Model Predictive Control for Discrete-Time Linear Systems with Time Delays and Unknown Input*

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Abstract. The paper deals with Model Predictive Control synthesis based on the system output tracking with control and state delays. Input and state constraints are taken into account when solving the MPC problem for systems with unknown input. A prediction is carried out on the base of object states estimates that is obtained by the Kalman filter. The criteria function is assumed to be convex quadratic. The proposed algorithm allows to get around the state space extension.

Keywords: Model Predictive Control, discrete systems, state delay, input delay, unknown input, Kalman filter.

1 Introduction

One of the modern formalized approaches to the system control synthesis based on mathematical methods of optimizadtion is Dynamic object control theory with predictive models - Model Predictive Control (MPC).

This approach began to develop in the early 1960s. It was destined for the process control in petrochemical and energy industries for which the application of traditional synthesis methods was extremely complicated according to mathematical models complication. For the last years, field of MPC application has been considerably extended covering technologic fields for object with time delay [1-6], inventory control [7-8]; and portfolio control and optimization [9].

The paper is devoted to Model Predictive Control synthesis based on the system output tracking allowing for input and state delays. It has been suggested to make a synthesis of predictive control using estimates of unknown input that can be evaluated on the base of modified LSM [10-12].

A new algorithm proposed in the paper allows to include control and state delays into the model getting around the state space extension. This reduces the dimension of the block matrices used in the algorithm significantly.

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2 Problem Statement

Suppose the object can be described by the following system of linear-difference equations:

$$x_{t+1} = Ax_t + \sum_{i=1}^{s} A_i x_{t-i} + Bu_{t-h} + Ir_t + w_t,$$
$$x_k = \bar{x}_k, (k = \overline{-s, 0}), u_i = \bar{u}_i, (i = \overline{-h, -1}), \tag{1}$$
$$\psi_t = Hx_t + v_t, \tag{2}$$
$$y_t = Gx_t. \tag{3}$$

Here $x_t \in R^n$ is the object state ($x_k = \bar{x}_k, k = -s, \ldots, -1, 0, \bar{x}_k$ is considered to be given), $u_k \in R^m$ is the control input ($u_k = \bar{u}_k, k = -h, \ldots, -1, \bar{u}_k$ is given), $r_t \in R^q$ is the unknown input, $y_t \in R^p$ is the output (to be controlled), $\psi_t \in R^l$ is the observation (measured output), s, h are the state and conrol input delay values respectively. Further, the state noise w_t and measurement noise v_t are assumed to be Gaussian distributed with zero mean and covariances W and V respectively, i.e. $M\{w_t w_k^{\mathrm{T}}\} = W\delta_{t,k}$, $M\{v_t v_k^{\mathrm{T}}\} = V\delta_{t,k}$, where $\delta_{t,k}$ is the Kronecker delta.

In the simple case, when r_t is a zero-mean random vector with the known variance, the optimal filtering problem for the model (1)-(3) comes to the Kalman filtering algorithm. If the input r_t is a deterministic component and its evolution in time is governed by the known linear system, the optimal estimates of r_t and x_t can be obtained using the extended state Kalman filter. In this paper we consider the case when prior knowledge about the time evolution of r_t is not available. Vector r_t is supposed to be completely unknown.

The model under consideration is used to make predictions about the plant behavior over the prediction horizon denoted by N using information (measurements of inputs and outputs) up to and including the current time t. The plant is supposed to operate under the constrained conditions:

$$a_1 \leq S_1 x_t \leq a_2, \tag{4}$$
$$\phi_1(x_{t-h}) \leq S_2 u_{t-h} \leq \phi_2(x_{t-h}). \tag{5}$$

Here S_1 and S_2 are structural matrices that are composed of zeros and units, identifying constrained components of vectors x_t and u_t ; a_1, a_2, $\phi_1(x_t)$, $\phi_2(x_t)$ are given constant vectors and vector-functions. The problem is to determine an acting strategy on the base of the observation ψ_t according to which the output vector of the system y_t will be close to the reference taking into account constraints on the state and control input.

