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Comparative study of standard *k*–ε and *k*–ω turbulence models by giving an analysis of turbulent natural convection in an enclosure

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Abstract. Turbulent natural convection in a square cavity has been numerically studied. The mathematical model has been formulated in terms of the dimensionless stream function, vorticity and temperature using two standard turbulence models (k–ε and k–ω). For an obtaining of more accurate values of governing parameters close to the walls a special coordinate transformation has been used. Formulated partial differential equations along with the corresponding boundary conditions have been solved by the finite difference method. It has been shown that standard k–ε model is more accurate for the considered problem.

1. Introduction

Convective heat and mass transfer processes play a significant role in nature and many technical fields. Now these processes are associated with a wide range of problems [1]. Taking into account an importance of these processes, control of heat and mass transfer is a key problem. Therefore it is necessary to understand nature of this phenomenon and simulation methods.

An objective of this paper is a mathematical simulation of turbulent natural convection in a square enclosure having adiabatic horizontal walls and isothermal vertical walls using two standard turbulence models (k–ε and k–ω).

2. Governing equations and numerical results

The boundary-value problem of turbulent convective heat transfer in a square enclosure has been studied. The medium inside the cavity is assumed to be heat-conducting, Newtonian and the Boussinesq approximation is valid. The vertical walls ($x = 0$ and $x = L$) are kept at constant and different temperatures T_1 and T_2 ($T_1 > T_2$) while the horizontal walls are adiabatic.

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Taking into account this physical description of the problem, the heat transfer inside the domain of interest is modeled on the basis of unsteady two-dimensional equations of turbulent natural convection in terms of the dimensionless variables such as stream function, vorticity and temperature [2].

For an obtaining of more accurate values of governing parameters close to the walls a special coordinate transformation is used for a densening of the computational grid, allowing to pass from non-uniform grid in physical domain to a uniform grid in computational domain. Such transformation has the following form:

$$\xi = \frac{1}{2}\left\{1 + \mathrm{tg}\left[\frac{\pi}{2}(2X-1)\varepsilon\right]\Big/\mathrm{tg}\left[\frac{\pi}{2}\varepsilon\right]\right\};\ \eta = \frac{1}{2}\left\{1 + \mathrm{tg}\left[\frac{\pi}{2}(2Y-1)\varepsilon\right]\Big/\mathrm{tg}\left[\frac{\pi}{2}\varepsilon\right]\right\}.$$

As a result, the transfer processes of mass, momentum and energy in dimensionless variables such as stream function, vorticity and temperature taking into account the abovementioned algebraic coordinate transformation are described by the following differential equations:

$$\frac{d^2\xi}{dX^2}\frac{\partial\Psi}{\partial\xi} + \left(\frac{d\xi}{dX}\right)^2\frac{\partial^2\Psi}{\partial\xi^2} + \frac{d^2\eta}{dY^2}\frac{\partial\Psi}{\partial\eta} + \left(\frac{d\eta}{dY}\right)^2\frac{\partial^2\Psi}{\partial\eta^2} = -\Omega, \tag{1}$$

