Comparison of the physical optics code with the GOIE method and the direct solution of Maxwell equations obtained by FDTD

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ABSTRACT

A comparison of the physical optics code and GOIE method to solve the problem of light scattering by hexagonal ice crystals has been presented. It was found that in the case of diffraction on a hole in the perpendicular screen, both methods give the same diffraction scattering cross section for the diffraction angles up to 60 degrees. The polarization elements of the Mueller matrix in this case differ significantly even for the angles of 15-30 degrees. It is also shown that in the case of diffraction on the tilted screen, the difference between these methods may be significant. The comparison of the results with the exact solution obtained by FDTD has confirmed that the difference between these methods is not significant for the case of diffraction on the perpendicular screen, but it is slightly preferable to use the GOIE for the calculations. The good agreement with the exact solution confirms the possibility of using the method of physical optics to solve the problem of light scattering by particles with characteristic size greater than 10 microns.

Keywords: physical optics approximation, beam-splitting technique, light scattering, ice crystals, FDTD

1. INTRODUCTION

At present, there is no generally accepted method to solve the problem of light scattering by ice crystals of cirrus clouds. However, the cirrus clouds are the main source of uncertainty in the development of numerical models of the radiation balance of the planet[1-3]. For this reason, attempts to solve this problem with completely different approaches are being made within two past decades[4-10]. There are high expectations on the method of physical optics[11-13].

Despite the active development of physical optics approximations[14-19], the international optical scientific community still has no commonly used definition of this approximation. It is obvious that the physical optics approximations are extensions of geometrical optics by an approximate calculation of the wave properties of light. This does not specify how the wave properties of light are taken into account.

Nevertheless, there are three diffraction theories: *E*-theory, *M*-theory and (*E*,*M*)-theory[20]. These lead to three formulae:

$$\mathbf{E}_{I}^{sct}(\mathbf{r}) = ik\frac{e^{ikr}}{2\pi r}\mathbf{s}\times\oint_{S_p^+}\left(\mathbf{n}\times\mathbf{E}(\mathbf{r}')\right)e^{-ik\mathbf{s}\cdot\mathbf{r}'}ds', \tag{1}$$

$$\mathbf{E}_{II}^{sct}(\mathbf{r}) = -ik\frac{e^{ikr}}{2\pi r}\mathbf{s}\times\left(\mathbf{s}\times\oint_{S_p^+}\left(\mathbf{n}\times\left(\mathbf{r}_b\times\mathbf{E}(\mathbf{r}')\right)\right)e^{-ik\mathbf{s}\cdot\mathbf{r}'}ds'\right), \tag{2}$$

$$\mathbf{E}^{sct}(\mathbf{r}) = ik\frac{e^{ikr}}{4\pi r}\left(-\mathbf{s}\times\left(\mathbf{s}\times\oint_{S_p^+}\left(\mathbf{n}\times\left(\mathbf{r}_b\times\mathbf{E}(\mathbf{r}')\right)\right)e^{-ik\mathbf{s}\cdot\mathbf{r}'}ds'\right)+\mathbf{s}\times\oint_{S_p^+}\left(\mathbf{n}\times\mathbf{E}(\mathbf{r}')\right)e^{-ik\mathbf{s}\cdot\mathbf{r}'}ds'\right), \tag{3}$$

respectively.

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The first two expressions are mathematically correct formulation, while the last one, strictly speaking, corresponds to a saltus problem. The solution in the form (1) was used in [16, 21-23], let us refer to this solutions as the physical optics approximation. The solution in the form (3) was used in [17, 24, 25], let us refer to this solutions as the GOIE method.

This paper compares these methods for solving the problem of light scattering by quasi-horizontally oriented hexagonal ice plates of cirrus clouds in the vicinity of the backscattering direction, which are the subject of research in a number of papers[26, 27].

2. COMPARISON OF NUMERICAL SOLUTIONS OBTAINED BY THE PHYSICAL OPTICS CODE AND GOIE METHOD

The physical optics code[16] was implemented by the authors by use of the beam-splitting algorithm, which calculated a field inside the crystal in the framework of the geometrical optics approximation. The beam-splitting algorithm is described in detail in [28, 29]. The modified algorithm with the physical optics approximation is available at [30]. The difference of this algorithm from the one described in [28, 29] consists only of diffraction formulas (1)-(3).

Let us compare the three diffraction formulas (1)-(3). As a test problem, we consider the plane-parallel wave going out from the hexagonal face of the ice crystal. The mathematical formulation in this case is equivalent to the diffraction on a perpendicular screen with a hexagonal hole (see. Fig. 1).

