A methodology to assess the economic impact of power storage technologies.

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Abstract

We present a methodology for assessing the economic impact of power storage technologies. The methodology is founded on classical approaches to the optimal stopping of stochastic processes but involves an innovation that circumvents the need to, *ex ante*, identify the form of a driving process and works directly on observed data, avoiding model risks. Power storage is regarded as a complement to the intermittent output of renewable energy generators and is therefore important in contributing to the reduction of carbon intensive power generation. Our aim is to present a methodology suitable for use by policy makers that is simple to maintain, adaptable to different technologies and is easy to interpret. The methodology has benefits over current techniques and is able to value, by identifying a viable optimal operational strategy, a conceived storage facility based on compressed air technology operating in the UK.

1 Introduction

Many renewable power sources (wind, solar, tidal) are naturally controlled, sometimes described as being intermittent or variable, and it is often difficult to predict their output. This means that in areas where there is a high

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proportion of generation capacity provided by renewable sources there are new problems for those tasked with balancing supply and demand. A solution, suggested in ([1], [2]), is to use power storage as a 'liquidity reserve' that holds excess power generated at one time that can then be released at a subsequent time of power scarcity. This immediately raises the question of how to optimally control a power storage facility so as to maximise its value and suggests the problem is one of optimal stopping.

It is profitable for technology operators to invest in developing detailed, complex, representational models of the specific plant they manage to address the specific issue of optimally managing a single implementation of a power storage technology. Policy makers and regulators[1], on the other hand, must address the broader question of what portfolio of generation/storage technologies will deliver a low cost, resilient power system that meets policy objectives, such as the reduction of carbon emissions. The choices facing policy makers, as distinct from technology operators, are characterised by: long time horizons with radical uncertainty, meaning that statistical inference is often inappropriate; political considerations, so that some factors are impossible to quantify; conflicting objectives with a strong normative dimension (profit/safety); and options involving high upfront investments and so require robust justification. In general, the future state of the world is ambiguous for policy makers and so models need to be signifiers of the policy options, rather than accurate representations of the system [3, 4].

Energy policy can be delivered via market mechanisms such as feed in tariffs and capacity payments and these will need to be tested by multiple runs on different model scenarios. The consequence is that policy makers need to employ models that are adaptable to different realisations of what might occur, are relatively easy to maintain and are straightforward to interpret at the expense of being accurate representations. The methodology developed here is aimed specifically at supporting these types of decisions and goes some way to addressing a requirement for the development of models to assist policy-makers [5, 5.2.2].

The methodology presented identifies the residual power load (demand net of renewable generation) at which an optimally operated storage facility would be charged (filled) and discharged (emptied), indicating how the tech-

[1]Here we use the term 'regulator' to cover all parties that can influence the behaviour of the power market. In the UK this would include the regulator (Ofgem and GEMA). Policy makers are the parts of government that set the aims of the regulators, in the UK this is the Department of Energy and Climate Control (DECC).

nology would impact the existing power system. In doing this it values an implementation of the technology on the implicit assumption that a storage facility has a negligible impact on the overall system. The methodology is novel in that it offers a solution to the optimal stopping problem rooted in the classical theory of optimal stopping but does not require the *ex ante* identification of the Itō process driving the system. Rather the analysis is based directly on the observed data. This reduces the problem of 'model risk' resulting from basing decisions on a theoretical, canonical processes that have no relation to the phenomenon of interest. While developed in relation to the power markets, this innovative methodology has the potential to be applied to a range of real-world problems.

Despite a proliferation in papers describing representational models to address the optimal operation of power storage facilities in the engineering literature, summaries can be found in [5] or [6], there has been relatively little interest in the contemporary economics literature. There is a larger body of economic and financial research investigating gas storage, such as [7], [8], [9], [10], [11] and [12]. An explanation for the imbalance in research into gas storage and electricity storage could lie in the fact that electricity prices, unlike gas prices, are typically regarded as exhibiting jumps [13] and the mathematical theory of the control of jump diffusions is still immature. Jumps in the electricity price can be explained as occurring when the load crosses one of the discontinuities in the, so called, merit order curve. The 'merit order curve' (MOC), or 'stack', is a mapping of load to price. It is constructed by ranking different power generators based on the ascending order of the price the generators bid to deliver a specific quantity of power; it is a monotone increasing non-continuous ('staircase') function (see Figure 1).

The relationship between the actual price paid for power and the merit order curve is not straightforward, however it provides a useful mechanism for converting load, which can be modelled as a continuous process, into an estimate of the average price paid to power suppliers for a given power load; this is an approach taken in [14]. The pay-off of charging/discharing a storage facility will be based on the time integral of the product of the charge/discharge rate and the instantaneous price of power until it is full/empty. This will not deliver smooth (C^2) pay-off functions because of the discontinuities in the MOC. However, the theory developed [15] and [16] and applied in [17] is able to accommodate non-smooth pay-offs and this theory was employed in [18] where the pay-offs were defined by passing an Itō diffusion through the MOC. This approach did not deliver useful results because the load data could not be modelled using a canonical Itō diffusion. This failure prompted the innovative approach taken in this paper: rather than identifying a canonical process representing the demand, and then calibrating the data, solve the problem directly from the data. This empirical approach involved calculating the pay-offs based on data and then fitting these observations to a polynomial, this delivered smooth pay-offs and so the approach taken in [15, 16, 17] was not needed but these papers provided the theoretical basis of the innovative empirical approach used here.

