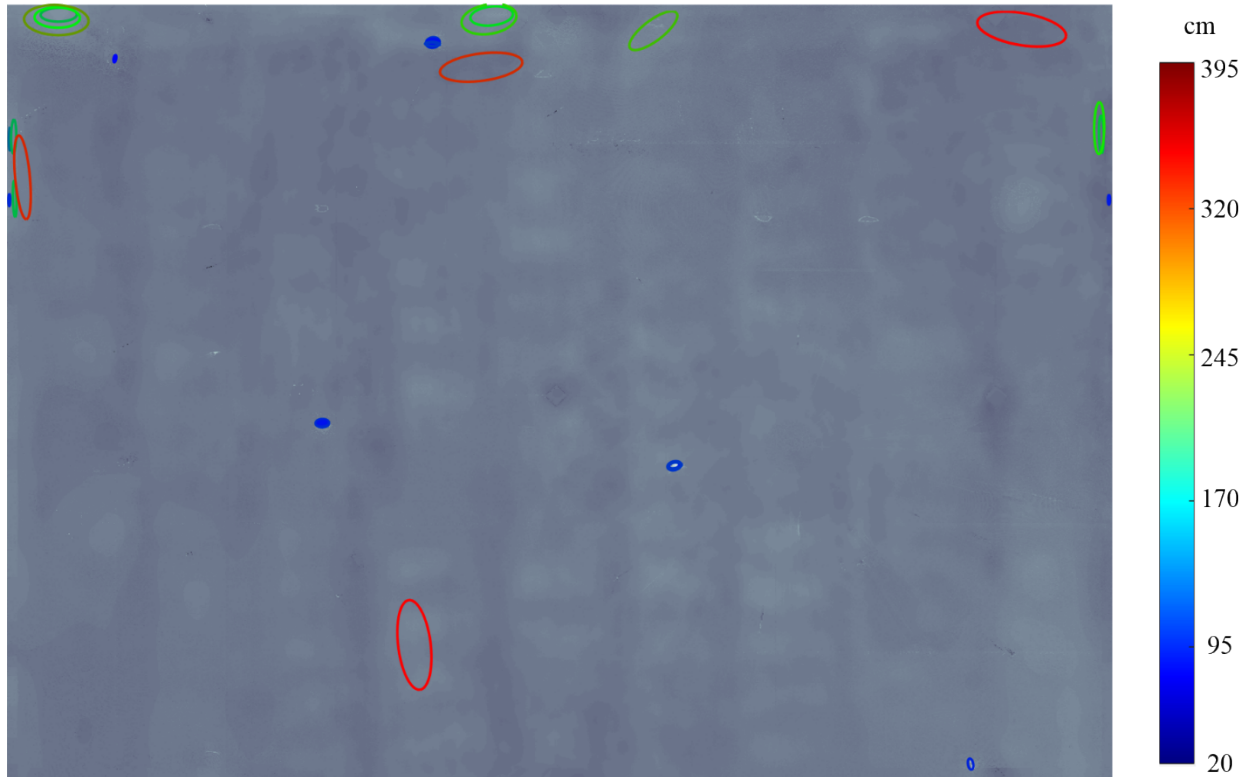
364 4.3.2 Surface analysis and automatic defect detection

365 Figure 8 illustrates regions, enclosed by ellipses, where potential defects have been
 366 identified for two of the five scales in Figure 7. Note that no defective regions were found for the
 367 other three scales.



368 (a) (b)
 369 Figure 8 Potential defective areas for a) 61cm (± 2 cm) and b) 244cm (± 2 cm).

370 The advantage of our approach is that it combines dense 3D data from TLS with the 2D CWT that
 371 can support the analysis of waviness with essentially any characteristic period (i.e. wavelength).
 372 This enables our approach to study waviness not just for a few discrete wavelengths (like the 5
 373 above), but for dense and large ranges of wavelengths. This is demonstrated in Figure 9 that
 374 summarizes the potential defects on the slab for any wavelength within the continuous range of 20
 375 to 400 cm.



