Performance Analysis of Physical Layer Security Over k-μ Shadowed Fading Channels

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Abstract: In this paper, the secrecy performance of the classic Wyner's wiretap model over k-μ shadowed fading channels is studied. More specifically, we derive two analytical expressions for the lower bound of secure outage probability at high signal-to-noise ratio regime and the probability of strictly positive secrecy capacity over k-μ shadowed fading channels, respectively. As there exist infinite series in the two derived expressions, we further obtain two simple and explicit approximate expressions for the lower bound of secure outage probability and the probability of strictly positive secrecy capacity with the aid of the moment matching method. It is shown that the match between the analytical results and simulations is very excellent for all parameters under considerations.

1 Introduction

Physical layer security (PLS) has emerged as a promising technique to provide trustworthiness and reliability for the future wireless communication [1, 2]. Unlike the traditional encryption algorithms, as well as the conventional security mechanisms with encryption processing, the key feature of PLS is to exploit the physical layer characteristics of wireless channels. Recently, the secure performance of communication systems over fading channels has been studied in the corresponding literatures [3-7]. Authors in [3] defined the secrecy capacity and characterized the secrecy capacity of a quasi-static Rayleigh fading channel in terms of outage probability. The exact closed-form expressions for the secure outage probability (SOP) and the probability of the strictly positive secrecy capacity (SPSC) of multiple-input multiple-output (MIMO) system over the Rayleigh fading channels were derived in [4]. Pan *et al.* [5] derived the analytical expressions for the exact and asymptotic SOP over Rician fading channels based on a hybrid visible light communication (VLC)-radio frequency (RF) system model. [6] derived a simple expression of the secrecy outage probability over Rician fading channels when the eavesdropper's location and the channel state information (CSI) of the main channel were known. In [7], zhao *et al.* derived the exact closed-form expressions of the probability of non-zero secrecy capacity and SOP over Nakagami-m fading channels. Unfortunately, the channel models investigated in the aforementioned studies are limited to small scale fading channels while ignoring the effects of shadowing fading. In practice, the secure performance suffers from not only small fading, but also shadowing fading which is a crucial factor of communication system [8, 9].

In this context, the performances of secure communications over non-small-scale fading channels and α-μ fading channels were studied in [10] and [11], respectively. Authors in [12] performed a secrecy analysis of PLS over generalized-K (G_K) fading channels using a mixture gamma distribution, where the closed-form expressions for the average secrecy capacity (ASC), SOP, and SPSC were derived. The work of [12] was extended to the case of single-input multiple-output (SIMO) system where the exact ASC, SOP and SPSC of SIMO systems over G_K fading channels were investigated in [13]. As a general composite channel model, the G_K model encompasses Nakagami-m multipath fading and Gamma shadowing fading and can characterize the fluctuations of homogeneous scattering environments [14]. However, the channel model is incapable of characterizing the fluctuations of inhomogeneous scattering environments. To this end, a new composite channel model referred to k-μ shadowed model was proposed in [15], which can capture randomly fluctuations for the general line of sight (LOS) propagation scenario. Moreover, the k-μ shadowed model can be reduced to the specific model such as Rician and Gamma shadowing fading, one-side Gaussian, Rayleigh, Nakagami-m, Rician and k-μ multipath fading based on different parameter settings. Recently, a few studies have been carried out based on k-μ shadowed fading channels. [16] derived approximate expressions for outage probability and channel capacity when the desired signals and interference signals both experience k-μ shadowed fading. The analytic expression of the ergodic capacity over k-μ shadowed fading was presented in [17]. Authors in [18] investigated the effective rate of multiple-input single-output (MISO) systems over independent and identically distributed (i.i.d.) k-μ shadowed fading channels. The performance of energy detection over k-μ shadowed fading was analysed in [19]. To the best of the authors' knowledge, the performance of the physical layer security over k-μ shadowed fading channels has not been presented in the open technical literature, which motivates us to develop this treatise.

Motivated by the above discussion, in this study, we investigate the secrecy performance over the k-μ shadowed fading channels. The analytical expressions of the SOP lower bound and exact SPSC for the classic Wyner's wiretap model are derived. However, the derived expressions contain infinite series, which makes challenge to analyze the influence of fading parameters on system performance. To solve this problem, the approximate expressions of the asymptotic lower bound of SOP and the asymptotic SPSC are derived with the aid of a moment matching method. Two explicit asymptotic expressions are easy to be numerically evaluated because they provide a unified form in terms of well-known Gamma function and Meijer G-function. Moreover, our theoretical analysis is confirmed by Monte-Carlo simulation results.

