Research of Mathematical Model of Insurance Company in the Form of Queueing System with Unlimited Number of Servers Considering "Implicit Advertising"

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Abstract. This paper is devoted to the research of the model of insurance company with an unlimited insurance field and the parameter of arrival process of insurance risks, which depends on the risks that are already insured in the company. Using method of characteristic functions we got joint probability distribution of a two-dimensional stochastic process of a number of risks that are insured in the company and a number of benefit payments. We also got expressions for the expected values and variances of components of a two-dimensional process. Total benefit payments is reviewed and its distribution and numerical characteristic are found.

Keywords: Mathematical model · Insurance company · Benefit payments · Queueing system · Characteristic function

1 Introduction

In modern economics mathematical methods are widely used, both for solving practical tasks and for theoretical modeling of sociology-economic process. These models and their researches are getting pretty much of attention nowadays. Models of actuarial mathematics, which studies insurance, are not left aside either. Generally, all the papers devoted to the research of insurance company's mathematical models have such characteristics of a company's work as: expected values of risk's number, capital, bankruptcy possibility and so on. Thus, paper [1] is about model of insurance company takes into account advertising expenses, paper [2] is about model with possibility of reassurance of some company's risks. In [3] we got the distribution of number of benefit payments with random variable of the duration of the contract and the stationary Poisson arrival process of insurance risks. In [4] by using method of asymptotic analysis we have found probability distribution of two-dimensional process of a number benefit payments and a number of insurance risks, given that the arrival process of insurance risks is stationary Poisson. In this paper we research two-dimensional process of a

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number of benefit payments and a number of risks, that are insured in the company, in case when the parameter of the arrival process of insurance risks depends on the risks that are already insured in the company, which considers possibility of an implicit advertising, which is no doubt present in real life. Models with this arrival process of insurance risks are reviewed in [5], but methods of model's research are of different nature and the process of a number of benefit payments is ignored.

2 Mathematical Model and Formulation of the Problem

Let's review the model of insurance company with an unlimited insurance field [6] in the form of queuing system with an unlimited number of servers (Fig. 1). The validity of the insurance contract matches the server's duration of request handling. We will assume that risks are flowing into company, forming the arrival process with intensity that depends on a number of insured risks. Intensity of that arrival process will be determined by two components: parameter λ, which determines the arrival process of risks that come independently from insured ones, and parameter α, which determines the arrival process of risks that are under the influence of "implicit advertising". Every risk that has been in the company for the period of insurance police validity regadless of other risks generate a demand for insurance payment with γ intensity. And these requests also form the stationary Poisson process of events. Its natural to assume that benefit payment is determined by insured accident. We will assume that duration of the insurance contract for each risk located in the company will be random variable that is distributed by exponential law with parameter μ.

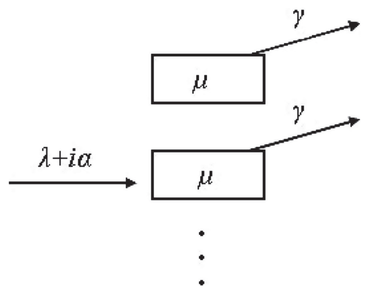

Fig. 1. Model of the insurance company in form of queuing system with an unlimited number of servers.

Designations: $n(t)$ — number of benefit payments during the time interval $[0, t]$, $i(t)$ — number of insurance risks located in the company at instant of time t, $P(i,n,t) = \mathbf{P}\{i(t) = i, n(t) = n\}$ — probability distribution of a two-dimensional process of a number of benefit payments and a number of insurance risks at instant of time t. The task is to find this distribution.