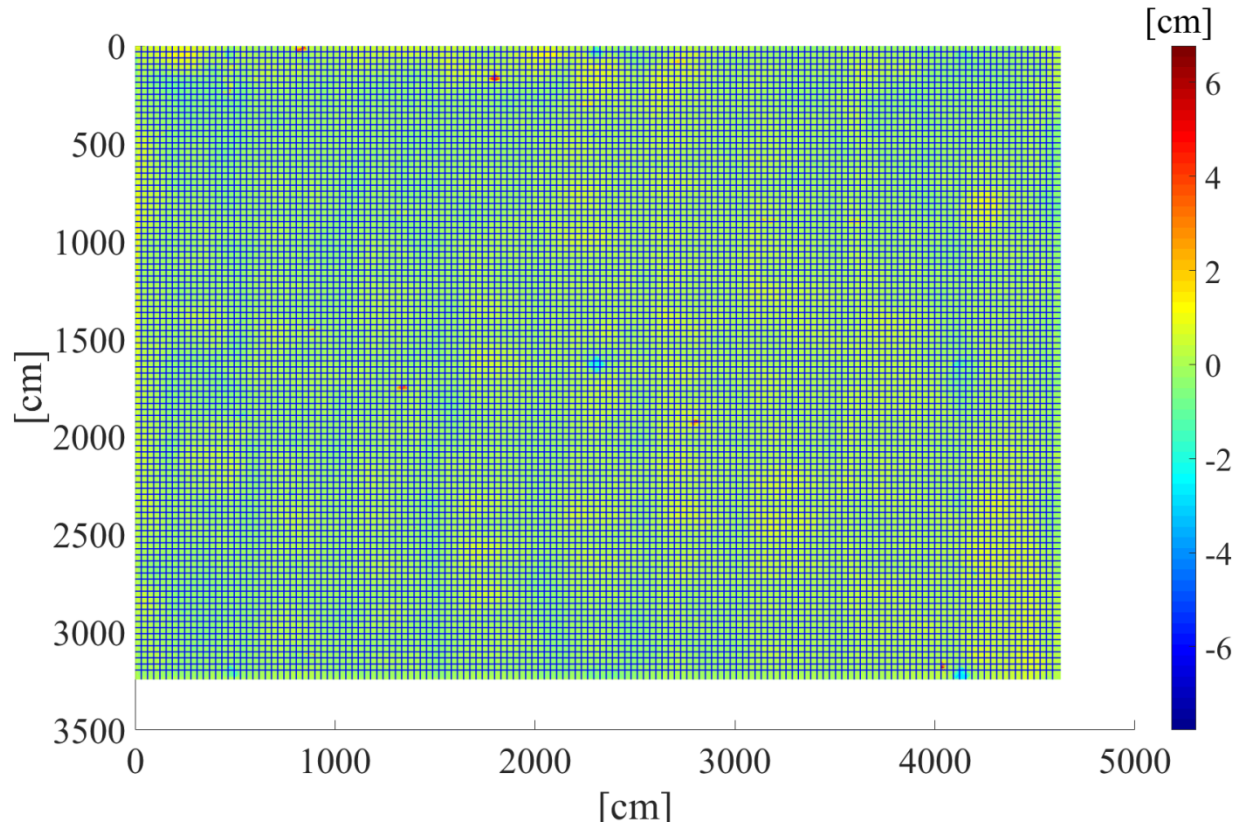
376

377

Figure 9 Detected defects for periods between 20 and 400 cm.

378 4.4 Comparison of Results with WI method

379 The ASTM E1486-14 standard describes the test method for measuring the waviness of
 380 concrete floors using the WI method. In a similar way to [8], we propose to apply the WI method
 381 as defined in this standard, but using the digitized slab surface as the surface of application (instead
 382 of the real slab). Referring to this standard, 103 survey lines along the x-axis and 148 lines along
 383 the y-axis were defined on the slab, as shown in Figure 10. The lines are spaced at 1-ft intervals.
 384 This is much denser than what would normally be achieved in normal practice, but is useful to
 385 conduct the comparison with the proposed CWT approach.



386

387 Figure 10 The 251 survey lines (103 along the x-axis and 148 along the y-axis) that are defined
388 across the slab surface.

389 The length adjusted RMS deviation (LAD) responses are calculated for each line. The survey lines
390 are parallel to each other and are spaced at a distance of 30.5 cm (1ft). Survey points with a spacing
391 $s=30.5$ cm (1ft) are measured along those lines. The standard defines chord length as the length of
392 the imaginary line joining two points on the surface of the concrete floor. The chord length is equal
393 to $2ks$, where $k= \{1, 2, 3, 4 \text{ and } 5\}$. The vertical distance between the midpoint of the chord and
394 the survey point on the surface, D_{kj} , is calculated using the following formula,

$$395 \quad D_{kj} = h_{j+k} - 0.5(h_j + h_{j+2k}) \quad (6)$$

396 where, h_{j+k} , h_j and h_{j+2k} represent the heights of the survey point and the two end points of the
397 chord, respectively. These heights are obtained from the depth map calculated in section 4.2.