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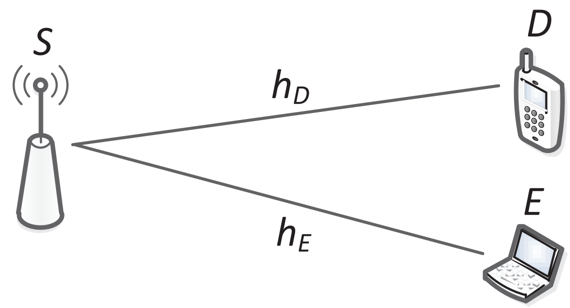

Fig. 1: System model

The remainder of this paper is organised as follows: In Section 2, we introduce the system model and the statistics for the k-μ shadowed distribution. Section 3 derives the exact analytical expressions for the lower bound of the SOP and the SPSC over the k-μ shadowed fading channels based on the classic Wyner's wiretap model, respectively. In Section 4, we provide asymptotic expressions of the lower bound of SOP and SPSC by using the moment matching method. Section 5 showcases numerical results according to the Monte-Carlo simulations to validate the correctness of our analyses. Finally, the paper is concluded in Section 6.

2 System model and statistics for the k-μ shadowed distribution

2.1 System Model

In this subsection, the classic Wyner's wiretap model consisting of three entities is considered: a legitimate transmitter (*S*), a legitimate receiver (*D*), and an eavesdropper (*E*). As shown in Fig. 1, the communication occurs over the main channel between *S* and *D*, while *E* is able to intercept the signal from the eavesdropper channels. We have the following assumption that both the main and eavesdropper channels experience independent and non-identical k-μ shadowed fading. Two channels are block fading channels where the channels vary independently from one block to another while remaining constant during a block period. Additionally, it is assumed that *S* has the perfect CSI of both main and eavesdropper channels. In practice, it is very difficult to obtain perfect CSI, which is obtained by channel estimation.This problem will be as our future research content. The received signals at the receiver (*D* and *E*) can be written as

$$y_i = h_i x + n, i \in \{D, E\} \tag{1}$$

where h_i denotes the k-μ shadowed fading channels between the transmitter and the single receiver, $i \in \{D, E\}$ means the parameter belongs to the main channel or the eavesdropper channel; x is the transmitted signal; n represents the additive white Gaussian noise having zero mean and fixed variance σ_n^2.

2.2 Statistics for The k-μ Shadowed Distribution

In the following, the main and eavesdropper channels undergo independent and non-identical k-μ shadowed fading, and the probability density function (PDF) of the SNR over k-μ shadowed fading channels is given as [15]

$$\begin{aligned} f_i(\gamma) = &\frac{{\mu_i}^{\mu_i} {m_i}^{m_i} (1+k_i)^{\mu_i}}{\Gamma(\mu_i)(\mu_i k_i + m_i)^{m_i} {\Omega_i}^{\mu_i}} \gamma^{\mu_i - 1} \\ &\times \exp\left(-\frac{\mu_i(1+k_i)}{\Omega_i}\gamma\right) \\ &\times {}_1F_1\left(m_i, \mu_i; \frac{\mu_i^2 k_i (1+k_i)}{(\mu_i k_i + m_i)\Omega_i}\gamma\right), i \in \{D, E\} \end{aligned} \tag{2}$$

where $\Omega_i = E[\gamma_i]$ is the average SNR at D or E, k_i, μ_i and m_i are channel's parameters of D or E with the meanings of the ratio between the total power of the dominant components and the total power of the scattered waves, the number of clusters, and the shaping parameter of the Nakagami-m random variable (RV), respectively. $\Gamma(\cdot)$ and ${}_1F_1(\cdot)$ are the Gamma function [20, Eq. (8.310.1)] and the confluent hypergeometric function[20, Eq. (9.14.1)], respectively.

Then, the cumulative distribution function (CDF) of k-μ shadowed distribution of SNR is given as [15]

$$\begin{aligned} F_i(\gamma) = &\frac{{\mu_i}^{\mu_i - 1} {m_i}^{m_i} (1+k_i)^{\mu_i}}{\Gamma(\mu_i)(\mu_i k_i + m_i)^{m_i} {\Omega_i}^{\mu_i}} \gamma^{\mu_i} \\ &\times \Phi_2\left(\mu_i - m_i, m_i, \mu_i + 1\,; \right. \\ &\left. -\frac{\mu_i(1+k_i)\gamma}{\Omega_i}, -\frac{\mu_i m_i (1+k_i)\gamma}{(\mu_i k_i + m_i)\Omega_i}\right), i \in \{D, E\} \end{aligned} \tag{3}$$

where $\Phi_2(\cdot)$ is the confluent multivariate hypergeometric function [20].

3 Analyses for secure outage probability and probability of strictly positive secrecy capacity

In this section, the exact analytical closed-form expressions for the lower bound of the SOP and the SPSC over k-μ shadowed fading channels are derived in the following theorems, respectively.

3.1 SOP Analysis

As an important performance metric to characterize wireless communications, SOP is the probability that the instantaneous secrecy capacity is smaller than the target rate [3, 21], which is defined as

$$P_{out}(R_S) = P(C_S \leq R_S) \tag{4}$$

where $C_S = C_m - C_w$, C_S is the instantaneous secrecy capacity for the wireless fading channels; C_m and C_w denote the capacity of the main channel and the eavesdropper channel, respectively; R_S represents a target secrecy rate.