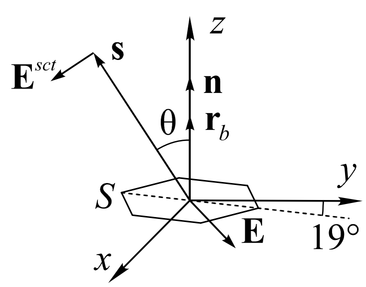

Figure 1. Diffraction on a perpendicular screen.

We carried out a number of calculations for various sizes and shapes of the particle, as well as for various orientations of the polarization vector of the incident wave with respect to the hole. Below, as illustrations, only borderline cases will be presented that allow us to formulate the basic regularities.

Let the plane-parallel wave with unit intensity **E** propagating in the direction $\mathbf{r}_b$ falls to the hexagonal hole S with unit normal **n**. We find the scattered field $\mathbf{E}^{sct}$ in the direction **s**. We assume that the wave is incident on the hole normally: $\mathbf{n} = \mathbf{r}_b$. The hole's diameter is 10 μm, the wavelength is 0.532 μm.

To get rid of the symmetry the particle is rotated by 19 degrees. In this case, the scattering matrix has the view

$$\mathbf{M} = M_{11}\begin{pmatrix} 1 & m_{12} & 0 & 0 \\ m_{12} & 1 & 0 & 0 \\ 0 & 0 & m_{33} & 0 \\ 0 & 0 & 0 & m_{33} \end{pmatrix}. \quad (4)$$

The results of calculations are presented in Fig. 2. They show that element M_{11} of the Mueller matrix obtained by the formulas (1) agrees well with the element M_{11} obtained by the formula (3) in the range of diffraction angles from 0 to 60 degrees. It is also clearly seen that the solution of the integral (1) is symmetric with respect to the plane of the screen, consequently this solution is correct only "behind the screen". It is worth noting that all three diffraction formulas satisfy the energy conservation law.

The calculations also show that the polarization elements m_{12} and m_{33} begin to diverge much faster. Thus, element m_{33} agrees in the range from 0 to 30 degrees, while the element m_{12} agrees only in the range from 0 to 15 degrees. The solution of 10 micron apertures was used as an example. If the apertures will increase, it leads to a change of only M_{11}

element in the way that the size of the diffraction fringes will decrease. Consequently, for a hole larger than 10 microns, all three formulas coincide well within the first five diffraction fringes.

