Gaussian Approximation of Distribution of States of the Retrial Queueing System with r-Persistent Exclusion of Alternative Customers

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Abstract. In this paper, we study the retrial queueing system with two arrival processes and two orbits with r-persistent exclusion of alternative customers by method of asymptotic analysis under condition of long delay. Stationary probability distribution of server states and values of asymptotic means of the number of customers in the orbits are obtained.

Keywords: Retrial queuing system · r-persistent exclusion of alternative customers · Orbit · Asymptotic analysis

1 Introduction

Queueing systems, in which arriving customers who find all servers and waiting positions (if any) occupied may retry for service after a period of time, are called Retrial queues [1–3]. A review of the main results on this topic can be found in [4]. Retrial queues have been widely used as mathematical models of different communication systems: shared bus local area networks operating under transmission protocols like CSMA/CD (Carrier Sense Multiple Access with Collision Detection), cellular mobile networks, computer and communications networks, IP networks. Priority control is also wildely used in production practice, transportation management, etc. Several authors including Choi, B.D. [6–10], Rengnanathan, N. [11], Krishna Reedy, G.V. [12], Zhu, Y.J. [13] have studied priority queues. These authors and several others have studied single server or multi-server queues with two or more priority classes under preemptive or non-preemptive priority rules. Choi, B.D. We analyzed a M/G/1 retrial queueing systems with two types of calls and finite capacity, Moreno, P. considered an M/G/1 retrial queue with recurrent customers and general retrial times [14]. In [15] retrial queue system M/G/1 with queue length r and the priority of the primary customers is studied. In [16], generalization of [15] is implemented.

In this paper, we study the retrial queueing system $M^{(2)}/M^{(2)}/1$ with r-persistent exclusion of alternative customers.

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2 Problem Statement

We consider retrial queueing system with two arrival processes and two orbits with r-persistent collision of alternative customers (Fig. 1).

We assume that two arrival processes to the system are described by the stationary Poisson process with intensity λ_1 and λ_2, respectively. Customer, which finds the free server, occupies it during a random time which is exponentially distributed with intensity μ_1 and μ_2, respectively. If, at the moment of arrival, customer of the first type finds the server busy with a customer of the first type, then it goes to the orbit 1 (the orbit for customer of the first type), where it performs a random delay with duration determined by exponential distribution with intensity σ_1. From the orbit 1, after the random delay, the customer tries to occupy the server again. If at the time of arrival, customer of the first type finds the server busy with a customer of the second type, then the arrived customer with probability r_1 replaces the customer, which was in service, and occupies the server, and with probability $1 - r_1$ it goes to the orbit 1.

The same goes for the second type customer. If at the moment of arrival, customer of the second type finds the server busy with a customer of the second type, then it goes to the orbit 2 (the orbit for customer of the second type), where it performs a random delay with duration determined by exponential distribution with intensity σ_2. From the orbit 2, after the random delay, the customer tries to occupy the server again. If, at the time of arrival, customer of the second type finds the server busy with a customer of the first type, then an arrived customer

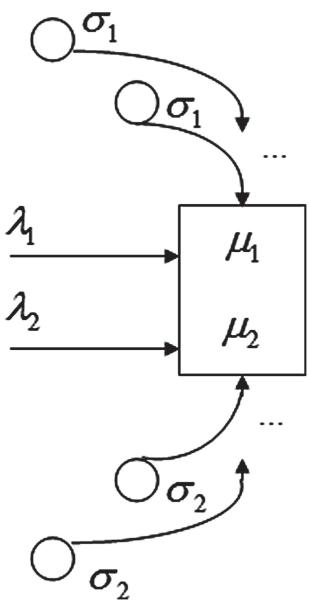

Fig. 1. Retrial queueing system $M^{(2)}/M^{(2)}/1$.

with probability r_2 replaces the customer, which was in service, and occupies the server, and with probability $1 - r_2$ it goes to the orbit 2.

Let $i_1(t)$ be the number of customers in the orbit 1 and $i_2(t)$ be the number of customers in the orbit 2, and the process $k(t)$ defines the server state at the moment t in the following way:

$$k(t) = \begin{cases} 0, & \text{if server is free,} \\ 1, & \text{if server is busy with a customer of the first type,} \\ 2, & \text{if server is busy with a customer of the second type.} \end{cases}$$

We would like to solve a problem of computation of stationary probability distribution of the number of customers in the orbits 1 and 2 and server state.

