DELAYED FEEDBACK CONTROL IN STOCHASTIC EXCITABLE NETWORKS

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Abstract

A simplified model of a stochastic neural network is considered, being a system of a large number of identical excitable FitzHugh-Nagumo oscillators coupled via the mean field. The possibility to control the global dynamics of this network is investigated. The control tool being probed is Pyrgas delayed feedback constructed and applied through the mean field. It is shown that one can to destroy or diminish stochastic synchronization in a partially synchronized network by a weak delayed feedback under the appropriate choice of delay.

Key words

network, neruon model, stochastic, delay, control

1 Introduction

We consider the collective behavior of a network of excitable stochastic units coupled through the mean field. Each network element is represented by the FitzHugh-Nagumo system in the excitable regime under the influence of noise, which is a paradigmatic model of a single excitable unit (Lindner *et al.*, 2004). This system serves as a rough model of a neural network. It has been earlier shown (Zaks *et al.*, 2005) that such a network is capable of demonstrating various kinds of collective behavior: from non-synchronized independently spiking units, through a few distinct stages when spiking of different units is synchronized only partially, to the perfectly synchronized network. The detection of different stages of synchronization is possible through the mean field, which demonstrates periodic or chaotic small oscillations around the only fixed point in the absence of synchronization, or periodic or aperiodic spiking. The effect of synchronization in a real neural network is two-fold. On the one hand, synchronization is believed to help better processing of information and is thus advantageous (Samonds *et al.*, 2004; Benucci *et al.*, 2004). On the other hand, synchronization is suggested to be responsible for inducing a regular rhythmic activity in the brain, which is associated with Parkinson's disease, essential tremor and epilepsy (Dreifuss and et al, 1981; Tass, 2002; Tass *et al.*, 1998; Grosse *et al.*, 2002). With this, it remains an important clinical challenge to develop an efficient control technique with the ability to manipulate the neural synchrony. Recently, a number of methods have been proposed for the suppression of synchrony of the arrays of coupled oscillators in which oscillations are self-sustained, i.e. exist regardless of the applied noise (Rosenblum and Pikovsky, 2004; Popovych *et al.*, 2005). The purpose of this paper is to demonstrate the possibility to manipulate the properties of the collective behavior of a network of units in which there is no dynamics without external perturbation, and any oscillations are induced merely by external sources of random noise.

2 Stochastic excitable network with delayed feedback

The control technique being probed is the Pyragas delayed feedback (Pyragas, 1992; Pyragas, 1995). The controlling signal is constructed from the macroscopic mean field of the network, and the same signal is applied to all units. The model equations read (Patidar *et al.*, 2009):

$$\begin{aligned} \epsilon\dot{x}_i &= x_i - \frac{x_i^3}{3} - y_i + \gamma(M_X - x_i), \qquad (1)\\ \dot{y}_i &= x_i + a + \sqrt{2T}\xi_i(t) + K(M_Y(t-\tau) - M_Y(t)),\\ M_X &= \frac{1}{N}\sum_{i=1}^{N} x_i, \quad M_Y = \frac{1}{N}\sum_{i=1}^{N} y_i. \end{aligned}$$

Here, N is the total number of units in the network; parameters $a = 1.05$ and $\epsilon = 0.01$ are fixed to ensure excitability of each unit; $\xi_i(t)$ is Gaussian white noise with zero mean and unity variance, and the noise sources in different elements are uncorrelated, i.e. $\xi_i(t)\xi_j(t+s) = \delta_{ij}\delta(s)$, where δ_{ij} is Kronecker delta

