Филиал Кемеровского государственного университета
в г. Анжеро-Судженске

Национальный исследовательский
Томский государственный университет

Кемеровский государственный университет

Институт проблем управления им. В. А. Трапезникова РАН

Институт вычислительных технологий СО РАН

ИНФОРМАЦИОННЫЕ ТЕХНОЛОГИИ И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ (ИТММ–2015)

Материалы XIV Международной конференции имени А. Ф. Терпугова

18–22 ноября 2015 г.

Часть 1

Издательство Томского университета

2015

ОПТИМИЗАЦИОННЫЕ МОДЕЛИ И ИССЛЕДОВАНИЕ ОПЕРАЦИЙ

DYNAMIC LOCALLY OPTIMAL CONTROL OF DISCRETE STATE DELAY SYSTEMS WITH RANDOM PARAMETERS

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Control of the discrete systems with random parameters is considered in [1–4]. In this paper we propose for object with random parameters to realize synthesis of dynamic tracking system control on output based on optimization of the local criterion under indirect measurements taking into account state delay. To improve the quality control of objects practice of insertion in control law Luenberger observer [5] or dynamic feedback reduced dimensionality [6] is used. Control is defined as a function of the measured variables and the tracking signal. Asymptotic behavior of the system, construct estimates for asymptotic tracking accuracy are researched. An example is given to illustrate the usefulness of the proposed results.

1. Problem statement. Consider the following discrete-time system with time delay and random parameters:

$$x(k+1) = (A + \sum_{i=1}^{r} A_i \theta_i(k))x(k) + (\tilde{A} + \sum_{i=1}^{r} \tilde{A}_i \theta_i(k))x(k-h) +$$

$$+(B + \sum_{i=1}^{r} B_i \theta_i(k))u(k) + q(k);$$

$$x(\tau) = \varphi(\tau), \tau = -h, 1-h, 2-h, \ldots, 0;\ k = 0,1,2,\ldots, \quad (1)$$

$$y(k) = Sx(k) + v(k). \quad (2)$$

In (1), (2) $x(k) \in R^n$ is state vector; $h > 0$ is positive integer time delay; $u(k) \in R^m$ is control input; $y(k) \in R^l$ is observations vector; $A, A_i, \widetilde{A}, \widetilde{A}_i, B, B_i, i = \overline{1,r}$ are constant matrices of appropriate dimensions; S is matrix of measurement channel; matrix B and S are of full rank; pairs of matrices (A, B) and $(\widetilde{A}, B)$ are controllable; pairs of matrices (S, A) and $(S, \widetilde{A})$ are observable; $\varphi(\tau)$ is prescribed determinate function of initial conditions on the interval $[-h, 1-h, \ldots, -1]$, as this takes place $\varphi(0) = x(0) = x_0$ is random vector with characteristics: $M\{x_0\} = \bar{x}_0$, $M\{x_0 x_0^{\mathrm{T}}\} = P_{x_0}$; $q(k)$, $v(k)$ are Gaussian random sequences of input disturbances and measurement errors with characteristics: $\mathrm{M}\{q(k)\} = 0$, $\mathrm{M}\{v(k)\} = 0$, $\mathrm{M}\{q(k)v^{\mathrm{T}}(j)\} = 0$, $\mathrm{M}\{q(k)q^{\mathrm{T}}(j)\} = Q(k)\delta_{kj}$, $\mathrm{M}\{v(k)v^{\mathrm{T}}(j)\} = V(k)\delta_{kj}$ ($\delta_{k,j}$ is Kronecker delta), $Q(k) = Q^{\mathrm{T}}(k) \geq 0$,

$V(k)=V^{\mathrm{T}}(k)\geq 0$ are nonnegative definite matrices); $\theta_i(k)$ is Gaussian random sequences ($M\{\theta_i(k)\}=0$; $M\{\theta_i(k)\theta_i^{\mathrm{T}}(j)\}=\delta_{k,j}$).

