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Effect of viscous dissipation on temperature, viscosity, and flow parameters while filling a channel*

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A non-steady, non-isothermal flow while filling a channel is studied with account for dissipation of mechanical energy, dependency of viscosity on temperature, and existence of free surface. Simulation results are presented for fields of temperature, viscosity, dynamic and kinematic parameters of flow as a function of key dimensionless parameters.

Key words: filling, fountain flow, viscous dissipation, free surface, simulation results.

Introduction

Filling of cavities by liquids is often used in industry whenever the production needs molding technique. In particular, in producing polymer filling of cavities is a key step in articles mold using the injection molding method. The challenges at this stage are possibly forming of air cavities inside the flow and at flow interface, weld lines after merging of folds in free surface, etc.: these features may cause faults in the produced items. The proper arrangement of production process needs the detailed study of physical and chemical processes in fluid flow occurring during processing of polymer compositions.

In most general case, the polymer fluid during filling is unsteady, non-isothermal flow with complex rheology, chemical transitions with different rates. Besides, presence of free surfaces and a multitude of possible cavity shapes make the problem more complicated [1]. If we take into account all these factors in mathematical model for process quantitative description, this makes the problem more complex not only from the point of view of obtaining its solution, but also in the formulation of criteria relations of main process parameters. Thus, we have to single out a selected factor in a mathematical model for detailed study of impact of this factor in the molding process.

In recent decades, many studies of physical-chemical hydrodynamics occurring during filling cavities by injection molding have been carried out. Both physical and mathematical modeling was used in these studies.

Initially, simplified mathematical models without account for free surface were considered: this produced approximate solutions in analytical form or numerical solutions with a simple

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algorithm [2, 3]. In later years, many researchers had applied the finite difference method and finite element method for study of isothermal flow while filling of cavities in flat or axisymmetric problem statement; the key results have been presented in [4–6]. The modern level of research for isothermal flow was discussed in [7].

Polymer compositions are thermoplastic and thermoreactive materials with rheology and phase state depending on temperature. Non-isothermal process of cavity filling with polymeric fluid is driven by energy dissipation in the flow, chemical transitions, heat transfer boundary conditions, and initial conditions. Viscous dissipation (as a mechanical source of heat) changes the medium temperature and, thus, changes the rheology and degree of chemical transformation, which, in turn, varies the kinematic and dynamic parameters of the flow. One simple case that helps in understanding of viscous dissipation effect is a flow through a channel. The results of studies for non-isothermal flow of Newtonian and non-Newtonian fluids in channels without free surface were outlined in reviews [8–11]. The non-Newtonian flow while filling of cavities with account for free interface was studied in [12–15].

The goal of this research is studying of the effect of dissipation warming on kinematic and dynamic parameters of flow, temperature field, and viscosity (for filling of a channel with Newtonian fluid). The research uses a proprietorship simulation method with approximation of natural boundary conditions on explicitly distinguished interface.

1. Problem statement

Here we consider the process of filling a vertical flat channel with incompressible fluid under action of gravity with account for dissipation heating and known dependency of viscosity vs. temperature, and existence of free surface. The solution domain is depicted in Fig. 1. The flow is described by equations of motion, continuity, and energy which are formulated in dimensionless form:

$$\mathrm{Re}\frac{d\mathbf{V}}{dt} = -\nabla p + \nabla\cdot(2B\mathbf{E}) + \mathbf{W}, \tag{1}$$

$$\nabla\cdot\mathbf{V} = 0, \tag{2}$$

$$\mathrm{Pe}\frac{d\theta}{dt} = \Delta\theta + C_1\cdot 2B\cdot I_2. \tag{3}$$

The dependency of viscosity on temperature is given by a ratio which is a dimensionless analog of Reynolds equation [16],

$$B = \mathrm{e}^{-C_2\theta}. \tag{4}$$

Here $\mathbf{V}$ is the velocity vector, p is the pressure, t is time, $\mathbf{W} = \{0, W\}$, $\theta = (T - T_0)/T_0$ is the temperature, where T and T_0 are dimensional temperatures for liquid and for solid wall, $I_2 = e_{ij}e_{ji}$ is the second invariant of shear rate tensor $\mathbf{E}$, $\mathrm{Re} = \rho UL/\mu$ is the Reynolds number, $W = \rho gL^2/\mu U$ is the parameter describing the ratio of gravitation and viscous forces, $\mathrm{Pe} = c\rho UL/\lambda$ is the Peclet number, $C_1 = \mu U^2/\lambda T_0$ is the parameter for the ratio of dissipative to conductive heat transfer, $C_2 = \alpha T_0$ is the dimensionless parameter in exponential formula for viscosity on temperature, ρ is the density, μ is the viscosity at temperature T_0, g is the gravity acceleration, α is a constant, c is the heat capacity, and λ is the thermal conductivity

