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Corresponding Author: Dr. Qing-Ping Zou, Ph.D.
Corresponding Author's Institution:
First Author: xuelian jiang, Dr.
Order of Authors: xuelian jiang, Dr.; Qing-Ping Zou, Ph.D.; Na Zhang, Dr.
Abstract: Laboratory experiments were conducted to investigate the characteristics of wave loading on submerged circular-front breakwaters due to irregular waves. The wave force spectrum for a semicircular breakwater is similar to that for a quarter-circular breakwater. The peak wave force normalized by the incident wave height for irregular waves is less than that for regular waves. The performance of our theoretical wave load model is improved significantly by incorporating the effect of wave transmission and flow separation. A RANS-VOF model was used to investigate the effect of local hydrodynamic disturbances by submerged breakwaters on the pressure distribution around the breakwater and total wave load. The numerical results reveal that wave-induced vortex at the structure have a substantial influence on the wave loading on the submerged quarter-circular breakwater but not on the semicircular breakwater. A parametric analysis is used to further improve the relationship between the wave load and vortex.

- Submerged quarter-circular and semicircular breakwaters have similar force spectra.
- Wave load under irregular waves is smaller than that under regular waves.
- Improved wave load model by incorporating wave transmission and flow separation and vortex effect.
- The effect of vorticity on the dynamic pressure is examined using RANS-VOF model.
- Correlation between vortex and wave load was found

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# Wave load on submerged quarter-circular and semicircular breakwaters under irregular waves 

Xue-Lian Jiang ${ }^{\text {a, }}$, Qing-Ping Zou ${ }^{\text {bd }}$, Na Zhang ${ }^{\text {c }}$<br>${ }^{\text {a }}$ State Key Laboratory of Hydraulic Engineering Simulation and Safety, Tianjin University, 92 Weijin Road, Nankai District, Tianjin 300072, China,<br>${ }^{\mathrm{b}}$ Department of Civil and Environmental Engineering, The University of Maine, 5711 Boardman Hall, Orono, ME 04469-5711, USA.<br>${ }^{c}$ Tianjin Key Laboratory of Soft Soil Characteristics \& Engineering Environment, Tianjin Chengjian University, 26 Jinjing Road, Xiqing District, Tianjin 300384, China<br>${ }^{d}$ State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, 610065, Chengdu, China<br>* Corresponding author: qingping.zou@ gmail.com


#### Abstract

Laboratory experiments were conducted to investigate the characteristics of wave loading on submerged circular-front breakwaters due to irregular waves. The wave force spectrum for a semicircular breakwater is similar to that for a quarter-circular breakwater. The peak wave force normalized by the incident wave height for irregular waves is less than that for regular waves. The performance of our theoretical wave load model is improved significantly by incorporating the effect of wave transmission and flow separation. A RANS-VOF model was used to investigate the effect of local hydrodynamic disturbances by submerged breakwaters on the pressure distribution around the breakwater and total wave load. The numerical results reveal that wave-induced vortex at the structure have a substantial influence on the wave loading on the submerged quarter-circular breakwater but not on the semicircular breakwater. A parametric analysis is used to further improve the relationship between the wave load and vortex.


Keywords: Submerged circular-front breakwaters; Irregular waves; Wave load model; Pressure; Vorticity.

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## 1. Introduction

Breakwaters play an important role in mitigating wave damage and protecting shorelines from erosion. Traditionally, emerged breakwaters have been used for this purpose. However, designing emerged breakwaters has become increasingly difficult since the construction often takes place in a severe environment caused by fierce waves and a poor seabed [Mizutani et al., 1998; Jeng et al., 2013]. According to Rambabu and Mani [2005], submerged breakwaters have recently gained popularity in order to reduce pressures on the sheltered structures and retain sediments in the sheltered harbor through premature wave breaking [Burcharth et al., 2006]. In addition, submerged breakwaters provide more aesthetically pleasing view of the sea and better water quality in a harbor than emerged breakwaters [Johnson, 2006; Kobayashi et al., 2007].

Various types of submerged breakwaters have been employed in engineering practice, such as vertical, rubble mound, and circular-front breakwaters. They have different hydrodynamic performances due to the variation of wave reflection, dissipation, and transmission in response to the structure geometry. Young \& Testik [2011] reported that semicircular breakwaters reflect less and transmit more energy than vertical breakwaters at the same relative submergence depth. Reduced wave reflection minimizes the seaside scour at the structure and benefits the navigation of vessels near the structure but greater wave transmission may deteriorate harbor tranquility and encourage beach erosion behind the structure. However, the submerged semicircular breakwaters with perforated walls may work well resulting in a smaller transmission coefficient because of additional energy dissipation by the turbulence inside the hollow chamber [Dhinakaran et al., 2009; Liu \& Li, 2012]. Regarding wave loading, extensive studies on vertical breakwaters have been carried out in the past decades. Examples include the effect of wave breaker on dynamic pressures by Kirkgöz [1982, 1992], Ergin \& Abdalla [1993], Hattori et al. [1994], Cooker \& Peregrine [1992], Prabhkar \& Sundar [2001], the influence of aeration and scale on wave impacts by Blackmore \& Hewson [1984], Bullock et al. [2001, 2007], and dynamic response under wave attack by Oumeraci [1994], Franco [1994], Goda \& Takagi [2000], Takahashi et al [2001], Li et al. [2006], Cuomo et al. [2011]. In comparison with vertical breakwaters, the pressure on a circular wall acts towards the center of the circle and therefore circular breakwaters generally have smaller horizontal
force, overturning moment and soil subgrade reaction [Tanimoto \& Takahashi, 1994] that results in better stability and lower engineering cost [Dhinakaran et al., 2012].

This study focuses on the characteristics of wave loads on the circular-front breakwaters due to irregular waves. Two types of circular-front breakwaters have been used in coastal protection, i.e. quarter-circular breakwaters (QCB) and semicircular breakwaters (SCB). They consist of a curved concrete caisson founded on a rubble mound (see figure 2). At low water levels, circular breakwaters act as a rubble mound breakwater while serve as a composite breakwater at high water levels.

Since the first SCB was constructed at Miyazaki Port in Japan in 1993, efforts have been dedicated to characterize the dynamic pressure on SCBs. In summary, three wave load models for SCBs have been proposed. First in Tanimoto et al [1987], Goda's formulation for vertical breakwaters was modified by incorporating the phase and center angle difference to calculate the pressures on semicircular breakwaters. More wave energy will pass over submerged structures than emerged structures and therefore generate substantial surface fluctuation on the leeside, which can significantly alter the total wave loads. Yu et al. [1999] indicated that irregular wave forces on submerged SCBs exhibit very different behaviors from those on emerged SCBs. Xie [1999] concluded that submerged semicircular structures may be at risk if Tanimoto's formula was used in design. Instead, he proposed the second wave load model accordingly by including a pressure distribution on the inside circumference of the rear wall and introducing a new phase modification coefficient in Tanimoto's model. Yuan and Tao [2003] presented the third wave load model to predict the total wave forces on submerged, alternately submerged, and emerged SCBs based on a potential flow model. Figure 1 shows the freeboard ranges that these models can be applied [Li et al., 2003]. More discussions about the dynamic pressure on SCBs exposed to normally incident and oblique waves can be found in Sundar \& Ragu [1997, 1998], Zhang et al. [2005] and Liu \& Li [2013], respectively.