Local criterion described by:

$$I(k)=M\{(w(k+1)-z(k))^{\mathrm{T}}C(w(k+1)-z(k))+u^{\mathrm{T}}(k)Du(k)\}, \quad (3)$$

where $w(k)=Hx(k)$ is controlled output of the system (H is matrix of the output of the system); $C=C^{\mathrm{T}}, D=D^{\mathrm{T}}\geq 0$ are weighting matrices; $z(k)\in R^n$ is the reference input which described by the equation

$$z(k+1)=Fz(k)+q_z(k),\ z(0)=z_0, k=0,1,2,\dots. \quad (4)$$

In (4) $q_z(k)$ is Gaussian random sequence with characteristics $\mathrm{M}\{q_z(k)\}=0$, $\mathrm{M}\{q_z(k)q^{\mathrm{T}}(j)\}=0$, $\mathrm{M}\{q_z(k)v^{\mathrm{T}}(j)\}=0$, $\mathrm{M}\{q_z(k)q_z{}^{\mathrm{T}}(j)\}=Q_z(k)\delta_{k,j}$, z_0 are initial conditions ($\mathrm{M}\{z_0z_0^{\mathrm{T}}\}=P_{z_0}, \mathrm{M}\{z_0x_0^{\mathrm{T}}\}=P_{z_0x_0}$, $\mathrm{M}\{x_0z_0^{\mathrm{T}}\}=P_{x_0z_0}$), F is matrix of the dynamics model of the reference input.

It is required to construct a control of system (1), using observations (2) and minimizing the criterion (3).

2. Locally criterion optimization. Let the control law of the system (1) under observations (2) is defined as:

$$u(k)=K_0(k)\omega(k)+K_1(k)y(k)+K_2(k)y(k-h)+K_3(k)z(k)=K_0(k)\omega(k)+$$
$$+K_1(k)Sx(k)+K_1(k)v(k)+K_2(k)Sx(k-h)+K_2(k)v(k-h)+K_3(k)z(k), \quad (5)$$

where transfer coefficients are $K_0(k), K_1(k), K_2(k), K_3(k)$ to be determined and variable $\omega(k)$ is defined with by the reduced dimensionality equation [6].

Theorem 1. If for object (1), observations (2) and local criterion (3) matrices

$$\overline{C}(k)=(B^{\mathrm{T}}H^{\mathrm{T}}CHB+D+\sum_{i=1}^{r}B_i^{\mathrm{T}}H^{\mathrm{T}}CHB_i)>0,$$

$$\overline{P}(k)=\begin{bmatrix} P_{\omega}(k) & SP_{x\omega}(k) & SP_{x\omega}(k-h,k) & P_{z\omega}(k) \\ P_{\omega x}(k)S^{\mathrm{T}} & SP_x(k)S^{\mathrm{T}}+V(k) & SP_x(k-h,k)S^{\mathrm{T}} & P_{zx}(k)S^{\mathrm{T}} \\ P_{\omega x}(k,k-h)S^{\mathrm{T}} & SP_x(k,k-h)S^{\mathrm{T}} & SP_x(k-h)S^{\mathrm{T}}+V(k-h) & P_{zx}(k,k-h)S^{\mathrm{T}} \\ P_{\omega z}(k) & SP_{xz}(k) & SP_{xz}(k-h,k) & P_z(k) \end{bmatrix}>0 \quad (6)$$

are positive definite for all $k=1,2,\dots$, then optimal in the sense of minimum criteria (3) transfer coefficients for control (5) are determined by the formulas:

$$K_0^*(k)=aK_1^*(k)+bK_2^*(k)+cK_3^*(k)+d; \quad (7)$$

$$K_1^*(k)=eK_0^*(k)+fK_1^*(k)+gK_2^*(k)+h; \quad (8)$$

$$K_2^*(k)=mK_0^*(k)+nK_1^*(k)+pK_3^*(k)+r; \quad (9)$$

$$K_3^*(k)=sK_0^*(k)+tK_1^*(k)+lK_2^*(k)+k, \quad (10)$$

where $a,b,c,d,e,f,g,h,m,n,p,r,s,l,k$ are variables depending on $P_z(k,r)=M\{z(k)z^{\mathrm{T}}(r)\}$; $P_\omega(k,r)=M\{\omega(k)\omega^{\mathrm{T}}(r)\}$; $P_{z\omega}(k,r)=$ $=P_{\omega z}^{\mathrm{T}}(r,k)=M\{z(k)\omega^{\mathrm{T}}(r)\}$, which are defined by the system of difference matrix equations without delays with initial conditions: $P_z(0)=P_{z_0}$; $P_\omega(0)=P_{z\omega}(0)=P_{\omega z}(0)=0$ and $P_x(k,r)=M\{x(k)x^{\mathrm{T}}(r)\}$; $P_{zx}(k,r)=P_{xz}^{\mathrm{T}}(r,k)=$ $=M\{z(k)x^{\mathrm{T}}(r)\}$, which are defined by the system of difference matrix equations with time delays with initial and boundary conditions: $P_x(t-h,j-h)=\varphi(t-h)\varphi^{\mathrm{T}}(j-h)$ for $t,j=\overline{0,h-1}$; $P_x(0,\tau)=\overline{x}_0\varphi^{\mathrm{T}}(\tau)$; $P_x(\tau,0)=\varphi(\tau)\overline{x}_0^{\mathrm{T}}$; $P_{zx}(0,\tau)=\overline{z}_0\varphi^{\mathrm{T}}(\tau)$; $P_{xz}(\tau,0)=\varphi(\tau)\overline{z}_0^{\mathrm{T}}$; $P_{x\omega}(\tau,0)=$ $=P_{\omega x}(0,\tau)=0$ для $\tau=-h,1-h,2-h,\ldots,-1$; $P_x(0)=P_{x_0}$; $P_{zx}(0)=P_{z_0x_0}$; $P_{xz}(0)=P_{x_0z_0}$; $P_{zx}(0)=P_{z_0x_0}$; $P_{\omega x}(0)=P_{x\omega}(0)=0$.

The proof of the theorem is performed similar paper [7].

3. Asymptotic behavior. Asymptotic tracking accuracy for the object (1) is defined by calculating criterion estimation:

$$J=\lim_{k\to\infty}\mathrm{M}\{\|x(k+1)-z\|^2\}, \quad (11)$$

where $\|.\|$ is Euclidean norm of vector, z is constant reference input.

Theorem 2. Let in description of system (1), observations (2), criterion (3) and model of reference input (4) matrices $A,A_i,\widetilde{A},\widetilde{A}_i,B,B_i,Q,S,V,C,D$, $i=\overline{1,r}$ are constant; $F=E; q_z(k)=0$. Then, if the condition (6) theorem 1 is satisfied, there exist steady-state solutions of difference equations for $P_x(t,j)$, $P_{zx}(t,j)$, $P_{xz}(t,j)$, $P_{x\omega}(t,j)$, $P_{\omega x}(t,j)$ and condition is satisfied:

$$\alpha_1^2+\Phi_1^2<1, \quad (12)$$

then for criterion (11) estimate is valid:

$$J\le\frac{[(G+R)^2+(g+r_2)^2+tr\tilde{Q}][(\alpha_1^2+\Phi_1^2)+(\alpha_2^2+\Phi_2^2)]}{1-(\alpha_1^2+\Phi_1^2)}+$$
$$+2(\alpha_1\alpha_2+\Phi_1\Phi_2)\frac{(G+R)^2+(g+r_2)^2+tr\tilde{Q}_1}{1-(\alpha_1^2+\Phi_1^2)}+(g+r_1)^2+(G+R)^2+$$
$$+2(g+r_2)\frac{(\alpha_1+\alpha_2)(g+r_1)+(\Phi_1+\Phi_2)(G+R)}{1-\alpha_1}+tr\widetilde{Q}, \quad (13)$$

where

$$\|\xi\|_s=\alpha_1;\ \|\xi_2\|_s=\alpha_2;\ \|\varphi_1\|_s=\Phi;\ \|\varphi_2\|_s=\Phi_2;\ \xi=A+BK_1^*S;\ \xi_2=\tilde{A}+BK_2^*S;$$

$$\xi_3=BK_3^*-E;\ \varphi_1=\sum_{i=1}^{r}(A_i\theta_i(k)+B_i\theta_i(k)K_1^*S);\varphi_2=\sum_{i=1}^{r}(\tilde{A}_i\theta_i(k)+B_i\theta_i(k)K_2^*S);$$

$$K_1^*=\lim_{k\to\infty}K_1^*(k);\ K_2^*=\lim_{k\to\infty}K_2^*(k);\ K_3^*=\lim_{k\to\infty}K_3^*(k);\ r_1=\|\xi_3z\|;\ r_2=\|BK_3^*z\|;$$

$$R=\left\|\sum_{i=1}^{r}B_i\theta_i(k)K_3^*z\right\|;\ g=\|BK_0^*\omega\|;\ G=\left\|\sum_{i=1}^{r}B_i\theta_i(k)K_0^*\omega\right\|;$$

$$\tilde{Q}_1=BK_1^*VK_2^{*\mathrm{T}}B^{\mathrm{T}}+\sum_{i=1}^{r}B_iK_1^*VK_2^{*\mathrm{T}}B_i^{\mathrm{T}};$$

$$\tilde{Q}=Q+BK_1^*VK_1^{*\mathrm{T}}B^{\mathrm{T}}+\sum_{i=1}^{r}B_iK_1^*VK_1^{*\mathrm{T}}B_i^{\mathrm{T}}+BK_2^*VK_1^{*\mathrm{T}}B^{\mathrm{T}}+\sum_{i=1}^{r}B_iK_2^*VK_1^{*\mathrm{T}}B_i^{\mathrm{T}}.$$