398 After the deviation D_{kj} is calculated, the length adjusted RMS deviation (LAD_k) is calculated using
399 equation 7.

$$400 \quad LAD_{l,k} = \sqrt{\frac{L_r}{2ks} [\sum_{i=1}^{jmax_{l,k}} (D_{l,k,j})^2]} \quad (7)$$

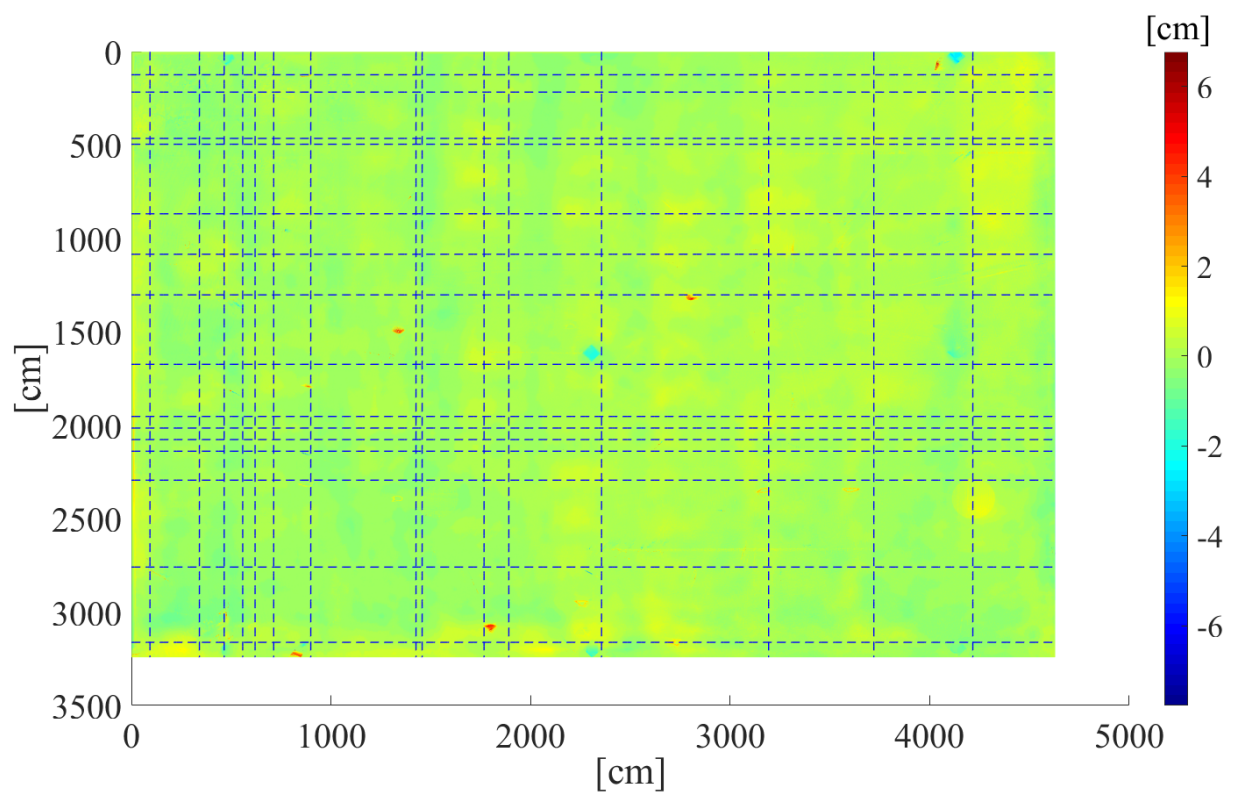
401 where L_r corresponds to the reference length of 1 ft. $jmax_k$ corresponds to the total number of
 402 deviation calculations with a chord length $2ks$ along a survey line and l denotes the survey line
 403 being tested.

404 Similarly, the 2D CWT responses at the j^{th} sampled location, $CWT_{l,a,j}$, for scales 15, 30, 45, 60 and
 405 75 were obtained in section 4.3.1. These scales correspond to the WI k-values 1, 2, 3, 4 and 5
 406 respectively. It is proposed that the CWT responses for each of the 210 lines for those 5 scales,
 407 $CWT_{l,a}$, be calculated using the similar formula:

$$408 \quad CWT_{l,a} = \sqrt{\frac{\sum_{i=1}^{jmax_{l,a}} CWT_{l,a,j}^2}{jmax_{l,a}}} \quad (8)$$

409 where $jmax_k$ corresponds to number of locations at which the 2D CWT response have been
 410 calculated.

