Томский государственный университет
Механико-математический факультет

Молодежная научная конференция

«Все грани математики и механики»

24–30 апреля 2015 г.

Сборник тезисов

Томск – 2015

Prediction with guaranteed accuracy for Ornstein-Uhlenbeck process

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The process $x(t)$ is supposed to be stable and the parameter $a \in [-\Delta; -\delta]$ with known Δ and δ :

$$dx(t) = a \cdot x(t)dt + dw(t) \qquad (1)$$

The problem is to construct the predictions of the process $x(t)$ with the given mean square accuracy by discrete-time observations $y_n = x_{nh}$, $n = \overline{0, T/h}$; h is a discretisation step.

Using the solution of the process (1) we obtain the autoregressive-type equation for observations y_n: $y_n = \lambda y_{n-1} + \eta_n$, where $\lambda = e^{ah}$ and η_n are i.i.d. Gaussian noises with finite variance.

According to the condition on the minimal value of the variance equals to: $\sigma^2 = \dfrac{\left(1 - e^{-2\Delta(t-nh)}\right)^2}{2\Delta}, nh \le t < (n+1)h$.

If the parameter a is known, the optimal predictor $x^0(t)$ of $x(t)$ is defined: $x^0(t) = e^{a(t-nh)} y_n, nh \le t < (n+1)h$.

For the definition of adaptive predictor (the case of unknown a) the parameter a should be replaced on some estimator. We use the truncated estimator $\left(a_n\right)_{n \ge 1}, a_0 = 0$, constructed in [1] by observations $\left(y_k\right)_{k=\overline{0,nh}}$,

and having the property: $M\left(a_n - a\right)^2 \le \dfrac{C}{n}, n = \overline{1, T/h}$.

It is proved that adaptive predictor $\hat{x}(t) = e^{a_n(t-nh)} y_n, nh \le t < (n+1)h$ is closed to the optimal one in the following non-asymptotic sense:

$$M\left(\hat{x}(t) - x^0(t)\right)^2 \le \frac{C}{t}, t = \overline{0, T}.$$

References

1 Vasiliev\,V.\,A. A truncated estimation method with guaranteed accuracy // Annals of the Institute of Statistical Mathematics. 2013 . Vol. 66, Iss. 1. P. 141-163