3 Joint Probability Distribution of Two-Dimensional Stochastic Process of a Number of Insurance Risks and a Number of Benefit Payments

Let's set up a system of Kolmogorov differential equations [7] for probability distribution $P(i,n,t)$ using the Δt method. First, the prelimit equalities:

$$\begin{aligned} P(i,n,t+\Delta t) = P(i,n,t)(1-(\lambda+i\alpha)\Delta t)(1-i\gamma\Delta t)(1-i\mu\Delta t) \\ +(\lambda+(i-1)\alpha)\Delta t P(i-1,n,t) \\ +i\gamma\Delta t P(i,n-1,t)+(i+1)\mu\Delta t P(i+1,n,t)+o(\Delta t). \end{aligned} \tag{1}$$

System of differential equations will have this form:

$$\begin{aligned} \frac{\partial P(i,n,t)}{\partial t} &= -[\lambda+i(\alpha+\mu+\gamma)]P(i,n,t)+(\lambda+(i-1)\alpha)P(i-1,n,t) \\ &\quad +(i+1)\mu P(i+1,n,t)+i\gamma P(i,n-1,t). \end{aligned} \tag{2}$$

To solve system (2) let's introduce function:

$$\sum_{i=0}^{\infty}\sum_{n=0}^{\infty} e^{jui} z^n P(i,n,t) = H(u,z,t), \tag{3}$$

that is characteristic by u and generating by z, where j is the imaginary unit. We will continue solving the task of determining the form of this function. Then, form system (2), considering properties of characteristic functions, we will get partial differential equation of first order for the function $H(u,z,t)$:

$$\begin{aligned} \frac{\partial H(u,z,t)}{\partial t} = -\lambda H(u,z,t)(1-e^{ju}) \\ +j\frac{\partial H(u,z,t)}{\partial u}(\alpha+\mu+\gamma-\alpha e^{ju}-\mu e^{-ju}-\gamma z). \end{aligned} \tag{4}$$

Solution for this differential equation is determined by solving of the following system of ordinary differential equations for characteristic curves [8]:

$$\frac{dt}{1} = \frac{du}{-j(\alpha+\mu+\gamma-\alpha e^{ju}-\mu e^{-ju}-\gamma z)} = \frac{dH(u,z,t)}{H(u,z,t)\lambda(e^{ju}-1)}. \tag{5}$$

We will start by finding the two first integrals of this system. First, let's take a look at this equation:

$$dt = \frac{du}{j(\alpha(e^{ju}-1)+\mu(e^{-ju}-1)-\gamma(1-z))}. \tag{6}$$

We will change variables $e^{ju}-1=v$, and, considering

$$u = \frac{\ln(v+1)}{j}, \quad du = \frac{dv}{j(v+1)}, \quad e^{-ju} = \frac{1}{v+1}, \quad j^2=-1, \tag{7}$$

the Eq. (6) will have this form:

$$dt = \frac{dv}{-(\alpha v^2 + (\alpha - \mu - \gamma(1-z))v - \gamma(1-z))}. \tag{8}$$

Let's take a look at right part of the last equation. We can write down

$$\alpha v^2 + (\alpha - \mu - \gamma(1-z))v - \gamma(1-z) = \alpha(v - v_1)(v - v_2), \tag{9}$$

where v_1 and v_2 are the roots of said quadratic equation. Let's write down expressions for v_1 and v_2:

$$\begin{aligned} v_1 &= \frac{1}{2}\left[\left(1 - \frac{\mu}{\alpha} - \frac{\gamma}{\alpha}(1-z)\right) + \sqrt{D}\right], \\ v_2 &= \frac{1}{2}\left[\left(1 - \frac{\mu}{\alpha} - \frac{\gamma}{\alpha}(1-z)\right) - \sqrt{D}\right], \end{aligned} \tag{10}$$

where discriminant is

$$D = \left(1 - \frac{\mu}{\alpha} - \frac{\gamma}{\alpha}(1-z)\right)^2 + 4\frac{\gamma}{\alpha}(1-z) > 0. \tag{11}$$

Therefore, roots v_1 and v_2 are real and different. Besides, given that natural condition $\alpha < \mu$, roots $v_1 > 0$ and $v_2 \leq 0$.

