Национальный исследовательский
Томский государственный университет
Кемеровский государственный университет
Кемеровский научный центр СО РАН
Институт вычислительных технологий СО РАН
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# ИНФОРМАЦИОННЫЕ ТЕХНОЛОГИИ И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ (ИТММ–2014)

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Table 2

**Results of modeling change point procedure**

| $H$ | $\delta$ | $E(\theta_1)$ | $E(\theta_2)$ |
|---|---|---|---|
| 20 | 0.25 | 101910 | 100890 |
| 50 | 0.20 | 101180 | 103090 |
| 100 | 0.20 | 105540 | 104510 |
| 150 | 0.15 | 107340 | 103880 |
| 200 | 0.10 | 114740 | 107300 |
| 400 | 0.05 | 122410 | 113170 |

It should be noted that with the growth of $H$ there is bigger length of delay in change point detection. It is connected with the accuracy of estimation.

References

1. *Tong H.* On aThreshold Model in Pattern Recognition and Signal Processing. – Amsterdam: C. H. Chen (Editor), Sijhoff&Noordhoff, 1978. – P. 101–141.

2. *Rabemanajara R.* Threshold ARCH models and asymmetries in volatility / R. Rabemanajara, J. M. Zakoïan // Journal of Applied Econometrics. – 1993. – Vol. 8. – P. 31–49.

3. *Lange T.* Estimation and Asymptotic Inference in the AR-ARCH Model / T. Lange, A. Rahbek, S. T. Jensen // Econometric Reviews. – 2011. – Vol. 30, I. 2. – P. 129–153.

4. *Hamadeh T.* Asymptotic properties of LS and QML estimators for a class of nonlinear GARCH processes / T. Hamadeh, J. N. Zakoian // J. Statist. Plann. Inference. – 2011. – Vol. 141. – P. 488–507.

5. *Sergeeva K.* An efficient algorithm for detecting a change point of autoregressive parameters of AR(p)/ARCH(q) process / K. Sergeeva, S. Vorobejchikov // Proceedings of the 11th International Conference of Pattern Recognition and Information Processing – Minsk, Belarus, May 18–20, 2011. – Belarus: Minsk, PRIP'2011. – P. 156–159.

6. *Burkatovskaya Y.* Guaranteed estimation of parameters of threshold autoregressive process with conditional heteroskedasticity / Y. Burkatovskaya, S. Vorobeychikov // Journal of Control and Computer Science. – 2013. – Vol. 2. – P. 32–41.

## NONPARAMETRIC ESTIMATION OF NET PREMIUMS IN COLLECTIVE LIFE INSURANCE FOR THE M INDIVIDUALS

***G. M. Koshkin, Ya. N. Lopukhin***

*National Research Tomsk State University, Tomsk, Russia*

**Introduction.** One of the main problem in actuarial mathematics is to find the «right» ratio between premiums and benefits. In this case, the calculation of net premiums allows to cover damages and to give the zero average income of the insurance company. Note that in the section, devoted to this area in the known book «Actuarial Mathematics» [1], is used the calculation of net premiums on the base of mortality tables. Interesting results by this approach have been presented in [2–6]. At present, the theory and practice of insurance is strongly required the using complex mathematical models and processes. Such results are obtained in [7–10]. In this paper, we develop the ideas from [11–15]. The nonparametric estimators of the net premium functionals for the different statues of collective life insurance are constructed, and the asymptotic properties of the proposed estimators are studied.

## Collective Life Insurance

The concept of status is the useful abstraction in the collective life insurance according to [1]. Consider **m** members of ages $x_1, x_2, \dots, x_m$ who desire to buy insurance policy. Let us denote the future lifetime of the **k**-th individual by $T(x_k) = X - x_k$. Let us put in a correspondence a status $U$ with its future lifetime $T(U)$ and a set of numbers $T(x_1), T(x_2), \dots, T(x_m)$ [11]. In [12–15], there were considered the cases of the joint-life status and the last-survivor status. In this paper, we consider the general case of the **k**-survivor status, which is denoted $U := \frac{k}{x_1 : x_2 : \ldots : x_m}$ and exists as long as at least alive a **k** among the **m** individuals $x_1, x_2, \dots, x_m$, i.e., it is considered destroyed upon the occurrence of the **(m-k+1)** death.

