View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Tomsk State University Repository

Coherent and incoherent additions of light beams at solutions of the light scattering problem by use the beam tracing method within the framework of physical optics

Alexander V. Konoshonkin[a,b], Natalia V. Kustova[*a], Anatoli G. Borovoi[a,b], Hajime Okamoto[c]
[a]National Research Tomsk State University, 36, Lenina Avenue, Tomsk, 634050, Russia;
[b]V.E. Zuev Institute of Atmospheric Optics, Rus. Acad. Sci., 1 Academician Zuev Sq., Tomsk, 634021, Russia;
[c]Kyushu University, Kasuga, Fukuoka 816-8580, Japan.

ABSTRACT

Interference effects between the scattered beams in the problem of light scattering by atmospheric ice crystals have been studied. Since the crystals are much larger than the wavelength, it is shown that the interference effects can be neglected if the crystal sizes are statistically varied more than 5% of the mean size. As a result, any calculations of the optical properties of the crystals performing an averaging over crystal sizes can be effectively replaced by the procedure of the incoherent addition of the scattered beams. This procedure allows us decrease the execution time up to 100 times.

Keywords: physical optics, interference, light scattering, cirrus clouds

1. INTRODUCTION

At present, none of the numerical methods is capable to solve effectively the problem of light scattering by ice crystals of cirrus clouds. On the one hand, there is the classical approach where a strict solution is obtained starting from the Maxwell equations [1]. However, such an approach demands great computer resources where complexity of the calculations increases dramatically with particle size. On the other hand, there is the geometric-optics approximation [2,3]. Advantages of this approach are its simplicity and calculation speed and the drawback is appearance of singularities in its solution. Therefore the physical-optics method developed by the authors for last 5 years [4,5] looks as a good choice solving the problem of light scattering by the submillimeter particles. Of course, the physical-optics method ranks below the geometric-optics approach concerning the calculation speed.

In this paper, we discuss the possibilities to add the scattered beams either coherently or incoherently within the physical-optics approximation that impact essentially on the calculation speed of the method.

2. HEXAGONAL PLATE

At solutions of the problem of light scattering by ice hexagonal plates, the following equation connecting their heights (h) and diameters (D) is often used [6-8]

$$h = 2.02 D^{0.449}, \tag{1}$$

that was obtained in 1970 [9] as an approximation of the experimental data (see Fig. 1).

Let us consider an impact of the coherent and incoherent addition of the scattered beams on optical properties of such crystals. The scattered beams are defined and described by us in details in the papers [10].

As a test problem, we use a horizontally oriented hexagonal ice plate that is illuminated perpendicularly. The diameter was taken as $D = h/0.568$ at the wavelength $\lambda = 0.532\ \mu m$ and the refractive index $n = 1.3116$.

*kustova@iao.ru; phone +7 3822 492864; fax +7 3822 492086; iao.ru

21st International Symposium on Atmospheric and Ocean Optics: Atmospheric Physics,
edited by G. G. Matvienko, O. A. Romanovskii, Proc. of SPIE Vol. 9680, 96802X
© 2015 SPIE · CCC code: 0277-786X/15/$18
doi: 10.1117/12.2204879

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 11/25/2015 Terms of Use: http://spiedigitallibrary.org/ss/TermsOfUse.aspx