2 Theoretical basis

We assume that the demand is driven by a one-dimensional, time homogeneous Itō diffusion given by the stochastic differential equation

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dW_t, \quad X_0 = x \in \mathcal{I}. \tag{1}$$

Here, $\mathcal{I}$ is a given open interval with left endpoint $-\infty \leq \alpha$ and right endpoint $\beta \leq \infty$. The functions $b, \sigma : \mathcal{I} \to \mathbb{R}$ satisfy Assumptions 1–3 in [15] such that there is a weak solution to (1), the diffusion is non-explosive and that the boundaries α and β are inaccessible. Since we are concerned with an economic question, all future pay-offs are discounted by a state-dependent factor defined by

$$\Lambda_t = \int_0^t r(X_s)\,ds.$$

We require that $r : \mathcal{I} \to]0, \infty[$ is $\mathcal{B}(\mathcal{I})$-measurable, it is locally bounded and there exists $r_0 > 0$ such that $r(x) \geq r_0$, for all $x \in \mathcal{I}$.

On this basis, denote by τ_y the first hitting time of the point $y \in \mathcal{I}$ by the process, X_t

$$\tau_y := \inf\{t \geq 0 \,|\, X_t = y\}.$$

With regard to [15, Eqns (6)–(10)] and [19, 11.10], given any points $y < z$ in $\mathcal{I}$ we can define the function ψ by

$$\psi(y) = \psi(z)\mathbb{E}_y\left[e^{-\Lambda_{\tau_z}}\right], \tag{2}$$

and the function, ϕ, by

$$\phi(z) = \phi(y)\mathbb{E}_z\left[e^{-\Lambda_{\tau_y}}\right]. \tag{3}$$

These functions have absolutely continuous first derivatives, are unique, modulo multiplicative constants,

$$0 < \phi(x) \quad \text{and} \quad \phi'(x) < 0, \quad \text{for all } x \in \mathcal{I}, \tag{4}$$

$$0 < \psi(x) \quad \text{and} \quad \psi'(x) > 0, \quad \text{for all } x \in \mathcal{I}, \tag{5}$$

and

$$\lim_{x \downarrow \alpha} \phi(x) = \lim_{x \uparrow \beta} \psi(x) = \infty. \tag{6}$$

The stochastic system that we consider is switched between two modes, 'charged' (full) or 'discharged' (empty). As in [15] we use a controlled finite variation process Z that takes values in $\{0, 1\}$ to keep track of the system's operating mode over time. If $Z_t = 1$ (resp., $Z_t = 0$), then the system is in its charged (resp., discharged) operating mode at time t. The jumps of Z occur at the sequence of stopping-times (T_n), with respect to the weak solution of (1), when system is switched between its two operating modes. Assuming that the system is initially in operating mode $z \in \{0, 1\}$, the decision maker's objective is to select a switching strategy, $\mathbb{Z}_{z,x}$, that maximises the performance criterion (compare with [15, Eqn (5)])

$$J(\mathbb{Z}_{z,x}) := \lim_{n \to \infty} \mathbb{E}_x \left[\sum_{j=1}^{n-1} e^{-\Lambda_{T_j}} \left[E(X_{T_j}) \mathbf{1}_{\{\Delta Z_{T_j} = -1\}} - F(X_{T_j}) \mathbf{1}_{\{\Delta Z_{T_j} = 1\}} \right] \mathbf{1}_{\{T_j < \infty\}} \right], \tag{7}$$

where E represents income gained by discharging the facility and F the cost of charging the facility.

To ensure that our optimisation problem is well-posed (see [15, Lemma 1]), we assume

$$E(x) - F(x) < 0, \quad \text{for all } x \in \mathcal{I},$$

meaning that the 'round trip' efficiency of the storage technology is less than 100%. In the problem considered here, the MOC is bounded and the functions E and F are smooth and so the requirements of Assumption 4 in [15] are satisfied. This means that we have the identity [15, Eqn (19)] for F, with corresponding expression for E,

$$F(x) = -\left(\phi(x) \int_\alpha^x \frac{2\,\psi(s)}{\sigma^2(s)\mathcal{W}(s)} \mathcal{L}F(s)ds + \psi(x) \int_x^\beta \frac{2\,\phi(s)}{\sigma^2(s)\mathcal{W}(s)} \mathcal{L}F(s)ds \right). \tag{8}$$

Here $\mathcal{W}$ is defined as

$$\mathcal{W}(x) := \phi(x)\psi'(x) - \phi'(x)\psi(x) > 0, \quad \text{for all } x \in \mathcal{I}.$$

and the operator $\mathcal{L}$ is defined by

$$\mathcal{L}f(x) := \frac{1}{2}\sigma^2(x)f_{xx}(x) + b(x)f_x(x) - r(x)f(x). \tag{9}$$

This operator is determined by b and σ, which are not specified unless (1) is defined. However, (8) yields

$$F'(x)\phi(x) - F(x)\phi'(x) = -\mathcal{W}(x)\int_x^\beta \frac{2\,\phi(s)}{\sigma^2(s)\mathcal{W}(s)}\,\mathcal{L}F(s)ds,$$
$$F'(x)\psi(x) - F(x)\psi'(x) = \mathcal{W}(x)\int_\alpha^x \frac{2\,\psi(x)}{\sigma^2(x)\mathcal{W}(x)}\,\mathcal{L}F(s)ds,$$

implying

$$-\left(\frac{F(x)}{\phi(x)}\right)'\frac{\phi^2(x)}{\mathcal{W}(x)} = \int_x^\beta \frac{2\,\phi(s)}{\sigma^2(s)\mathcal{W}(s)}\,\mathcal{L}F(s)ds, \tag{10}$$
$$\left(\frac{F(x)}{\psi(x)}\right)'\frac{\psi^2(x)}{\mathcal{W}(x)} = \int_\alpha^x \frac{2\,\psi(x)}{\sigma^2(x)\mathcal{W}(x)}\,\mathcal{L}F(s)ds. \tag{11}$$

revealing that the integrals in (8) are related to the slopes F/ϕ and F/ψ and the sign of $\mathcal{L}F$ is given by the signs of the derivatives of (10)–(11). In particular, we have the result that $\mathcal{L}\psi = \mathcal{L}\phi = 0$.