3 Prediction

With the Gaussian assumptions on the state and the measurement noise it is possible to make optimal (in the minimum variance sense) predictions of state and output using a Kalman filter, see e.g. [13].

Let $\hat{x}_{i|j}$ and $\hat{y}_{i|j}$ to be estimates of the state and the output at time i giving information up to and including time j where $j \leq i$. Then

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t-1} + \sum_{i=1}^{s} A_i\hat{x}_{t-i|t-i-1} + Bu_{t-h} + I\hat{r}_t + K_t(\psi_t - H\hat{x}_{t|t-1}),$$
$$\hat{x}_{t|t-1} = \bar{x}_k, k = \overline{-s, 0},$$
$$\hat{y}_{t+1|t} = G\hat{x}_{t+1|t},$$
$$K_t = AP_tH^{\mathrm{T}}(HP_tH^{\mathrm{T}} + V)^{-1},$$
$$P_{t+1} = W + AP_tA^{\mathrm{T}} + AP_tH^{\mathrm{T}}(HP_tH^{\mathrm{T}} + V)^{-1}HP_tA^{\mathrm{T}}, P_0 = P_{x_0}, \quad (6)$$

where P_{x_0} is the given initial value of the variance matrix. Equation (6) for P_t is known as the discrete-time Riccati-equation.

Evaluate estimates of the unknown input using LSM [10] in order to develop a pedictive model. In this case there is no need to know a behavioral model of the unknown input. Let evaluate state predictions as a result of solving a new optimal control problem where by "control" we will mean the unknown input $\hat{r}_t$. The following quadratic function is proposed to use as an optimal criterion:

$$J(\hat{r}_{t-1}) = \sum_{i=1}^{t}\{\|\psi_t - H\hat{x}_{i|i-1}\|_{C_R}^2 + \|\hat{r}_{i-1}\|_{D_R}^2\}, \quad (7)$$

where C_R and D_R are symmetric positive definite matrices.

Optimization of the criterion (7) up to the current time t comes to the criterion minimization in each time $i = \overline{1, t}$.

$$J(\hat{r}_{t-1}) = \min_{\hat{r}_0}\min_{\hat{r}_1}\ldots\min_{\hat{r}_{t-1}}\sum_{i=1}^{t}\{\|\psi_t - H\hat{x}_{i|i-1}\|_{C_R}^2 + \|\hat{r}_{i-1}\|_{D_R}^2\}.$$

An optimal estimate of the unknown input at the first step ($t = 1$):

$$J(\hat{r}_0) = \min_{\hat{r}_0}\{\|\psi_1 - H\hat{x}_{1|0}\|_{C_R}^2 + \|\hat{r}_0\|_{D_R}^2\}.$$

Taking into account $\hat{x}_{1|0} = Ax_0 + Bu_0 + I\hat{r}_0$, we get the following:

$$J(\hat{r}_0) = \min_{\hat{r}_0}\{\|\psi_1 - HAx_0 - HBu_0 - HI\hat{r}_0\|_{C_R}^2 + \|\hat{r}_0\|_{D_R}^2\}. \quad (8)$$

After some manipulations, we have:

$$J(\hat{r}_0) = \min_{\hat{r}_0}\{\hat{r}_0^{\mathrm{T}}(I^{\mathrm{T}}H^{\mathrm{T}}C_RHI + D_R)\hat{r}_0 -$$
$$-2\hat{r}_0^{\mathrm{T}}I^{\mathrm{T}}H^{\mathrm{T}}C_R(\psi_1 - HAx_0 - HBu_0) + \alpha_0\},$$

where α_0 - variable independent of $\hat{r}_0$.