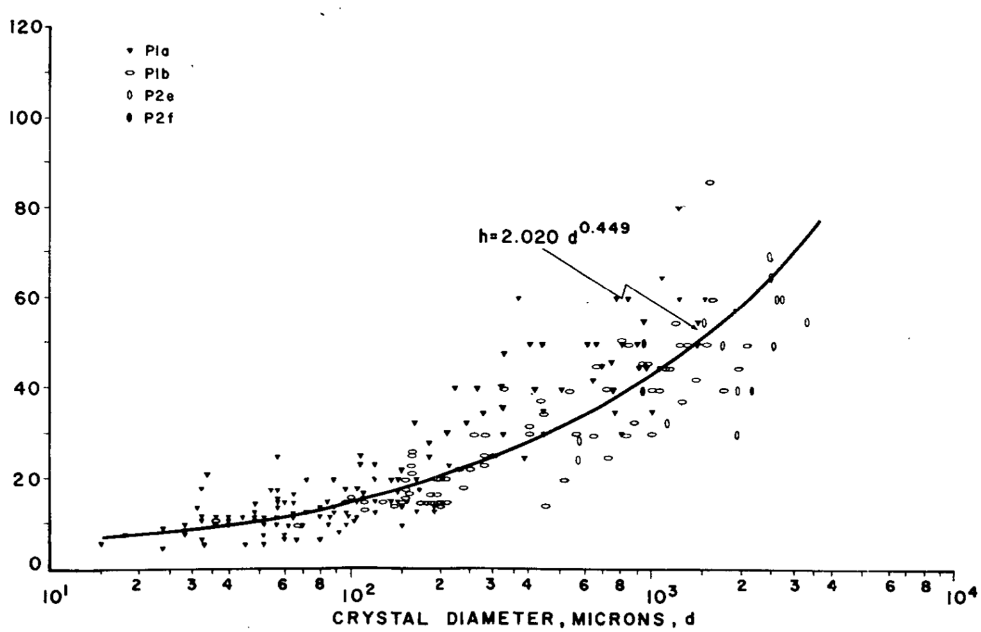

Figure 1. Dependence of heights of the hexagonal ice plates on their diameters obtained experimentally in [9].

It is convenient to consider several cases where either coherent or incoherent additions were applied for the specular reflected optical beams (see Fig. 2(a)) in the case of the plate with varying heights. In Fig. 2(b), the solid curves represent the differential scattering cross sections obtained by the coherent addition of two hexagonal beams in the vicinity of the backward direction $\theta = 0$. The data of the incoherent addition are shown in Fig. 2(b) by dots.

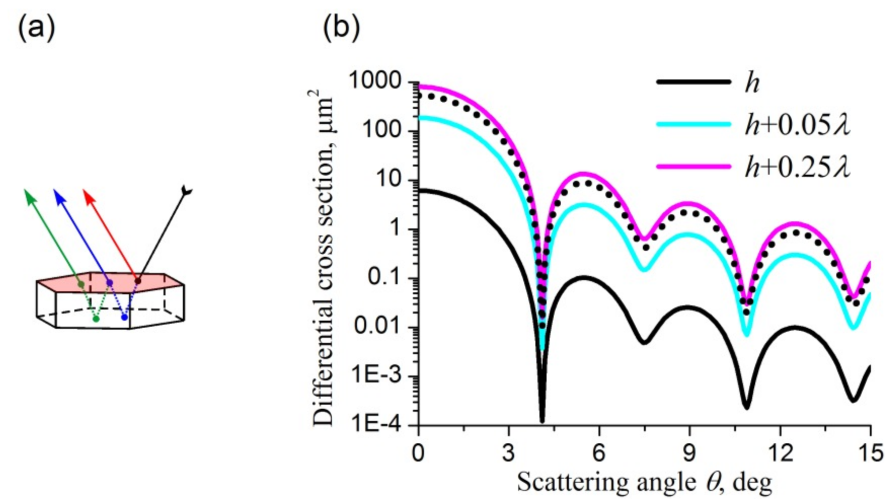

Figure 2. The specular optical beams giving main contributions to backscattering (a); the differential scattering cross section obtained at the coherent addition of two hexagonal optical beams in the vicinity of the backward direction $\theta = 0$ for the plate with varying height. The data of the incoherent addition are presented by the dots.

It is obvious that Eq. (1) concerns only the mean values of the height. Therefore we can use the Gaussian distribution to take into account the variance of the heights

$$f(\tilde{h}, s, h) = \frac{1}{s\sqrt{2\pi}} exp\left(-\frac{(\tilde{h}-h)^2}{2s^2}\right), \tag{2}$$

where f is the distribution function for the real heights $\tilde{h}$, h – the mean height obeying Eq. (1) and s is the variance.