We have the result, [15, Defn 3, Thrm 4], that a function $v : \{0,1\}\times\mathcal{I} \mapsto \mathbb{R}$ defines the optimal strategy in relation to the performance criterion, (7), if

$$-\mathcal{L}v \text{ is positive on } \mathcal{I}, \tag{12}$$
$$v(1-z,x) - v(z,x) + zE(x) - (1-z)F(x) \le 0, \quad \text{for all } x \in \mathcal{I}, z \in \{0,1\}, \tag{13}$$

and

$$\mathcal{L}v(0,\cdot)(\mathcal{C}_\mathrm{c}) = \mathcal{L}v(1,\cdot)(\mathcal{C}_\mathrm{d}) = 0. \tag{14}$$

Here, $\mathcal{C}_\mathrm{c}$ and $\mathcal{C}_\mathrm{d}$ are the open sets defined by

$$\mathcal{C}_\mathrm{c} = \big\{x \in \mathcal{I} \mid \ v(0,x) > v(1,x) - F(x)\big\}, \tag{15}$$
$$\mathcal{C}_\mathrm{d} = \big\{x \in \mathcal{I} \mid \ v(1,x) > v(0,x) + E(x)\big\}. \tag{16}$$

The objective of the storage problem is to identify the load, a, below which we should charge the system by buying energy and the load, b, at which we discharge system, earning income that maximises the expected discounted value of the storage facility. This is analogous to the 'Switching Case' in [15] suggesting that the value function has the form given by the expressions

$$v(0,x) = \begin{cases} B\psi(x) - F(x), & \text{if } x \in \,]\alpha, a], \\ A\phi(x), & \text{if } x \in \,]a, \beta[= \mathcal{C}_{\mathrm{c}}, \end{cases} \tag{17}$$

$$v(1,x) = \begin{cases} B\psi(x), & \text{if } x \in \,]\alpha, b[= \mathcal{C}_{\mathrm{d}}, \\ A\phi(x) + E(x), & \text{if } x \in [b, \beta[, \end{cases} \tag{18}$$

for some constants A, B and free-boundary points a, b such that $\alpha < a < b < \beta$.

With reference to [15, 115–135], (17)–(18) and (10)–(11) imply that at a and b the equality

$$q_\psi(a,b) = 0 = q_\phi(a,b) \tag{19}$$

should hold, where

$$q_\psi(u,v) = \left[\frac{d}{dx}\left(\frac{E(x)}{\psi(x)}\right)\bigg|_{x=v} \frac{\psi^2(v)}{\mathcal{W}(v)}\right] - \left[\frac{d}{dx}\left(\frac{F(x)}{\psi(x)}\right)\bigg|_{x=u} \frac{\psi^2(u)}{\mathcal{W}(u)}\right], \tag{20}$$

$$q_\phi(u,v) = \left[\frac{d}{dx}\left(\frac{E(x)}{\phi(x)}\right)\bigg|_{x=v} \frac{\phi^2(v)}{\mathcal{W}(v)}\right] - \left[\frac{d}{dx}\left(\frac{F(x)}{\phi(x)}\right)\bigg|_{x=u} \frac{\phi^2(u)}{\mathcal{W}(u)}\right]. \tag{21}$$

In addition, (17)–(18) must satisfy (12), which precludes the optimal strategy being identified solely by solving (19). Checking (12) can be done by using the derivatives of (10)–(11). This means that the set of possible charging points can be defined by

$$\mathcal{D}_{\mathrm{c}} := \left\{ x \in]\alpha, \beta[\;\middle|\; \operatorname{sgn}\left(\left[\left(\frac{F(x)}{\psi(x)}\right)' \frac{\psi^2(x)}{\mathcal{W}(x)}\right]'\right) = \operatorname{sgn}\left(\left[\left(\frac{F(x)}{\phi(x)}\right)' \frac{\phi^2(x)}{\mathcal{W}(x)}\right]'\right) \geq 0 \right\} \tag{22}$$

and possible discharging points by

$$\mathcal{D}_{\mathrm{d}} := \left\{ x \in]\alpha, \beta[\;\middle|\; \operatorname{sgn}\left(\left[\left(\frac{E(x)}{\psi(x)}\right)' \frac{\psi^2(x)}{\mathcal{W}(x)}\right]'\right) = \operatorname{sgn}\left(\left[\left(\frac{E(x)}{\phi(x)}\right)' \frac{\phi^2(x)}{\mathcal{W}(x)}\right]'\right) \leq 0 \right\}. \tag{23}$$

Having identified the sets (22)– (23) a solution to (19) is sought by defining the functions,

$$l_A : \mathcal{D}_\mathrm{c} \mapsto \mathcal{D}_\mathrm{d} \quad \text{and} \quad l_B : \mathcal{D}_\mathrm{c} \mapsto \mathcal{D}_\mathrm{d} \tag{24}$$

such that

$$q_\psi(u, l_A(u)) = 0 \quad \text{and} \quad q_\phi(u, l_B(u)) = 0, \tag{25}$$

and

$$u < l_A(u) \quad \text{and} \quad u < l_B(u). \tag{26}$$

It is possible that $\mathcal{D}_\mathrm{c}$ and $\mathcal{D}_\mathrm{d}$ are not contiguous intervals requiring the construction of various pairs of l_A and l_B. In addition, for each pair of (sub-) intervals of $\mathcal{D}_\mathrm{c}$ and $\mathcal{D}_\mathrm{d}$ they will be unique and continuous so long as the pay-offs are smooth, as is the case here (see Figure 3).