Figure 1. The valid range of freeboard of the empirical wave load models for semicircular breakwaters. $R_{c}$ is crest freeboard height.

Xie et al. [2006] introduced the concept of QCBs based on SCBs in order to cut the construction cost since the bulk volume of QCBs is smaller than SCBS at a given crest height,. Literature on wave loads exerted on QCBs is rare. Xie et al. [2006] estimated the regular wave loads on QCBs by adding an amplification factor to Tanimoto's model for SCBs. Liu et al. [2006] discussed the effects of wave steepness, relative wave height, and water depth on the regular wave forces acting on QCBs. However, the irregular wave loads on QCBs have not been investigated previously.

Shi et al. [2011] found that with the same curved front wall, emerged QCBs and SCBs have the same hydrodynamic performance and wave loading. However, when a coastal structure is submerged, the presence of the structure changes the flow field adjacent to the structure for example, the generation and shedding of vortices around the structure [Young et al., 2009]. These vortices are expected to cause the oscillation of forces on submerged structures that cannot be neglected in design [Poupardin et al., 2012]. The intensity of vortex would depend on the wave parameters and structure geometry. Jiang et al. [2012] reported that submerged QCBs have greater energy dissipation than submerged SCBs partly due to stronger trailing vortices. It is therefore necessary to examine the effect of vortex dynamics on wave loads.

The main objective of this study is to improve the wave load model for submerged circular-front breakwaters exposed to irregular waves, and to examine the influence of wave-induced vortices on dynamic pressure. Firstly, the experimental setup is described
in section 2. Then, the characteristics of wave loads due to irregular waves are discussed in section 3. Subsequently, the predictions of a previous wave load model are compared with the measurements in section 4 and an improved model is proposed in section 5. In section 6, the effect of transmission on wave loads is further examined based on the measured pressure distributions and RANS-VOF modeling results of vortex dynamics. Finally, some concluding remarks are summarized in section 7.

## 2. Experiment Set-up

This experiment was carried out in the wave flume of the State Key Laboratory of Hydraulic Engineering Simulation and Safety at Tianjin University in China. The flume is 35 m long, 0.5 m wide and 0.8 m deep. At the inlet, a wave generator with perforated absorbers is used to produce desired incident waves. At the outlet, a permeable beach with a $1: 4$ slope is used to absorb the incoming wave energy. Structures were placed 20 m away from the wave generator to minimize the secondary reflection. The experiment configuration is shown in figure 2 .

The test scale was set to $1: 40$ following geometric similarity, Froude law, and the wave conditions that can be generated in the flume. Impermeable concrete caissons of 0.15 m in radius, 0.175 m in height and the same width as the flume were erected on a rubble mound of 0.125 m high filled with coarse gravel (see figure 2 ). The rubble mound has a seaward slope of 1:2 and a shoreward slope of 1:1.5.

The free surface displacement was measured using six wave gauges (gauge 1~gauge 6), as shown in figure 2 (a). The first three gauges were placed in front of structures for the wave reflection analysis. The distances among them were adjusted immediately before each run so that three-probe method for wave reflection analysis by Mansard and Funke [1980] can be applied. The fourth and fifth gauges were placed over the toe and heel of caissons to record the surface displacement near the caissons. The sixth gauge was one wavelength behind the structures to record the transmitted wave profile. The variation in pressure was measured by thirteen diaphragm-type pressure transducers fixed on the QCB and fourteen on the SCB (see figure 2 (b), P1~P14). All wave gauges and pressure transducers were calibrated immediately before each run to avoid zero drift. The same personal computer was used to provide the control signals to the wave generator
and acquire the signals from wave gauges and pressure transducers through an amplifier. During each run, data were recorded simultaneously from a 24 -channel data acquisition card for 160 seconds at a sampling frequency of 50 Hz .
(a)

(b)


Figure 2. (a) Diagram of the experiment setup in the wave flume; (b) Pressure transducers on caissons.

Structures were subject to irregular waves defined by the JONSWAP-type spectra with a constant shape factor of $\gamma=3.3$. Significant wave heights in the range of $0.085 \mathrm{~m}-0.117 \mathrm{~m}$ and significant wave periods in the range of $(1.24 \mathrm{~s}-1.51 \mathrm{~s})$ were combined to generate the incident wave spectra. Tests 1-5 have a same period but different wave heights, while Tests 3 and $6-8$ have a same wave height but different periods. During the entire set of tests, the water depth was set to 0.383 m representing the submerged condition. Table 1 lists the wave conditions for this experiment. To reduce the
error in measurements, each test was performed three times and the mean value was used in the analysis. The total number of runs was 24 .

Table 1. Wave conditions.

|  | Significant wave <br> height, <br> Test No. | Significant wave <br> period, <br> $H_{s}(\mathrm{~m})$ | Peak wave <br> period, <br> $T_{s}(\mathrm{~s})$ | Significant <br> wavelength, |
| :---: | :---: | :---: | :---: | :---: |
|  | 0.117 | 1.24 | 1.33 | 2.01 |
| Test-1 | 0.100 | 1.24 | 1.33 | 2.01 |
| Test-2 | 0.093 | 1.24 | 1.33 | 2.01 |
| Test-3 | 0.085 | 1.24 | 1.33 | 2.01 |
| Test-4 | 0.078 | 1.24 | 1.33 | 2.01 |
| Test-5 | 0.093 | 1.44 | 1.54 | 2.45 |
| Test-6 | 0.093 | 1.48 | 1.58 | 2.52 |
| Test-7 | 0.093 | 1.51 | 1.61 | 2.59 |
| Test-8 |  |  |  |  |

To verify the performance of the wave generator and wave flume, a preliminary experiment was performed for all tests in Table 1 before structures were placed in the flume. Figure 3 exemplifies the time histories of the surface displacement and the incident spectrum. The agreement between the measured and desired wave spectra is very good except for small deviations at some frequencies.




Figure 3. Time series of measured and desired wave elevation (a, b) and incident wave spectrum (c, d): (a) \& (c) are for test $3\left(d=0.383 m, H_{s}=0.093 m, T_{p}=1.33 s\right)$; (b) \& (d) are for test $7\left(d=0.383 m, H_{s}=0.093 m, T_{p}=1.58 s\right) . \eta$ refers to the surface displacement and $d$ refers to the water depth.