Also, separately consider the case of the **[k]**-deferred survivor status $U := \frac{[k]}{x_1 : x_2 : \ldots : x_m}$. Here, if a **k** of the **m** individuals $x_1, x_2, \dots, x_m$ are exactly alive, i.e., the status starts at the **(m-k)**-th death and lasts until the **(m-k+1)**-th death. This status is widely used in the calculation of sequences payments of limited duration [11].

Note that the new statuses can also be combined from the above base status. Mixed status is a condition in which basis is a combination of statuses, and at least one of them asked for more than one individual. For example, $\overline{x_1 : x_2} : \overline{x_3 : x_4}$ it condition persists if there are alive at least one of $x_1$ and $x_2$ and at least one of $x_3$ and $x_4$. The destruction moment of status $\overline{x_1 : x_2} : \overline{x_3 : x_4}$ is

$$T(U) = \min\left[\max(T(x_1), \mathrm{T}(x_2)), \max(T(x_3), \mathrm{T}(x_4))\right].$$

Similarly, the fracture point may be found for a combination of statuses.

## Functionals of Net Premiums in Collective Life Insurance

Consider the random variables $Z_i = X_i - x_i,\ i = \overline{1,m}$. Order them in ascending and obtain the order statistics $Z_{(i)},\ i = \overline{1,m}$. By analogy with the joint-life status [15] the net premium is defined as

$$\overline{A}_{\frac{k}{x_1 : x_2 : \ldots : x_m}} = \frac{\int_0^\infty e^{-\delta t} dP\{T(U) < t\}}{S(x_1, x_2, \dots, x_m)} = \frac{\int_0^\infty e^{-\delta t} dP\{Z_{(m-k+1)} < t\}}{P\{Z_{(1)} > 0\}}, \tag{1}$$

where $S(x_1, x_2, \dots, x_m)$ is the survival function of **m** individuals.

In the case of the **[k]**-deferred survivor status, we have

$$\overline{A}_{\overline{x_1:x_2:\ldots:x_m}}^{[k]} = \frac{\int_0^\infty e^{-\delta t} dP\left\{Z_{(m-k+1)} < t < Z_{(m-k)}\right\}}{S(x_1, x_2, \ldots, x_m)} = \overline{A}_{\overline{x_1:x_2:\ldots:x_m}}^{k-1} - \overline{A}_{\overline{x_1:x_2:\ldots:x_m}}^{k}.$$

Similarly, the net premiums functionals for mixed cases can be written. For example, the net premium for $\overline{x_1 : x_2} : \overline{x_3 : x_4}$ is defined by the formula

$$\overline{A}_{\overline{\overline{x_1:x_2}:\overline{x_3:x_4}}} = \frac{\int_0^\infty e^{-\delta t} dP\left\{\min\left[\max(Z_1, Z_2), \max(Z_3, Z_4)\right]\right\}}{S(x_1, x_2, x_3, x_4)}.$$

**Nonparametric Estimates of Net Premiums in Collective Life Insurance**

Let $(Z_{11}, \ldots, Z_{m1})$, $(Z_{12}, \ldots, Z_{m2}), \ldots, (Z_{1n}, \ldots, Z_{mn})$ be an **m**-dimensional random sample of a size **n.** Denote the corresponding ordered set as $(Z_{(1)1}, \ldots, Z_{(m)1})$, $(Z_{(1)2}, \ldots, Z_{(m)2}), \ldots, (Z_{(1)n}, \ldots, Z_{(m)n})$. Estimate the distribution function $P\left\{Z_{(m-k+1)} < t\right\}$, the survival function $P\left\{Z_{(1)} > 0\right\}$ by the nonparametric estimates $\frac{1}{n}\sum_{i=1}^{n} I(Z_{(m-k+1)i} < t)$, $\frac{1}{n}\sum_{i=1}^{n} I(Z_{(1)i} > 0)$. Then nonparametric estimate of (1) is