3 System of Kolmogorov Differential Equations

We consider Markovian process $\{k(t), i_1(t), i_2(t)\}$, $t \geq 0$.

Let us denote by $P\{k(t) = k, i_1(t) = i_1, i_2(t) = i_2\} = P_k(i_1, i_2, t)$ a probability that, at the moment t , the server in the state k and i_1 customers are in the orbit 1, i_2 customers are in the orbit 2.

We write system of differential Kolmogorovs equations for the probability distribution $\{P_0(i_1, i_2, t), P_1(i_1, i_2, t), P_2(i_1, i_2, t)\}$:

$$\begin{aligned}
\frac{\partial P_0(i_1, i_2, t)}{\partial t} =& - (\lambda_1 + \lambda_2 + i_1\sigma_1 + i_2\sigma_2)P_0(i_1, i_2, t) + \mu_1 P_1(i_1, i_2, t) + \mu_2 P_2(i_1, i_2, t), \\
\frac{\partial P_1(i_1, i_2, t)}{\partial t} =& - (\lambda_1 + \lambda_2 + \mu_1 + r_2 i_2\sigma_2)P_1(i_1, i_2, t) + (1 - r_2)\lambda_2 P_1(i_1, i_2 - 1, t) \\
&+ \lambda_1 P_0(i_1, i_2, t) + (i_1 + 1)\sigma_1 P_0(i_1 + 1, i_2, t) + \lambda_1 P_1(i_1 - 1, i_2, t) \\
&+ r_1\lambda_1 P_2(i_1, i_2 - 1, t) + r_1(i_1 + 1)\sigma_1 P_2(i_1 + 1, i_2 - 1, t), \qquad (1) \\
\frac{\partial P_2(i_1, i_2, t)}{\partial t} =& - (\lambda_1 + \lambda_2 + \mu_2 + r_1 i_1\sigma_1)P_2(i_1, i_2, t) + (1 - r_1)\lambda_1 P_2(i_1 - 1, i_2, t) \\
&+ \lambda_2 P_0(i_1, i_2, t) + (i_2 + 1)\sigma_2 P_0(i_1, i_2 + 1, t) + \lambda_2 P_2(i_1, i_2 - 1, t) \\
&+ r_2\lambda_2 P_1(i_1 - 1, i_2, t) + r_2(i_2 + 1)\sigma_2 P_1(i_1 - 1, i_2 + 1, t).
\end{aligned}$$

4 Equations for Partial Characteristic Function

We introduce the partial characteristic function in the following form:

$$H_k(u_1, u_2, t) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} e^{ju_1 i_1} e^{ju_2 i_2} P_k(i_1, i_2, t), \quad k = 0, 1, 2,$$

where $j = \sqrt{-1}$ is imaginary unit. We rewrite the system (1) for partial characteristic function.

We can rewrite system (1) as:

$$
\begin{aligned}
&-(\lambda_1+\lambda_2)H_0(u_1,u_2)+j\sigma_1\frac{\partial H_0(u_1,u_2)}{\partial u_1}+j\sigma_2\frac{\partial H_0(u_1,u_2)}{\partial u_2} \\
&\quad+\mu_1 H_1(u_1,u_2)+\mu_2 H_2(u_1,u_2)=0, \\
&-(\lambda_1+\lambda_2+\mu_1)H_1(u_1,u_2)+j\sigma_2 r_2\frac{\partial H_1(u_1,u_2)}{\partial u_2}-j\sigma_1 e^{-ju_1}\frac{\partial H_0(u_1,u_2)}{\partial u_1} \\
&\quad+(1-r_2)\lambda_2 e^{ju_2}H_1(u_1,u_2)+\lambda_1 H_0(u_1,u_2)+\lambda_1 e^{ju_1}H_1(u_1,u_2) \\
&\quad+r_1\lambda_1 e^{ju_2}H_2(u_1,u_2)-jr_1\sigma_1 e^{j(u_2-u_1)}\frac{\partial H_2(u_1,u_2)}{\partial u_1}=0, \\
&-(\lambda_1+\lambda_2+\mu_2)H_2(u_1,u_2)+j\sigma_1 r_1\frac{\partial H_2(u_1,u_2)}{\partial u_1}-j\sigma_2 e^{-ju_2}\frac{\partial H_0(u_1,u_2)}{\partial u_2} \\
&\quad+(1-r_1)\lambda_1 e^{ju_1}H_2(u_1,u_2)+\lambda_2 H_0(u_1,u_2)+\lambda_2 e^{ju_2}H_2(u_1,u_2) \\
&\quad+r_2\lambda_2 e^{ju_1}H_1(u_1,u_2)-jr_2\sigma_2 e^{j(u_1-u_2)}\frac{\partial H_1(u_1,u_2)}{\partial u_2}=0.
\end{aligned}
\tag{2}
$$