$$\begin{aligned}
&\frac{\partial\Omega}{\partial\tau} + \left(U - \frac{d\xi}{dX}\frac{\partial\nu_t}{\partial\xi}\right)\frac{d\xi}{dX}\frac{\partial\Omega}{\partial\xi} + \left(V - \frac{d\eta}{dY}\frac{\partial\nu_t}{\partial\eta}\right)\frac{d\eta}{dY}\frac{\partial\Omega}{\partial\eta}\\
&= \frac{d\xi}{dX}\frac{\partial}{\partial\xi}\left[\left(\sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} + \nu_t\right)\frac{d\xi}{dX}\frac{\partial\Omega}{\partial\xi}\right] + \frac{d\eta}{dY}\frac{\partial}{\partial\eta}\left[\left(\sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} + \nu_t\right)\frac{d\eta}{dY}\frac{\partial\Omega}{\partial\eta}\right]\\
&+ \left(\frac{d^2\xi}{dX^2}\frac{\partial\nu_t}{\partial\xi} + \left(\frac{d\xi}{dX}\right)^2\frac{\partial^2\nu_t}{\partial\xi^2} - \frac{d^2\eta}{dY^2}\frac{\partial\nu_t}{\partial\eta} - \left(\frac{d\eta}{dY}\right)^2\frac{\partial^2\nu_t}{\partial\eta^2}\right)\\
&\times\left(\Omega + 2\frac{d\eta}{dY}\frac{\partial U}{\partial\eta}\right) + 4\frac{d\xi}{dX}\left(\frac{d\eta}{dY}\right)^2\frac{\partial^2\nu_t}{\partial\xi\partial\eta}\frac{\partial V}{\partial\eta} + \frac{d\xi}{dX}\frac{\partial\Theta}{\partial\xi},
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\frac{\partial\Theta}{\partial\tau} + U\frac{d\xi}{dX}\frac{\partial\Theta}{\partial\xi} + V\frac{d\eta}{dY}\frac{\partial\Theta}{\partial\eta} = \frac{d\xi}{dX}\frac{\partial}{\partial\xi}\left[\left(\frac{1}{\sqrt{\mathrm{Ra\,Pr}}} + \frac{\nu_t}{\mathrm{Pr}_t}\right)\frac{d\xi}{dX}\frac{\partial\Theta}{\partial\xi}\right]\\
&\quad + \frac{d\eta}{dY}\frac{\partial}{\partial\eta}\left[\left(\frac{1}{\sqrt{\mathrm{Ra\,Pr}}} + \frac{\nu_t}{\mathrm{Pr}_t}\right)\frac{d\eta}{dY}\frac{\partial\Theta}{\partial\eta}\right].
\end{aligned} \tag{3}$$

As a turbulence model we used the standard k–ε model [2]:

$$\begin{aligned}
&\frac{\partial K}{\partial\tau} + U\frac{d\xi}{dX}\frac{\partial K}{\partial\xi} + V\frac{d\eta}{dY}\frac{\partial K}{\partial\eta} = \frac{d\xi}{dX}\frac{\partial}{\partial\xi}\left[\left(\sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} + \frac{\nu_t}{\sigma_k}\right)\frac{d\xi}{dX}\frac{\partial K}{\partial\xi}\right]\\
&\quad + \frac{d\eta}{dY}\frac{\partial}{\partial\eta}\left[\left(\sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} + \frac{\nu_t}{\sigma_k}\right)\frac{d\eta}{dY}\frac{\partial K}{\partial\eta}\right] + \bar{P}_k + \bar{G}_k - E,
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\frac{\partial E}{\partial\tau} + U\frac{d\xi}{dX}\frac{\partial E}{\partial\xi} + V\frac{d\eta}{dY}\frac{\partial E}{\partial\eta} = \frac{d\xi}{dX}\frac{\partial}{\partial\xi}\left[\left(\sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{d\xi}{dX}\frac{\partial E}{\partial\xi}\right]\\
&\quad + \frac{d\eta}{dY}\frac{\partial}{\partial\eta}\left[\left(\sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}}} + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{d\eta}{dY}\frac{\partial E}{\partial\eta}\right] + \left[c_{1\varepsilon}\left(\bar{P}_k + c_{3\varepsilon}\bar{G}_k\right) - c_{2\varepsilon}E\right]\frac{E}{K}.
\end{aligned} \tag{5}$$

and standard k–ω model

$$\frac{\partial K}{\partial \tau} + U\frac{d\xi}{dX}\frac{\partial K}{\partial \xi} + V\frac{d\eta}{dY}\frac{\partial K}{\partial \eta} = \frac{d\xi}{dX}\frac{\partial}{\partial \xi}\left[\left(\sqrt{\frac{\text{Pr}}{Ra}} + \frac{\nu_t}{\sigma_k}\right)\frac{d\xi}{dX}\frac{\partial K}{\partial \xi}\right]$$
$$+ \frac{d\eta}{dY}\frac{\partial}{\partial \eta}\left[\left(\sqrt{\frac{\text{Pr}}{Ra}} + \frac{\nu_t}{\sigma_k}\right)\frac{d\eta}{dY}\frac{\partial K}{\partial \eta}\right] + \bar{P}_k + \bar{G}_k - \beta^* W K, \quad (6)$$

$$\frac{\partial W}{\partial \tau} + U\frac{d\xi}{dX}\frac{\partial W}{\partial \xi} + V\frac{d\eta}{dY}\frac{\partial W}{\partial \eta} = \frac{d\xi}{dX}\frac{\partial}{\partial \xi}\left[\left(\sqrt{\frac{\text{Pr}}{\text{Ra}}} + \frac{\nu_t}{\sigma_k}\right)\frac{d\xi}{dX}\frac{\partial W}{\partial \xi}\right]$$
$$+ \frac{d\eta}{dY}\frac{\partial}{\partial \eta}\left[\left(\sqrt{\frac{\text{Pr}}{\text{Ra}}} + \frac{\nu_t}{\sigma_k}\right)\frac{d\eta}{dY}\frac{\partial W}{\partial \eta}\right] + \alpha\frac{W}{K}\bar{P}_k + \frac{W}{K}\bar{G}_k - \beta W^2. \quad (7)$$

