Национальный исследовательский
Томский государственный университет
Кемеровский государственный университет
Кемеровский научный центр СО РАН
Институт вычислительных технологий СО РАН
Филиал Кемеровского государственного университета
в г. Анжеро-Судженске

ИНФОРМАЦИОННЫЕ ТЕХНОЛОГИИ И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ (ИТММ–2014)

Материалы XIII Международной научно-практической конференции имени А. Ф. Терпугова

20–22 ноября 2014 г.

Часть 1

Издательство Томского университета

2014

ВЕРОЯТНОСТНЫЕ И ЭКОНОМЕТРИЧЕСКИЕ МЕТОДЫ И МОДЕЛИ

CONTINUOUS LIMIT OF TIME-CHANGED BRANCHING PROCESSES

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It is well-known that the class of continuous-state branching processes arises as limit of sequences of Galton-Watson processes (eventually in continuous time) in which the size of the population goes to infinity and time is suitably rescaled.

In this talk we present a large-population limit process for sequences of suitably normalized branching processes characterized by heavy-tails waiting times. We prove that the non-Markov limit process is a continuous-state branching process time-changed through an independent right-inverse process of a stable subordinator. Properties and relations to fractional calculus are derived and discussed in general or in specific cases. Extensions with different random changes of time are also considered. Finally, some connections to networks models are suggested.

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ON GUARANTEED SEQUENTIAL CHANGE POINT DETECTION FOR TAR(1)/ARCH(1) PROCESS

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Introduction. The TAR models were first proposed by Tong in [1]. Autoregressive heteroscedastic (ARCH) models has proved to be very useful for describing changing in volatility of econometric processes and other time series. The first efforts to combine aforementioned models and to research properties were made in [2].

Estimation of unknown parameters of models with mixed structure, which consists of both linear and nonlinear parts, is very interesting for applications but quite difficult task. A modified quasi-maximum likelihood estimator for AR(1)/ARCH(1) model was proposed in [3]. Estimators based on least squares method were considered in [4]. In [5] a method for estimation unknown autoregressive parameters of AR/ARCH model with guaranteed accuracy based on weighted least squares method was proposed.

The problem of change point detection arises often in different applications connected with time series analysis, financial mathematics, image processing, etc. In [5] we proposed to detect the instant of parameter change by making use of guaranteed sequential estimators. In this study such approach is applied to the TAR(1)/ARCH(1) model. The properties are investigated. Simulation experiments were conducted and the result showed good performance of the proposed procedure.

Problem Statement. We consider TAR(1)/ARCH(1) process specified by the equation

$$\begin{aligned} x_{k+1} &= \lambda_1 x_k^+ + \lambda_2 x_k^- + \sqrt{\omega + \alpha x_k^2}\,\xi_{k+1}; \\ x_k^+ &= \max\{0, x_k\};\ x_k^- = \min\{0, x_k\}, \end{aligned} \tag{1}$$

where $\{\xi_k\}_{k\geq 0}$ is a sequence of i.i.d. random variables with zero mean and unit variance, $\omega > 0$. The value of the parameter vector $\lambda = [\lambda_1, \lambda_2]$ changes from $\mu^0 = [\mu_1^0, \mu_2^0]$ to $\mu^1 = [\mu_1^1, \mu_2^1]$ at the change point θ, so as $\left(\mu^0 - \mu^1\right)^2 \geq \Delta$, where Δ is the known value. Values of parameters before and after θ are supposed to be unknown. The problem is to detect the change point θ from observations x_k.

Estimation procedure. In [6] we obtained sufficient conditions of the ergodicity for process (1), which are as follows

$$\begin{gathered} \lambda_1 < 1, \lambda_2 < 1, \lambda_1\lambda_2 < 1, \mu = \max\left\{ \int_0^{\infty} z f_\xi(z)dz - \int_{-\infty}^{0} z f_\xi(z)dz \right\}, \\ \alpha^2 < \frac{\min\left\{(1-\lambda_1)^2, (1-\lambda_2)^2, (1-\lambda_1)^2(1-\lambda_2)^2, (1-\lambda_1\lambda_2)^2\right\}}{(2-\lambda_1-\lambda_2)^2\mu^2}. \end{gathered} \tag{2}$$

We construct the guaranteed estimator of the parameter vector λ, using estimator which was proposed in [6] and modify it for weaker assumptions about the noise distributions. It should be noted that process (1) is an autoregressive process with unknown and unbounded from above noise variance. To obtain a process with bounded noise variance we rewrite the process (1) in the form

$$y_{k+1} = \lambda_1 y_{k,1} + \lambda_2 y_{k,2} + \gamma_k \varepsilon_{k+1};\ m_k = \max\{1,|x_k|\};$$

$$y_{k+1} = \frac{x_{k+1}}{m_k},\ y_{k,1} = \frac{x_k^+}{\mathrm{m}_k},\ y_{k,2} = \frac{x_k^-}{\mathrm{m}_k},\ \gamma_k = \frac{\sqrt{\omega + \alpha x_k^2}}{m_k}.$$