If the pair of functions l_A and l_B exist and there is a point $\bar{a}$ such that $l_A(\bar{a}) = l_B(\bar{a}) := \bar{b}$ then the pair $(\bar{a}, \bar{b})$ is a candidate to solve (19). If either of the sets $\{x \in]\alpha, \bar{a}] \,\big|\, x \notin \mathcal{D}_\mathrm{c}\}$ or $\{x \in]\bar{b}, \beta] \,\big|\, x \notin \mathcal{D}_\mathrm{d}\}$ is not empty, then these sets need to be 'covered' by a continuation region using the approach described in [17, Case VI]. If (a, b) are located then A and B can be identified by noting (8) with (10)–(11), and the equivalent expression for E, imply that

$$A = -\left(\frac{F(a)}{\psi(a)}\right)' \frac{\psi^2(a)}{\mathcal{W}(a)} = \left(\frac{E(b)}{\psi(b)}\right)' \frac{\psi^2(b)}{\mathcal{W}(b)}$$

while

$$B = \left(\frac{F(a)}{\phi(a)}\right)' \frac{\phi^2(a)}{\mathcal{W}(a)} = -\left(\frac{E(b)}{\phi(b)}\right)' \frac{\phi^2(b)}{\mathcal{W}(b)}.$$

These facts enable the innovation taken here, where we solve a practical, complex, optimisation problem based only on knowledge of E, F, ψ and ϕ, without any presumption of the SDE (1). The assumption that ψ and ϕ are as (2)–(6) is substantial, though weaker than the assumption the process is given by a particular choice of b and σ which deliver ψ and ϕ. In [15] conditions are specified on the pay-off and diffusion parameters that ensure that the pair of functions l_A and l_B and there is a unique pair, (a, b) such that $l_A(a) = l_B(a) = b$.

In the practical problem of the storage facility, the Assumptions underpinning the verification theorem summarised by (12)–(14) might hold but this will not guarantee the existence of (a, b); and even if these exist, the value function might be more complex than that defined by (17)–(18); and even if such a value function can be constructed, it might not satisfy (12)–(14). An example of a complex case where no valid value function can be identified is mentioned in the following section and discussed in the Supplementary Material.

3 Data and method

There are four inputs to the model: load data; the MOC; the operating characteristics of the storage facility made up of its capacity (in Watts), rate of charging/discharging of the facility and round trip efficiency of the facility; and the discount rate. These are combined to deliver functions that map load to cost of charging the facility, F, and to the revenue from discharging the facility, E. The load data and discount rate are combined to deliver the functions ϕ and ψ.

This paper seeks to be a 'proof of concept' of applying the approach described in Section 2 to realistic data rather than accurately model a specific technology.

The load data was taken from the 'Gridwatch' website[2] that publishes details of UK load supplied by Elexon, the UK system operator (system balancer) at five minute intervals split into different classes of generation and enabled large scale wind generation to be distinguished. The primary driver of gross load is economic activity and the weather. Load net of wind has also been impacted by the rapid increase in large scale wind generation and in order to study a relatively stable period, the study was restricted to in the period 1 August 2013 to 31 July 2015. This period was chosen based on European Wind Energy Agency data that after 1 August 2013 the UK had an installed wind generation capacity of 10.5 GW while from 1 July 2014 it had a wind generation capacity of 12.4 GW. Over this period the actual average daily wind generation did not exceed 6.24 GW and wind generation did not exceed 20% of total generation during the night-time and 17% during the day. Small scale renewable, including all solar, generation cannot be distinguished in the Gridwatch data; this is important because there was a significant

[2] `www.gridwatch.templar.co.uk`

increase in domestic solar generation that affected daytime load recorded in the Gridwatch data over the period of interest. Overall, these factors indicate that the data was not stationary and it would be difficult to perform sound statistical analysis on it. To mitigate seasonal non-stationarity, in modelling UK power load it is common to split the year into 3 'seasons' with distinctive demand characteristics. During the winter: November, December, January and February, load is consistently high (apart from the Christmas week running 24 December-1 January), while in the summer: May, June, July and August, load is consistently low. In the 'shoulder' months of March, April, September and October, load is less predictable; it can be mild and windy, or cold and still. In order to simplify matters, without losing realism, the analysis we undertake is restricted to Winter months, excluding weekends and the Christmas week (25 December – 1 January).

The merit-order-curve was supplied by the energy markets consultancy, ÅF-Mercados EMI[3]. The MOC was constructed by cataloguing and classifying all large scale hydro and thermal generators and for each class of generator a short-run production (marginal) cost based on market prices of input commodities, technical parameters, historical running characteristics and fixed operating costs was calculated. The MOC will be constantly changing as market prices change meaning that the data employed in this study was realistic for a period in 2014 but not generally accurate.

The discount rate was set at a constant 5% per annum continuously compounded and there were no 'working day' adjustments. In the empirical approach taken here, the discount factor is only relevant in the estimation of ϕ and ψ and it is straight forward to apply state dependent discounting, unlike in the standard approach. In regard to the power storage problem this is not necessary, though it may be appropriate when a firm's cost of capital is related to the price of its products, as with commodity producers.

The data for the storage facility was based on, but does not reproduce, the Huntorf Compressed Air Energy (CAES) storage plant in Germany. CAES technology was chosen because it is between chemical (battery) technology and pumped hydro technology in scale. Battery storage is important in managing short-term imbalances ('frequency response') but does not have the capacity to deliver 'peak-load' balancing, between night-time and day-time demand, which can be provided by pumped-storage facilities, such as the Ffestiniog Power Station. Wind variability poses new challenges as there can

[3]Personal communication.

be periods of time lasting days when wind-generation capacity falls, so supplementary storage capacity needs to be on the scale of Gigawatt-hours. To realistically model the longer durations of over/under-supply, Huntdorf was chosen because it has limited capacity that would be fully charged/discharged in a single operation. If Ffestiniog had been modelled, with a capacity of around 360 MW over 5.5 hours, it might only be partially emptied/filled in a single operation.