Here we have the following parameters:

$$\bar{P}_k = \nu_t\left[2\left(\frac{d\xi}{dX}\frac{\partial U}{\partial \xi}\right)^2 + 2\left(\frac{d\eta}{dY}\frac{\partial V}{\partial \eta}\right)^2 + \left(\frac{d\eta}{dY}\frac{\partial U}{\partial \eta} + \frac{d\xi}{dX}\frac{\partial V}{\partial \xi}\right)^2\right],$$

$$\bar{G}_k = -\frac{\nu_t}{\text{Pr}_t}\frac{d\eta}{dY}\frac{\partial \Theta}{\partial \eta}, \ \nu_t = c_\mu K^2 / E.$$

The standard k–ε model parameters are:

$$\sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3, \quad \text{Pr}_t = 1.0, \quad c_\mu = 0.09, \quad c_{1\varepsilon} = 1.44, \quad c_{2\varepsilon} = 1.92, \quad \sigma_{3\varepsilon} = 0.8.$$

The standard k–ω model parameters are:

$$\alpha = 0.56, \ \beta = 0.075, \ \beta^* = 0.09, \ \sigma_k = 0.5, \ \sigma_\omega = 0.5.$$

Initial and boundary conditions for the formulated differential Eqs. (1)–(5) or (1)–(3), (6), (7) have been described in detail previously in [2, 3].

The formulated boundary-value problem has been solved by the finite difference method [2] on uniform mesh (ξ, η) of 200×200 points. The parabolic equations have been solved using the locally one-dimensional Samarsky scheme. The obtained system of linear algebraic equations with special matrix has been solved by Thomas algorithm. Convective terms have been approximated using the monotone Samarsky scheme and the diffusive terms have been approximated by central differences. The difference Poisson equation for the stream function has been solved by successive over relaxation method.

Numerical simulation has been conducted in a wide range of the Rayleigh and Prandtl numbers ($10^7 \leq \text{Ra} \leq 1.58 \cdot 10^9$, $\text{Pr} = 0.7, 7.0, 0.0115$). The distributions of isolines of stream function, temperature, turbulent kinetic energy and dissipation rate of turbulent kinetic energy have been defined. It has been shown that an increase in the Rayleigh number leads to both a decrease in the thermal boundary layer thickness and a narrowing of convective core. Three-core convective structure forms in the central part of the cavity at small Prandtl number values ($\text{Pr} = 0.0115$).

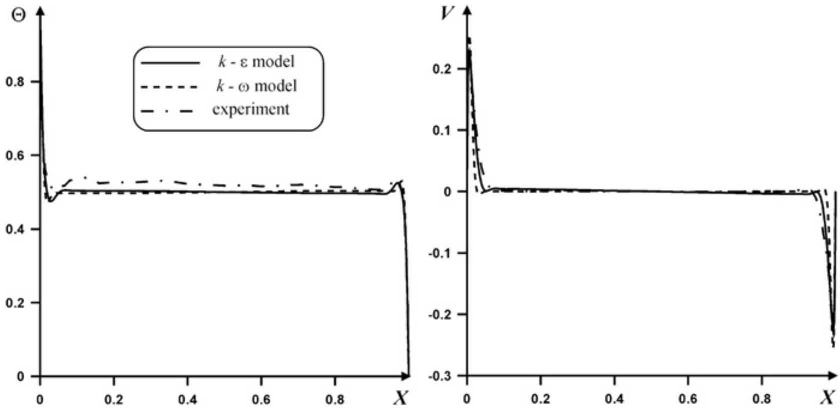

Figure 1. Profiles of temperature and vertical velocity at middle cross-section of the cavity in comparison with experimental data [3].

The obtained results allowed to analyze two turbulence models (standard k–ε and k–ω models) and to compare them with experimental data [3] (Fig. 1). It has been shown that standard k–ε model is more accurate for the considered problem and has good agreement with data of other authors [3, 4].

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