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Numerical Simulation of Oil Pool Boundary Evolution

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Abstract. The study of spatial distribution of hydrocarbon resources and forecasting their geographical location is of great importance for the most complete recovery of hydrocarbons from deposits. The present study gives new mathematical results in the theory of stratified fluid flow in a porous medium. This paper analyzes the evolution of oil pool boundary basing on vortex numerical model for movement of the boundary separating fluids of different densities. It presents the investigation of how the location of light fluid regarding the heavier fluid influences on the changes in the boundary between two media in case of various shifting of the well.

INTRODUCTION

The part of oil reserves difficult to recover is constantly increasing. According to expert evaluation globally such reserves exceed 1 trillion tons and the developed industrial economies consider them not only a reserve for oil production but also a main source of it in the future. The detection of non-hydrocarbons in oil pools becomes crucial, i. e. underground water, which is often used as a displacement agent for injection into oil pools, thus enhancing the oil recovery out of the pool.

Naturally, management of new technological processes and rigorous assessment of their effectiveness is almost impossible without knowing the laws of oil, gas and water motion to wells. In addition, a science-based system of developing oil fields of various types is required.

In the 40-s of the last century Polubarinova-Kochina [1, 2] and Kufarev [3] investigated the issue of the evolution of the oil pool surrounded by water when oil was being pumped out of the well located inside the pool. Their mathematical investigation showed that through time the oil boundary remained to be an algebraic curve of the forth degree. In 1981 Richardson found the infinite series of integrals for oil region movement equation, and thus he obtained the effective ways of solution.

Recently, with the growth of the oil industry, predicting spatial distribution of hydrocarbons using modern methods of evaluation attracts attention of oil companies and researchers.

Thus, for example, spatial distribution of hydrocarbons in the length of an oil reservoir is modelled by means of the fast Fourier transformation (FFT) method [4]. This fractal model allows obtaining not only the size of undiscovered petroleum accumulations, but also their possible whereabouts. In the study of wells it is also necessary to determine physical and geological characteristics of layers [5, 6], as solving the problem of oil fields development can only be based on a comprehensive geological and hydrodynamic analysis.

In the present paper we introduce a new method for predicting the evolution path of an oil bed. Namely, we suggest a numerical method based on replacing the interface between two fluids by an equivalent fluidized bed with a variable time circulation. To apply this model, it is necessary to know the intensity of vortices in the layer replacing the interface between two immiscible liquids. The formula of generating vortices in the layer is obtained

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from the fundamental equations of hydrodynamics. The first calculations of fluidized beds were carried out by Lagally [7] and Rosenhead [8]. At that time, a flow at a constant rate suffering a break when passing through a fluidized bed was studied. Such interface between two fluids flowing at different rates was not sustainable [9]. In case this problem is solved analytically, assuming small perturbations, it turns out to be only the initial stage of the instability of the interface development which develops over time exponentially. Vortex numerical simulation [10] presented in this article shows how the movement of the corresponding vortex layer within considerable periods of time occurs in reality.

One should note that this oil well task was not considered to be the task about movement of the line separating fluids of different densities. This task was solved as a simplified mathematical assignment when the velocity potential at the oil boundary was considered to be zero; all movements were caused by the influence of the source (drainage). Regarding vortex numerical model [10] the oil well task is considered to be the task of double-layer fluid motion, which is more relevant to the reality. The gravity has no significance for this task since it is perpendicular towards the motion plane. But there is a field of convective accelerations from the source and it makes the line separating fluids of different densities to swirl.

NUMERICAL SIMULATION

Let the source be located at the point of reference, and its power equals to J, then the complex potential (for two-dimensional task) will be the following:

$$W(z) = \frac{J}{2\pi} \ln(z). \tag{1}$$

The velocity and convective acceleration $A = (V, \Delta)V$ from the source will equal to

$$\dot{z} = \left(\frac{dW}{dz}\right)^* = \frac{J}{2\pi}\frac{z}{|z|^2}; \qquad A = \frac{dW}{dz}\left(\frac{d^2W}{dz^2}\right)^* = -\frac{J^2}{4\pi^2}\frac{z}{|z|^4}. \tag{2}$$

Since A convective acceleration is proportional to squared power of the source, then it will be the same for the source and drainage and it will be always forwarded towards the center of the source. Should z distances be dimensionless values relevant to the typical dimensions of R_0 oil pool, then the equation of motion (Darcy law [11]) will be the following:

$$\mathbf{V} = -k\left[\frac{\nabla p}{\rho\tilde{g}} + \frac{z}{|z|^4}\right], \tag{3}$$

where $\tilde{g} = J^2 / 4\pi^2 R_0^3$; k – coefficient describing the filtration properties of soil and it is of the velocity dimensions [2, 12]. Basing on equation (3) one can estimate vortex circulation within the boundary of separation of densities. For this purpose (3) is divided by ρ density and integrated over the outline covering the line of separation. When the outline is contracted to the point, the following is obtained:

$$\frac{d\Gamma}{ds} = -2kR_0 \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\left(\boldsymbol{\tau}, \frac{z}{|z|^4}\right), \tag{4}$$

where τ – tangent vector to the line of separation.

