UDK 539.3
DOI 10.17223/19988621/35/8

S. Darya zadeh, G.I. Lvov

THE CALCULATION OF EFFECTIVE ELASTIC CONSTANTS IN A COMPOSITE WITH 3D ORTHOGONAL NONWOVEN FIBERS

The paper is devoted to the research of the effective characteristics of 3D orthogonal nonwoven fibers composites. The results were received via ANSYS software package. In this research a volume element of fibers in cubic unit cell is considered. The effective elastic properties of fiber reinforced composite have been defined by the numerical stress analysis of the unit cell.

Keywords: *composite*, *fiber*, *effective elastic constants*, *numerical method.*

1. Introduction

Composite materials which consist of two or more constituent materials are commonly used in advanced structural applications, e.g. in the marine and aerospace industry. This is because of appropriate mechanical properties such as high specific strength and stiffness, low density and high resistance to corrosion. However, the limited understanding of the composite material behavior requires more research. This is further complicated by the fact that these materials behavior is dependent on lay-up, loading direction, specimen size and environmental effects such as temperature and moisture. Research on determination of effective elastic constants for anisotropic materials is very important in composite structures.

A orthogonal nonwoven fibers reinforced resin matrix composites are used in some structural applications, due to their various reasons especially to their excellent mechanical behavior in terms of their specific stiffness in the direction of the fibers. The prediction of the mechanical properties of composites has been the main objective of many researchers. The well-known models that have been proposed and used to evaluate the properties of cross-ply laminate composites are Voigt [1], 1989 and Reuss [2], 1829 models. The Voigt model is also known as the rule of mixture model or the iso-strain model, while the Reuss model is also known as the inverse of mixture model or the iso-stress model. The study will be using complex functions to determine the effective elastic coefficients of the unidirectional plates as presented by Vanin [3] (1961). Semi empirical models have emerged to correct the rule of mixture model where correcting factors are introduced. Under this category, it is noticed three important models: the modified rule of mixture, the Halpin –Tsai [4] model (Halpin et al., 1976) and Chamis [5] model (Chamis, 1989). The Halpin – Tsai model emerged as a semi-empirical model that tends to correct the transverse Young’s modulus and longitudinal shear modulus. The Chamis micromechanical model is the most used and trusted model which give a formulation for all five independent elastic properties. Hashin and Rosen [6] (Hashin et al., 1964) initially proposed a composite cylinder assemblage model to evaluate the elastic properties of cross-ply laminate laminate composites. Alfootov [7] determined the mechanical properties of cross-ply laminate reinforced composites with perpendicular fibers. Christensen [8], 1990 proposed a generalized self-consistent model in order to better evaluate the transversal shear modulus. Also the Mori – Tanaka model

[9] (Mori et al., 1990) is a famous model which is widely used for modeling different kinds of composite materials. This is an inclusion model where fibers are simulated by inclusions embedded in a homogeneous medium. The self-consistent model has been proposed by Hill [10], 1965 and Budiansky [11], 1965 to predict the elastic properties of composite materials reinforced by isotropic spherical particulates. Later the model was presented and used to predict the elastic properties of short fibers composites [12] (Chou et al., 1980). Recently, a new micromechanical model has been proposed by Huang [13, 14], 2001. The model is developed to predict the stiffness and the strength of cross-ply laminate composites.

Assuming cubic symmetry structure and using ANSYS software, effective characteristics of this composite are studied. Numerical studies are performed for some stress states in a representative unit cell for determination the effective elastic properties of fibers reinforced orthogonal nonwoven composite.

