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FINITE ELEMENT MODELLING OF STRUCTURAL CONCRETE

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Abstract. *Over the years a large number of finite element analysis programs have been developed in order to investigate the behaviour of reinforced concrete (RC) elements and structures. These are based on the use of a wide range of concrete material laws, the majority of which can be classified as empirical, plastic, visco-plastic, damage and hybrid, depending on the theory or combination of theories upon which their analytical formulation is based. The formulation of most, if not all, of these material models relies heavily on a number of empirical parameters, the inclusion of which is essential for defining material behaviour. These parameters are usually linked to post-peak concrete characteristics such as, for example, strain softening, tension stiffening, shear-retention ability, etc, coupled with stress- and/or strain-rate sensitivity when high-rate loading problems are considered; their values often vary depending on the type of problem investigated.*

Three widely used packages, (LS-DYNA, ANSYS and ABAQUS), are adopted in the present work in order to investigate analytically the experimental response of simply supported RC beams under monotonic loading applied at various rates, ranging from static and earthquake to rates encountered in impact and blast problems. A fundamental assumption adopted in the case studies investigated herein, is that for the case of high-rate-loading, concrete constitutive behaviour is essentially independent of the loading rate and that the effect of the latter on structural response can be primarily attributed to inertia forces. The predictions obtained are compared with published experimental data as well as the predictions of a specialized in concrete structures analysis package (RC-FINEL), which, in contrast with the above packages, incorporates a fully brittle material law for the constitutive description of concrete behaviour under triaxial loading. The aim of the present investigation is to explore the generality and applicability of the FE models presently adopted and their ability to yield realistic predictions of structural concrete behaviour.

1 INTRODUCTION

A large number of constitutive models have been developed to date aiming at realistically predicting the nonlinear response of concrete structural forms under various types of loading, ranging from static and seismic to more extreme loading conditions such as those encountered in blast and high velocity impact problems. The inclusion of such models into various finite-element analysis (FEA) schemes has led to the development of powerful tools (FEA packages) for the numerical investigation of reinforced concrete (RC) structures.

The models of concrete behaviour may be broadly classified into two categories: those directly derived from regression analyses of experimental data (so called empirical models) and those relying on continuum mechanics theories for their development, although the latter also remain dependent on the use of experimental data for their calibration. Continuum mechanics theories (such as, nonlinear elasticity, plasticity, viscoplasticity, damage mechanics) were initially adopted for describing the behaviour of near-homogeneous and isotropic materials such as steel. With time, however, the use of these theories was extended for describing the phenomenological behaviour of essentially heterogeneous materials such as concrete, which also includes discontinuities. The resulting formulations usually incorporate a number of parameters, the evaluation of which is essential for achieving a close correlation between the model predicted behaviour and its experimentally-established counterpart. These parameters, which are usually linked to post-peak strength concrete characteristics in both compression and tension, such as, for example, strain softening, tension stiffening, shear-retention ability, etc, coupled with stress- and/or strain-rate sensitivity when high-rate loading conditions are considered, are often established through calibration based on the use of experimental information at the structural, rather than at the material level.

A FEA package is usually considered to be capable of yielding realistic predictions of the response of a concrete structural form when the deviation of the predicted from the experimentally measured values of particular structural characteristics does not exceed a value of the order of 20% of the corresponding measured quantity. Such structural characteristics usually include the load-carrying capacity, the relation between applied load and corresponding displacements, reactions or first order deformation derivatives (e.g. rotations); furthermore, qualitative behaviour pattern matches are also considered, such as the crack patterns at various load stages and the mode of structural failure. Moreover, a FEA package is considered to be characterised by objectivity and generality when it is capable of providing realistic predictions of structural behaviour for any type of structural concrete configuration, without the need of recalibrating the constitutive model adopted or its parameters.

The present article sets out to compare the numerical predictions obtained from three widely used commercial FEA packages, namely ANSYS, LS-DYNA and ABAQUS, as well as the predictions of a specialized in concrete structures FEA model (RC-FINEL). All these packages are capable of carrying out three dimensional (3D) nonlinear (NL) static and dynamic analyses, but they adopt different material models and numerical solution strategies. In particular the work presented herein is the first step of an ongoing research project aiming to investigate the generality and objectivity of the packages available through a comparative study of the solutions obtained for a number of different problems. The case studies selected in the present article are initially concerned with the investigation of the behaviour of two simply supported beams under static monotonic loading applied at their mid-span. For both RC beams published experimental information is available: The first of the two beams, investigated experimentally by Hughes and Spiers [1], is characterised by *ductile* behaviour, whereas the second – investigated experimentally by Bresler and Scordelis [2] – failed in a *brittle* manner. The behaviour of the RC beam exhibiting ductile behaviour under static loading is further investigated for the case of monotonic loading applied at its mid-span at various rates of loading, in accordance with published experimental test results [1].

2 FEA PACKAGES USED

Two are the main requirements in concrete modelling, for the application of FEA to concrete structures: (a) to employ a material constitutive model capable of describing the nonlinear behaviour of concrete, steel reinforcement and their interaction; (b) to adopt a nonlinear numerical procedure capable of implementing the redistribution of the internal stress and/or strain state imposed by the material nonlinearity under external loading and the numerical description of the cracking processes that concrete undergoes.

Although the assumption of a gradual and controlled reduction of the material residual strength beyond a peak stress value is implicit, the nonlinear numerical schemes adopted by most FEA packages have been developed independently from the material models adopted. Most commonly, the majority of FEA packages employed to predict the behaviour of concrete structural forms adopt:

- an iterative procedure based on well established numerical techniques such as the Newton-Raphson method in order to account for the stress redistributions during which the crack formation and closure checks as well as convergence checks are carried out simultaneously in each iteration. Iterations are repeated until the residual forces – calculated form the equilibrium equations – attain a predefined minimum value (convergence criteria), and.
- an implicit or explicit integration scheme used to solve numerically the governing equation of motion for the case of dynamic problems.

Although, a more detailed discussion of the adopted numerical iterative solution strategies is beyond the scope of the present work, a brief description will be given in connection with the iterative process in RC-FINEL, used herein, since in this case, the development of the nonlinear strategy is dependent on the constitutive model of concrete behaviour adopted and the assumptions upon which its analytical formulation is based. As a result of this dependency a unique iterative procedure is formulated in which crack formation and closure as well as convergence are checked separately during each iteration, following a predefined sequence of events described later on in paragraph 2.4. This unique iterative procedure has been found to effectively counteract the numerical instabilities that stem from the brittle concrete material model adopted in RC-FINEL [10-12].

When comparing the various concrete material models available, it is possible to identify significant differences in their analytical formulation depending on the experimental data used to calibrate the models, the interpretation of the available test data and the theory upon which the formulation of the models is based. As a result, in what follows, emphasis will be placed on the effect of particular parameters of the constitutive models adopted on the predicted structural behaviour.

2.1 ANSYS [3]

ANSYS is a well known commercial FEA package that is widely used in a variety of complex structural problems. For the purpose of the present investigation, its application to concrete structures relies on a model of concrete behaviour, the derivation of which has been based on regression analysis of test data on concrete cylinders and cubes subjected to uniaxial compression and tension under quasi static load rates. The resulting stress-strain curves, which describe concrete behaviour in compression (Fig. 1a) and tension (Fig. 1b) consist of an ascending (strain-hardening) and a descending (strain-softening) branch which describe the behaviour of concrete both prior and after the peak-stress value. By assuming that concrete is an orthotropic material, the uniaxial stress-strain curves described above are applied to each principal-stress axis, thus leading to a formulation of a general constitutive model describing the behaviour of concrete under triaxial loading conditions. Localized failure of the concrete medium, which occurs in the form of cracking, is modelled by adopting the smeared crack approach and is controlled by the well known failure criteria of William and Wranke [20].