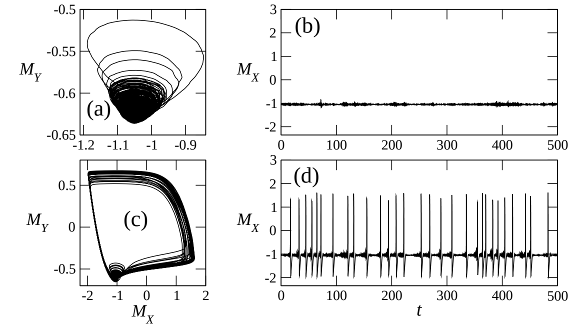

Figure 1. Phase portraits and realizations of the mean field of Eqs. (1) without feedback at $\gamma = 0.1$: (a,b) non-synchronous spiking with chaotic subthreshold mean field at $T = 0.00027$; (c,d) partially synchronous spiking with chaotic superthreshold mean filed at $T = 0.00028$.

and $\delta(s)$ is Dirac delta-function. T is the noise intensity which is the same in all units. Coupling between the units is realized through the mean field, when each unit experiences the same averaged input $M_X(t)$ from the rest of the network with the coupling strength γ. To allow for direct comparison of the effects of coupling and of the feedback with those in the earlier works, the network is coupled through $M_X(t)$ in the first of Eqs. (1) like in (Zaks *et al.*, 2005), while the delayed feedback $K(M_Y(t-\tau) - M_Y(t))$ is applied in the second of Eqs. (1) like in (Janson *et al.*, 2004; Balanov *et al.*, 2004). In the above, τ is the feedback delay and K is the feedback strength. There are two possible kinds of chaotic behavior of the mean field: those related to non-synchronous spiking in the network and those related to partially synchronized units. At $\gamma = 0.1$ these regimes can be observed e.g. at $T = 0.00027$ and at $T = 0.00028$, respectively (see Fig. 1).

We switch the delayed feedback on, fix its strength at $K = 0.1$ and examine the response of the network as a function of τ in the original states at $T = 0.00027$ and at $T = 0.00028$ with $\gamma = 0.1$. Eqs. (1) are numerically integrated with $N = 10000$ and the dynamics of the ensemble averages M_X and M_Y are studied. The collective response is characterized by the average interspike interval T and the amplitude A of the mean field M_X and is illustrated in Fig. 2.

One can see that where initially the network was desynchronized ($T = 0.00027$ (a,b)), there is a range of τ values at which the feedback is capable of inducing synchronization. The latter is detected by the finite value of T, as opposed to the ifinitely large value when the mean field does not spike, and by the large amplitude A. The domains of synchronous spiking are shown as patterned areas in Fig. 2. At the same time, where the network was initially partially synchronized, ($T = 0.00028$ (c,d)), there are ranges of τ at which synchronization is suppressed (infinitely large $\langle T \rangle$). One can see that inside the domains of synchronization the time scale of the mean field oscillations depend on the value of τ and thus can be controlled by

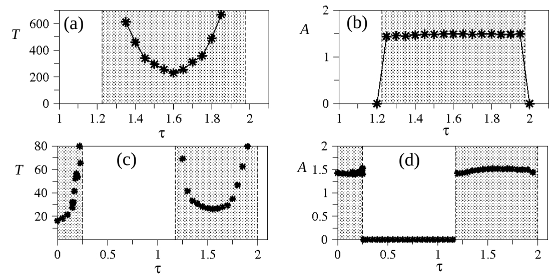

Figure 2. Mean interspike intervals $\langle T \rangle$ and mean spiking amplitudes A as functions of delay τ as the feedback is applied with $\gamma = 0.1$ when the original states were as illustrated by Fig. 1: (a,b) $T = 0.00027$ (c,d) $T = 0.00028$.

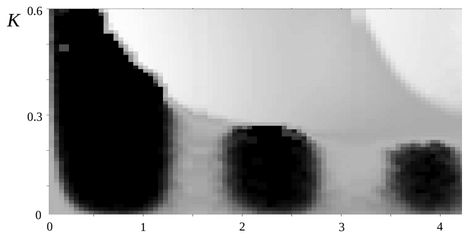

Figure 3. Mean spiking frequency as a function of feedback strength K and delay τ. Without feedback the system demonstrated partial synchronization with chaotically spiking mean field at $T = 0.00028$ and $\gamma = 0.1$.

the feedback.