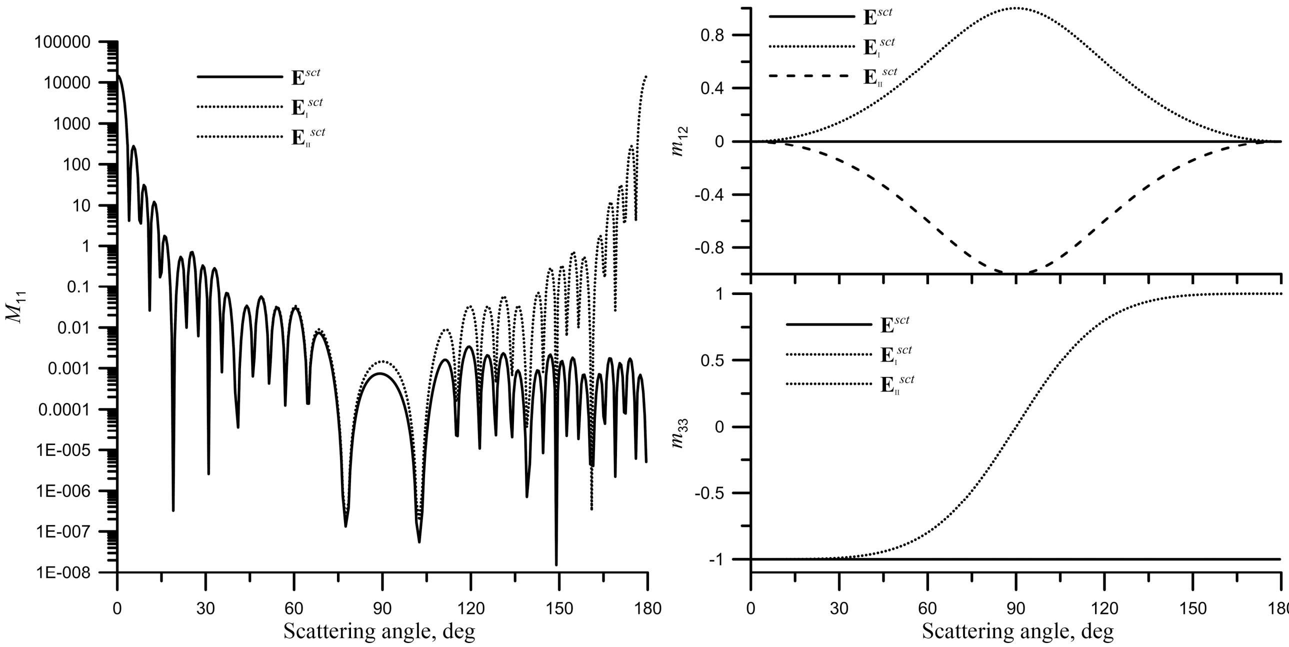

Figure 2. Mueller matrix elements for the case of diffraction on a perpendicular screen.

Let us consider the case of light is incident on a screen at angle β (see. Fig. 3). In this case, it is useful to consider the solution obtained on the imaginary screen with a hole S', which is perpendicular to the incident wave. Let us define this solution as $\mathbf{E}_{\perp}^{sct}$. The hole S' is defined as a projection of the hole S on the imaginary screen. The Mueller matrix elements are shown in Fig. 4.

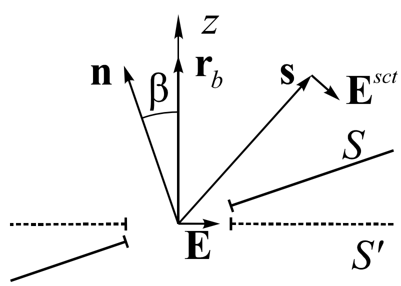

Figure 3. Diffraction on a tilted screen.

The results of calculations show that the element M_{11} has a good agreement for all solutions at small diffraction angles, but it is significantly different at diffraction angles close to 180 degrees. It is seen that $\mathbf{E}_{I}^{sct}$ creates a false diffraction peak as the mirror image of the existing one, therefore, the solution must be cut off. The cropping should occur at the angles of 110 and -70 degrees, in this case. At these angles the scattering cross section sharply vanishes.

It is evident that the polarization elements of the Mueller matrix in case of a tilted screen diverge faster, but, at the direction of beam propagation $\mathbf{r}_b$, all four solutions are equal. When the diffraction angle is less than 10 degrees the divergence of the elements m_{12} and m_{33} is less than 8% and 2%, respectively.

The results of calculations show that the integrals (1) and (2) give bad solutions for the case of a tilted screen, with the tilt angle more than 40 degrees. The polarization elements m_{12} and m_{33} are dramatically changed in the vicinity of the first diffraction fringe. By the way, $\mathbf{E}_{\perp}^{sct}$ and $\mathbf{E}^{sct}$ demonstrate relative agreement within the first three diffraction fringes.

This is $\mathbf{E}_{\perp}^{sct}$ that is used in the physical optics code in the case of diffraction on a tilted screen, while GOIE method uses $\mathbf{E}^{sct}$ to evaluate the scattered field.