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 11/25/2015 Terms of Use: http://spiedigitallibrary.org/ss/TermsOfUse.aspx

Let us calculate the mean intensity of the backscattered light $I(\tilde{h},D)$ using Eq. (2). For simplicity, the averaging was performed in the limits from h-4s to h+4s that provided the accuracy of 99.994%

$$< I(D) >= \int_{h(D)-4s}^{h(D)+4s} I\left(\tilde{h},D\right) f(\tilde{h},s,h(D))d\tilde{h}\,, \tag{3}$$

Fig. 3 shows the initial value of the intensity (red curve) and its mean values for $s = \{0.005h, 0.01h, 0.05h\}$ that are the green, yellow, and blue curves, respectively. We see that the oscillations disappear if the height variance is more than 5%.

The oscillations presented in Fig. 3(a) are the results of the coherent addition of two beams. They are of the same origination as the interference fringes at light reflection from a plane-parallel plate.

The incoherent addition of these beams should give the mean value. As an example, Fig. 3(b) represents the solutions obtained by the incoherent addition of two beams (black curve) and the values obtained by the coherent addition with the height variance of 0.5% (green curve) and 5% (red curve).

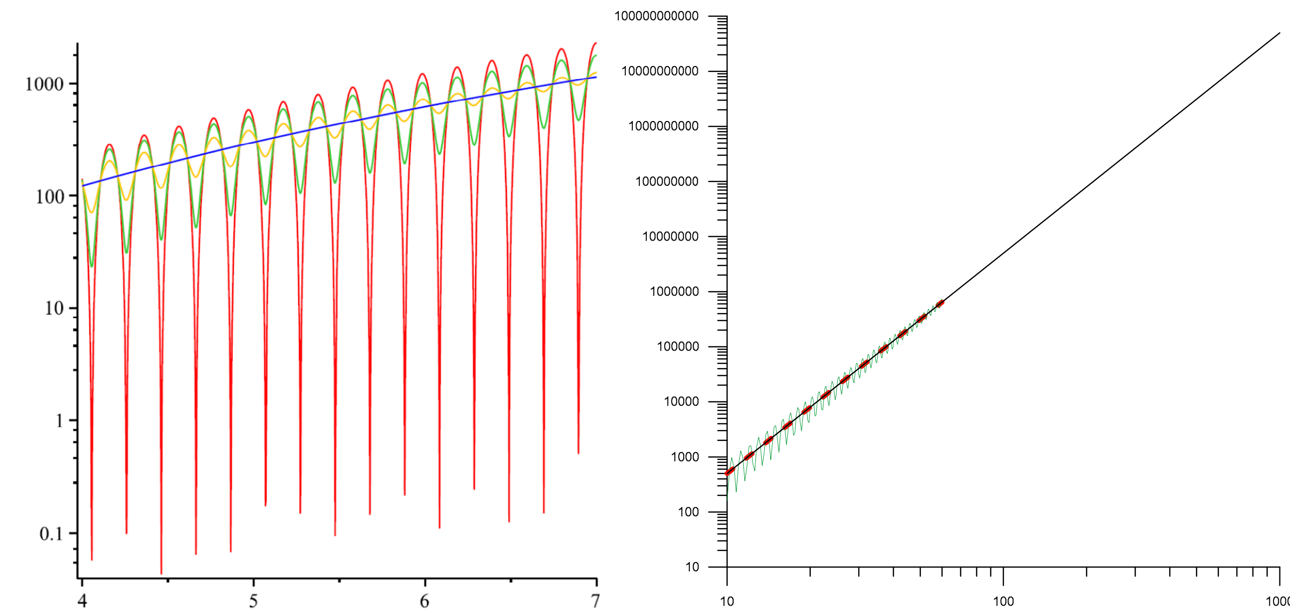

Figure 3. Dependence of the backscattered intensity on the plate height for different variances of the heights (a), the dependence of the intensity on the plate diameter at the coherent and incoherent additions of the light beams (b).