Since the velocity of fluid flow equals to the sum of velocities from the source and from the vortex system of the separation line, then the kinematic equation will be the following:

$$\frac{dz}{dt} = \frac{J}{2\pi R_0}\frac{z}{|z|^2} + \frac{1}{2\pi R_0} Q\Gamma \,. \tag{5}$$

Components of Q matrix binding the vortex circulations and velocity points on the separation lines are defined in this case by dimensionless formula:

$$Q_{n,m} = \left(\frac{1}{i}\frac{1}{z_n - z_m}\right)^* . \tag{6}$$

Equations (4) and (5) can be put into dimensionless formula and can be made standard through it for them to be free from J and k values. For this kR_0 is taken as the velocity circulation unit, and the distance unit will be: $R_0 = J/k$. Then the standard equations of the task will be the following:

$$\frac{d\Gamma}{ds} = -2\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\left(\boldsymbol{\tau}, \frac{z}{|z|^4}\right), \qquad \frac{dz}{d\tau} = \pm\frac{z}{|z|^2} + Q\Gamma \,. \tag{7}$$

Here «+» is used for the source and «–» is used for drainage. τ dimensionless time is related to t real time through correlations (8):

$$\tau = \frac{k^2}{2\pi J} t, \qquad t = \frac{2\pi J}{k^2}\tau \,. \tag{8}$$

RESULTS OF CALCULATIONS

Standard equations (7) allow investigating various modes of oil pool outlines movement. Additionally to the mode of the source and drainage they allow considering cases when there is light or heavy fluid nearby the source.

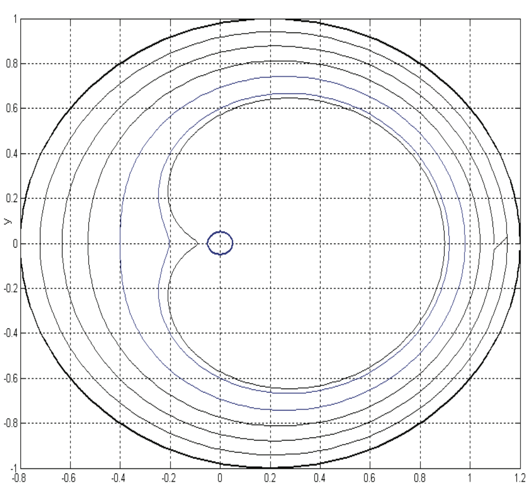

FIGURE 1. Changing the round outline of the fluid influenced only by drainage in the point of reference, ρ=const, $\tau_1 = \tau_2 = .. = \tau_7 = 0.05$

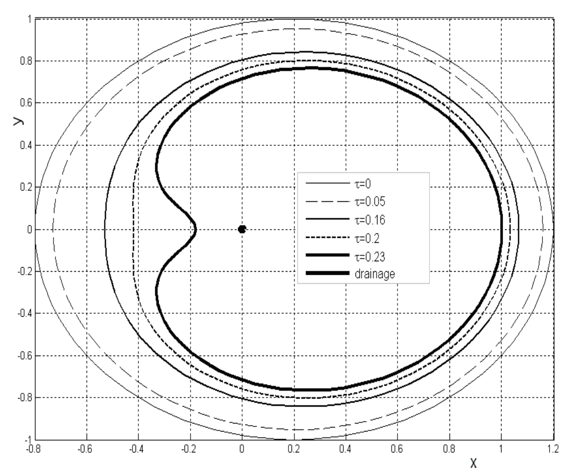

FIGURE 2. Evolution of the separation line for density ratio $\rho_1/\rho_2 = 0.8$

Figure 2 shows time positions of the interface line for the case when a lighter fluid (oil) is interfacing the well. Water breaks into the well much earlier than the oil is being pumped out. Figure 3 and Figure 4 show how the water is pumped out from under oil in case of different location of the well in respect to the pool center. Vortexes on the separation line are whirling to the different direction and the evolution of the boundary is different: water is fully pumped out from under oil. Thus the premature water breakthrough happens and water finds the shortest way to the production well through a narrow channel (Fig. 3).