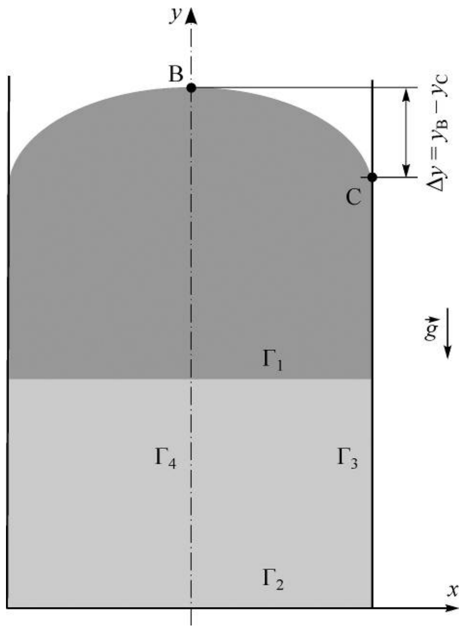

Fig. 1. Solution domain.

coefficient. The dimension scale parameters are the following: channel half-width L for the length, the mean-flow-rate velocity at the inlet cross section U for the velocity, $\mu U / L$, viscosity μ.

The free surface Γ_1 (Fig. 1) has boundary conditions as zero tangential shear stress, equality between the normal and external pressures (which is zero without additional restrictions), and zero heat flux. Besides, the free interface follows the kinematic condition. The inlet boundary Γ_2 has the distribution of velocity and temperature (according to actual physical problem statement). The solid wall Γ_3 has a non-slip condition, and the temperature coincides with the wall temperature. This kind of wall conditions is quite common in study of nonisothermal fluids, including the case of polymer molding process [10]. The symmetry line Γ_4 expresses the symmetry conditions. The surface tension is considered to be much lower than viscosity forces and can be discarded [5, 7].

Thus, the boundary conditions are the following:

$$\Gamma_1: \ \frac{\partial v_n}{\partial s} + \frac{\partial v_s}{\partial n} = 0, \ \ p = 2B\frac{\partial v_n}{\partial n}, \ \ \frac{\partial \theta}{\partial n} = 0, \tag{5}$$

$$\Gamma_2: \ v_x = 0, \ \ v_y = f(x), \ \ \theta = \varphi(x), \tag{6}$$

$$\Gamma_3: \ v_x = 0, \ \ v_y = 0, \ \ \theta = 0, \tag{7}$$

$$\Gamma_4: \ v_x = 0, \ \ \frac{\partial v_y}{\partial x} = 0, \ \ \frac{\partial p}{\partial x} = 0, \ \ \frac{\partial \theta}{\partial x} = 0, \tag{8}$$

where v_n, v_s are the normal and tangential components of velocity on the free surface, $f(x)$ and $\varphi(x)$ are assigned functions.

Conditions (5) are written in a local Cartesian coordinate system which is linked normally to the free surface. The motion of free surface Γ_1 occurs with kinematic condition written in the form

$$\frac{dx}{dt} = v_x, \quad \frac{dy}{dt} = v_y. \tag{9}$$

At the initial time moment, the channel is filled partly, and the free surface is far enough from the inlet boundary Γ_2 (this was made to eliminate the impact of free surface on the flow near the inlet zone). The physical problem statement gives us the initial fields of velocity and temperature.

2. Solution method

Simulation of the stated problem was accomplished with a finite-difference scheme [17], which is based on invariants method for simulation of flow on free surface [18] and the algorithm SIMPLE for computation of searched variables in the inner nodes of staggered grid [19]. The values for velocities are calculated with the exponential scheme; the temperature field is calculated with upstream approximation of convective components. According to [18], the first condition from (5) is written together with continuity equation; this enables us for using the running calculation method in computation of velocity components for marker particles at free surface. The values of pressure and temperature are calculated from the difference analogs of the second and third conditions from (5), correspondingly. Evolution of free surface is found from difference analogs of equations (9) with the use of Euler scheme. Computation methods were tested with a test problem of fluid flow through a flat channel with a given flow rate (with account for dissipative warming and known experimental dependency of viscosity on temperature (4)). A parabolic velocity profile and zero temperature were assigned at the channel inlet; soft boundary conditions were assigned at the channel outlet. Conditions (7) are fulfilled at the solid wall. The channel length is taken long enough for establishing a steady flow at the outlet cross section. The simulation results are compared with semi-analytical solution of equivalent 1D problem [20]. Figure 2 presents comparison of velocity and temperature distributions at the channel outlet obtained using our numerical method and the solution of 1D problem. A good compliance confirms the validity of method. Besides, while testing of the solution algorithm for the case of flow with free surface, we carried out parametric computations of isothermal filling of the channel. As known from [5–7], at the isothermal filling flow at a constant flow rate, a free surface shape is steady, and the key characteristic for this shape is parameter $\chi = \Delta y / L,$ indicating the position of point B in the symmetry line relative the point C in the three-phase contact line (Fig. 1). The dependency of χ on W at low Reynolds numbers is plotted in Fig. 3. The results obtained (curve *1*) are compared with data from [7] (curve *2*), where the finite element method was used in flow simulation. Good compliance of results was observed. The approximation convergence was tested on different rectangular grids; this allowed us to use the grid mesh equal to 1/40. The time step was limited by the Courant condition [21]. The discrepancy in mass conservation law in all simulations was less than 1 %.