411 The correlation between the $LAD_{l,k}$ and $CWT_{l,a}$ responses is calculated to compare the surface
 412 waviness results obtained using the WI and 2D CWT methods. 15 survey lines along the x-axis
 413 and 15 survey lines along the y-axis, as shown in Figure 11, are randomly selected (out of the
 414 previously defined 251 lines), to illustrate the correlation results presented in Figure 12 and Figure
 415 13. The correlation coefficients, denoted by r^2 , are included in the top left-hand corner of each
 416 graph. The values indicate a strong correlation between the results obtained using the WI and 2D
 417 CWT methods. This strongly validates the value of the proposed approach, which has the
 418 additional advantage of being able to more precisely define defects' wavelengths and locations
 419 (including actual orientation).

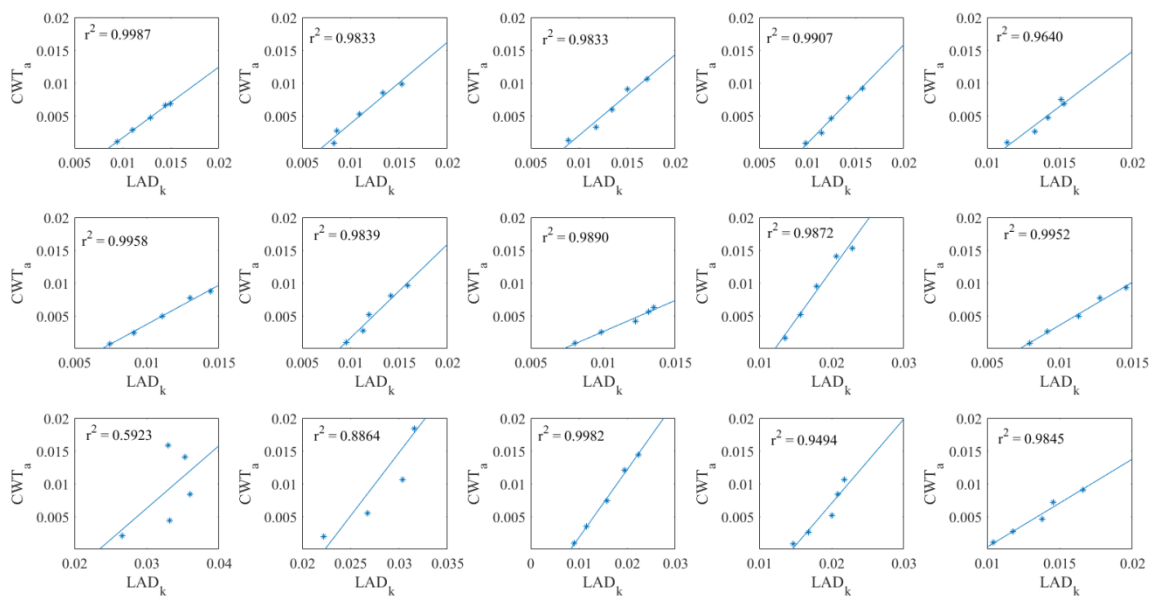


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421

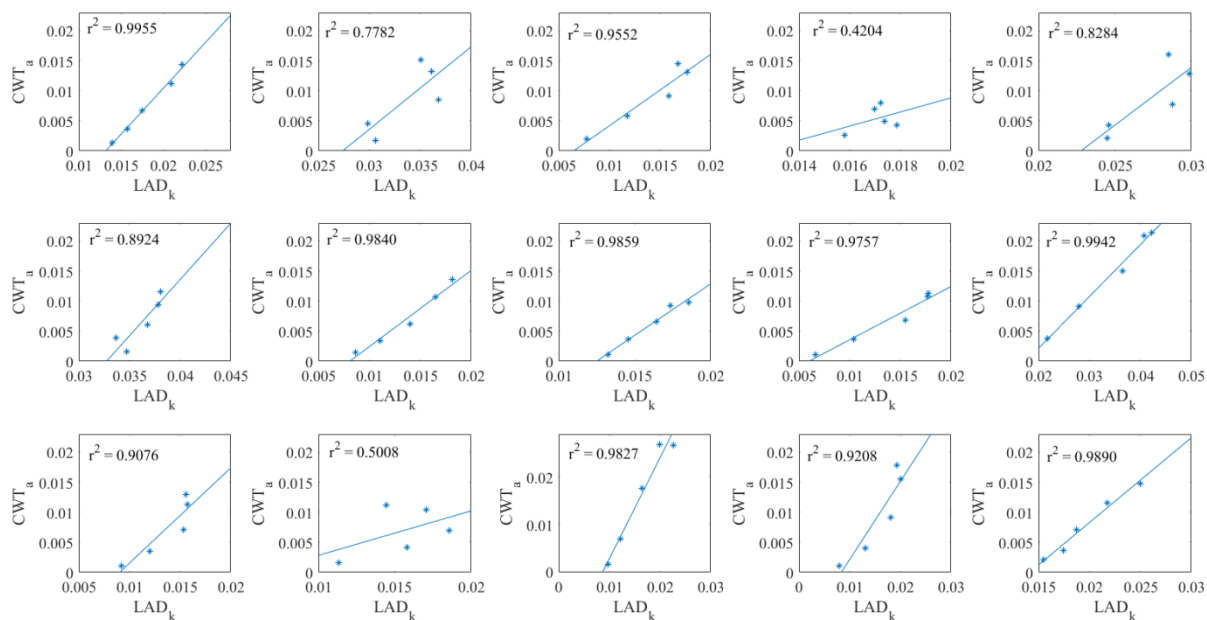
422

Figure 11 The 30 survey lines (15 along each axis) that were selected for the generation of correlation results.



423

424 Figure 12 Correlation between $LAD_{l,k}$ and $CWT_{l,a}$ responses for the five characteristic undulation
 425 periods [61, 121.9, 182.9, 243.8, 304.8] cm, along 16 lines along the x-axis shown in Figure 11.



426
 427 Figure 13 Correlation between $LAD_{l,k}$ and $CWT_{l,a}$ responses for the five characteristic undulation
 428 periods [61, 121.9, 182.9, 243.8, 304.8] cm, along 16 lines along the y-axis shown in Figure 11.

429
 430 The proposed approach overcomes all of the drawbacks of traditional waviness assessment
 431 methods mentioned earlier. The 2D CWT method enables a comprehensive analysis of the
 432 waviness of 2D surfaces, both in the spatial and wavelength domains. In contrast to the F-number
 433 and WI methods, the output of the CWT-based approach enables easy visual representations of
 434 where waviness defects are located on the surface (as shown in Figure 7). This can help users with
 435 minimal knowledge about 3D TLS or 2D CWT to quickly determine where surface corrections
 436 should be applied, for example.