The noise variance of the process y_k is bounded by unknown value $\omega + \alpha$. To eliminate the influence of the unknown constant the special factor Γ_N by first N observations is used. Let $\tilde{m}_n = \min(1,|x_n|)$, we select such interval, where all m_n are sufficiently differ from zero, so process (1) can be written as follows

$$\tilde{y}_{n+1} = \lambda_1 \tilde{y}_n^+ + \lambda_2 \tilde{y}_n^- + \tilde{\gamma}_n \varepsilon_n;\ \tilde{\gamma}_n = \frac{\sqrt{\omega + \alpha x_n^2}}{\tilde{m}_n};\ \tilde{y}_{n,1} = \frac{x_n^+}{\tilde{m}_n};\ y_{n,2} = \frac{x_n^-}{\tilde{\mathrm{m}}_n}.$$

For a process (1) with $|\lambda_1| < 1, |\lambda_2| < 1$ we take compensating factor Γ_N in a next form

$$\Gamma_N = C_N \sum_{n=1}^{N} \left\{ \frac{|\tilde{y}_{n+1}| + |\tilde{y}_n|}{\tilde{m}_n} \right\}^2,\ C_N = E\left(\sum_{n=1}^{N} \varepsilon_n^2 \right)^{-1}.$$

If $\{\xi_k\}$ have standard normal distribution then the sum $\sum_{k=1}^{N} \varepsilon_k^2$ has $\chi^2(N)$ distribution and $C_N = 1/((N-2)(N-4))$.

This constant is defined for $N \geq 5$. The proposed estimators of the parameter vector λ is written in the following form

$$\hat{\lambda}_i = \hat{\lambda}_i(H) = \frac{1}{\Gamma_N H} \sum_{k=N+1}^{\tau_i} v_{k,i} y_{k,i} y_{k+1},\ i = 1,2, \tag{3}$$

where stopping time $\tau_i = \min\left\{ t > N : \sum_{k=N+1}^{t} y_{k,i}^2 > \Gamma_N H \right\},\ i = 1,2.$ We take weighted coefficients $\{v_{n,i}\}_{n \geq 1}$ equal to 1 on $[1, \tau_i - 1]$. The last coefficients `for stopping moments $[\tau_1, \tau_2]$ can be found from following conditions $\sum_{n=N+1}^{\tau_i} v_{n,i} y_{n,i}^2 = \Gamma_N H.$

Theorem 1. For process (1) satisfying conditions (2) stopping time is finite with probability one. Estimators (3) are unbiased and the variance of the estimators is bounded from above

$$E\left(\hat{\lambda}_i - \lambda_i\right)^2 \leq \frac{1}{H},\ i = 1,2.$$

Theorem 2. If process (1) is ergodic, and compensating factor Γ_N satisfies the following conditions $N \to \infty$, $N/H \to \infty$ as $H \to \infty$, then for sufficiently large H

$$P\left\{\left(\hat{\lambda}_i-\lambda_i\right)^2>x\right\}\le 2\left(1-\Phi\left(\sqrt{xH}\right)\right),$$

where $\Phi(\cdot)$ is the standard normal distribution function.

Change point detection procedure. Since the parameters are unknown we use theirs estimators in the change point procedure. We construct a set of sequential estimation plans

$$\left(\tau_i^n,\hat{\lambda}_i^n\right)=\left(\tau_i^n(H),\hat{\lambda}_i^n(H)\right),\, n\ge 1,\, i=1,2,$$

where $\left\{\tau_i^n\right\}, n\ge 0$ is the increasing sequence of stopping instances and $\hat{\lambda}_i^n$ is guaranteed parameter estimator on the $\left[\tau_i^{n-1}+1,\tau_i^n\right]$. We associate the statistic J_i^n with interval $\left[\tau_i^{n-1}+1,\tau_i^n\right]$ for all $n\ge l$

$$J_i^n=\left(\hat{\lambda}_i^n-\hat{\lambda}_i^{n-l}\right)^2.$$

This statistic is squared deviation of the estimators with number n and $n-l$. Properties of the statistics are given in the following theorems.

Theorem 3. The probability of false alarm P_0 and the probability of delay P_1 in any observation cycle $\left[\tau_i^{n-1}+1,\tau_i^n\right]$ are bounded from above

$$P_0\le\frac{2}{\delta H},\, P_1\le\frac{2}{\left(\sqrt{\Delta}-\sqrt{\delta}\right)^2 H}.$$

Theorem 4. If process (1) is ergodic, and the compensating factor Γ_N satisfies the following conditions $N\to\infty$, $N/H\to\infty$ as $H\to\infty$, than for sufficiently large H the probability of false alarm P_0 and the probability of delay P_1 are bounded from above

$$P_0\le 2\left(1-\Phi\left(\sqrt{\delta H/2}\right)\right);\, P_1\le 1\left(1-\Phi\left(\left(\sqrt{\Delta}-\sqrt{\delta}\right)/\sqrt{H/2}\right)\right),$$

where $\Phi(\cdot)$ is the standard normal distribution function.