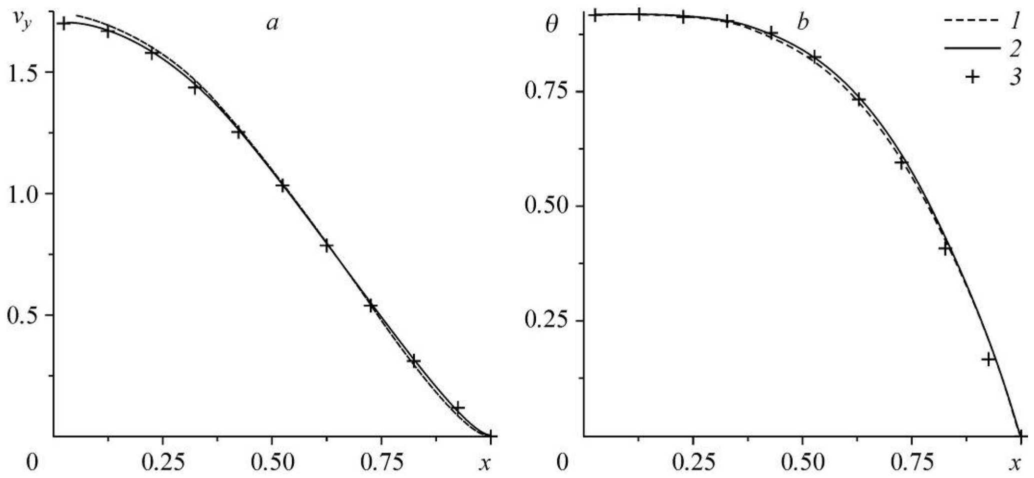

Fig. 2. Profiles of velocity (*a*) and temperature (*b*) at the outlet cross section.

Re = 0.001, Pe = 1000, C_1 = 2, C_2 = 1.33; the mesh size 1/10 (*1*), 1/20 (*2*); analytical solution (*3*).

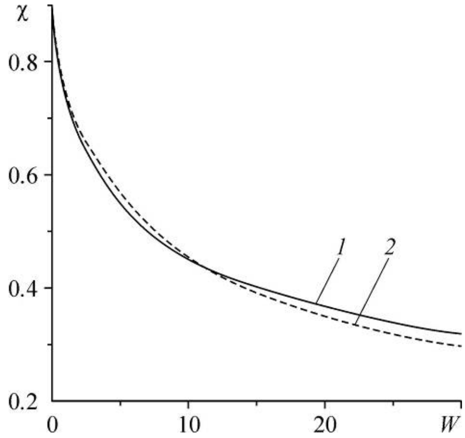

Fig. 3. Dependency of parameter χ on W for isothermal flow at Re = 0.01.
1 — simulation, *2* — results from paper [7].

3. Simulation results

Two physical problem statements were considered for estimating the effect of viscous dissipation on medium temperature and flow characteristics. Firstly, the constant-rate liquid flows into a channel; the temperature is a temperature of steady non-isothermal liquid flow in infinite channel (with account for mechanical energy dissipation and viscosity vs. temperature dependency). The boundary conditions for velocity at the inlet and initial distributions for velocity and temperature correspond to this kind of flow. Meanwhile, the values for key parameters have to ensure existence of stationary solution which eliminates the phenomenon of hydrodynamic heat explosion [22]. A typical distribution for longitudinal velocity and temperature for steady flow is plotted in Fig. 2.

The rate of viscous dissipation in a flow is determined by parameter C_1. Figure 4 demonstrates the impact of this parameter on flow characteristics (under other conditions equal). The picture shows the distributions for temperature, viscosity, pressure, and velocity at time moment t = 5.