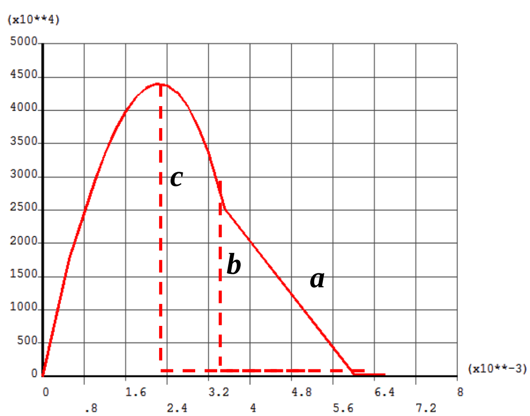
The analytical formulation of the material model includes a number of parameters which are mainly linked to post-peak concrete characteristics such as, for example, the strain softening, the tension stiffening, and the shear-retention ability. The effect of these parameters on the numerical predictions is investigated through a parametric investigation as described in the following. Initially, the parameter investigated is the shape of the post-peak branch of the stress-strain curve of concrete in compression; three different stress-strain curves are considered for this purpose (Fig.1a). The first curve (curve A) consists of an ascending and a descending branch which is identical with that described in Eurocode 2 up to a strain value of 0.35%; beyond this value, the stress values decrease linearly until a small residual value is attained (presently set at 0.1MPa) and remains constant thereafter. The second curve (curve B), is identical to the first (curve A) for values of strain up to 0.35%; beyond this value, the stress drops abruptly to 0.1MPa and remains constant thereafter. Finally, in the case of the third stress strain curve (curve C), the descending branch describes a complete and immediate loss of load-carrying capacity (concrete is considered to be a brittle material).

A second parameter, the effect of which is investigated herein, is the shear retention factor, SRF, which is used to ascribe to cracked concrete a reduction in shear capacity. A low value of SRF would suggest small frictional stresses developing on the crack faces due to their shearing movement, with the frictional forces increasing with increasing values of SRF. In addition ANSYS employs a parameter/multiplier which describes the manner in which the residual strength in tension decreases with increasing strain, and, in essence, defines the slope of the descending – softening – branch of the stress-strain curve of concrete in tension (Fig.1b). The default value set by ANSYS is 0.6; the effect of the variation of this parameter on the predictions obtained is not investigated in the present article.

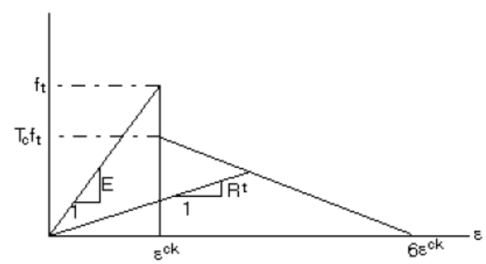
2.2 ABAQUS [4]

The material model selected for the purposes of this study is the brittle cracking model, which is available in ABAQUS/Explicit. The model is designed for cases in which the behavior is dominated by tensile cracking. In this model, the behaviour of concrete in compression is assumed to be always linear elastic, which also defines the material behavior completely prior to cracking. ABAQUS /Explicit uses a smeared crack model to represent the discontinuous brittle behavior in concrete. For purposes of crack detection, a simple Rankine criterion is used to detect crack initiation (i.e. a crack forms when the maximum principal tensile stress exceeds the specified tensile strength of concrete). As soon as the Rankine criterion for crack formation is met, a first crack is assumed to form. The crack surface is taken to be normal to the direction of the maximum tensile principal stress. Subsequent cracks may form with their surface orthogonal to the directions of any existing crack surface at the same point. Crack closing and reopening is allowed for (i.e. cracks can close completely when the stress across them becomes compressive).

The model includes two modes of fracture: Mode I (tension softening/stiffening) and Mode II (shear softening/retention) behaviour. For the present study, a simple linear ascending branch followed by a linear softening branch was adopted. The strain softening after failure was assumed to reduce the stress linearly to zero at a total strain about ten times the strain at failure. In the cracking model, crack initiation is based on Mode I fracture only, while post-cracking behaviour includes both tension-stiffening and shear-retention modes of fracture (i.e. Modes I and II). To model Mode II shear behavior, the post-cracking shear modulus is expressed as a fraction of the uncracked shear modulus. ABAQUS /Explicit offers a shear retention model in which the post-cracking shear stiffness is defined as a function of the opening strain across the crack. For the purposes of the present study, the retention factor is assumed to vary linearly (Fig.2) with the strain across the crack (i.e. from 1.0 at the cracking stress to 0.0 at the maximum strain).



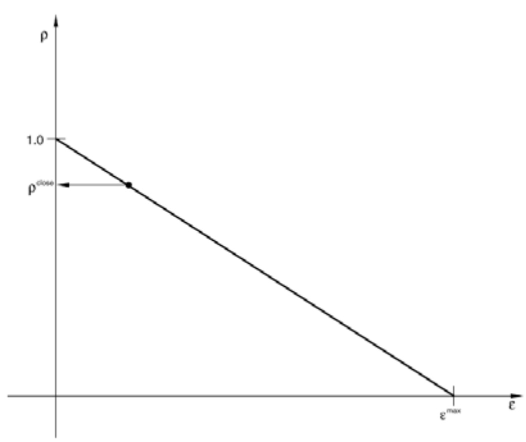
(a) Typical uniaxial compression stress-strain curve



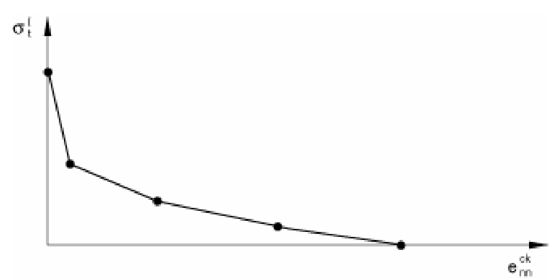
f_t = uniaxial tensile cracking stress
T_c = multiplier for amount of tensile stress relaxation

(b) Typical uniaxial tensile stress-strain curve.

Figure 1 Modelling concrete behaviour with ANSYS [3].



(a) Shear retention factor as adopted by ABAQUS

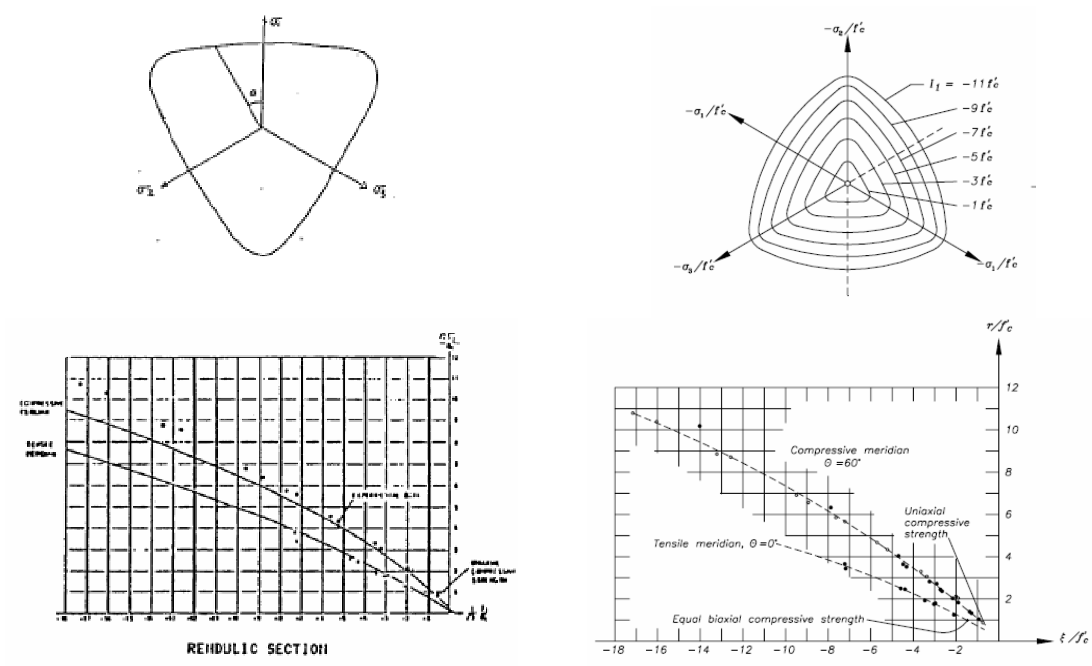


(b) Typical uniaxial tensile stress-strain curve adopted by ABAQUS

Figure 2 Modelling concrete behaviour with ABAQUS [4].