Theorem 1. *For k-μ shadowed fading channels, the closed-form expression for the lower bound of the SOP is given as*

$$\begin{aligned} SOP^L = &\frac{a_D^{\mu_D}}{\mu_D \Gamma(\mu_D) b_D^{m_D} \Gamma(c_E)} \\ &\times \sum_{p,q=0}^{\infty} \frac{(c_D)_p (m_D)_p b_E^{d-m_E}}{(\mu_D+1)_{p+q} p! q! {b_D}^q a_E^{p+q+\mu_D}} \\ &\times (-a_D)^{p+q} \Theta^{p+q+\mu_D} G_{2,2}^{1,2}\left[\frac{k_E \mu_E}{m_E} \middle|_{0, 1-\mu_E}^{1+m_E-\mu_E, 1-d}\right] \end{aligned} \tag{5}$$

where $a_i = \frac{\mu_i(1+k_i)}{\Omega_i}$, $b_i = \frac{\mu_i k_i + m_i}{m_i}$, $c_i = \mu_i - m_i$, $d = p + q + \mu_D + \mu_E$, and $\Theta = \exp(C_{th})$, C_{th} is the target secrecy capacity threshold, $G(\cdot)$ is the Meijer G-function [20, Eq. (9.301)], and $(\cdot)!$ is the factorial operation.

Proof: The detailed proof is provided in Appendix □

3.2 SPSC Analysis

In this subsection, we consider another benchmark, SPSC, which is the probability of existence of strictly positive secrecy capacity [3, 21]. In secure communications, SPSC is an essential metric to characterize the system performance, which is defined as

$$P_{out} = P(C_S > 0) \tag{6}$$

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Theorem 2. *For k-μ shadowed fading channels, the closed-form expression for SPSC is given as*

$$SPSC = 1 - \frac{a_D^{\mu_D}}{\mu_D \Gamma(\mu_D) b_D^{m_D} \Gamma(c_E)} \times \sum_{p,q=0}^{\infty} \frac{(c_D)_p (m_D)_p b_E^{d-m_E}}{(\mu_D+1)_{p+q} p! q! {b_D}^q a_E^{p+q+\mu_D}} (-a_D)^{p+q} \times G_{2,2}^{1,2}\left[\frac{k_E \mu_E}{m_E}\middle|_{0,1-\mu_E}^{1+m_E-\mu_E,1-d}\right] \quad (7)$$

Proof: According to (6), SPSC can be obtained as

$$\begin{aligned} SPSC &= P\{C_S(\gamma_D,\gamma_E) > 0\} \\ &= 1 - P\{\ln(1+\gamma_D) - \ln(1+\gamma_E) \le 0\} \\ &= 1 - P\{\gamma_D \le \gamma_E\} \\ &= 1 - \int_0^{\infty} F_D(\gamma_E) f_E(\gamma_E) d\gamma_E \\ &= 1 - \frac{a_D^{\mu_D} a_E^{\mu_E}}{\mu_D \Gamma(\mu_D) b_D^{m_D} b_E^{m_E} \Gamma(c_E)} \\ &\times \sum_{p,q=0}^{\infty} \frac{(c_D)_p (m_D)_p}{(\mu_D+1)_{p+q} p! q! {b_D}^q} (-a_D)^{p+q} \\ &\times \int_0^{\infty} {\gamma_E}^{d-1} G_{0,1}^{1,0}\left[\frac{a_E}{b_E}\gamma_E\middle|_{0}^{-}\right] \\ &\times G_{1,2}^{1,1}\left[\frac{a_E k_E \mu_E}{b_E m_E}\gamma_E\middle|_{0,1-\mu_E}^{1+m_E-\mu_E}\right] d\gamma_E \end{aligned} \quad (8)$$

Using (A.14), we can derive SPSC as (7) after some simplifications. □

Obviously, the exact analytical expressions for the lower bound of the SOP and the SPSC over k-μ shadowed fading channels can be efficiently evaluated by standard mathematical software, e.g. MATLAB and/or MATHEMATICA. However, the analytical expressions both involve an infinite series which is adverse to the analysis. To handle this problem, we further provide two simple and explicit approximate expressions for the probability of the SOP and the SPSC over k-μ shadowed fading channels by employing the moment matching method in the following section.

4 Approximate secure outage probability and probability of strictly positive secrecy capacity

In this section, we perform the approximate analysis by employing the moment matching method, where this approximate method is adopted for the following reasons:

(1) Gamma distribution is friendlier and mathematically tractable, which is extensively used to approximate many types of fading channels [14, 16, 18, 22];

(2) Some research contributions have shown that $\kappa-\mu$ shadowed distribution can be effectively approximated by the Gamma distribution [16, 18].

Based on the moment matching method, the PDF of squared k-μ shadowed RV is expressed as

$$f_i(\gamma) = \frac{1}{\Gamma(\Delta_i)}\left(\frac{\Omega_i}{\Delta_i}\right)^{-\Delta_i} \gamma^{\Delta_i - 1} \exp\left(-\frac{\Delta_i}{\Omega_i}\gamma\right) \quad (9)$$

where $\Delta_i = \frac{m_i \mu_i (1+k_i)^2}{m_i + \mu_i {k_i}^2 + 2 m_i k_i}$.