4. Illustrative example. Let the system and local criterion are described by the following matrices and vectors:

$$A=\begin{bmatrix}0{,}05 & 1\\ -0{,}025 & 1\end{bmatrix};\tilde{A}=\begin{bmatrix}0 & 0\\ 0{,}03 & 0\end{bmatrix};B=\begin{bmatrix}0{,}1\\ 1\end{bmatrix};A_1=\begin{bmatrix}0{,}05 & 0\\ 0 & 0\end{bmatrix};A_2=\begin{bmatrix}0 & 0\\ 0{,}005 & 0\end{bmatrix};$$

$$A_3=\begin{bmatrix}0 & 0{,}1\\ 0 & 0\end{bmatrix};A_4=\begin{bmatrix}0 & 0\\ 0 & 0{,}01\end{bmatrix};A_5=A_6=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix};\tilde{A}_1=\begin{bmatrix}0{,}03 & 0\\ 0 & 0\end{bmatrix};$$

$$\tilde{A}_2=\begin{bmatrix}0 & 0\\ 0{,}05 & 0\end{bmatrix};\ \tilde{A}_3=\begin{bmatrix}0 & 0{,}001\\ 0 & 0\end{bmatrix};\tilde{A}_4=\begin{bmatrix}0 & 0\\ 0 & 0{,}002\end{bmatrix};\ \tilde{A}_5=\tilde{A}_6=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix};$$

$$B_1=B_2=B_3=B_4=\begin{bmatrix}0\\ 0\end{bmatrix};B_5=\begin{bmatrix}0{,}005\\ 0\end{bmatrix};B_6=\begin{bmatrix}0\\ 0{,}025\end{bmatrix};\ F=1;z=\begin{bmatrix}10\\ 10\end{bmatrix};$$

$$Q=\begin{bmatrix}0{,}02 & 0\\ 0 & 0{,}02\end{bmatrix};S=[0\quad 1];H=[1\quad 0];\ C=1;D=0{,}2;h=1.$$

Parameters $\theta_i(k)$ are modeled as Gaussian random variable with mathematical expectation is 0 and mean-square deviation is 1.

To substantiate of the utility of introduction of the dynamic element $\omega(k)$ in control law we modeled four control systems (algorithms): algorithm 1 is control built on the nominal values of the parameters; algorithm 2 is locally-optimal control calculated with regard to random parameters; algorithm 3 is control synthesized on the nominal values of the parameters and with the introduction of the dynamic element in the control law; algorithm 4 is locally-optimal control with the dynamic element, calculated taking into account the random parameters.

In the table the values of quality criterion of the convergence of the state vector $x(k)$ to the reference input $z(k)$ for the four control systems is cited.

Average error

Algorithm	1	2	3	4
e	2,325	2,103	1,806	1,638

The table shows that minimum average error deviation is reached at 4th algorithm with locally-optimal control with dynamic element, constructed on the transfer coefficients calculated with regard to the random parameters.

Conclusion. The problem of controlling the output of discrete systems with state delays and random parameters based on the synthesis of locally optimal linear tracking control system of discrete systems with indirect observations and introduction dynamic element in control law has been solved. The asymptotic behavior of the system has been analyzed. It is shown that introduction of the dynamic element in system greatly improves system performance and reduces the sensitivity to disturbing influences. Also it is shown that the optimal dynamic control system, built on random parameters with constant transfer coefficients has the property of robustness.

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ИССЛЕДОВАНИЕ ЭФФЕКТИВНОСТИ ГЕНЕТИЧЕСКОГО АЛГОРИТМА БЕЗУСЛОВНОЙ ОПТИМИЗАЦИИ

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Оптимизационные задачи заключаются в нахождении экстремума заданной целевой функции. Как правило, это сложная функция, зависящая от многих переменных (входных параметров). Требуется найти значения этих параметров, при которых целевая функция достигает своего экстремума.

Одним из наиболее часто встречающихся на практике типов задач являются задачи безусловной оптимизации. В задачах такого типа требуется