It is seen from Fig. 3 that already at the variance of 5% the light intensity coincides with the result obtained by the incoherent addition of the beams. Analyzing the experimental data obtained in [9] we find that the ice crystals occurring in nature reveals the height variance larger than 15%. Here, if anybody is finding a strict solution, he has to get the exact solutions with the mesh less than 0.02λ and then to make the averaging. Such an approach is very labor-consuming, it is necessary to get, at least, 100 additional solutions of the light scattering problem needed to make the averaging over the heights. The incoherent addition of these beams essentially shortens the execution time [10-17]. Thus, the specularly reflected beams should be added incoherently.

3. HEXAGONAL COLUMN

Unlike the case of the hexagonal plates, the hexagonal columns create backscattering by other types of ray trajectories. We show below that these types of the trajectories should be added coherently.

We choose the hexagonal column as the basic shape since it is the typical shape of the crystals in cirrus clouds. Besides, it is the hexagonal column that is used in majority of the papers dealing with any test calculations of the optical

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 11/25/2015 Terms of Use: http://spiedigitallibrary.org/ss/TermsOfUse.aspx

properties [18-20]. In this paper, we consider the hexagonal column of the typical diameter of $D = 30\,\mu\text{m}$. Heights of the columns h in cirrus clouds are not statistically independent of the diameters, as seen in Fig. 4. The height is taken from the model of [8] as $h = 73.64\,\mu\text{m}$. The refractive index is taken as 1.3116 for $\lambda = 0.532\,\mu\text{m}$. The main axis is assumed to pass through centers of the hexagonal faces. Orientation of a column is determined by three Euler angles α , β , and γ : α defines rotation of the crystal about the incident direction; β is the crystal tilt, i.e. the angle between the incident direction and the main axis; and γ describes crystal rotation about the main axis.

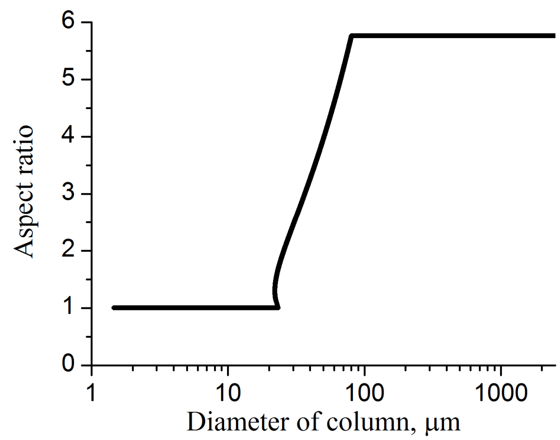

Figure 4. Relation between aspect ratio Q=h/D and D for the ice columns according to [8].

Our previous numerical calculations performed for randomly oriented regular columns [21-24] proved that predominant contribution to both the backward direction $\theta = 0$ and its vicinity $\theta \neq 0$ is obtained from beams shown in Fig. 5. These beams are associated with the corner-reflection effect produced by the dihedral angle of 90° between crystal faces [2,25]. The beams 1-4 leaving the skew rectangular faces are called the skew ones while the other beams 5-6 are called the straight ones. It is important that all the beams reveal a sharp maximum of their contributions to the backscatter if a column is orientated as ($\beta \approx 32°$, $\gamma = 0°$).

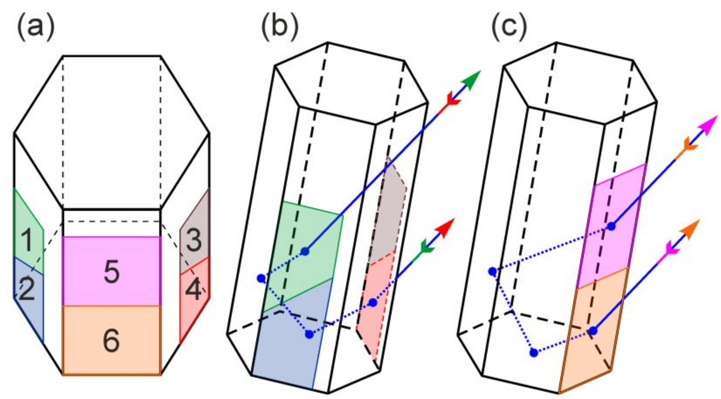

Figure 5. Skew (1-4) and straight (5-6) plane-parallel beams giving predominant contributions to backscatter. Their ray trajectories are shown in Fig. 5(b) and Figs. 5(c), respectively.