The efficiency of Huntorf is around 42%. This efficiency is too low to deliver an expected profit and so an efficiency of $\eta = 0.9$ was chosen such that $E(x) = \eta F(x)$. We construct F by combining the discount rate, the actual load data with the merit order curve and the fact that Huntorf's capacity is 290MW and fully charges/discharges over two hours. The results, and Merit Order Curve, are shown in Figure 1 with

$$\begin{aligned} F(x) = \;& 5.1926 \times 10^{-35}\, x^9 - 1.7997 \times 10^{-29}\, x^8 + 2.7287 \times 10^{-24}\, x^7 \\ & - 2.3728 \times 10^{-19}\, x^6 + 1.3025 \times 10^{-14}\, x^5 - 4.6751 \times 10^{-10}\, x^4 \\ & + 1.0960 \times 10^{-5}\, x^3 - 0.16169\, x^2 + 1361.3 x^1 - 4.9723 \times 10^6. \end{aligned}$$

Further details of how all results were obtained are in the Supplementary Material.

The functions ϕ and ψ were estimated, as $\hat{\phi}$ and $\hat{\psi}$, by applying the load data and discount rate to (2)–(3). By observing a scatter plot of the preliminary results shown in Figure 2 it was decided to fit the data to the functions

$$\hat{\phi}(x) := H_\phi + \frac{J_\phi}{(x - P_\phi)^{K_\phi}} + \frac{L_\phi}{(x - Q_\phi)^{M_\phi}}, \tag{27}$$

$$\hat{\psi}(x) := H_\psi + \frac{J_\psi}{(x - P_\psi)^{K_\psi}} + \frac{L_\psi}{(Q_\psi - x)^{M_\psi}}. \tag{28}$$

This choice was based on the fact that the hyperbolic functions ensure (6) at the boundaries $\alpha \equiv P_\phi$ and $\beta \equiv Q_\psi$. The parameter values were calculated to be as in Table 1 and shown, along with the estimations of data points, $\hat{\phi}$ and $\hat{\psi}$, in Figure 2. It should be noted that since $M_\phi = 1$ then the conditions of (4) are satisfied while the fact that $J_\psi < 0$ ensures (5). Note that there is no suggestion that $\phi = \hat{\phi}$ only that $\hat{\phi}$ is an approximation of ϕ, similarly for ψ.

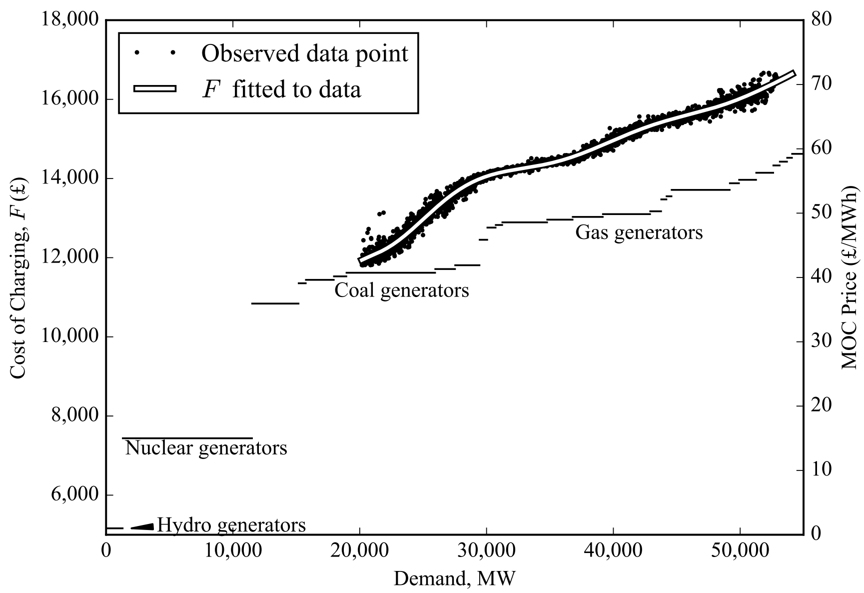

Figure 1: Construction of F, left-hand axis, based on observed load and the Merit Order Curve, right hand axis.

	H	J	K	L	M	P	Q
ϕ	1.00021	256.23253	1.65366	2.87953	1.00000	20156	54096.6
ψ	0.99986	-35.79520	1.38294	9.44348	1.15731	20156.	54093

Table 1: Parameters for $\hat{\phi}$ and $\hat{\psi}$. $\hat{\alpha} = P_\phi = 20156, \hat{\beta} = Q_\psi = 54093$

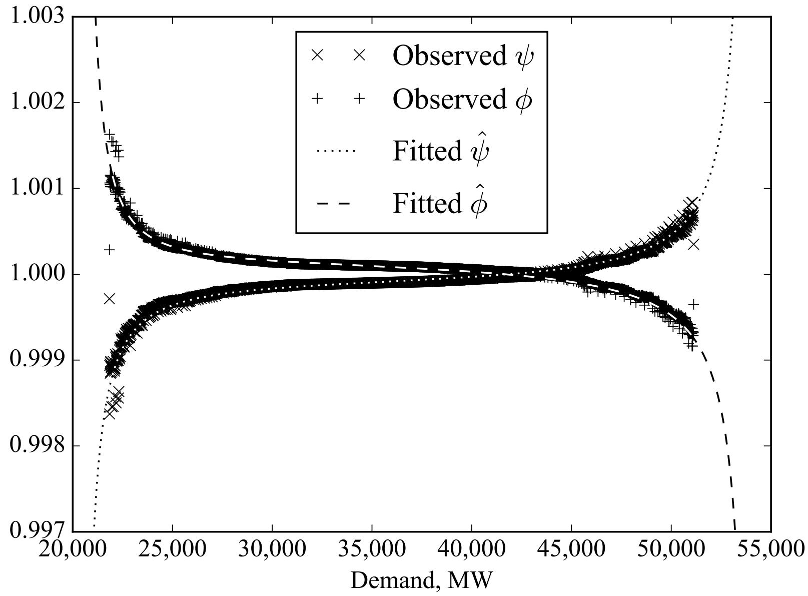

Figure 2: Construction of $\hat{\phi}$ and $\hat{\psi}$.