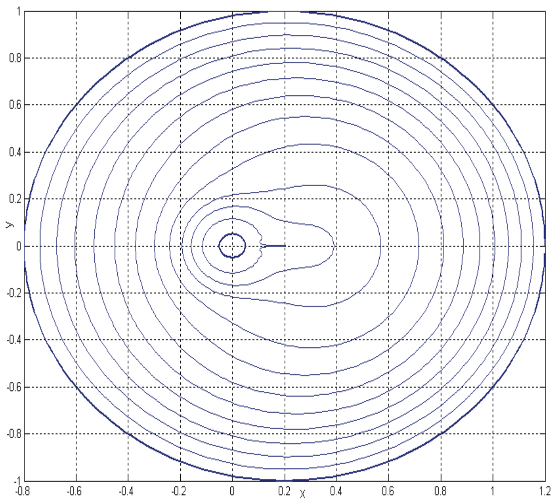

FIGURE 3. Changes in the boundary through time when heavier fluid is pumped out $\rho_2/\rho_1 = 0.7$

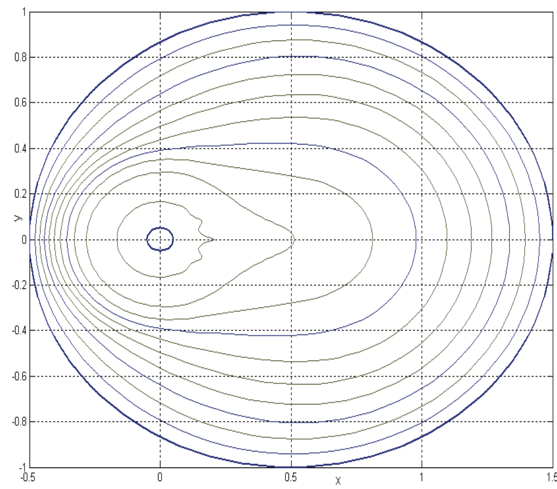

FIGURE 4. Position of the separation lines when the well is shifted greater from the pool center ($\rho_2/\rho_1 = 0.7$)

The similar situation is observed if the ratio of densities makes $\rho_2/\rho_1 < 0.75$. Should this ratio approach one, there will not be enough time for water to be pumped out fully before oil breaks through into the well.

For oil and water the density ratio is within $0.79 < \rho_2/\rho_1 < 0.95$, and within these ranges the interface line behaves in the way shown on Fig. 5 and Fig. 6.

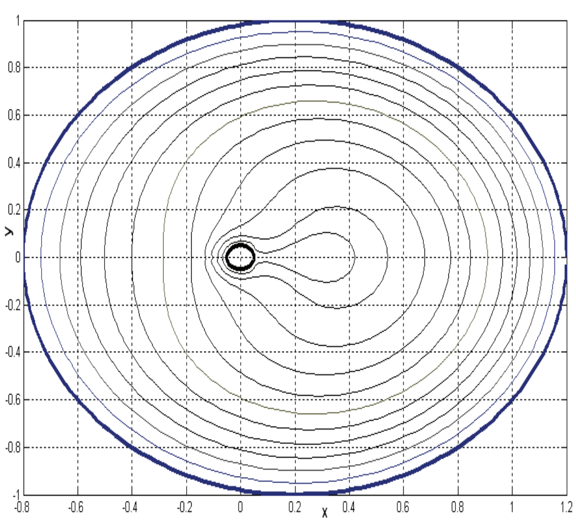

FIGURE 5. Positions of the interface when the water is pumped out from under oil ($\rho_2/\rho_1 = 0.93$)

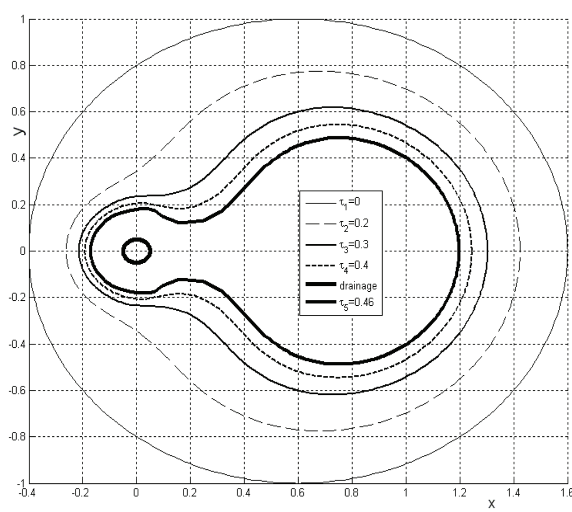

FIGURE 6. Interface positions when water is pumped out from under oil when the well is shifted greater in the respect of the pool center ($\rho_2/\rho_1 = 0.93$)

CONCLUSION

A new method for modelling distribution of hydrocarbons proposed in this article is based on understanding the theory of ground water motion in a porous medium and is obtained by replacing the interfaces between filtered fluids with different densities by moving vortex layers. In comparison with the theory operating the concept of a free boundary which is difficult to realize, the proposed vortex model is characterized by simplicity of calculations. The results of numerical modelling suggest the applicability of the conducted research in the study of gas and chemical advance recovery methods and can be recommended for use in predicting locations of wells in order to reduce the likelihood of premature water breakthroughs.

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