Consider the skew beams at the fixed column orientation of ($\beta = 32.22°$, $\gamma = 0°$). Here phases of the beams are the same because of similarity of their ray trajectories. Therefore contributions of the beams to the backscatter should be summarized coherently. The calculation results obtained for the skew trajectories at their coherent and incoherent additions are presented in Figs. 6(a) and 6(b), respectively. The similar results for the straight trajectories are shown in Figs. 7(a) and 7(b), respectively.

As seen in Figs. 6 and 7, the coherent and incoherent additions of the beams lead to different results both qualitatively and quantitatively. It is important to mention that the variance of the crystal sizes does not create any oscillations of the

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 11/25/2015 Terms of Use: http://spiedigitallibrary.org/ss/TermsOfUse.aspx

differential scattering cross section. Fig. 8 shows the differential scattering cross section averaged over the rotation angle γ for the skew and straight trajectories at the size variance of 0.5λ. Contrary to Fig. 2(b), we see that the averaging over the sizes does not equalize the differential scattering cross sections for the coherent and incoherent additions. The reason of this is that the optical paths of the beams are saved if the particle sizes are changed. Therefore, the incoherent addition would be not correct. Moreover, we don't need in the averaging since the strong oscillations aggravating the solutions don't appear at all.

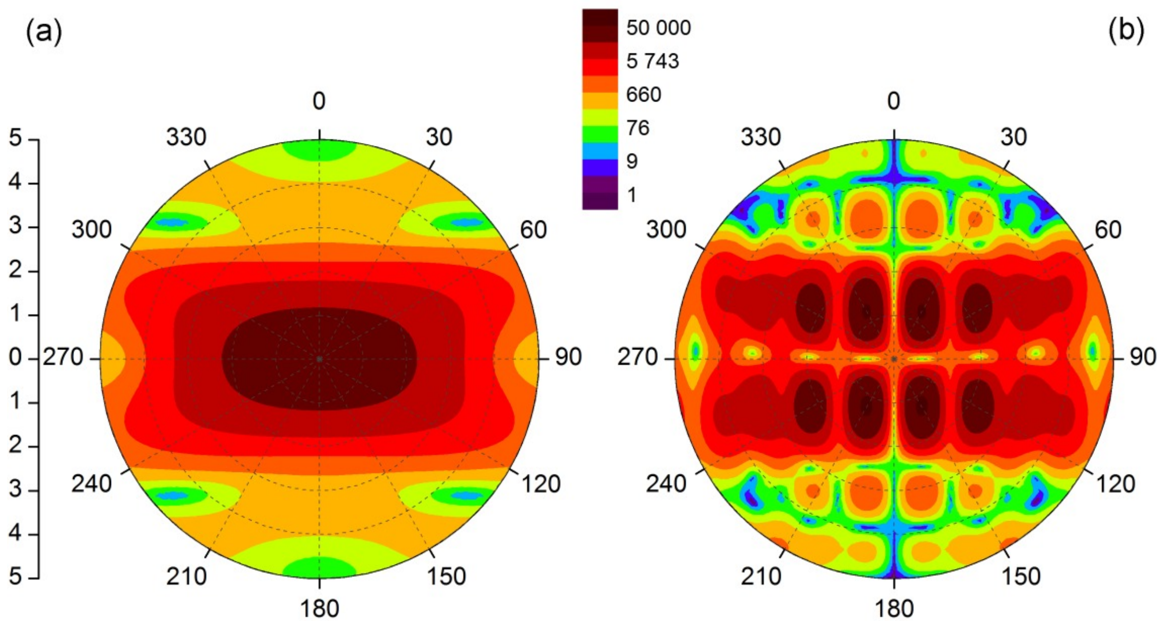

Figure 6. The differential scattering cross section of the skew beams 1-4 within the cone of 5° for column with the fixed orientation ($\beta = 32.22°$, $\gamma = 0°$) at $\lambda = 0.532\ \mu m$. The left patterns correspond to the incoherent addition of the beams while the patterns for the coherent addition of the beams are shown to the right.