## 3. Experimental results

Caissons must be designed properly in order to sustain the potential sliding and overturning motions induced by waves. For circular-front caissons, the overturning moment is small, and hence the sliding failure is the major concern in its design. Oumeraci et al. (2001) pointed out the sliding failure of vertical breakwaters often takes place when the shoreward horizontal force reaches its peak. However, Rao et al. [2001] came to a different conclusion for the submerged SCBs that the sliding failure most likely occurs at the time of the maximum seaward horizontal force. Therefore, we will reexamine these findings in this study.

Figure 4 shows the time series of the horizontal force $\left(F_{h}\right)$, the vertical force $\left(F_{v}\right)$ and the safety margin against sliding $\left(K_{s}\right)$. At each time step, the total wave forces are calculated by integrating the instantaneous pressure distribution along the surface of the structures using the Simpson rule (Polyanin and Manzhirov, 1998). Then, the total wave forces are decomposed into horizontal $\left(F_{h}\right)$ and vertical components $\left(F_{v}\right) . F_{h}$ is positive
shoreward and $F_{v}$ is positive upward. It should be noted that the vertical force ( $F_{v}$ ) includes both the vertical component of pressures on the wall and the uplift pressure on the base of caissons. The safety margin against sliding is proposed by

$$
\begin{equation*}
K_{s}=\frac{f\left[\gamma_{G} G-\gamma_{v} F_{v}\right]}{\gamma_{h} F_{h}}, \tag{1}
\end{equation*}
$$

where $G$ is the weight of caissons in still water $(0.493 \mathrm{kN} / \mathrm{m}$ for the QCB and $0.551 \mathrm{kN} / \mathrm{m}$ for the SCB), $f=0.6$ is the friction coefficient between the caisson and the rubble mound, $\gamma_{G}=1.0, \gamma_{v}=1.25, \gamma_{h}=1.25$ are the partial safety factors to account for the uncertainties in $G, F_{v}, F_{h}$ and their relative importance in the sliding failure. There is a phase shift between the horizontal and vertical forces, which is expected to change with the geometry of the breakwater relative to wavelength, freeboard and local water depth. The minimum safety margin against sliding ( $K_{s \text { min }}$ ) occurs at the peak shoreward horizontal force. The shoreward horizontal force ( $F_{+h \max }$ ) plays a crucial role in the sliding instability of the submerged caissons. Hence, we will next focus on $F_{+h \max }$.


Figure 4. Typical time histories of the horizontal and vertical forces, and the safety margin against sliding for the quarter-circular breakwater (test 2: $d=0.383 \mathrm{~m}, H_{s}=$ $\left.0.10 \mathrm{~m}, T_{s}=1.24 \mathrm{~s}\right)$.

Figure 5 compares the spectra of the horizontal force on the QCB and on the SCB. It can be observed from figure 5 (a) that the force spectra exerted on the QCB is similar to
those of the SCB. This result implies that the wave load model for a submerged SCB can be applied to a submerged QCB. Moreover, the force spectra are similar to the incident wave spectra (see figure 3), which means little energy transference between higher and lower harmonies when waves pass over well-submerged structures. It also shows that the peak value of force spectra increases with wave height but decreases with wave period. Goda [1970] suggested a spectral peakedness parameter $\left(Q_{p}=\frac{2}{m_{0}^{2}} \int_{0}^{\infty} \omega S^{2}(\omega) d \omega\right)$ to describe the peakedness of spectral peak and spectral width that correlates with wave groups and the corresponding damage to coastal structures. Figure 5(b) demonstrates the horizontal wave forces for the QCB and the SCB have a similar spectral profile to the incident wave. The spectral peakedness parameter of the horizontal force $\left(Q_{p}\right)_{F h}$ increases linearly with that of the incident wave $\left(Q_{p}\right)_{\eta}$. On average, the spectral width for the QCB is $9 \%$ narrower than that for the SCB and $6 \%$ narrower than that for the incident wave.

In addition, the observed dimensionless maximum horizontal force for irregular waves in this study ( $F_{h \text { max }} / \gamma_{w} H_{i} R, R$ refers to the radius of the circular wall) are compared with the observation of Liu et al. [2006] for regular waves. We found that the former is $20 \%$ for the SCB and $35 \%$ for the QCB smaller bthan the latter.


Figure 5. Spectral analysis: (a) Spectral density of the horizontal force (test 3: $d$ $=0.383 \mathrm{~m}, H_{s}=0.093 \mathrm{~m}, T_{p}=1.33 \mathrm{~s}$; test 7: $\left.d=0.383 \mathrm{~m}, H_{s}=0.093 \mathrm{~m}, T_{p}=1.58 \mathrm{~s}\right)$; (b) Spectral peakedness parameters for the horizontal force and the surface displacement.
the diagonal line denotes equal spectral peakedness.

## 4. Model-data comparison

The results in Section 3 show that submerged quarter- and semi- circular breakwaters have a similar dynamic response to irregular waves. In this section, the predictions of a wave load model for submerged SCBs subject to regular waves are compared with the measurements of irregular waves.

As reviewed in the introduction, there are three wave load models for SCBs (see figure 1). Xie's model (1999) for submerged SCBs (see Appendix A) is adopted here since the structures were submerged during the entire test. First, we need to define design wave parameters. Goda [2010] suggested $H_{\max }=1.8 H_{s}$ and the Chinese Code of Hydrology for Sea Harbor defined $H_{1 \%}$ as the design wave. Both recommendations have been tested in this study. Using $H_{1 \%}$.gives better agreement between model and data. Therefore, $H_{1 \%}$ was chosen as the design wave in our calculation. The design wave period is taken as that of significant wave according to Goda [2010], i.e. $T_{i}=T_{1 / 3}$. Besides, it should be noted that caissons in this study are impermeable as classified in Sasajima et al. [1994], i.e. we substitute $\varepsilon_{b}=0$ in the equation (A.6) and $p_{0}{ }^{\prime}=0$ in the equation (A.7). Other parameters are specified in figure A-1.

Figure 6 shows that the ratio of Xie's (1999) model predictions to our measurements has a mean value of 1.15 and standard deviations of 0.18 for the QCB and 0.14 for the SCB. The mean value implies Xie's (1999) model overestimate the observed wave load. Sasajima et al. [1994] drawn the same conclusion about Tanimoto's model based on their field tests at a rear-perforated SCB constructed at Miyazaki Port in Japan. Figure 6 also shows that the ratio of the measured maximum horizontal forces on QCB and SCB is about 1.05 , which is much lower than the value of 1.30 reported by Xie (2006) for regular waves. Thus, Xie's model is suitable for a deterministic design but not for irregular waves.