We will solve system (2) using the method of asymptotic analysis under condition of long delay ($\sigma \to 0$).

5 The First-Order Asymptotic Analysis

In system (2) we make substitutions:

$\sigma_m = \sigma\gamma_m$; $\sigma = \varepsilon$;$u_m = \varepsilon w_m, m = 1, 2$; $H_k(u_1, u_2) = F_k(w_1, w_2, \varepsilon)$, $k = 0, 1, 2$.

We can rewrite system (2) in the following form:

$$
\begin{aligned}
&-(\lambda_1+\lambda_2)F_0(w_1,w_2,\varepsilon)+j\gamma_1\frac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_1}+j\gamma_2\frac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_2} \\
&\quad+\mu_1 F_1(w_1,w_2,\varepsilon)+\mu_2 F_2(w_1,w_2,\varepsilon)=0, \\
&-(\lambda_1+\lambda_2+\mu_1)F_1(w_1,w_2,\varepsilon)+j\gamma_2 r_2\frac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_2}-j\gamma_1 e^{-j\varepsilon w_1}\frac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_1} \\
&\quad+(1-r_2)\lambda_2 e^{j\varepsilon w_2}F_1(w_1,w_2,\varepsilon)+\lambda_1 F_0(w_1,w_2,\varepsilon)+\lambda_1 e^{j\varepsilon w_1}F_1(w_1,w_2,\varepsilon) \\
&\quad+r_1\lambda_1 e^{j\varepsilon w_2}F_2(w_1,w_2,\varepsilon)-jr_1\gamma_1 e^{j\varepsilon(w_2-w_1)}\frac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_1}=0, \\
&-(\lambda_1+\lambda_2+\mu_2)F_2(w_1,w_2,\varepsilon)+j\gamma_1 r_1\frac{\partial F_2(w_1,w_2,\varepsilon)}{\partial w_1}-j\gamma_2 e^{-j\varepsilon w_2}\frac{\partial F_0(w_1,w_2,\varepsilon)}{\partial w_2} \\
&\quad+(1-r_1)\lambda_1 e^{j\varepsilon w_1}F_2(w_1,w_2,\varepsilon)+\lambda_2 F_0(w_1,w_2,\varepsilon)+\lambda_2 e^{j\varepsilon w_2}F_2(w_1,w_2,\varepsilon) \\
&\quad+r_2\lambda_2 e^{j\varepsilon w_1}F_1(w_1,w_2,\varepsilon)-jr_2\gamma_2 e^{j\varepsilon(w_1-w_2)}\frac{\partial F_1(w_1,w_2,\varepsilon)}{\partial w_2}=0.
\end{aligned}
\tag{3}
$$

Theorem 1. *Limiting values* $\{F_k(w_1, w_2)\}$ *of the solution* $\{F_k(w_1, w_2, \varepsilon)\}$ *of the system (3) have the following form:*