Thus, based on the foregoing, Eq. (8) could be written in this form:

$$dt = \frac{dv}{-\alpha(v - v_1)(v - v_2)}. \tag{12}$$

Solution for Eq. (12) will have this form:

$$t = \frac{1}{\alpha(v_1 - v_2)} \ln\left(\frac{v - v_2}{v - v_1}\right) - \ln(\tilde{C}_1), \tag{13}$$

which will be determining our first integral. Lets write down expression for constant $\tilde{C}_1$, we have:

$$\tilde{C}_1 = e^{-t}\left(\frac{v - v_2}{v - v_1}\right)^{\frac{1}{\alpha(v_1 - v_2)}}. \tag{14}$$

We denote $C_1 = \tilde{C}_1^{\alpha(v_1 - v_2)}$, then

$$C_1 = e^{-t\alpha(v_1 - v_2)}\left(\frac{v - v_2}{v - v_1}\right). \tag{15}$$

Other first integral will be found from equation:

$$\frac{dH(u,z,t)}{H(u,z,t)\lambda(e^{ju} - 1)} = \frac{du}{-j(\alpha + \mu + \gamma - \alpha e^{ju} - \mu e^{-ju} - \gamma z)}. \tag{16}$$

Let's make similar change of variables $e^{ju} - 1 = v$. We will introduce function $H_1(v, z, t) = H(u, z, t)$. Lets write down equation (16) for the function $H_1(v, z, t)$ while splitting variables:

$$\frac{dH_1(v, z, t)}{H_1(v, z, t)} = \frac{\lambda v dv}{-(\alpha v^2 + (\alpha - \mu - \gamma(1 - z))v - \gamma(1 - z))}, \tag{17}$$

or considering (9)

$$\frac{dH_1(v, z, t)}{H_1(v, z, t)} = \frac{\lambda v dv}{-\alpha(v - v_1(z))(v - v_2(z))}, \tag{18}$$

where v_1 and v_2 are determined by expressions (10). Let's write down the solution for Eq. (18), assuming that $v_1 = v_1(z)$ and $v_2 = v_2(z)$:

$$H_1(v, z, t) = C_2 \left[\frac{(v - v_2)^{v_2}}{(v - v_1)^{v_1}} \right]^{\frac{\lambda}{\alpha(v_1 - v_2)}}. \tag{19}$$

We will introduce arbitrary differentiable function $\phi(C_1) = C_2$. Then the general solution of Eq. (18) considering (15) will have this form:

$$H_1(v, z, t) = \phi \left[e^{-\alpha(v_1 - v_2)t} \left(\frac{v - v_2}{v - v_1} \right) \right] \left[\frac{(v - v_2)^{v_2}}{(v - v_1)^{v_1}} \right]^{\frac{\lambda}{\alpha(v_1 - v_2)}}. \tag{20}$$

We define particular solution with the help of initial conditions. To do this, we will write down value of function $H(u, z, t)$ at $t = 0$. Then

$$H(u, z, 0) = \sum_{i=0}^{\infty} \sum_{n=0}^{\infty} e^{jui} z^n P(i, n, 0) = \sum_{i=0}^{\infty} e^{jui} P(i), \tag{21}$$

because at the initial time (i.e. at the moment when insurance company starts their work) there were no benefit payment, which means $P(i, n, 0) = P(i)$, if $n = 0$, and $P(i, n, 0) = 0$, if $n > 0$.

Let's denote $H(u, z, 0) = G(u)$, then by using equation (4) we can write down the equation for function $G(u)$:

$$j(\mu - \alpha e^{ju}) \frac{dG(u)}{du} + \lambda e^{ju} G(u) = 0. \tag{22}$$

Solution will have this form:

$$G(u) = C_3 \left(e^{ju} - \frac{\mu}{\alpha} \right)^{-\frac{\lambda}{\alpha}}. \tag{23}$$

We will find constant C_3 from condition $G(0) = 1$. We have:

$$C_3 = \left(1 - \frac{\mu}{\alpha} \right)^{\frac{\lambda}{\alpha}}, \tag{24}$$

then

$$G(u) = \left(\frac{1 - \frac{\alpha}{\mu} e^{ju}}{1 - \frac{\alpha}{\mu}} \right)^{-\frac{\lambda}{\alpha}}. \tag{25}$$

Considering (20) we can write down

$$H_1(v, z, 0) = \phi \left(\frac{v - v_2}{v - v_1} \right) \left[\frac{(v - v_2)^{v_2}}{(v - v_1)^{v_1}} \right]^{\frac{\lambda}{\alpha(v_1 - v_2)}}, \tag{26}$$

or

$$\left(\frac{1 - \frac{\alpha}{\mu}(v + 1)}{1 - \frac{\alpha}{\mu}} \right)^{-\frac{\lambda}{\alpha}} = \phi \left(\frac{v - v_2}{v - v_1} \right) \left(\frac{(v - v_2)^{v_2}}{(v - v_1)^{v_1}} \right)^{\frac{\lambda}{\alpha(v_1 - v_2)}}. \tag{27}$$