Figure 6. Comparison between the predicted maximum horizontal force by Xie's (1999) wave load model and measurements for the quarter- (solid square) and semi(open square) circular breakwaters. The diagonal line represents the prediction is equal to the observation.

## 5. Improved wave load model

The wave load model by Xie (1999) was derived from Goda's formulation for vertical breakwaters with small overtopping and transmission. As a result, the larger waves on the seaside of structures cause a greater force on the front wall than the back wall, therefore, the contribution of the pressure on the rear wall is negligible. However, for a submerged structure, more incident wave energy is transmitted to the leeside and generates a larger wave behind the structure than an emerged one does. The transmitted wave is expected to cause a significant wave impact on the rear wall [van der Meer et al., 2005]. Figure 7 compares the percentage of contribution of the force components on the front $\left(F_{h}\right)_{\text {front }}$ and rear wall $\left(F_{h}\right)_{\text {rear }}$ to the maximum horizontal force $\left(F_{h \text { max }}\right)$. The average ratio $\frac{\left(F_{h}\right)_{\text {front }}}{F_{h \max }}$ is $72 \%$ and $\frac{\left(F_{h}\right)_{\text {rear }}}{F_{h \max }}$ is $28 \%$. This result suggests that the contribution of pressures on the rear wall needs to be included in calculating the wave force on a submerged breakwater.


Figure 7. The percentage of contribution from the wave force acting on the front $\frac{\left(F_{h}\right)_{\text {front }}}{F_{h \max }}$ (solid circle) and rear wall $\frac{\left(F_{h}\right)_{\text {rear }}}{F_{h \max }}$ (empty circle) to the total peak horizontal force $\left(F_{h \max }\right)=\left(F_{h}\right)_{\text {front }}+\left(F_{h}\right)_{\text {rear }}$ for the quarter-circular breakwater.

We will next extend Xie's model (See Appendix A) by including the effect of wave transmission and form drag due to the wake behind the structure on the wave loads over submerged circular breakwaters. As listed in table 1, eight test cases were conducted in the experiment and each case was repeated three times. The measurements in tests $1,3,5$ and 7 representing various wave conditions were used to derive the correction coefficients ( 2 runs in each case and 8 runs in total) and the other sixteen runs were used to verify the improved model. When waves encounter a submerged breakwater, the instantaneous force on the structure consists of the potential flow component mainly due to the free surface displacement and the form drag component due to the wake around the structure [Dean \& Dalrymple, 1991]. Accordingly, the maximum horizontal force is given by

$$
\begin{equation*}
F_{h \max }=F_{h W}+F_{h D} \tag{2}
\end{equation*}
$$

where $F_{h W}$ represents the potential component of horizontal force and $F_{h D}$ represents the drag component.

In potential wave theory, pressures acting on a submerged structure mostly depend on the free surface displacement around the structure. The instantaneous surface
displacement in figure 8 shows that when the horizontal force reaches its peak, a wave crest is passing over the toe of caisson (recorded by gauge 4) and a surface elevation below the still water level is at the heel (recorded by gauge 5).


Figure 8. Time evolution of the horizontal wave force (upper panel) and surface displacements at gauge 4 (middle panel) and 5 (lower panel) for the semicircular breakwater (test 1: $d=0.383 m, H_{s}=0.117 m, T_{p}=1.33 s$ ). $\eta_{4}$ and $\eta_{5}$ are the surface displacement recorded by gauge 4 at the toe and gauge 5 at the heel of the caisson. The vertical dash dot line corresponds to the time of the maximum horizontal wave force.

Therefore, the surface displacement at the structure consists of the reflection and transmission components,

$$
\begin{equation*}
\eta_{h \text { max }}=\left|\eta_{r}\right|+\left|\eta_{t}\right| . \tag{3}
\end{equation*}
$$

The amplitude of surface elevation $\left(\eta_{r}\right)$ due to reflection can be expressed as $\left|\eta_{r}\right|=0.5 \lambda_{r}(1+\cos \beta)\left(\alpha_{1}+\alpha_{2} \cos ^{2} \beta\right) H_{i}$ based on the equation (A.2). The correction factor $\lambda_{r}$ varies with the structure type (for upright walls, $\lambda_{r}=1.0$ ). The surface profile $\left(\eta_{t}\right)$ due to transmission is related to the relative submerged depth $\left(\frac{R_{c}}{H_{i}}\right)$ and the wave
breaking parameter $\left(\xi_{\mathrm{p} 0}\right)$ (van der Meer et al., 2005), where $R_{c}$ the freeboard, $\xi_{0 p}=\tan \alpha / \sqrt{2 \pi H_{i} / g T_{p}^{2}}$ the peak Iribarren parameter, $\tan \alpha$ the fictional slope connecting toe and crown ( 0.43 in this study), $T_{p}$ the peak period of incident waves. We modify the expression by van der Meer et al. (2005) for wave transmission at smooth and impermeable low-crested structures as $\left|\eta_{t}\right|=\left\{\lambda_{t 1}\left(\frac{\left|R_{c}\right|}{H_{i}}\right)^{\lambda_{12}}+\lambda_{t 3}\left[1-\exp \left(-0.5 \xi_{\mathrm{p} 0}\right)\right]\right\} H_{i}$ by including three correction coefficients $\lambda_{t 1}, \lambda_{t 2}, \lambda_{t 3}$. Using the surface profile in equation (3), we rewrite the equation (A.2) for the pressure at still wave level as

$$
\begin{align*}
p_{s}^{\prime} & =\gamma_{w} \eta_{h \max } \\
& =\left\{0.5 \lambda_{r}(1+\cos \beta)\left(\alpha_{1}+\alpha_{2} \cos ^{2} \beta\right)+\lambda_{t 1}\left(\frac{\left|R_{c}\right|}{H_{i}}\right)^{\lambda_{12}}+\lambda_{t 3}\left[1-\exp \left(-0.5 \xi_{\mathrm{p} 0}\right)\right]\right\} \gamma_{w} H_{i} \tag{4}
\end{align*}
$$

Correction coefficients in equation (4) have been tentatively estimated with the measurements by gauge 4 and gauge 5 . We obtain $\lambda_{r}=0.52, \lambda_{t 1}=0.06, \lambda_{t 2}=0.52, \lambda_{t 3}$ $=0.09$ for the QCB, and $\lambda_{r}=0.72, \lambda_{t 1}=0.05, \lambda_{t 2}=0.48, \lambda_{t 3}=0.12$ for the SCB.