Having defined the functions E, F, $\hat{\phi}$ and $\hat{\psi}$ problem is solved if the pair (a, b) that solve (19) are found and when used in (17)–(18) satisfies (12)–(14). The first step in doing this is to plot different components in (21) and (20), as in Figure 3.

Note that $\mathcal{D}_\mathrm{c}$ is made up of two intervals, $\mathcal{D}_\mathrm{c}^1 \cup \mathcal{D}_\mathrm{c}^2$ and similarly $\mathcal{D}_\mathrm{d} = \mathcal{D}_\mathrm{d}^1 \cup \mathcal{D}_\mathrm{d}^2$. This means that candidates for (a, b) need to be found by on the basis of

$$l_A^{11} : \mathcal{D}_\mathrm{c}^1 \mapsto \mathcal{D}_\mathrm{d}^1 \quad \text{and} \quad l_B^{11} : \mathcal{D}_\mathrm{c}^1 \mapsto \mathcal{D}_\mathrm{d}^1,$$
$$l_A^{22} : \mathcal{D}_\mathrm{c}^2 \mapsto \mathcal{D}_\mathrm{d}^2 \quad \text{and} \quad l_B^{22} : \mathcal{D}_\mathrm{c}^2 \mapsto \mathcal{D}_\mathrm{d}^2$$

and

$$l_A^{12} : \mathcal{D}_\mathrm{c}^1 \mapsto \mathcal{D}_\mathrm{d}^2 \quad \text{and} \quad l_B^{12} : \mathcal{D}_\mathrm{c}^1 \mapsto \mathcal{D}_\mathrm{d}^2.$$

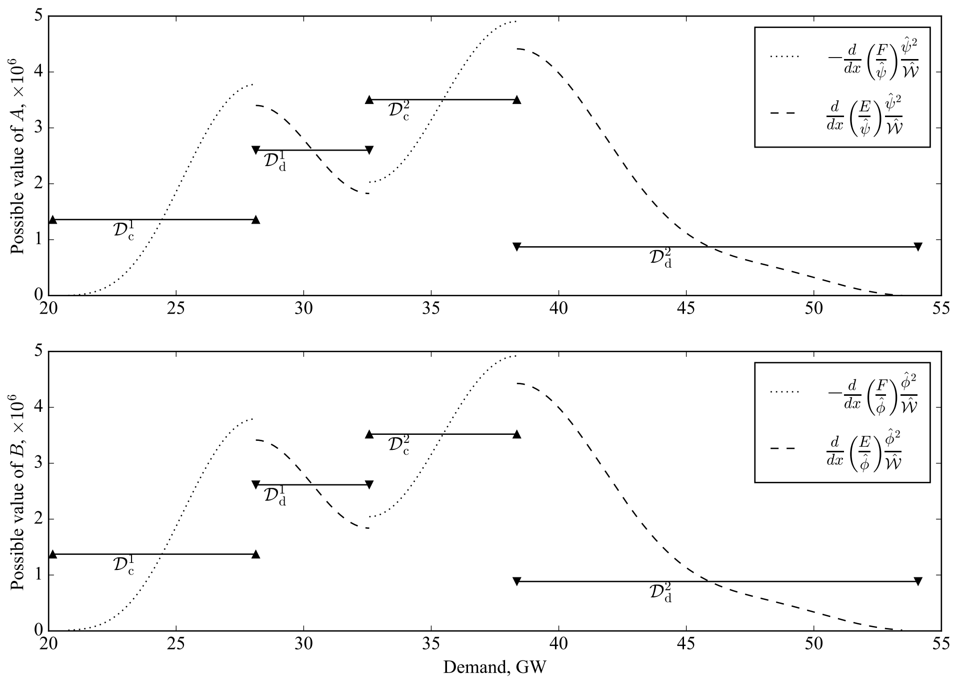

Figure 3: Preliminaries to finding the solution. The top plot relates to (20) and l_A, the bottom plot relates to (21) and l_B.

Observe that the function l_A satisfying (24)–(26) can be represented as a horizontal line starting on the curve $-\frac{d}{dx}\left(\frac{F(x)}{\phi(x)}\right)\frac{\phi^2(x)}{\mathcal{W}(x)}$ and finishing on $\frac{d}{dx}\left(\frac{E(x)}{\phi(x)}\right)\frac{\phi^2(x)}{\mathcal{W}(x)}$ in the top panel of Figure 3, while l_B can be similarly represented in the bottom panel. A candidate pair, (a, b), exists when the two lines start and finish at exactly the same point, so that (19) is satisfied. Only $\mathcal{D}^1_{\mathrm{c}} \mapsto \mathcal{D}^2_{\mathrm{d}}$ delivers a candidate pair: $a = 24,292.04$ MW and $b = 44,681.32$ MW with $A = £1,229,881$ and $B = £1,243,656$ as shown in Figure 4.

The value function is therefore

$$v(0,x) = \begin{cases} 1,243,656\ \psi(x) - F(x), & \text{if } x \in]\alpha, 24,292.0], \\ 1,229,881\ \phi(x), & \text{if } x \in]24,292.0, \beta[= \mathcal{C}_{\mathrm{c}}, \end{cases}$$

$$v(1,x) = \begin{cases} 1,243,656\ \psi(x), & \text{if } x \in]\alpha, 44,681.3[= \mathcal{C}_{\mathrm{d}}, \\ 1,229,881\ \phi(x) + E(x), & \text{if } x \in [44,681.3, \beta[, \end{cases}$$

as presented in Figure 5.