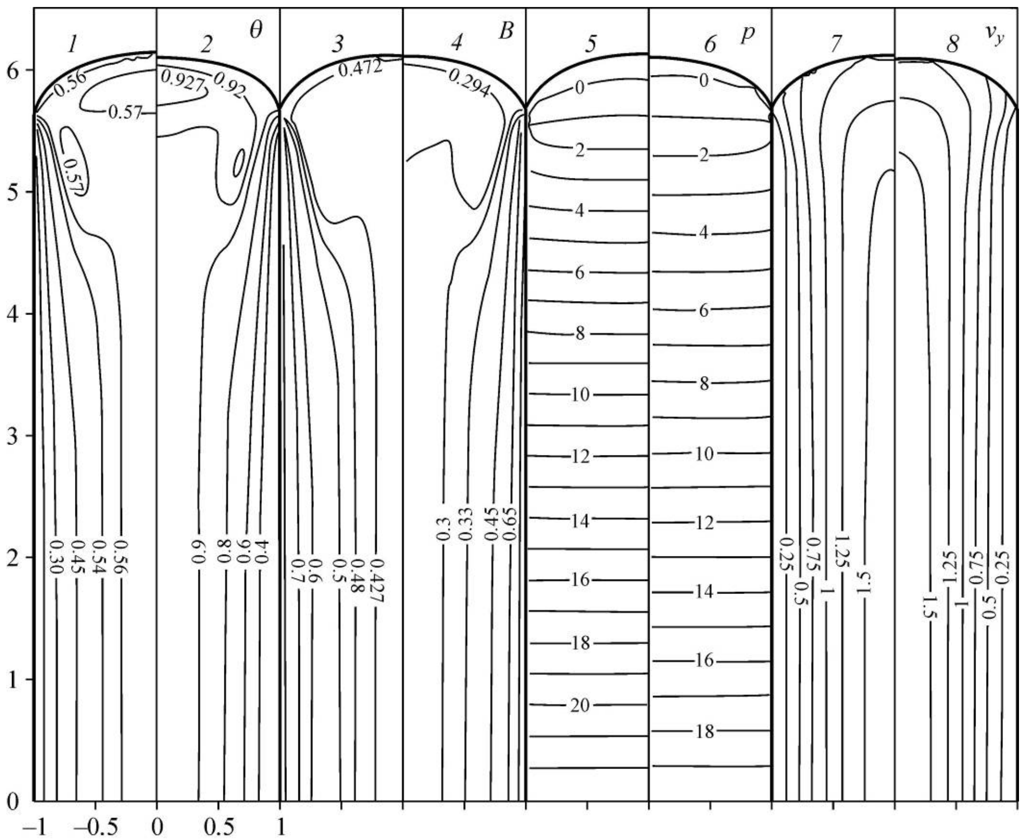

Fig. 4. Isolines for temperature, viscosity, pressure, and velocity.
Re = 0.01, W = 2, Pe = 100, C_2 = 1.33; C_1 = 1 (*1*, *3*, *5*, *7*), 2 (*2*, *4*, *6*, *8*).

Known studies of isothermal filling flow at constant rate and low Reynolds number have demonstrated formation of a quasi-steady flow mode with steady free interface [4–7]. Two zones can be distinguished in this flow: 1D flow zone far away from free interface and 2D fountain flow in vicinity of interface. The pattern of isolines distribution in Fig. 4 demonstrates that the flow can be divided into two zones of flow (similar to isothermal approximation). The size of 2D flow zone increases with increase in parameter C_1. A small increase in temperature is observed in the vicinity of free surface. With decline in viscosity, the intensity of flow spreading on solid walls near the free surface becomes stronger, therefore, the value of χ decreases with growing C_1. The pressure gradient in the flow also decreases with growing C_1 due to decline in viscosity.

The ratio of convective to conductive heat transfer in a flow is described by the Peclet number. Figure 5 presents distributions of temperature, viscosity, pressure, and velocity at Pe = = 1000, but the rest of parameters are the same as for the case of Fig. 4. Comparison of data in Figs. 4 and 5 demonstrates a smaller zone of 1D flow at a higher Peclet number and qualitative changes in temperature and viscosity distribution. The temperature and velocity profiles in cross sections y = const at time t = 6 for different values of Pe are shown in Figs. 6 and 7, correspondently. For both tested levels of Peclet number, the liquid layer in vicinity of symmetry line has almost the same temperatures along the entire channel, and this is due to low values of invariant for shear rate tensor in this area and due to low contribution of conductive heat transfer at high Pe. However, we observe a qualitative change in temperature profile for the near-wall part of flow. The decrease of the one-dimensional flow zone and higher convective heat transfer