437 5 Conclusions

438 TLS technology has a promising future in the construction industry owing to its ability to
 439 rapidly and accurately capture and record as-built conditions. Research efforts are being
 440 concentrated to identify specific areas, which could particularly benefit from the application of
 441 TLS. Dimensional QC is one such area. The analysis performed using the 2D CWT for QC
 442 provides great flexibility for examining the surface undulations with a wide range of characteristic

443 periods. The localization property of 2D CWT highlights regions on the surface and helps in
444 compliance assessment and corrective work planning.

445 The proposed approach in this paper demonstrates how TLS data of a concrete surface can be used
446 to characterize waviness by implementing 2D CWT, using Mexican Hat Wavelet as the mother
447 wavelet. The comparative analysis of the various methods of measuring waviness in concrete slabs
448 reveals that the 2D CWT method provides results that strongly correlate with those of the WI
449 method (the current state of the art), but has numerous advantages over it and other existing
450 methods.

451 Future research efforts can be directed toward improving the practicality of implementing laser
452 scanning for measuring floor surface waviness. The method proposed in this paper can be used in
453 conjunction with augmented reality devices to enable the visualization of undulations
454 corresponding to various characteristic periods on site. The proposed method can be further
455 improved by developing algorithms for automatically removing noise and generating scan plans
456 to estimate optimal scanning positions. For this study, manual effort was required for preparing
457 scan plans, setting up the scanner and collecting the point cloud data. A LiDAR integrated
458 Unmanned Aerial Vehicle (UAV) can also be used to collect the scans and automated registration
459 of laser scans can be further explored. Finally, a comparative analysis on using TLS and LiDAR
460 equipped UAVs for surface quality assessment can be conducted.

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