Note that if the candidate pair were such that $a \in \mathcal{D}^1_{\mathrm{c}}$ and $b \in \mathcal{D}^1_{\mathrm{d}}$ then (18) does not satisfy (12) on $\mathcal{D}^2_{\mathrm{c}}$ and the value function would have to be enhanced to include an interval where

$$v(1,x) = A^{11}\phi(x) + B^{11}\psi(x)$$

on a superset of $\mathcal{D}^2_{\mathrm{c}}$ that does not include b. An example of this issue occurring is given below, where it results in no strategy being identified, which is discussed in the Supplementary Material.

This is a naive strategy, in that there is a single charging and a single discharging interval. The optimality of this simple strategy can be easily checked by comparing it to all possible, similar, strategies applied to the data set. A contour plot of the present value of all such strategies applied to the load data is presented in Figure 6.

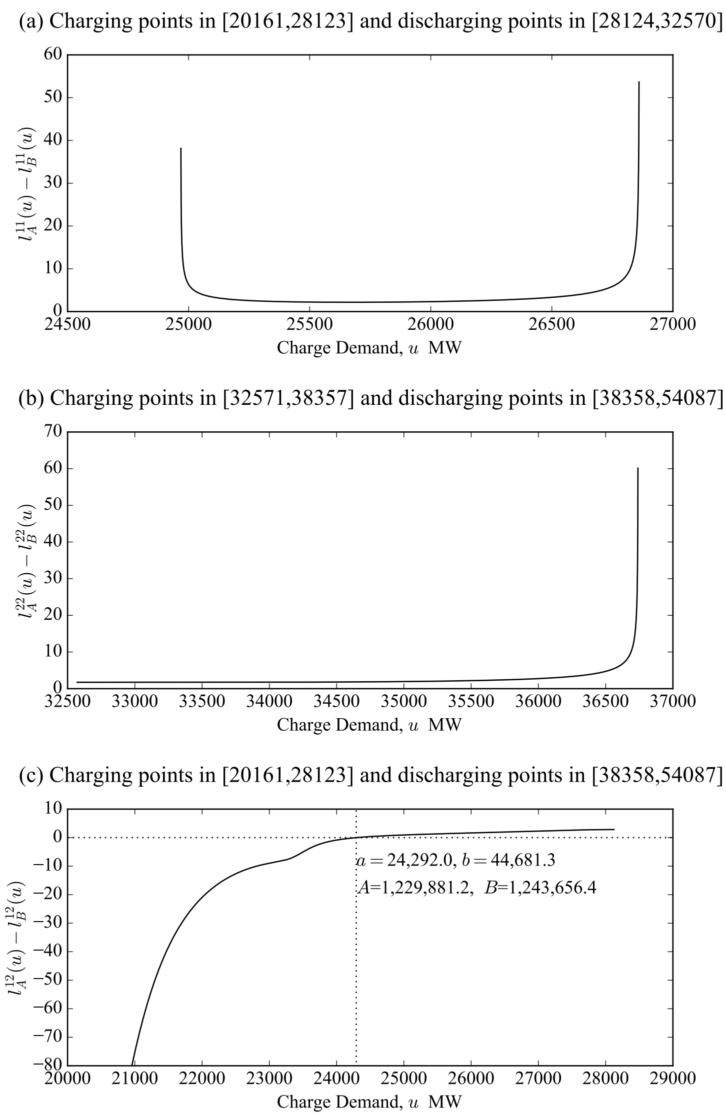

Figure 4: The solution to the stopping problem. There is a single solution in (c) at the point $u = 24,292 \in \mathcal{D}_\mathrm{c}^1$ and $l_A^{12}(u) = l_B^{12}(u) = 44,681 \in \mathcal{D}_\mathrm{d}^2$.

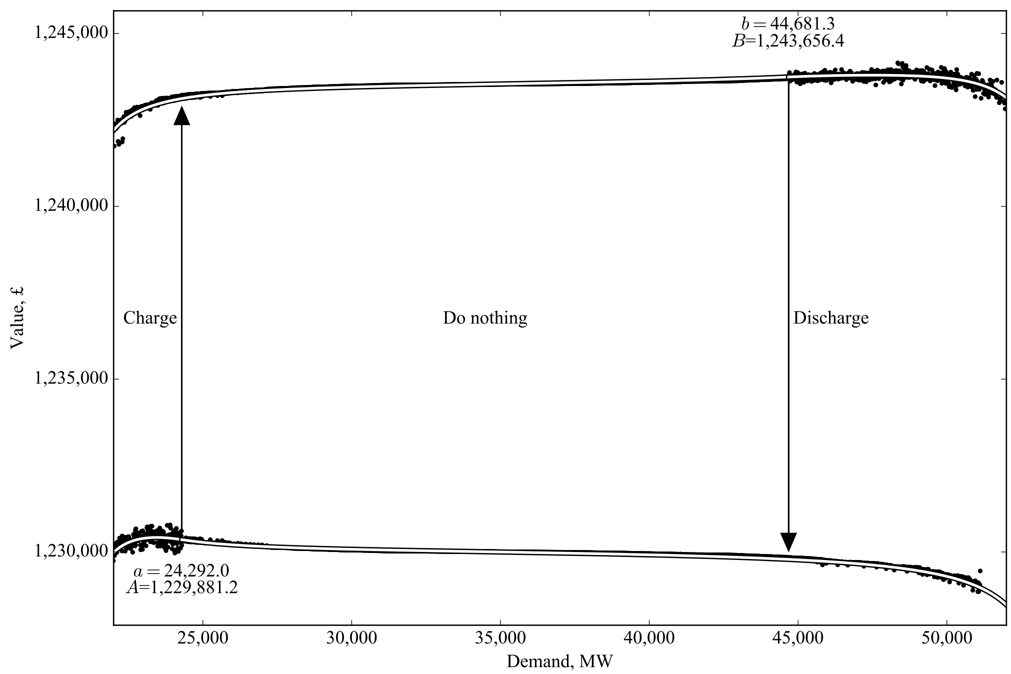

Figure 5: The identified strategy and value function (in white on a black background that represents values of E, F based on the data and estimates of $\hat{\phi}$ and $\hat{\psi}$).