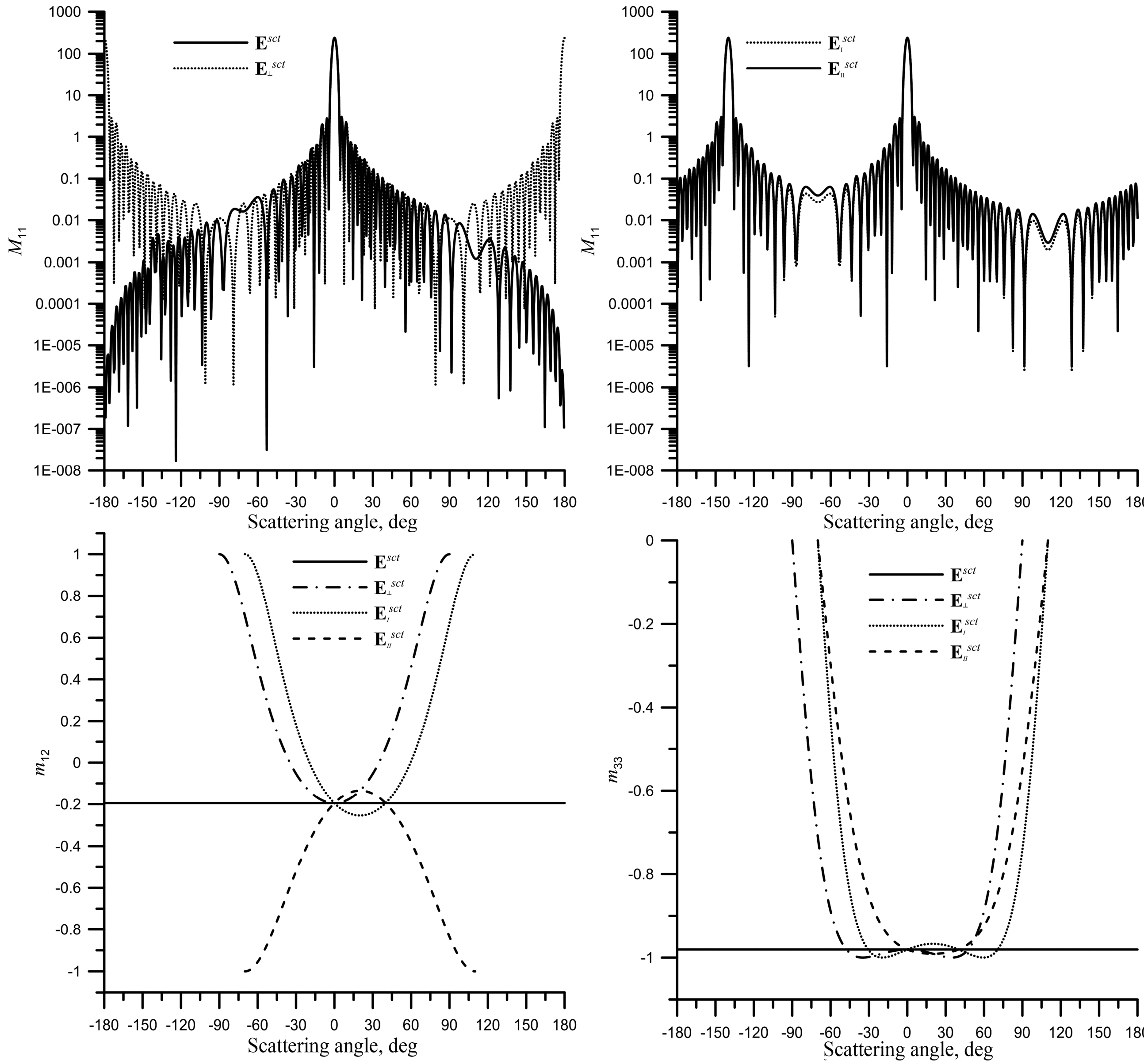

Figure 4. Mueller matrix elements for the case of diffraction on a tilted screen.

3. COMPARISON WITH THE EXACT SOLUTION OBTAINED BY FDTD

The physical optics code is inherently approximate, so it has to be compared with an exact solution. This comparison is difficult to carry out because of computing resources requirements of FDTD method for such kind of problem. These requirements significantly increase with increasing particle size. Cooperating with Meteorological Research Institute we managed to get a solution for the problem of light scattering by hexagonal ice plate with diameter of 10 µm and height of 5.79 µm. It was assumed that the light is incident to the hexagonal face of a plate normally. The solution has been obtained over all scattering directions and then uniformly averaged over the azimuthal angle. The refractive index was assumed to be 1.3116 for the wavelength of 0.532 microns. Due to computing resources requirements, the exact solution has been obtained with the mesh of 1 degree.