2. Computational procedure

2.1. Definition and elasticity effective parameters in cubic symmetry composite

The present approach is based on the theory anisotropic elasticity. A numerical method is able to simplify the problem by satisfy the stress – strain boundary conditions directly into the expression for defining the elastic properties in a composite material. This study considers a composite material with 3D orthogonal nonwoven fibers. In this structure of material fibers are parallel to x, y and z directions as follows and are said to define a cubic symmetry array. Theory of elasticity can be used for investigating the stress – strain state of fiber reinforced composite materials. The generalized Hook's law relating strains to stresses can be written as follows:

$$\langle \varepsilon_{ij} \rangle = \left[a_{ijkl} \right] \langle \sigma_{kl} \rangle , \quad i, j = 1, 2, 3. \tag{1}$$

Where $[A] = \left[a_{ijkl} \right]$ is the compliance matrix and $\langle \varepsilon_{ij} \rangle$, $\langle \sigma_{ij} \rangle$ are the strain and stress components, respectively. In this study, composites with orthogonal nonwoven fibers and constant radius are investigated as cubic symmetry materials.

The simplest anisotropic case, that of cubic symmetry has three independent elements.

These materials with volume V, stress and strain are described as follows:

$$\langle \sigma_{ij} \rangle = \frac{1}{V} \int_V \sigma_{ij} dV \quad \text{and} \quad \langle \varepsilon_{ij} \rangle = \frac{1}{V} \int_V \varepsilon_{ij} dV . \tag{2}$$

In Cartesian coordinates, Hook's law for cubic symmetry material is as follows:

$$\langle \sigma_x \rangle = b_{11} \langle \varepsilon_x \rangle + b_{12} \langle \varepsilon_y \rangle + b_{12} \langle \varepsilon_z \rangle ,$$

$$\langle \sigma_y \rangle = b_{21} \langle \varepsilon_x \rangle + b_{11} \langle \varepsilon_y \rangle + b_{12} \langle \varepsilon_z \rangle ,$$

$$\langle \sigma_z \rangle = b_{21} \langle \varepsilon_x \rangle + b_{21} \langle \varepsilon_y \rangle + b_{11} \langle \varepsilon_z \rangle ,$$

$$\langle \tau_{xy} \rangle = b_{44} \langle \gamma_{xy} \rangle , \ \langle \tau_{yz} \rangle = b_{44} \langle \gamma_{yz} \rangle , \ \langle \tau_{zx} \rangle = b_{44} \langle \gamma_{zx} \rangle . \tag{3}$$

Where b_{ij} are the coefficients of stiffness matrix $[B]$ for composite material. The stiffness matrix (inverse of the elastic compliance matrix) is symmetric.

$$b_{12} = b_{21}. \qquad (4)$$

Hence, the three unknown coefficients b_{11}, b_{12}, b_{44} need to be defined for determining properties of for cubic symmetry material.

2.2. Finite Element Modeling

A regular three-dimensional arrangement of fiber in a matrix was adequate to describe the overall behavior of the composite, and was modeled as a regular uniform arrangement, as illustrated in Fig. 1*a*. In this paper a composite in Cartesian coordinates system is considered. This model assumed that the fiber was a perfect cylinder of radius 0.4, in a cubic unit cell (1 × 1× 1) of the matrix. It is assumed that the geometry, material and loading of the cell are symmetrical with respect to $x-y$, $y-z$ and $z-x$ planes of coordinate system as shown in Fig. 1*a*. In this work, a cubic cell is considered and the three sides of a cell have equal lengths to $d_0 = 1$. Therefore, 37.7 % volume fraction fibers were inserted.

The numerical finite element modeling is widely used in predicting the mechanical properties of composites. In this paper for numerical analysis, a volume element of fibers is considered which plane symmetric exists on all of its planes. In order to investigate the numerical finite element modeling, the modeling of a unit cell for a cubic array is considered using ANSYS software as shown in Fig. 1*b*.

The usage of the designated volume as a representative cell is substantiated by the following: when dealing with infinite composite material that consists of infinite number of repetitive cells, the stress conditions of each cell are identical. At that, the specified boundary conditions for the designated volume correspond to the homogeneous state of equivalent homogeneous material. For determining the components of the stiffness matrix (b_{ij}), stress analysis is performed for considered volume with noting to boundary conditions. In the present procedure, normal strain is applied to one direction and shear strain is applied to one plane as follows. For numerical analysis, finite element software ANSYS is used and 36321 SOLID 95 elements with 20 nodes are utilized as shown in Fig. 1*b*. All the geometrical parameters and displacements are used in a dimensionless form based on the length of the edge of the representative volume. This allows receiving elastic averaged characteristics of the investigated type of composite, assigning only relative volume contents of fibers. The stresses are indicated in Pascal.