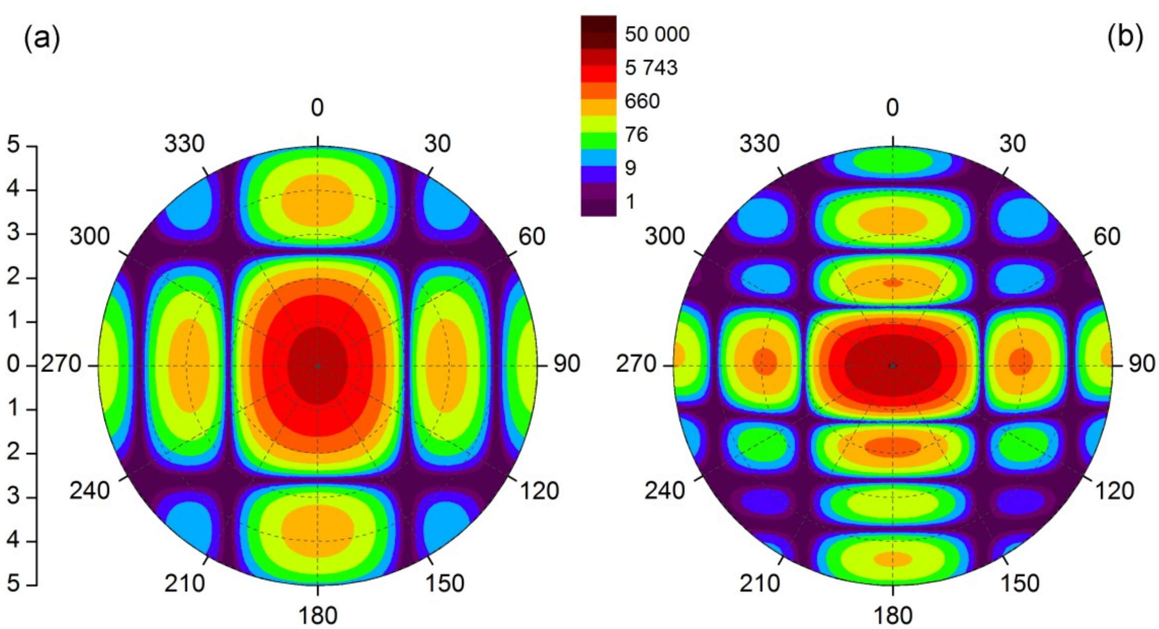

Figure 7. The same as Fig. 6 for the straight beams 5-6.

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 11/25/2015 Terms of Use: http://spiedigitallibrary.org/ss/TermsOfUse.aspx