$$\hat{\overline{A}}_{\overline{x_1:x_2:\ldots:x_m}}^{k} = \frac{\int_0^\infty e^{-\delta t} d\left(\frac{1}{n}\sum_{i=1}^{n} I(Z_{(m-k+1)i} < t)\right)}{\frac{1}{n}\sum_{i=1}^{n} I(Z_{(1)i} > 0)} = \frac{\frac{1}{n}\sum_{i=1}^{n}\int_0^\infty e^{-\delta t}\, \overline{\delta}\left(t - Z_{(m-k+1)i}\right)}{\frac{1}{n}\sum_{i=1}^{n} I(Z_{(1)i} > 0)} =$$

$$= \frac{\frac{1}{n}\sum_{i=1}^{n} e^{-\delta Z_{(m-k+1)i}}\, I(Z_{(m-k+1)i} > 0)}{\frac{1}{n}\sum_{i=1}^{n} I(Z_{(1)i} > 0)} = \frac{\Phi_n(x_1, x_2, \ldots, x_m, \delta, k)}{S_n(x_1, x_2, \ldots x_m)}, \qquad (2)$$

where $\overline{\delta}(u)$ is the Dirac delta-function. Obtaining the formula (2), we use the filtering property of the delta-function: $\int_{-\infty}^{+\infty} \varphi(x)\, \overline{\delta}(x-a) dx = \varphi(a)$.

In the case of the **[k]**-deferred survivor status, the nonparametric plug-in estimate of net premium is defined as

$$\hat{\overline{A}}_{\overline{x_1:x_2:\ldots:x_m}}^{[k]} = \frac{\Phi_n(x_1, x_2, \ldots, x_m, \delta, k-1) - \Phi_n(x_1, x_2, \ldots, x_m, \delta, k)}{S_n(x_1, x_2, \ldots x_m)}.$$

Similarly, the nonparametric estimates of the net premiums functionals for mixed cases can be found. For the status $\overline{x_1 : x_2} : \overline{x_3 : x_4}$, we have

$$\hat{\bar{A}}_{\overline{x_1:x_2}:\overline{x_3:x_4}} = \frac{\frac{1}{n}\sum_{i=1}^{n} e^{-\delta\min[\max(Z_1,Z_2),\max(Z_3,Z_4)]} I(\min[\max(Z_1,Z_2),\max(Z_3,Z_4)]>0)}{\frac{1}{n}\sum_{i=1}^{n} I(Z_{(1)i}>0)}.$$

Let $\Rightarrow$ be the symbol of the convergence in distribution.

**Theorem 1.** If the survival function $S(x_1,x_2,\ldots,x_m)\neq 0,$ then as $n\to\infty$

$$\left[\hat{\bar{A}}_{\overline{x_1:x_2:\ldots:x_m}}^{\;k} - \bar{A}_{\overline{x_1:x_2:\ldots:x_m}}^{\;k}\right]\Rightarrow$$

$$\Rightarrow N\left\{0,\frac{\Phi(x_1,x_2,\ldots,x_m,2\delta,\mathrm{k})S(x_1,x_2,\ldots x_m)-\Phi^2(x_1,x_2,\ldots,x_m,\delta,k)}{S(x_1,x_2,\ldots x_m)}\right\},$$

and the MSE of nonparametric estimate (2) is

$$u^2\left(\hat{\bar{A}}_{\overline{x_1:x_2:\ldots:x_m}}^{\;k}\right)=$$

$$=\frac{\Phi(x_1,x_2,\ldots,x_m,2\delta,k)\,S(x_1,x_2,\ldots,x_m)-\Phi^2(x_1,x_2,\ldots,x_m,\delta,k)}{n\,S^3(x_1,x_2,\ldots,x_m)}+O\left(n^{-3/2}\right).$$

This theorem is proved as the theorem for the nonparametric estimators of the joint life status (see Theorem 5 from [14]).