2.3 LS-DYNA [5]

In the case of LS-DYNA two different material models have been employed: (i) MAT_WINFRITH_CONCRETE (also denoted as MAT_084, 085), which is a plasticity model (developed by Broadhouse and Neilson [6] and Broadhouse [7] for S&ESD, Winfrith), and (ii) MAT_CONCRETE_DAMAGE_REL3 (also denoted as MAT_072R3 or K&C concrete model), a coupled/hybrid plasticity-damage model, based on the material model developed by Malvar et al. [8] of Karagozian & Case and later on extended by Schwer et al. [9]. Both models are classified under the term smeared crack models, with distributed reinforcement capability (not used herein). The models have been used extensively in LS-DYNA, with the former being the earlier model (also denoted herein as the Winfrith model) developed for this program, with successful analytical prediction performance strictly in high impact rate and explosive type of loading. The latter model, has been recently provided in the 971 release of LS-DYNA, and, in its current release form, has the advantage that the entire set of model parameters characterising the material are fully defined (as default) by the uniaxial compressive strength of concrete (f_c) and the tensile strength capacity, which is independently specified. Both material models decouple the volumetric and deviatoric response. For the volumetric response an Equation-of-State defines the pressure - volume response of the material which is either defined by the user (MAT_84) or directly generated using f_c (MAT_072R3); in this case, the default definition of the compaction curve (for model 072R3) was adopted in both model definitions.



(a) (b)

Figure 3 Modelling concrete behaviour with LS-DYNA (a) MAT_084 (Winfrith) and (b) MAT_072R3 model failure surfaces in three-dimensional space.

Material MAT_084 follows an elastic - plastic behaviour in deviatoric stress space, with the triaxial yield surface shape evolving from triangular at low stresses to circular at higher compressive stresses, following an experimental fitting. The material is assumed to flow in compression while exhibiting cracking in tension, in three orthogonal directions, using on a tensile cuttoff. Material MAT_072R3 adopts a multiyield surface approach, defining the yield, maximum and the residual material strength as a function of damage accumulation in compression and tension. While MAT_072R3 only outputs stress and strain related information and damage scale variables, the Winfrith model has the added capability of outputting the crack growth history (length, width and orientation) as well, which can be viewed later on the model mesh as a post processing capability (as depicted in the analysis results later on).

2.4 RC-FINEL [10-12]

RC-FINEL has been found to provide realistic predictions of a wide range of different concrete structural forms under static (monotonic and cyclic) [10-13] and dynamic (earthquake and impact) [14-18] loading conditions. It incorporates a brittle material model, which describes the behaviour of concrete under triaxial loading conditions [10], as well as a unique nonlinear strategy the formulation of which allows for the brittle nature of the material model employed, while at the same time it provides a realistic description of the cracking process and minimizes the likelihood of numerical instabilities associated with this process [10-12].

The material model of concrete behaviour adopted is characterised by both simplicity (fully brittle, with neither strain-rate nor load-path dependency, fully defined by a single material parameter - the uniaxial cylinder compressive strength f_c) and attention to the actual physical behaviour of concrete in a structure (unavoidable triaxiality which is described on the basis of experimental data of concrete cylinders under definable boundary conditions).

The nonlinear analysis is based on an iterative procedure, known as the modified Newton-Raphson method which is used to calculate stresses, strains and residual forces [10-12]. Initially, every Gauss point is checked, at first, in order to determine whether loading or unloading takes place, and then in order to establish whether any cracks close or form. During the crack-closure procedure only Gauss points with cracks formed in previous load steps are checked. For a crack to close the strains normal to the plane of the crack must become compressive. In the course of each iteration, the program singles out the crack with the largest compressive strain and closes it. It has been observed that, after the closure of one crack, there is usually a drastic drop in the number of cracks that need to close next. Because of the closure of a crack, changes need to be made to the element stress-strain matrix and, consequently, to the stiffness matrix of the structure, leading to redistribution of the stresses inside the structure. It should be noted that, during the crack-closing procedure, convergence is not checked (the residual stresses and forces are not eliminated during this stage but are only calculated and added to those calculated in previous iterations). The crack-closing procedure is repeated until all cracks that fulfil the crack-closure criterion close.

The crack-opening procedure commences after the completion of the crack-closure procedure. During each iteration (of this procedure), all Gauss points are checked in order to determine if any new cracks form. This is achieved by using the failure criterion since the opening of a crack corresponds to localized failure of the material. In order to avoid numerical instabilities during the solution of the problem, only a limited number of cracks (no more than three) are allowed to form per iteration. Should the number of cracks that need to open exceed this predefined number, then only the most critical cracks will be allowed to form. As for the case of crack closure, after the formation of the most critical cracks the number of cracks that need to form in the next iteration reduces rapidly due to the redistribution of stress achieved during this process. The formation of a crack leads to the modification of the element stress-strain matrix and the stiffness matrix of the structure, thus causing redistribution of the internal stresses. Unlike the crack-closure procedure, convergence of the residual forces is now checked after all cracks have opened. If the maximum value of the residual forces evaluated is greater than a certain predefined value, then these residual forces are re-imposed onto the structure in the form of an external loading.

Similar to the rest of the programs considered, crack formation is modelled by using the smeared-crack approach. A crack forms when the stress developing in a given part of the structure corresponds to a point in the principal stress space that lies outside the surface defining the failure criterion for concrete, thus resulting in localized material failure. This failure takes the form of a crack and is followed by immediate loss of load-carrying capacity in the direction normal to the plane of the crack. Concurrently, the shear stiffness is also considered to reduce drastically to a small percentage of its previous (i.e. uncracked) value (SRF = 5%). However, it is not set to zero in order to minimize the risk of numerical instabilities during the execution of the solution procedure, as explained elsewhere [10].

3 REINFORCEMENT MODELING

Steel reinforcement is explicitly included in the FE model using one-dimensional truss bars under uniaxial tension and compression only, with full deformation compatibility at the concrete-reinforcement interface nodes. Full bond is assumed therefore between steel and concrete, with local bond transfer bounded by the tensile capacity of the concrete Gauss points near the reinforcement. Steel constitutive behaviour follows a simple bilinear hardening model accounting for the initial elastic and an averaged post-yield behaviour of the bars.

4 LOADING-RATE DEPENDENCY OF CONCRETE PROPERTIES

The vast majority of constitutive models describing the behaviour of concrete under dynamic loading assume that there is a link between the constitutive properties of concrete and the strain rate at which the material is loaded and, consequently, the external loading rate. Although this seems to be the case for low loading rates, where creep plays a significant role in material behaviour, it has been proposed that for the case of high loading rates there is no need to change the static value of Young's modulus [12, 15-19].

In the present investigation the assumption is adopted, for the case of high-rate-loading problems, that the material properties of concrete and steel reinforcement are essentially independent of the loading rate and that the effects of the latter on structural response are primarily attributed to the inertia forces which develop within the structural member, and not to the loading-rate sensitivity of the mechanical characteristics of the materials involved. Adopting the above assumption (which stems from earlier work investigating the response of prismatic laboratory concrete samples [12 ,15-17] as well as RC beams under high rates of loading [18, 19]) results in a significant reduction of the number of parameters often required to fully define such analysis problems. Furthermore, this hypothesis constitutes a major departure from currently accepted design and numerical modelling practices which adopt exactly the opposite view, thus providing an alternative explanation as to the causes that affect the complex inelastic response of RC structural elements under high loading rates, as well as the cracking patterns, observed during testing. The validity of this assumption is investigated through the comparison of the predictions of the different FEA packages presently employed with the available experimental data.

5 STRUCTURAL FORMS INVESTIGATED

As discussed earlier, the case studies presently selected are concerned with the investigation of the behaviour of two simply supported beams under monotonic loading applied at their mid-span. Under static monotonic loading, the first of the two beams (investigated experimentally by Hughes and Spiers [1] – referred to as case study 1) is characterised by *ductile* behaviour whereas the second (investigated experimentally by Bresler and Scordelis [2] – referred to as case study 2) fails in a *brittle* manner. Furthermore, the response of the beam exhibiting ductile behaviour under static loading, is further investigated under monotonic loading applied at various rates of loading (referred to in the present article as case study 3)[1].