The pressure at the bottom level of caisson ( $p_{b}^{\prime}$ ) can be calculated by the equation (A.3)

$$
\begin{equation*}
p_{b}^{\prime}=p_{b}=\alpha_{3} p_{s}^{\prime} . \tag{5}
\end{equation*}
$$

The phase modification coefficient $\lambda_{p}^{\prime}$ in the equation (A.3) is neglected here in equation (5) for the following reasons. An assumption for the equation (A.3) is that the horizontal force should reach its peak value when a wave crest is passing over the crown of circular walls. As a result, the pressure at the toe of caisson ( $p_{b}^{\prime}$ ) should be reduced due to a wave phase discrepancy between the toe and crown. However, in this study, we observed that the horizontal force reaches its peak value when a wave crest is passing over the toe instead of the crown of circular wall. For example, the pressure recorded by P1 near the toe (see figure 2) is close to its peak value because it is at the wave crest phase at this point. Meanwhile, the pressure acting on the other part of the front circumference is smaller than its peak value due to its phase lags from P1. Thus, we
obtain the expression for pressures acting on the front circumference by including the phase modification coefficient $\lambda_{p}^{\prime}$

$$
\begin{equation*}
p(\theta)=\left(\alpha_{3}-\alpha_{3} \frac{z}{d_{1}}+\frac{z}{d_{1}}\right) \lambda_{p}^{\prime} p_{s}^{\prime} \cos \theta \tag{6}
\end{equation*}
$$

where $p(\theta)$ pressure after angle correction, $\theta$ the central angle, $d_{1}$ the water depth above wall bottom, $\lambda_{p}^{\prime}$ the phase correction coefficient and $z$ the vertical coordinate with $z=0$ located at the still water level and positive upward. Xie's model adopted $\lambda_{p}{ }^{\prime}=\cos \left[\frac{2 \pi(\Delta l)^{\prime}}{L_{s}}\right]$ based on the linear wave theory by neglecting the wave transformation over submerged structures, where $(\Delta l)^{\prime}$ the horizontal distance between toe and crown, $L_{s}$ the significant wavelength. Assuming $\lambda_{p}^{\prime}=\lambda_{p 1} \cos \left(\lambda_{p 2} \cdot \frac{2 \pi(\Delta l)^{\prime}}{L_{s}}\right)$, we obtain $\lambda_{p 1}=0.76, \lambda_{p 2}=3.0$ for the QCB and $\lambda_{p 1}=0.81, \lambda_{p 2}=3.6$ for the SCB based on the measurements of pressure on the front wall.

The potential flow component of horizontal force due to the free surface displacement is given by

$$
\begin{equation*}
F_{h W}=\int_{\text {frontwall }} p(\theta) \cos \theta d l . \tag{7}
\end{equation*}
$$

The wave force $F_{h w}$ calculated from equation (3) $\sim(7)$ is the potential component of horizontal force rather than the total force $F_{h \text { max }}$. It is in a good agreement with the measurement for the SCB but underestimate that for the QCB by $16 \%$. The reason could be that the measured surface displacement $\eta_{h \max }$ for QCB is $20 \%$ lower than the SCB. However, the measured horizontal forces for both structures are close to each other (see figure 6). This inconsistence implies that the total wave force $F_{h \text { max }}$ on the QCB attributes to not only the potential flow component $F_{h W}$ but also the drag component $F_{h D}$ due to wake effects. The length of structures in this study is smaller than the wave length ( $\frac{R}{L_{s}}$ varies from 0.058 to 0.075 , where $R$ is the radius of the circular wall, $L_{s}$ is the wave
length) and therefore a considerable wake effect is expected. Xie's study (2006) indicated that strong trailing vortices are formed on the leeside of submerged QCBs during the wave-structure interaction. QCBs are less streamlined than SCBs so that larger flow separations therefore drag are expected for QCBs. Thus it is important to include the drag component in the wave load calculation of QCBs and the following equation (8) instead of (7) should be used, even though the Keulegan-Carpenter number ( $K C$ ) in this study varies from 1.59 to 2.40 indicating non-dominant drag component. We rewrite equation (2) to include the wake effects

$$
\begin{equation*}
F_{h \text { max }}=\left(1+\lambda_{D}\right) F_{h W}, \tag{8}
\end{equation*}
$$

where $\lambda_{D}$ represents the fraction of the drag component in the maximum horizontal force, which is related to wave height $\left(H_{i}\right)$, wave period $\left(T_{s}\right)$, water depth $(d)$ and structure geometry (e.g. shape and size). According to Qiu (1986), $\lambda_{D} \propto(K C) \cdot \frac{2 k d+\sinh 2 k d}{8 \sinh (2 k d) \tanh (k d)}$. Note that the flow velocity $U$ is proportional to $\left(\frac{H_{i}}{T}\right)$ and $K C$ is proportional to $\left(\frac{H_{i}}{R}\right)$. Furthermore, if we assume $d \cong R_{c}+H_{i}$ when a wave crest is passing over the structure ( $R_{c}$ is the crest freeboard height), we have $\frac{2 k d+\sinh 2 k d}{8 \sinh (2 k d) \tanh (k d)} \cong \frac{1}{8 \pi} \frac{L_{s}}{d}$. As a result, we can give the expression of $\lambda_{D}$ as

$$
\begin{equation*}
\lambda_{D}=a \cdot\left(\frac{H_{i}}{R} \frac{L_{s}}{d}\right)+b \tag{9}
\end{equation*}
$$

By analyzing the measurements, we obtain $\mathrm{a}=-0.27$ and $\mathrm{b}=1.60$ for the QCB with a correlation coefficient close to 0.92 (see figure 9). The contribution of the drag component to the maximum horizontal force for the more streamlined SCB is negligible, i.e. $\lambda_{D}=0$ for the SCB .


Figure 9. $\lambda_{D}$ versus $\left(H_{i} L_{s} / R d\right)$ for the quarter-circular breakwater.

We calculate the maximum horizontal forces ( $F_{h \max }$ ) using equations (2) to (9) and compare with the measurements (see figure 10). The ratio of predictions to measurements has a mean value of 1.0 and standard deviations of 0.09 for the QCB and 0.13 for the SCB. Comparison of figure 10 with figure 6 suggests that the present model capture the observation better than Xie's (1999) model. In addition, the uncertainties in wave loading prediction (e.g. bias and spread) display a nearly normal stochastics in the present model and hence it is more suitable for a probabilistic design [Burcharth, 1999].


Figure 10. Comparison of the predicted maximum horizontal forces by the present wave load model equations (2)-(9) and measurements. The diagonal line denotes the prediction equal to the measurement.

## 6. Effect of wave transmission on dynamic pressure

As discussed previously, wave transmission affects the hydrodynamic loads on submerged SCBs and QCBs. In this section, we will explore the mechanism behind this based on the measured pressure distributions and the numerical results of vortex dynamics at the structure.