Now the task is to define the form of function $\phi(.)$. Let's denote

$$x = \frac{v - v_2}{v - v_1}. \tag{28}$$

Then

$$\phi(x) = \left[\frac{\left(1 - \frac{\alpha}{\mu}\right)(v_2 - v_1)}{(1 - x) - \frac{\alpha}{\mu}\left(1 + v_2 - x(1 + v_1)\right)} \right]^{\frac{\lambda}{\alpha}} x^{\frac{\lambda v_2}{\alpha(v_2 - v_1)}}, \tag{29}$$

where it is considered that

$$v = \frac{v_2 - x v_1}{1 - x}. \tag{30}$$

Now we can write down the expression for function $\phi(.)$:

$$\begin{aligned} \phi \left[e^{-\alpha(v_1 - v_2)t} \left(\frac{v - v_2}{v - v_1} \right) \right] &= e^{\lambda v_2 t} \left[\left(1 - \frac{\alpha}{\mu}\right)(v_2 - v_1)(v - v_1) \right]^{\frac{\lambda}{\alpha}} \\ &\times \Big[(v - v_1) - (v - v_2) e^{\alpha(v_2 - v_1)t} - \\ -\frac{\alpha}{\mu} &\left((v - v_1)(1 + v_2) - (v - v_2)(1 + v_1) e^{\alpha(v_2 - v_1)t} \right) \Big]^{-\frac{\lambda}{\alpha}} \\ &\times \left(\frac{v - v_2}{v - v_1} \right)^{\frac{\lambda v_2}{\alpha(v_2 - v_1)}}. \end{aligned} \tag{31}$$

Accordingly, we will write down the expression for function $H_1(v, z, t)$, taking into account that v_1 and v_2 are functions of z and have the form (10). We have:

$$\begin{aligned} H_1(v, z, t) &= e^{\lambda v_2(z) t} \left[\left(1 - \frac{\alpha}{\mu}\right)(v_1(z) - v_2(z)) \right]^{\frac{\lambda}{\alpha}} \\ &\times \Big\{ (v_1(z) - v) \left[1 - \frac{\alpha}{\mu}(1 + v_2(z)) \right] \\ -(v_2(z) - v) &e^{\alpha(v_2(z) - v_1(z))t} \left[1 - \frac{\alpha}{\mu}(1 + v_1(z)) \right] \Big\}^{-\frac{\lambda}{\alpha}}. \end{aligned} \tag{32}$$

By passing from variable v to variable u, let's write down the expression for function $H(u,z,t)$:

$$\begin{aligned} H(u,z,t) = e^{\lambda v_2(z)t} \left[\left(1-\frac{\alpha}{\mu}\right)(v_1(z)-v_2(z))\right]^{\frac{\lambda}{\alpha}} \\ \times \left\{(v_1(z)-e^{ju}+1)\left[1-\frac{\alpha}{\mu}(1+v_2(z))\right]\right. \\ \left. -(v_2(z)-e^{ju}+1)e^{\alpha(v_2(z)-v_1(z))t}\left[1-\frac{\alpha}{\mu}(1+v_1(z))\right]\right\}^{-\frac{\lambda}{\alpha}}. \end{aligned} \tag{33}$$

Thus, resulting function (33) is characteristic function of two-dimensional stochastic process of a number of risks that are insured in the company and a number of benefit payments. Knowing this function, we can find one-dimensional marginal distributions of processes $i(t)$ and $n(t)$.