It is interesting to examine to the effect of the feedback on the full range of the feedback strength K. Consider a partially synchronized network at $\gamma = 0.1$ and $T = 0.00028$ and apply the delayed feedback to it. In Fig. 3 one can see the mean spiking frequency $1/\langle T \rangle$ as a function of K and τ shown by the tint of grey color. Inside black regions the spiking frequency is zero, and therefore there is no synchronization. Interestingly, the synchrony suppression can be achieved here at the tiny values of K if τ is close to 0.8. It is also worth noting that outside the full suppression regions the spiking frequency varies considerably, which provides the room for the time scale manipulation in the system.

3 Cumulant equations with delay

Eqs. (1) without feedback ($K = 0$) were studied in detail in (Zaks *et al.*, 2005), and their behavior was qualitatively explained using the cumulant expansion of the probability density distribution (PDD) of the units, which was based on the assumption of the molecular chaos. Gaussian approximation was used to truncate the system of cumulant equations to five coupled cumulant equations whose dynamical behavior was qualitatively similar to that of the original stochastic network.

In this work we provide a qualitative explanation of the response of the stochastic network to delayed feedback control by deriving and analysing the cumulant equations with delay using the same two approximations. Molecular chaos approximation assumes that the oscillations in different units are uncorrelated, and thus the joint $2N$-dimensional PDD of the whole system can be represented as a product of the identical 2-dimensional PDDs of the individual units. Gaussian approximation assumes that each of the 2-dimensional PDDs is a Gaussian function of two variables, and can therefore be characterized by only five non-zero cumulants explained below. The cumulant equations were derived using the method proposed in (Desai and Zwanzig, 1978) and improved in (Rodriguez and Tuckwell, 1996; Tanabe and Pakdaman, 2001), and read

$$\begin{aligned}
\epsilon\frac{dm_X}{dt} &= m_X - \frac{{m_X}^3}{3} - m_Y - m_X D_X,\\
\frac{dm_Y}{dt} &= m_X + a + K(m_Y(t-\tau) - m_Y),\\
\epsilon\frac{dD_X}{dt} &= 2\Big[D_X(1-\gamma-{m_X}^2 - D_X) - D_{XY}\Big],\\
\frac{dD_Y}{dt} &= 2(D_{XY} + T), \qquad (2)\\
\epsilon\frac{dD_{XY}}{dt} &= \epsilon D_X + D_{XY}(1-{m_X}^2 - D_X - \gamma) - D_Y.
\end{aligned}$$

Here m_X and m_Y are the mean values of the distributions of the variables x and y, respectively, D_X and D_Y are their variances, and D_{XY} is their cross-variance which is the second moment of their joint distribution. As indicated in (Zaks *et al.*, 2005), the Gaussian approximation only provides a qualitative description of the eects in the network, while there is no quantitative agreement. With this, to compare the eects induced by the feedback in cumulant and in stochastic equations, we had to apply the feedback to topologically equivalent regimes. To do so, in Eqs. (2) with $K = 0$ we chose such parameters T and γ with which they had the regimes similar to those of stochastic Eqs. (1). Namely, at $\gamma = 0.1$ and $T = 0.001585$ Eqs. (2) demonstrated subthreshold chaos similar to that in Fig. 1 (a,b), while at $T = 0.001586$ they exhibited chaotic spiking.

It is known that the skeleton of a chaotic attractor is formed by the infinite number of unstable periodic orbits. Pyragas delayed feedback can stabilize such orbits if τ is equal to the orbit period and K is chosen appropriately. In addition, in (Balanov *et al.*, 2005) the eects of delayed feedback on a typical chaotic system were revealed for a large range of values of τ and K. It was found that in the plane (τ, K) domains can be found within which there are no stable oscillations at all, i.e. the fixed point is stabilized. Also, a range of bifurcations occurs which arise as a result of feedback application.