### 6.1. Instantaneous pressure distribution

The instantaneous pressure distribution along caisson at the time of the maximum horizontal force is shown in figure 11. The pressure at each pressure transducer for each run are first normalized using the toe pressure ( $p_{b}{ }^{\prime}$ ), and then averaged over all runs. At the front wall, the pressure decreases from toe to crown due to phase shift and then becomes negative at the leeside corner. In general, the QCB has a slightly lower pressure over the front wall than the SCB. On the rear wall, the pressure has an approximately uniform distribution over the QCB while it increases from the crown to the mid-point and then decreases towards the bottom for the SCB. The magnitude of the leeside pressure on the QCB is smaller than that on the SCB and the difference becomes smaller towards the bottom. The negative pressure behind the crest of the breakwater in figure 11 is mainly due to the negative surface elevation displacement as indicated in figure 8 . The wake effect may further modify the negative pressures on rear wall. The uplift pressure on the base slab increases linearly from the heel to the toe for both structures. The negative pressure at the heel can therefore be obtained through a linear extrapolation. In general, the heel pressure of emerged breakwaters is assumed to be zero in the design of emerged breakwaters. Therefore, the submerged circular-front breakwaters may have a lower uplift force than emerged vertical breakwaters. Goda [2010] also came to the same conclusion that a very small freeboard will result in a reduced uplift pressure due to wave overtopping.


Figure 11. Instantaneous pressure distribution at the time of the peak horizontal force: (a) for the quarter-circular breakwater; (b) for the semicircular breakwater. The dotted line indicates a linear extrapolation from the measurements.

### 6.2. Effect of flow disturbance

The difference between wave loads on submerged SCBs and QCBs mainly comes from the difference in the pressure on the leeside wall. Compared with semicircular and vertical breakwaters, the quarter-circular breakwater has asymmetry geometry. Hur et al. [2004] concluded that a complicated vortex generated by structural asymmetry greatly affects the time variation of the transverse wave force exerted on an asymmetric structure.

A numerical wave flume is developed in this section to examine the effect of vortex dynamics on pressure. This numerical flume is based on a free surface flow model developed by Lin \& Liu [1998]. In this model, the motion of an incompressible fluid is described by the 2D Reynolds-Averaged-Navier-Stokes equations (RANS) along with a nonlinear $k-\varepsilon$ turbulent model (see Appendix B.). The spatially averaged Navier-Stokes equations are used to describe the mean motion of flow in porous media by including additional frictional forces [Lin and Karunarathna, 2007]. Finite difference method is used for discretization, in which the advection terms are discretized with a mixed upwind and central difference scheme while the dissipation terms are discretized with a central difference scheme. A two-step projection algorithm is adopted to obtain the flow fields, pressures, and turbulence parameters. The volume of fluid function (VOF) by Hirt and Nichols [1981] is used to track the free surface displacement. This type of approach has been used extensively to study surf zone problems such as wave breaking
(Lubin et al 2003; Zhao et al 2004; Christensen, E.D., 2006; Wang et al 2009; Zhang et al 2009, Bakhtyar et al 2010 and 2012, Xie 2012).

This model has been applied widely to wave-structure interaction problems. Examples include the wave interaction with composite porous structures [Liu et al., 1999], with a rectangular obstacles [Lin, 2004], wave overtopping over a smooth impermeable seawall [Reeve et al., 2008], over rubble mound breakwaters [Losada et al., 2008], and over a seaward sloping dike [Peng and Zou, 2011], wave transformation over circular breakwaters [Jiang et al., 2008], over vertical breakwaters [Jiang et al., 2010], and over a low-crested structure [Zou and Peng, 2011].

In this study, the numerical flume is set up following the experiment configuration in figure 2. The combination of second-order polynomial function and sinusoidal function is employed to construct the structure boundary in the model. The partial cell technique is used to simplify the treatment of internal obstacles and solid boundaries [Lin, 2007 2008]. In this method, an openness function $(\beta)$ is introduced to represent the ratio of the fluid-occupied volume to total volume of a computation cell. As a result, the whole computational domain is divided into fluid cells ( $\beta=1$ ), solid cells ( $\beta=0$ ) and partial cells $(0<\beta<1)$. The mean value of a variable for a particular cell in the model is defined as the product of the value of openness function and the variable in that cell. Since the grid resolution used in the model is not fine enough to resolve the viscous boundary layer at the solid boundary, free slip boundary conditions are applied at the solid boundary to avoid underestimating flow velocity there. The pressure variation across the thin viscous boundary layer adjacent to a solid boundary is negligible [Zou, 1998, 2002] and the pressure boundary condition $\partial p / \partial n=0$ is often imposed on the solid boundary in the immersed boundary method [Zhang et al., 2010]. Thus the free slip boundary condition has little effect on the model results of wave loading on the breakwater while avoiding using excessive CPU time to resolve the very thin viscous boundary layers. A combination of zero-pressure and zero normal gradient of mean tangential velocity is imposed on the free surface. The turbulent kinetic energy $(k)$ and the dissipation rate ( $\varepsilon$ ) are assumed to be a function of frictional velocity at grid points immediately adjacent to the solid boundaries and have a zero normal gradient on the free surface. An open boundary condition is applied at the inlet and outlet boundaries. An internal wave-maker
developed by Lin and Liu (1999) is used to generate irregular waves in the numerical flume. The inverse Fourier transformation is first used to decompose a specific JONSWAP spectrum into wave components. Then, an irregular wave train is reconstructed by adding a mass source function in the equation of mass conservation in a source region. The mass source function is the summation of the different wave components. The source region is a rectangle with the size of 0.05 m (width) $\times 0.14 \mathrm{~m}$ (height). The source region is located 1.5 m away from the inlet boundary and is 0.22 m below the still water level. A non-uniform mesh was used in this study with the cell size from 0.001 m near the solid boundaries to 0.01 m away from the structures in both x - and z -directions.

As shown in figure 12 (a) and (b), the predicted and measured significant wave height agree well with each other. The significant wave heights display a mild increase between the first and fourth gauge stations. Then, the significant wave height is reduced considerably between the fourth and fifth gauge stations due to wave breaking over the submerged structure. In figure 12 (c), the good agreement between the predicted and measured wave spectra demonstrates that the incident wave spectrum is reproduced well in the numerical flume. In figure 12 (d), the predicted and measured maximum horizontal forces are normalized by $\gamma_{w} H_{i} h_{c}$, where $\gamma_{w}$ is the specific weight of water, $H_{i}$ the incident wave height, and $h_{c}$ the caisson height. The mean ratio of prediction to measurement is 1.01 for the QCB and 1.04 for the SCB . The standard deviation of the ratio is 0.17 for the QCB and 0.07 for the SCB. The deviation between the predicted and observed wave loads in figure 12 (d) is partly due to the deviation between predicted and observed incident wave spectra in figure 12 (c). The larger scatter in the case of QCB may be caused by stronger flow separation and vortices behind the structure.