**Synthesis of Estimators of Net Premiums for Other Forms of Insurance**

The above considered estimators of net premiums were constructed for the whole insurance; now we consider other forms of insurances.

**The p-years term life insurance.** In this case, the benefit to pay if the insured will die during of the contract validity. The company does not pay the benefit if the insured has lived the **p** years. Then

$$\hat{\bar{A}}_{\overline{x_1:x_2:\ldots:x_m}:p}^{\;k} = \frac{\frac{1}{n}\sum_{i=1}^{n} e^{-\delta Z_{(m-k+1)i}} I(0<Z_{(m-k+1)i}\le \mathrm{p})}{\frac{1}{n}\sum_{i=1}^{n} I(Z_{(1)i}>0)}.$$

**The r years deferred life insurance.** This form of insurance provides for a benefit following the death of the insured when he dies at least **r** years following policy issue. Here, the net premium is expressed in the form

$$\hat{\bar{A}}_{r|:\overline{x_1:x_2:\ldots:x_m}}^{\;k} = \frac{\frac{1}{n}\sum_{i=1}^{n} e^{-\delta Z_{(m-k+1)i}} I(Z_{(m-k+1)i}>\mathrm{r})}{\frac{1}{n}\sum_{i=1}^{n} I(Z_{(1)i}>0)}.$$

These nonparametric estimators properties are similar to properties of (2).

## Simulations Results

The nonparametric estimates show their adaptability if the distribution is changed and exceed parametric estimates, oriented on the best result only for its own distributions. Often, the MSE of nonparametric estimates are less than the MSE of parametric estimates in 2-3 times. The main modeling results obtained using data from the Makeham and de Moivre distributions.

References


1. *Bowers N.L., Gerber H.U., Hickman J.C. et al.* Actuarial Mathematics. – Itasca, Illinois: The Society of Actuaries, 1986. – 624 p.

2. *Dong Y.* Fair Valuation of Life Insurance Contracts under a Correlated Jump Diffusion Model // ASTIN Bulletin. – 2011. – Vol. 41(2). – P. 429–447.

3. *Faust R., Schmeiser H., Zemp A.* A Performance Analysis of Participating Life Insurance Contracts, Insurance // Mathematics and Economics. – 2013. – Vol. 51(1). – P. 158–171.

4. *Maurer R., Rogalla R., Siegelin I.* Participating Payout Life Annuities: Lessons from Germany // ASTIN Bulletin. – 2013. – Vol. 43(2). – P. 159–187.

5. *Schmeiser H., Wagner J.* A Joint Valuation of Premium Payment and Surrender Options in Participating Life Insurance Contracts, Insurance // Mathematics and Economics. – 2011. – Vol. 49(3). – P. 580–596.

6. *Sliwinski A., Michalski T., Rószkiewicz M.* Demand for Life Insurance–An Empirical Analysis in the Case of Poland // The Geneva Papers on Risk and Insurance. – 2013. – Vol. 38. – P. 62–87.

7. *Arora N., Arora P.* Insurance Premium Optimization// Perspective of Insurance Seeker and Insurance Provider // Journal of Management and Science. – 2014. – Vol. 4, No.1. – P. 43–53.

8. *Bohnert A., Gatzert N.* Analyzing Surplus Appropriation Schemes in Participating Life Insurance from the Insurer's and the Policyholder's Perspective, Insurance // Mathematics and Economics. – 2012. – Vol. 50(1). – P. 64–78.

9. *Gatzert N., Wesker H.* The Impact of Natural Hedging on a Life Insurer's Risk Situation // Journal of Risk Finance. – 2012. – Vol. 13(5). – P. 396–423.

10. *Rocio P., Aguilar C., Xu C.* Design Life Insurance Participating Policies using Optimization Techniques // International Journal of Innovative Computing, Information and Control. – 2010. – Vol. 6(4). – P. 1655–1666.