An optimal estimate of the unknown input at the 1st instant can be found from the following condition

$$\frac{\partial J(\hat{r}_0)}{\partial \hat{r}_0} = 2(I^{\mathrm{T}}H^{\mathrm{T}}C_RHI + D_R)\hat{r}_0 - 2I^{\mathrm{T}}H^{\mathrm{T}}C_R(\psi_1 - HAx_0 - HBu_0) = 0,$$

and have the following expression:

$$\hat{r}_0 = S_R(\psi_1 - HAx_0 - HBu_0),$$

where $S_R = (I^{\mathrm{T}} H^{\mathrm{T}} C_R HI + D_R)^{-1} I^{\mathrm{T}} H^{\mathrm{T}} C_R$. We can get criterion's value at the instant $t = 1$ using the obtained expression for $\hat{r}_0$ in (8), ,

$$J(\hat{r}_0) = (\psi_1 - HAx_0 - HBu_0)^{\mathrm{T}} M_R(\psi_1 - HAx_0 - HBu_0),$$

where $M_R = C_R - 2C_R HIS_R + S_R^{\mathrm{T}}(I^{\mathrm{T}} H^{\mathrm{T}} C_R HI + D_R)S_R$.

At the instant $t = 2$ an optimal estimate of the unknown input is found by optimizing the following function:

$$J(\hat{r}_1) = \min_{\hat{r}_0} \min_{\hat{r}_1} \{\|\psi_2 - H\hat{x}_{2|1}\|^2_{C_R} + \|\hat{r}_1\|^2_{D_R} + \|\psi_1 - H\hat{x}_{1|0}\|^2_{C_R} + \|\hat{r}_0\|^2_{D_R}\}.$$

Expression for $J(\hat{r}_1)$ can be rearranged in the following way using the Bellman's optimality principle,

$$\begin{gathered} J(\hat{r}_1) = \min_{\hat{r}_1} \{\|\psi_2 - H\hat{x}_{2|1}\|^2_{C_R} + \|\hat{r}_1\|^2_{D_R} + J(\hat{r}_0)\} = \\ = \min_{\hat{r}_1} \{\|\psi_2 - HA\hat{x}_{1|0} - HBu_1 - HI\hat{r}_1\|^2_{C_R} + \\ + \|\hat{r}_1\|^2_{D_R} + \|\psi_1 - HAx_0 - HBu_0\|^2_{M_R}\} = \\ = \min_{\hat{r}_1} \{\hat{r}_1^{\mathrm{T}}(I^{\mathrm{T}} H^{\mathrm{T}} C_R HI + D_R)\hat{r}_1 - 2\hat{r}_1^{\mathrm{T}} I^{\mathrm{T}} H^{\mathrm{T}} C_R(\psi_2 - HA\hat{x}_{1|0} - HBu_1) + \alpha_1\}, \end{gathered}$$

where α_0 - variable independent of $\hat{r}_1$. Differentiate with respect to $\hat{r}_1$ like in the first step and get the following:

$$\begin{gathered} \hat{r}_1 = S_R(\psi_2 - HA\hat{x}_{1|0} - HBu_1), \\ J(\hat{r}_1) = (\psi_2 - HA\hat{x}_{1|0} - HBu_1)^{\mathrm{T}} M_R(\psi_2 - HA\hat{x}_{1|0} - HBu_1). \end{gathered}$$

Applying the Bellman's optimality principle for the next steps and using a method of mathematical induction, we get $\hat{r}_t$:

$$\hat{r}_t = S_R(\psi_t + 1 - HA\hat{x}_{t|t-1} - HBu_t). \tag{9}$$

So, taking into account expressions for unknow input estimates, state and output prediction can be performed in accordance with the following formulas

$$\begin{gathered} \hat{x}_{t+i|t} = A^{i-1}\hat{x}_{t+1|t} + \sum_{k=1}^{i-1} A^{i-k-1} Bu_{t+k-h} + \sum_{k=1}^{i-1} A^{i-k-1} I\hat{r}_{t+k}, \\ \hat{y}_{t+i|t} = G\hat{x}_{t+i|t}, i = \overline{1, N}, \end{gathered} \tag{10}$$

where $u_{t+k|t}$ - the control input used for prediction, $\hat{r}_{t+k}$ - predicted unknown input estimates that can be obtained on the base of time series forecasting methods [14].