Figure 12. Comparison of the predicted and measured significant wave heights at six gauge stations indicated in figure 2 for test 6 (a) the quarter-circular breakwater, (b) the semicircular breakwater; (c) incident wave spectrum for test 1; (d) dimensionless maximum horizontal forces for the QCB and the SCB.

Regular waves are used next in the simulation to explore the relationship between the wave load and vorticity. Figure 13 (a) and (b) demonstrate that both QCB and SCB have similar flow pattern before the crown of the caisson. After the crown, the flow is smooth for the SCB but with an obvious trailing vortex present for the QCB. Figure 13 (c) and (d) show that both structures have an identical interaction between the vorticity and the wave force at point 1 but a great difference at point 2. At point 2, the submerged QCB displays a greater scatter than the submerged SCB due to larger variation in the vorticity. At point 3, the magnitude of the vorticity for both structures is small. As indicated in Chang et al. (2005), the rotational flow is confined to the region about 2 times wave particle excursion from a submerged obstacle and the center of the downstream vortices is located at about half of wave particle excursion away from the leeside corner. In our study, the wave particle excursion is between 4.7 cm and 7.0 cm , and the distance between point 3 and the leeside corner is 13.5 cm that reaches the edge of the rotational flow. The QCB has a greater vorticity than the SCB at point 3 that attributes to stronger vortex diffusion. Figure 13 (e) and (f) illustrate the variation of pressure with vorticity at point 2 . The pressure has a similar fluctuation period to the vorticity but a phase lag to the latter. It also reveals the structural asymmetry at the QCB generates a larger variation in the
vorticity, which considerably affects the magnitude and phase of the pressure on the structure.



Figure 13. Variations of velocity, force and pressure due to regular waves ( $d=0.383$ $m, H=0.117 m, T=1.24 s$ ): (a) and (b) flow fields, vorticity (color bar) and surface elevation; (c) and (d) total horizontal force as a function of vorticity at point 1, 2 and 3 indicated in (a) and (b); (e) and (f) time series of surface elevation, pressure and vorticity at point 2 near the crest of the caisson.

## 7. Conclusions

In this study, the characteristics of the wave loads acting on submerged semi--and quarter-circular breakwaters under irregular waves have been investigated. The main conclusions can be summarized as follows.

Submerged semi- and quarter-circular breakwaters have a similar anti-sliding response and wave force spectra. Comparison with previous observations indicates that the normalized wave force on submerged circular-front breakwaters under irregular waves is less than that under regular waves.

Xie's model gives a conservative estimation of the wave loads on submerged circular-front breakwaters. We improve the model by including the effect of wave transmission on the pressures on submerged circular structures.

The instantaneous pressures at the time of the maximum horizontal force for both semi- and quarter-circular breakwaters exhibit the same behaviors on the front wall but great differences on the rear wall due to distinct wake behaviors behind structures. RANS-VOF modeling results show that wave-induced vortices have a considerable effect on the wave loads of the quarter-circular breakwater but not the semicircular breakwater.

The wave load model was further improved by incorporating the effect of flow separation and vorticity due to wave transmission.

According to Sasajima et al. [1994], the semicircular breakwaters are categorized as impermeable type, front dissipating type (seaside wall perforated), rear dissipating type (leeside wall perforated) and fully dissipating type (both seaside and leeside walls perforated). As reviewed in Dhinakaran et al. [2012], a submerged perforated quartercircular and semicircular structure with a suitable permeability is subject to a smaller wave load than an impermeable breakwater with the same geometry. The reduction in the wave loading is due to wave energy loss to generate the turbulent flow within the hollow chamber. The amount of reduction is worth further investigation. However, the present study of impermeable breakwater provides a benchmark for both types of breakwater.

The behaviors of vortices are controlled by many factors, for example, the geometry and permeability of a structure, the ratio of structure dimension over wavelength, the relative freeboard height, wave height, and wave period. Further study is required to understand the relationship between vorticity and pressure for submerged structures, which will lead to great improvement of the formulas calculating wave loads on submerged obstacles.

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Appendix A: The wave load formula for submerged semicircular breakwaters by Xie (1999).

(a)
(b)
(c)


Figure A-1. Sketch of wave load model for submerged semicircular breakwater: (a) pressures on a fictional upright section; (b) pressures after phase correction; (c) pressures after center angle correction.

$$
\alpha_{1}=0.6+0.5\left[\frac{4 \pi d / L}{\sinh (4 \pi d / L)}\right]^{2}, \quad \alpha_{2}=\min \left\{\frac{h_{b}-d_{2}}{3 h_{b}}\left(\frac{H}{d_{2}}\right)^{2}, \frac{2 d_{2}}{H}\right\}, \quad \alpha_{3}=1-\frac{d_{1}}{d}\left[1-\frac{1}{\cosh (2 \pi d / L)}\right]
$$

Where

$$
\begin{array}{ll}
\eta \text { and } \eta^{\prime} \quad \text { elevation of zero dynamic pressure point above the still water level } \\
(\mathrm{m}) .
\end{array}
$$

$\beta \quad$ angle between the wave direction and the normal vector of structure

$$
\begin{align*}
& \eta^{\prime}=\eta=0.75(1+\cos \beta) H_{i}  \tag{A.1}\\
& p_{s}^{\prime}=p_{s}=0.5(1+\cos \beta)\left(\alpha_{1}+\alpha_{2} \cos ^{2} \beta\right) \gamma_{w} H_{i}  \tag{A.2}\\
& p_{b}{ }^{\prime}=\lambda_{p}{ }^{\prime} p_{b}=\lambda_{p}{ }^{\prime} \alpha_{3} p_{s} ; \lambda_{p}{ }^{\prime}=\cos \left[\frac{2 \pi(\Delta l)^{\prime}}{L}\right]  \tag{A.3}\\
& p^{\prime}(z)=p_{b}{ }^{\prime}+\left(p_{s}{ }^{\prime}-p_{b}{ }^{\prime}\right) \cdot z / d_{1}  \tag{A.4}\\
& p^{\prime}(\theta)=p^{\prime}(z) \cos (\theta)  \tag{A.5}\\
& p_{u}{ }^{\prime}=\left\{\begin{array}{lll}
p_{b}{ }^{\prime}, & \text { for } & \varepsilon_{b}<0.1 \\
0, & \text { for } & \varepsilon_{b} \geq 0.1
\end{array}\right.  \tag{A.6}\\
& p_{0}{ }^{\prime}= \begin{cases}0, \text { for } & \varepsilon_{b}<0.1 \\
p_{b}{ }^{\prime} / 2, & \text { for } \quad \varepsilon_{b} \geq 0.1\end{cases} \tag{A.7}
\end{align*}
$$

surface $\left({ }^{\circ}\right)$.