In Fig. 4 the bifurcation diagrams of Eqs. (2) in the plane (τ, K) are shown, which were obtained with the help of continuation technique using the free software DDEBIFTOOL (Engelborghs *et al.*, 2001) and also numerical simulation of oscillating solutions. The common feature of the diagrams is the large parabola-like curves of Andronov-Hopf bifurcation of the fixed point, above which the fixed point is stable. In the white areas below these curves the oscillations of the cumulants are subthreshold, either periodic or chaotic. Patterned green areas denote the regimes of spiking which can be periodic or chaotic. These regimes were found by numerical integration of the cumulant equations rather than by means of continuation technique. When comparing Figs. 3 and 4, one can notice that in both figures along the lines of fixed K there are domains in which all spiking of the mean field is suppressed, and those where spiking occurs. However, the agreement between the maps of regimes is only qualitative because Gaussian approximation does not give an accurate description of the stochastic network, and also because the initial regimes without the feedback were similar only in their nature but not in the dynamical detail.

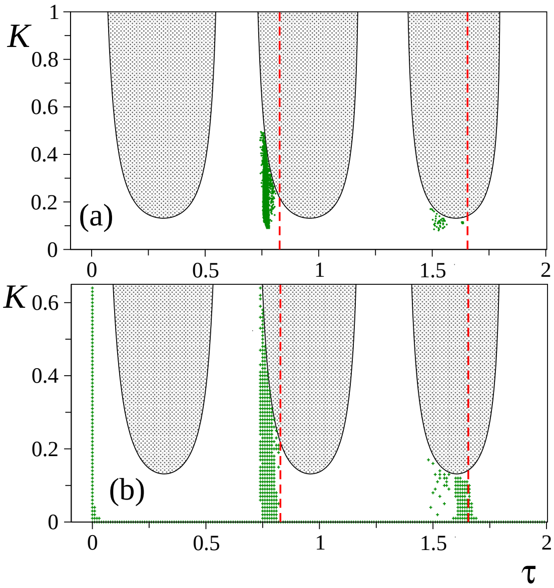

Figure 4. Maps of regimes of cumulant equations with delayed feedback Eqs. (2) on the plane τ and K as the feedback is applied when the original states of the cumulant equations were: (a) subthreshold chaotic oscillations at $T = 0.001585$; (b) chaotic spiking at $T = 0.001586$. Light grey areas inside parabolas indicate the regions of stability of the only fixed point; patterned green areas indicate spiking of cumulants; white areas indicate the absence of spiking in the cumulants.

4 Summary and Conclusions

A simplified model of a stochastic neural network was considered in the form of a large number of excitable FitzHugh-Nagumo units subjected to uncorrelated sources of noise and coupled through the mean field. As it was known from an earlier work, such a network could demonstrate a range of forms of collective behavior, from non-synchronously spiking units, through the units synchronous only partially, to the fully synchronized network. Distinct regimes could be characterized and detected by the behavior of the mean field, which would spike in partly of fully synchronous cases, or demonstrate subthreshold oscillations or no oscillations at all in the absence of synchronization. The purpose of the work was to find out if it was possible to manipulate the properties of the collective spiking of the network by using some macroscopic feedback. As a candidate for the control technique Pyragas delayed feedback was probed, which was constructed from, and applied through the mean field. It is demonstrated that the delayed feedback is capable of destroying synchronization in a partially synchronized network, such that with the appropriately chosen delay time the strength of the feedback is very small. The feedback can also shift the mean spiking frequency of the mean field.

The action of the delayed feedback control on the large stochastic network is explained on a qualitative level by considering cumulant equations. Two main approximations were used for their derivation: molecular chaos and Gaussian approximation. The system of five cumulant equations with delay have reproduced the main feature of the delayed feedback effect on the network: the possibility to stabilize the originally unstable fixed point related to the mean field. The shapes of the stability domains in stochastic and in cumulant equations were qualitatively similar. However, Gaussian approximation has proved quite crude since the maps of regimes differed in quantitative detail.