The first numerical testing is unidirectional tension in x direction. In this condition, tensor of average values for strains is obtained the following forms:

$$\langle \varepsilon_x \rangle = 10^{-3}, \ \langle \varepsilon_y \rangle = 0, \ \langle \varepsilon_z \rangle = 0, \ \langle \gamma_{xy} \rangle = 0, \ \langle \gamma_{yz} \rangle = 0, \ \langle \gamma_{xz} \rangle = 0. \qquad (5)$$

The case of unidirectional tension in x direction the boundary conditions for this structural analysis are as follows:

In plane $x = 1$:

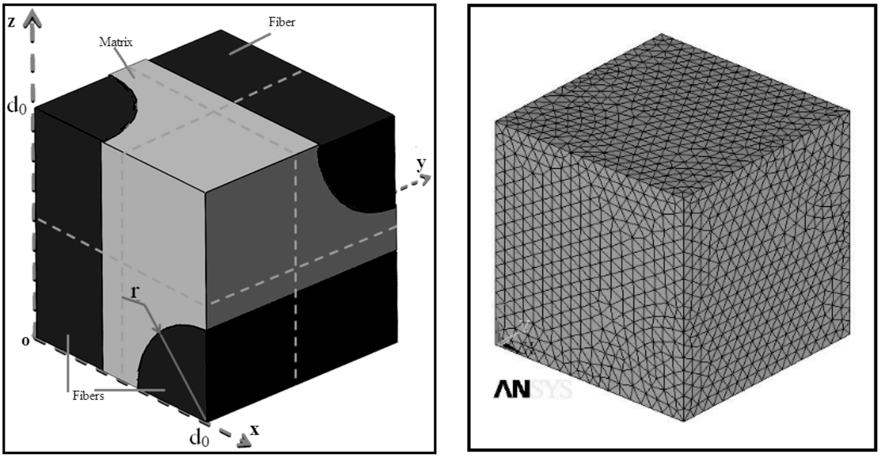
$$u_x(x=1, y, z) = 10^{-3} \text{ and } \tau_{xy} = \tau_{xz},$$

In this situation, there are symmetric conditions in other planes and the boundary conditions are therefore

$$u_x(x=0, y, z) = 0, \ u_y(x, y=0, z) = 0, \ u_y(x, y=1, z) = 0,$$

$$u_z(x, y, z=0) = 0 \text{ and } u_z(x, y, z=1) = 0,$$

where u_i ($i = x, y, z$) is displacement in x, y and z directions, respectively.



a *b*

Fig. 1. Volume: *a* – representative unit cell model; *b* – element in mesh formed

Stress components are determined as

$$\langle \sigma_x \rangle = \int_0^1 \int_0^1 \sigma_x dydz \, , \quad \langle \sigma_y \rangle = \int_0^1 \int_0^1 \sigma_y dxdz \, . \tag{6}$$

Therefore, according to Eq. 3, by the first numerical testing two coefficients of elasticity can be determined as follows:

$$b_{11} = \frac{\langle \sigma_x \rangle}{\langle \varepsilon_x \rangle} , \quad b_{12} = \frac{\langle \sigma_y \rangle}{\langle \varepsilon_x \rangle} \, . \tag{7}$$

In this analysis polymer epoxy is considered as matrix and its mechanical properties are as follows [15]:

$$E_m = 4200 \text{ MPa}; \quad G_m = 1500 \text{ MPa}; \quad \nu_m = 0.4.$$

Mechanical properties of glass fibers are as follows:

$$E_a = 74800 \text{ MPa}; \quad G_a = 31000 \text{ MPa}; \quad \nu_a = 0.2.$$

For composite with fibers in constant radius as $0 < r < 1$, fiber-volume fraction is calculated as follows:

$$\xi = \frac{V'}{V} \, , \tag{8}$$

where $V = d_0^3$ – the volume of a cubic cell and $V' = \frac{3}{4} \pi r^2 d_0$ – fiber volume content in material.