11. *Gerber H.U.* Principles of Premium Calculation and Reinsurance // Transactions of the 21st International Congress of Actuaries. – 1980. – P. 137–142.

12. *Koshkin G.M.* Nonparametric Estimation of Moments of Net Premium in Life Insurance // Computer Data Analysis and Modeling // Proceedings of the Fifth International Conference (June 8–12 1998, Minsk). – Vol. 1: A–M / Editing by Prof. S.A. Aivazyan and Prof. Yu. S. Kharin. – Minsk: BSU, 1998. – P. 148–153.

13. *Koshkin G.M., Lopukhin Ya.N.* On estimation of net premium in collective life insurance // The 5th Korea-Russian International Symposium on Science and Technology. (June 26 – July 3, 2001, Tomsk, Russia). Proceedings. KORUS 2001. – Vol. 2. – Tomsk: Tomsk Polytechnic University, 2001. – P. 296–299.

14. *Koshkin G.M., Lopukhin Ya.N.* Estimation of Net Premiums in Collective Models of Life Insurance // XIth Annual International AFIR Colloquium (September 6–7, 2001). – Vol. 2. – Toronto, Canada: Canadian Institute of Actuaries, 2001. – P. 447–457.

15. *Koshkin G.M., Lopukhin Ya.N.* Nonparametric Estimation of Net Premiums in Collective Insurance // Computer Data Analysis and Modeling: Robustness and Computer Intensive Methods: Proceedings of the Sixth International Conference (September 10–14, 2001,

Minsk). – Vol. 1: A–K / Edited by Prof. Dr. S. Aivazyan, Prof. Dr. Yu. Kharin and Prof. Dr. H. Rieder. – Minsk: BSU, 2001. – P. 236–241.

## USE OF REGRESSION DEPENDENCES BETWEEN THE HUMAN DEVELOPMENT INDEX AND DOMESTIC INVESTMENTS FOR THE MODELING OF OPTIMAL DEVELOPMENT OF THE COUNTRY

***V. V. Meshechkin, G. V. Turchaninov***

*Kemerovo State University, Kemerovo, Russia*

The aim of the work is to model the relationships between domestic investments and the Human Development Index and to use them in constructing of mathematical model to optimize the development of the country (on the example of the Russian Federation).

The Human Development Index (HDI) is an aggregate characteristic of the quality of life, based on a set of basic indicators of longevity, education and level of life itself as the main characteristics of the human potential of investigated territory. The index is calculated annually and is a standard tool for the general comparing of standards of living in various countries and regions. It is published in the annual United Nations Human Development Reports since 1990 [1].

Different factors affect on the total value of the HDI, some of them directly and others – indirectly. In this paper, the dynamics of the HDI is associated by means of regression analysis with domestic investments into such branches as mining and manufacturing industries, construction, agriculture, etc. According to collected statistical data for the Russian Federation regression and autoregression models of different types are built, their comparative analysis is carried out and the model, which reflects dependences between considered indicators better than others, is selected. On the basis of this model the problem of maximizing the HDI in the country due to the optimization and redistribution of domestic investments in the conditions of limited investment budget is formed [2]. The variables in this problem are values of domestic investments into various branches by years, the limitations are imposed on their maximum permissible values, and the target function is the average for all the annual values of the HDI.

For the constructed mathematical model which uses the Human Development Index of the country as an optimality criterion, numerical calculations are carried out, the optimal (in the sense of the problem) solution is found, and conclusions about the desired changes of the amounts of branches financing for the HDI improvement are drawn.

References

1. Human Development Reports [Electronic resource] // URL: http://hdr.undp.org/en/ (date of access 30.06.2014).

2. *Turchaninov G.V., Meshechkin V.V.* Postroenie optimizatsionnoy modeli razvitiya strany na osnove indeksa razvitiya chelovecheskogo potentsiala // Obrazovanie, nauka, inno-