| $H_{i}$ | incident wave height ( $\mathrm{m}, H_{1 \%}$ for irregular waves). |
| :--- | :--- |
| $L$ | incident wavelength (m, the significant wavelength $L_{s}$ for irregular |
| waves). |  |
| $d_{1}$ | water depth in front of breakwater (m). |
| $d_{2}$ | water depth above wall bottom (m). |
| $h_{b}$ | water depth above armor layer (m). |
|  | water depth at the location at a distance $5 H_{1 / 3}$ seaward of the |
| breakwater $(\mathrm{m})$. |  |

$\gamma_{w} \quad$ water density $\left(\mathrm{kN} / \mathrm{m}^{3}\right)$.
$p_{s}^{\prime}$ and $p_{s} \quad$ pressure at the still wave level (kpa).
$p_{b}$ and $p_{b} \quad$ pressure at the bottom level of caisson (kpa).
$p^{\prime}(z)$
pressure at the water depth of $d_{1-z}(\mathrm{kpa})$.
pressure after angle correction to $p^{\prime}(z)(\mathrm{kpa}) ; \theta$ is the central angle at the acting point $\left({ }^{\circ}\right)$.
$p_{u}{ }^{\prime} \quad$ wave uplift pressure at the toe of caisson (kpa).
$p_{0}{ }^{\prime}$
$\varepsilon_{b}$
$(\Delta l)^{\prime}$
$\lambda_{p}{ }^{\prime}$
$\lambda_{p}{ }^{\prime} \quad$ phase correction coefficient (m).
coefficient representing the variation of pressure with the wave period.
coefficient representing the variation of pressure with the height of
$\alpha_{2}$
$\alpha_{3} \quad$ coefficient representing a linear relationship between $p_{s}^{\prime}$ and $p_{d}^{\prime}$.
For $R / L \leq 0.085$ only, where $R$ is the radius of circular wall.

Appendix B: Reynolds Averaged Navier-Stokes equations with nonlinear turbulent kinetic-dissipation equations.

$$
\begin{array}{ll}
\text { Continuity } & \frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{w}}{\partial z}=0 \\
\text { equation } & \frac{\partial \bar{u}}{\partial t}+\frac{-\bar{\partial} \bar{u}}{\partial x}+\bar{w} \frac{\partial \bar{u}}{\partial z}=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}+\frac{\partial}{\partial x}\left(v(v \bar{u})+\frac{\partial}{\partial x}\right)\left(v\left(v \overline{\partial \bar{u}} \frac{\partial z}{\partial z}\right)-\frac{\partial}{\partial x}\left\langle u^{\prime} u^{\prime}\right\rangle-\frac{\partial}{\partial z}\left\langle u^{\prime} w^{\prime}\right\rangle\right. \\
\text { Momentum } & \frac{\partial \bar{w}}{\partial t}+\frac{-\bar{w}}{\partial x}+\bar{w} \frac{\partial \bar{w}}{\partial z}=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z}+\frac{\partial}{\partial x}\left(v \frac{\partial \bar{w}}{\partial x}\right)+\frac{\partial}{\partial z}\left(v \frac{\partial \bar{w}}{\partial z}\right)-\frac{\partial}{\partial x}\left\langle u^{\prime} w^{\prime}\right\rangle-\frac{\partial}{\partial z}\left\langle w^{\prime} w^{\prime}\right\rangle-g  \tag{B.3}\\
\text { equations }
\end{array}
$$

Turbulent $\quad \frac{\partial k}{\partial t}+u \frac{\partial k}{\partial x}+\bar{w} \frac{\partial k}{\partial z}$
kinetic energy
equation $=\frac{\partial}{\partial x}\left[\left(v+\frac{v_{t}}{\sigma_{k}}\right) \frac{\partial k}{\partial x}\right]+\frac{\partial}{\partial z}\left[\left(v+\frac{v_{t}}{\sigma_{k}}\right) \frac{\partial k}{\partial z}\right]-\left\langle u^{\prime} u^{\prime}\right\rangle \frac{\partial \bar{u}}{\partial x}-\left\langle u^{\prime} w^{\prime}\right\rangle \frac{\bar{u}}{\partial z}-\left\langle u^{\prime} w^{\prime}\right\rangle \frac{\partial \bar{w}}{\partial x}-\left\langle w^{\prime} w^{\prime}\right\rangle \frac{\partial \bar{w}}{\partial z}-\varepsilon$

where

| $x$ and $z$ | Cartesian coordinates $(\mathrm{m})$ in horizontal and vertical direction. |
| :--- | :--- |
| $t$ | Time (s). |
| $\bar{u}$ and $\bar{w}$ | mean velocity components in x- and z-directions $(\mathrm{m} / \mathrm{s})$. |
| $\bar{P}$ | mean pressure $(\mathrm{kPa})$. |
| $\rho$ | fluid density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$. |
| $g$ | gravitational acceleration $\left(\mathrm{m} / \mathrm{s}^{-2}\right)$. |
| $v_{t}$ | eddy viscosity, $v_{t}=C_{d} \frac{k^{2}}{\varepsilon}$. |
| $v$ | kinematic viscosity, $v=1.3 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}$. |
| $\left\langle u^{\prime} u^{\prime}\right\rangle,\left\langle u^{\prime} w^{\prime}\right\rangle,\left\langle w^{\prime} w^{\prime}\right\rangle$ | Reynolds stresses. |
| $k$ | turbulent kinetic energy $(\mathrm{kJ})$. |
| $\varepsilon$ | energy dissipation rate. |
| Constants | $\sigma_{k}=1.0, \sigma_{\varepsilon}=1.3, C_{d}=0.09, C_{\varepsilon 1}=1.44, C_{\varepsilon 2}=1.92$. |

$x$ and $z \quad$ Cartesian coordinates (m) in horizontal and vertical direction.
$\bar{u}$ and $\bar{w} \quad$ mean velocity components in x - and z -directions $(\mathrm{m} / \mathrm{s})$.
mean pressure ( kPa ).
fluid density ( $\mathrm{kg} / \mathrm{m}^{3}$ ).
gravitational acceleration $\left(\mathrm{m} / \mathrm{s}^{-2}\right)$.
eddy viscosity, $v_{t}=C_{d} \frac{k^{2}}{\varepsilon}$.
kinematic viscosity, $v=1.3 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}$.
$\sigma_{k}=1.0, \quad \sigma_{\varepsilon}=1.3, \quad C_{d}=0.09, \quad C_{\varepsilon 1}=1.44, C_{\varepsilon 2}=1.92$.

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[^0]:    *Corresponding author.
    E-mail address: qingping.zou@maine.edu (Q.-P. Zou).