$$F_k(w_1, w_2) = R_k e^{jw_1x_1 + jw_2x_2},$$

where values R_0, R_1, R_2, x_1, x_2 *is the solution of the following system:*

$$\begin{aligned}
&-(\lambda_1 + \lambda_2 + \gamma_1 x_1 + \gamma_2 x_2)R_0 + \mu_1 R_1 + \mu_2 R_2 = 0, \\
&(\lambda_1 + \gamma_1 x_1)R_0 - (\lambda_2 + \mu_1 + r_2\gamma_2 x_2 - (1 - r_2)\lambda_2)R_1 + (r_1\lambda_1 + r_1\gamma_1 x_1)R_2 = 0, \\
&(\lambda_2 + \gamma_2 x_2)R_0 + (r_2\lambda_2 + r_2\gamma_2 x_2)R_1 - (\lambda_1 + \mu_2 + r_1\gamma_1 x_1 - (1 - r_1)\lambda_1)R_2 = 0, \\
&-\gamma_1 x_1 R_0 + (\lambda_1 + r_2\lambda_2 + r_2\gamma_2 x_2)R_1 + (r_1\gamma_1 x_1 + (1 - r_1)\lambda_1)R_2 = 0, \\
&-\gamma_2 x_2 R_0 + (r_2\gamma_2 x_2 + (1 - r_2)\lambda_2)R_1 + (\lambda_1 + r_1\lambda_1 + r_1\gamma_1 x_1)R_2 = 0.
\end{aligned} \tag{4}$$

6 The Second-Order Asymptotic Analysis

To find the asymptotic of the second order we must execute following substitute at system (2):

$$H_k(u_1, u_2) = H_k{}^{(2)}(u_1, u_2) \exp\left\{j\frac{u_1}{\sigma}x_1 + j\frac{u_2}{\sigma}x_2\right\}$$

$$\sigma_k = \gamma_k\sigma, \quad \sigma = \varepsilon^2, \quad u_k = \varepsilon w_k, \quad H_k{}^{(2)}(u_1, u_2) = F_k(w_1, w_2, \varepsilon).$$

We can rewrite system (2) as:

$$\begin{aligned}
&-(\lambda_1 + \lambda_2)F_0(w_1, w_2, \varepsilon) + j\gamma_1\varepsilon\frac{\partial F_0(w_1, w_2, \varepsilon)}{\partial w_1} + j\gamma_2\varepsilon\frac{\partial F_0(w_1, w_2, \varepsilon)}{\partial w_2} \\
&\quad + \mu_1 F_1(w_1, w_2, \varepsilon) + \mu_2 F_2(w_1, w_2, \varepsilon) - \gamma_1 x_1 F_0(w_1, w_2, \varepsilon) - \gamma_2 x_2 F_0(w_1, w_2, \varepsilon) = 0, \\
&-(\lambda_1 + \lambda_2 + \mu_1)F_1(w_1, w_2, \varepsilon) + j\gamma_2 r_2\varepsilon\frac{\partial F_1(w_1, w_2, \varepsilon)}{\partial w_2} - r_2\gamma_2 x_2 F_1(w_1, w_2, \varepsilon) \\
&\quad - j\gamma_1 e^{-j\varepsilon w_1}\varepsilon\frac{\partial F_0(w_1, w_2, \varepsilon)}{\partial w_1} + \gamma_1 x_1 e^{-j\varepsilon w_1}F_0(w_1, w_2, \varepsilon) \\
&\quad + (1 - r_2)\lambda_2 e^{j\varepsilon w_2}F_1(w_1, w_2, \varepsilon) + \lambda_1 F_0(w_1, w_2, \varepsilon) \\
&\quad + \lambda_1 e^{j\varepsilon w_1}F_1(w_1, w_2, \varepsilon) + r_1\lambda_1 e^{j\varepsilon w_2}F_2(w_1, w_2, \varepsilon) \\
&\quad - jr_1\gamma_1\varepsilon e^{j\varepsilon(w_2 - w_1)}\frac{\partial F_2(w_1, w_2, \varepsilon)}{\partial w_1} + r_1\gamma_1 x_1 e^{j\varepsilon(w_2 - w_1)}F_2(w_1, w_2, \varepsilon) = 0, \\
&-(\lambda_1 + \lambda_2 + \mu_2)F_2(w_1, w_2, \varepsilon) + j\gamma_1 r_1\varepsilon\frac{\partial F_2(w_1, w_2, \varepsilon)}{\partial w_1} - r_1\gamma_1 x_1 F_2(w_1, w_2, \varepsilon) \\
&\quad - j\gamma_2 e^{-j\varepsilon w_2}\varepsilon\frac{\partial F_0(w_1, w_2, \varepsilon)}{\partial w_2} + \gamma_2 x_2 e^{-j\varepsilon w_2}F_0(w_1, w_2, \varepsilon) \\
&\quad + (1 - r_1)\lambda_1 e^{j\varepsilon w_1}F_2(w_1, w_2, \varepsilon) + \lambda_2 F_0(w_1, w_2, \varepsilon) \\
&\quad + \lambda_2 e^{j\varepsilon w_2}F_2(w_1, w_2, \varepsilon) + r_2\lambda_2 e^{j\varepsilon w_1}F_1(w_1, w_2, \varepsilon) \\
&\quad - jr_2\gamma_2\varepsilon e^{j\varepsilon(w_1 - w_2)}\frac{\partial F_1(w_1, w_2, \varepsilon)}{\partial w_2} + r_2\gamma_2 x_2 e^{j\varepsilon(w_1 - w_2)}F_1(w_1, w_2, \varepsilon) = 0.
\end{aligned} \tag{5}$$