Effective elasticity properties for $\xi = 0.377$ (r = 0.4) is determined by Numerical Method. In Fig. 2 the result of the first condition is shown.

The second numerical testing is shearing in *xz* plane. In this condition, tensor of average values for strains is obtained:

$$\langle \varepsilon_x \rangle = 0 \, , \ \langle \varepsilon_y \rangle = 0 \, , \ \langle \varepsilon_z \rangle = 0 \, , \ \langle \gamma_{xy} \rangle = 0 \, , \ \langle \gamma_{yz} \rangle = 0 \, , \ \langle \gamma_{xz} \rangle = 10^{-3} \, . \tag{9}$$

Therefore, the displacement in plane $z = 1$: $u_x(x, y, z = 1) = 10^{-3}$ and $\sigma_z = \tau_{zy} = 0$.

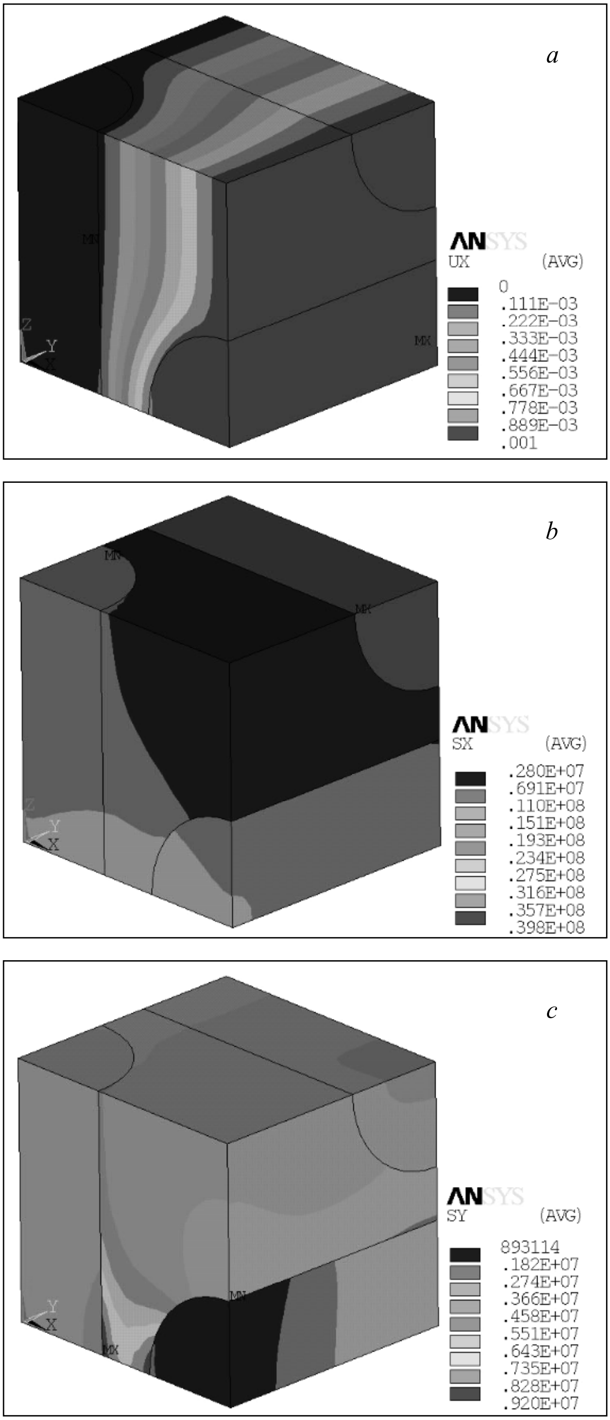


Fig. 2. The result of: *a* – displacement in *x* direction when $u_x = 10^{-3}$;
b and *c* – stress distribution of normal stresses σ_x, σ_y