5.1 CASE STUDY I: Ductile beam under Static Monotonic Loading

The beam selected for the present case study is a simply supported beam tested by Hughes and Speirs **[1]**. The beam with a clear span of 2700 mm and a rectangular cross-section of 200 mm (height) x 100 mm (width), is reinforced with two 12 mm diameter tension bars, two 6 mm diameter compression bars, and 6 mm diameter stirrups at an approximately 180 mm centre-to-centre spacing (Figs 5 and 8). The modulus of elasticity (E_S), yield stress (f_y), and ultimate strength (f_u) of both the longitudinal and transverse reinforcement bars used are 206 GPa, 460 MPa and 560 MPa, respectively, with the cylinder compressive strength (f_c) of the concrete used being approximately 45 MPa and the uniaxial tensile strength (f_t) being approximately 3 MPa. During testing the beam exhibited ductile behaviour: its load-carrying capacity and corresponding mid-span displacement were 29kN and 20mm, respectively, while the maximum displacement recorded was 50mm. Cracking initiated in the mid-span region and gradually extended throughout the length of the beam with increasing levels of loading (see Fig 6). Failure of the specimen was caused by yielding of the longitudinal reinforcement bars in the mid-span region of the specimen, resulting in the formation of extensive flexural cracking in that region that ultimately led to failure of the compressive zone.

In the case of ANSYS, ABAQUS and LS-DYNA the concrete medium is modelled by using a dense mesh of 8-node brick elements with an edge size between 1cm and 3cm (see Fig. 7a-c); the element formulation adopts a reduced integration scheme to avoid numerical problems due to locking. Reinforcement bars are modelled by 2-node single Gauss point truss elements with sectional areas distributed to the relevant nodes of the beam's cross-section so as to be equivalent, in terms of both cross-sectional area and location, to the actual reinforcement of the beams. Because of the double symmetry of the problem at hand, one quarter of the actual specimen was modelled with suitable symmetry boundary conditions.

In the case of RC-FINEL, concrete is modelled by using 27-node brick Lagrangian elements, since the use of such elements combined with a 3x3x3 integration rule leads to a significant reduction of the number of elements required to model realistically the structural forms investigated. The beam was subdivided into 2x18 27-node Lagrangian brick elements (Fig. 8). Again, only a quarter of the beam was modelled. Truss elements representing the steel reinforcement were placed along successive series of nodal points in both vertical and horizontal directions, in order to model the steel reinforcement. Since the spacing of these line elements was predefined by the location of the brick elements' nodes, their cross-sectional area was adjusted so that the total amount of both longitudinal and transverse reinforcement to be equal to the design values.

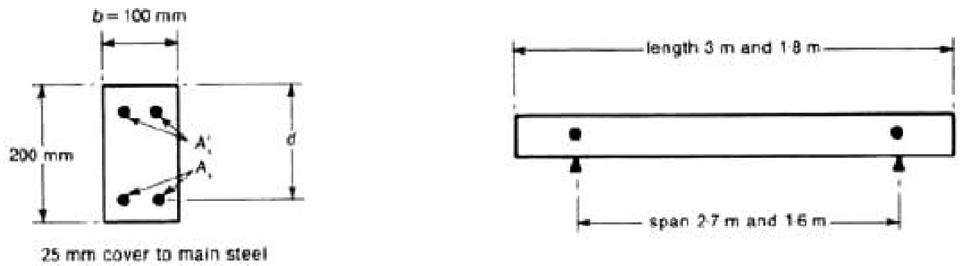


Figure 5 Presentation geometry and reinforcement details of the of the ductile beam [1].

Figure 6 Crack patterns recorded during testing of the ductile beam [1].

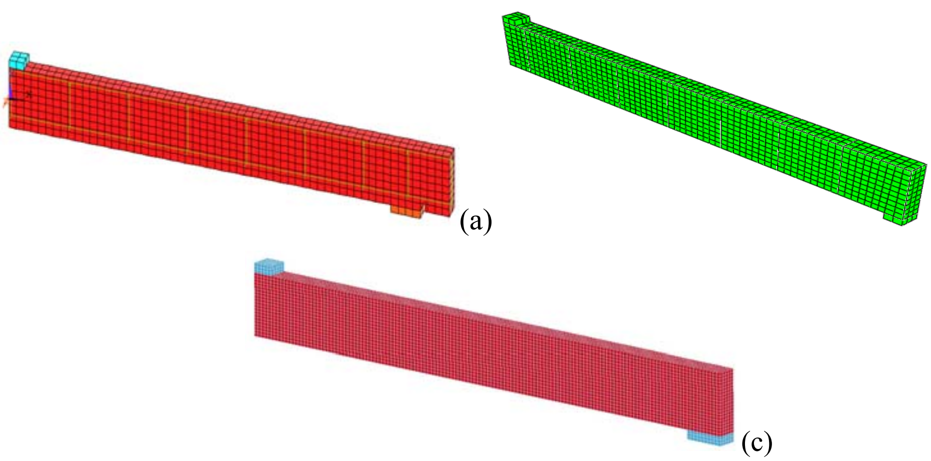


Figure 7 FE Models adopted by (**a**) ANSYS, (**b**) ABAQUS and (**c**) LS-DYNA.

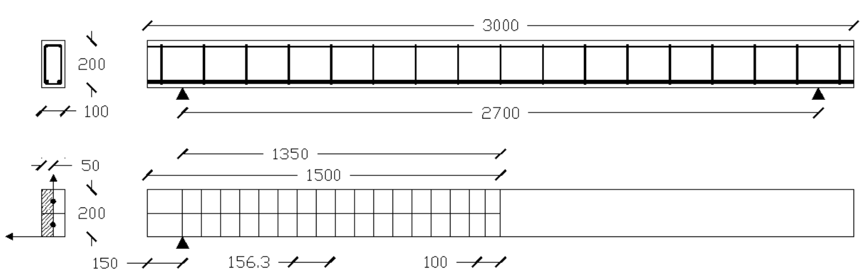


Figure 8 FE Model adopted in the RC-FINEL analysis.

The external load was applied to the model in the form of displacement increments through rigid elements (similar in shape and size to the steel platens/plates used in the experiment) situated on the top face of the beam at its mid-span. Rigid elements were used to form a vertical support on the bottom face close to the edge of the beam. The rigid elements were used in order to distribute the applied point and reaction loads to an area on the mid-span and support regions of the RC beam and to avoid the development of high stress concentrations in either location.

5.2 CASE STUDY II: Brittle beam under Static Monotonic Loading

As in the case of the ductile beam presented in the previous section, the beam selected for the present case study is also a simply supported beam with a concentrated load applied at it's mid-span tested by Bresler and Scordelis [2]. The beam has a clear span of approximately 3660 mm and a rectangular cross-section of 556 mm (height) x 310 mm (width). It is reinforced with only four 29 mm diameter longitudinal bars (Fig. 9). The modulus of elasticity (E_S), yield stress (f_y), and ultimate strength (f_u) of the reinforcement bars used are 200 GPa, 555 MPa and 958 MPa, respectively, with the cylinder compressive strength (f_c) of the concrete used being approximately 22.5 MPa. From the experiment, the beam was found to fail in a brittle manner at a load approximately 334 kN corresponding to a mid-spam deflection of 6.6mm. As in the previous case study only a quarter of the beam is analysed, whereas the concrete and steel reinforcement are modelled as in the previous case study, with the external load being applied in the form of displacement increments. Furthermore, rigid elements were used in order to apply the external load and to form the supports of the beams