Simulation experiments. For the first step of our experiments two realization of the process (1) were simulated. Parameters $\{\lambda_1,\lambda_2,\alpha\}$ were chosen from condition of ergodicity of the TAR/ARCH process [13]. Parameters were takenequal $\{0,5;\,0,3;\,0,5\}$ for simulation ergodic process and $\{0,5;\,0,3;\,0,8\}$ for non-ergodic respectively. Results of simulation are illustrated on graphs 1.a and 1.b.

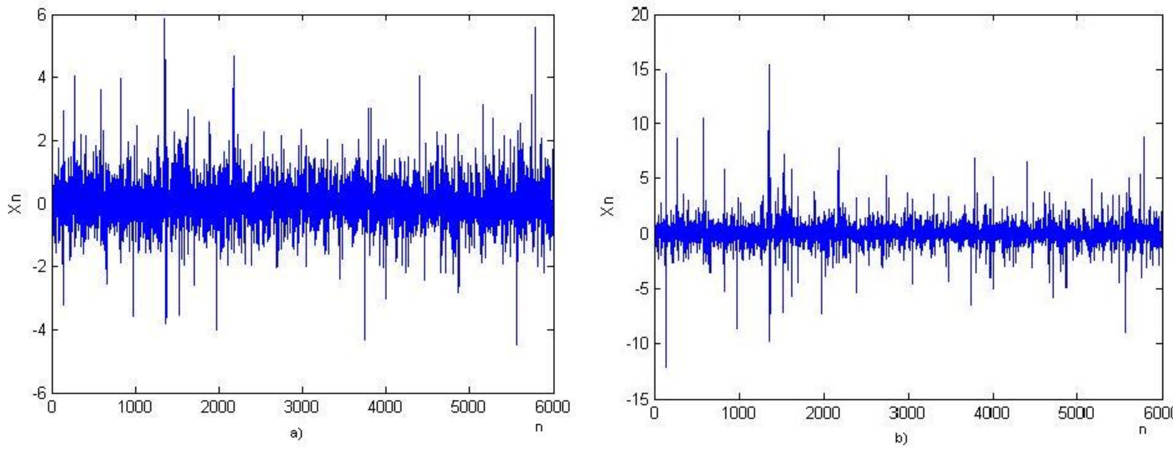

Fig. 1. Realizations of the TAR/ARCH process

For testing parameter estimation procedure the process (1) with parameters $\lambda_1 = 0,5,\ \lambda_2 = 0,3,\ \omega = 0,2,\ \alpha = 0,5$ was generated. The results of simulations are given in the following table

Table 1

Results of modeling estimation procedure

H	$E(\hat{\lambda}_1)$	$E(\hat{\lambda}_2)$	T_1	T_2	$\frac{1}{H}$	$E(\hat{\lambda}_1 - \lambda_1)^2$	$E(\hat{\lambda}_2 - \lambda_2)^2$
20	0,498	0,2977	754	1049	0,05	0,0036	0,0035
50	0,4995	0,2989	1882	2617	0,02	0,0015	0,0015
100	0,5004	0,2986	3760	5240	0,01	0,00068	0,00072
150	0,5005	0,2989	5630	7860	0,0067	0,00043	0,00047
200	0,5005	0,2988	7503	10468	0,005	0,00032	0,00033
400	0,4991	0,3005	15046	20865	0,0025	0,00015	0,00015

To sum up, one can see that the variances of the estimators do not exceed the theoretical upper bound $1/H$ and decrease with the growth of H. The mean numbers of observations T_1 and T_2 increase linearly by H. This property is important for sequential estimators.

For simulating change point detection procedure the process (1) with parameters $\lambda_1 = 0,5,\ \lambda_2 = 0,3,\ \omega = 0,2,\ \alpha = 0,5$ was generated. At the moment $\theta_1 = 100000$ the parameter λ_1 has been changed to $\lambda_1 = 0,3$, and at the moment $\theta_2 = 100000$ the parameter λ_2 has been changed to $\lambda_2 = 0,5$. The parameter δ was chosen in order to avoid false alarms. The results of simulations are given in the following table.

Table 2

Results of modeling change point procedure

H	δ	$E(\theta_1)$	$E(\theta_2)$
20	0.25	101910	100890
50	0.20	101180	103090
100	0.20	105540	104510
150	0.15	107340	103880
200	0.10	114740	107300
400	0.05	122410	113170

It should be noted that with the growth of H there is bigger length of delay in change point detection. It is connected with the accuracy of estimation.

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NONPARAMETRIC ESTIMATION OF NET PREMIUMS IN COLLECTIVE LIFE INSURANCE FOR THE M INDIVIDUALS

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Introduction. One of the main problem in actuarial mathematics is to find the «right» ratio between premiums and benefits. In this case, the calculation of net premiums allows to cover damages and to give the zero average income of the insurance company. Note that in the section, devoted to this area in the known book «Actuarial Mathematics» [1], is used the calculation of net premiums on the base of mortality tables. Interesting results by this approach have been presented in [2–6]. At present, the theory and practice of insurance is