Now, as the boundary conditions are symmetrical for this structural analysis are as follows:

In plane $z = 0$ the displacement in the direction of x axis is zero ($u_x(x, y, z = 0) = 0$) and two are symmetric conditions in planes $y = 0$, $y = 1$ therefore, in these planes: $u_y(x, y = 0, z) = 0$ and $u_y(x, y = 1, z) = 0$,

In planes $x = 0$ and $x = 1$: $u_z(x = 0, y, z) = 0$, $u_z(x = 1, y, z) = 0$; and $\sigma_x = \tau_{xy} = 0$. The shear stress component in xz plane is given by

$$\langle \tau_{xz} \rangle = \int_0^1 \int_0^1 \tau_{xz} dydz \; . \tag{10}$$

Therefore, the unknown coefficient b_{44}, can then be calculated by substituting value $\langle \tau_{xz} \rangle$ into Eq. 3 and take following form:

$$b_{44} = \langle \tau_{xz} \rangle / \langle \gamma_{xz} \rangle \; . \tag{11}$$

In this case for solving the problem, Hook's law is used directly:

$$\langle \varepsilon \rangle = [A] \cdot \langle \sigma \rangle , \tag{12}$$

where $[A]$ is the compliance matrix: $[A] = [B]^{-1}$.

Obviously, considering matrix $[A]$, elasticity coefficient such as poison ratio and shear modulus of composite material can be obtained as follows:

$$\langle E_x \rangle = \langle E_y \rangle = \langle E_y \rangle = \frac{1}{a_{11}} , \quad \langle \nu_{xy} \rangle = \langle \nu_{xz} \rangle = \langle \nu_{yz} \rangle = -a_{12} . \langle E_y \rangle$$

$$\text{and} \quad \langle G_{xy} \rangle = \langle G_{xz} \rangle = \langle G_{yz} \rangle = \frac{1}{a_{44}} = b_{44} \; . \tag{13}$$

3. Results and discussion

In this approach work, effective elasticity properties for $\xi = 0.377$ is determined by numerical procedure proposed in this research. Table 1 shows numerical results for the effective elastic constants of composite material.

Table 1

Numerical results of effective elasticity properties for $\xi = 0.377$

Elasticity properties	Modulus of elasticity, MPa	Modulus of shear, MPa	Poisson's coefficient
	$\langle E_x \rangle = \langle E_y \rangle = \langle E_z \rangle$	$\langle G_{xy} \rangle = \langle G_{xz} \rangle = \langle G_{yz} \rangle$	$\langle \nu_{xy} \rangle = \langle \nu_{xz} \rangle = \langle \nu_{yz} \rangle$
Numerical Method	17501	3098	0.26

The variation of $E_1 = \langle E_x \rangle / E_m$, $G = \langle G_{xy} \rangle / G_m$ versus different values of ξ are obtained for cubic symmetry material glass fibers. Mechanical properties of composite are determined by the proposed method in this paper (numerical method) with finite element method. Numerical values are calculated by ANSYS. It is obvious that $\langle E_x \rangle = \langle E_y \rangle = \langle E_z \rangle$, $\langle G_{xy} \rangle = \langle G_{xz} \rangle = \langle G_{yz} \rangle$ and $\langle \nu_{xy} \rangle = \langle \nu_{xz} \rangle = \langle \nu_{yz} \rangle$.

Fig. 3 shows the variation of E_i (i = 1,2,3) with respect to different values of ξ for orthogonal nonwoven glass fibers composite. E_i is the ratio modulus of elasticity of composite to E_m in fibers direction and E_m is modulus of elasticity of matrix. In this figure, the curve is obtained by the method of this paper. As it can be seen, the behavior of curve is nonlinear. In this result, should be reminded that the curve E_i is linear for the unidirectional composite. Fig. 3 shows that in small value of ξ, the value of E_1 (i = 1,2,3) is near to 1. Also for the maximum value of ξ (ξ = 0.58875), the value of E_1 is near to the modulus of elasticity of fibers E_a/E_m, as it is predicted. E_a is modulus of elasticity of fiber.