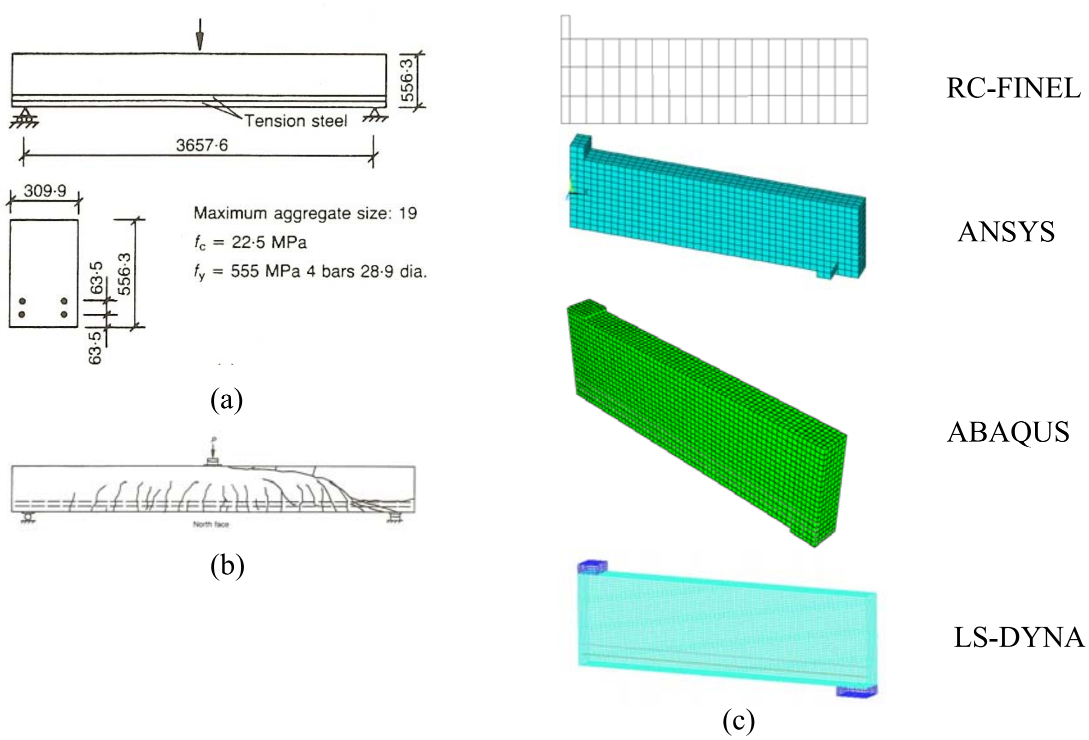


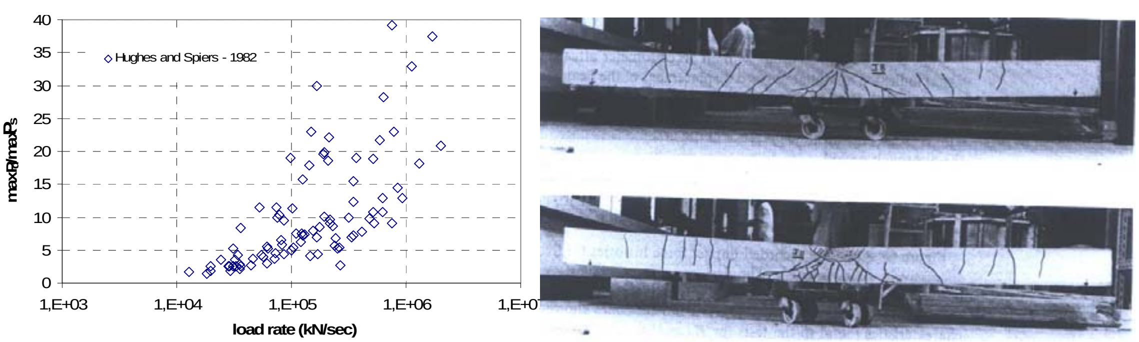
Figure 9 Brittle beam [2]: **(i)** Presentation geometry and reinforcement details of the beam, **(ii)** crack patterns recorded during testing (**iii**) FE meshes adopted by the various models.

5.3 CASE STUDY III: Ductile Beam under monotonic high rate loading

The ductile beam investigated in the first case study under static loading is typical of a number of beams tested **[1]** under loading applied at rates which vary from 2 kN/msec (static loading) to 2000 kN/msec (impact loading). For the case of impact loading, the load is applied by means of a steel mass left to fall onto the specimen from a certain height, depending on the desired rate of loading **[1]**. The value of the applied load increases linearly at a constant rate until the load-carrying capacity of the RC beam is reached and failure occurs. Various rates of loading are investigated ranging from 2kN/msec to 200kN/msec (which corresponds to 0.5kN/msec to 50kN/msec when considering the quarter of the specimen actually modelled). As regards the FE modelling the FE mesh and boundary conditions adopted are similar to those described in the case of case study 1.

It has been established both experimentally and numerically in previous work [18, 19] that an increase in the loading rate beyond a limit value leads to an increase in load-carrying capacity (see Fig 10a) and to a stiffer structural response (i.e. smaller deflection at a given load level). Furthermore, as the rate of the applied load increases, the portion of the RC beam mostly affected by the application of the external load tends to concentrate at the mid-span region of the specimen (see Fig 10b), where the load is exerted. In addition, at a certain distance from the mid-span, cracks begin to form that initiate from the upper face of the specimen and extend downwards (see also the observed crack patterns in Fig 10b). The cracking process described above differs considerably from that exhibited under static loading in that the latter initiates at the bottom face and gradually extends upwards as the load increases, at the midspan only.

Finally, it is interesting to note in Fig. 10a the very large scatter exhibited by the test data. The cause of this scatter appears to predominantly reflect the difficulty in establishing experimentally the specimen load-carrying capacity under impact loading, with most values indicated in the figure usually exceeding the “true” load-carrying capacity by a significant margin. Hence, the trends of behaviour described by data such as those in the figure can only provide a qualitative, rather than quantitative, description of structural behaviour.



(a) (b)

Figure 10 Case study III [1]: (a) experimental data expressing the variation of load-carrying capacity of the RC beams with the applied loading rate, (b) typical crack patterns under impact loading.

6 NUMERICAL PREDICTIONS

6.1 Case Study I

In the case of the ductile beam subjected to static monotonic loading at its mid-span, the numerical predictions obtained from the FEA packages employed are compared in Figs. 11 and 12 in the form of curves describing the relation between the displacement exhibited at mid-span and the applied load while the predicted crack patterns (from ANSYS and RC-FINEL) which develop at various levels of loading are presented in Figs 13. Similar form of crack patters were obtained from LS-DYNA (the Winfrith model) and are not shown. A comparison of the cracking process established during testing with the crack patterns predicted by ANSYS and RC-FINEL show a realistic correlation. Cracking initiates in the mid-span region (where the bending moment attains its higher values) gradually spreading throughout the full length of the RC beam concurrently extending upwards with increasing levels of loading. This leads to the yielding of the reinforcement bars in tension and ultimately to failure of the concrete in the compressive zone at the mid-span region.

By comparing the numerical predictions with their experimental counterparts (Fig. 11) it can be observed that the concrete material models (Winfrith and Schwer) adopted by LS-DYNA appear capable of predicting ductile RC beam behaviour and maximum values of mid-spam displacement that correlate closely to their experimental counterparts. On the other hand, they overestimate the load carrying capacity as well as the stiffness of the RC beam. ABAQUS appears to show similar prediction trends of the behaviour to those of LS-DYNA, however, it is noted that for the purpose of the present investigation, ABAQUS employs a concrete model that assumes that concrete behaviour in compression is elastic. This assumption explains the differences exhibited between the numerical predictions and the test data in terms of stiffness, maximum mid-span deflection and load-carrying capacity. RC-FINEL provides much better predictions in terms of stiffness and load-carrying capacity, however it slightly underestimates the ductility exhibited by the beam as the predicted maximum value of the mid-span deflection is less than its experimental counterpart.

The parametric analysis carried out with ANSYS revealed that for the present case study the best predictions in terms of load-carrying capacity and ductility were obtained when stress-strain curve A was adopted for describing the behaviour of concrete under uniaxial compression and when the shear retention factor was equal to 20%. The use of curves B and C, as well as the use of smaller values for the shear retention factor, resulted in a reduction of predicted ductility, while, at the same time, the predicted load-carrying capacity and the initial stiffness remain practically unaffected, closely correlating to their experimental counterparts.

6.2 Case Study II

In the case of the brittle beam subjected to static monotonic loading at its mid-span, the numerical predictions obtained from the various FEA packages employed are presented in Figs 14 and 15 in the form of curves describing the relation between the displacement exhibited at mid-span and the applied load. At the same time the predicted crack patterns (obtained from ANSYS and RC-FINEL) which develop at various levels of loading are presented in Fig 16.

By comparing the numerical predictions with their experimental counterparts (Fig. 14) it can be concluded that when adopting Winfrith's material model to describe concrete behaviour, LS-DYNA predicts a response much stiffer than that established experimentally. At the same time, the predicted values for load carrying capacity and ductility – expressed by the maximum mid-span deflection – are much higher than their experimental counterparts. On the other hand, when adopting Schwer's material model (with the default values), in spite the fact that LS-DYNA again overestimates the load-carrying capacity and mid-span deflection, it is noted that the predicted stiffness is much closer to that established by testing. In the case of ABAQUS the predictions for stiffness and maximum deflection are realistic whereas load-carrying capacity is overestimated. RC-FINEL provides realistic predictions of load-carrying capacity while it tends to overestimate the deformational response (see Fig. 14).