4 Probability Distributions of a Number of Insurance Risks and a Number of Benefit Payments

Let's suppose that in (33) $u = 0$, now we can get generating function of process $n(t)$:

$$\begin{aligned} H(0,z,t) = F(z,t) = e^{\lambda v_2(z)t} \left[\left(1-\frac{\alpha}{\mu}\right)(v_1(z)-v_2(z))\right]^{\frac{\lambda}{\alpha}} \\ \times \left\{v_1(z)\left[1-\frac{\alpha}{\mu}(1+v_2(z))\right]\right. \\ \left. -v_2(z)e^{\alpha(v_2(z)-v_1(z))t}\left[1-\frac{\alpha}{\mu}(1+v_1(z))\right]\right\}^{-\frac{\lambda}{\alpha}}. \end{aligned} \tag{34}$$

We write down characteristic function of process $i(t)$ by assuming that in (33) $z = 1$. Because of

$$v_1(1) = \frac{\mu}{\alpha} - 1,\ v_2(1) = 0,$$

we have

$$H(u,1,t) = G(u) = \left(\frac{1-\dfrac{\alpha}{\mu}}{1-\dfrac{\alpha}{\mu}\,e^{ju}}\right)^{\frac{\lambda}{\alpha}}. \tag{35}$$

Since the resulting characteristic function (35) does not depend of time, we can say that process of a number of insurance risks is stationary.

Let's find probability distributions for a number of insurance risks $P_1(i)$ and a number of benefit payments $P_2(n,t)$, by looking at numerical example. Figures 2 and 3 show distributions $P_1(i)$ and $P_2(n,t)$ for the following parameters: $\lambda = 0.6$, $\mu = 1$, $\alpha = 0.9$, $\gamma = 0.1$, $t = 1$.

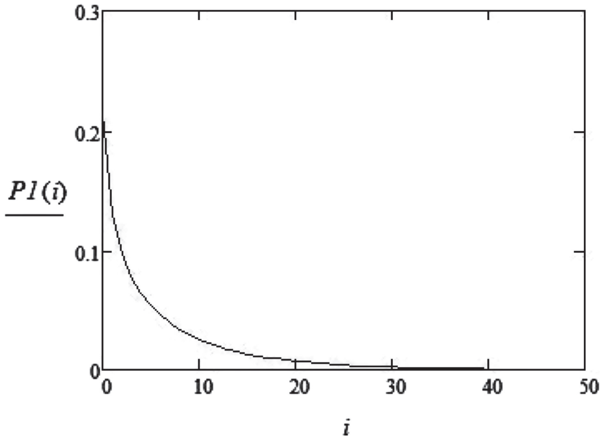

Fig. 2. Probability distribution of a number of insurance risks

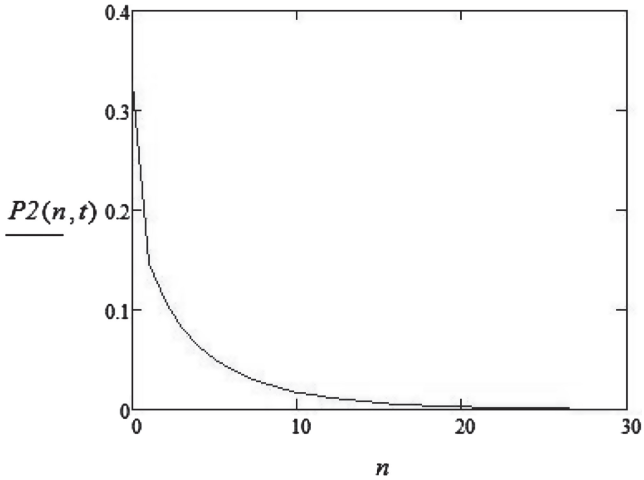

Fig. 3. Probability distribution of a number of benefit payments

5 Numerical Characteristics of a Number of Insured Risks and a Number of Benefit Payments

Now we can write down expected values for a number of risks and a number of benefit payments:

$$\mathsf{E}\{i(t)\} = \frac{1}{j}\frac{dG(u)}{du}\bigg|_{u=0} = \frac{\lambda}{\mu-\alpha}, \tag{36}$$

and

$$\mathsf{E}\{n(t)\} = \frac{\partial F(z,t)}{\partial z}\bigg|_{z=1} = \frac{\lambda\gamma}{\mu-\alpha}t. \tag{37}$$

Following expressions are for variances:

$$\mathsf{D}\{i(t)\} = \frac{\lambda\mu}{(\mu-\alpha)^2}, \tag{38}$$

and

$$\mathsf{D}\{n(t)\} = 2\frac{\lambda\mu\gamma^2}{(\mu-\alpha)^3}t + \frac{\lambda\gamma}{\mu-\alpha}t - 2\frac{\lambda\mu\gamma^2}{(\mu-\alpha)^4}\left(1 - e^{-(\mu-\alpha)t}\right). \tag{39}$$

Formulas (36) and (38) match with the result we got in [5], where one-dimensional process of a number of insured risks considering "implicit advertising" is researched.