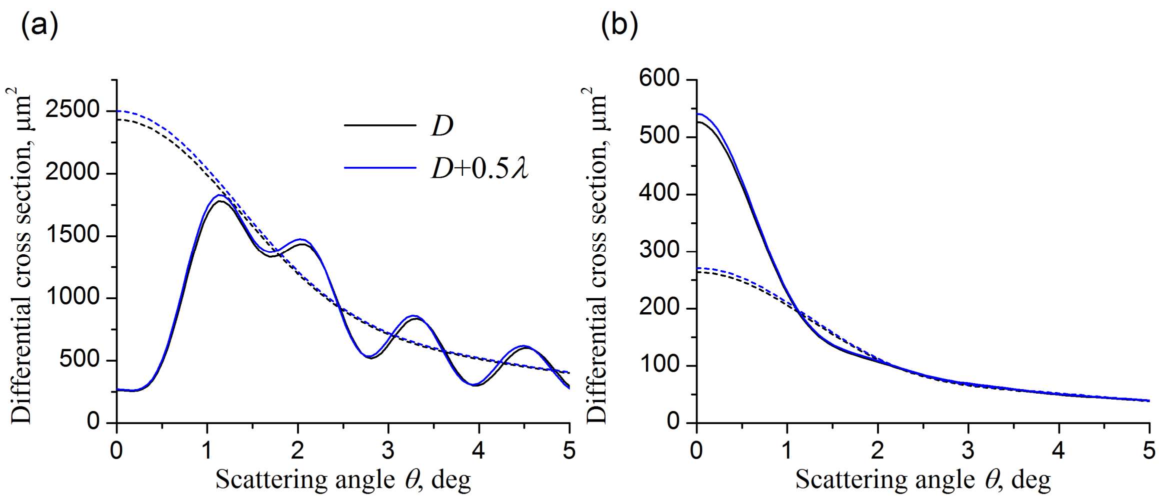

Figure 8. The differential scattering cross section averaged over the rotation angle γ for the skew (a) and straight (b) trajectories at the size variance of 0.5λ for column with the fixed orientation ($\beta = 32.22°$) within the cone of 15°. The solid curves correspond to the coherent addition of the beams; the data of the incoherent addition are shown by the dashed curves.

4. CONCLUSIONS

It is shown that the calculations of the backscattering intensity by means of the beam-tracing algorithm of the physical-optics approximation can be much shortened if the coherent addition of the beams is replaced by the incoherent addition. This statement is true only for the beams whose phase difference essentially depends on particle size. Therefore the method of the physical optics includes, as a preliminary step, a study of the phase differences. If the phase differences of the given beams don't change with a crystal size variance, such beam should be added coherently.

ACKNOWLEDGMENTS

This work is supported by the Russian Foundation for Basic Research under Grants no. 15-05-06100 and no. 15-55-53081, by the RF President grant on the support of leading scientific schools NSH-4714.2014.5, by the RF President grant MK-6680.2015.5, by the RF Ministry of Education and Science under the program rising competitiveness of the TSU and supported in part by the Russian Science Foundation (Agreement no. 14-27-00022).

REFERENCES

[1] Yurkin, M. A., Hoekstra, A. G., "The discrete-dipole-approximation code ADDA: Capabilities and known limitations," J. Quant. Spectrosc. Radiat. Transfer 112, 2234-2247 (2011).
[2] Borovoi, A., Grishin, I., "Scattering matrices for large ice crystal particles," J. Opt. Soc. Am. 20(11), 2071-2080 (2003).
[3] Konoshonkin, A. V., Kustova, N. V., Borovoi, A. G., "Limits of applicability of geometrical optics approximation for light backscattering on a quasihorizontally oriented hexagonal ice plate," Optica Atmosfery i Okeana 27(8), 705-712 (2014).
[4] Borovoi, A., Konoshonkin, A., Kustova, N., "The physics-optics approximation and its application to light backscattering by hexagonal ice crystals," J. Quant. Spectrosc. Radiat. Transfer 146, 181-189 (2014).
[5] Borovoi, A., Konoshonkin, A., Kustova, N., "Backscattering reciprocity for large particles," Opt. Letters 38(9), 1485-1487 (2013).
[6] Mitchell, D. L., "Use of mass- and area-dimensional power laws for determining precipitation particle terminal velocities," J. Atm. Sci. 53, 1710-1723 (1996).
[7] Heymsfield, A. J., and Miloshevich, L.M., "Parameterization for the cross-sectional area and extinction of cirrus and stratiform ice particles," J. Atmos. Sci. 60, 936-956 (2003).

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 11/25/2015 Terms of Use: http://spiedigitallibrary.org/ss/TermsOfUse.aspx

[8] Sato, K., Okamoto, H., “Characterization of Ze and LDR of nonspherical and inhomogeneous ice particles for 45-GHz cloud lidar: Its implication to microphysical retrievals,” J. Geophys. Res. 111(D22), D22213 (2006).

[9] Auer, A. H., Veal, D. L., “The dimension of ice crystals in natural clouds,” J. Atmos. Sci. 29, 311-317 (1970).

[10] Borovoi, A., Konoshonkin, A., Kustova, N., Okamoto, H., “Backscattering Mueller matrix for quasi-horizontally oriented ice plates of cirrus clouds: application to CALIPSO signals,” Opt. Express 20(27), 28222-28233 (2012).