According to the theory of probability, the CDF of squared k-μ shadowed RV is derived as

$$F_i(\gamma) = \frac{1}{\Gamma(\Delta_i)} \Upsilon\left(\Delta_i, -\frac{\Delta_i}{\Omega_i}\gamma\right) \quad (10)$$

where $\Upsilon(\alpha,x) = \int_0^x \mathrm{e}^{-t} t^{\alpha-1} dt$ is the lower incomplete Gamma function [20, Eq. (8.350.1)].

Theorem 3. *For k-μ shadowed fading channels, the approximate expression for the lower bound of SOP is given as*

$$SOP^L = \frac{1}{\Gamma(\Delta_D)\Gamma(\Delta_E)} G_{2,2}^{1,2}\left[\frac{\Omega_E \Delta_D}{\Omega_D \Delta_E}\Theta\middle|_{\Delta_D,0}^{1,1-\Delta_E}\right] \quad (11)$$

Proof: SOP can be presented as

$$\begin{aligned} SOP &= Pr\{C_S(\gamma_D,\gamma_E) \le C_{th}\} \\ &= P\{\gamma_D \le \gamma_E \Theta + \Theta - 1\} \\ &= \int_0^{\infty} F_D(\Theta\gamma_E + \Theta - 1) f_E(\gamma_E) d\gamma_E \\ &= \frac{\Omega_E^{-\Delta_E}}{\Gamma(\Delta_D)\Gamma(\Delta_E)\Delta_E^{-\Delta_E}} \int_0^{\infty} {\gamma_E}^{\Delta_E - 1} \exp(-\frac{\Delta_E}{\Omega_E}\gamma_E) \\ &\times \Upsilon(\Delta_D, \frac{\Delta_D(\Theta\gamma_E + \Theta - 1)}{\Omega_D}) d\gamma_E \end{aligned} \quad (12)$$

For $\Upsilon(\Delta_D, \frac{\Delta_D(\Theta\gamma_E+\Theta-1)}{\Omega_D})$, $\Theta = \exp(C_{th})$ is a finite value. As disscussed in Section 3, we can obtain

$$\Upsilon(\Delta_D, \frac{\Delta_D(\Theta\gamma_E + \Theta - 1)}{\Omega_D}) \approx \Upsilon(\Delta_D, \frac{\Delta_D \Theta \gamma_E}{\Omega_D}), \gamma_E \to \infty \quad (13)$$

Then, using (A.11) , the lower bound of SOP can be given by

$$\begin{aligned} SOP^L &= P\{\gamma_D \le \Theta\gamma_E\} \\ &= \frac{\Omega_E^{-\Delta_E}}{\Gamma(\Delta_D)\Gamma(\Delta_E)\Delta_E^{-\Delta_E}} \int_0^{\infty} {\gamma_E}^{\Delta_E - 1} \exp(-\frac{\Delta_E}{\Omega_E}\gamma_E) \\ &\times \Upsilon(\Delta_D, \frac{\Delta_D \Theta \gamma_E}{\Omega_D}) d\gamma_E \end{aligned} \quad (14)$$

The integral in (14) contains a power function, an exponential function and a lower incomplete gamma function. Using [23, Eq. (11)] and the integral in [24, Eq. (8.4.16.1)], we can rewrite the exponential function and the lower incomplete gamma function in the form of MeijerG-function as

$$\exp(-\frac{\Delta_E}{\Omega_E}\gamma_E) = G_{0,1}^{1,0}\left[\frac{\Delta_E}{\Omega_E}\gamma_E\middle|_{0}^{-}\right] \quad (15)$$

$$\Upsilon(\Delta_D, \frac{\Delta_D\Theta\gamma_E}{\Omega_D}) = G_{1,2}^{1,1}\left[\frac{\Delta_D\Theta\gamma_E}{\Omega_D}\middle|_{\Delta_D,0}^{1}\right] \quad (16)$$

Then, by substituting (15) and (16) into (14), and utilizing (A.14) , we can finally derive the approximate expression for the lower bound of SOP as

$$\begin{aligned} SOP^L &= P\{\gamma_D \le \Theta\gamma_E\} \\ &= \frac{\Omega_E^{-\Delta_E}}{\Gamma(\Delta_D)\Gamma(\Delta_E)\Delta_E^{-\Delta_E}} \int_0^{\infty} {\gamma_E}^{\Delta_E - 1} G_{0.1}^{1,0}\left[\frac{\Delta_E}{\Omega_E}\gamma_E\middle|_{0}^{-}\right] \\ &\times G_{1,2}^{1,1}\left[\frac{\Delta_D\Theta\gamma_E}{\Omega_D}\middle|_{\Delta_D,0}^{1}\right] d\gamma_E \\ &= \frac{1}{\Gamma(\Delta_D)\Gamma(\Delta_E)} G_{2,2}^{1,2}\left[\frac{\Omega_E\Delta_D}{\Omega_D\Delta_E}\Theta\middle|_{\Delta_D,0}^{1,1-\Delta_E}\right] \end{aligned} \quad (17)$$