Theorem 2. *Limiting values* $\{F_k(w_1, w_2)\}$ *of the solution* $\{F_k(w_1, w_2, \varepsilon)\}$ *of the system (5) have the following form:*

$$F_k(w_1, w_2) = R_k \Phi(w_1, w_2),$$

where values R_0, R_1, R_2, x_1, x_2 *is the solution of the system (4).*

We write function $\Phi(w_1, w_2)$ *in the following form:*

$$\Phi(w_1, w_2) = \exp\left\{\frac{(jw_1)^2}{2}Q_{11} + \frac{(jw_2)^2}{2}Q_{22} + jw_1jw_2Q_{12}\right\},$$

where values Q_{11}, Q_{12}, Q_{22} *is the solution of the following system:*

$$\begin{aligned}
&Q_{11}(\gamma_1R_0y_0 - \gamma_1R_0y_1 - r_1\gamma_1R_2y_1 + r_1\gamma_1R_2y_2 - \gamma_1R_0 - r_1\gamma_1R_2)\\
&+ Q_{12}(\gamma_2R_0y_0 + r_2\gamma_2R_1y_1 - \gamma_2R_0y_2 - r_2\gamma_2R_1y_2 + r_2\gamma_2R_1)\\
&= \lambda_1R_1y_1 - x_1\gamma_1R_0y_1 - r_1\gamma_1x_1R_2y_1 + (1-r_1)\lambda_1R_2y_2 + r_2\lambda_2R_1y_2\\
&+ r_2\gamma_2x_2R_1y_2 - \frac{1}{2}\gamma_1x_1R_0 - \frac{1}{2}\lambda_1R_1 - \frac{1}{2}r_2\lambda_2R_1 - \frac{1}{2}r_2\gamma_2x_2R_1\\
&- \frac{1}{2}r_1\gamma_1x_1R_2 - \frac{1}{2}(1-r_1)\lambda_1R_2,\\
&Q_{22}(\gamma_2R_0d_0 - \gamma_2R_0d_2 - r_2\gamma_2R_1d_2 + r_2\gamma_2R_1d_1 - \gamma_2R_0 - r_2\gamma_2R_1)\\
&+ Q_{12}(\gamma_1R_0d_0 + r_1\gamma_1R_2d_2 - \gamma_2R_0d_2 - r_1\gamma_1R_2d_1 + r_1\gamma_1R_2)\\
&= \lambda_2R_2d_2 - x_2\gamma_2R_0d_2 - r_2\gamma_2x_2R_1d_2 + (1-r_2)\lambda_2R_1d_1 + r_1\lambda_1R_2d_1\\
&+ r_1\gamma_1x_1R_2d_1 - \frac{1}{2}\gamma_2x_2R_0 - \frac{1}{2}\lambda_2R_2 - \frac{1}{2}r_1\lambda_2R_2 - \frac{1}{2}r_1\gamma_1x_1R_2\\
&- \frac{1}{2}r_2\gamma_2x_2R_1 - \frac{1}{2}(1-r_2)\lambda_2R_1,\\
&Q_{11}(\gamma_1R_0z_0^{(0)} - \gamma_1R_0z_1^{(0)} - r_1\gamma_1R_2z_1^{(0)} + r_1\gamma_1R_2z_2^{(0)} + r_1\gamma_1R_2)\\
&+ Q_{12}(\gamma_2R_0z_0^{(0)} + r_2\gamma_2R_1z_1^{(0)} - \gamma_2R_0z_2^{(0)} - r_2\gamma_2R_1z_2^{(0)} + r_2\gamma_2R_1 + \gamma_1R_0z_0^{(1)}\\
&- \gamma_1R_0z_1^{(1)} - r_1\gamma_1R_2z_1^{(1)} + r_1\gamma_1R_2z_2^{(1)} + r_1\gamma_1R_2 - \gamma_1R_0 - \gamma_2R_0)\\
&+ Q_{22}(\gamma_2R_0z_0^{(1)} - \gamma_2R_0z_2^{(1)} - r_2\gamma_2R_1z_2^{(1)} + r_2\gamma_2R_1z_1^{(1)} + r_2\gamma_2R_1)\\
&= \lambda_1R_1z_1^{(0)} - x_1\gamma_1R_0z_1^{(0)} - r_1\gamma_1x_1R_2z_1^{(0)} + (1-r_1)\lambda_1R_2z_2^{(0)} + r_2\lambda_2R_1z_2^{(0)}\\
&+ r_2\gamma_2x_2R_1z_2^{(0)} + \lambda_2R_2z_2^{(1)} - x_2\gamma_2R_0z_2^{(1)} - r_2\gamma_2x_2R_1z_2^{(1)} + (1-r_2)\lambda_2R_1z_1^{(1)}\\
&+ r_1\lambda_1R_2z_1^{(1)} + r_1\gamma_1x_1R_2z_1^{(1)} + r_1\gamma_1x_1R_2 + r_2\gamma_2x_2R_1,
\end{aligned}$$