The delayed feedback applied macroscopically to a stochastic network could be a promising technique for the control of synchronization in real neural networks in various medical conditions believed to be related to synchronization: epilepsy, Parkinson disease, and essential tremor. This method could be particularly attractive in view of the weakness of the feedback signal that is necessary to desynchronize the system, under the suitable choice of time delay.

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References

Balanov, A.G., N.B. Janson and E. Schöll (2004). Control of noise-induced oscillations by delayed feedback. *Physica D* **199**, 1.

Balanov, A.G., N.B. Janson and E. Schöll (2005). Delayed feedback control of chaos: Bifurcation analysis. *Physical Review E* **71**, 016222.

Benucci, A., P.F.M. J. Verschure and P. König (2004). High-order events in cortical networks: A lower bound. *Phys. Rev. E* **70**, 051909.

Desai, R. C. and R. Zwanzig (1978). Statistical mechanics of a nonlinear stochastic model. *J. Stat. Phys.* **19**, 1.

Dreifuss, F.E. and et al (1981). Proposal for revised clinical and electroencephalographic classification of epileptic seizures : From the commission on classification and terminology of the international league against epilepsy*. *Epilepsia* **22**, 489.

Engelborghs, K., T. Luzyanina and G. Samaey (2001). Dde-biftool v. 2.00: a matlab package for bifurcation analysis of delay differential equations. *Technical Report TW-330, Department of Computer Science, K.U. Leuven, Leuven, Belgium.*

Grosse, P., M. J. Cassidy and P. Brown (2002). Eeg-emg, meg-emg and emg-emg frequency analysis: physiological principles and clinical applications. *Clin. Neurophysiol.* **113**, 1523.

Janson, N.B., A.G. Balanov and E. Schöll (2004). Delayed feedback as a mean of control of noise-induced motion. *Phys. Rev. Lett.* **93**, 010601.

Lindner, B., J. Garcia-Ojalvo, A. Neiman and L. Schimansky-Geier (2004). Effects of noise in excitable systems. *Physics Reports* **392**, 321–424.

Patidar, S., A. Pototsky and N.B. Janson (2009). Controlling noise-induced behavior of excitable networks. *New J. Physics* **11**, 073001.

Popovych, O., C. Hauptmann and P.A. Tass (2005). Effective desynchronization by nonlinear delayed feedback. *Phys. Rev. Lett* **94**, 164102.

Pyragas, K. (1992). Continuous control of chaos by self-controlling feedback. *Phys. Lett. A* **170**, 421.

Pyragas, K. (1995). Control of chaos via extended delay feedback. *Phys. Lett. A* **206**, 323.

Rodriguez, R. and H. Tuckwell (1996). Statistical properties of stochastic nonlinear dynamical models of single spiking neurons and neural networks. *Phys. Rev. E* **54**, 5585.

Rosenblum, M. and A. Pikovsky (2004). Delayed feedback control of collective synchrony: An approach to suppression of pathological brain rhythms. *Phys. Rev. E* **70**, 041904.

Samonds, J.M., J.D. Allison, H.A Brown and A.B. Bonds (2004). Cooperative synchronized assemblies enhance orientation discrimination. *Proc. Natl. Acad. Sci. U.S.A.* **101**, 6722.

Tanabe, S. and H. Pakdaman (2001). Dynamics of moments of fitzhugh-nagumo neuronal models and stochastic bifurcations. *Phys. Rev. E* **63**, 031911.

Tass, P. (2002). Effective desynchronization with bipolar double-pulse stimulation. *Phys. Rev. E* **66**, 036226.

Tass, P., M. G. Rosenblum, J. Weule, J. Kurths, A. Pikovsky, J. Volkmann, A. Schnitzler and H.-J. Freund (1998). Detection of n:m phase locking from noisy data: Application to magnetoencephalography. *Phys. Rev. Lett.* **81**, 3291.

Zaks, M.A., X. Sailer, L. Schimansky-Geier and A.B. Neiman (2005). Noise induced complexity: From subthreshold oscillations to spiking in coupled excitable systems. *CHAOS* **15**, 026117.