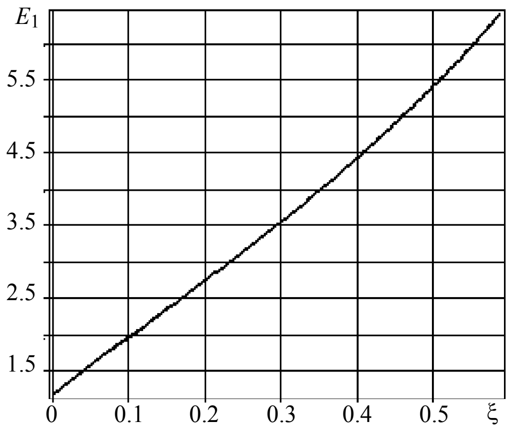


Fig. 3. The variation of E_i (i = 1,2,3) versus values of ξ for orthogonal nonwoven glass fibers

Fig. 4 shows the variation of G_i (i = 1,2,3) versus different values of ξ for orthogonal nonwoven glass fibers composite in a cubic pattern. G_i is the ratio shear modulus of composite to G_m in xy, xz and yz planes and G_m is the shear modulus of matrix. As it can be seen, the behavior of the present work is sowed that curve G_i is nonlinear.

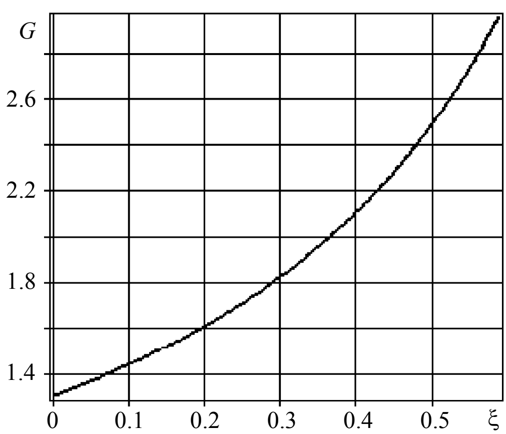


Fig. 4. The variation of G_i (i = 1,2,3) versus values of ξ for orthogonal nonwoven glass fibers

Fig. 4 shows that in small value of ξ , the value of G_i (i = 1,2,3) is near to 1. Also for the maximum value of ξ ($\xi = 0.58875$), G_i is near to a value that is smaller than the shear modulus of fibers G_a/G_m, where G_a is the shear modulus of fiber.

The results of the numerical calculation of effective elastic properties were not compared with theoretical or experimental methods. The authors do not have the complete set of the necessary data at their disposal as there are no such data in open literature. The similar approach was used by the authors [16, 17] for unidirectional fibrous composite, where it was managed to receive the evidence of the results accuracy. The following technique was applied for the evaluation of the results accuracy in this research – finite elements grid clustering and intrinsic criteria parameters of error in ANSYS.

Conclusions

The procedure of finding the effective elastic characteristics for 3D orthogonal nonwoven fiber composites has been developed. In this research a volume element of fibers in cubic symmetry cell is considered which plane symmetric exists on all of its planes. The effective elastic properties of fiber-glass composite which polymer epoxy have been defined via ANSYS software package. The finite-element method has been used for the research.

After considering the results set out in this section the following observations could be made:

Minimum requirement unit cells have been defined for different types of reinforcement as well as the boundary conditions for them, which enable composite behavior modeling under basic experiments conditions.

The procedure of finding effective elastic constants for reinforced composites allows receiving results with the reasonable degree of accuracy for practical application.

Apart from effective elastic constants finding, the advantage of the developed procedure is the ability to investigate local stress concentration in the unit cell area.

The developed procedure enables to carry out the multivariate analysis of the elastic properties of composite material at the design stage as well as to set and solve the tasks associated with the optimal design of composites.