Let's review correlation coefficient of processes $i(t)$ and $n(t)$. Knowing function $H(u, z, t)$, we can find joint moment of studied processes. We have:

$$\frac{1}{j}\frac{\partial^2 H(u,z,t)}{\partial u \partial z}\bigg|_{u=0,\, z=1} = \mathsf{E}\{i(t)n(t)\}, \tag{40}$$

then, considering characteristics we got earlier, let's write down the expression for correlation coefficient:

$$\mathsf{r}_{in}(t) = \frac{\lambda\gamma\mu(1 - e^{(\alpha-\mu)t})}{\sqrt{\lambda\mu[2\lambda\mu\gamma^2(\mu-\alpha)t + \lambda\gamma(\mu-\alpha)^3 t - 2\lambda\mu\gamma^2\left(1 - e^{-(\mu-\alpha)t}\right)]}}. \tag{41}$$

Nonzero correlation coefficient shows the presence of dependence between processes $i(t)$ and $n(t)$.

6 Numerical Characteristics of Value of the Total Benefit Payments

We will denote $S(t)$ as a value of the total benefit payments for all insured accidents during the time interval $[0, t]$, ξ — the value of the payment for one insured accident. Let's introduce characteristic function of the value $S(t)$:

$$\Psi(\eta, t) = \mathsf{E}\{e^{-\eta S(t)}\}. \tag{42}$$

Let's take a closer look at this function. We have:

$$\begin{aligned}\Psi(\eta,t) &= \mathsf{E}\left\{e^{-\eta S(t)}\right\} = \mathsf{E}\left\{e^{-\eta\sum\limits_{i=0}^{n(t)}\xi_i}\right\} \\ &= \sum_{n=0}^{\infty}\mathsf{E}\left\{e^{-\eta\sum\limits_{i=0}^{n(t)}\xi_i}\,\middle|\, n(t)=n\right\}P(n,t) \\ &= \sum_{n=0}^{\infty}\mathsf{E}\left\{\prod_{i=0}^{n}e^{-\eta\xi_i}\,\middle|\, n(t)=n\right\}P(n,t) = \sum_{n=0}^{\infty}\theta^n(\eta)P(n,t),\end{aligned} \tag{43}$$

where $\theta(\eta) = \mathsf{E}\{e^{-\eta\xi}\}$ is the characteristic function of the value ξ. With this in mind we can write down:

$$\Psi(\eta, t) = \sum_{n=0}^{\infty} \theta^n(\xi) P(n, t) = F(\theta(\eta), t). \tag{44}$$

Let's introduce functions

$$\begin{aligned} w_1(\eta) = v_1(\phi(\eta)) &= \frac{1}{2}\left[\left(1 - \frac{\mu}{\alpha} - \frac{\gamma}{\alpha}(1 - \phi(\eta))\right) + \sqrt{D(\theta(\eta))}\right], \\ w_2(\eta) = v_2(\phi(\eta)) &= \frac{1}{2}\left[\left(1 - \frac{\mu}{\alpha} - \frac{\gamma}{\alpha}(1 - \phi(\eta))\right) - \sqrt{D(\theta(\eta))}\right], \end{aligned} \tag{45}$$

where

$$D(\theta(\eta)) = \left[1 - \frac{\mu}{\alpha} - \frac{\gamma}{\alpha}(1 - \phi(\eta))\right]^2 + 4\frac{\gamma}{\alpha}(1 - \phi(\eta)). \tag{46}$$