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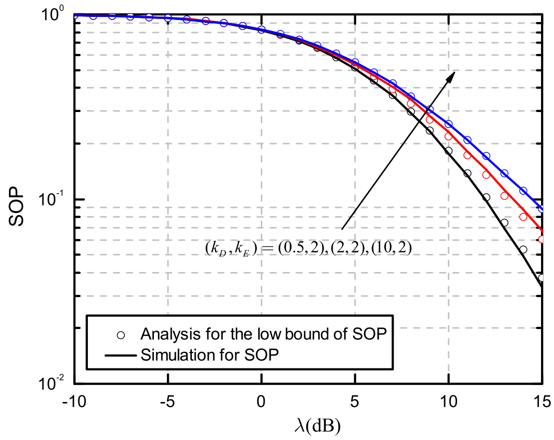

Fig. 2: SOP with changing (k_D, k_E) versus λ, $(k_D, k_E) = \{(0.5, 2), (2, 2), (10, 2)\}$

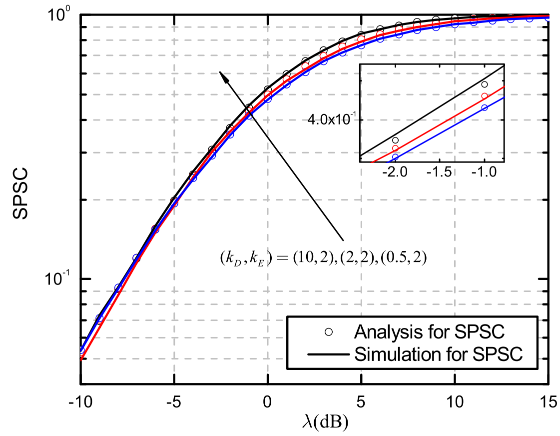

Fig. 3: SPSC with changing (k_D, k_E) versus λ, $(k_D, k_E) = \{(10, 2), (2, 2), (0.5, 2)\}$

Theorem 4. *For k-μ shadowed fading channels, the approximate expression for the SPSC is given as*

$$SPSC = 1 - \frac{1}{\Gamma(\Delta_D)\Gamma(\Delta_E)} G_{2,2}^{1,2}\left[\frac{\Omega_E \Delta_D}{\Omega_D \Delta_E}\middle|_{\Delta_D, 0}^{1, 1-\Delta_E}\right] \tag{18}$$

Proof: Referring to (8), SPSC can be obtained as

$$\begin{aligned} SPSC &= P\left\{C_S(\gamma_D, \gamma_E) > 0\right\} \\ &= 1 - P\left\{\gamma_D \le \gamma_E\right\} \\ &= 1 - \int_0^\infty F_D(\gamma_E) f_E(\gamma_E) d\gamma_E \\ &= 1 - \frac{\Omega_E^{-\Delta_E}}{\Gamma(\Delta_D)\Gamma(\Delta_E)\Delta_E^{-\Delta_E}} \int_0^\infty \gamma_E^{\Delta_E - 1} \\ &\times G_{0.1}^{1,0}\left[\frac{\Delta_E}{\Omega_E}\gamma_E \middle|_0^-\right] G_{1,2}^{1,1}\left[\frac{\Delta_D \gamma_E}{\Omega_D}\middle|_{\Delta_D, 0}^{1}\right] d\gamma_E \end{aligned} \tag{19}$$

In (19), the integral consists of three terms: a power function and two exponential functions. As suggested by (A.14), we can finally derive the expression of (18) after some algebraic operations. □

5 Numerical results

In this section, the proposed analytical derivations are validated against the simulation results, and the effects of various channel parameters on the secrecy performance are discussed. According to Eq. (2), the k-μ shadowed RVs are generated in 10^5 realizations. These random RVs are used to obtain the simulation results of the SOP and SPSC. Unless other specified, we have the following assumption: $C_{th} = 1$dB, $\Omega_D = \lambda \Omega_E$, SOP simulation: $\Omega_E = 20$dB, SPSC simulation: $\Omega_E = 1$dB, where λ represents the ratio of the received SNR between the main and the eavesdropper channels.

In Figs. 2 and 3, the approximate lower bound of the SOP in (11) and approximate SPSC in (18) are compared with the simulation results versus λ for parameters (k_D, k_E), respectively. The specific parameters are set as follows: $\mu_D = \mu_E = 2$, $m_D = m_E = 1$. It is clearly shown that the simulation results sufficiently match with analysis results, A higher λ causes a smaller SOP and a bigger SPSC since a higher λ represents that the quality of main channel outperforms the one of eavesdropper channel. In addition, when $\lambda > -2$dB, SOP increases and SPSC decreases while k_D increasing, because k is the ratio between the total power of the dominant components and the total power of the scattered waves. Finally, we can observe that small k_D is beneficial to enhance the secrecy performance of the considered system.