REFERENCES

1. Voigt W. Über die Beziehung zwischen den beiden Elastizitätskonstanten Isotroper Körper. *Wied. Ann.*, 1889, vol. 38, pp. 573–587.
2. Reuss A. Berechnung der Fliessgrense von Mischkristallen auf Grund der Plastizitätsbedingung für Einkristalle. *Zeitschrift Angewandte Mathematik und Mechanik*, 1929, no. 9, pp. 49–58.
3. Vanin G.A. *Micro-Mechanics of Composite Materials*. Kiev, Nauka Dumka Publ., 1985.
4. Halpin J.C, Kardos J.L. The Halpin-Tsai equations: A review. *Polymer Engineering and Science*, 1976, May, vol. 16, no. 5.
5. Chamis C.C. Mechanics of composite materials: past, present and future. *J. Compos. Technol. Res. ASTM*, 1989, no. 11, pp. 3–14.
6. Hashin Z., Rosen B.W. The elastic moduli of fiber reinforced materials. *Journal of Applied Mechanics*, *Trans ASME*, 1964, vol. 31, pp. 223–232.
7. Alfootov N.A. *Calculation of laminated plates and shells made of composite materials*. Moscow, Mashinostroitelni Publ., 1984.
8. Christensen R.M. A critical evaluation for a class of micromechanical models. *Journal of Mechanics and Physics of Solids*, 1990, vol. 38, no. 3, pp. 379–404.
9. Mori T., Tanaka K. Average stress in matrix and average elastic energy of materials with misfitting inclusions. *Acta Mettall.*, 1973, vol. 21, pp. 571–574.

10. Hill R. Theory of mechanical properties of fiber-strengthen materials-3. Self-consistent model. *Journal of Mechanics and Physics of Solids*, 1965, vol. 13. pp. 189–198.
11. Bubiansky B. On the elastic modulli of some heterogeneous materials. *Journal of Mechanics and Physics of solids*, 1965, vol. 13, pp. 223–227.
12. Chou Tw., Nomura S., Taya M. A self-consistent approach to the elastic stiffness of short-fiber composites. *J. Compos. Mater.*, 1980, vol. 14, pp. 178–188.
13. Huang Z.M. Simulation of the mechanical properties of fibrous composites by the bridging micromechanics model. *Composites: Part A*, 2001, vol. 32, pp. 143–172.
14. Huang Z.M. Micromechanical prediction of ultimate strength of transversely isotropic fibrous composites. *International Journal of Solids and Structures*, 2001, vol. 38, pp. 4147–4172.
15. Carpeenosa D.M. Composite Materials. Kiev, Nauka Dumka Publ., 1985.
16. Darya zadeh S. A numerical method for determination of the effective characteristics of unidirectionally reinforced composites. *Vestnik KhPI*, 2013, vol. 58, pp. 71–77.
17. Darya zadeh S., Lvov G.I. A numerical method for determination of the effective characteristics of unidirectionally reinforced composites with hexagonal structure. *Aviation – Aerospace Technology and Technology "KhAI"* 2014, no. 2(109), pp. 59–65.

Статья поступила 18.02.2015 г.

Darya zadeh S., *Lvov G.I.* THE CALCULATION OF EFFECTIVE ELASTIC CONSTANTS IN A COMPOSITE WITH 3D ORTHOGONAL NONWOVEN FIBERS

DOI 10.17223/19988621/35/8

The paper is devoted to the research of the effective characteristics of 3D orthogonal nonwoven fibers composites. The results were received via ANSYS software package. In this research a volume element of fibers in cubic unit cell is considered. The effective elastic properties of fiber reinforced composite have been defined by the numerical stress analysis of the unit cell.

Keywords: composite, fiber, effective elastic constants, numerical method.

DARYA ZADEH SAEED Massaieb (PhD, National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine)

E-mail: s_daryazadeh@yahoo.com

LVOV Gennadii Ivanivich (PhD, National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine)
E-mail: lvovgi@list.ru