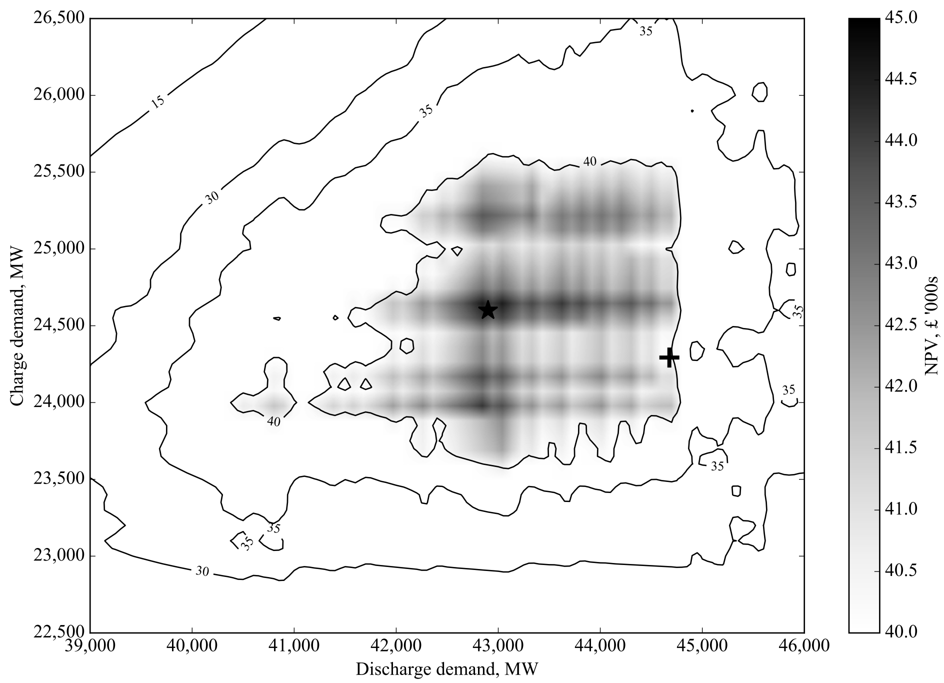

Figure 6: NPV contour plot of the value of naive strategies applied to input load data. The star ($a^* = 24,600$ MW, $b^* = 42,900$ MW) is the, *ex-ante*, optimal strategy, the cross is the candidate strategy identified by the methodology ($a = 24,292$ MW, $b = 44,681$ MW).

The candidate strategy that has been identified ($a = 24,292$ MW, $b = 44,681$ MW) resulted in a NPV over the two winters of £40,149 that is close to what would have been the optimal strategy of £44,514 ($a^* = 24,600$ MW, $b^* = 42,900$ MW). This is reasonable given that the identified strategy maximises the expected payoff based on an assumption of stationarity of data, which is not the case. Both these strategies lie on an 'NPV plateau' discharging in the load range of 42-45 GW, which corresponds to power supplied by similar Combined Cycle Gas Turbines[4] and charging load range of 24-25.5 GW corresponding to the four Ratcliffe on Soar coal powered turbines.

The robustness of the solution was investigated by considering a storage facility that could charge/discharge four times faster, in half-an-hour. This delivered a candidate optimal strategy of ($a = 24,021.9$ MW, $b = 43,434.9$ MW), which is still on the 'plateau', with $A =$£1,499,921 and $B =$£1,513,763. However, as discussed in the Supplementary Material, this candidate requires that the discharge interval must be discontinuous and a value function that conforms to (12)–(14) does not exist.

Reducing the round-trip efficiency has a more dramatic effect. Intuitively, reducing the efficiency will induce a larger separation between a and b, reducing the potential number of cycles resulting in an overall loss of value. The results of reducing the round-trip efficiency from 0.90 to 0.85 changes delivers a candidate solution ($a = 23,345.8$ MW, $b = 47,628.7$ MW) with $A =$£569,797 and $B =$£582,949.

4 Discussion

The aim of this paper was to develop a methodology for assessing the economic impact of power storage facilities that can be used by policy makers, in the spirit of the recommendations in [5, 5.2.2]. The methodology delivers reasonable results in a manner more accessible to policy maker than, for example [20, 21, 10, 22, 23]. The results are not predictive, in that they would not accurately value the storage facility, but they are indicative, suggesting that the storage facility considered would store coal generated power and discharge it in competition with flexible CCGT turbines. This might be sub-optimal in broader policy terms. The identified strategy is naive, in-

[4] Specifically, Damhead Creek, Rocksavage, Shoreham, Spalding, Great Yarmouth, Enfield, South Humberbank 1 & 2.

volving a single stopping region for each regime. However the methodology is capable of identifying more sophisticated strategies, involving a number of stopping regions in each regime, a possibility discussed in relation to the fast charging/discharging case. While capable of delivering complex strategies the methodology does not require scarce mathematical expertise that would be costly for policy makers to access.

The innovation in this paper is removing the requirement to first identify a driving diffusion (1). Rather, it assumes that the data is driven by a suitable diffusion and on this basis estimates of ϕ, ψ, F and E are made directly from the data. This innovation is made possible by realising that the critical conditions (12)–(14) can be verified just with reference to the slopes of functions given explicitly by ϕ, ψ, F and E. On this basis an optimal strategy is identified that is reasonable when compared to the optimal strategy identified with perfect hindsight. The solution is found with reference to calculating the slopes of polynomials and does not require detailed knowledge of special functions, such as in [24, 25, 26, 27, 28]. This novel approach would be useful to a wide range of problems beyond power systems economics.

Having delivered the methodology, future work should focus on modelling time series of load net of renewable supply which could be applied to various technologies and policies to support power storage. The methodology presented shifts the workload from delivering results to considering different scenarios to enable the identification of the best incentive policies for power storage providers to address wind-variability.

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