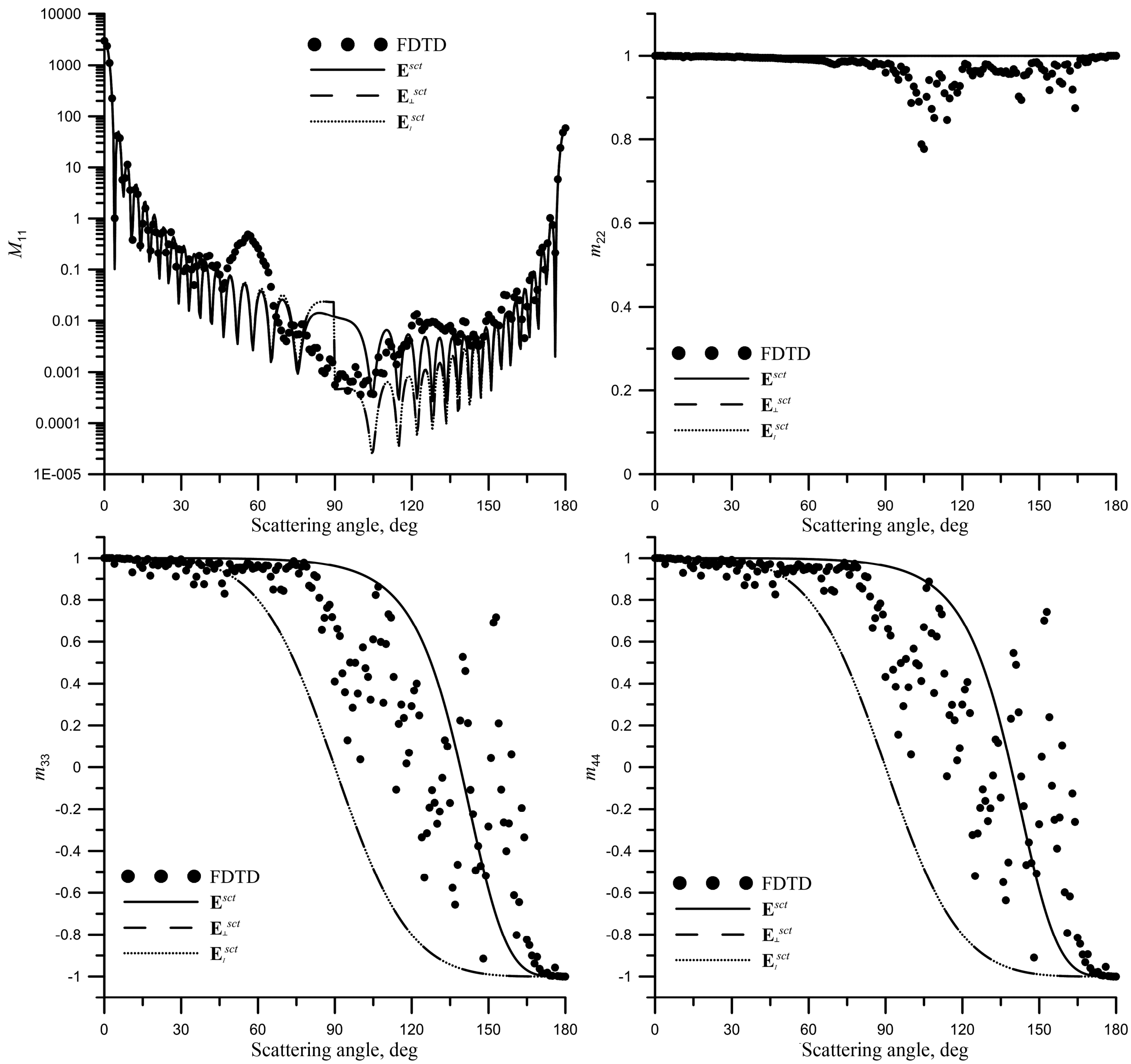

Figure 5. Comparison the physical optics code to the exact solution obtained by FDTD.

The results of a comparison are shown in Fig. 5. It is seen that the intensity of the scattered light is equal within the first five diffraction fringes in the forward scattering direction and within the first three diffraction fringes in the backward scattering direction. This is the very good result. In the region of scattering angles from 30 to 150 degrees total scattered energy is negligible, so differences in the results can be ignored. However, absence of the jump of intensity at 90 degrees for GOIE method makes it preferred for the calculation of intensity. According to the diagonal elements of the Mueller matrix, it can be concluded that the results of FDTD method was obtained with a coarse mesh. This is particularly evident in Fig. 4 of a range of scattering angles between 80 and 130 degrees. It is also worth bearing in mind that in this area the intensity value is 100000 times lower than the value at 0 degree. It leads to additional computational errors. However, in Fig. 4 the clear correlation of the exact solution with the solution for $\mathbf{E}^{sct}$ is seen while the solution for $\mathbf{E}_{\perp}^{sct}$ significantly diverge from the exact solution at the scattering region from 40 to 160 degrees.

4. CONCLUSION

A comparison of the approximate solutions obtained by equations (1)-(3) with the exact solution obtained by FDTD method showed that in the case of scattering by a large particles at small diffractions angle, generally speaking, all three formulas are suitable. The physical optics code based on equation $\mathbf{E}_{\perp}^{sct}$ is very close to GOIE method based on equation $\mathbf{E}^{sct}$, which corresponds to a saltus problem.

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