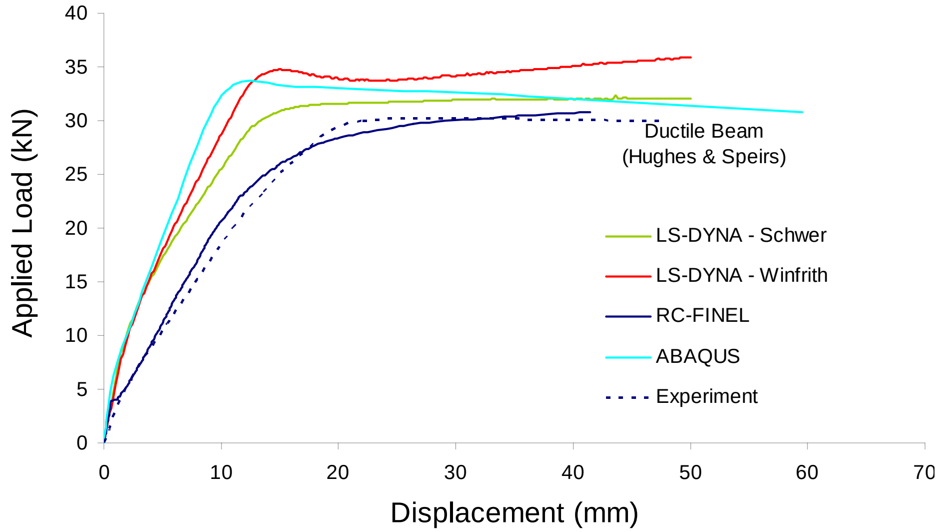


Figure 11 Comparison of load-deflection curves obtained by LS-DYNA, ABAQUS and RC-FINEL with the experimentally established one.

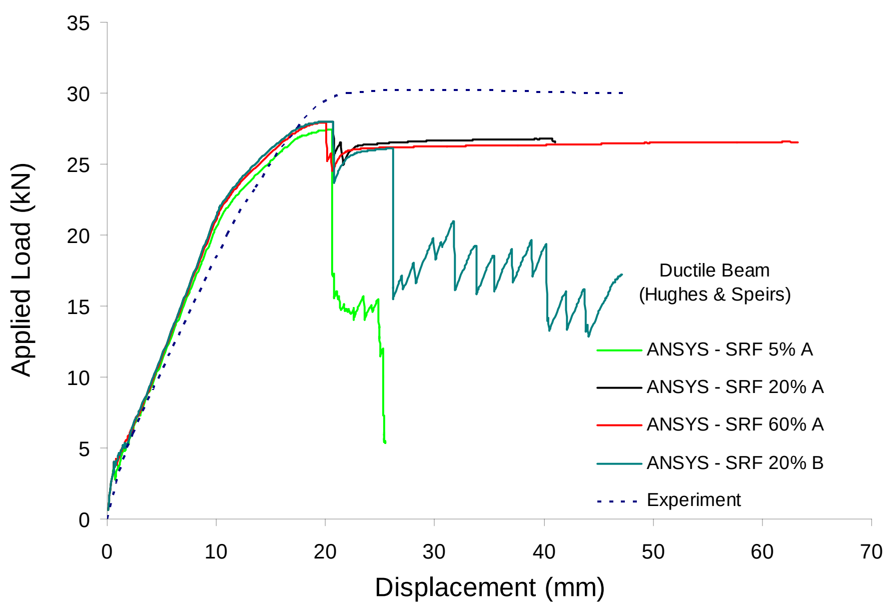


Figure 12 Comparison of load-deflection curves obtained by ANSYS and the experimentally established one.

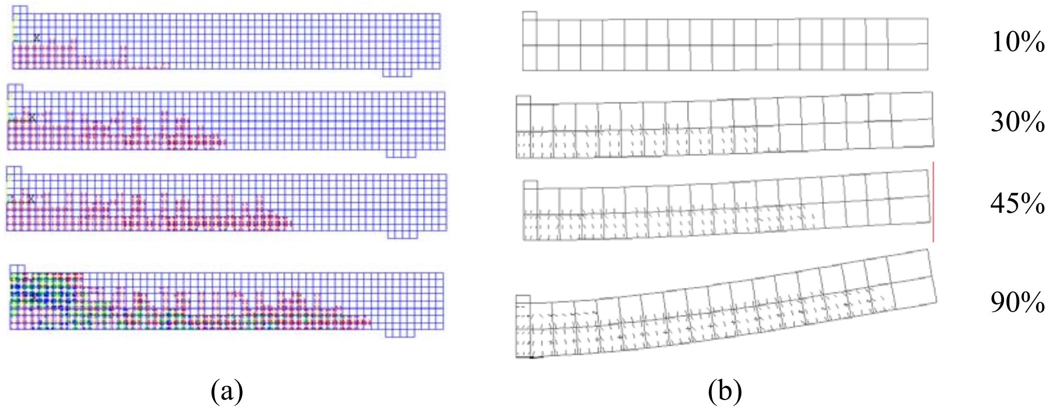


Figure 13 Development of crack patterns as predicted by (a) ANSYS and (b) the brittle concrete model of RC-FINEL.

The parametric analysis carried out with ANSYS revealed that for the present brittle case study the best predictions in terms of load-carrying capacity and ductility were obtained when stress-strain curve C is adopted for describing the behaviour of concrete (it is noted that in curve C the descending branch of the of the stress strain curve was fully ignored) and the shear retention factor is set to 20%. The use of curves A and B as well as the use of higher values for the shear retention factor resulted mainly in the model overestimating the load-carrying capacity and the deflection at mid-span.

Finally, from of the crack patterns predicted by ANSYS and RC-FINEL (Fig.16) it can be seen that cracks form in the mid-span region of the RC beam and spread rapidly throughout the beam length concurrently extending upwards, ultimately resulting in a brittle type of failure. In both cases the predicted crack patterns are similar to those experimentally established

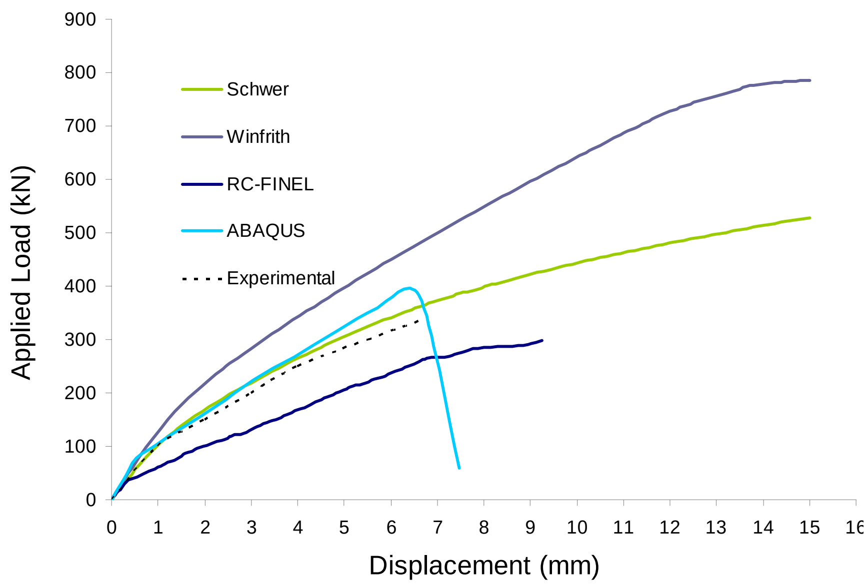


Figure 14 Comparison between the numerical predictions obtained from LS-DYNA, ABAQUS and RC-FINEL and their experimental counterpart.