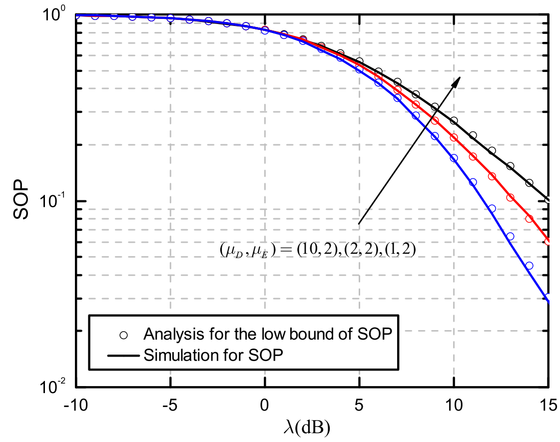

Fig. 4: SOP with changing (μ_D, μ_E) versus λ, $(\mu_D, \mu_E) = \{(10, 2), (2, 2), (1, 2)\}$

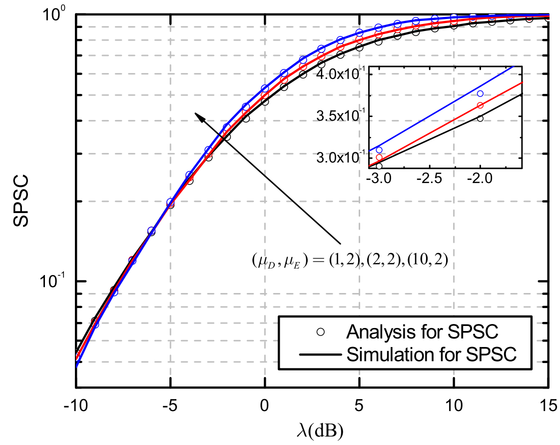

Fig. 5: SPSC with changing (μ_D, μ_E) versus λ, $(\mu_D, \mu_E) = \{(1, 2), (2, 2), (10, 2)\}$

Figs. 4 and 5 compare the approximate analysis results with the simulations of SOP and SPSC versus λ for different parameters (μ_D, μ_E). In this simulation, the parameters are set as follows: $k_D = k_E = 2$, $m_D = m_E = 1$. The figures illustrate that the simulated and analytical curves are closely matched for all parameters under consideration. Furthermore, when $\lambda > -2$dB, we observe that

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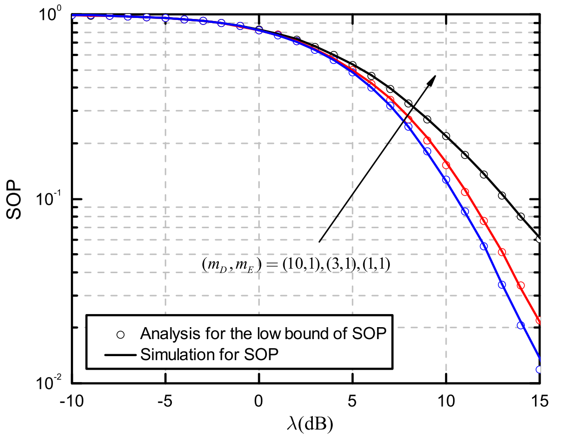

Fig. 6: SOP with changing (m_D, m_E) versus λ, $(m_D, m_E) = \{(10,1), (3,1), (1,1)\}$

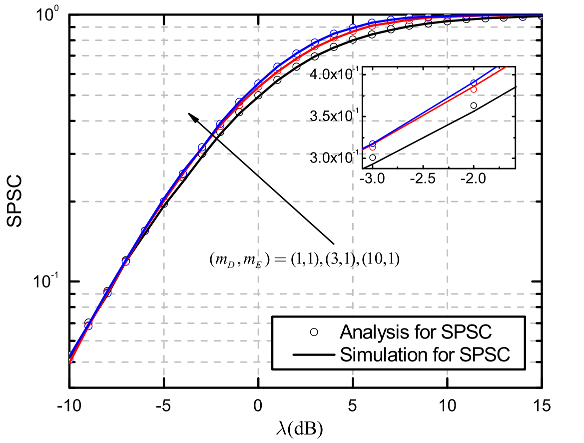

Fig. 7: SPSC with changing (m_D, m_E) versus λ, $(m_D, m_E) = \{(1,1), (3,1), (10,1)\}$

large value of μ_D yields lower SOP and higher SPSC, which means that large μ_D is beneficial for enhancing SOP and SPSC, since μ is the number of clusters.

In Figs. 6 and 7, we compare simulation and approximate analysis results of the SOP and SPSC over k-μ shadowed fading channels versus λ for different parameters (m_D, m_E). The parameter sets are given as follows: $k_D = k_E = 2$, $\mu_D = \mu_E = 2$. The close match between analysis and simulation results verifies the correctness of our proposed analytical models. Moreover, we can see that SOP decreases and SPSC increases by increasing m_D with $\lambda > -2$dB, where m is the shaping parameter of Nakagami-m RV. Finally, we can observe that large m_D is helpful in improving the performance of PLS.