Values y_0, y_1, y_2; d_0, d_1, d_2; $z_0^{(0)}$, $z_1^{(0)}$, $z_2^{(0)}$; $z_0^{(1)}$, $z_1^{(1)}$, $z_2^{(1)}$ *are the solutions of the system (6)-(9), respectively.*

$$\begin{aligned}
&-(a_1 + a_2)y_0 + a_1y_1 + a_2y_2 = \lambda_1 - a_1, \qquad (6)\\
&\mu_1y_0 - (\mu_1 + a_2r_2)y_1 + r_2a_2y_2 = \lambda_1 + r_2a_2,\\
&\mu_2y_0 + r_1a_1y_1 - (\mu_2 + a_1r_1)y_2 = \lambda_1 - r_1a_1.
\end{aligned}$$

$$
\begin{aligned}
&-(a_1+a_2)d_0+a_1d_1+a_2d_2=\lambda_2-a_2,\\
&\mu_1d_0-(\mu_1+a_2r_2)d_1+r_2a_2d_2=\lambda_2-r_2a_2,\\
&\mu_2d_0+r_1a_1d_1-(\mu_2+a_1r_1)d_2=\lambda_2+r_1a_1.
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&-(a_1+a_2)z_0^{(0)}+a_1z_1^{(0)}+a_2z_2^{(0)}=\lambda_2-a_2,\\
&\mu_1z_0^{(0)}-(\mu_1+a_2r_2)z_1^{(0)}+r_2a_2z_2^{(0)}=\lambda_2-r_2a_2,\\
&\mu_2z_0^{(0)}+r_1a_1z_1^{(0)}-(\mu_2+a_1r_1)z_2^{(0)}=\lambda_2+r_1a_1.
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&-(a_1+a_2)z_0^{(1)}+a_1z_1^{(1)}+a_2z_2^{(1)}=\lambda_1-a_1,\\
&\mu_1z_0^{(1)}-(\mu_1+a_2r_2)z_1^{(1)}+r_2a_2z_2^{(1)}=\lambda_1+r_2a_2,\\
&\mu_2z_0^{(1)}+r_1a_1z_1^{(1)}-(\mu_2+a_1r_1)z_2^{(1)}=\lambda_1-r_1a_1.
\end{aligned} \tag{9}
$$

7 Numerical Realization

For example, we take the parameters of arrival processes as:

$$\lambda_1=3, \lambda_2=2.$$

If the parameters of exponential law service are fixed as follow is:

$$\mu_1=10, \mu_2=20.$$

The parameters of a random delay with duration determined by exponential distribution are fixed in following form:

$$\sigma_1=0.02, \sigma_2=0.03.$$

So as $\sigma_1=\gamma_1\sigma$, $\sigma_2=\gamma_2\sigma$, then we will take $\gamma_1=2$, $\gamma_2=3$. Probability of displacement of the customer from the server by the customer of the first type $r_1=1$. Probability of displacement of the customer from the server by the customer of the second type $r_2=1$.