MPC usually requires estimates of the state and/or output over the entire prediction horizon N from time $t + 1$ until time $t + N$, and can only make these predictions based on information up to and including the current time

t. Equations (6) can be used to obtain $\hat{x}_{t+1|t}$, $\hat{y}_{t+1|t}$. Optimal state/output estimates from instant $t+2$ to$t+N$ can be obtained as follows

$$\hat{x}_{t+i+1|t} = A\hat{x}_{t+i|t} + \sum_{j=1}^{s} A_j \hat{x}_{t+i-j|t-j-1} + Bu_{t-h+i|t} + I\hat{r}_{t+i}, \tag{11}$$

$$\hat{y}_{t+i|t} = G\hat{x}_{t+i|t}, i = \overline{1, N}. \tag{12}$$

In the above the notation $u_{t-h+i|t}$ is used to distinguish the actual input at the instant $t+i$, namely u_{t-h+i}, from that used for prediction purposes, namely $u_{t-h+i|t}$.

Equation (11) can be expanded in terms of the initial state $\hat{x}_{t+1|t}$ and future control actions $u_{t-h+i|t}$ as follows

$$\hat{x}_{t+i|t} = A^{i-1}\hat{x}_{t+1|t} + \sum_{k=1}^{i-1} A^{i-k-1} \sum_{j=1}^{s} A_j \hat{x}_{t+k-j|t-j-1} + \\ + \sum_{k=1}^{i-1} A^{i-k-1} Bu_{t-h+k|t} + \sum_{k=1}^{i-1} A^{i-k-1} I\hat{r}_{t+k}, i = \overline{1, N}. \tag{13}$$

Now in terms of predicting the output, equation (12) can be expanded in terms of the above expression for $\hat{x}_{t+i|t}$, which results in series of equations that provide optimal output predictions. The key point to note is that each output prediction is a function of the initial state $\hat{x}_{t+1|t}$ and future inputs $u_{th+i|t}$ only:

$$\hat{y}_{t+i|t} = GA^{i-1}\hat{x}_{t+1|t} + G\sum_{k=1}^{i-1} A^{i-k-1} \sum_{j=1}^{s} A_j \hat{x}_{t+k-j|t-j-1} + \\ +G\sum_{k=1}^{i-1} A^{i-k-1} Bu_{t-h+k|t} + G\sum_{k=1}^{i-1} A^{i-k-1} I\hat{r}_{t+k}, i = \overline{1, N}. \tag{14}$$

These series of prediction equations can be stated in an equivalent manner using matrix vector notation. Denote

$$\hat{X}_t = \begin{bmatrix} \hat{x}_{t+1|t} \\ \vdots \\ \hat{x}_{t+N|t} \end{bmatrix}, \hat{X}_i^0 = \begin{bmatrix} \hat{x}_{t+1-i|t-i} \\ \vdots \\ \hat{x}_{t+N-i|t-i} \end{bmatrix}, i = \overline{1, s}, \hat{Y}_t = \begin{bmatrix} \hat{y}_{t+1|t} \\ \vdots \\ \hat{y}_{t+N|t} \end{bmatrix}, \hat{R}_t = \begin{bmatrix} \hat{r}_{t+1} \\ \vdots \\ \hat{r}_{t+N} \end{bmatrix},$$

$$U_{t-h} = \begin{bmatrix} u_{t-h+1|t} \\ \vdots \\ u_{t-h+N|t} \end{bmatrix}, \Psi = \begin{bmatrix} E_n \\ A \\ A^2 \\ \vdots \\ A^{N-1} \end{bmatrix}, \Lambda = \begin{bmatrix} G \\ GA \\ GA^2 \\ \vdots \\ GA^{N-1} \end{bmatrix},$$

$$\Psi_i^0 = \begin{bmatrix} 0 & 0 & 0 \dots 0 \\ A_i & 0 & 0 \dots 0 \\ AA_i & A_i & 0 \dots 0 \\ \vdots & \vdots & \vdots \ddots \vdots \\ A^{N-2}A_i & A^{N-3}A_i & \dots A_i \; 0 \end{bmatrix}, \Lambda_i^0 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ GA_i & 0 & 0 & \dots & 0 \\ GAA_i & GA_i & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ GA^{N-2}A_i & GA^{N-3}A_i & \dots & GA_i & 0 \end{bmatrix},$$