6 Conclusion

In this paper, we investigate the secrecy performance of the classic Wyner's wiretap model over k-μ shadowed channels. The closed-form expressions for the SOP lower bound followed by high SNR analysis and the exact SPSC are derived. However, the derived expressions involve infinite series which is a challenge to further analysis. In order to solve this problem, we obtain two simple and explicit approximate expressions for the lower bound of the SOP and the SPSC by using a moment matching method. Finally, the asymptotic expressions are validated through the simulation results.

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9 Appendix

In this section, the proof of theorem 1 is given.

Proof: Based on the definition in (4), SOP can be provided as

$$\begin{aligned} SOP &= P\left\{C_S(\gamma_D,\gamma_E) \le C_{th}\right\} \\ &= P\left\{\ln(1+\gamma_D) - \ln(1+\gamma_E) \le C_{th}\right\} \\ &= P\left\{\frac{1+\gamma_D}{1+\gamma_E} \le \Theta\right\} \\ &= P\left\{\gamma_D \le \gamma_E\Theta + \Theta - 1\right\} \\ &= \int_0^\infty \int_0^{\Theta\gamma_E+\Theta-1} f_D(\gamma_D)d\gamma_D f_E(\gamma_E)d\gamma_E \\ &= \int_0^\infty F_D(\Theta\gamma_E+\Theta-1)f_E(\gamma_E)d\gamma_E \end{aligned} \tag{A.1}$$

According to [25], we can obtain the following formula as

$$ {}_pF_q\left(x\left|{}^{a_p}_{b_q}\right.\right) = \frac{\Gamma(b_q)}{\Gamma(a_p)} G^{1,p}_{p,q+1}\left[-x\left|{}^{1-a_p}_{0,1-b_q}\right.\right] \tag{A.2}$$

Employing the following identity [24, Eq. (7.11.1.2)],

$$ {}_1F_1\left(a,b;x\right) = e^x\, {}_1F_1\left(b-a,b;-x\right) \tag{A.3}$$

and after some algebraic manipulations, the confluent hypergeometric function in (2) can be rewritten as

$$\begin{aligned} {}_1F_1\left(m_i,\mu_i;\frac{\mu_i^2 k_i(1+k_i)}{(\mu_i k_i+m_i)\Omega_i}\gamma\right) &= \exp\left(\frac{a_i k_i \mu_i}{b_i m_i}\gamma\right)\frac{\Gamma(\mu_i)}{\Gamma(c_i)} \\ &\times G^{1,1}_{1,2}\left[\frac{a_i k_i \mu_i}{b_i m_i}\gamma\left|{}^{1+m_i-\mu_i}_{0,1-\mu_i}\right.\right] \end{aligned} \tag{A.4}$$

Then, (2) can be rewritten as

$$\begin{aligned} f_i(\gamma) &= \frac{{a_i}^{\mu_i}}{{b_i}^{m_i}\Gamma(c_i)}\gamma^{\mu_i-1}\exp\left(-\frac{a_i}{b_i}\gamma\right) \\ &\times G^{1,1}_{1,2}\left[\frac{a_i k_i \mu_i}{b_i m_i}\gamma\left|{}^{1+m_i-\mu_i}_{0,1-\mu_i}\right.\right] \end{aligned} \tag{A.5}$$

Substituting the following identity [20, Eq. (9.261.2)] into (3),

$$\Phi_2\left(\alpha,\beta,\gamma;x,y\right) = \sum_{p,q=0}^{\infty}\frac{(\alpha)_p(\beta)_p}{(\gamma)_{p+q}p!q!}x^p y^q \tag{A.6}$$

the CDF of k-μ shadowed distribution can be simplified as

$$\begin{aligned} F_i(\gamma) &= \frac{{a_i}^{\mu_i}}{\mu_i\Gamma(\mu_i){b_i}^{m_i}}\sum_{p,q=0}^{\infty}\frac{(c_i)_p(m_i)_p}{(\mu_i+1)_{p+q}p!q!{b_i}^q} \\ &\times (-a_i)^{p+q}\gamma^{p+q+\mu_i} \end{aligned} \tag{A.7}$$

Then, substituting (A.5) and (A.7) into (A.1), SOP can be further expressed as

$$\begin{aligned} SOP &= \frac{a_D^{\mu_D}a_E^{\mu_E}}{\mu_D\Gamma(\mu_D)b_D^{m_D}b_E^{m_E}\Gamma(c_E)} \\ &\times \sum_{p,q=0}^{\infty}\frac{(c_D)_p(m_D)_p}{(\mu_D+1)_{p+q}p!q!{b_D}^q}(-a_D)^{p+q} \\ &\times \int_0^\infty(\Theta\gamma_E+\Theta-1)^{p+q+\mu_D}{\gamma_E}^{\mu_E-1}\exp\left(-\frac{a_E}{b_E}\gamma_E\right) \\ &\times G^{1,1}_{1,2}\left[\frac{a_E k_E \mu_E}{b_E m_E}\gamma_E\left|{}^{1+m_E-\mu_E}_{0,1-\mu_E}\right.\right]d\gamma_E \end{aligned} \tag{A.8}$$