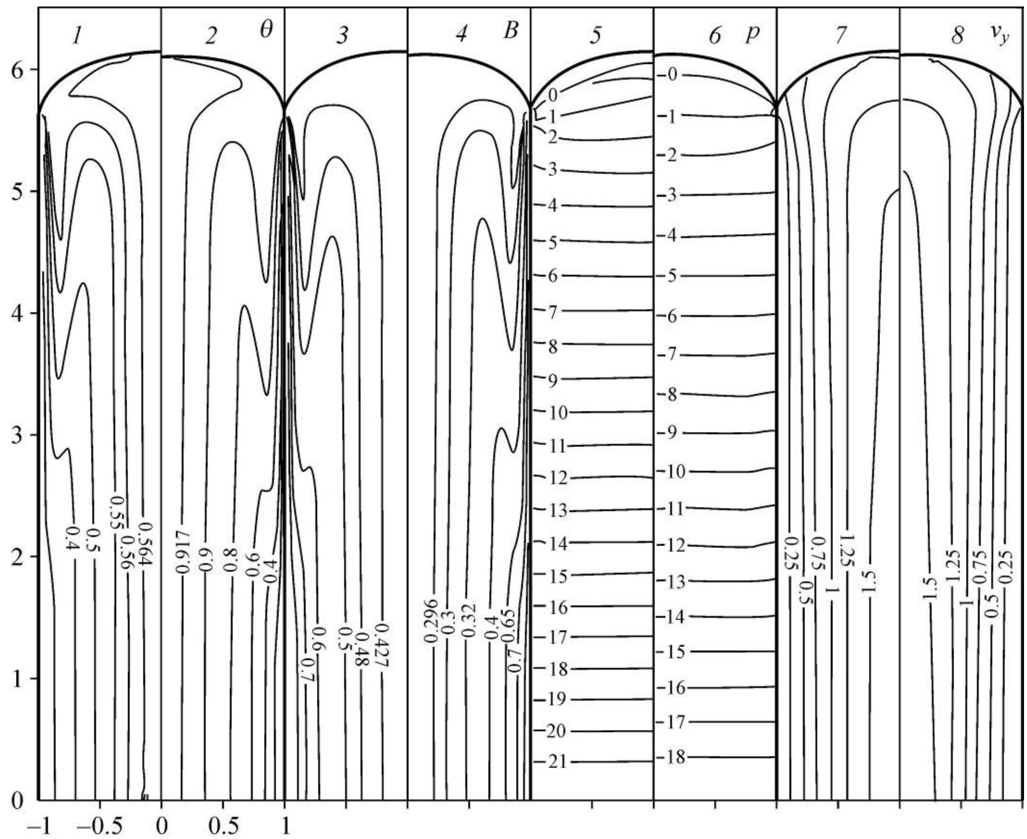

Fig. 5. Isolines for temperature, viscosity, pressure and velocity .
Re = 0.01, W = 2, Pe = 1000, C_2 = 1.33, C_1 = 1 (*1*, *3*, *5*, *7*), 2 (*2*, *4*, *6*, *8*).

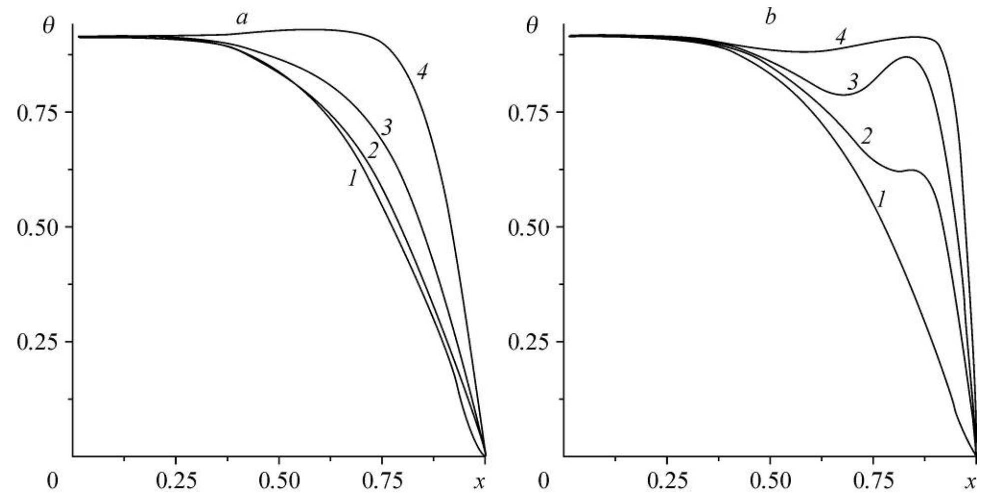

Fig. 6. Temperature distribution in cross sections y = const.
Re = 0.01, W = 2, C_1 = 2, C_2 = 1.33, t = 6; Pe = 100 (*a*), 1000 (*b*); y = 0 (*1*), 3 (*2*), 4.5 (*3*), 6 (*4*).

at higher Peclet numbers makes higher the influence of free surface on temperature distribution. As consequence, the temperature profile deviates from the profile for 1D flow in the infinite channel (Fig. 2). With a higher Peclet number, the distributions of isotherms become more similar to the case of kinematics of fountain flow [20]. The profile of longitudinal velocity in Fig. 7 demonstrates qualitative similarity of curves behavior at both values of Pe and minor qualitative differences.

The increase in number Re up to 1 and parameter W up to 10 does not bring any qualitative changes in flow behavior and the observed quantitative changes are insignificant.