$$\mathrm{P} = \begin{bmatrix} 0 & 0 & 0 \dots 0 \\ B & 0 & 0 \dots 0 \\ AB & B & 0 \dots 0 \\ \vdots & \vdots & \vdots \ddots \vdots \\ A^{N-2}B & A^{N-3}B & \dots \; B \; 0 \end{bmatrix}, \Phi = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ GB & 0 & 0 & \dots & 0 \\ GAB & GB & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ GA^{N-2}B & GA^{N-3}B & \dots & GB & 0 \end{bmatrix},$$

$$S = \begin{bmatrix} 0 & 0 & 0 \dots 0 \\ I & 0 & 0 \dots 0 \\ AI & I & 0 \dots 0 \\ \vdots & \vdots & \vdots \ddots \vdots \\ A^{N-2}I & A^{N-3}I \dots & I \;\; 0 \end{bmatrix}, Q = \begin{bmatrix} 0 & 0 & 0 \dots 0 \\ GI & 0 & 0 \dots 0 \\ GAI & GI & 0 \dots 0 \\ \vdots & \vdots & \vdots \ddots \vdots \\ GA^{N-2}I & GA^{N-3}I \dots & GI \; 0 \end{bmatrix}. \quad (15)$$

Here E_n is the n-by-n identity matrix.

The predictive model (13)-(14) in matrix notation is as follows

$$\hat{X}_t = \Psi \hat{x}_{t+1|t} + \sum_{i=1}^{s} \Psi_i^0 \hat{X}_i^0 + \mathrm{P} U_{t-h} + S\hat{R}_t,$$
$$\hat{Y}_t = \Lambda \hat{x}_{t+1|t} + \sum_{i=1}^{s} \Lambda_i^0 \hat{X}_i^0 + \Phi U_{t-h} + Q\hat{R}_t. \quad (16)$$

4 Model Predictive Control Synthesis

It is proposed to use the following criterion in order to solve the posed problem

$$J(t) = \frac{1}{2} \sum_{k=1}^{N} \{ \| \hat{y}_{t+k|t} - \bar{y}_t \|_C^2 + \| u_{t-h+k|t} - u_{t-h+k-1|t} \|_D^2 \}, \quad (17)$$

where weighing matrices C and D are assumed to be symmetric and positive definite.

In case when the reference trajectory $\bar{y}_{t+k}$ is unknown for $k \geq 0$ it is reasonable to assume that $\bar{y}_{t+k} = \hat{y}_t$, i.e. the same reference point is held throughout the entire prediction horizon.

The summation terms in (17) can be expanded to offer a quadratic objective function in terms of $\bar{x}_{t+1|t}$ and U_{t-h}. Let

$$\bar{Y}_t = \begin{bmatrix} \bar{y}_{t+1} \\ \vdots \\ \bar{y}_{t+N} \end{bmatrix}.$$

Then using (16) we can get the following expression

$$\tfrac{1}{2} \textstyle\sum_{k=1}^{N} \| \hat{y}_{t+k|t} - \bar{y}_t \|_C^2 = \tfrac{1}{2} \| \hat{Y}_t - \bar{Y}_t \|_{\bar{C}}^2 = \tfrac{1}{2} U_{t-h}^{\mathrm{T}} \Phi^{\mathrm{T}} \bar{C} \Phi U_{t-h} +$$
$$+ U_{t-h}^{\mathrm{T}} [\Phi^{\mathrm{T}} \bar{C} \Lambda \hat{x}_{t+1|t} + \Phi^{\mathrm{T}} \bar{C} \textstyle\sum_{i=1}^{s} \Lambda_i^0 \hat{X}_i^0 - \Phi^{\mathrm{T}} \bar{C} \bar{Y}_t] + c_1, \quad (18)$$

where c_1 is a constant term that does not depend either on U_{t-h} or $\hat{x}_{t+1|t}$; and $\bar{C}$ is given by

$$\bar{C} = \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C \end{bmatrix}.$$