We have values of asymptotic means of the number of customers in the orbits with these values of parameters

$$x_1=100, x_2=44$$

and variance

$$Q_{11}=1.152, Q_{22}=0.308$$

and correlation coefficient

$$r=0.421.$$

8 Conclusion

In the paper we study the retrial queueing system $M^{(2)}$—$M^{(2)}$— 1 with r-persistent exclusion of alternative customers by method of asymptotic analysis under condition of long delay. Stationary probability distribution of server states and values of asymptotic means of the number of customers in the orbits are obtained. Two-demension marginal distribution of the number of customers in the orbit 1, in the orbit 2 is asymptotically Gaussian. We obtain the numerical realization for the condidered parameters.

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References

1. Artalejo, J.R.: A classified bibliography of research on retrial queues: progress in 1990–1999. Top **7**(2), 187–211 (1999)
2. Artalejo, J.R.: Accessible bibliography on retrial queues. Math. Comput. Model **30**(1–2), 1–6 (1999)
3. Artalejo, J.R.: Accessible bibliography on retrial queues: progress in 2000–2009. Math. Comput. Model **51**, 1071–1081 (2010)
4. Falin, G.I.: A survey of retrial queues. Queuing Syst. **7**, 127–167 (1990)
5. Falin, G.I., Artalejo, J.R., Martin, M.: On the single retrial queue with priority customers. Queueing Syst. **14**(3–4), 439–455 (1993)
6. Choi, B.D., Chang, Y.: Single server retrial queues with priority calls. Math. Comput. Model. **30**(3–4), 7–32 (1999)
7. Choi, B.D., Choi, K.B., Lee, Y.W.: M/G/1 retrial queueing systems with two types of calls and finite capacity. Queueing Syst. **19**, 215–229 (1995)
8. Choi, B.D., Park, K.K.: The M/G/1 retrial queue with bernoulli schedule. Queueing Syst. **7**(2), 219–227 (1990)
9. Choi, B.D., Shin, Y.W., Ahn, W.C.: Retrial queues with collision arising from unslotted CSMA/CD protocol. Queueing Syst. **11**(4), 335–356 (1992)
10. Choi, B.D., Park, K.K., Pearce, C.E.M.: An M/M/1 retrial queue with control policy and general retrial times. Queueing Syst. **14**(3–4), 275–292 (1993)
11. Rengnanathan, N., Kalayanaraman, R., Srinivasan, B.: A finite capacity single server retrial queue with two types of calls. Int. J. Inf. Manage. Sci. **13**(3), 47–56 (2002)
12. Reedy, G.V.K., Nadarajan, R.: A non-preemptive priority multi-server queueing system with general bulk service and hertergeneous arrivals. Comput. Oper. Res. **4**, 447–453 (1993)
13. Zhu, Y.J., Zhou, Z.H., Feng, Y.G.: M/G/1 retrial queue system with priority and repair. Zidonghua Xuebao/Acta Automatica Sin **34**(2), 195–201 (2008)
14. Moreno, P.: An M/G/1 retrial queue with recurrent customers and general retrial times. Appl. Math. Comput. **159**(3), 651–666 (2004)
15. Bocharov, P.P., Pavlova, O.I., Puzikova, D.A.: M/G/1r retrial queueing systems with priority of primary customers. Math. Comput. Model. **30**(3–4), 89–98 (1999)
16. DApice, C., De Simone, T., Manzo, R., Rizelian, G.: Priority service of primary customers in the M/G/1/r retrial queueing system with server searching for customers. Informacionny Processy **4**(1), 13–23 (2004)

17. Nazarov, A.A., Chernikova, J.E.: Study of RQ system M/ GI/ 1 with replacement in condition of long delay. Bull. Tomsk Polytech. Univ. Manage. Comput. Eng. Inf. **323**(5), 16–20 (2013)
18. Nazarov, A.A., Chernikova, J.E.: Research of the RQ- system M/ GI/ 1 with the priority of arriving customers by method of asymptotic cumulants. In: Proceedings of the 2nd Russian National Youth Scientific Conference with International Participation: Mathematics and Program Supporting of Information, Technical and Economical Systems, Tomsk (2014)