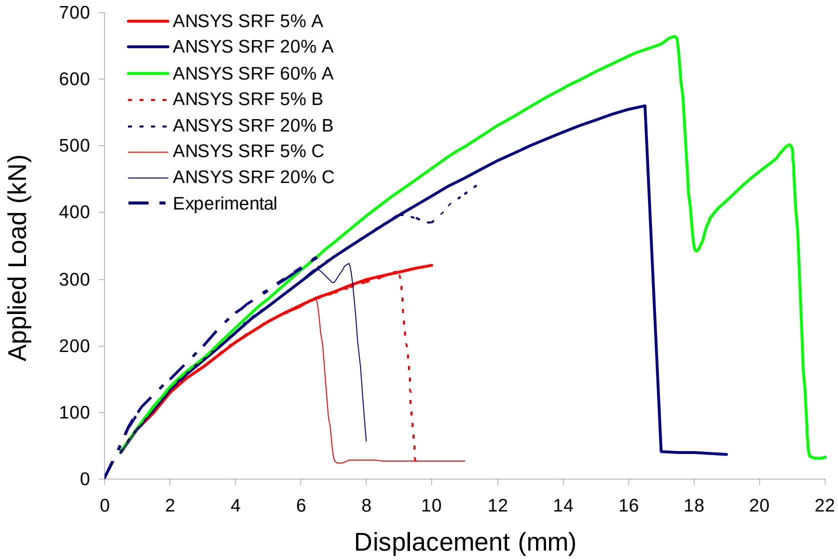


Figure 15 Comparison between the numerical predictions obtained from ANSYS and their experimental counterpart.

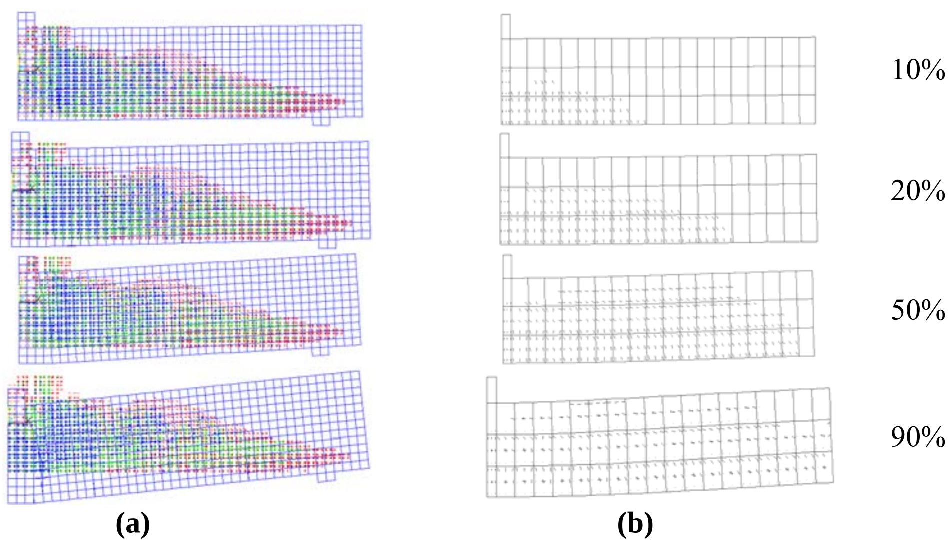


Figure 16 Comparison between the numerical predictions obtained from (i) ANSYS and (ii) RC-FINEL.

6.3 Case study III

The RC beam investigated in case study 1 is typical of a number of beams tested under loading applied at rates which vary from 2 kN/msec (static loading) to 200 kN/msec (impact loading) [1]. The numerical results obtained show that under high rates of loading there is a significant change in the RC beam's response when compared to its counterpart under static loading. A comparison between numerical predictions and their experimental counterparts is presented in Fig 17 which shows the variation of load-carrying capacity under dynamic (high rate) loading (maxP_d) normalised with respect to its counterpart under static loading (maxP_s), with applied loading rate. From the figure, it can be seen that the predictions obtained from ANSYS form an upper bound to the experimental data, whereas the predictions obtained from LS-DYNA (both material models) and RC-FINEL provide a closer fit of the experimental measurements.

Figures 18 to 20 depict the deflected shape and crack pattern of the beam exhibited when subjected to different imposed rates of loading. From these figures it can be seen that for low rates of loading the deflected shape is similar to that exhibited under static loading, in that it is a near-parabolic form. However, as the rate of loading increases, the deflected shape progressively attains a narrower bell-shaped form in the vicinity of the loading point, with its convex portions near the supports gradually increasing at the expense of the middle concave portion, whose deflection becomes disproportionally large. The analysis further reveals that the deflected shape reflects the effect of the crack process on deformation, as these are depicted in Figs 19 (ANSYS, LS-DYNA - Winfrith concrete) and 20 (RC-FINEL).

An important feature of the crack distribution is that the cracks under static and low-rate impact loading form at the impact location, at the bottom part of the beam (where high tensile stresses develop) and extend upwards as the applied load increases, with an inclination that depends on the cross-sectional distance from the mid-span of the beam (the longer the distance, the more inclined the cracks are). In the case of high rate loading, cracking seems to form also in the upper part of the specimen, gradually extending (almost vertically) downwards.

A comparison of the numerical and experimental data reveals similar trends of behavior for the RC beams under high rate loading. In particular, it can be suggested that the agreement observed between numerical and experimental data validates the initial assumption that the effect of loading rates on the specimen behaviour can be attributed, at least for the rates considered, mainly to the inertia effect of the RC beams' mass and the reduced effective response length and not to the loading-rate sensitivity of the properties of concrete and steel.

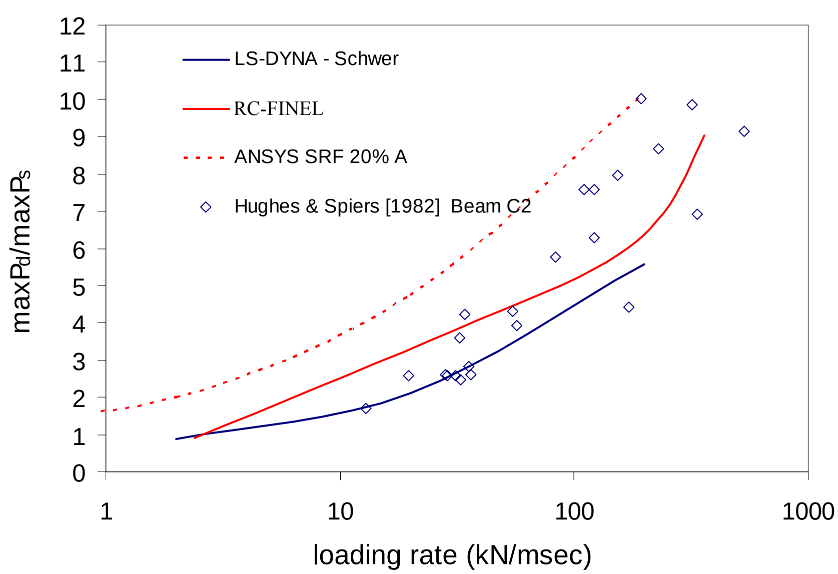


Figure 17 Experimental and predicted variations of the maxP_d/maxP_s ratio with the rate of loading (where maxP_d and maxP_s are the values of load-carrying capacity under dynamic and static, respectively, loading).

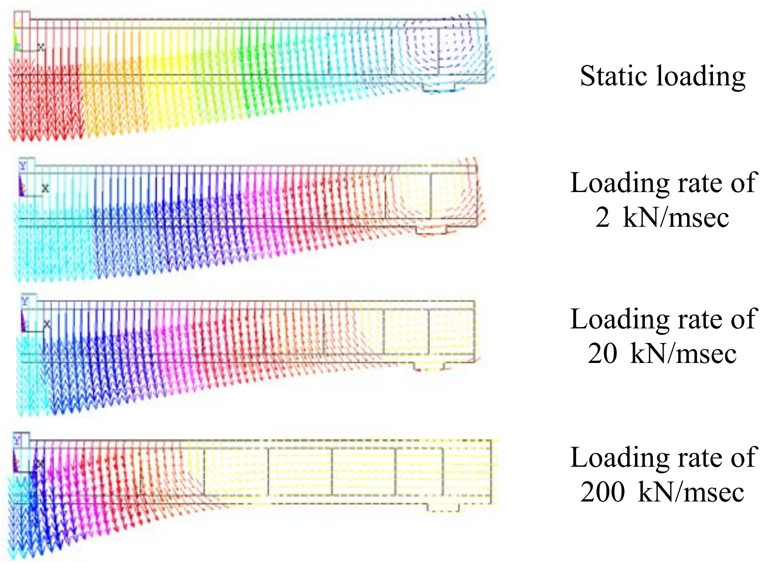


Figure 18 Deformed shape of the RC beam investigated under load applied at mid span at various rates of loading (the arrows show the displacement vectors): (a) static load, (b) 2kN/msec, (c) 20kN/msec and (d) 200kN/msec.