[11] Balin, Yu. S., Kaul, B. V., Kokhanenko, G. P., “Observations of specularly reflective particles and layers in crystal clouds,” Optica Atmosfery i Okeana 24(4), 293-299 (2011).

[12] Galileiskii, V. P., Borovoy, A. G., Matvienko, G. G., Morozov, A. M., “Specularly reflected component at light scattering by ice crystals with predominant orientation,” Optica Atmosfery i Okeana 21(9), 668-673 (2008).

[13] Galileiskii, V. P., Kaul, B. V., Matvienko, G. G., Morozov, A. M., “Angular structure of the light intensity near the angles of mirror reflection from the faces of ice crystalline particles,” Atmospheric and Oceanic Optics 22(5), 506-512 (2009).

[14] Konoshonkin, A. V., Borovoy, A. G., “Specular scattering of light on cloud ice crystals and wavy water surface,” Atmospheric and Oceanic Optics 26(5), 438-443 (2013).

[15] Burnashov, A. V., Konoshonkin, A. V., “Matrix of light scattering on a truncated plate-like droxtal preferably oriented in a horizontal plane,” Atmospheric and Oceanic Optics 26(3), 194-200 (2013).

[16] Samokhvalov, I. V., Kaul, B. V., Nasonov, S. V., Zhivotenyuk, I. V., Bryukhanov, I. D., “Backscattering matrix of the mirror-reflecting upper-level cloud layers formed by horizontally oriented crystal particles,” Optica Atmosfery i Okeana 25(5), 403-411 (2012).

[17] Borovoi, A., Balin, Y., Kokhanenko, G., Penner, I., Konoshonkin, A., Kustova, N., “Layers of quasi-horizontally oriented ice crystals in cirrus clouds observed by a two-wavelength polarization lidar,” Opt. Express 22(20), 24566-24573 (2014).

[18] Konoshonkin, A. V., Kustova, N. V., Borovoy, A. G., “Beam splitting algorithm for light scattering by atmospheric ice crystals. Part 1. Theory,” Optica Atmosfery i Okeana 28(4), 324-330 (2015).

[19] Konoshonkin, A. V., Kustova, N. V., Borovoy, A. G., “Beam splitting algorithm for light scattering by atmospheric ice crystals. Part 2. Comparison with the ray tracing algorithm,” Optica Atmosfery i Okeana 28(4), 324-330 (2015).

[20] Konoshonkin, A. V., Kustova, N. V., Borovoi, A. G., “Beam-splitting code for light scattering by ice crystal particles within geometric-optics approximation,” J. Quant. Spectrosc. Radiat. Transf., http://dx.doi.org/10.1016/j.jqsrt.2015.06.008 (2015).

[21] Konoshonkin, A. V., Kustova, N. V., Borovoy, A. G., “Peculiarities of the depolarization ratio in lidar signals for randomly oriented ice crystals of cirrus clouds,” Optica Atmosfery i Okeana 26(5), 385-387 (2013).

[22] Borovoi, A., Konoshonkin, A., and Kustova, N., “Backscattering by hexagonal ice crystals of cirrus clouds,” Opt. Lett. 38(15), 2881-2884 (2013).

[23] Borovoi, A., Konoshonkin, A., and Kustova, N., “Backscatter ratios for arbitrary oriented hexagonal ice crystals of cirrus clouds,” Opt. Lett. 39(19), 5788-5791 (2014).

[24] Borovoi, A., Konoshonkin, A., and Kustova, N., “The physical-optics approximation and its application to light backscattering by hexagonal ice crystals,” J. Quant. Spectrosc. Radiat. Transf. 146, 181-189 (2014).

[25] Borovoi, A. G., Kustova, N. V., Oppel, U. G., “Light backscattering by hexagonal ice crystal particles in the geometrical optics approximation,” Opt. Eng. 44(7), 1-10 (2005).

Downloaded From: http://proceedings.spiedigitallibrary.org/ on 11/25/2015 Terms of Use: http://spiedigitallibrary.org/ss/TermsOfUse.aspx