Along with the problem statement considered above, we also developed a mathematical model with a parabolic profile for the inlet velocity (which corresponds to 1D isothermal flow), and the temperature is assigned equal to the wall temperature. The initial conditions were zero distributions for velocity and temperature. The fields of temperature, viscosity, pressure, and velocity for this case at time moment t = 5 for two values of parameters C_1, (with other conditions identical) are plotted in Fig. 8. The liquid supplied into the channel is heated due to viscous

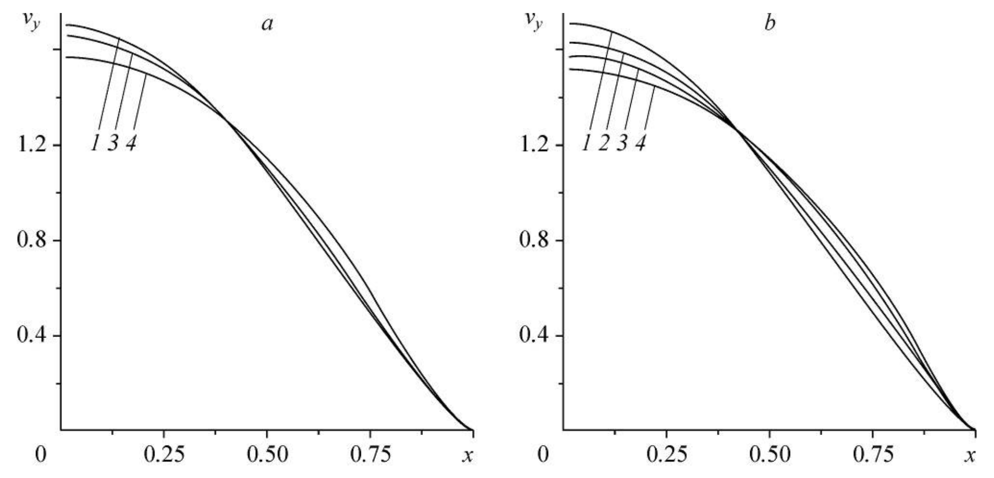

Fig. 7. Distribution of longitudinal velocity for cross sections y = const.
Re = 0.01, W = 2, C_1 = 2, C_2 = 1.33, t = 6; Pe = 100 (*a*), 1000 (*b*); y = 0 (*1*), 3 (*2*), 4.5 (*3*), 6 (*4*).

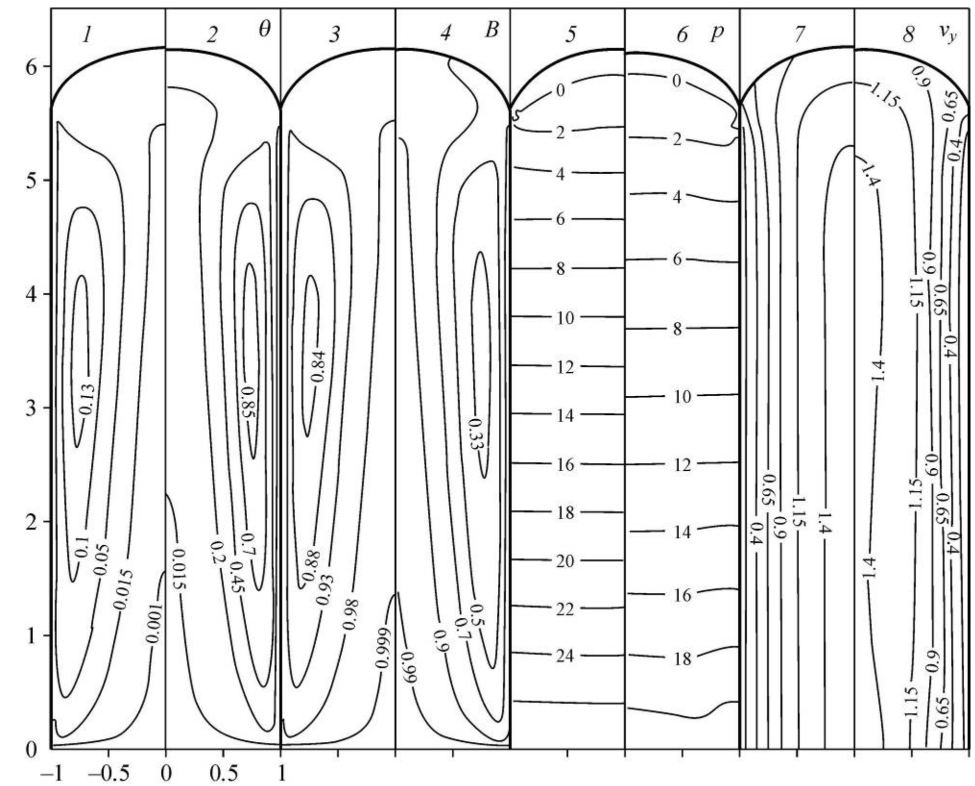

Fig. 8. Isolines for temperature, viscosity, pressure, and velocity.
Re = 0.01, W = 2, Pe = 100, C_2 = 1.33, C_1 = 1 (*1*, *3*, *5*, *7*), 10 (*2*, *4*, *6*, *8*).