In a similar manner rearrange the second term of sum in (17)

$$\frac{1}{2}\sum_{k=1}^{N}\|u_{t-h+k|t}-u_{t-h+k-1|t}\|_D^2=\frac{1}{2}U_{t-h}^{\mathrm{T}}\bar{D}U_{t-h}-u_{t-h+1|t}^{\mathrm{T}}Du_{t-h}+c_2, \quad (19)$$

where c_2 is a constant term that does not depend on u_{t-h+k} $(k=\overline{1,N})$; and $\bar{D}$ is given by

$$\bar{D}=\begin{bmatrix} 2D & -D & 0 & \dots & 0 \\ -D & 2D & -D & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & -D & 2D & -D \\ 0 & 0 & \dots & -D & 2D \end{bmatrix}.$$

Combining the above, the criteria function can be expressed as

$$J(t)=\frac{1}{2}U_{t-h}^{\mathrm{T}}FU_{t-h}+U_{t-h}^{\mathrm{T}}f+c_3. \quad (20)$$

Here c_3 is the combination of previous constant terms c_1 and c_2 and may be safely ignored. The terms F and f are given by

$$F=\Phi^{\mathrm{T}}\bar{C}\Phi+\bar{D}, f=\Gamma\begin{bmatrix}\hat{x}_{t+1|t}\\ \sum_{i=1}^{s}\Lambda_i^0\hat{X}_i^0\\ \bar{Y}_t\end{bmatrix}-\begin{bmatrix}Du_{t-h}\\0\\ \vdots\\0\end{bmatrix}, \Gamma=\left[\,\Phi^{\mathrm{T}}\bar{C}\;\Lambda\bar{C}Q\;-\Phi^{\mathrm{T}}\bar{C}\,\right].$$

In the absence of constraints an analytical solution of the posed problem can be obtained from the condition $\frac{dJ}{dU_{t-h}}=0$ using vector derivative formulas, see e.g. [15]:

$$\frac{\partial J}{\partial U_{t-h}}=\frac{\partial J}{\partial U_{t-h}}\left[\tfrac{1}{2}U_{t-h}^{\mathrm{T}}FU_{t-h}+U_{t-h}^{\mathrm{T}}f+c_3\right]=$$
$$=\frac{1}{2}\frac{\partial(trFU_{t-h}Ut-h^{\mathrm{T}})}{\partial U_{t-h}}+\frac{\partial(Ut-h^{\mathrm{T}}f)}{\partial U_{t-h}}=\frac{1}{2}[F^{\mathrm{T}}U_{t-h}+FU_{t-h}]+f=0. \quad (21)$$

As the matrix F is symmetric, the equation (21) can be expressed as follows

$$FU_{th}+f=0.$$

So, the criteria function can be rearranged as

$$U_{t-h}^{*}=-(\Phi^{\mathrm{T}}\bar{C}\Phi+\bar{D})^{-1}(\Phi^{\mathrm{T}}\bar{C}\Lambda\hat{x}_{t+1|t}+\Phi^{\mathrm{T}}\bar{C}Q\hat{R}_t-\Phi^{\mathrm{T}}\bar{C}\bar{Y}_t)-\begin{bmatrix}Du_{t-h}\\0\\ \vdots\\0\end{bmatrix},$$

and the optimal predictive control has the form:

$$u_{t-h+1|t}^{*}=\left[\,E_n\;0\dots0\,\right]U_{t-h}^{*}.$$

Optimization of the model with constraints (4), (5) can be performed numerically using Matlab function quadprog.

5 Conclusion

The Model Predictive Control of the system allowing to state and input delays with unknown input is solved, guaranteeing constraints satisfaction and feasibility. The problem of the MPC synthesis is solved without the extension of the state space. The extrapolator is offered to use in order to obtain predicted values of the system output.

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