The integral term contained in (A.8) is more complex, which will cause difficulty in integral operation. Considering that $\Theta = \exp(C_{th})$ is a finite value, we utilize the similar method adopted in [26], with the assumption of $\gamma_E \to \infty$, the following formula can be obtained as

$$(\Theta\gamma_E+\Theta-1)^{p+q+\mu_D} \approx (\Theta\gamma_E)^{p+q+\mu_D} \tag{A.9}$$

Then, by using the following inequality [21, Eq. (6)],

$$\begin{aligned} SOP &= P\left\{\gamma_D \le \Theta\gamma_E+\Theta-1\right\} \\ &\ge SOP^L = P\left\{\gamma_D \le \Theta\gamma_E\right\} \end{aligned} \tag{A.10}$$

the lower bound of SOP can be given by

$$\begin{aligned} SOP^L &= P\left\{\gamma_D \le \Theta\gamma_E\right\} \\ &= \frac{a_D^{\mu_D}a_E^{\mu_E}}{\mu_D\Gamma(\mu_D)b_D^{m_D}b_E^{m_E}\Gamma(c_E)} \\ &\times \sum_{p,q=0}^{\infty}\frac{(c_D)_p(m_D)_q}{(\mu_D+1)_{p+q}p!q!{b_D}^q} \\ &\times (-a_D)^{p+q}\Theta^{p+q+\mu_D}\int_0^\infty {\gamma_E}^{d-1} \\ &\times \exp\left(-\frac{a_E}{b_E}\gamma_E\right)G^{1,1}_{1,2}\left[\frac{a_E k_E \mu_E}{b_E m_E}\gamma_E\left|{}^{1+m_E-\mu_E}_{0,1-\mu_E}\right.\right]d\gamma_E \end{aligned} \tag{A.11}$$

Here, the remaining task is to calculate the integral in the (A.11). The integrand contains a power function, an exponential function and a lower incomplete gamma function. Using [23, Eq. (11)], we can rewrite the exponential function in the form of Meijer G-function as

$$\exp\left(-\frac{a_E}{b_E}\gamma_E\right) = G^{1,0}_{0,1}\left[\frac{a_E}{b_E}\gamma_E\left|{}^{-}_{0}\right.\right] \tag{A.12}$$

So the integral in Eq. (A.11) is transformed into

$$\begin{aligned} &\int_0^\infty {\gamma_E}^{d-1}G^{1,0}_{0,1}\left[\frac{a_E}{b_E}\gamma_E\left|{}^{-}_{0}\right.\right] \\ &\times G^{1,1}_{1,2}\left[\frac{a_E k_E \mu_E}{b_E m_E}\gamma_E\left|{}^{1+m_E-\mu_E}_{0,1-\mu_E}\right.\right]d\gamma_E \end{aligned} \tag{A.13}$$

With the aid of the identity [23, Eq. (21)], setting the parameters: $l = 1$ and $k = 1$, we can obtain:

$$\begin{aligned} &\int_0^\infty x^{\alpha-1}G^{s,t}_{u,v}\left[\sigma x\left|\begin{matrix}(c_u)\\(d_v)\end{matrix}\right.\right]G^{m,n}_{p,q}\left[\omega x\left|\begin{matrix}(a_p)\\(b_q)\end{matrix}\right.\right]dx \\ &= \sigma^{-\alpha}G^{m+t,n+s}_{p+v,q+u}\left[\frac{\omega}{\sigma}\left|\begin{matrix}\Delta(1,a_1),\ldots,\Delta(1,a_n),\\ \Delta(1,b_1),\ldots,\Delta(1,b_m),\end{matrix}\right.\right. \\ &\quad \begin{matrix}\Delta(1,1-\alpha-d_1)\ldots,\Delta(1,1-\alpha-d_v),\\ \Delta(1,1-\alpha-c_1),\ldots,\Delta(1,1-\alpha-c_u),\end{matrix} \\ &\quad \left.\begin{matrix}\Delta(1,a_{n+1}),\ldots,\Delta(1,a_p)\\ \Delta(1,b_{m+1}),\ldots,\Delta(1,a_q)\end{matrix}\right] \\ &= \sigma^{-\alpha}G^{m+t,n+s}_{p+v,q+u}\left[\frac{\omega}{\sigma}\left|\begin{matrix}a_1,\ldots,a_n,\\ b_1,\ldots,b_m,\end{matrix}\right.\right. \\ &\quad \left.\begin{matrix}1-\alpha-d_1,\ldots,1-\alpha-d_v,a_{n+1},\ldots,a_p\\ 1-\alpha-c_1,\ldots,1-\alpha-c_u,b_{m+1},\ldots,a_q\end{matrix}\right] \end{aligned} \tag{A.14}$$

where $\Delta(k,a) = \frac{a}{k},\frac{a+1}{k},\ldots,\frac{a+k+1}{k}$ is defined in [23], and thus we can derive the identity: $\Delta(1,a) = a$. Then, making use of (A.13) and (A.14), the integral in (A.11) can be solved. Finally, after some simple manipulations, we can derive the desired result as (5). □