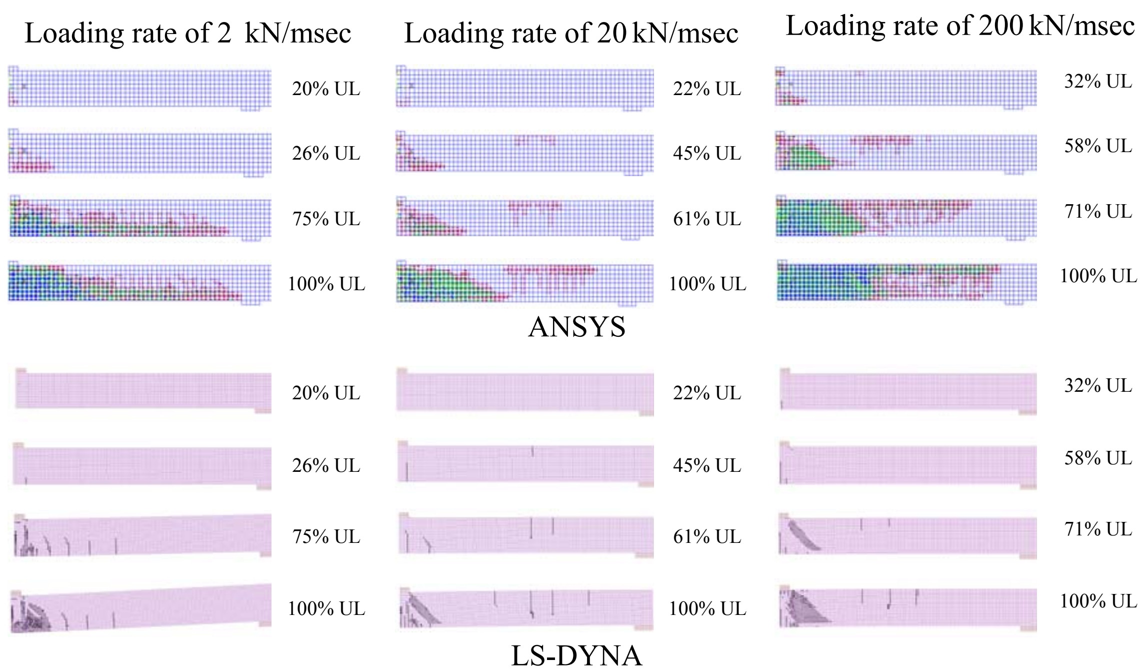


Figure 19 Crack patterns indicating the 'effective response length' L_{eff} of the RC beam investigated at various stages of the applied load for various rates of loading, for the ANSYS and LS-DYNA (Winfrith) models: (a) 2 kN/msec, (b) 20kN/msec and (c) 200 kN/msec.

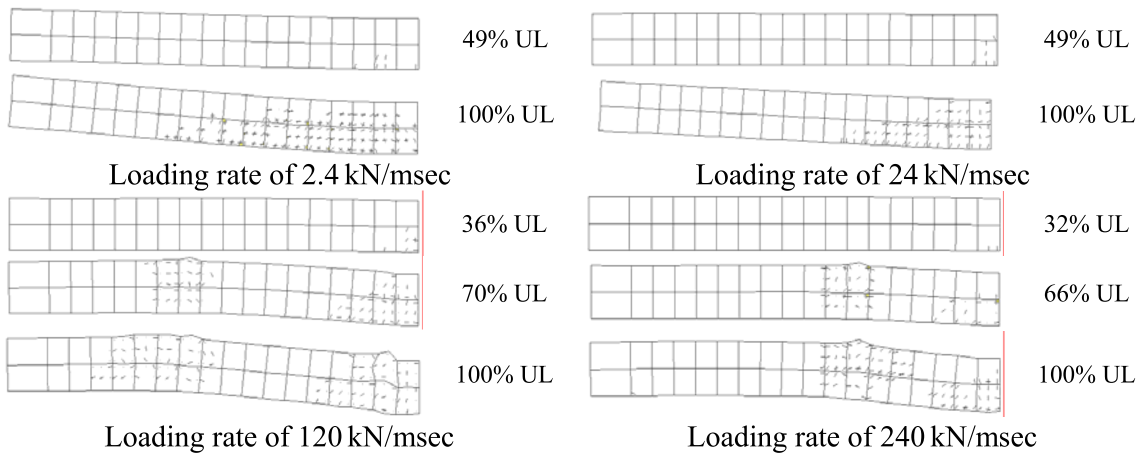


Figure 20. Deformation profile and crack patterns of the RC beam investigated under different rates of loading as predicted by the analysis, RC-FINEL model.

7 CONCLUSIONS

Based on the numerical predictions obtained from the various case studies presented herein it is possible to derive the following conclusions:

1. Although LS-DYNA is capable to predict the ductile response exhibited by the RC beam under static monotonic loading in case study 1, the same FEA package was unable to provide a realistic prediction of the brittle behaviour of the second RC beam in case study 2 when subjected to the same loading and boundary conditions.

2. Even though assuming that the behaviour of concrete under uniaxial compression is elastic, ABAQUS was still able to predict the ductile and brittle response of the RC beams examined in case studies 1 and 2 respectively.

3. In spite the fact that a fully brittle material model was employed to describe concrete behaviour, RC-FINEL was still able to predict the ductile and brittle response of the RC beams examined in case studies 1 and 2 respectively.

4. The parametric investigation carried out using ANSYS revealed that when adopting relatively high values for the shear retention factor (>20%) and stress-strain curve A to describe the behaviour to concrete under uniaxial compression (which accounts for both descending and ascending branch) enables the FEA model to yield realistic predictions of the response of the ductile beam investigated in case study 1. Adopting the same parameters in the case of the brittle beam resulted in the FEA model overestimating by a significant margin of error both the load carrying capacity and the maximum deflection exhibited by the brittle beam prior to failure.

5. On the other hand, when adopting relatively low values for the shear retention factor (<20%) and stress-strain curve C to describe the behaviour to concrete under uniaxial compression (which accounts only for the ascending branch) enables the FEA model to yield realistic predictions of the response of the brittle beam investigated in case study 2. Adopting these parameters in the case of the ductile beam resulted in the FEA

model considerably underestimating the ductility exhibited by the beam during testing while, at the same time, the predicted behaviour of the RC beam that was exhibited prior to it attaining its load-carrying capacity, remains practically unaffected.

6. The good correlation exhibited between the numerical predictions obtained from case study 3 with the available experimental data validates the assumption that for the case of high-rate-loading problems the material properties of concrete and steel reinforcement are essentially independent of the strain rate and that the effects of the latter on structural response are primarily attributed to the inertia forces which develop within the structural member, and not to the loading-rate sensitivity of the mechanical characteristics of the materials involved.

Based on the parametric analysis carried out with ANSYS and the predictions obtained from LS-DYNA, the various parameters required to fully define the majority of concrete material models often attribute to concrete behaviour a certain ductility which is often not justified by the available experimental evidence. In doing so, the NLFEA packages that employ these models often overestimate the ductility of the various RC structural forms investigated increasing the likelihood of them not being able to realistically identify brittle types of failure. On the other hand, the FEA packages employing concrete material models with brittle characteristics (i.e. ABAQUS and RC-FINEL) are found capable of yielding good predictions in both ductile and brittle types of problems. In addition, it can be concluded that, due to the use of various parameters in the formulation of the various concrete models, the FEA packages which incorporate them become case study-dependent and their ability to yield realistic predictions of structural behaviour is limited to the particular problems whose experimental information was used for calibrating the model. Applying these packages to different problem types requires recalibration of the model of concrete behaviour following tests with comparable experimental response.

Furthermore, the need emerges to re-evaluate whether the effect of loading rates on structural response at high rates is primarily attributed to inertia or the strain rate sensitivity of the mechanical properties of concrete and steel reinforcement. The work referred to in the present article provides evidence that even though strain-rate sensitivity is ignored, the numerical predictions obtained from the various packages exhibit similar trends with the available experimental data.

Finally, it should be noted at this point that the case studies investigated are presently limited to certain problem types, namely simply supported RC beams subjected to a concentrated load at mid-span, applied at various rates. The investigation is currently being extended to other problem types including structural indeterminacy.

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