dissipation; the high temperature zone is formed at a distance from the wall, where the dissipation function reaches the highest values. The corresponding distribution of viscosity produces 2D flow in the entire zone. For $C_1 = 10$ the (dimensional) temperature increases almost twice as compared with the initial temperature (with appropriate decline in viscosity). The profiles of temperature and velocity for cross sections y = const at time moment t = 5 for different

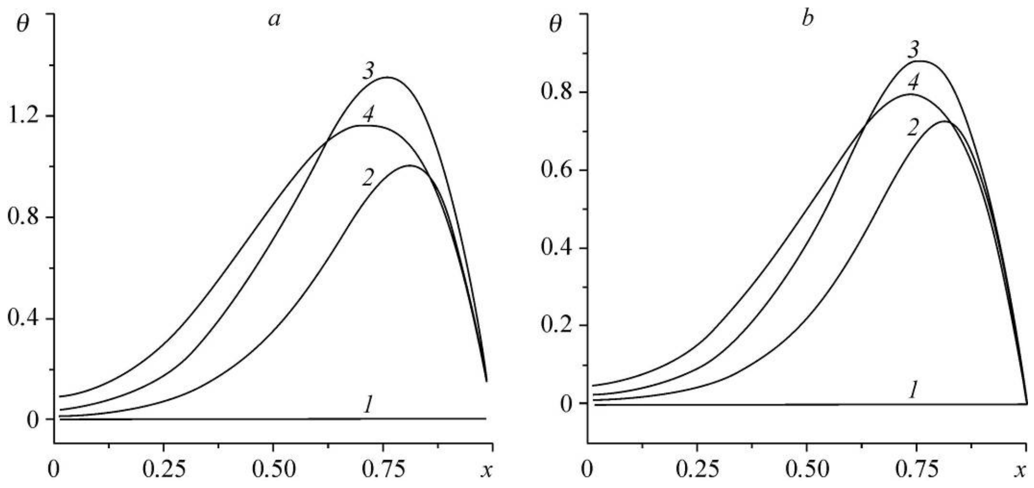

Fig. 9. Temperature distribution in cross sections y = const.
Re = 0.01, W = 2, Pe = 100, C_2 = 1.33, t = 5; C_1 = 1 (*a*), C_1 = 10 (*b*); y = 0 (*1*), 1.5 (*2*), 3 (*3*), 4.5 (*4*).

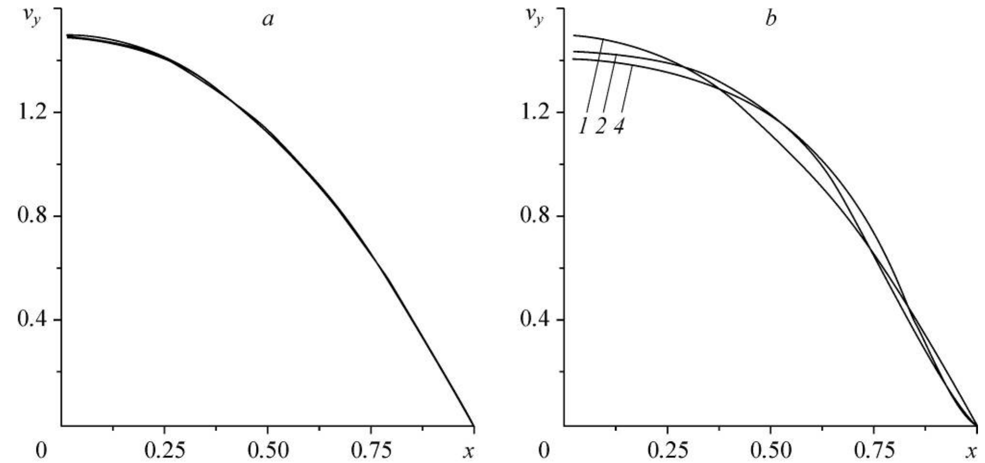

Fig. 10. Longitudinal velocity distribution in cross sections y = const.
Re = 0.01, W = 2, Pe = 100, C_2 = 1.33, t = 5, C_1 = 1 (*a*), 10 (*b*); y = 0 (*1*), 1.5 (*2*), 4.5 (*4*).

values C_1 (with other conditions identical) are plotted in Figs. 9 and 10. For both chosen values of C_1, we observe qualitative similarity of temperature profiles for different cross sections (although quantitative differences are significant). Meanwhile the velocity profiles in different cross sections at given time moment exhibit a minor change only. The impact of Peclet number on temperature and viscosity distributions is shown in Fig. 11 for time moment t = 5. The growth in the convective component of heat transfer is also exhibited in changes of temperature and viscosity. Therefore, for this problem statement and for chosen set of parameters, the simulated flow pattern is different from the pattern produced by the first mathematical model. However, if we assume existence of quasi-steady filling flow for key parameters, for the high times and for the front portion of flow we would observe the flow described in the first problem statement.