Expressions (45), (46) are written considering (10) and (11). Then function $\Psi(\eta, t)$ will have this form

$$\begin{aligned} \Psi(\eta, t) = F(\theta(\eta), t) = e^{\lambda w_2(\eta) t} &\left[\left(1 - \frac{\alpha}{\mu}\right)(w_2(\eta) - w_1(\eta))\right]^{\frac{\lambda}{\alpha}} \\ &\times \Big(-w_1(\eta) + w_2(\eta) e^{\alpha(w_2(\eta) - w_1(\eta)) t} \\ -\frac{\alpha}{\mu}\Big[(-w_1(\eta)(1 + w_2(\eta)) &+ w_2(\eta)(1 + w_1(\eta)) e^{\alpha(w_2(\eta) - w_1(\eta)) t}\Big]\Big)^{-\frac{\lambda}{\alpha}}. \end{aligned} \tag{47}$$

Now, that we know the form of the characteristic function of a value of the total benefit payments, we can obtain the expected value and the variance of value $S(t)$. Let's denote $\mathsf{E}\{\xi\} = a_1$, $\mathsf{E}\{\xi^2\} = a_2$. Because of

$$\left.\frac{\partial \Psi(\eta, t)}{\partial \eta}\right|_{\eta=0} = -\mathsf{E}\{S(t)\}, \tag{48}$$

after transformations we will get

$$\mathsf{E}\{S(t)\} = \frac{\lambda\gamma a_1}{\mu - \alpha} t. \tag{49}$$

For the second initial moment $S(t)$ we can write down

$$\left.\frac{\partial^2 \Psi(\eta, t)}{\partial \eta^2}\right|_{\eta=0} = \mathsf{E}\{S^2(t)\}. \tag{50}$$

Then the variance of the total benefit payments will have the following form:

$$\mathsf{D}\{S(t)\} = \frac{\lambda\gamma a_2}{\mu - \alpha} t + 2\frac{\lambda\mu\gamma^2 a_1^2}{(\mu - \alpha)^3} t - 2\frac{\lambda\mu\gamma^2 a_1^2}{(\mu - \alpha)^4}\left(1 - e^{-(\mu - \alpha)t}\right). \tag{51}$$

Let's take a look at another characteristic of $S(t)$ - coefficient of variation $V\{S(t)\}$. It is defined as the ratio of the standart devation to the expected value:

$$V\{S(t)\} = \frac{\sqrt{\mathsf{D}\{S(t)\}}}{\mathsf{E}\{S(t)\}}.$$

Behavior of coefficient of variation $V(t)$ is shown at Fig. 3 with the following parameters: $\lambda = 5$, $\mu = 1$, $\alpha = 0.8$, $\gamma = 0.1$, $a_1 = 10$, $a_2 = 60$. Numerical calculations show that $V\{S(t)\}$ is significantly decreasing with the passage of time, reaching value of 0.01 at $t = 540$, which allows us to find pretty accurate prognosed value of the capital of insurance company (Fig. 4).

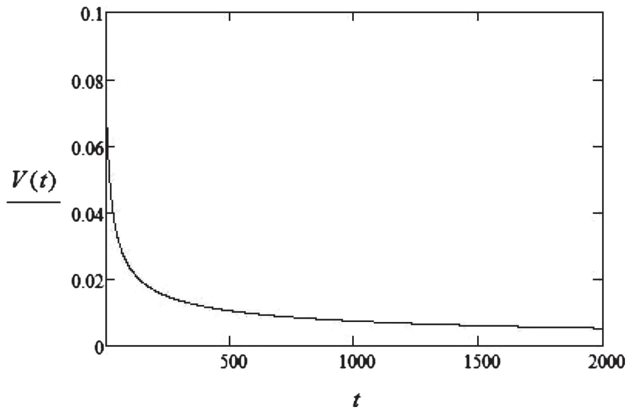

Fig. 4. Coefficient of variation of the total benefit payments

7 Conclusions

Thereby, in this paper we have researched mathematical model of the insurance company in the form of queueing system with an unlimited number of servers. We have found the expression for characteristic function of a two-dimensinal process of a number of benefit payments and a number of insurance risk. Also we have found expressions for numerical characteristics of said processes. It is shown that the results are the generalization of particular cases. Characteristic function, expected value and variance of a value of the total benefit payments have also been found. These results may be used for analysis of indicators of economic activity of insurance companies.

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