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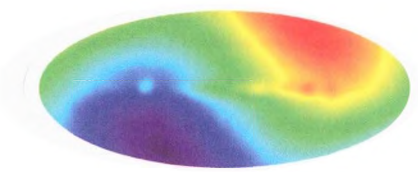

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Anisotropic exponential cosmological solutions in the 4th and 5th orders of Lovelock gravity

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In the 4th and 5th orders of Lovelock gravity exponential cosmological solutions of Bianchi-I type involving the lower orders are obtained.

Keywords: Modified gravity; metric of Bianchi-I type.

In Lovelock theory[1] gravitational field equations are

$$\sum_{p=0}^{\left[\frac{n-1}{2}\right]} \alpha_p G^{(p)\mu}{}_{\nu} = \frac{8\pi G}{c^4} T^{\mu}{}_{\nu},$$

where α_p are arbitrary constants, $G^{(p)\mu}{}_{\nu}$ is p-th Lovelock tensor:

$$G^{(p)\mu}{}_{\nu} = \delta^{\mu\sigma_1\sigma_2\ldots\sigma_{2p}}_{\nu\lambda_1\lambda_2\ldots\lambda_{2p}} \prod_{i=1}^{p} R^{\lambda_{2i-1}\lambda_{2i}}{}_{\sigma_{2i-1}\sigma_{2i}},$$

n is the space-time dimensionality, square brackets denote the integer part of a fraction, $\delta^{\cdots}_{\cdots}$ is the multi-index δ-symbol (equal to 1, when lower indices form an even transposition of upper ones, equal to -1, when odd, and equal to zero in other cases).

Such a theory have extra spatial dimensions as an obligate condition. But if we have extra dimensions, why are they invisible? For the explanation we should find anisotropic cosmological solutions. Power-law solutions[2,3] have shortcoming: they involve only one, p-th, order of Lovelock gravity, all the other orders, the upper and the lower, are excluded.

In opposition to this, exponential solutions involve p-th and the lower orders. For the 2nd and 3rd orders such solutions were obtained in Ref. 4.

In the 4th order equations $\alpha_1 G^{(1)}_{\mu\nu}+\alpha_2 G^{(2)}_{\mu\nu}+\alpha_3 G^{(3)}_{\mu\nu}+\alpha_4 G^{(4)}_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ in presence of matter with EoS $p = w\rho$ have solution

$$g_{\mu\nu} = \mathrm{diag}\{-1, e^{2H_1 t}, e^{2H_2 t}, ..., e^{2H_n t}\},$$

where Hubble parameters H_i meet the conditions (here $\sigma_s \equiv \sum_i H_i^s$, $\kappa_0 \equiv 8\pi G\varepsilon_0/c^4$)

$$\begin{aligned}
\sigma_1 &= 0,\\
\sigma_6 &= -\frac{1}{8}\sigma_2^3 + \frac{3}{4}\sigma_4\sigma_2 + \frac{1}{3}\sigma_3^2 + \frac{\alpha_2}{24\alpha_3}\left(\sigma_2^2 - 2\sigma_4\right) - \frac{\alpha_1}{32\alpha_3}\sigma_2 - \frac{1-7w}{384\alpha_3}\kappa_0,\\
\sigma_8 &= -\frac{1}{16}\sigma_2^4 + \frac{1}{4}\sigma_4\sigma_2^2 + \frac{\alpha_2}{36\alpha_3}\left(\sigma_2^3 - 2\sigma_4\sigma_2\right) - \frac{\alpha_1}{48\alpha_3}\sigma_2^2 - \frac{1-7w}{576\alpha_3}\sigma_2\kappa_0 +\\
&\quad + \frac{8}{15}\sigma_5\sigma_3 + \frac{1}{4}\sigma_4^2 - \frac{\alpha_2}{2880\alpha_4}\left(\sigma_2^2 - 2\sigma_4\right) + \frac{\alpha_1}{2880\alpha_4}\sigma_2 + \frac{1-5w}{23040\alpha_4}\kappa_0.
\end{aligned}$$

In the 5th order we have solution, where Hubble parameters meet the conditions

$$\begin{aligned}
\sigma_1 &= 0,\\
\sigma_8 &= \frac{9w-1}{23040\alpha_4}\kappa_0 - \frac{\alpha_1}{1440\alpha_4}\sigma_2 + \frac{\alpha_2}{960\alpha_4}\left(\sigma_2^2 - 2\sigma_4\right) + \frac{\alpha_3}{720\alpha_4}\left(-3\sigma_2^3 + \right.\\
&\quad \left. + 18\sigma_4\sigma_2 + 8\sigma_3^2 - 24\sigma_6\right) + \frac{1}{48}\sigma_2^4 - \frac{1}{4}\sigma_4\sigma_2 - \frac{2}{9}\sigma_3^2\sigma_2 + \frac{2}{3}\sigma_6\sigma_2 +\\
&\quad + \frac{8}{15}\sigma_5\sigma_3 + \frac{1}{4}\sigma_4^2,\\
\sigma_{10} &= -\frac{7w-1}{64\cdot 8!\alpha_5}\kappa_0 + \frac{3\alpha_1}{16\cdot 8!\alpha_5}\sigma_2 - \frac{\alpha_2}{4\cdot 8!\alpha_5}\left(\sigma_2^2 - 2\sigma_4\right) - \frac{\alpha_3}{4\cdot 8!\alpha_5}\left(-3\sigma_2^3 + \right.\\
&\quad \left. + 18\sigma_4\sigma_2 + 8\sigma_3^2 - 24\sigma_6\right) - \frac{105}{8!}\sigma_2^5 + \frac{2100}{8!}\sigma_4\sigma_2^3 + \frac{2800}{8!}\sigma_3^2\sigma_2^2 -\\
&\quad - \frac{8400}{8!}\sigma_6\sigma_2^2 - \frac{1}{3}\sigma_5\sigma_3\sigma_2 - \frac{6300}{8!}\sigma_4^2\sigma_2 + \frac{25200}{8!}\sigma_8\sigma_2 - \frac{5600}{8!}\sigma_4\sigma_3^2 +\\
&\quad + \frac{19200}{8!}\sigma_7\sigma_3 + \frac{16800}{8!}\sigma_6\sigma_4 + \frac{1}{5}\sigma_5^2.
\end{aligned}$$

Finally, I suggest similar conditions for the arbitrary order, but, unfortunately, I haven't prove this assumption yet.

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