At least, simulation of flow in a channel without account for free surface with a trend to a steady solution (Fig. 2) and the study within the framework of the first problem statement gives grounds for this assumption. Indeed, simulation supports this assumption. Figure 12 presents the distribution

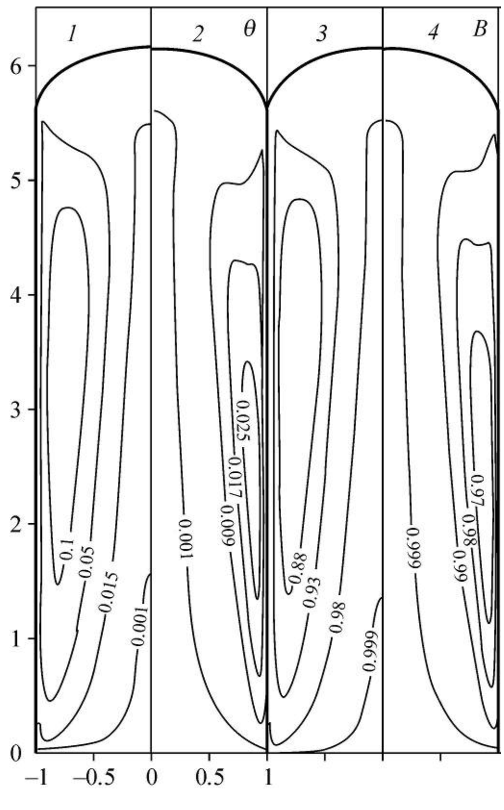

Fig. 11. Isolines for temperature and viscosity.
Re = 0.01, W = 2, C_1 = 1, C_2 = 1.33, Pe = 100 (*1*, *3*), 1000 (*2*, *4*).

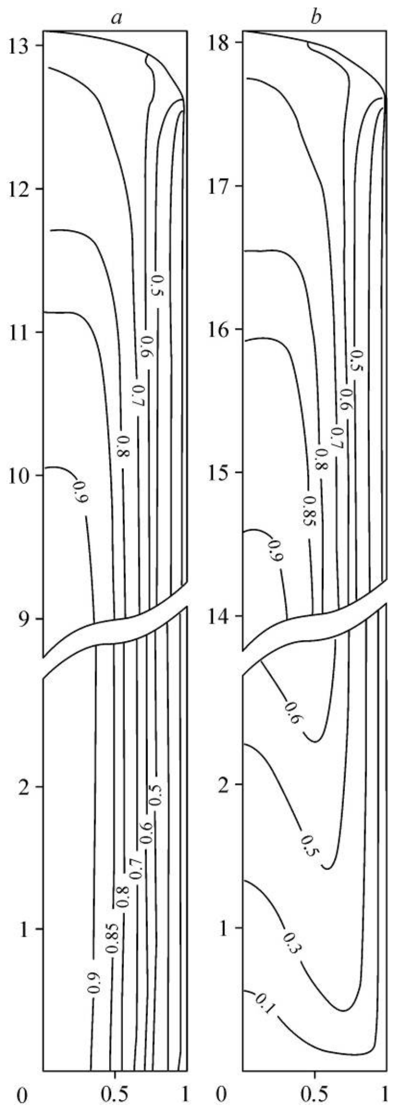

Fig. 12. Temperature isolines.
Re = 0.01, W = 2, Pe = 5, C_1 = 2, C_2 = 1.33.

of temperature calculated in the frames of the first (Fig. 12*a*) and second (Fig. 12*b*) problem statement. The upper part of flow for the case of second problem statement demonstrates for the length of several length units a temperature distribution identical to the distribution for the first case (thus, regularities in viscosity distributions and flow patterns are the same).

Conclusion

The performed study demonstrated the impact of viscous dissipation on temperature, viscosity, kinematic and dynamic parameters of flow that fills a flat channel. The simulated flow can be divided into the zone of 2D fountain flow in the vicinity of free surface and the zone of 1D flow away from the interface with nonisothermal flow: these results were obtained when the initial temperature and velocity distributions were taken in the form of a steady flow. The flow characteristics were considered as functions of different parameters: intensity of mechanical energy dissipation, viscosity vs. temperature, Reynolds and Peclet numbers, and parameters describing ratio of gravitational and viscous forces. For different initial and inlet boundary conditions, the peculiarities of flow development were considered.

In general, the considered fluid flow demonstrates complicated relations between the viscosity decline due to dissipative warming and, this, decrease in dissipation rate that controls the decrease in viscosity.

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